Metaheuristic Optimization

1. Introduction

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website

Transportation Planning: Task

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- So let us start by looking at some examples for optimization problems.

Section Outline

Transportation Planning: Task

- Build a system which tells a logistics company what it needs to do to fulfill all transportation orders at minimum costs. [1–5]

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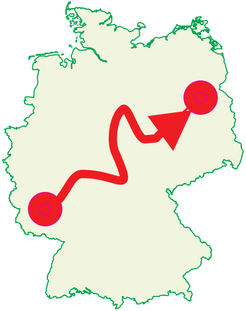
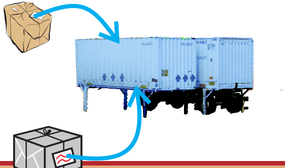
- Build a system which tells a logistics company what it needs to do to fulfill all transportation orders at minimum costs. [1–5]
- What does this mean?

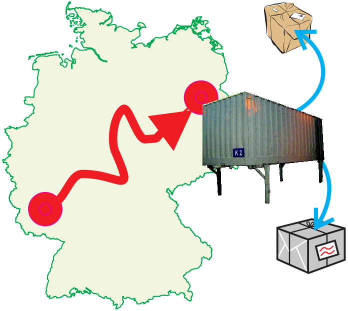
- Build a system which tells a logistics company what it needs to do to fulfill all transportation orders at minimum costs. [1–5]
  1. Find routes on the map



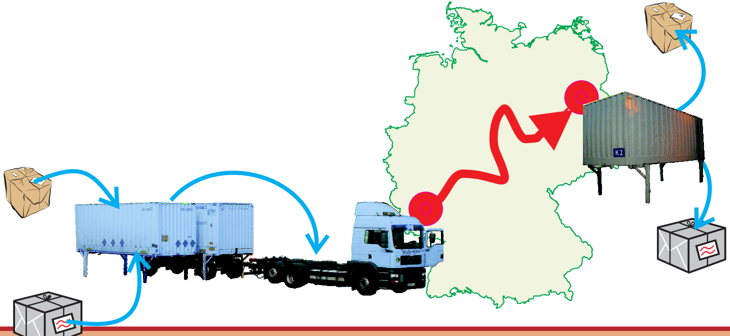
Transportation Planning: Task

- Build a system which tells a logistics company what it needs to do to fulfill all transportation orders at minimum costs. [1–5]
  1. Find routes on the map and assignments of orders to containers





- Build a system which tells a logistics company what it needs to do to fulfill all transportation orders at minimum costs. [1–5]
  1. Find routes on the map and assignments of orders to containers and containers to trucks



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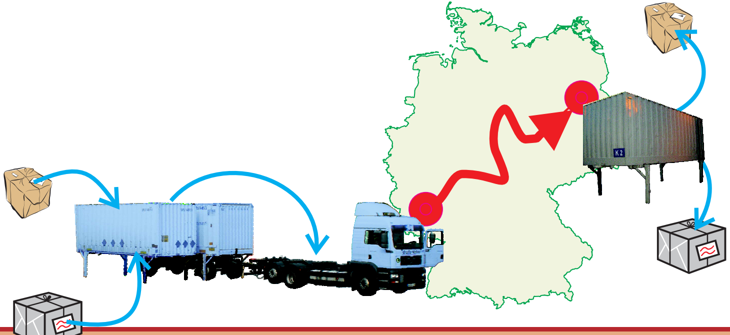

- Build a system which tells a logistics company what it needs to do to fulfill all transportation orders at minimum costs. [1–5]
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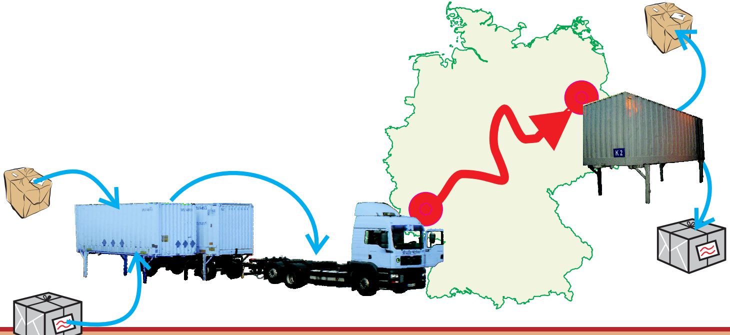

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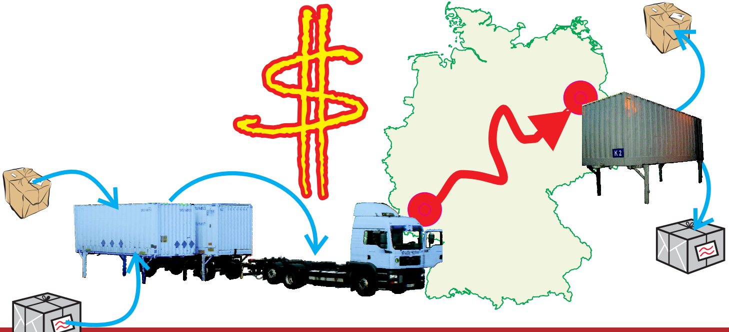

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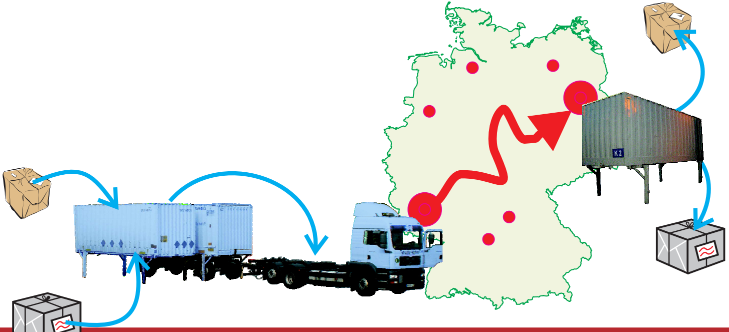

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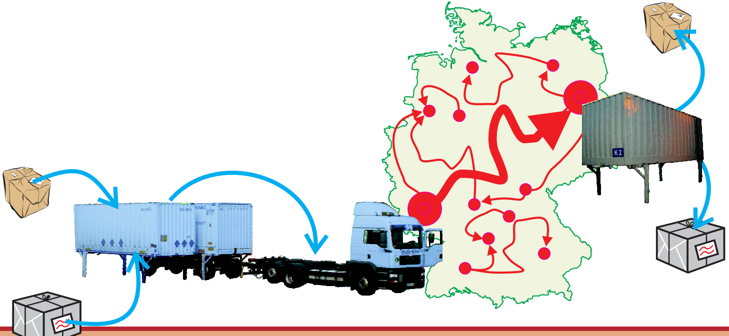

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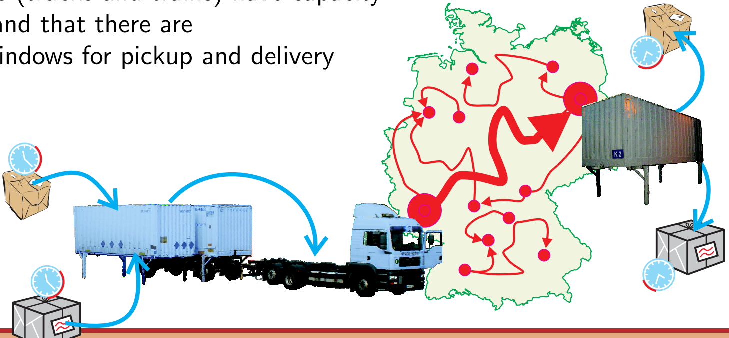

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  5. and constraints and laws.
  6. Time limit for optimization: 1 day

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- Before the problem was solved by hand, by manual planning with Excel sheets...
- With an optimization algorithm, we can get better solutions than that.
- In this course, you will learn how we can do such a thing.

Section Outline

What is optimization?

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Definition (Optimization)

Optimization is the process of solving an optimization problem, i.e., finding suitable solutions for it.

- What is optimization? [6]
- Ok, so what is an optimization problem?

Definition (Optimization Problem: Economical View)

An optimization problem is a situation which requires deciding for one choice from a set of possible alternatives in order to reach a predefined/required benefit at minimal costs.

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Definition (Optimization Problem: Economical View)

An optimization problem is a situation which requires deciding for one choice from a set of possible alternatives in order to reach a predefined/required benefit at minimal costs.

Definition (Optimization Problem: Simplified Mathematical View)

Solving an optimization problem requires finding an input value $x^{\star}$ for which a mathematical function f takes on the smallest possible value (while usually obeying to some restrictions on the possible values of $x^{\star}$).

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- Then, we want to find approximate solutions: solutions which are "as good as possible" within a feasible time
- This is what metaheuristic optimization algorithms do. This is what we will learn in this course.

Optimization

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  - Find the *shortest* tour for a salesman to visit a certain set of cities in China and return to Hefei!

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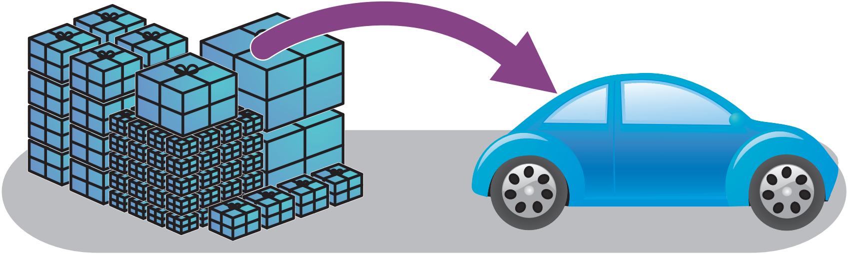
  - Find the *shortest* tour for a salesman to visit a certain set of cities
  - I need to transport n items from here to another city but they are too big to transport them all at once. How can I load them best into my car so that I have to travel back and forth the least times?



Optimization

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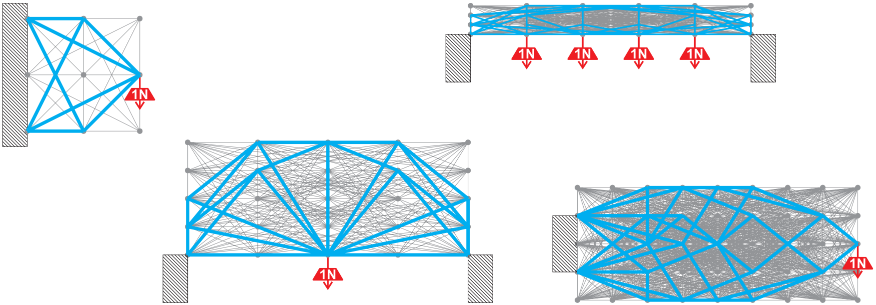
  - Find the *shortest* tour for a salesman to visit a certain set of cities
  - I need to transport n items from here to another city
  - How can I construct a truss which can hold a certain weight with at most a certain amount of iron?



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  - Find the *shortest* tour for a salesman to visit a certain set of cities
  - I need to transport n items from here to another city
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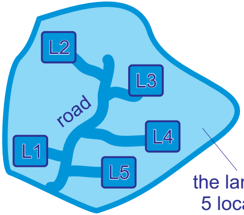
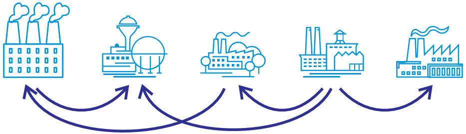
  - I want to build a large factory with n workshops. I know the flow of material between each two workshops and now need to choose the locations of the workshops such that the overall running cost incurred by material transportation is *minimized*.



the land with
5 locations



5 workshops and goods flows betwee them
which need to be assigned to locations

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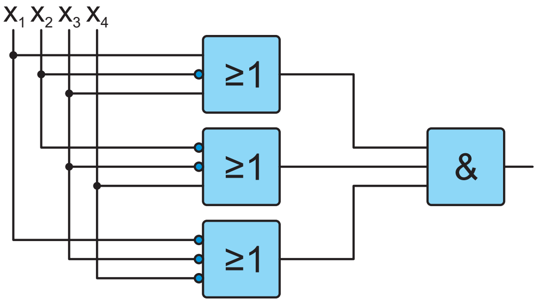
  - Find the *shortest* tour for a salesman to visit a certain set of cities
  - I need to transport n items from here to another city
  - Construct a truss which can hold a certain weight
  - Assign workshops to locations
  - Which setting of x_1, x_2, x_3, and x_4 can make $(x_1 \vee \neg x_2 \vee x_3) \wedge (\neg x_2 \vee \neg x_3 \vee x_4) \wedge (\neg x_1 \vee \neg x_3 \vee \neg x_4)$ become true? (or, at least, as *many* of its terms as possible)?



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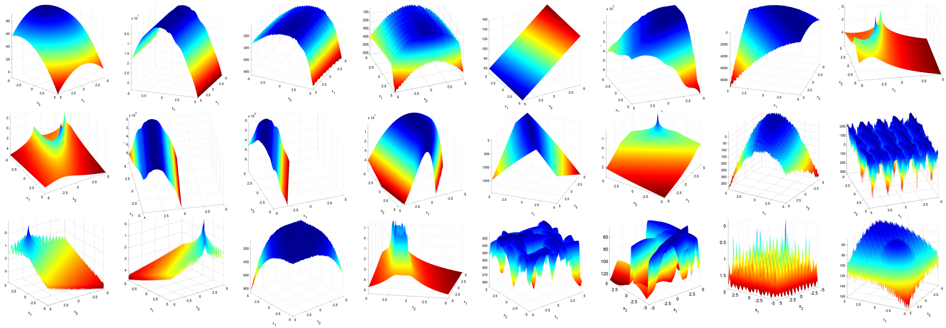
  - I need to transport n items from here to another city
  - Construct a truss which can hold a certain weight
  - Assign workshops to locations
  - Satisfy Boolean formula
  - Find the minima of complex, multi-dimensional mathematical formulas



More Examples for Optimization Problems

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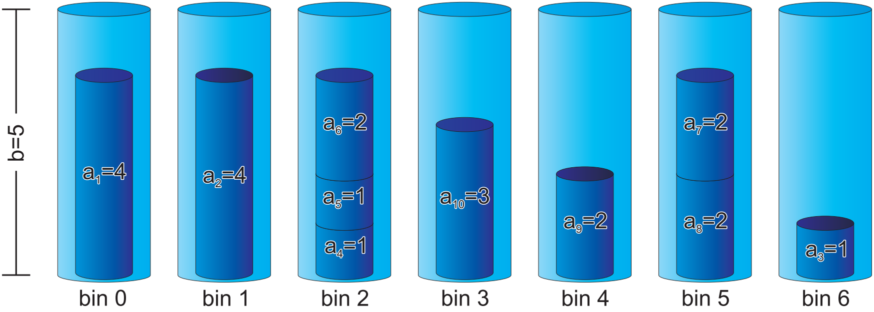

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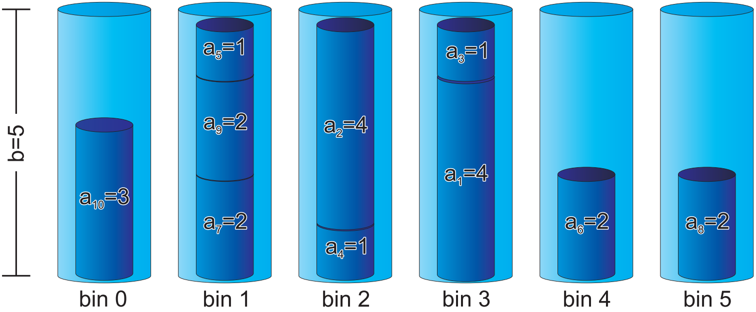

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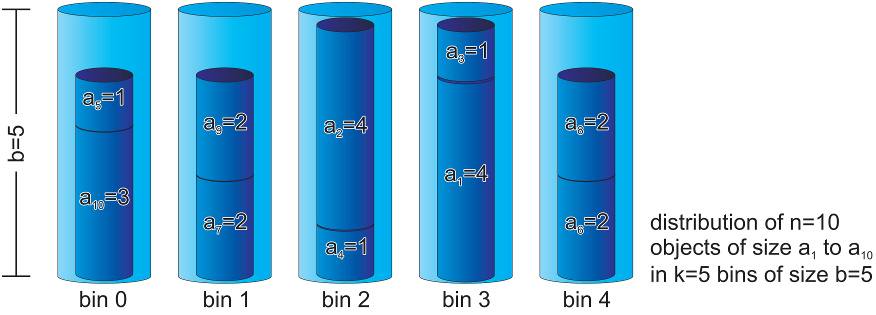

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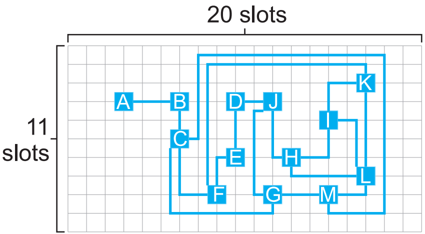
  1. Can we pack $n = 10$ objects of sizes $a_1, a_2, \dots, a_{10}$ into bins of size $b = 5$? [7]
  2. How to place chips on a circuit board while minimizing wire length? [8, 9]



S=20×11

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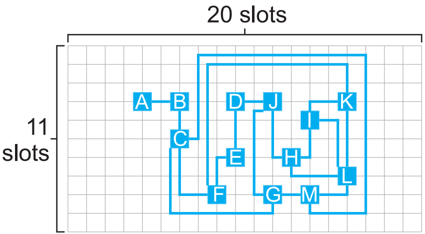

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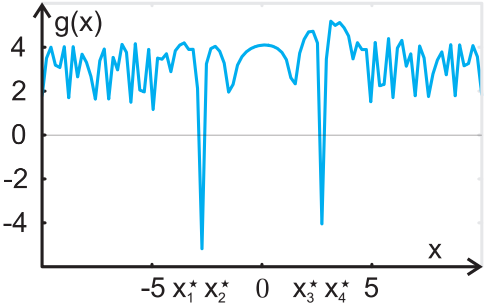

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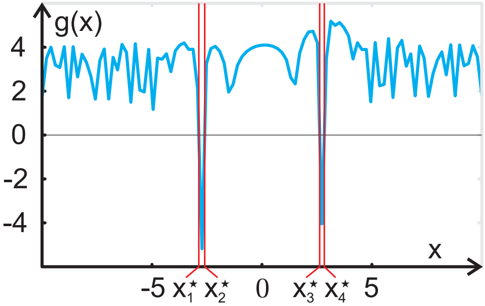

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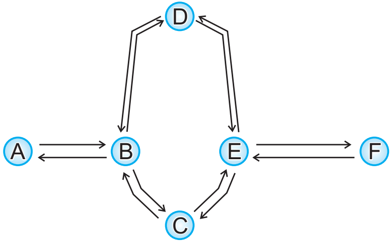
4. Find the shortest path between two nodes in a network! [14–16]



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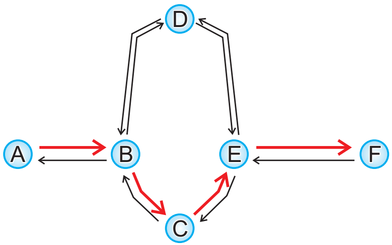

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  4. Find the shortest path between two nodes in a network! [14–16]
  5. Assign sub-jobs $j_i \in J$ of jobs J to machines $m \in M$ under order and deadline constraints! [17–19]

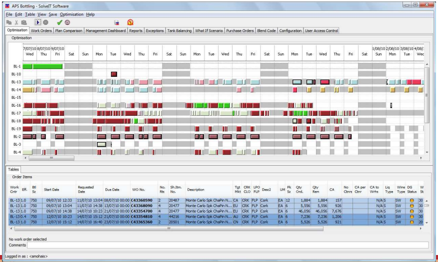

image source: [17]

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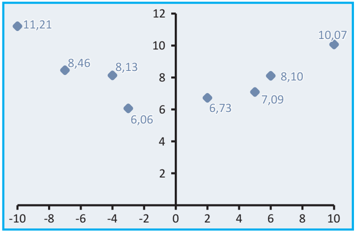

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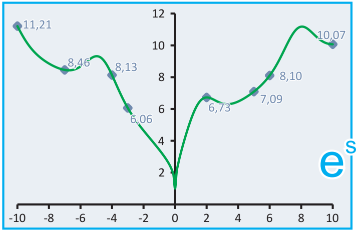

$$e^{\sin x} + 3\sqrt{|x|}$$

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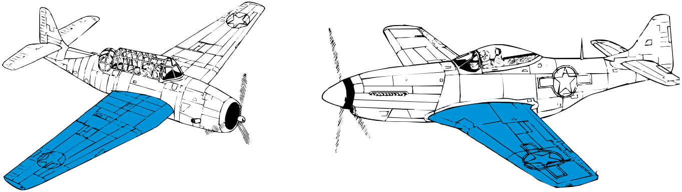

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Classification of Problem Types

- Are there different classes of optimization problems?

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- Many problems are either combinatorial or numerical:

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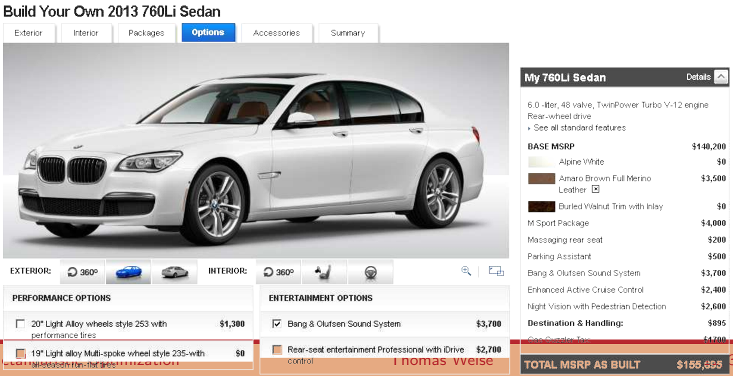
Definition (Combinatorial Optimization Problem)

Combinatorial optimization problems [32–38] are problems which have finitely many and discrete solutions.



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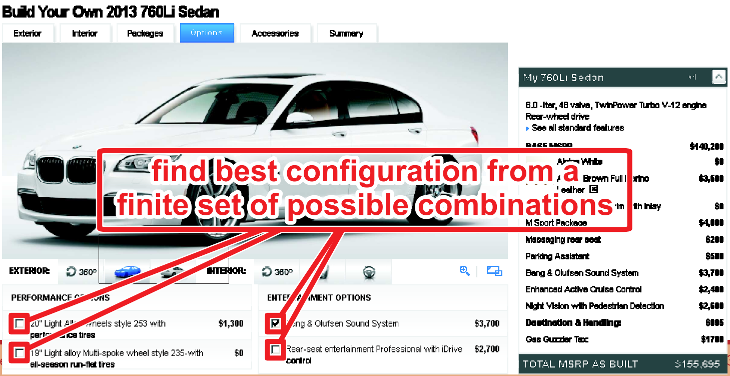

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Find the visiting sequence of cities corresponding to the shortest round-trip tour.

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1. bit strings (i.e., many yes-no decisions) or
2. permutation of elements (order something).

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Numerical optimization problems are problems that are defined over numerical domains or involve real-valued decision variables. [10, 11, 40–43]

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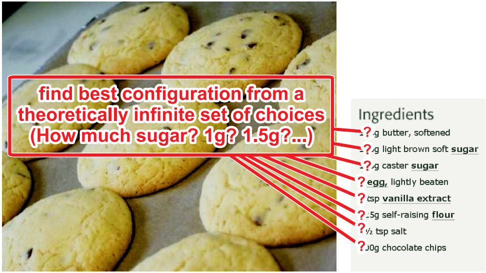

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- Are these problems combinatorial or numerical?

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3) Truss Optimization
4) Quadratic Assignment
5) MAX-SAT
6) Minimize Function
7) Circuit Layout
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Optimization algorithms tell us how to find solutions which are *rated best* (or at least well) from a set of possible solutions, for a general class of problems.

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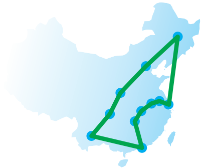

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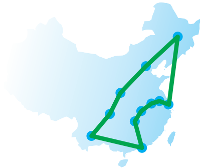

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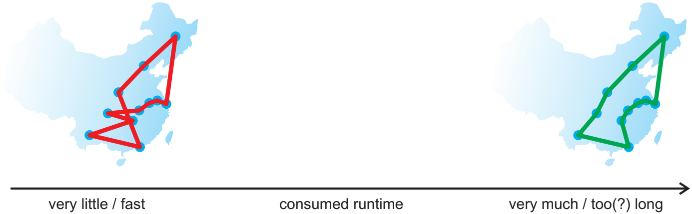

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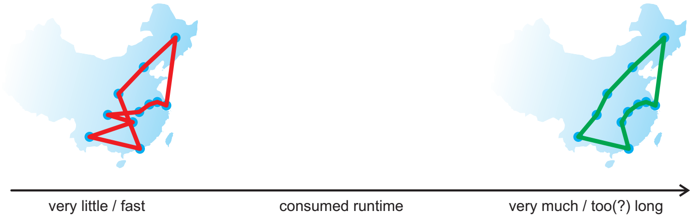

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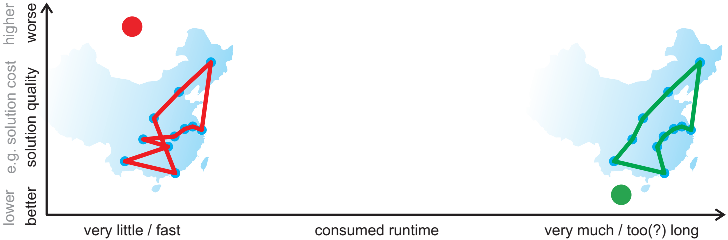

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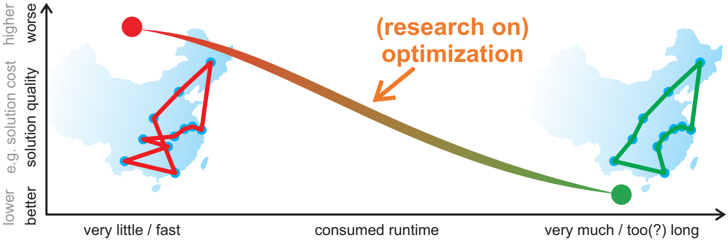

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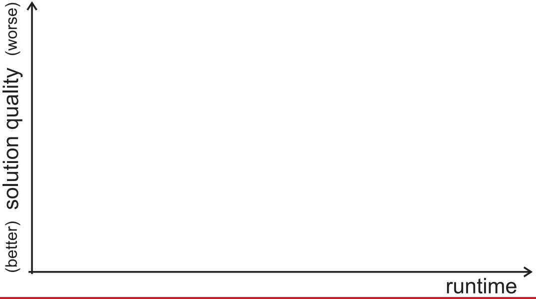

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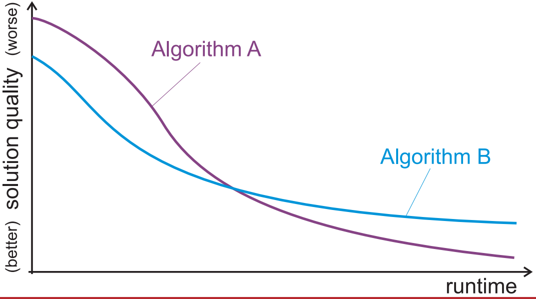

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- We can let them run arbitrarily long, there usually is no explicit, natural end point.

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1. Introductory Example
2. Optimization Problems
3. Optimization Algorithms
4. Runtime vs. Solution Quality
5. **Other Algorithm Features**
6. Summary

Deterministic and Randomized Algorithms

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Example:

- *Exhaustive Enumeration:* Test all possible solutions one by one and, thus, find the optimum.
- *Deterministic Heuristic:* Solve the bin packing problem by putting the largest remaining element into a bin until the bin is full, then take the next bin, until all objects are packed.

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Examples:

- *Random Sampling:* Keep creating solutions randomly until the time is up, remember the best.
- *Randomized Local Search:* Create a random solution, then try to improve it by modifying it a bit (randomly). If the new one is better, keep it. Modify again.

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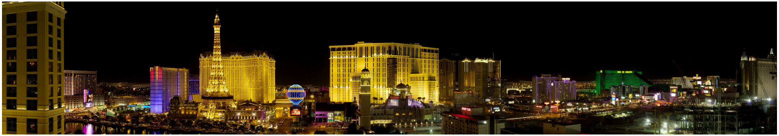

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- ...since we often have a limited computational budget and need something that finishes within that time

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Number of Objectives/Criteria

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Definition (Constraint Optimization)

Single- or multi-objective Optimization where the solutions also need to fullful additional constraints. [67–69]

Section Outline

Summary

- Wide variety of optimization problem types
- Wide variety of optimization algorithms
- Wide variety of applications and requirements

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- Wide variety of optimization algorithms
- Wide variety of applications and requirements
- One thing is clear: There is no single, perfect algorithm to solve all of them [70–73]
- Experience is needed!
- We will try to get a good perspective on all of that
- ...and conduct many experiments in order to get experience

谢谢

Thank you

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