**Context and Situation:**

ADEME (French Environment and Energy Management Agency) is seeking innovative projects to experiment with new solutions for **mobility of people and goods** that are environmentally friendly. Your team at **CesiCDP**, which has experience in the field of Smart Multimodal Mobility, is planning to submit a proposal in response to this call.

Your focus is on **optimizing delivery routes**. Specifically, the goal is to find the best possible route for delivering items between multiple cities, considering traffic patterns and time constraints. By developing an algorithmic solution for efficient delivery route planning, your team hopes to win new contracts with attractive funding from ADEME.

**Problem to Solve:**

You need to solve a classical **logistics optimization problem**—more precisely, a variant of the **Traveling Salesman Problem (TSP)**. In this problem, a delivery truck must:

* Visit a subset of cities.
* Return to the starting point.
* Minimize the **total duration** of the trip.

You’ll need to account for traffic variations depending on the time of day and perhaps other practical constraints like delivery time windows and truck capacities, depending on how realistic you want to make the problem.

**Deliverable 1 - Modelling:**

For this first phase, the goal is to **formally model the problem** and understand its theoretical properties. You are **not yet implementing** the solution or writing code for the algorithm—that comes later. Instead, you'll be setting up the groundwork to:

1. **Present the problem**: Understand and clearly define what the problem is, the context in which it occurs, and the industrial challenges it addresses.
2. **Rephrase the problem formally**: You'll need to model the problem mathematically. This involves:
   * Defining variables, objectives, and constraints clearly.
   * Detailing how traffic, time slots, delivery schedules, truck availability, and capacities could affect the optimization.
   * Incorporating additional constraints that might make the problem harder but more realistic (e.g., multiple trucks, capacity, variable traffic conditions, time windows, etc.).
3. **Theoretical analysis**: Study the complexity of the problem:
   * Show that the problem is a variant of known NP-hard problems like the TSP (Traveling Salesman Problem).
   * Discuss how adding constraints affects the problem’s complexity.
   * Reference relevant scientific literature to support your analysis.

**Breaking Down the Key Elements for the Modelling Deliverable:**

1. **Formal Representation**:
   * **Data Representation**:
     + Cities, roads, and traffic times should be represented as a graph.
     + Nodes represent cities, and edges represent roads with travel times (which could vary by time of day).
   * **Variables**:
     + Define decision variables such as the route taken by the truck, the time taken on each road segment, truck capacity, etc.
   * **Objective**:
     + Minimize the **total time** of delivery, which includes driving time, waiting time (if arriving early at a city), etc.
   * **Constraints**:
     + Example constraints you might include:
       - Each city must be visited once.
       - Trucks may have different capacities.
       - Time slots for deliveries (i.e., items must be delivered during a specific time window).
       - Multiple trucks may be available, and you must optimize their routes as well.
2. **Theoretical Properties**:
   * Explain the **complexity** of the problem, such as how it relates to TSP and Vehicle Routing Problems (VRP).
   * Discuss how adding constraints like **time slots, truck capacities**, and **traffic variations** makes the problem more complex and harder to solve.
   * Reference academic papers or algorithms that have been used to tackle similar problems.
3. **Optional Constraints**:
   * The more constraints you include, the closer your model will be to real-world scenarios, but this also makes the problem harder to solve. Examples:
     + **Traffic variation**: Adjust travel times dynamically based on time of day.
     + **Multiple trucks**: Optimize for multiple trucks and minimize the return time of the last truck.
     + **Truck capacities**: Ensure that trucks do not exceed their capacity with deliveries.

**Structure of the Deliverable:**

* **Introduction**: Context of the problem, why it is important, and how solving it impacts mobility and environmental sustainability.
* **Formal Problem Definition**: Define the problem mathematically, including:
  + Graph-based representation (nodes = cities, edges = roads with travel times).
  + Decision variables (route, truck, time).
  + Objective function (minimize total time).
  + Constraints (time windows, truck capacity, etc.).
* **Theoretical Analysis**:
  + Prove the complexity (e.g., NP-hardness) of the problem.
  + Reference any algorithms or metaheuristics (e.g., Genetic Algorithms, Simulated Annealing, etc.) used to solve similar problems.
  + Provide bibliographic references to support your analysis.

**Key Points to Keep in Mind:**

* Focus on **clarity** and **scientific rigor**: You are laying the foundation for the next steps of the project.
* Consider how adding realistic constraints (traffic variation, multiple trucks, time slots, etc.) makes the problem harder but more valuable from an industrial perspective.
* Ensure your deliverable is **well-structured and well-documented**: Present your formal model in a clear way, with comments and references to relevant literature.

By the end of this phase, you should have a solid, well-argued **mathematical model** of the problem, and a clear understanding of its theoretical challenges. This will set the stage for the next phase, where you'll design and implement algorithms to solve the problem.

Those two constraints—**specific collection points** and **varying travel times due to traffic flow**—are excellent choices for making your project stand out while remaining achievable. Let me explain why and how you can leverage them effectively in your deliverable:

**1. Each Item Has a Specific Collection Point:**

This constraint adds a layer of realism and complexity to your model, without drastically increasing the computational difficulty. In many real-world logistics scenarios, items are picked up from various suppliers or warehouses before being delivered to their final destinations.

* **Why it makes you stand out**: This brings your problem closer to **real-world Vehicle Routing Problems (VRP)** with pickups and deliveries, making it more relevant to industries like e-commerce, logistics, and supply chain management.
* **Impact on the model**:
  + You’ll need to specify that for each item to be delivered, there is a **pre-defined collection point** (warehouse, factory, or supplier).
  + The route will involve both **pickup locations** and **delivery points**, so the optimization must handle two kinds of visits—pickups and deliveries.
  + This changes your optimization goal slightly, as now you must plan both the **route from the warehouse to pick up each item** and **the route to deliver it**.
* **Achievability**: The extension is achievable because this is a relatively well-known problem in operations research called the **Pickup and Delivery Problem (PDP)**. There are well-established approaches and heuristics that you can adapt (e.g., metaheuristics like Genetic Algorithms or Simulated Annealing) to handle this.

**2. The Travel Time of an Edge Varies Over Time (Traffic Flow Variation):**

This constraint introduces **dynamic traffic flow** into your model, representing real-world conditions where travel times on roads vary by time of day. Traffic congestion in urban areas is a significant challenge for delivery routes, and incorporating this makes your solution more applicable to **urban logistics**.

* **Why it makes you stand out**: Adding **time-dependent travel times** elevates your problem to what is known as a **Time-Dependent Vehicle Routing Problem (TDVRP)**. It makes your solution more sophisticated because you are capturing the complexity of dynamic traffic patterns, which are critical for efficient delivery in cities.
* **Impact on the model**:
  + You’ll need to model **travel times as functions of time**. For example, an edge (road) connecting two cities may have different travel times depending on when the truck starts the journey on that road.
  + The optimization must account for these time-dependent travel times when planning the route to ensure that deliveries are done as efficiently as possible.
  + This also introduces an additional decision-making layer: trucks may have to delay starting on certain roads if traveling at a later time results in lower congestion and faster travel.
* **Achievability**: While this is more challenging than constant travel times, it is still very doable. You’ll need to represent the road network with **dynamic edge weights**, where the weight (travel time) changes based on the time of day. This can be tackled with **heuristics or metaheuristics** that take traffic conditions into account, and there is a lot of existing research on time-dependent TSP and VRP that you can reference.

**Constraints Chosen:**

* Each item has a specific collection point
* The travel time of an edge varies over time (which is equivalent to varying its length), to represent the variation in traffic flow

**Why These Constraints Make Sense Together:**

These two constraints naturally complement each other:

* **Collection points** add a dimension of route complexity (you have to plan both pickups and deliveries).
* **Varying travel times** introduce temporal complexity (the optimal route depends on traffic conditions at different times of day).

Together, they create a **real-world scenario** where delivery routes are impacted by both **geographic factors** (where items are collected and delivered) and **temporal factors** (how traffic affects travel times). This makes your project **more applicable to real logistics challenges** without over-complicating it.

**Additional Benefits:**

1. **Relevance to ADEME’s Environmental Goals**: By optimizing for traffic variations, your solution can reduce the **overall carbon footprint** of deliveries. Shorter travel times and more efficient routes lead to lower fuel consumption and emissions, aligning well with ADEME’s focus on **reducing energy consumption and greenhouse gas emissions**.
2. **Practicality for Urban Mobility Solutions**: ADEME’s call is focused on new mobility solutions, and urban areas suffer the most from **traffic congestion**. Your model addresses this by incorporating traffic flows, making it highly relevant to cities.
3. **Scalability**: These constraints are challenging but scalable. Even as the number of cities, collection points, and deliveries grows, the structure of your problem remains well-defined and solvable using existing optimization techniques (though performance may need to be tuned through experimentation and heuristic methods).

**Implementation Considerations:**

* **Data representation**: You will need to represent the graph’s edges (roads between cities) with time-varying weights. This means having a **time-dependent cost function** for each edge.
* **Algorithmic choice**: You could consider using **metaheuristic approaches** (such as Simulated Annealing, Tabu Search, or Genetic Algorithms) to handle the dynamic nature of the problem, especially since exact methods might struggle with large instances.
* **Computational study**: Since your model introduces two forms of complexity (geographic and temporal), your experimental study will need to carefully evaluate performance on **different time slots** and **varying levels of traffic** to see how well the algorithm adapts.

**Suggestions for Further Differentiation (Optional but Possible):**

* **Dynamic rerouting**: If you want to go further, consider exploring **dynamic rerouting**, where the algorithm can adjust the route in real-time based on traffic updates.
* **Multiple trucks**: If you have the time and resources, adding **multiple trucks** with capacity limits and varying traffic conditions can add more depth.

**Conclusion:**

The constraints you’ve chosen—**specific collection points** and **time-varying travel times**—are strong candidates for a realistic, impactful, and achievable project. They elevate the complexity just enough to stand out in terms of real-world relevance and technical challenge, but they are still manageable within the scope of your project.

By focusing on these constraints, you'll be able to demonstrate your team's ability to tackle a sophisticated logistics problem with real-world implications, while keeping the problem solvable within the given timeframe.