Learning Outcomes

Exercises Using First Shifting Property

Examples Using First Shifting Property

First Translation Property & Example

Exercise Using Direct Formula

Examples Using Direct Formula

Important Formulae

This Lecture Covers

Inverse Laplace Transformation

Complex Variable, Laplace & Z-Transformation

Lecture 04

Learning Outcomes

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Examples Using Direct Formula

Important Formulae

This Lecture Covers

1. Formula of Inverse Laplace Transformation.

2. Examples & Exercise of Inverse Laplace Transformation Using Direct Formula.

3. First Shifting Property of Inverse Laplace Transformation.

4. Examples & Exercises of Inverse Laplace Transformation Using First Shifting Property.

Inverse Laplace Transformation

1. $\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} = 1,$

2. $\mathcal{L}^{-1}\left\{\frac{1}{s^{n+1}}\right\} = \frac{t^n}{n!},$

3. $\mathcal{L}^{-1}\left\{\frac{1}{s-a}\right\} = e^{at},$

4. $\mathcal{L}^{-1}\left\{\frac{s}{s^2+a^2}\right\} = \cos at,$

5. $\mathcal{L}^{-1}\left\{\frac{a}{s^2+a^2}\right\} = \sin at,$

6. $\mathcal{L}^{-1}\left\{\frac{s}{s^2-a^2}\right\} = \cosh at,$

7. $\mathcal{L}^{-1}\left\{\frac{a}{s^2-a^2}\right\} = \sinh at.$

1. $\mathcal{L}^{-1}\left\{\frac{s^2+1}{s^3}\right\}$

$= \mathcal{L}^{-1}\left\{\frac{1}{s}+\frac{1}{s^3}\right\} = 1+\frac{t^2}{2!} = 1+\frac{t^2}{2}.$

2. $\mathcal{L}^{-1}\left\{\frac{1}{2s-5}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{2(s-\frac{5}{2})}\right\} = \frac{1}{2}e^{\frac{5}{2}t}$

3. $\mathcal{L}^{-1}\left\{\frac{2s}{s^2-9}\right\} = 2\mathcal{L}^{-1}\left\{\frac{s}{s^2-3^2}\right\} = 2\cosh 3t$

4. $\mathcal{L}^{-1}\left\{\frac{5}{s}-\frac{3s}{s^2+16}+\frac{2}{s^2+4}\right\}$

$= 5\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} - 3\mathcal{L}^{-1}\left\{\frac{s}{s^2+16}\right\} + \mathcal{L}^{-1}\left\{\frac{2}{s^2+2^2}\right\}$

$= 5 - 3\cos 4t + \sin 2t.$

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Exercise Using Direct Formula

Find The Inverse Laplace Transformation of the following functions:

1. $F(s) = \frac{1}{s-5}$,

2. $F(s) = \frac{1}{s^5}$,

3. $F(s) = \frac{s^3-5s^2+6}{s^4}$,

4. $F(s) = \frac{2+4s}{s^2+25}$,

5. $F(s) = \frac{3}{s^2+4}$,

6. $F(s) = \frac{3}{s^2-4}$.

Examples Using Direct Formula

Important Formulae

This Lecture Covers

Inverse Laplace Transformation

First translation property

If $\mathcal{L}^{-1}\{F(s)\} = f(t)$ then

$$\mathcal{L}^{-1}\{F(s-a)\} = e^{at}\mathcal{L}^{-1}\{F(s)\}.$$

Example: 01

$$\mathcal{L}^{-1}\left\{\frac{10}{(s+3)^4}\right\}$$
$$= 10\mathcal{L}^{-1}\left\{\frac{1}{(s+3)^4}\right\}$$
$$= 10e^{-3t}\mathcal{L}^{-1}\left\{\frac{1}{s^4}\right\}$$
$$= 10e^{-3t}\frac{t^3}{3!} = \frac{10}{6}e^{-3t}t^3.$$

Example: 02

$$\mathcal{L}^{-1}\left\{\frac{1}{(s-2)^2+1}\right\}$$
$$= e^{2t}\mathcal{L}^{-1}\left\{\frac{1}{s^2+1}\right\}$$
$$= e^{2t}\sin t\,.$$

Example 3.

$$\mathcal{L}^{-1}\left\{\frac{2s+1}{s^2+4s+13}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{2s+1}{s^2+2.s.2+4+9}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{2(s+2)-3}{(s+2)^2+9}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{2(s+2)}{(s+2)^2+3^2}-\frac{3}{(s+2)^2+3^2}\right\}$$

$$= 2e^{-2t}\cos 3t - e^{-2t}\sin 3t.$$

Learning Outcomes

Exercises Using First Shifting Property

Find Inverse Laplace of the following functions:

1. $F(s) = \frac{1}{(s-3)^4}$,

2. $F(s) = \frac{3}{(s+2)^2+9}$,

3. $F(s) = \frac{s-2}{(s-2)^2-16}$,

4. $F(s) = \frac{s}{s^2+4s-9}$,

5. $F(s) = \frac{5s-7}{s^2-6s+25}$,

6. $F(s) = \frac{s}{s^2-6s+10}$.

Examples Using First Shifting Property

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Inverse Laplace Transformation

Learning Outcomes

After completing this chapter you can easily evaluate the inverse Laplace transformation of function using direct formula & also using property.

Inverse Laplace Transformation

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Exercises Using First Shifting Property

Sample MCQ

1. $\mathcal{L}^{-1}\left\{\frac{s^2+1}{s^3}\right\}=?$

(a) $1+\frac{t}{2}$ (b) $1+\frac{t^2}{2}$ (c) $1-\frac{t^2}{2}$ (d) $\frac{t^2}{2}$

2. $\mathcal{L}^{-1}\left\{\frac{4}{s-2}-\frac{s}{s^2-16}+\frac{4}{s^2-4}\right\}=?$

(a) $e^{2t}-\cosh 4t+2\sinh 2t$ (b) $4e^{2t}+\cosh 4t+2\sinh 2t$

(c) $4e^{2t}-\cosh 4t+2\sinh 2t$ (d) $4e^{2t}-\cosh 4t$

3. $\mathcal{L}^{-1}\left\{\frac{s}{s^2+4s+13}\right\}=?$

(a) $e^{-2t}\cos 3t-\frac{2}{3}e^{-2t}\sin 3t$ (b) $e^{-2t}\cos 3t+2e^{-2t}\sin 3t$

(c) $e^{2t}\cos 3t-\frac{2}{3}e^{2t}\sin 3t$ (d) $e^{-2t}\cos 3t-2e^{-2t}\sin 3t$

4. $\mathcal{L}^{-1}\left\{\frac{s-2}{(s-2)^2-16}\right\}=?$

(a)$e^{2t}cosh4t$ (b)$e^{2t}sinh4t$ (c)$e^{-2t}cosh4t$ (d)$e^{2t}\frac{sinh4t}{2}$

THE END


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