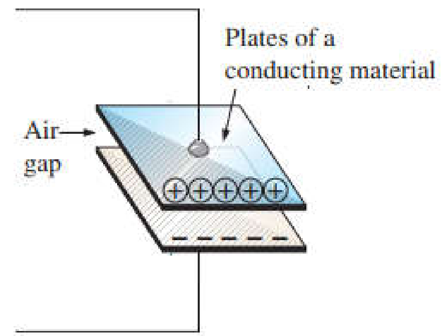
Chapter 10

Capacitors

10.4 Capacitor



Capacitor:

A capacitor is a passive element that is constructed simply of two conducting surfaces or plats separated by the air gap or insulator or dielectric materials**.** Capacitor is also called **condenser**.

Capacitance:

Capacitance is a measure the ability of a capacitor to store charge on its plates as well as to oppose the rate of change of voltage (dv/dt). Unit of capacitance is **Farad** (F).

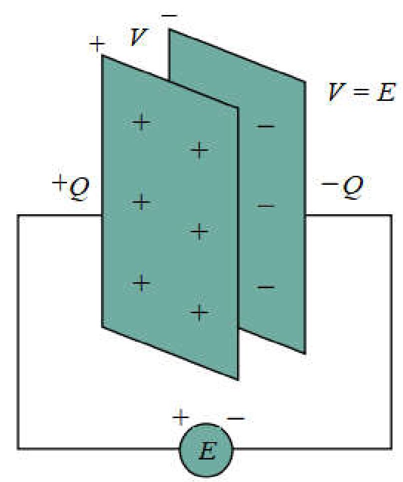
The higher the capacitance of a capacitor, the greater the amount of charge stored on the plates for the same applied voltage.

Relationship among the applied voltage (E=V), the charge on the plates (Q), and the capacitance level (C):

Capacitance is the ratio of the charge (Q) in one plate of a capacitor to the voltage difference (V) between the two plates.

$$\boxed{C = \frac{Q}{V}} \qquad \begin{array}{l} C = \text{farads (F)} \\ Q = \text{coulombs (C)} \\ V = \text{volts (V)} \end{array} \qquad \textbf{(10.5)}$$

$$\boxed{Q = CV} \qquad \text{(coulombs, C)} \qquad \textbf{(10.6)}$$



Problem 3 **[P453]** Find the capacitance of a parallel plate capacitor if 1200 μC of charge are deposited on its plates when 10 V are applied across the plates.

Solution: $Q = 1200\ \mu C = 1200\times10^{-6}\ \mu C$, $V = 10$ V

We know that:

$$C = \frac{Q}{V} = \frac{1200\times10^{-6}\ \text{C}}{10\ \text{V}} = \mathbf{120\times10^{-6}\ F}\ \text{or}\ \mathbf{120\ \mu F}$$

Problem 4 **[P453]** How much charge is deposited on the plates of a 0.15 μF capacitor if 45 V are applied across the capacitor?

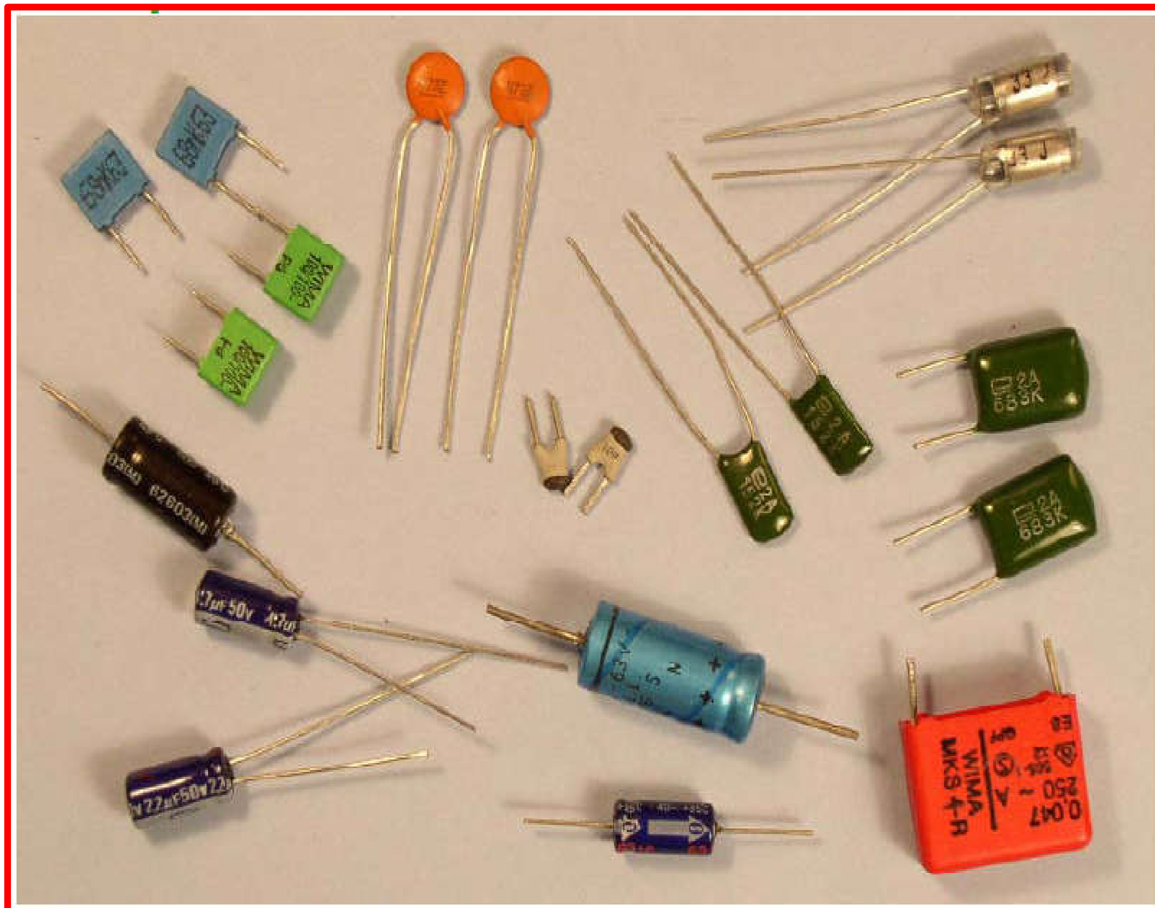
Solution: $C = 0.15\ \mu F = 0.15\times10^{-6}\ \mu F$, $V = 45$ V

We know that: $C = \frac{Q}{V}$

$$\therefore\quad Q = CV = (0.15\times10^{-6}\ \text{C})(45\ \text{V}) = \mathbf{6.75\times10^{-6}\ C}\ \text{or}\ \mathbf{6.75\ \mu C}$$

$$\boxed{C = \frac{Q}{V}} \quad \boxed{Q = CV}$$

Practice Example 10.1



1. Limit rate of change of voltage
2. Radio Receiver
3. Block dc
4. Pass ac
5. Shift phase
6. Store energy
7. Start motors
8. Suppress noise
9. Filter Circuits
10. Improve power factor

Capacitance Based on Physical Dimension

Equation of Capacitance:

$$\boxed{C = \epsilon \frac{A}{d}} \qquad \begin{aligned} C &= \text{farads (F)} \\ \epsilon &= \text{permittivity (F/m)} \\ A &= \text{m}^2 \\ d &= \text{m} \end{aligned} \qquad \textbf{(10.9)}$$

$$\boxed{C = \epsilon_o \epsilon_r \frac{A}{d}} \quad \text{(farads, F)} \qquad \textbf{(10.10)}$$

$$\boxed{C = 8.85 \times 10^{-12} \epsilon_r \frac{A}{d}} \quad \text{(farads, F)} \qquad \textbf{(10.11)}$$

$\epsilon_o = 8.85 \times 10^{-12}$ [F/m] and ϵ_r is called the relative **permittivity**.

FIG. 10.11 *(a) fixed; (b) variable.*

Let, C_o is the value of capacitance considering air as a dielectric material and C is the value of capacitance for any other dielectric material, then we have

$$\boxed{C = \epsilon_r C_o} \qquad \textbf{(10.12)}$$

$$\boxed{C_o = \epsilon_o \frac{A}{d}} \quad \text{(farads, F)}$$

TABLE 10.1
Relative permittivity (dielectric constant) ϵ_r of various dielectrics.

Dielectric	ϵ_r (Average Values)
Vacuum	1.0
Air	1.0006
Teflon®	2.0
Paper, paraffined	2.5
Rubber	3.0
Polystyrene	3.0
Oil	4.0
Mica	5.0
Porcelain	6.0
Bakelite®	7.0
Aluminum oxide	7
Glass	7.5
Tantalum oxide	30
Ceramics	20–7500
Barium-strontium titanite (ceramic)	7500.0

EXAMPLE 10.2 In Fig. 10.9(a) and (b), if each air capacitor in the left column is changed to the type appearing in the right column, find the new capacitance level. For each change, the other factors remain the same.

$$\boxed{C = \epsilon_o \epsilon_r \frac{A}{d}}$$

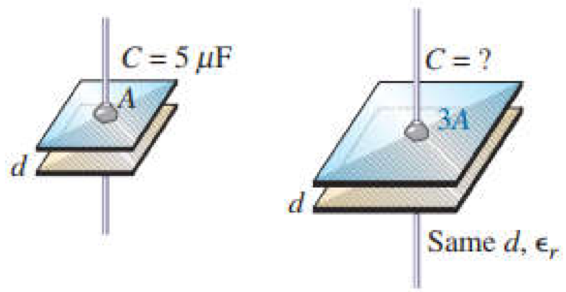


FIG. 10.9 *Example 10.2.* (a)

Solutions:

a. In Fig. 10.9(a), the area has increased by a factor of three, providing more space for the storage of charge on each plate. Since the area appears in the numerator of the capacitance equation, the capacitance increases by a factor of three. That is,

$$C = 3(C_o) = 3(5\ \mu\text{F}) = \mathbf{15\ \mu F}$$

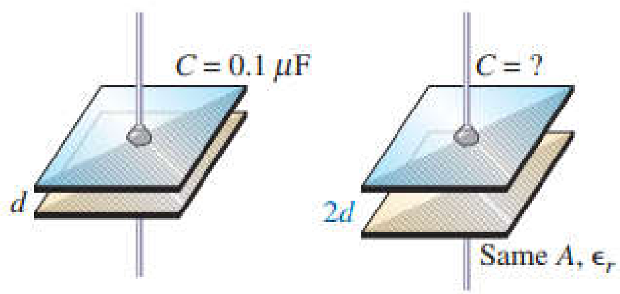


FIG. 10.9 *Example 10.2.* (b)

b. In Fig. 10.9(b), the area stayed the same, but the distance between the plates was increased by a factor of two. Increasing the distance reduces the capacitance level, so the resulting capacitance is one-half of what it was before. That is,

$$C = \frac{1}{2}(0.1\ \mu\text{F}) = \mathbf{0.05\ \mu F}$$

EXAMPLE 10.2 In Fig. 10.9(c) and (d), if each air capacitor in the left column is changed to the type appearing in the right column, find the new capacitance level. For each change, the other factors remain the same.

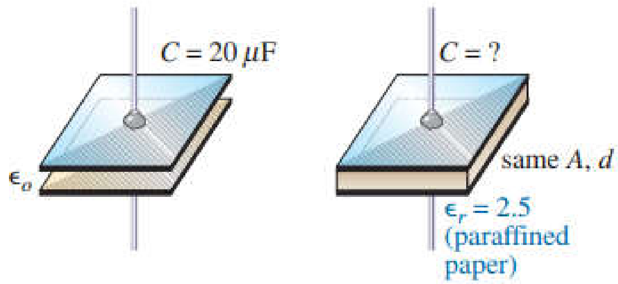


FIG. 10.9 *Example 10.2.* (c)

c. In Fig. 10.9(c), the area and the distance between the plates were maintained, but a dielectric of paraffined (waxed) paper was added between the plates. Since the permittivity appears in the numerator of the capacitance equation, the capacitance increases by a factor determined by the relative permittivity. That is,

$$C = \epsilon_r C_o = 2.5(20\ \mu F) = \mathbf{50\ \mu F}$$

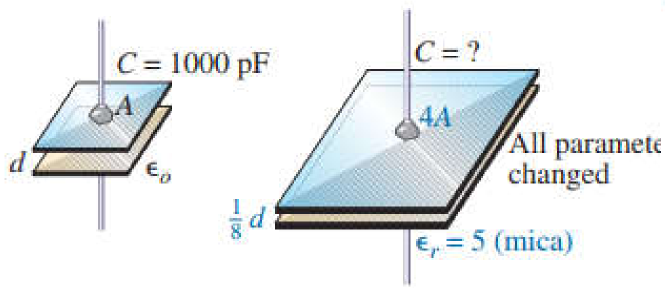


FIG. 10.9 *Example 10.2.*(d)

d. In Fig. 10.9(d), a multitude of changes are happening at the same time. However, solving the problem is simply a matter of determining whether the change increases or decreases the capacitance and then placing the multiplying factor in the numerator or denominator of the equation. The increase in area by a factor of four produces a multiplier of four in the numerator, as shown in the equation below. Reducing the distance by a factor of 1/8 will increase the capacitance by its inverse, or a factor of eight. Inserting the mica dielectric increases the capacitance by a factor of five. The result is

$$C = (5)\frac{4}{(1/8)}(C_o) = 160(1000\ \text{pF}) = \mathbf{0.16\ \mu F}$$

Practice Problem 14 [P453]

10.5 TRANSIENTS IN CAPACITIVE NETWORKS

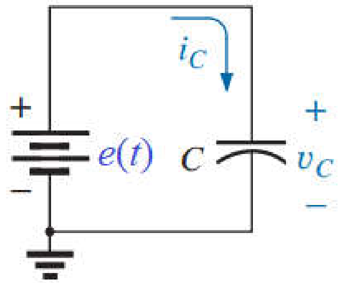
If the applied voltage is change with respect to time the storage charge and current are also change in a capacitor. The changeable voltage, current and charge are represented by small letter *i.e.* v, i and q.

Eq. (10.6) can be written as follows where voltage (v) and charge (q) are changing with respect to time:

$$q = Cv \qquad \textbf{(10.6.1)}$$

Similarly, Eq. (2.5) can be written as follows where current (i) and charge (q) are changing with respect to time:

$$i = \frac{dq}{dt} \qquad \textbf{(2.5.1)}$$



Combining Eq. (10.6.1) and Eq. (2.5.1) we have:

$$i_C = C\frac{dv_C}{dt} \qquad \textbf{(10.5.1)}$$

By integrating Eq. (10.5.1) in both sides we have:

$$v_C = \frac{1}{C}\int_{t_0}^{t} i_C\,dt + v_C(t_0) \qquad \textbf{(10.5.2)}$$

Energy storage by a capacitor can be calculated as follows:

$$W_C = \frac{1}{2}Cv_C^2 \quad [\text{J}] \qquad \textbf{(10.5.3)}$$

Charging and Discharging in a Capacitor

In Figure 10.39(a):

Charging Mode: When the switch is in **position 1 as shown in Figure 10.39 (b)**, capacitor is stored the charge.

Discharging Mode: When the switch is in **position 2 as shown in Figure 10.39 (c)**, capacitor is released the charge through the resistor.

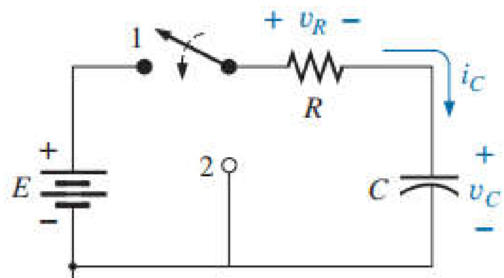


Figure 10.39 (a)

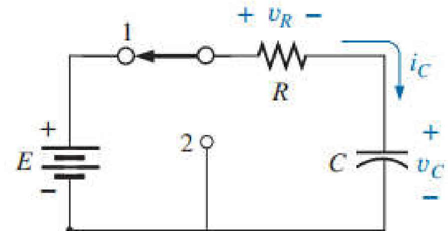


Figure 10.39 (b)

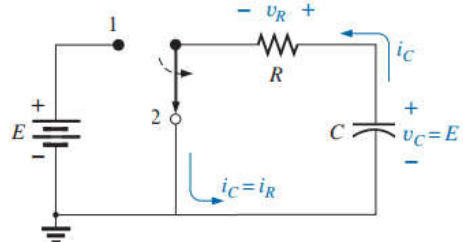


Figure 10.39 (c)

Charging Mode of Fig. 10.39(a):

According to KVL we have:

$$v_R + v_C = E \qquad Ri_C + v_C = E$$

$$RC\frac{dv_C}{dt} + v_C = E \qquad (i)$$

The solution of Eq. (i) is as follows:

$$v_C = E\left(1 - e^{-t/\tau}\right) \quad \text{(volt, V)} \qquad \mathbf{(10.13)}$$

Substitute Eq. (10.13) into Eq. (10.5.1), we have:

$$i_C = \frac{E}{R}e^{-t/\tau} \quad \text{(ampere, A)} \qquad \mathbf{(10.15)}$$

The voltage drop across the resistor will be:

$$v_R = Ee^{-t/\tau} \quad \text{(volt, V)} \qquad \mathbf{(10.16)}$$

The quantity τ is called **time constant**, which is given by:

$$\tau = RC \quad \text{(second, s)} \qquad \mathbf{(10.14)}$$

Prove that the unit of $\tau = RC$ is seconds.

$$RC = \left(\frac{V}{I}\right)\left(\frac{Q}{V}\right) = \left(\frac{Q}{I}\right) = \left(\frac{It}{I}\right) = t \quad \text{(seconds)}$$

At time $t = 0$ (at the instant of switch is closed), from Eqs. (10.13) and (10.15) we have:

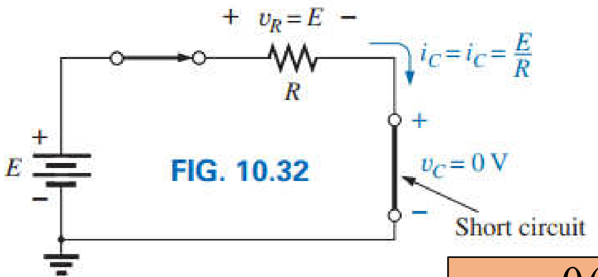


FIG. 10.32

$$v_C = 0 \text{ (C is shorted)}$$

$$i_C = \frac{E}{R} \qquad v_R = E$$

At time $t = \infty$ (steady-state condition), from Eqs. (10.13) and (10.15) we have:

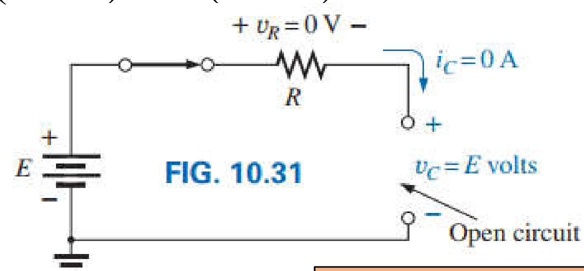


FIG. 10.31

$$v_C = E \qquad v_R = 0$$

$$i_C = 0 \text{ (C is open)}$$

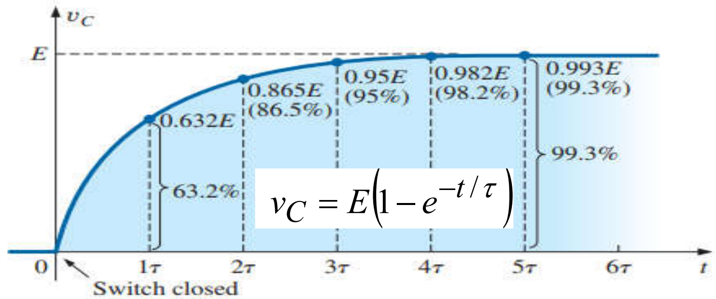


FIG. 10.29 v_C versus time (t).

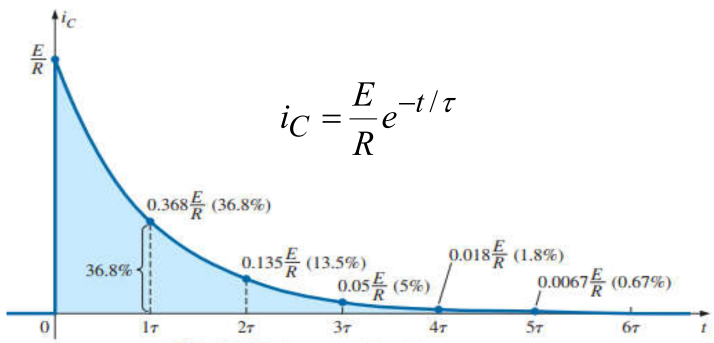


FIG. 10.30 i_C versus time (t).

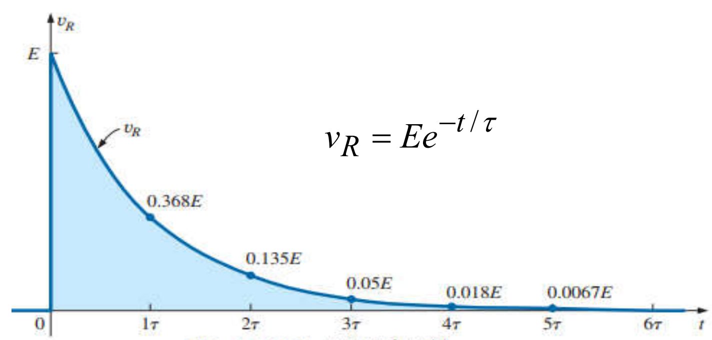


FIG. 10.33 v_R versus time (t).

It has been observed from graphs of Figures 10.29 and 10.30 that at $t = 5\tau$, the capacitor voltage becomes almost equal to supply voltage and the current becomes almost zero.

The transient or charging phase of a capacitor has essentially ended after five time constants.

After five time constants, circuit in steady state.

Discharging Phase

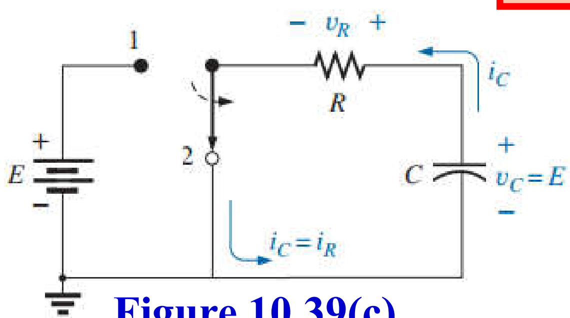


Figure 10.39(c)

According to KVL we have:

$$v_R = v_C \qquad Ri_C = E \qquad RC\frac{dv_C}{dt} = E \qquad (ii)$$

The solution of Eq. (ii) is as follows:

$$v_C = Ee^{-t/\tau} \quad \text{(volt, V)} \qquad \textbf{(10.17)}$$

Substitute Eq. (10.17) into Eq. (10.5.1), we have:

$$i_C = -\frac{E}{R}e^{-t/\tau} \quad \text{(ampere, A)} \qquad \textbf{(10.19)}$$

The voltage drop across the resistor will be:

$$v_R = -Ee^{-t/\tau} \quad \text{(volt, V)} \qquad \textbf{(10.20)}$$

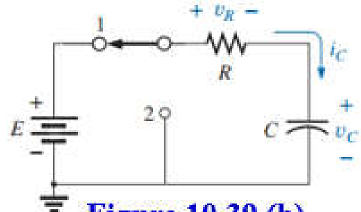


Figure 10.39 (b)

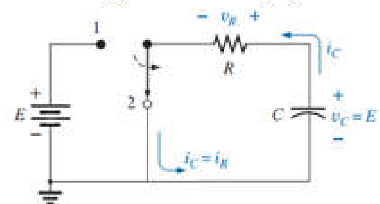
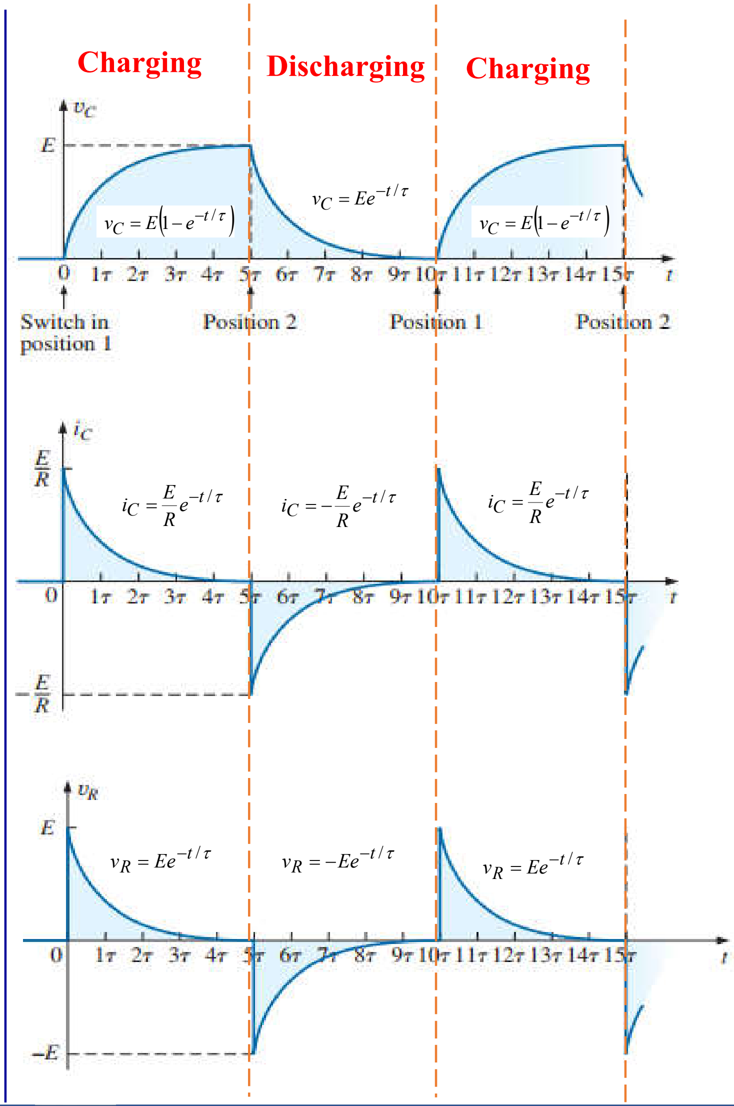


Figure 10.39 (c)

It has been observed from Figure 10.39(b) with Figure 10.39(c), the direction of current i_C and the polarity of voltage v_R are opposite in discharging mode as compare with charging mode.



EXAMPLE 10.6 For the circuit in Fig. 10.35:

a. Find the mathematical expression for the transient behavior of v_C, i_C, and v_R if the switch is closed at $t = 0$ s.

b. Plot the waveform of v_C versus the time constant of the network.

c. Plot the waveforms of i_C and v_R versus the time constant of the network.

d. What is the value of v_C at $t = 20$ ms?

e. On a practical basis, how much time must pass before we can assume that the charging phase has passed?

f. When the charging phase has passed, how much charge is sitting on the plates?

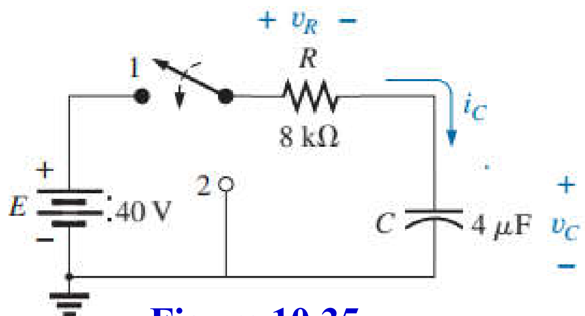


Figure 10.35

Solution: Given, $E = 40$ V, $R = 8\times10^3\ \Omega$, and $C = 4\times10^{-6}$ F,

a. The time constant of the network is

$$\tau = RC = (8\ \text{k}\Omega)(4\ \mu\text{F}) = 32\ \text{ms}$$

resulting in the following mathematical equations:

$$v_C = E(1 - e^{-t/\tau}) = \mathbf{40\ V(1 - e^{-t/32ms})}$$

$$i_C = \frac{E}{R}e^{-t/\tau} = \frac{40\ \text{V}}{8\ \text{k}\Omega}e^{-t/32\text{ms}} = \mathbf{5\ mAe^{-t/32ms}}$$

$$v_R = Ee^{-t/\tau} = \mathbf{40\ Ve^{-t/32ms}}$$

b. The resulting plot appears in Fig. 10.36.

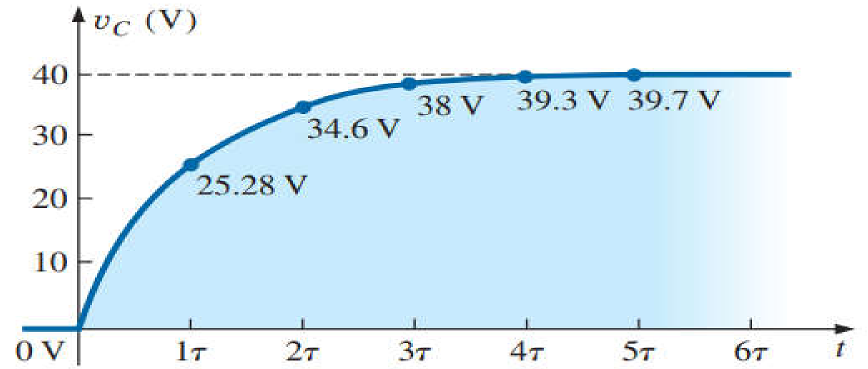


FIG. 10.36

v_C versus time for the charging network in Fig. 10.35.

c. The horizontal scale will now be against time rather than time constants, as shown in Fig. 10.37. The plot points in Fig. 10.37 were taken from Fig. 10.36.

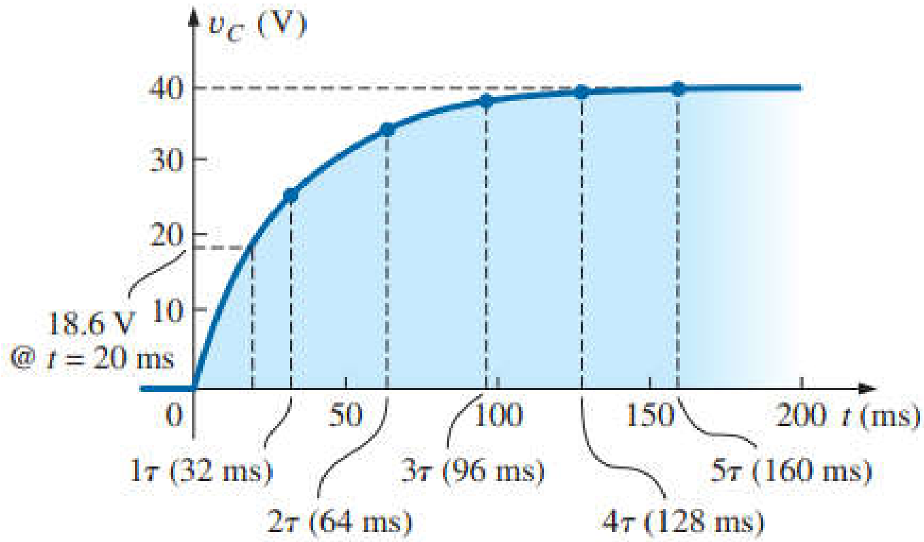


FIG. 10.37

Plotting the waveform in Fig. 10.36 versus time (t).

d. Both plots appear in Fig. 10.38.

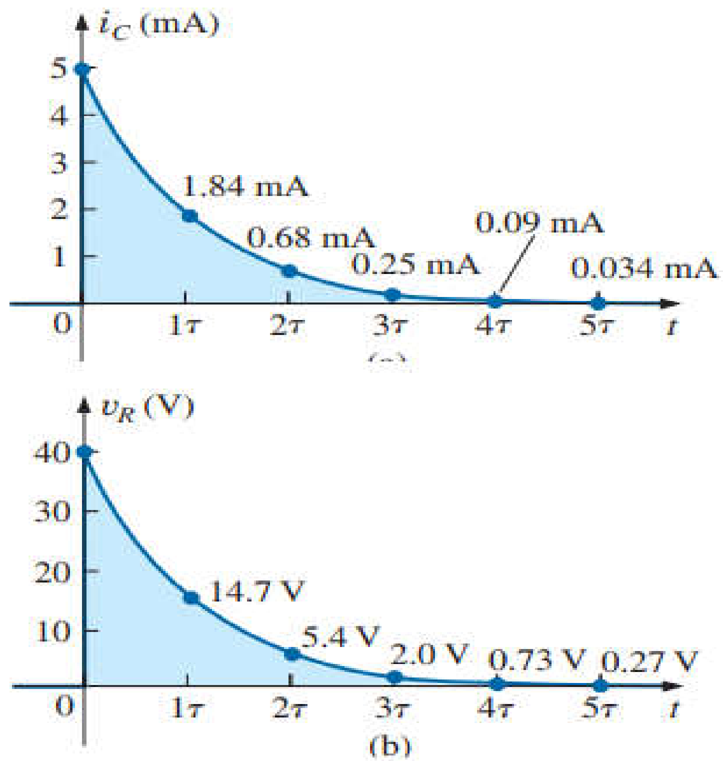


FIG. 10.38

i_C and v_R for the charging network in Fig. 10.36.

e. Substituting the time $t = 20$ ms results in the following for the exponential part of the equation:

$$e^{-t/\tau} = e^{-20\text{ms}/32\text{ms}} = e^{-0.625} = 0.535 \text{ (using a calculator)}$$

so that $\upsilon_C = 40\text{ V}(1 - e^{-t/32\text{ms}}) = 40\text{ V}(1 - 0.535)$

$= (40\text{ V})(0.465) = \mathbf{18.6\ V}$ (as verified by Fig. 10.37)

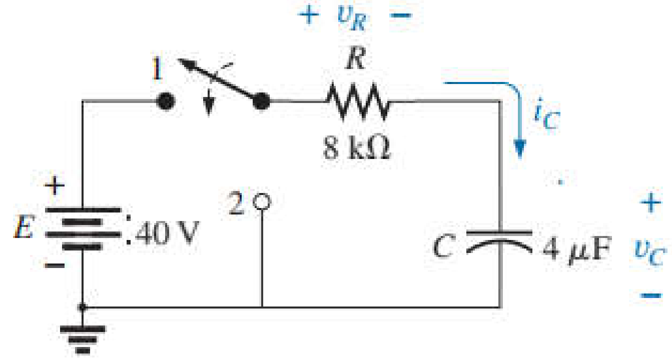
f. Assuming a full charge in five time constants results in

$$5\tau = 5(32\text{ ms}) = \mathbf{160\ ms} = \mathbf{0.16\ s}$$

g. Using Eq. (10.6):

$$Q = CV = (4\ \mu\text{F})(40\text{ V}) = \mathbf{160\ \mu C}$$

EXAMPLE 10.7 Plot the waveforms for v_C and i_C resulting from switching between contacts 1 and 2 in the following figure every five time constants.



Solution: Given, $E = 40$ V, $R = 8\times10^3\ \Omega$, and $C = 4\times10^{-6}$ F

a. The time constant of the network is

$$\tau = RC = (8\text{ k}\Omega)(4\ \mu\text{F}) = 32\text{ ms}$$

Charging Mode:

$$v_C = E(1 - e^{-t/\tau}) = \mathbf{40\ V(1 - e^{-t/32ms})}$$

$$i_C = \frac{E}{R}e^{-t/\tau} = \frac{40\text{ V}}{8\text{ k}\Omega}e^{-t/32\text{ms}} = \mathbf{5\ mAe^{-t/32ms}}$$

$$v_R = Ee^{-t/\tau} = \mathbf{40\ Ve^{-t/32ms}}$$

Discharging Mode:

$$v_C = Ee^{-t/\tau} = \mathbf{40\ Ve^{-t/32ms}}$$

$$i_C = -\frac{E}{R}e^{-t/\tau} = \frac{40\text{ V}}{8\text{ k}\Omega}e^{-t/32\text{ms}} = \mathbf{-5\ mAe^{-t/32ms}}$$

$$v_R = v_C = \mathbf{40\ Ve^{-t/32ms}}$$

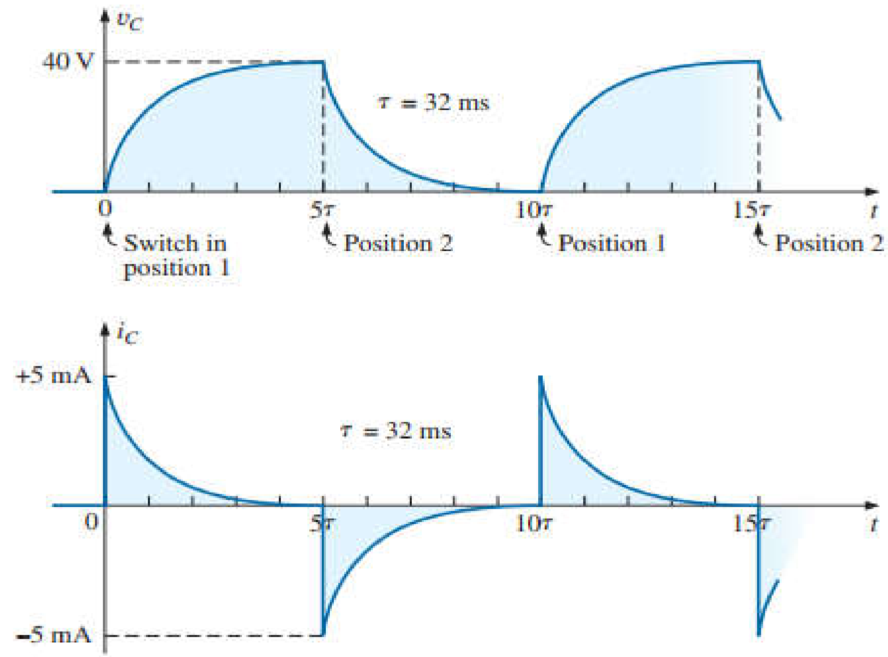


FIG. 10.41

v_C and i_C for the network in Fig. 10.39(a) with the values in Example 10.6.

EXAMPLE 10.8 For the circuit in Fig. 10.44:

a. Find the mathematical expressions for the transient behavior of the voltage υ_C and the current i_C if the capacitor was initially uncharged and the switch is thrown into position 1 at $t = 0$ s.

b. Find the mathematical expressions for the voltage υ_C and the current i_C if the switch is moved to position 2 at $t = 10$ ms. (Assume that the leakage resistance of the capacitor is infinite ohms; that is, there is no leakage current.)

c. Find the mathematical expressions for the voltage υ_C and the current i_C if the switch is thrown into position 3 at $t = 20$ ms.

d. Plot the waveforms obtained in parts (a)–(c) on the same time axis using the defined polarities in Fig. 10.44.

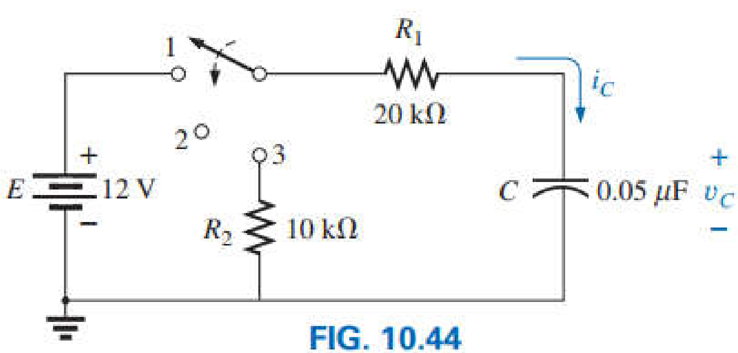


FIG. 10.44
Network to be analyzed in Example 10.8.

Solutions:

a. *Charging phase:*

$$\tau = R_1C = (20\text{ k}\Omega)(0.05\ \mu\text{F}) = 1\text{ ms}$$

$$\upsilon_C = E(1 - e^{-t/\tau}) = \mathbf{12\ V(1 - e^{-t/1ms})}$$

$$i_C = \frac{E}{R_1}e^{-t/\tau} = \frac{12\text{ V}}{20\text{ k}\Omega}e^{-t/1\text{ms}} = \mathbf{0.6\ mAe^{-t/1ms}}$$

b. *Storage phase:* At 10 ms, a period of time equal to 10τ has passed, permitting the assumption that the capacitor is fully charged. The result is that both υ_C and i_C will remain at a fixed value of

$$\upsilon_C = \mathbf{12\ V}$$
$$i_C = \mathbf{0\ A}$$

c. *Discharge phase* (using 20 ms as the new $t = 0$ s for the equations): The new time constant is

$$\tau' = RC = (R_1 + R_2)C = (20\text{ k}\Omega + 10\text{ k}\Omega)(0.05\ \mu\text{F}) = 1.5\text{ ms}$$

$$\upsilon_C = Ee^{-t/\tau'} = \mathbf{12\ Ve^{-t/1.5ms}}$$

$$i_C = -\frac{E}{R}e^{-t/\tau'} = -\frac{E}{R_1 + R_2}e^{-t/\tau'}$$

$$= -\frac{12\text{ V}}{20\text{ k}\Omega + 10\text{ k}\Omega}e^{-t/1.5\text{ms}} = \mathbf{-0.4\ mAe^{-t/1.5ms}}$$

d. See Fig. 10.45.

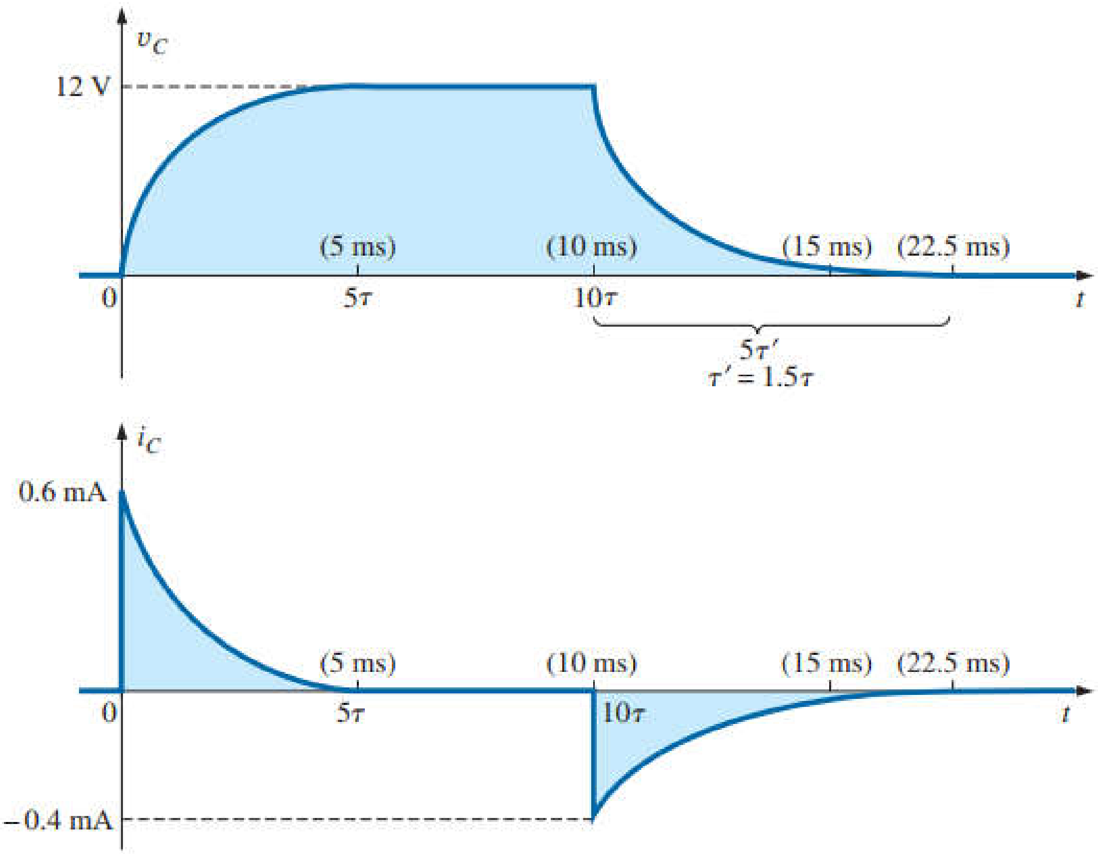


FIG. 10.45
v_C and i_C for the network in Fig. 10.44.

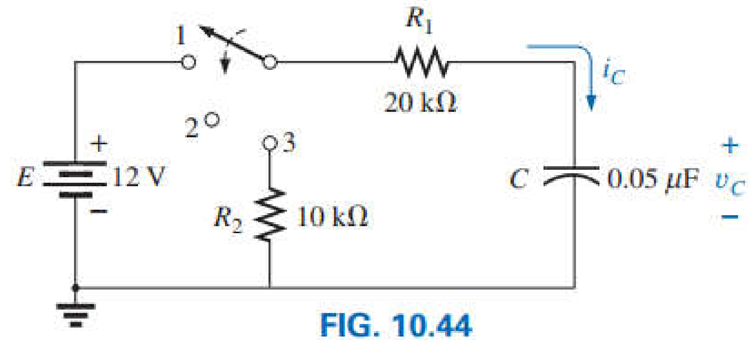


FIG. 10.44
Network to be analyzed in Example 10.8.

$$\tau = R_1C = (20\ \text{k}\Omega)(0.05\ \mu\text{F}) = 1\ \text{ms}$$
$$\tau' = RC = (R_1 + R_2)C = (20\ \text{k}\Omega + 10\ \text{k}\Omega)(0.05\ \mu\text{F}) = 1.5\ \text{ms}$$

27. For the *R-C* circuit in Fig. 10.90, composed of standard values:

a. Determine the time constant of the circuit when the switch is thrown into position 1.

b. Find the mathematical expression for the voltage across the capacitor and the current after the switch is thrown into position 1.

c. Determine the magnitude of the voltage v_C and the current i_C the instant the switch is thrown into position 2 at $t = 1$ s.

d. Determine the mathematical expression for the voltage v_C and the current i_C for the discharge phase.

e. Plot the waveforms of v_C and i_C for a period of time extending from 0 to 2 s from when the switch was thrown into position 1.

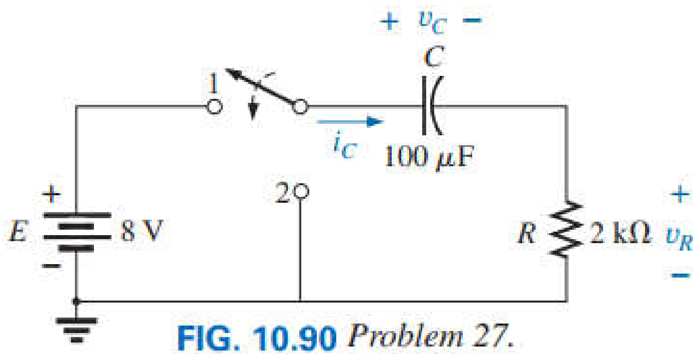


FIG. 10.90 *Problem 27.*

Solution: Given, $E = 8$ V, $R = 2\times10^3\ \Omega$, and $C = 100\times10^{-6}$ F

$$(a)\ \tau = RC = (2\times10^3)(100\times10^{-6}) = \mathbf{200\ ms}$$

$$(b)\ v_C = E\left(1-e^{-t/\tau}\right) = \mathbf{8V\left(1-e^{-t/200ms}\right)}$$

$$i_C = \frac{E}{R}e^{-t/\tau} = \frac{8\text{V}}{2\text{k}\Omega} = \mathbf{4mAe^{-t/200ms}}$$

$$(c)\ e^{-t/\tau} = e^{-1\text{s}/200\text{ms}} = e^{-5} = 0.0067$$

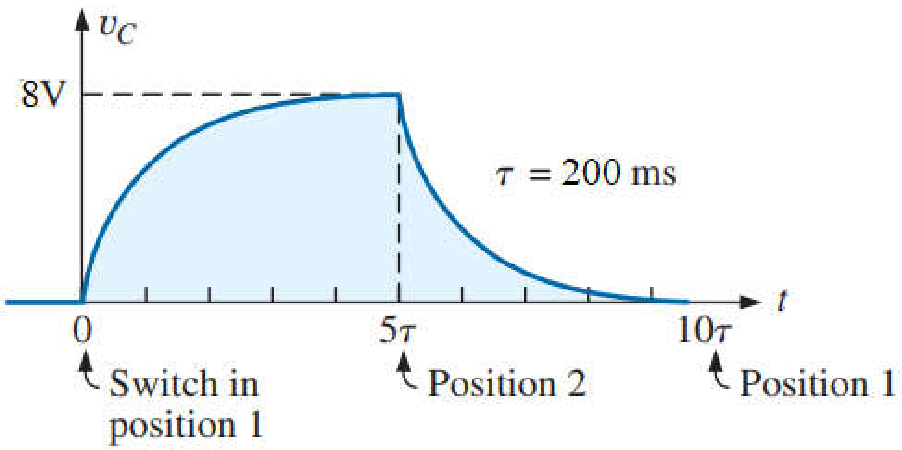
$$v_C = 8\text{V}\left(1-e^{-t/200\text{ms}}\right) = 8\text{V}\times(1-0.0067) = \mathbf{7.95\,V}$$

$$i_C = 4\text{mA}e^{-t/200\text{ms}} = 4\text{mA}\times0.0067 = \mathbf{0.0268mA}$$

$$(d)\ v_C = Ee^{-t/\tau} = \mathbf{8Ve^{-t/200ms}}$$

$$i_C = -\frac{E}{R}e^{-t/\tau} = -\frac{8\text{V}}{2\text{k}\Omega} = \mathbf{-4mAe^{-t/200ms}}$$

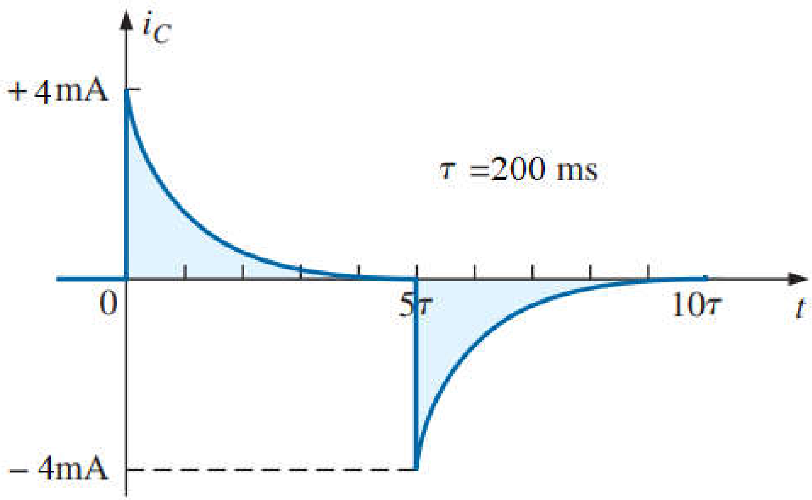
e. Plot the waveforms of v_C and i_C for a period of time extending from 0 to 2 s from when the switch was thrown into position 1.



$$\tau = 200\,\text{ms}$$

$$1\,\text{s} = 5\tau$$

$$2\,\text{s} = 10\tau$$



Practice Problems 21 ~ 30 [Ch. 10]

10.11 CAPACITORS IN SERIES AND IN PARALLEL

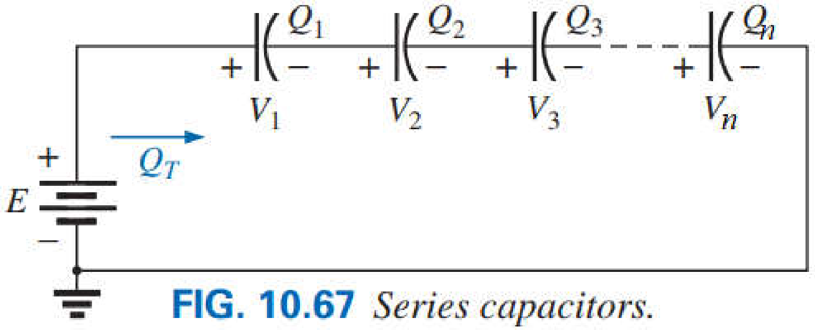
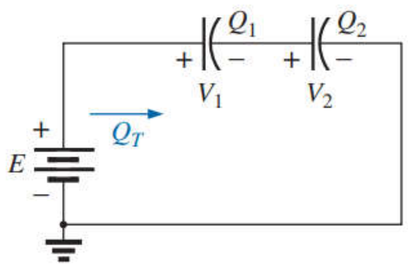


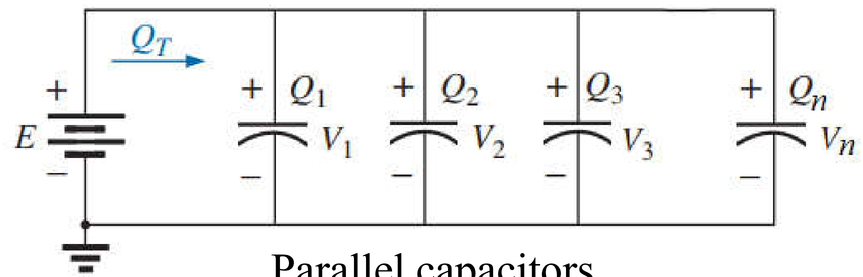
FIG. 10.67 *Series capacitors.*

$$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots\ldots + \frac{1}{C_n} \qquad \textbf{(10.29)}$$

When two capacitors are connected in series:



$$C_T = \frac{C_1 C_2}{C_1 + C_2} \qquad \textbf{(10.30)}$$



Parallel capacitors

$$C_T = C_1 + C_2 + C_3 + \ldots\ldots + C_n \qquad \textbf{(10.33)}$$

EXAMPLE 10.15 For the circuit in Fig. 10.69:

a. Find the total capacitance.
b. Determine the charge on each plate.
c. Find the voltage across each capacitor.

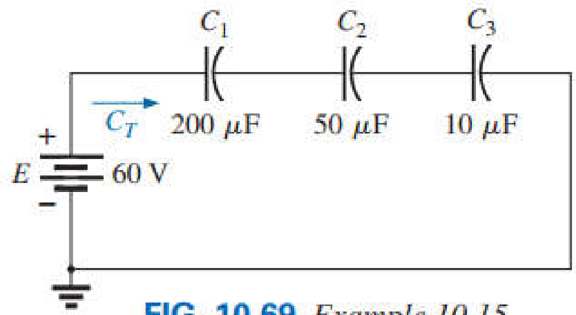


FIG. 10.69 *Example 10.15.*

Solutions:

a. $$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$= \frac{1}{200 \times 10^{-6}\,\text{F}} + \frac{1}{50 \times 10^{-6}\,\text{F}} + \frac{1}{10 \times 10^{-6}\,\text{F}}$$

$$= 0.005 \times 10^6 + 0.02 \times 10^6 + 0.1 \times 10^6$$

$$= 0.125 \times 10^6$$

and $$C_T = \frac{1}{0.125 \times 10^6} = \mathbf{8\ \mu F}$$

b. $$Q_T = Q_1 = Q_2 = Q_3$$

$$= C_T E = (8 \times 10^{-6}\,\text{F})\,(60\,\text{V}) = \mathbf{480\ \mu C}$$

c. $$V_1 = \frac{Q_1}{C_1} = \frac{480 \times 10^{-6}\,\text{C}}{200 \times 10^{-6}\,\text{F}} = \mathbf{2.4\ V}$$

$$V_2 = \frac{Q_2}{C_2} = \frac{480 \times 10^{-6}\,\text{C}}{50 \times 10^{-6}\,\text{F}} = \mathbf{9.6\ V}$$

$$V_3 = \frac{Q_3}{C_3} = \frac{480 \times 10^{-6}\,\text{C}}{10 \times 10^{-6}\,\text{F}} = \mathbf{48.0\ V}$$

and $E = V_1 + V_2 + V_3 = 2.4\,\text{V} + 9.6\,\text{V} + 48\,\text{V} = \mathbf{60\ V}$ (checks)

EXAMPLE 10.16 For the network in Fig. 10.70:

a. Find the total capacitance.
b. Determine the charge on each plate.
c. Find the total charge.

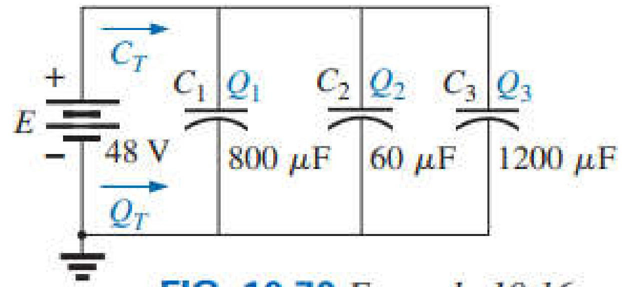


FIG. 10.70 *Example 10.16.*

Solutions:

a. $C_T = C_1 + C_2 + C_3 = 800\ \mu\text{F} + 60\ \mu\text{F} + 1200\ \mu\text{F} = \mathbf{2060\ \mu F}$

b. $Q_1 = C_1E = (800 \times 10^{-6}\,\text{F})(48\ \text{V}) = \mathbf{38.4\ mC}$

$Q_2 = C_2E = (60 \times 10^{-6}\,\text{F})(48\ \text{V}) = \mathbf{2.88\ mC}$

$Q_3 = C_3E = (1200 \times 10^{-6}\,\text{F})(48\ \text{V}) = \mathbf{57.6\ mC}$

c. $Q_T = Q_1 + Q_2 + Q_3 = 38.4\ \text{mC} + 2.88\ \text{mC} + 57.6\ \text{mC} = \mathbf{98.88\ mC}$

28. For the network in Fig. 10.91, composed of standard values:

a. Write the mathematical expressions for the voltages v_C, and v_{R_1} and the current i_C after the switch is thrown into position 1.

b. Find the values of v_C, v_{R_1}, and i_C when the switch is moved to position 2 at $t = 100$ ms.

c. Write the mathematical expressions for the voltages v_C and v_{R_2} and the current i_C if the switch is moved to position 3 at $t = 200$ ms.

d. Plot the waveforms of v_C, v_{R_2}, and i_C for the time period extending from 0 to 300 ms.

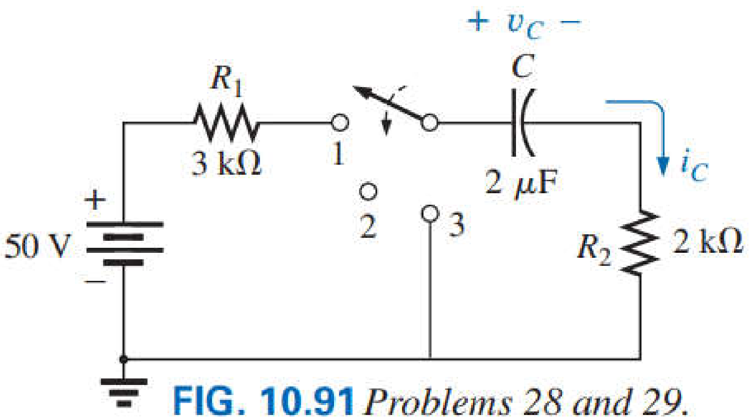


FIG. 10.91 *Problems 28 and 29.*

$$\tau_1 = (R_1 + R_2)C = (3\text{k}\Omega + 2\text{k}\Omega)\times(2\,\mu F)\text{ s}$$
$$= \left(5\times10^3\,\Omega\right)\times\left(2\times10^{-6}\,\text{F}\right)\text{s} = 10\text{ ms}$$

$$v_C = 50\text{V}\left(1 - e^{-t/10\text{ms}}\right)$$
$$i_C = 10\text{mA}e^{-t/10\text{ms}}$$
$$v_{R1} = 30\text{V}e^{-t/10\text{ms}}$$
$$v_{R2} = 20\text{V}e^{-t/10\text{ms}}$$

$$\tau_2 = R_2C = (3\text{k}\Omega)\times(2\,\mu F)\text{ s}$$
$$= \left(3\times10^3\,\Omega\right)\times\left(2\times10^{-6}\,\text{F}\right)\text{s} = 6\text{ ms}$$

$$v_C = 50\text{V}e^{-t/6\text{ms}}$$
$$i_C = -25\text{mA}e^{-t/6\text{ms}}$$
$$v_{R2} = -50\text{V}e^{-t/6\text{ms}}$$

30. For the network in Fig. 10.92, composed of standard values:

a. Find the mathematical expressions for the voltage v_C and the current i_C when the switch is thrown into position 1.

b. Find the mathematical expressions for the voltage v_C and the current i_C if the switch is thrown into position 2 at a time equal to five time constants of the charging circuit.

c. Plot the waveforms of v_C and i_C for a period of time extending from 0 to 30 μs.

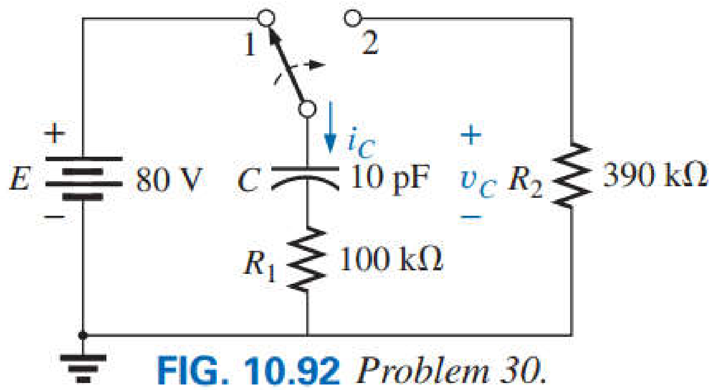
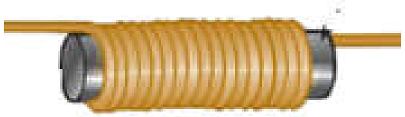


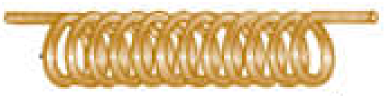
FIG. 10.92 *Problem 30.*

Chapter 11

Inductors

Inductor





Inductor:

An inductor is a passive element that is constructed simply raping the wire on air or iron core. The inductor is also called **coil** or **choke**.

Inductance:

Inductance is a measure the ability of an inductor to store energy in its magnetic field as well as to oppose the rate of change of current (*di*/*dt*). Unit of capacitance is **Henry** (H).

The higher the inductance of an inductor, the greater the amount of stored magnetic energy on the coil or winding.

Type of Inductors

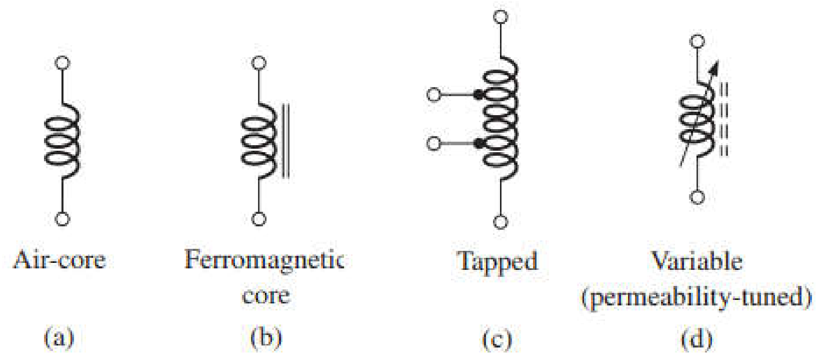
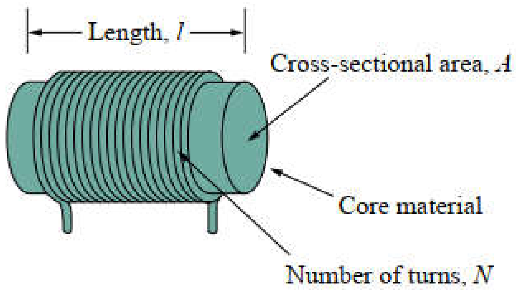


FIG. 11.20 *Inductor (coil) symbols.*

Inductance Based on Physical Dimension

Equation of inductance:



Typical form of an inductor.

$$L = \frac{\mu N^2 A}{l} \qquad (11.6)$$

μ = permeability (Wb/A · m)
N = number of turns (t)
A = m^2
l = m
L = henries (H)

$$L = \frac{\mu_o \mu_r N^2 A}{l} \text{ (henries, H)} \qquad \mathbf{(11.7)}$$

$$\mu_o = 4\pi \times 10^{-7} \text{ (Wb/A.m)} \qquad \mathbf{(11.4)}$$

μ_r is called the relative **permeability**.

Let, L_o is the value of inductance considering air core and L is the value of inductance for any other metal core, then we have

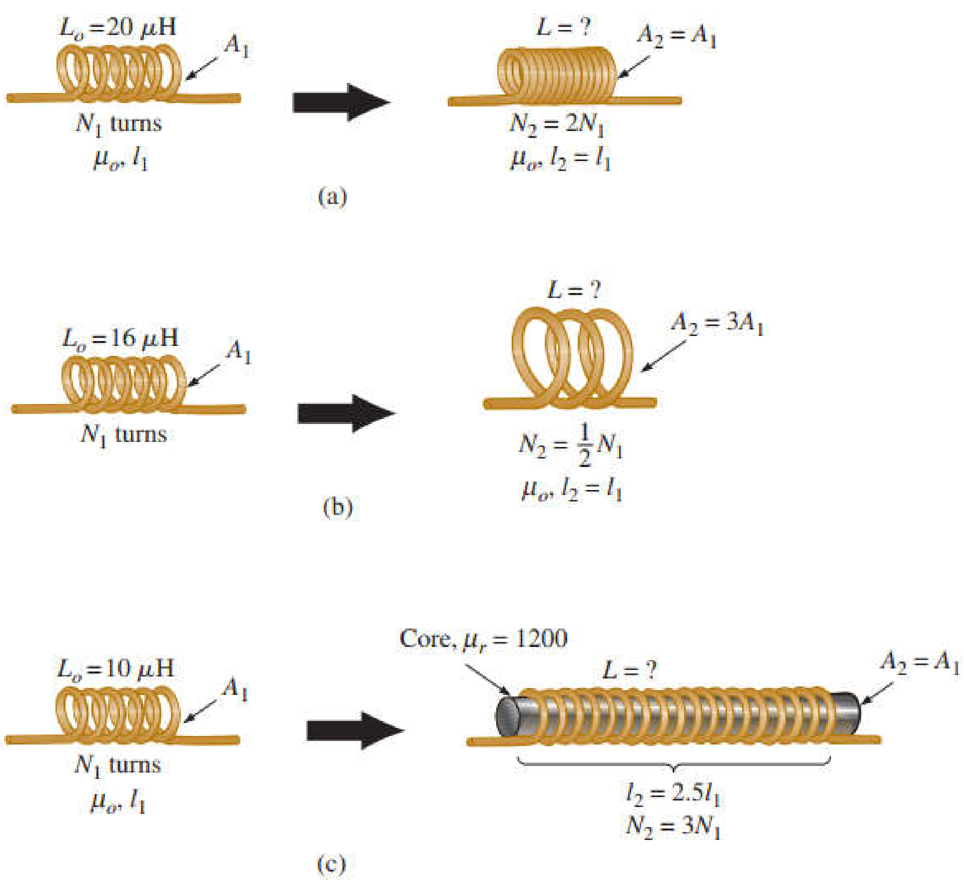
$$L = \mu_r L_o \text{ (henries, H)} \qquad \mathbf{(11.8)}$$

$$\text{where, } L_o = \frac{\mu_o N^2 A}{l} \text{ (henries, H)}$$

Relative permeability of metals.

Material	Relative permeability, μ_r	Material	Relative permeability, μ_r
Aluminum	1.00000065	Nickel	50–600
Cobalt	60	Palladium	1.0008
Copper	0.999994	Permalloy 45	2500
Ferrite (NiZn)	16–640	Platinum	1.000265
Gold	0.999998	Silver	0.99999981
Iron	5000–6000	Steel	100–40000
Lead	0.999983	Superconductors	0
Magnesium	1.00000693	Supermalloy	100000
Manganese	1.000125	Tungsten	1.000068
Mumetal	20000–1000000	Wood (dry)	0.99999942

EXAMPLE 11.2 In Fig. 11.19, if each inductor in the left column is changed to the type appearing in the right column, find the new inductance level. For each change, assume that the other factors remain the same.



Solutions:

a. The only change was the number of turns, but it is a squared factor, resulting in

$$L = (2)^2 L_o = (4)(20\ \mu\text{H}) = \mathbf{80\ \mu H}$$

b. In this case, the area is three times the original size, and the number of turns is 1/2. Since the area is in the numerator, it increases the inductance by a factor of three. The drop in the number of turns reduces the inductance by a factor of $(1/2)^2 = 1/4$. Therefore,

$$L = (3)\left(\frac{1}{4}\right)L_o = \frac{3}{4}(16\ \mu\text{H}) = \mathbf{12\ \mu H}$$

c. Both μ and the number of turns have increased, although the increase in the number of turns is squared. The increased length reduces the inductance. Therefore,

$$L = \frac{(3)^2(1200)}{2.5}L_o = (4.32 \times 10^3)(10\ \mu\text{H}) = \mathbf{43.2\ mH}$$

EXAMPLE 11.1 For the air-core coil in Fig. 11.18:

a. Find the inductance.
b. Find the inductance if a metallic core with $\mu_r = 2000$ is inserted in the coil.

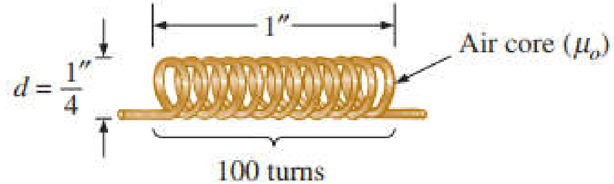


FIG. 11.18 *Air-core coil for Example 11.1.*

Solution: We know that, 39.37 inch = 1 meter

a. $d = \frac{1}{4}\cancel{\text{in.}}\left(\frac{1\text{ m}}{39.37\ \cancel{\text{in.}}}\right) = 6.35\text{ mm}$ $\quad A = \frac{\pi d^2}{4} = \frac{\pi(6.35\text{ mm})^2}{4} = 31.7\ \mu\text{m}^2$ $\quad l = 1\ \cancel{\text{in.}}\left(\frac{1\text{ m}}{39.37\ \cancel{\text{in.}}}\right) = 25.4\text{ mm}$

$$L = 4\pi \times 10^{-7}\frac{\mu_r N^2 A}{l} = 4\pi \times 10^{-7}\frac{(1)(100\text{ t})^2(31.7\ \mu\text{m}^2)}{25.4\text{ mm}} = \mathbf{15.68\ \mu H}$$

b. Eq. (11.8): $\quad L = \mu_r L_o = (2000)(15.68\ \mu\text{H}) = \mathbf{31.36\ mH}$

Practice Problem 1 ~ 5 [P504]

Problems [P504]

2. For the inductor in Fig. 11.82, find the inductance L in henries.

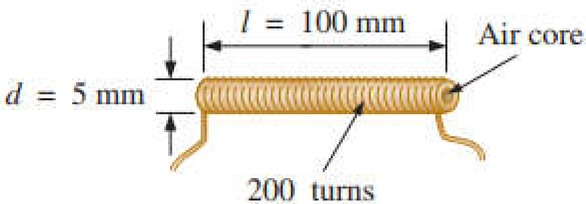


FIG. 11.82 *Problems 2 and 3.*

Solution:

$$A = \frac{\pi d^2}{4} = \frac{\pi(5\text{ mm})^2}{4} = 19.63 \times 10^{-6}\text{ m}^2$$

$$L = \frac{N^2 \mu A}{\ell} = \frac{(200\text{ t})^2(4\pi \times 10^{-7})(19.63 \times 10^{-6}\text{ m}^2)}{100\text{ mm}} = \mathbf{9.87\ \mu H}$$

3. Repeat Problem 2 with $l = 1.6$ in., $d = 0.2$ in., and a ferromagnetic core with $\mu_r = 500$.

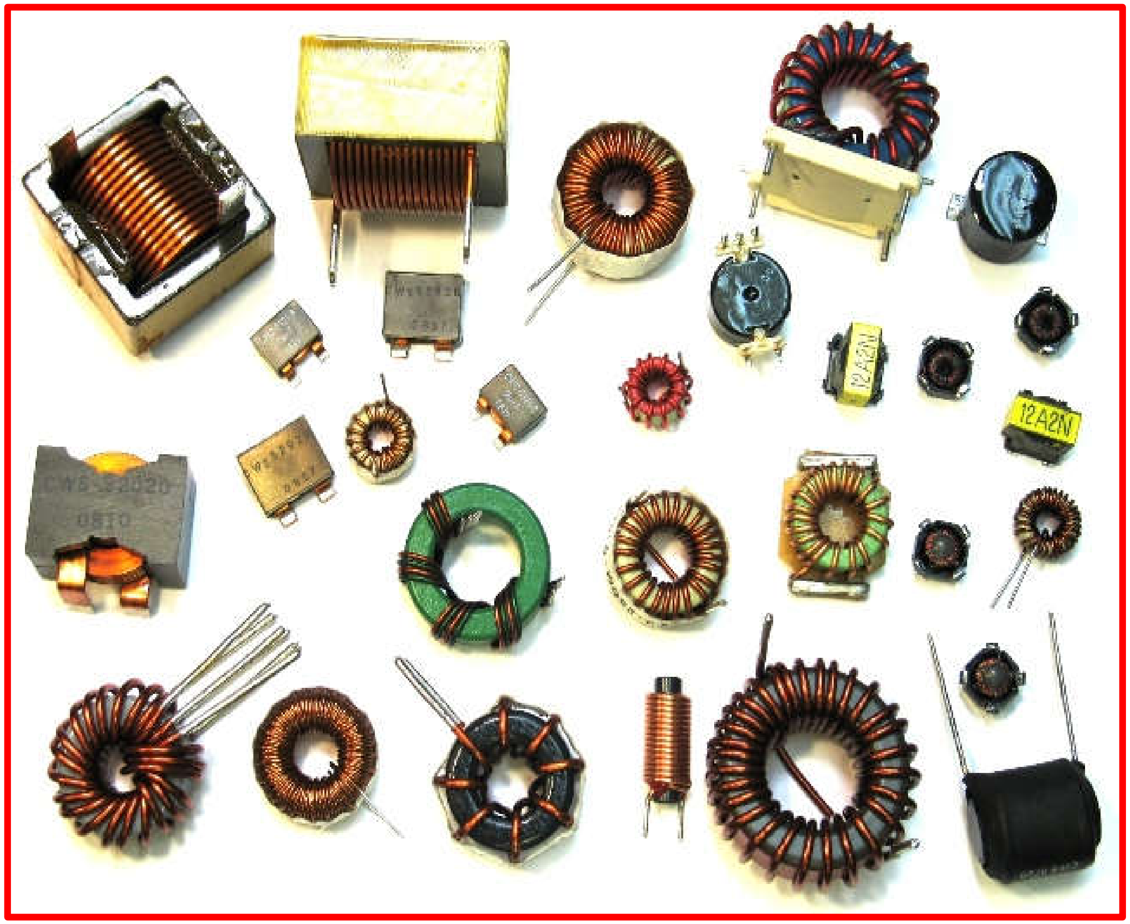
Solution:

$$d = 0.2\ \cancel{\text{in.}}\left[\frac{1\text{ m}}{39.37\ \cancel{\text{in.}}}\right] = 5.08\text{ mm}$$

$$A = \frac{\pi d^2}{4} = \frac{(\pi)(5.08\text{ mm})^2}{4} = 20.27 \times 10^{-6}\text{ m}^2$$

$$\ell = 1.6\ \cancel{\text{in.}}\left(\frac{1\text{ m}}{39.37\ \cancel{\text{in.}}}\right) = 40.64\text{ mm}$$

$$L = \frac{N^2 \mu_r \mu_o A}{\ell} = \frac{(200\text{ t})^2(500)(4\pi \times 10^{-7})(20.27 \times 10^{-6}\text{ m}^2)}{40.64\text{ mm}} = \mathbf{12.54\ mH}$$



1. Limit rate of change of current
2. Power Supplies
3. Transformers
4. Radios
5. Televisions (TVs)
6. Radars
7. Electric Heater
8. Electric motors and generators

TRANSIENTS IN INDUCTIVE NETWORKS

If current is allowed to pass through an inductor, it is found that the voltage across the inductor is directly proportional to the time rate of change of the current.

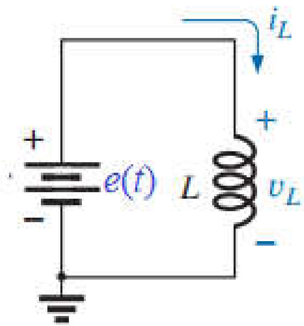
$$v_L = L\frac{di_L}{dt} \qquad \textbf{(11.12)}$$

By integrating Eq. (11.12) in both sides we have:

$$i_L = \frac{1}{L}\int_{t_0}^{t} v_L\,dt + i_L(t_0) \qquad \textbf{(11.12.1)}$$

Energy storage by an inductor can be calculated as follows:

$$W_L = \frac{1}{2}Li_L^2 \quad [\text{J}] \qquad \textbf{(11.12.2)}$$



Effect of Opening a Switch in Series with an Inductor with a Steady-State Current

In *R-L* circuits, the energy is stored in the form of a magnetic field established by the current through the coil. If the series *R-L* circuit in Fig. 11.41 reaches steady state conditions and the switch is quickly opened, a ***spark*** will occur across the contacts due to the rapid change in current from a maximum value to zero amperes. ***The change in current establishes a high voltage drop across the coil*** that, in conjunction with the applied voltage *E*, appears across the points of the switch.

The solution is to use a network like that in Fig. 11.42.

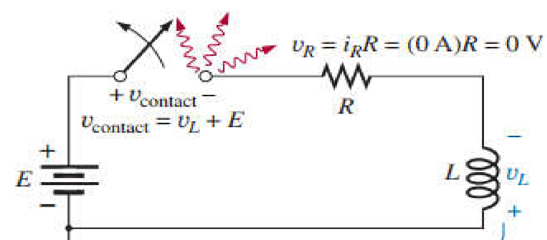


FIG. 11.41

Demonstrating the effect of opening a switch in series with an inductor with a steady-state current.

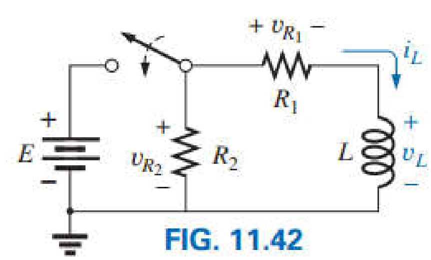
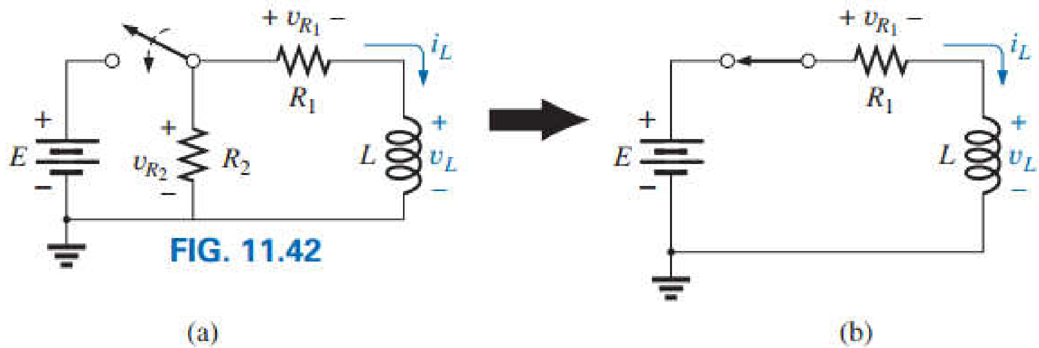


FIG. 11.42

11.5 *R-L* TRANSIENTS: THE STORAGE PHASE



Initiating the storage phase for an inductor by closing the switch.

Charging phase of Fig. (b): According to KVL we have:

$$v_{R1} + v_L = E$$

$$R_1 i_L + L\frac{di_L}{dt} = E$$

$$L\frac{di_L}{dt} + R_1 i_L = E \qquad (i)$$

The solution of Eq. (*i*) is as follows:

$$i_L = \frac{E}{R_1}\left(1 - e^{-t/\tau}\right) \quad \text{(ampere, A)} \qquad \textbf{(11.13)}$$

Substitute Eq. (11.13) into Eq. (11.12), we have:

$$v_L = Ee^{-t/\tau} \quad \text{(volt, V)} \qquad \textbf{(11.15)}$$

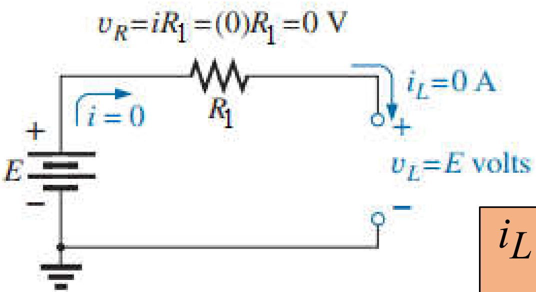
The voltage drop across the resistor will be:

$$v_{R1} = E\left(1 - e^{-t/\tau}\right) \quad \text{(volt, V)} \qquad \textbf{(11.16)}$$

The quantity τ is called **time constant**, which is given by:

$$\tau = \frac{L}{R} = \frac{L}{R_1} \quad \text{(second, s)} \qquad \textbf{(11.14)}$$

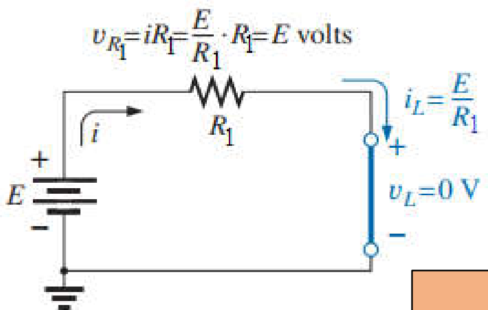
At time $t = 0$ (at the instant of switch is closed), from Eqs. (11.13) and (11.15) we have:



$$i_L = 0 \ \ \textbf{(L is open)}$$

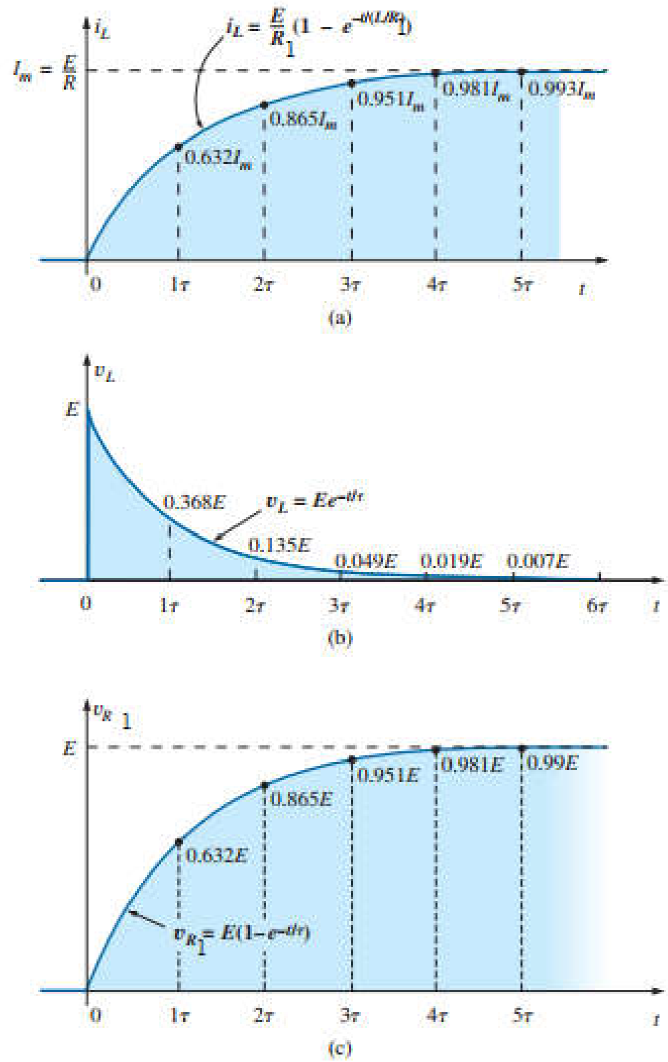
$$v_L = E \qquad v_{R1} = 0$$

At time $t = \infty$ (steady-state condition), from Eqs. (11.13) and (15.15) we have:



$$i_L = \frac{E}{R_1} \qquad v_{R1} = E$$

$$v_L = 0 \ \ \textbf{(L is shorted)}$$



It has been observed from graphs of Figures (a) and (b) that at $t = 5\tau$, the inductor current becomes almost equal to its maximum value and the voltage becomes almost zero.

The transient or storage phase of an inductor has essentially ended after five time constants.

After five time constants, circuit in steady state.

EXAMPLE 11.3 Find the mathematical expressions for the transient behavior of i_L and v_L for the circuit in Fig. 11.36 if the switch is closed at $t = 0$ s. Sketch the resulting curves.

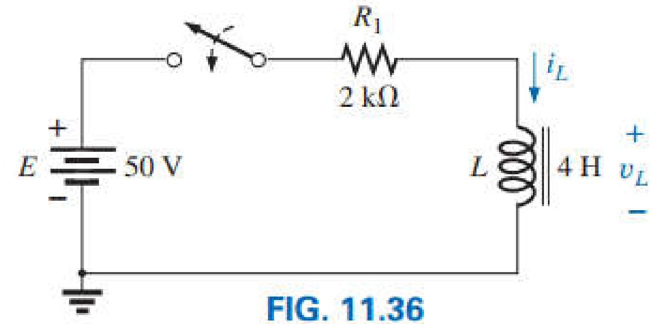


FIG. 11.36
Series R-L circuit for Example 11.3.

Solution: First, the time constant is determined: $\tau = \frac{L}{R_1} = \frac{4\text{ H}}{2\text{ k}\Omega} = 2\text{ ms}$

Then the maximum or steady-state current is

$$I_m = \frac{E}{R_1} = \frac{50\text{ V}}{2\text{ k}\Omega} = 25 \times 10^{-3}\text{A} = 25\text{ mA}$$

Substituting into Eq. (11.13): $i_L = \mathbf{25\ mA\ (1 - e^{-t/2ms})}$

Using Eq. (11.15): $v_L = \mathbf{50\ Ve^{-t/2ms}}$

The resulting waveforms appear in Fig. 11.37.

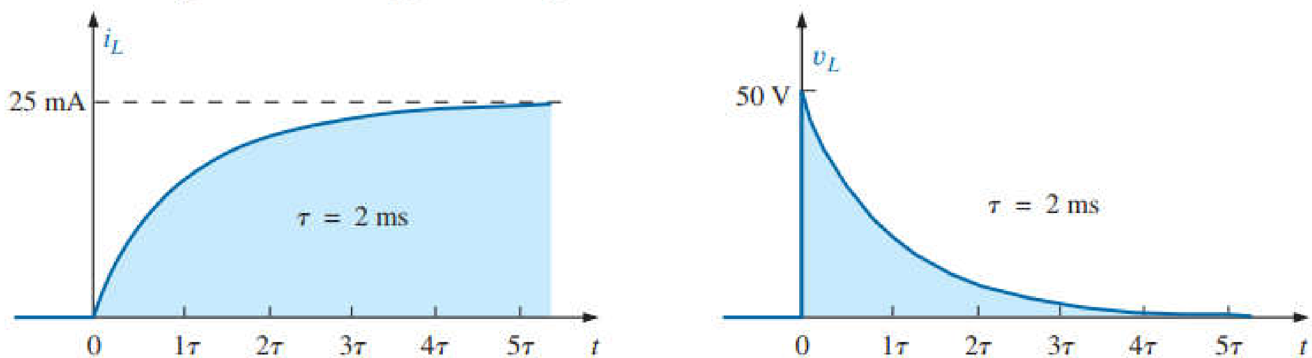


FIG. 11.37 *i_L and v_L for the network in Fig. 11.36.*

11.7 *R-L* TRANSIENTS: THE RELEASE PHASE

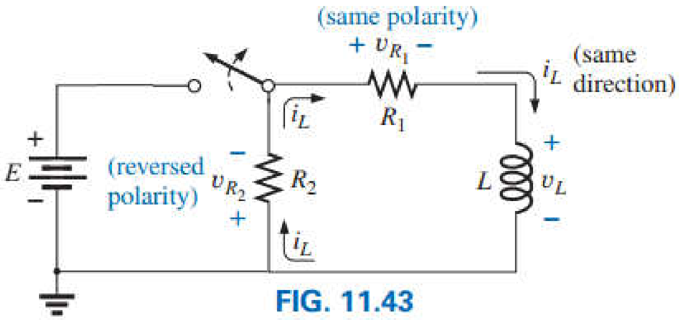


FIG. 11.43
Network in Fig. 11.42 the instant the switch is opened.

According to KVL we have:

$$v_L = -(v_{R1} + v_{R2}) = -(R_1 + R_2)i_L$$

$$L\frac{di_L}{dt} + (R_1 + R_2)i_L = 0 \qquad (ii)$$

At the moment of switch open, $i_{L0} = \dfrac{E}{R_1}$ thus:

$$V_i = (R_1 + R_2)i_{L0} = \left(1 + \frac{R_2}{R_1}\right)E \qquad v_{L0} = -V_i$$

The inductor current exponentially decays from the initial value of i_{L0} to the final value of zero. Thus, the solution of Eq. (ii) yields:

$$i_L = \frac{E}{R_1}e^{-t/\tau'} \quad \text{(ampere, A)} \qquad \textbf{(11.21)}$$

$$\tau' = \frac{L}{R_1 + R_2} \quad \text{(second, s)} \qquad \textbf{(11.14.1)}$$

Substitute Eq. (11.21) into Eq. (11.12), we have:

$$v_L = -V_i e^{-t/\tau'} \quad \text{(volt, V)} \qquad \textbf{(11.20)}$$

The voltage drop across the resistor R_1 will be:

$$v_{R1} = Ee^{-t/\tau'} \quad \text{(volt, V)} \qquad \textbf{(11.22)}$$

The voltage drop across the resistor R_2 will be:

$$v_{R2} = -\frac{R_2}{R_1}Ee^{-t/\tau'} \quad \text{(volt, V)} \qquad \textbf{(11.22)}$$

EXAMPLE 11.5 Resistor R_2 was added to the network in Fig. 11.36 as shown in Fig. 11.44.

a. Find the mathematical expressions for i_L, v_L, v_{R_1}, and v_{R_2} for five time constants of the storage phase.
b. Find the mathematical expressions for i_L, v_L, v_{R_1}, and v_{R_2} if the switch is opened after five time constants of the storage phase.
c. Sketch the waveforms for each voltage and current for both phases covered by this example. Use the defined polarities in Fig. 11.43.

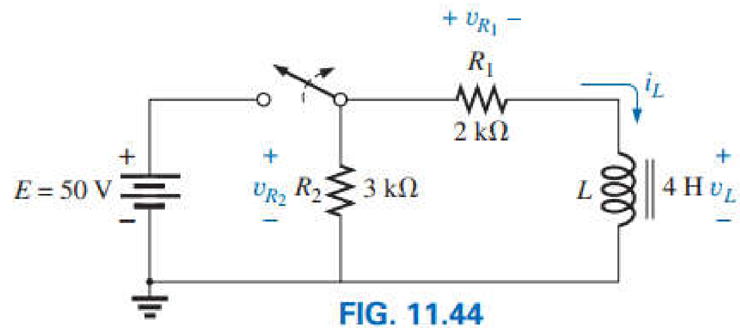


FIG. 11.44

Defined polarities for v_{R_1}, v_{R_2}, v_L, and current direction for i_L for Example 11.5.

Solution:

(a) First, the time constant is determined: $\tau = \dfrac{L}{R_1} = \dfrac{4\text{ H}}{2\text{ k}\Omega} = 2\text{ ms}$

Then the maximum or steady-state current is $I_m = \dfrac{E}{R_1} = \dfrac{50\text{ V}}{2\text{ k}\Omega} = 25 \times 10^{-3}\text{A} = 25\text{ mA}$

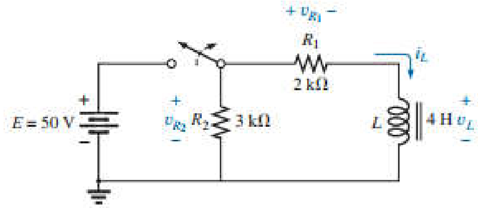
Substituting into Eq. (11.13): $i_L = \mathbf{25\text{ mA}\,(1 - e^{-t/2\text{ms}})}$

Using Eq. (11.15): $v_L = \mathbf{50\text{ V}e^{-t/2\text{ms}}}$

$$v_{R_1} = \mathbf{50\text{ V}}(1 - e^{-t/2\text{ms}})$$
$$v_{R_2} = E = \mathbf{50\text{ V}}$$

b. $\tau' = \dfrac{L}{R_1 + R_2} = \dfrac{4\text{ H}}{2\text{ k}\Omega + 3\text{ k}\Omega} = \dfrac{4\text{ H}}{5 \times 10^3\,\Omega}$

$= 0.8 \times 10^{-3}\text{ s} = 0.8\text{ ms}$



By Eqs. (11.19) and (11.20):

$$V_i = \left(1 + \frac{R_2}{R_1}\right)E = \left(1 + \frac{3\text{ k}\Omega}{2\text{ k}\Omega}\right)(50\text{ V}) = 125\text{ V}$$

and $\qquad v_L = -V_i e^{-t/\tau'} = \mathbf{-125\text{ V}e^{-t/-0.8\text{ms}}}$

By Eq. (11.21):

$$I_m = \frac{E}{R_1} = \frac{50\text{ V}}{2\text{ k}\Omega} = 25\text{ mA}$$

and $\qquad i_L = I_m e^{-t/\tau'} = \mathbf{25\text{ mA}e^{-t/0.8\text{ms}}}$

By Eq. (11.22):

$$v_{R_1} = Ee^{-t/\tau'} = \mathbf{50\text{ V}e^{-t/0.8\text{ms}}}$$

By Eq. (11.23):

$$v_{R_2} = -\frac{R_2}{R_1}Ee^{-t/\tau'} = -\frac{3\text{ k}\Omega}{2\text{ k}\Omega}(50\text{ V})e^{-t/\tau'} = \mathbf{-75\text{ V}e^{-t/0.8\text{ ms}}}$$

c. See Fig. 11.45:

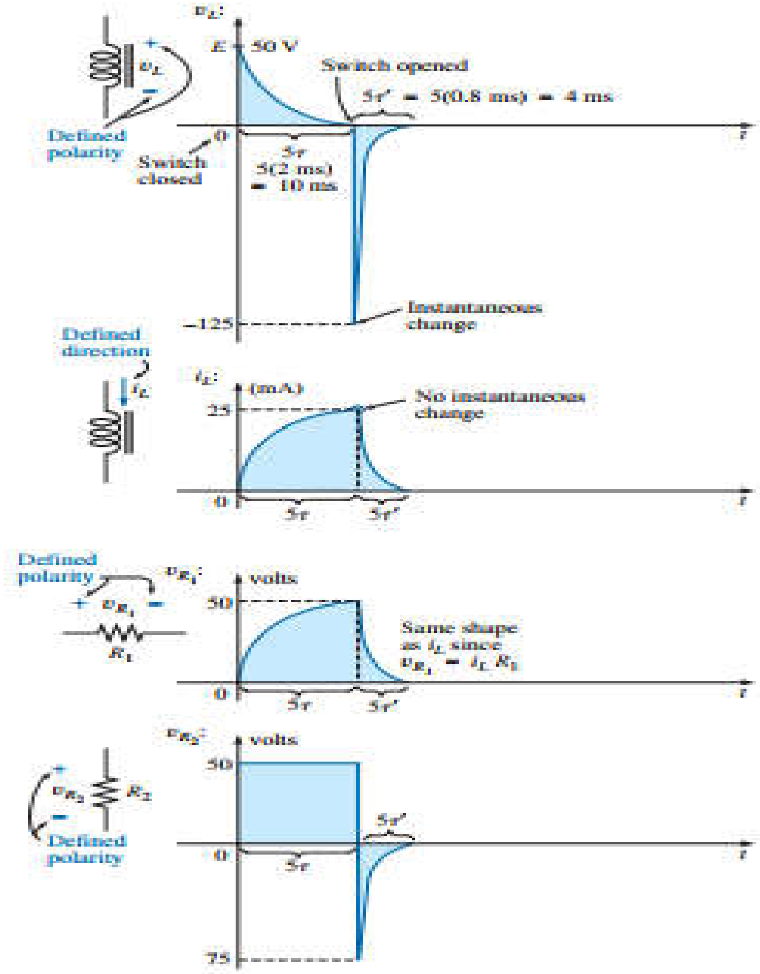


FIG. 11.45
The various voltages and the current for the network in Fig. 11.44.

Problem 12 [P505]:

12. For the circuit in Fig. 11.84:

a. Determine the time constant.

b. Write the mathematical expression for the current i_L after the switch is closed.

c. Repeat part (b) for v_L and v_R.

d. Determine i_L and v_L at one, three, and five time constants.

e. Sketch the waveforms of i_L, v_L, and v_R.

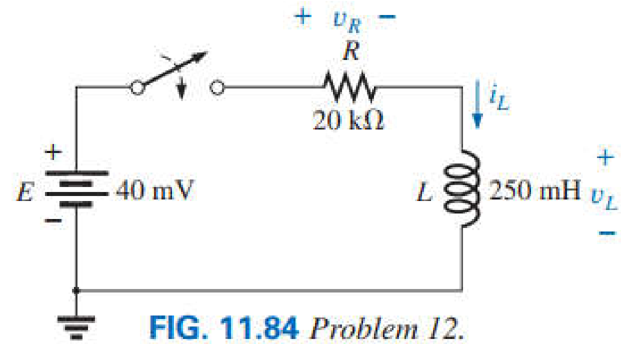


FIG. 11.84 *Problem 12.*

Solution:

$$(a)\ \tau = \frac{L}{R} = \frac{250\,\text{mH}}{20\,\text{k}\Omega} = \mathbf{12.5\ \mu s}$$

$$(b)\ i_L = \frac{E}{R}\left(1 - e^{-t/\tau}\right)$$

$$= \frac{40\,\text{mV}}{20\,\text{k}\Omega}\left(1 - e^{-t/12.5\mu s}\right) = \mathbf{2\ \mu A\left(1 - e^{-t/12.5\mu s}\right)}$$

$$(c)\ v_L = Ee^{-t/\tau} = \mathbf{40\,mV}\boldsymbol{e^{-t/12.5\mu s}}$$

$$v_R = Ri_L = E\left(1 - e^{-t/\tau}\right) = \mathbf{40\,mV\left(1 - e^{-t/12.5\mu s}\right)}$$

$$(d)\ i_L(t = \tau) = 2\,\mu\text{A}\left(1 - e^{-\tau/\tau}\right)$$

$$= 2\,\mu\text{A}\left(1 - e^{-1}\right) = \mathbf{1.26\ \mu A}$$

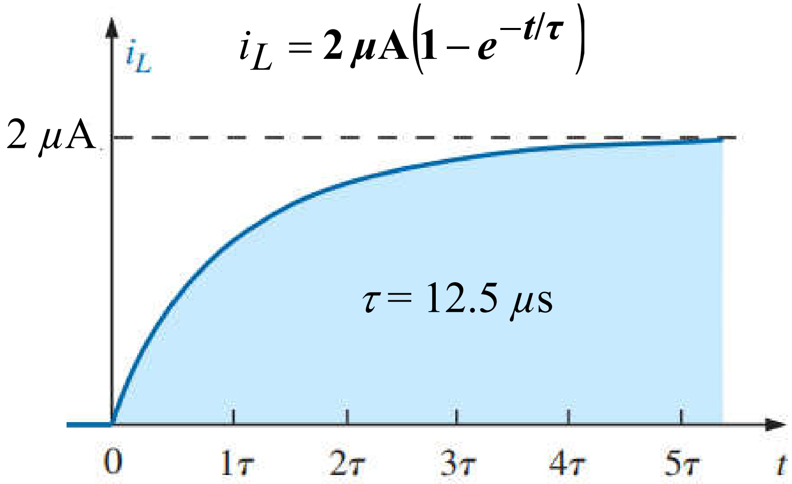
$$i_L(t = 3\tau) = 2\,\mu\text{A}\left(1 - e^{-3}\right) = \mathbf{1.9\ \mu A}$$

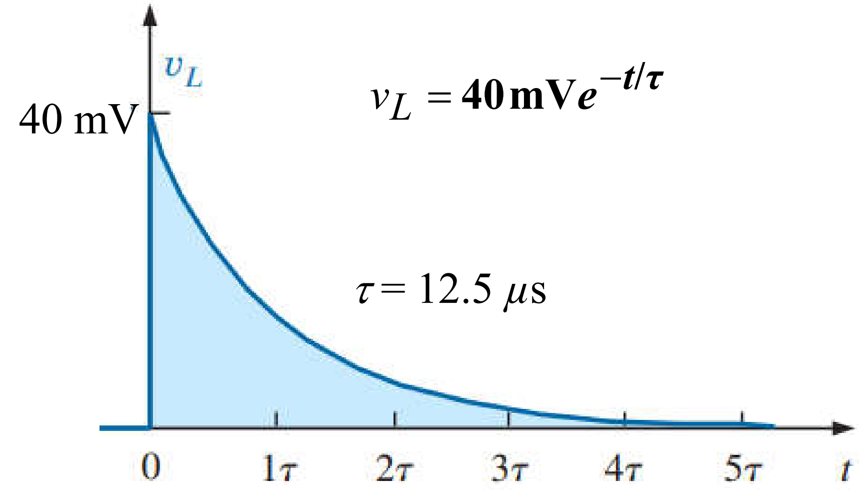
$$i_L(t = 5\tau) = 2\,\mu\text{A}\left(1 - e^{-3}\right) = \mathbf{1.99\ \mu A}$$

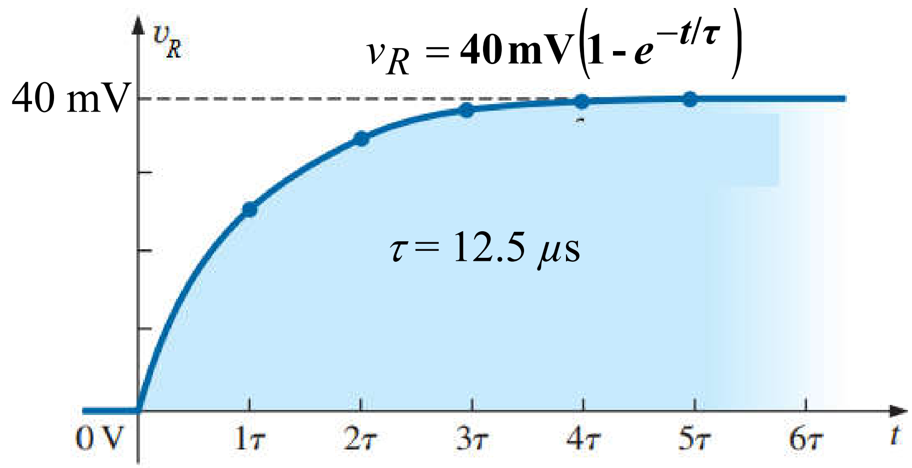
$$v_L(t = \tau) = 40\text{mV}\left(e^{-t/\tau}\right) = 40\text{mV}\left(e^{-1}\right) = \mathbf{14.72\,V}$$

$$v_L(t = 3\tau) = 40\text{mV}\left(e^{-3}\right) = \mathbf{1.99\,V}$$

$$v_L(t = 5\tau) = 40\text{mV}\left(e^{-5}\right) = \mathbf{0.27\,V}$$


i_L
$i_L = 2\,\mu\mathbf{A}\left(1 - e^{-t/\tau}\right)$
2 μA
τ = 12.5 μs
0
1τ
2τ
3τ
4τ
5τ
t


v_L
40 mV
$v_L = 40\,\mathbf{mV}e^{-t/\tau}$
τ = 12.5 μs
0
1τ
2τ
3τ
4τ
5τ
t


v_R
$v_R = 40\,\mathbf{mV}\left(1 - e^{-t/\tau}\right)$
40 mV
τ = 12.5 μs
0 V
1τ
2τ
3τ
4τ
5τ
6τ
t

Problem 20 [P505]:

20. For the network in Fig. 11.92:

a. Determine the mathematical expressions for the current i_L and the voltage v_L following the closing of the switch.

b. Repeat part (a) if the switch is opened at $t = 1\ \mu s$.

c. Sketch the waveforms of parts (a) and (b) on the same set of axes.

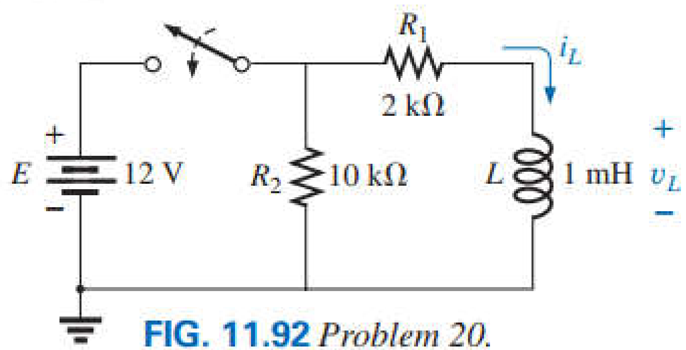


FIG. 11.92 *Problem 20.*

Solution:

$$(a)\ \tau = \frac{L}{R_1} = \frac{1\,\text{mH}}{2\,\text{k}\Omega} = \mathbf{0.5\ \mu s}$$

$$i_L = \frac{E}{R}\left(1 - e^{-t/\tau}\right) = \frac{12\,\text{V}}{2\,\text{k}\Omega}\left(1 - e^{-t/0.5\mu s}\right)$$

$$= \mathbf{6\,mA\left(1 - e^{-t/0.5\mu s}\right)}$$

$$v_L = Ee^{-t/\tau} = \mathbf{12\,V e^{-t/0.5\mu s}}$$

(b) At $t = 1\ \mu s$,

$$\tau' = \frac{L}{R_1 + R_2} = \frac{1\,\text{mH}}{12\,\text{k}\Omega} = \mathbf{8.33\ ns}$$

$$i_L = 6\,\text{mA}\left(1 - e^{-t/0.5\mu s}\right) = 6\,\text{mA}\left(1 - e^{-1\mu s/0.5\mu s}\right) = 5.19\,\text{mA}$$

$$I_m = 5.19\,\text{mA}$$

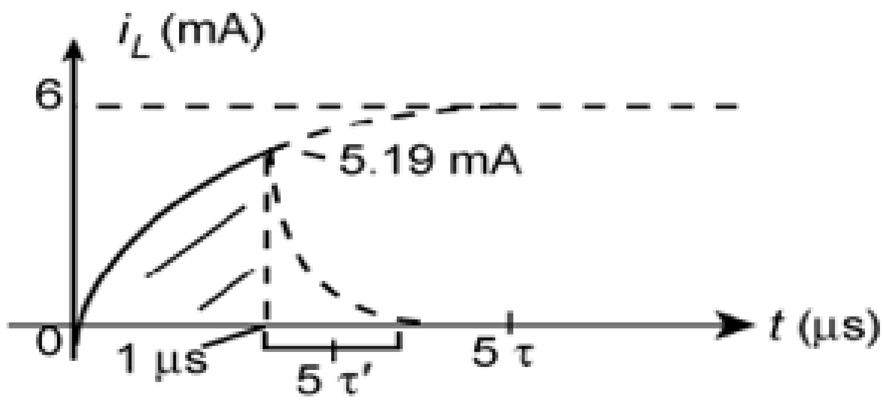
$$i_L = I_m e^{-t/\tau} = \mathbf{5.19\,mA e^{-t/8.33ns}}$$

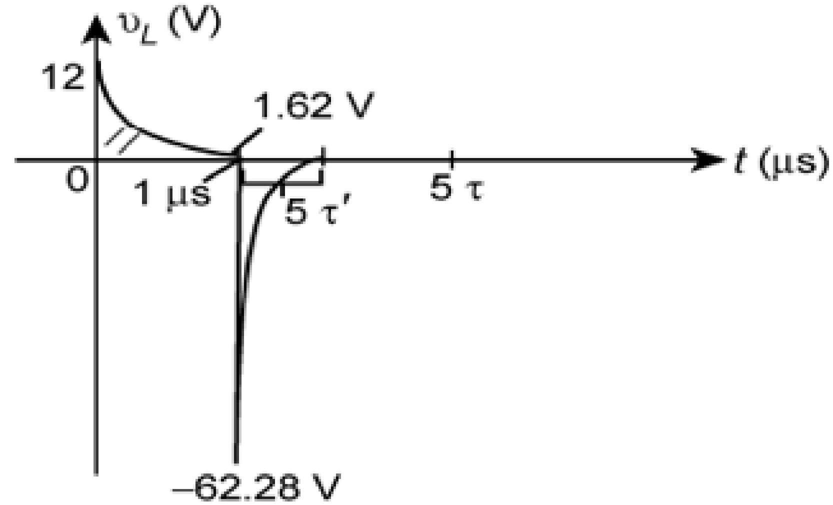
$$v_L = 12\,\text{V}e^{-t/0.5\mu s} = 12\,\text{V}e^{-1\mu s/0.5\mu s} = 1.62\,\text{V}$$

$$V_i = (R_1 + R_2)I_m = (5.19\,\text{mA})(12\,\text{k}\Omega) = 62.28\ \text{V}$$

$$v_L = -V_i e^{-t/\tau} = \mathbf{62.28\,V e^{-t/8.33ns}}$$

c.





Practice Problem 19 ~ 21 [P506]

11.11 INDUCTORS IN SERIES AND IN PARALLEL

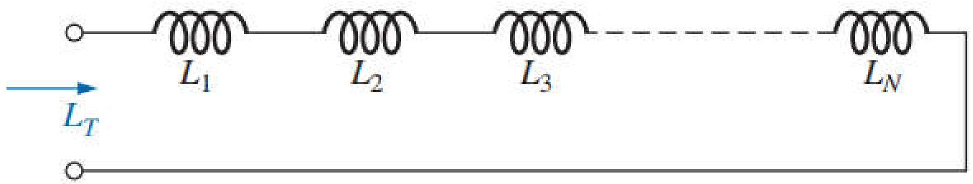


FIG. 11.56
Inductors in series.

$$L_T = L_1 + L_2 + L_3 + \ldots\ldots + L_N \qquad \textbf{(11.30)}$$

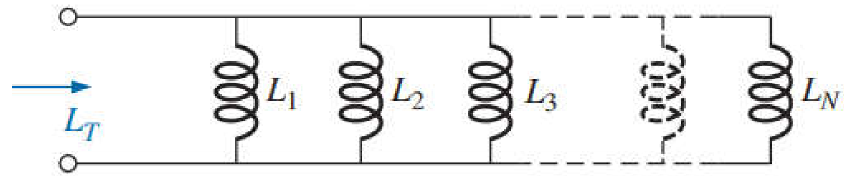
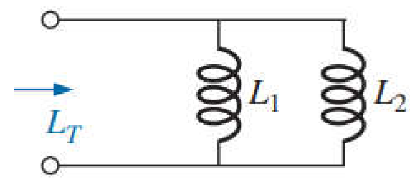


FIG. 11.57
Inductors in parallel.

$$\frac{1}{L_T} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \ldots\ldots + \frac{1}{L_N} \qquad \textbf{(11.31)}$$



When two inductors are connected in parallel:

$$L_T = \frac{L_1 L_2}{L_1 + L_2} \qquad \textbf{(11.32)}$$

35. Find the total inductance of the circuits in Fig. 11.106.

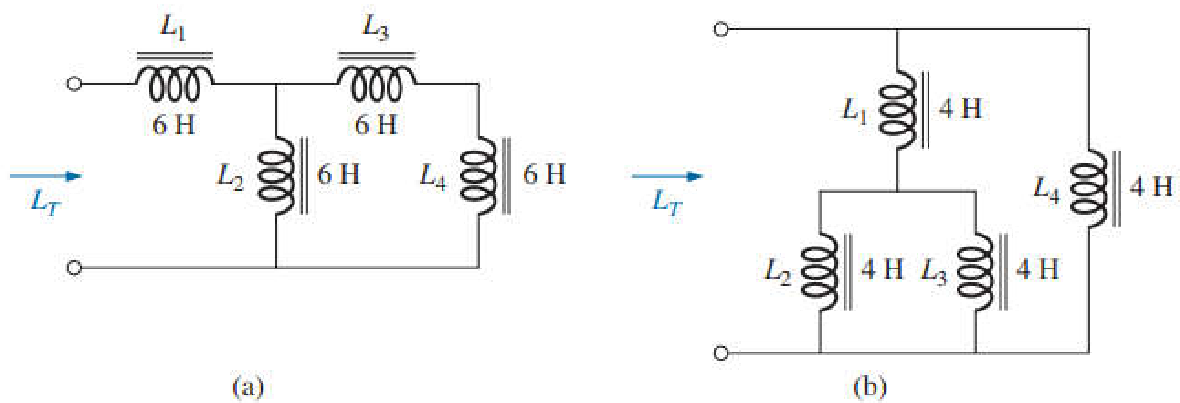


FIG. 11.106 *Problem 35.*

(a) $L_T = 6+[6//(6+6)] = 6 + [6//12] = 6 + 4 =$ **10 H**

(b) $L_T = 4//[4+(4//4)] = 4//[4+2] = 4//6 =$ **2.4 H**