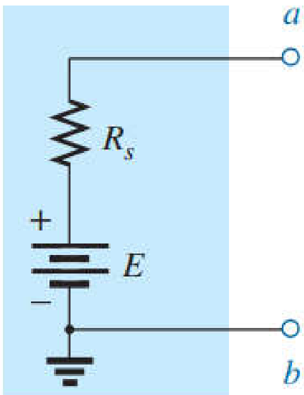
Chapter 8 (DC)

Methods of Analysis

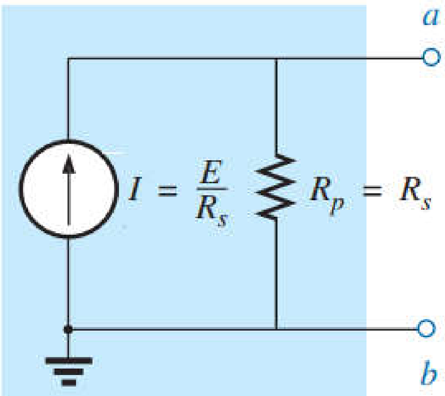
And Selected Topics

8.3 SOURCE Transformation

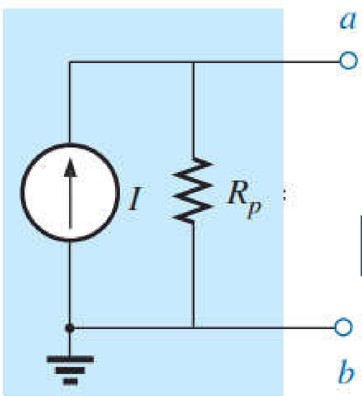
Voltage Source Convert to Current Source



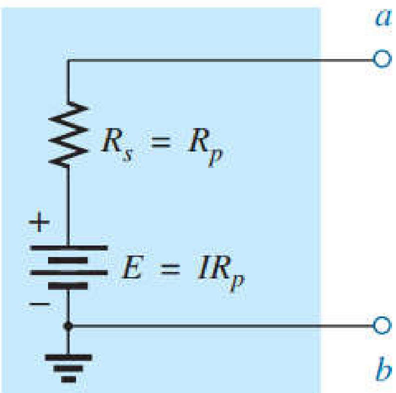
$$I = \frac{E}{R_s} \qquad R_p = R_s$$



Current Source Convert to Voltage Source



$$E = IR_p \qquad R_s = R_p$$



13. Convert the voltage sources in Fig. 8.114 to current sources.

a. Find the voltage V_{ab} and the polarity of points a and b.

b. Find the magnitude and direction of the current I_3.

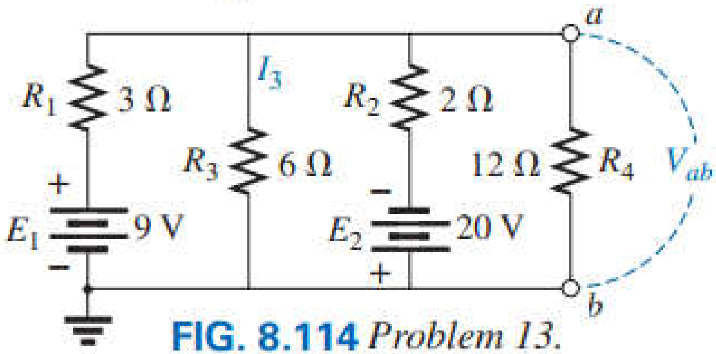
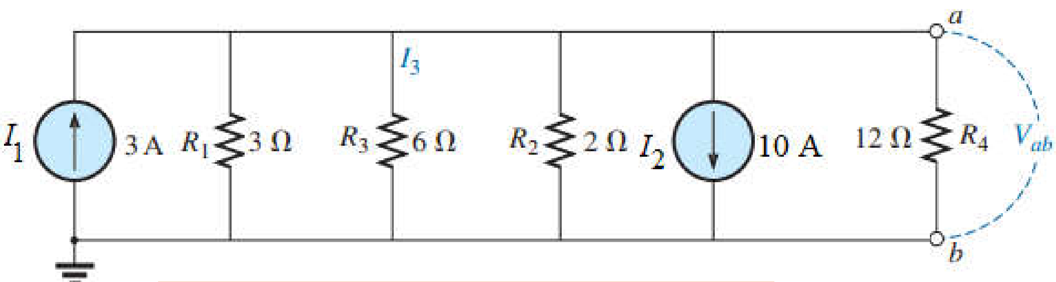
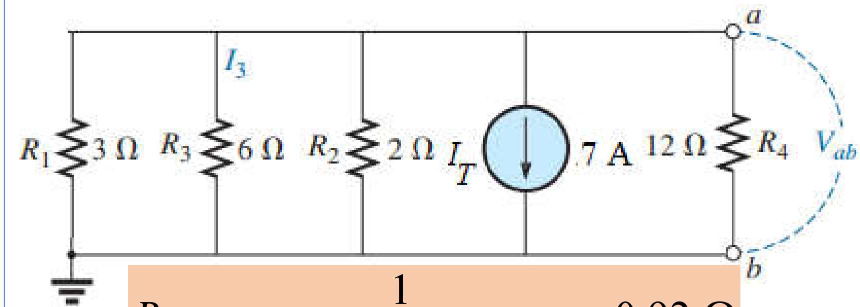


FIG. 8.114 *Problem 13.*

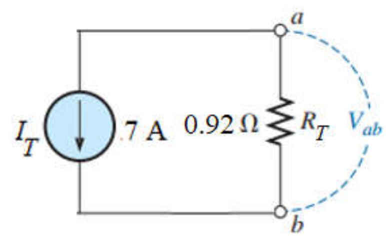
$$I_1 = \frac{E_1}{R_1} = \frac{9\ \text{V}}{3\ \Omega} = 3\ \text{A} \qquad I_2 = \frac{E_2}{R_2} = \frac{20\ \text{V}}{2\ \Omega} = 10\ \text{A}$$



$$I_T \downarrow = I_2 - I_1 = 10\ \text{A} - 3\ \text{A} = 7\ \text{A}$$



$$R_T = \frac{1}{\frac{1}{R_1} + \frac{1}{R_6} + \frac{1}{R_2} + \frac{1}{R_4}} = 0.92\ \Omega$$



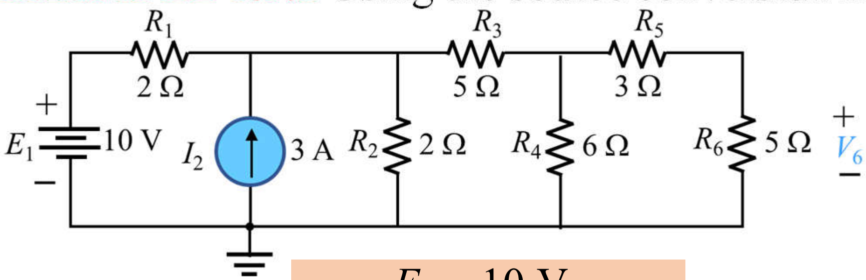
$$V_{ab} = -I_T R_T = \mathbf{-6.44\ V}$$

$$I_3 \uparrow = \frac{-V_{ab}}{R_3} = \frac{6.44\ \text{V}}{6\ \Omega} = \mathbf{1.07\ A}$$

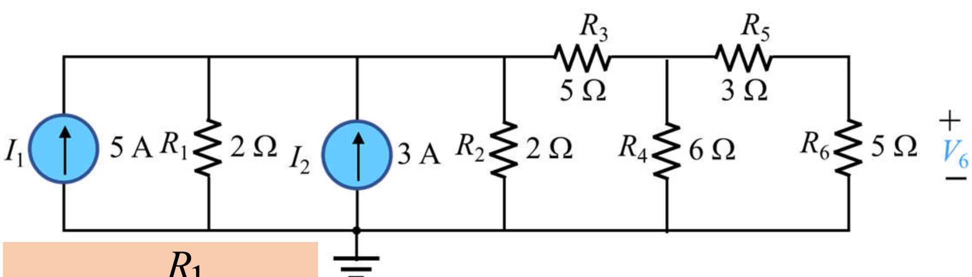
Practice Book [Ch 8]

Problem: 7 ~ 10 and 14

EXAMPLE 8.3.1 Using the source conversion find the value of voltage V_6.

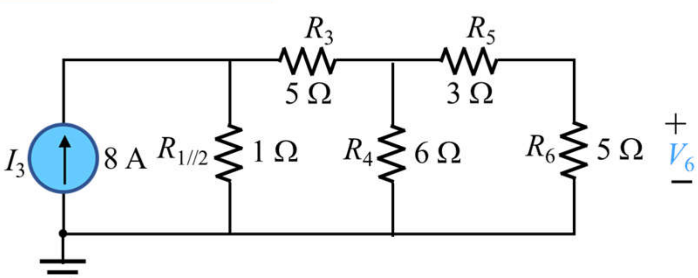


$$I_1 = \frac{E_1}{R_1} = \frac{10\ \text{V}}{2\ \Omega} = 5\ \text{A}$$

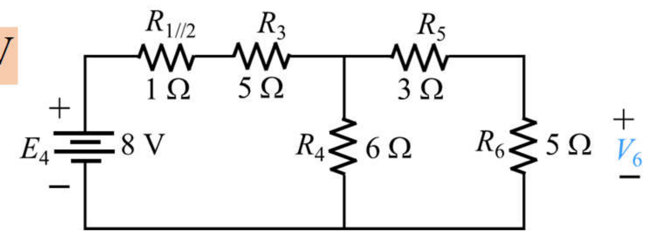


$$R_{1//2} = \frac{R_1}{2} = 1\ \Omega$$

$$I_3 = I_1 + I_2 = 8\ \text{A}$$

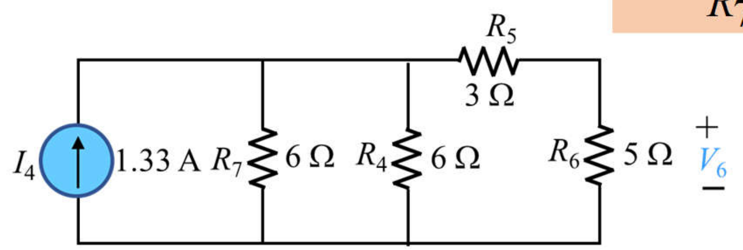


$$E_4 = I_3 R_{1//2} = 8\ \text{V}$$



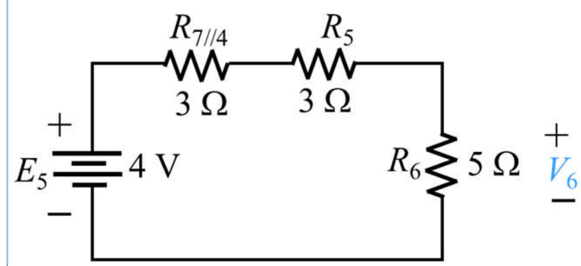
$$R_7 = R_{1//2} + R_3 = 6\ \Omega$$

$$I_4 = \frac{E_4}{R_7} = 1.33\ \text{A}$$



$$R_{7//4} = \frac{R_7}{2} = 3\ \Omega$$

$$E_5 = I_4 R_{7//4} = 4\ \text{V}$$



$$V_6 = \frac{R_6}{R_{7//4} + R_5 + R_6} E_5$$
$$= 1.1\ \text{V}$$

KVL for Mesh/Loop Analysis

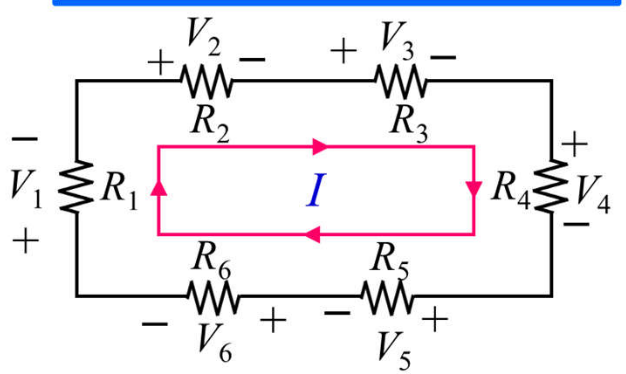
Kirchhoff's Voltage Law [KVL]

In closed path, according to KVL: Summation of Voltage Drop = Summation of Voltage Rise

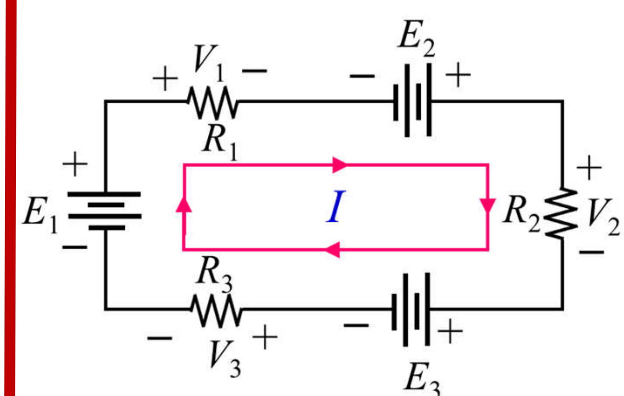
Positive if current entering through positive terminal.
Negative if current entering through negative terminal.

$$\sum_{\circlearrowright} V_{\text{drops}} = \sum_{\circlearrowright} V_{\text{rises}}$$

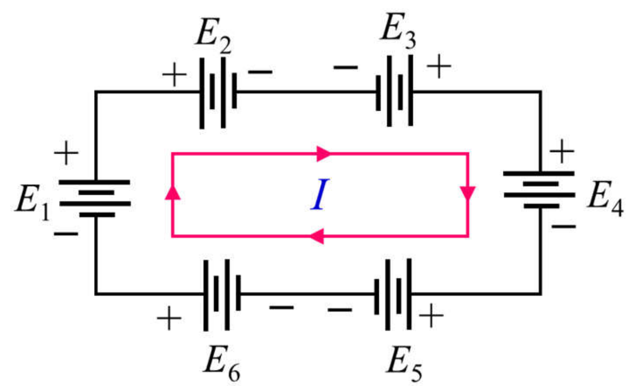
Positive if current entering through negative terminal.
Negative if current entering through positive terminal.



$$V_1 + V_2 + V_3 + V_4 + V_5 + V_6 = 0$$
$$IR_1 + IR_2 + IR_3 + IR_4 + IR_5 + IR_6 = 0$$
$$(R_1 + R_2 + R_3 + R_4 + R_5 + R_6)I = 0$$



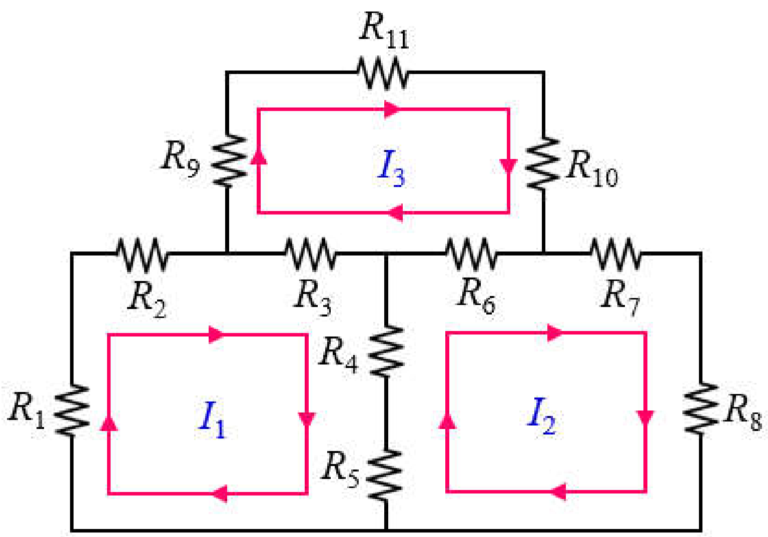
$$V_1 + V_2 + V_3 = E_1 + E_2 - E_3$$
$$(R_1 + R_2 + R_3)I = E_1 + E_2 - E_3$$



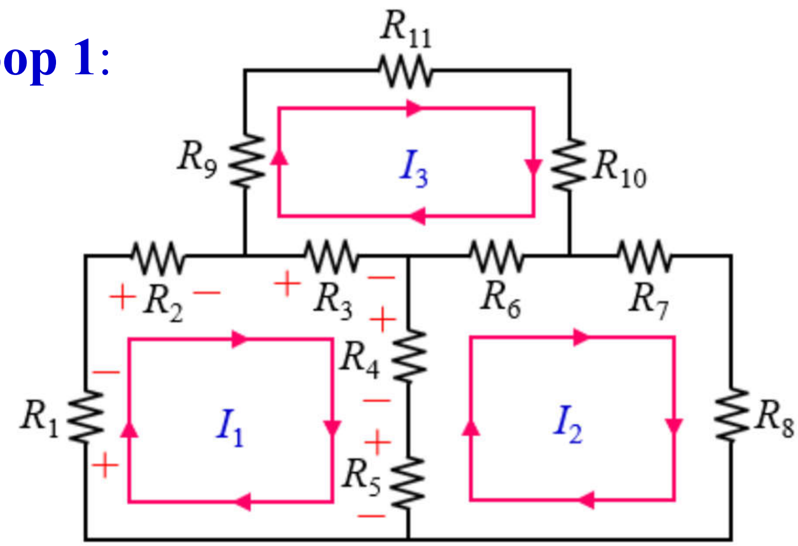
$$0 = E_1 - E_2 + E_3 - E_4 - E_5 + E_6$$
$$0 = (E_1 + E_3 + E_6) - (E_2 + E_4 + E_5)$$

If a resistor has two or more assumed currents through it, the total current through the resistor is the assumed current of the loop in which Kirchhoff's voltage law is being applied, **plus the assumed currents of the other loops passing through in the same direction**, **minus the assumed currents through in the opposite direction.**

Example: **Write the loop equations for the following circuit.**



Loop 1:

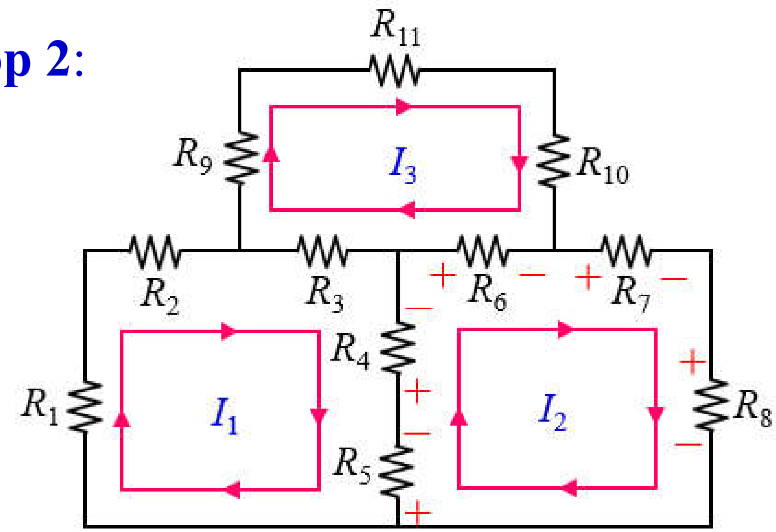


$$R_1 I_1 + R_2 I_1 + R_3(I_1 - I_3) + R_4(I_1 - I_2) + R_5(I_1 - I_2) = 0$$

$$(R_1 + R_2 + R_3 + R_4 + R_5)I_1 - R_3 I_3 - R_4 I_2 - R_5 I_2 = 0$$

$$(R_1 + R_2 + R_3 + R_4 + R_5)I_1 - (R_4 + R_5)I_2 - R_3 I_3 = 0$$

Loop 2:

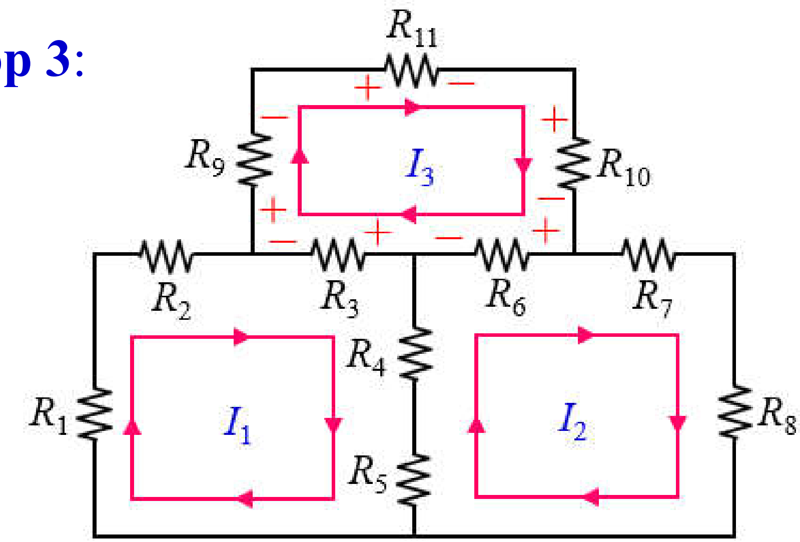


$$R_5(I_2 - I_1) + R_4(I_2 - I_1) + R_6(I_2 - I_3) + R_7 I_2 + R_8 I_2 = 0$$

$$(R_5 + R_4 + R_6 + R_7 + R_8)I_2 - R_5 I_1 - R_4 I_1 - R_6 I_3 = 0$$

$$-(R_5 + R_4)I_1 + (R_5 + R_4 + R_6 + R_7 + R_8)I_2 - R_6 I_3 = 0$$

Loop 3:

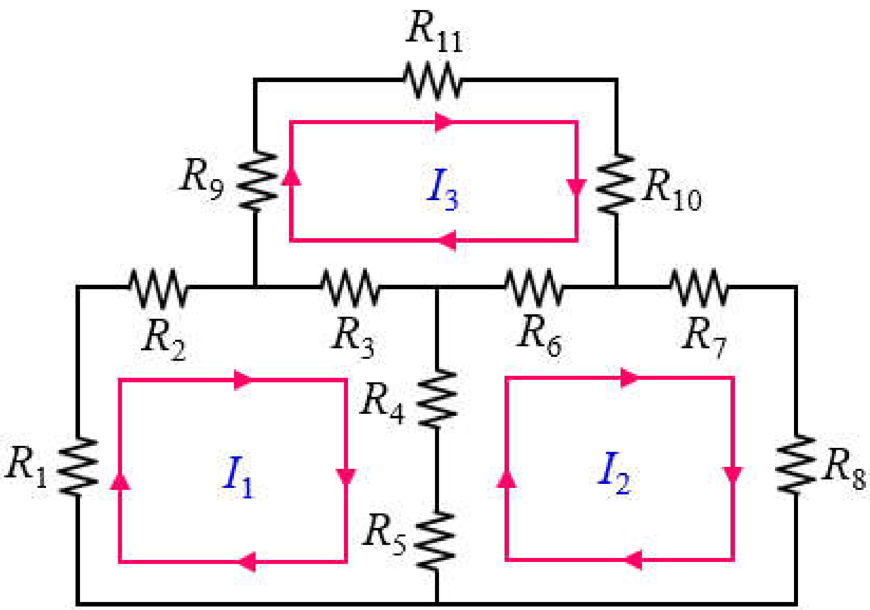


$$R_9 I_3 + R_{11} I_3 + R_{10} I_3 + R_6(I_3 - I_2) + R_3(I_3 - I_1) = 0$$

$$(R_9 + R_{11} + R_{10} + R_6 + R_3)I_3 - R_6 I_2 - R_3 I_1 = 0$$

$$-R_3 I_1 - R_6 I_2 + (R_9 + R_{11} + R_{10} + R_6 + R_3)I_3 = 0$$

Three the loop equations for the following circuit.



Loop 1:

$$(R_1 + R_2 + R_3 + R_4 + R_5)I_1 - (R_4 + R_5)I_2 - R_3 I_3 = 0$$

Loop 2:

$$-(R_5 + R_4)I_1 + (R_5 + R_4 + R_6 + R_7 + R_8)I_2 - R_6 I_3 = 0$$

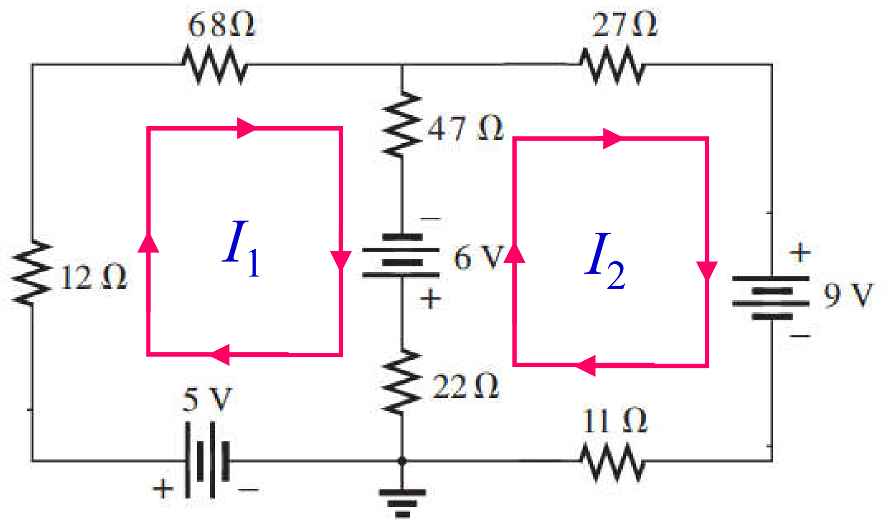
Loop 3:

$$-R_3 I_1 - R_6 I_2 + (R_9 + R_{11} + R_{10} + R_6 + R_3)I_3 = 0$$

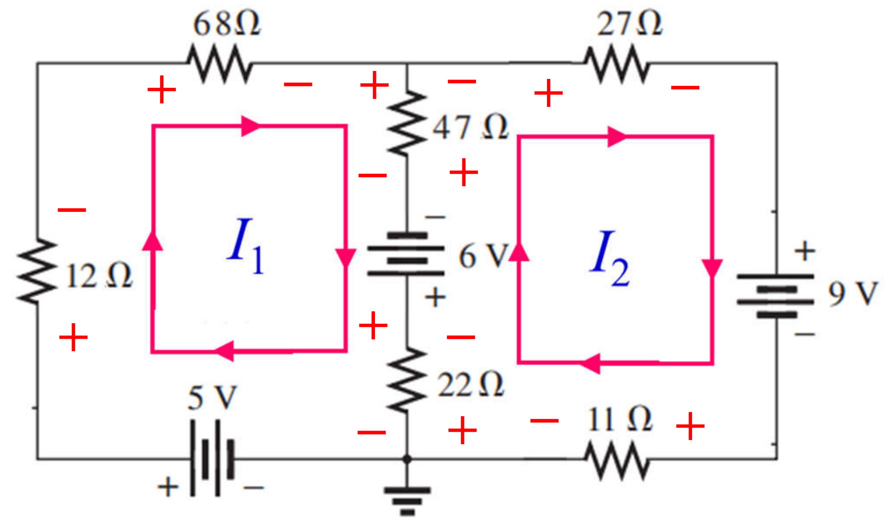
8.7 MESH ANALYSIS

8.7 MESH ANALYSIS

Step 1: Assign a distinct current in the clockwise direction to each independent, closed loop of the network. It is not absolutely necessary to choose the clockwise direction for each loop current.



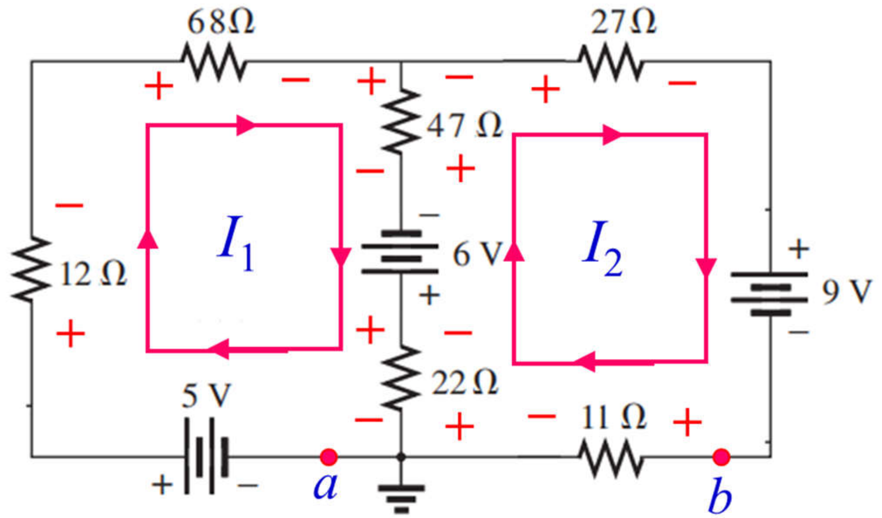
Step 2: 2. Indicate the polarities within each loop for each resistor as determined by the assumed direction of loop current for that loop.



Step 3: Apply Kirchhoff's voltage law around each closed loop in the clockwise direction. [Consider voltage is positive if current entering through negative terminal of an elements and voltage is negative if current entering through positive terminal of an elements]

a. If a resistor has two or more assumed currents through it, the total current through the resistor is the assumed current of the loop in which Kirchhoff's voltage law is being applied, plus the assumed currents of the other loops passing through in the same direction, minus the assumed currents through in the opposite direction.

b. The polarity of a voltage source is unaffected by the direction of the assigned loop currents.



Loop 1: From *a* terminal

$$12I_1 + 68I_1 + 47(I_1 - I_2) + 22(I_1 - I_2) = 5\text{ V} + 6\text{ V}$$

$$(12 + 68 + 47 + 22)I_1 - 47I_2 - 22I_2 = 11\text{V}$$

$$149I_1 - 69I_2 = 11\text{V}$$

Loop 2: From *b* terminal

$$11I_2 + 22(I_2 - I_1) + 47(I_2 - I_1) + 27I_2 = -6\text{V} - 9\text{V}$$

$$(11 + 22 + 47 + 27)I_2 - 22I_1 - 47I_1 = -15\text{V}$$

$$-69I_1 + 107I_2 = -15\text{V}$$

$$149I_1 - 69I_2 = 11\text{V}$$

$$-69I_1 + 107I_2 = -15\text{V}$$

Step 4: Solve the resulting simultaneous linear equations for the assumed loop currents.

$$\begin{bmatrix} 149 & -69 \\ -69 & 107 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 11\text{V} \\ -15\text{V} \end{bmatrix}$$

$$D = \begin{vmatrix} 149 & -69 \\ -69 & 107 \end{vmatrix} = 149 \times 107 - (-69)(-69) = 11182$$

$$D_1 = \begin{vmatrix} 11\text{V} & -69 \\ -15\text{V} & 107 \end{vmatrix} = 11\text{V} \times 107 - (-15\text{V})(-69) = 142$$

$$D_2 = \begin{vmatrix} 149 & 11\text{V} \\ -69 & -15\text{V} \end{vmatrix} = 149 \times (-15\text{V}) - (-69)(11\text{V}) = -1476$$

$$I_1 = \frac{D_1}{D} = \frac{142}{11182} = \mathbf{12.7\ mA}$$

$$I_2 = \frac{D_2}{D} = \frac{-1476}{11182} = \mathbf{-131.99\ mA}$$

EXAMPLE 8.16 Write the mesh equations for the network in Fig. 8.40, and find the current through the 8 Ω and 7 Ω resistors.

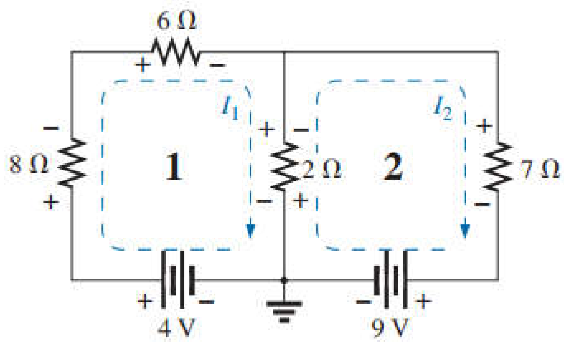


FIG. 8.40 *Example 8.16.*

Loop 1: $(8+6+2)I_1 - 2I_2 = 4$

Loop 2: $(2+7)I_2 - 2I_1 = -9$

$$16I_1 - 2I_2 = 4$$
$$-2I_1 + 9I_2 = -9$$

$$D = \begin{vmatrix} 16 & -2 \\ -2 & 9 \end{vmatrix} = 16\times 9 - (-2)\times(-2) = 140$$

$$D_1 = \begin{vmatrix} 4 & -2 \\ -9 & 9 \end{vmatrix} = 4\times 9 - (-2)\times(-9) = 18$$

$$D_2 = \begin{vmatrix} 16 & 4 \\ -2 & -9 \end{vmatrix} = 16\times(-9) - (-2)\times 4 = -136$$

$$I_1 = I_{8\Omega} = \frac{D_1}{D} = \frac{18}{140} = \mathbf{0.13\ A}$$

$$I_2 = I_{7\Omega} = \frac{D_2}{D} = \frac{-136}{140} = \mathbf{-0.97\ A}$$

EXAMPLE 8.13 Write the mesh equations for the network in Fig. 8.32, and find the branch currents.

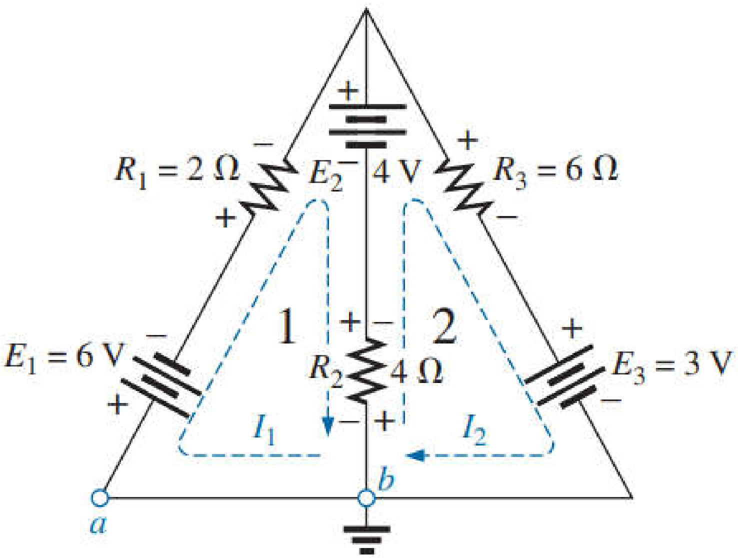


FIG. 8.32 *Example 8.13.*

$$D = \begin{vmatrix} 6 & -4 \\ -4 & 10 \end{vmatrix} = 6\times10-(-4)\times(-4) = 44$$

$$D_1 = \begin{vmatrix} -10 & -4 \\ 1 & 10 \end{vmatrix} = (-10)\times10-(1)\times(-4) = -96$$

$$D_2 = \begin{vmatrix} 6 & -10 \\ -4 & 1 \end{vmatrix} = 6\times1-(-4)\times(-10) = -34$$

Loop 1: $(2+4)I_1 - 4I_2 = -6-4$

Loop 2: $(4+6)I_2 - 4I_1 = 4-3$

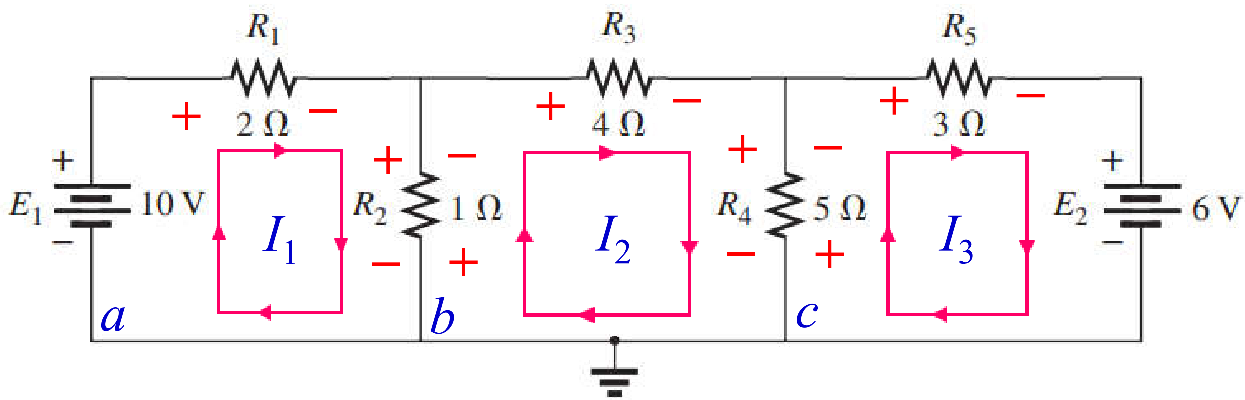
$$6I_1 - 4I_2 = -10$$
$$-4I_1 + 10I_2 = 1$$

$$I_1 = I_{2\Omega} = \frac{D_1}{D} = \frac{-96}{44} = \mathbf{-2.18\ A}$$

$$I_2 = I_{6\Omega} = \frac{D_2}{D} = \frac{-34}{44} = \mathbf{-0.77\ A}$$

$$I_4 = I_1 - I_2 = -2.18\text{ A} - (-0.77\text{ A}) = \mathbf{-1.41\ A}$$

EXAMPLE 8.7.1 (**a**) Write the mesh equations for each loop of the networks. (**b**) Using determinants, solve for the loop currents. (**b**) Find the current of each branch.



Loop 1: Start from point *a*

$$R_1 I_1 + R_2 (I_1 - I_2) = E_1$$

$$(R_1 + R_2) I_1 - R_2 I_2 = E_1$$

Loop 2: Start from point *b*

$$R_2 (I_2 - I_1) + R_3 I_2 + R_4 (I_2 - I_3) = 0$$

$$-R_2 I_1 + (R_2 + R_3 + R_4) I_2 - R_4 I_3 = 0$$

Loop 3: Start from point *c*

$$R_4 (I_2 - I_3) + R_5 I_3 = -E_2$$

$$-R_4 I_2 + (R_4 + R_5) I_3 = -E_2$$

Putting the values of resistances and voltages:

$$3I_1 - I_2 = 10\text{V}$$

$$-I_1 + 10 I_2 - 5 I_3 = 0\text{V}$$

$$-5 I_2 + 8 I_3 = -6\text{V}$$

$$3I_1 - I_2 = 10\text{V}$$
$$-I_1 + 10I_2 - 5I_3 = 0\text{V}$$
$$-5I_2 + 8I_3 = -6\text{V}$$

$$\begin{bmatrix} 3 & -1 & 0 \\ -1 & 10 & -5 \\ 0 & -5 & 8 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 10\text{V} \\ 0\text{V} \\ -6\text{V} \end{bmatrix}$$

$$D = \begin{vmatrix} 3 & -1 & 0 \\ -1 & 10 & -5 \\ 0 & -5 & 8 \end{vmatrix}$$

$$D = \left|\begin{matrix} 3 & -1 & 0 \\ -1 & 10 & -5 \\ 0 & -5 & 8 \end{matrix}\right| \begin{matrix} 3 & -1 \\ -1 & 10 \\ 0 & -5 \end{matrix}$$

$$\begin{aligned} D &= (3)(10)(8) + (-1)(-5)(0) + (0)(-1)(-5) \\ &\quad - (0)(10)(0) - (3)(-5)(-5) - (-1)(-1)(8) \\ &= 240 - 75 - 8 = 157 \end{aligned}$$

$$D_1 = \begin{vmatrix} 10\text{V} & -1 & 0 \\ 0\text{V} & 10 & -5 \\ -6\text{V} & -5 & 8 \end{vmatrix}$$

$$D_1 = \left|\begin{matrix} 10\text{V} & -1 & 0 \\ 0\text{V} & 10 & -5 \\ -6\text{V} & -5 & 8 \end{matrix}\right| \begin{matrix} 10\text{V} & -1 \\ 0\text{V} & 10 \\ -6\text{V} & -5 \end{matrix}$$

$$\begin{aligned} D_1 &= (10)(10)(8) + (-1)(-5)(-6) + (0)(0)(-5) \\ &\quad - (0)(10)(-6) - (10)(-5)(-5) - (-1)(0)(8) \\ &= 800 - 30 - 250 = 520 \end{aligned}$$

$$D_2 = \begin{vmatrix} 3 & 10\text{V} & 0 \\ -1 & 0\text{V} & -5 \\ 0 & -6\text{V} & 8 \end{vmatrix}$$

$$D_2 = \left|\begin{matrix} 3 & 10\text{V} & 0 \\ -1 & 0\text{V} & -5 \\ 0 & -6\text{V} & 8 \end{matrix}\right| \begin{matrix} 3 & 10\text{V} \\ -1 & 0\text{V} \\ 0 & -6\text{V} \end{matrix}$$

$$\begin{aligned} D_2 &= (3)(0)(8) + (10)(-5)(0) + (0)(-1)(-6) \\ &\quad - (0)(0)(0) - (3)(-5)(-6) - (10)(-1)(8) \\ &= -90 + 80 = -10 \end{aligned}$$

$$D_3 = \begin{vmatrix} 3 & -1 & 10V \\ -1 & 10 & 0V \\ 0 & -5 & -6V \end{vmatrix}$$

$$D_3 = \begin{vmatrix} 3 & -1 & 10V \\ -1 & 10 & 0V \\ 0 & -5 & -6V \end{vmatrix} \begin{matrix} 3 & -1 \\ -1 & 10 \\ 0 & -5 \end{matrix}$$

$$\begin{aligned} D_3 &= (3)(10)(-6) + (-1)(0)(0) + (10)(-1)(-5) \\ &\quad - (10)(10)(0) - (3)(0)(-5) - (-1)(-1)(-6) \\ &= -180 + 50 + 6 = -124 \end{aligned}$$

$$I_1 = \frac{D_1}{D} = \frac{520}{157} = \mathbf{3.31\ A}$$

$$I_2 = \frac{D_2}{D} = \frac{-10}{157} = \mathbf{-0.0637\ A} \quad \text{or} \quad \mathbf{-63.7\ mA}$$

$$I_3 = \frac{D_3}{D} = \frac{-124}{157} = \mathbf{-0.79\ A} \quad \text{or} \quad \mathbf{-790\ mA}$$

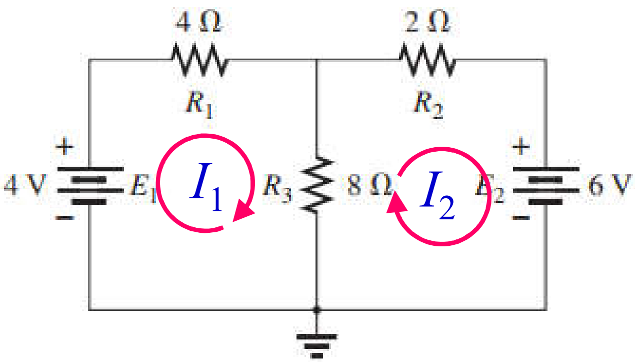
Check or Justification of Results:

$$\begin{aligned} 3I_1 - I_2 &= 10V \\ -I_1 + 10I_2 - 5I_3 &= 0V \\ -5I_2 + 8I_3 &= -6V \end{aligned}$$

$$\begin{aligned} 3I_1 - I_2 &= 3\times3.31 - 0.0637 = 10V \\ -I_1 + 10I_2 - 5I_3 &= -3.31 + 10\times0.0637 - 5\times0.79 = 0V \\ -5I_2 + 8I_3 &= -5\times0.0637 + 8\times0.79 = -6V \end{aligned}$$

(Justified)

Write the mesh/loop equations for the following networks.



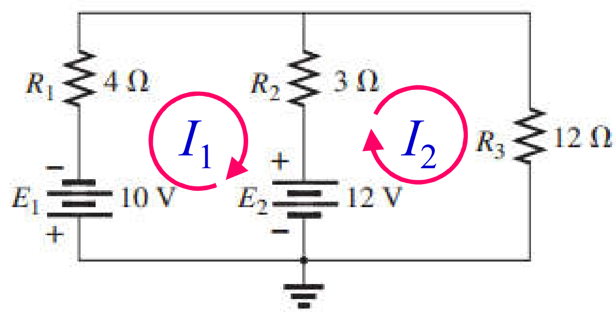
Loop 1:

$$(4+8)I_1 - 8I_2 = 4$$

Loop 2:

$$(8+2)I_2 - 8I_1 = -6$$

$$12I_1 - 8I_2 = 4$$
$$-8I_1 + 10I_2 = -6$$



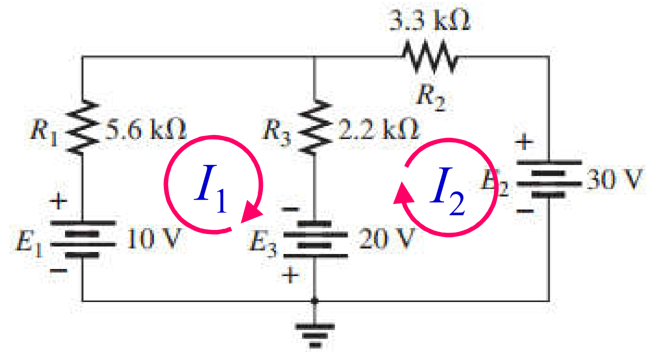
Loop 1:

$$(4+3)I_1 - 3I_2 = -10-12$$

Loop 2:

$$(3+12)I_2 - 3I_1 = 12$$

$$7I_1 - 3I_2 = -22$$
$$-3I_1 + 15I_2 = 12$$



Loop 1:

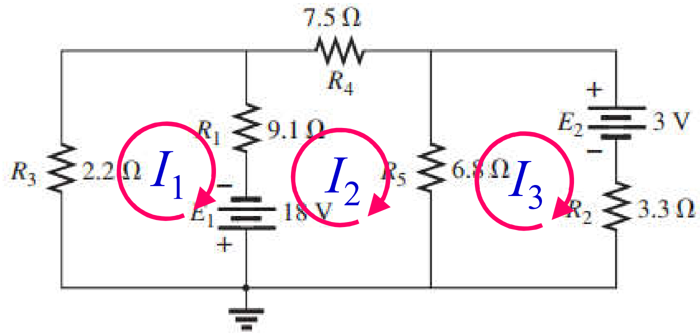
$$(5.6+2.2)(\text{k}\Omega)I_1 - 2.2(\text{k}\Omega)I_2 = 10+20$$

Loop 2:

$$(2.2+3.3)(\text{k}\Omega)I_2 - 2.2(\text{k}\Omega)I_1 = -20-30$$

$$(7.8\text{k}\Omega)I_1 - (2.2\text{k}\Omega)I_2 = 30$$
$$-(2.2\text{k}\Omega)I_1 + (5.5\text{k}\Omega)I_2 = -50$$

Write the mesh/loop equations for the following networks.

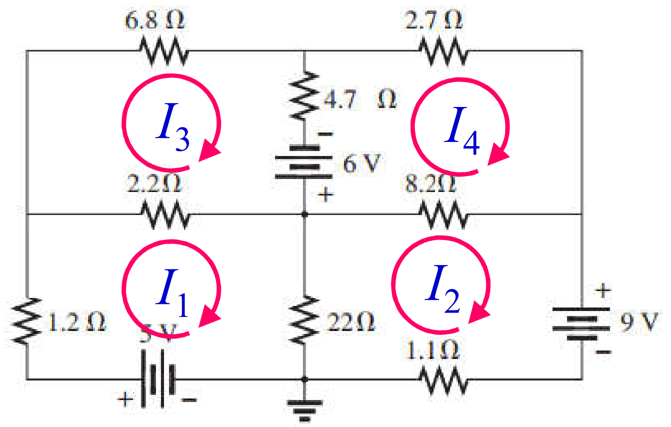


Loop 1: $(2.2+9.1)I_1-9.1I_2=18$

Loop 2: $(9.1+7.5+6.8)I_2-9.1I_1-6.8I_3=-18$

Loop 3: $(6.8+3.3)I_3-6.8I_2=-3$

$$11.3I_1-9.1I_2=18$$
$$-9.1I_1+23.4I_2-6.8I_3=-18$$
$$-6.8I_2+10.1I_3=-3$$



Loop 1: $(1.2+2.2+22)I_1-22I_2-2.2I_3=5$

Loop 2: $(22+8.2+1.1)I_2-22I_1-8.2I_4=-18$

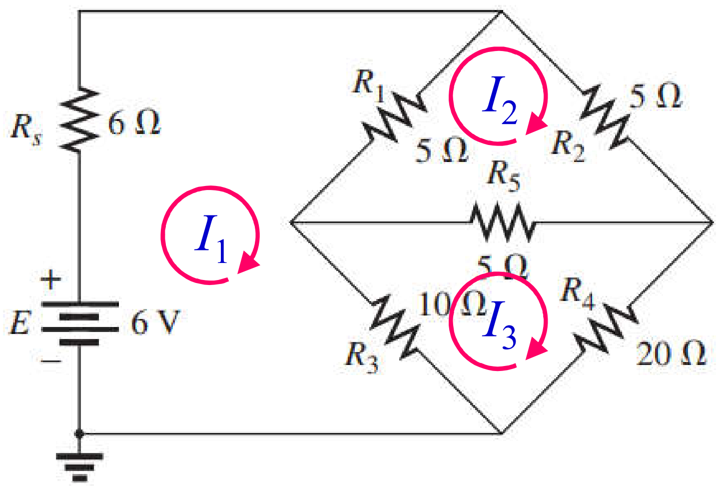
Loop 3: $(2.2+6.8+4.7)I_3-2.2I_1-4.7I_4=6$

Loop 4: $(4.7+2.7+8.2)I_4-8.2I_2-4.7I_3=-6$

$$25.4I_1-22I_2-2.2I_3=5$$
$$-22I_1+31.3I_2-8.2I_4=-18$$
$$-2.2I_1+13.7I_3-4.7I_4=6$$
$$-8.2I_2-4.7I_3+15.6I_4=-6$$

Practice Book [Ch 8] Problem: 28 ~ 34

Write the mesh/loop equations for the following networks.



Loop 1: $(6+5+10)I_1 - 5I_2 - 10I_3 = 6$

Loop 2: $(5+5+5)I_2 - 5I_1 - 5I_3 = 0$

Loop 3: $(10+5+20)I_3 - 10I_1 - 5I_2 = 0$

NODAL ANALYSIS

8.9 NODAL ANALYSIS

Steps of Nodal Analysis:

1. **Convert voltage sources to current sources.**
2. **Determine the number of nodes within the network.**

2. **Pick a reference node, and label each remaining node with a subscripted value of voltage: V_1, V_2, and so on.**

3. **Apply Kirchhoff's current law (KCL) at each node except the reference. Assume that:**

   (*a*) all unknown currents leave the node for each application of Kirchhoff's current law (KCL).

   (*b*) Each node is to be treated as a separate entity, independent of the application of KCL to the other nodes.

4. **Solve the resulting equations for the nodal voltages.**

EXAMPLE 8.20 Apply nodal analysis to the network in Fig. 8.49.

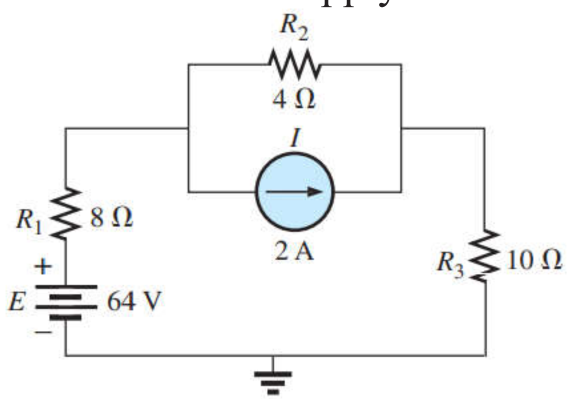
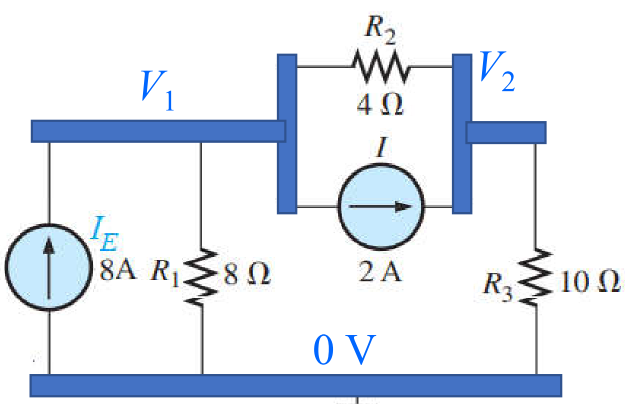


FIG. 8.49 *Example 8.20.*

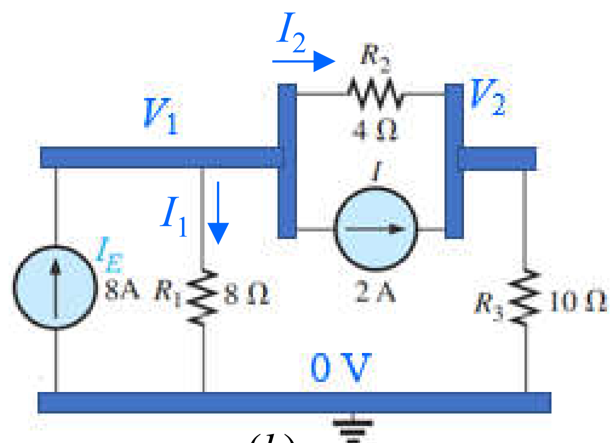
Solution: Convert the voltage sources to current sources as shown in Figure (a).



(*a*)

Step 1 and 2: The network has three nodes with the bottom node defined as the reference node (at ground potential, or zero volts), and the other nodes as V_1 and V_2.

Step 3: For node V_1, the currents are defined as shown in the following Figure (b) and Kirchhoff's current law is applied:



(*b*)

$$I_1 + I_2 + I = I_E$$

$$I_1 + I_2 = I_E - I$$

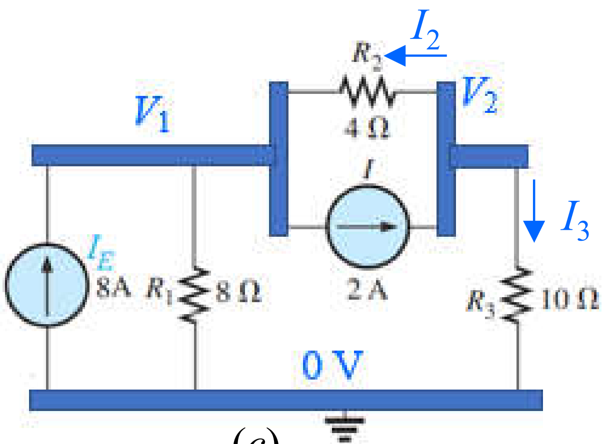
$$\frac{V_1}{R_1} + \frac{V_1 - V_2}{R_2} = I_E - I$$

$$\left(\frac{1}{R_1} + \frac{1}{R_2}\right)V_1 - \left(\frac{1}{R_2}\right)V_2 = I_E - I$$

$$\left(\frac{1}{8} + \frac{1}{4}\right)V_1 - \left(\frac{1}{4}\right)V_2 = 8 - 2$$

$$3V_1 - 2V_2 = 48$$

For node V_2, the currents are defined as shown in the following Figure (c) and Kirchhoff's current law is applied:



(c)

$$I_2 + I_3 = I$$

$$\left(\frac{1}{R_2} + \frac{1}{R_3}\right)V_2 - \left(\frac{1}{R_2}\right)V_1 = I$$

$$\left(\frac{1}{4} + \frac{1}{10}\right)V_2 - \left(\frac{1}{4}\right)V_1 = 2$$

$$7V_2 - 5V_1 = 40$$

$$3V_1 - 2V_2 = 48$$

$$-5V_2 + 7V_2 = 40$$

$$D = \begin{vmatrix} 3 & -2 \\ -5 & 7 \end{vmatrix} = 21 - 10 = 11$$

$$D_1 = \begin{vmatrix} 48 & -2 \\ 40 & 7 \end{vmatrix} = 336 + 80 = 416$$

$$D_2 = \begin{vmatrix} 3 & 48 \\ -5 & 40 \end{vmatrix} = 120 + 240 = 360$$

$$V_1 = \frac{D_1}{D} = \frac{416}{11} = \mathbf{37.82\ V}$$

$$V_2 = \frac{D_2}{D} = \frac{360}{11} = \mathbf{32.72\ V}$$

$$I_{R_1} = \frac{E - V_1}{R_1}$$

$$= \frac{64\ \text{V} - 37.82\ \text{V}}{8\ \Omega} = \mathbf{3.27\ A}$$

$$I_{R_3} = \frac{V_{R_3}}{R_3} = \frac{V_2}{R_3}$$

$$= \frac{32.73\ \text{V}}{10\ \Omega} = \mathbf{3.27\ A}$$

$$I_{R_2} = \frac{V_1 - V_2}{R_2}$$

$$= \frac{37.82\ \text{V} - 32.73\ \text{V}}{4\ \Omega} = \mathbf{1.27\ A}$$

EXAMPLE 8.21 Determine the nodal voltages for the network in Fig. 8.54.

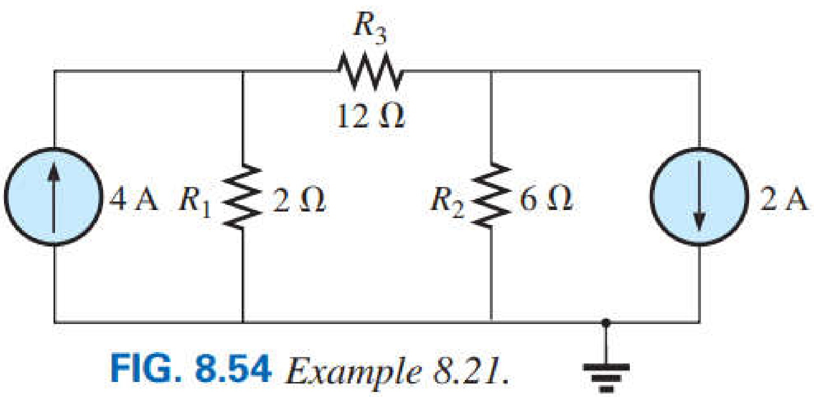
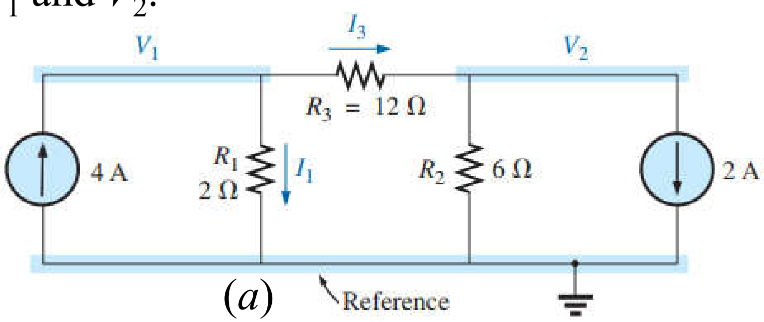


FIG. 8.54 *Example 8.21.*

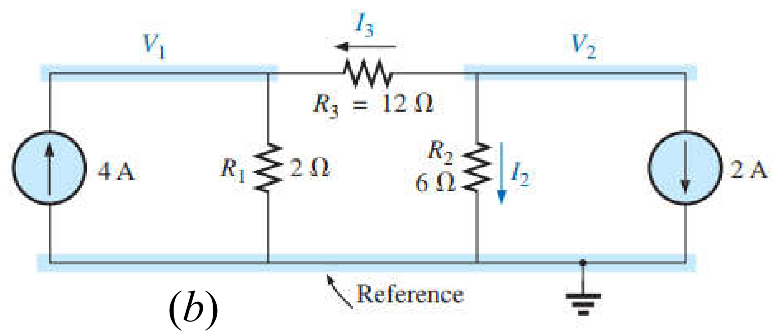
Step 1 and 2: The network has three nodes with the bottom node defined as the reference node (at ground potential, or zero volts), and the other nodes as V_1 and V_2.



Step 3: For node V_1, the currents are defined as shown in the following Figure (a) and Kirchhoff's current law is applied:

$$I_1 + I_3 = 4 \qquad \left(\frac{1}{2}+\frac{1}{12}\right)V_1 - \left(\frac{1}{12}\right)V_2 = 4 \qquad 7V_1 - V_2 = 48$$

For node V_2, the currents are defined as shown in the following Figure (b) and Kirchhoff's current law is applied:



$$I_2 + I_3 = -2$$

$$\left(\frac{1}{12}+\frac{1}{6}\right)V_2 - \left(\frac{1}{12}\right)V_1 = -2 \qquad 3V_2 - V_1 = -24$$

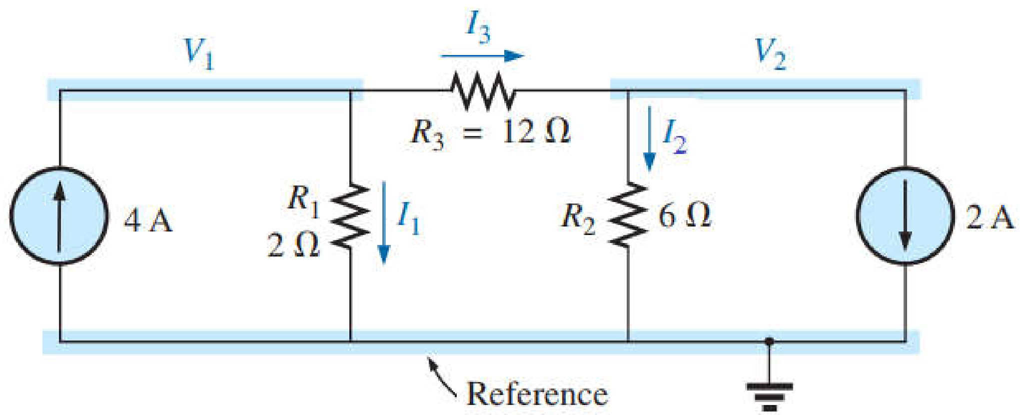
$$7V_1 - V_2 = 48$$
$$-V_1 + 3V_2 = -24$$

$$D = \begin{vmatrix} 7 & -1 \\ -1 & 3 \end{vmatrix} = 21 - 1 = 20$$

$$D_1 = \begin{vmatrix} 48 & -1 \\ -24 & 3 \end{vmatrix} = 144 - 24 = 120$$

$$D_2 = \begin{vmatrix} 7 & 48 \\ -1 & -24 \end{vmatrix} = -168 + 48 = -120$$

$$V_1 = \frac{D_1}{D} = \frac{120}{20} = \mathbf{6\ V}$$
$$V_2 = \frac{D_2}{D} = \frac{-120}{20} = \mathbf{-20\ V}$$



Here, $V_1 > V_1$

$$I_{R_1} = \frac{V_{R_1}}{R_1} = \frac{V_1}{R_1} = \frac{6\ \text{V}}{2\ \Omega} = \mathbf{3\ A}$$

$$I_{R_2} = \frac{V_{R_2}}{R_2} = \frac{V_2}{R_2} = \frac{6\ \text{V}}{6\ \Omega} = \mathbf{1\ A}$$

$$I_{R_3} = \frac{V_1 - V_2}{R_3} = \frac{6\ \text{V} - (-6\ \text{V})}{12\ \Omega} = \frac{12\ \text{V}}{12\ \Omega} = \mathbf{1\ A}$$

EXAMPLE 8.24 Find the voltage across the 3 Ω resistor in Fig. 8.61 by nodal analysis.

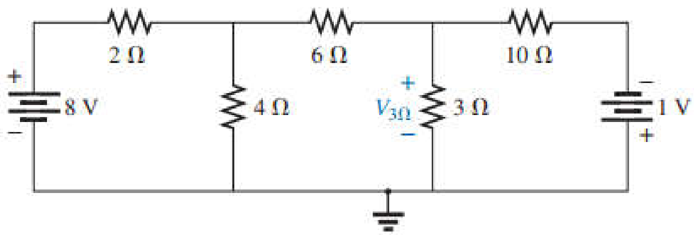
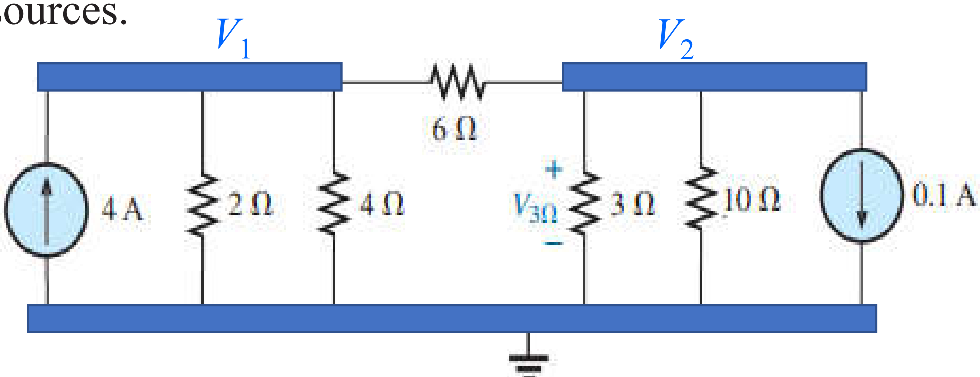


FIG. 8.61 *Example 8.24.*

Solution: First convert two voltage sources to current sources.



Step 1 and 2: The network has three nodes with the bottom node defined as the reference node (at ground potential, or zero volts), and the other nodes as V_1 and V_2.

Step 3: For node V_1, the currents are defined as shown in the following Figure (b) and Kirchhoff's current law is applied:

$$\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{6}\right)V_1-\left(\frac{1}{6}\right)V_2=4 \qquad 11V_1-2V_2=48$$

For node V_2:

$$\left(\frac{1}{6}+\frac{1}{3}+\frac{1}{10}\right)V_2-\left(\frac{1}{6}\right)V_1=-0.1 \qquad 18V_2-5V_1=-3$$

Simplified form:

$$11V_1-2V_2=48$$
$$-5V_1+18V_2=-3$$

$$D=\begin{vmatrix}11 & -2\\ -5 & 18\end{vmatrix}=198-10=188$$

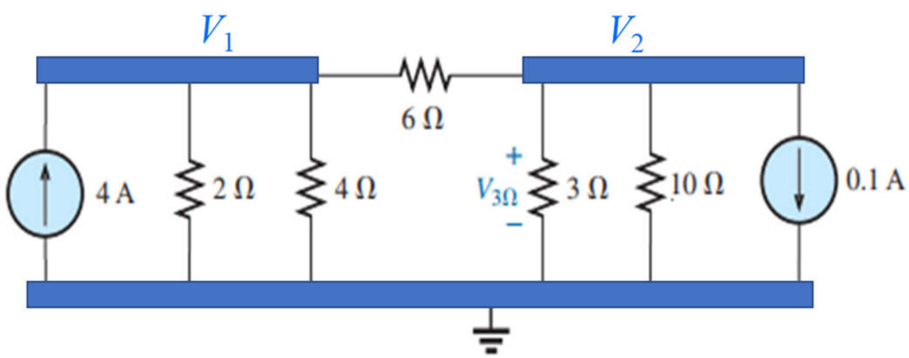
$$11V_1 - 2V_2 = 48$$
$$-5V_1 + 18V_2 = -3$$

$$D_1 = \begin{vmatrix} 48 & -2 \\ -3 & 18 \end{vmatrix} = 864 - 6 = 858$$

$$D_2 = \begin{vmatrix} 11 & 48 \\ -5 & -3 \end{vmatrix} = -33 + 240 = 207$$

$$V_1 = \frac{D_1}{D} = \frac{858}{188} = \mathbf{4.56\ V}$$

$$V_2 = V_{3\Omega} = \frac{D_2}{D} = \frac{207}{188} = \mathbf{1.1\ V}$$



Current of branches:

$$I_{2\Omega} = \frac{V_1}{2\Omega} = \frac{4.56\,\text{V}}{2\Omega} = \mathbf{2.28\ A}$$

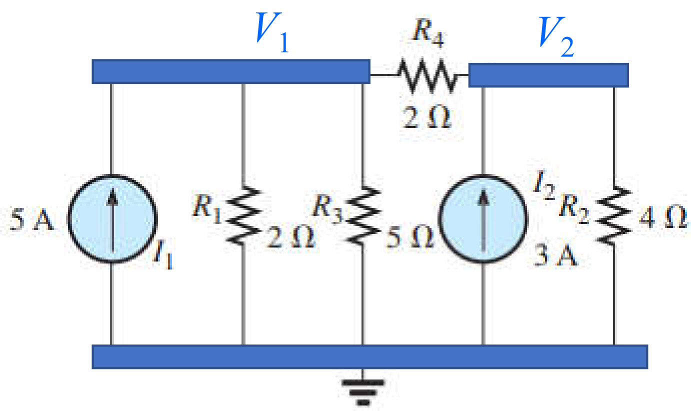
$$I_{4\Omega} = \frac{V_1}{4\Omega} = \frac{4.56\,\text{V}}{4\Omega} = \mathbf{1.14\ A}$$

$$I_{6\Omega} = \frac{V_1 - V_2}{6\Omega} = \frac{4.56\,\text{V} - 1.1\,\text{V}}{6\Omega} = \mathbf{0.577\ A}$$

$$I_{3\Omega} = \frac{V_2}{3\Omega} = \frac{1.1\text{V}}{3\Omega} = \mathbf{0.367\ A}$$

$$I_{10\Omega} = \frac{V_2}{10\Omega} = \frac{1.1\text{V}}{10\Omega} = \mathbf{0.11\ A}$$

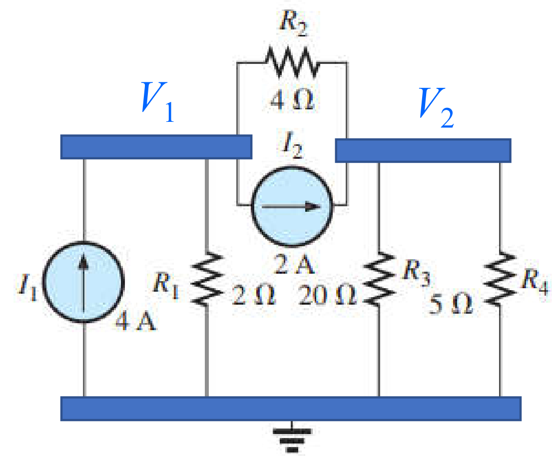
Write the nodal equations for the following networks.



(a)

$$\left(\frac{1}{2}+\frac{1}{5}+\frac{1}{2}\right)V_1-\left(\frac{1}{2}\right)V_2=5 \qquad 12V_1-5V_2=50$$

$$\left(\frac{1}{2}+\frac{1}{4}\right)V_2-\left(\frac{1}{2}\right)V_1=3 \qquad 3V_2-2V_1=6$$

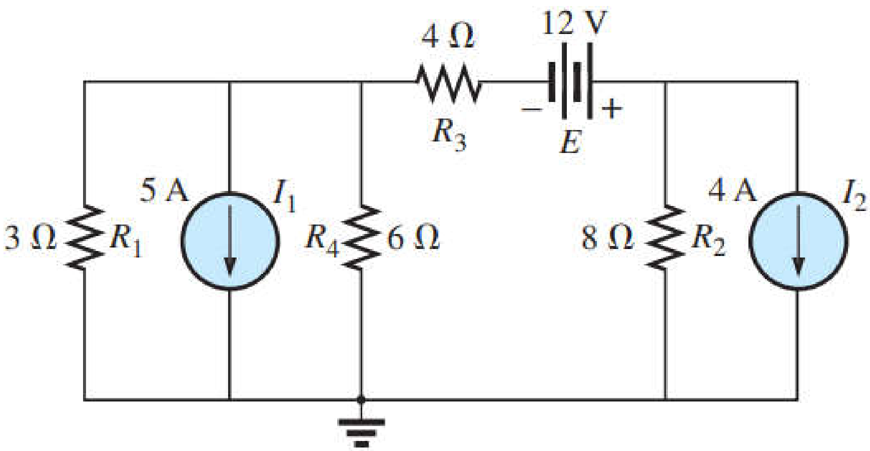


(b)

$$\left(\frac{1}{2}+\frac{1}{4}\right)V_1-\left(\frac{1}{4}\right)V_2=4-2 \qquad 3V_1-V_2=8$$

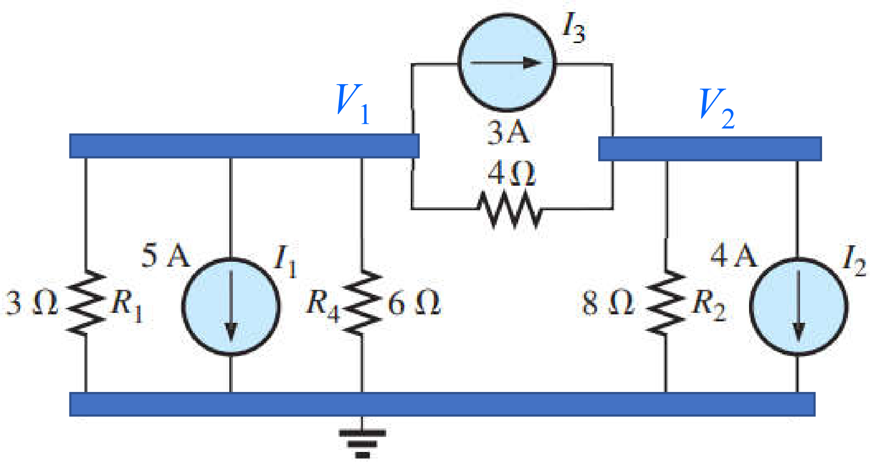
$$\left(\frac{1}{4}+\frac{1}{20}+\frac{1}{5}\right)V_2-\left(\frac{1}{4}\right)V_1=2 \qquad 10V_2-5V_1=40$$

Write the nodal equations for the following networks.

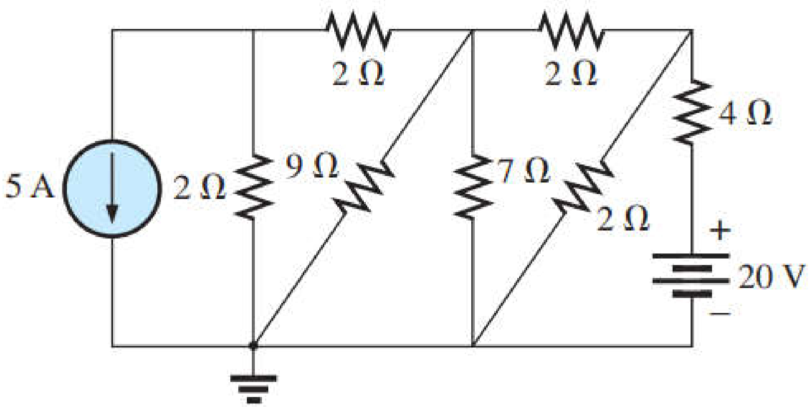


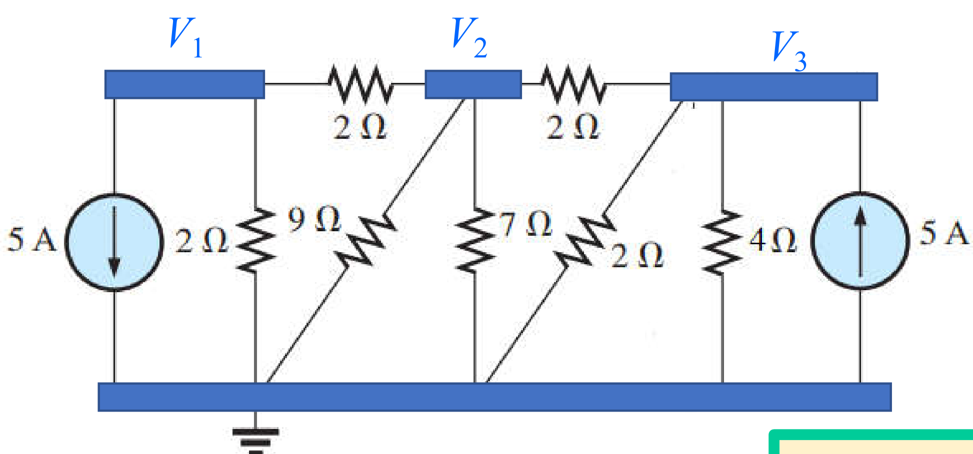
$$\left(\frac{1}{3}+\frac{1}{6}+\frac{1}{4}\right)V_1-\left(\frac{1}{4}\right)V_2=-5-3 \qquad 9V_1-3V_2=-48$$

$$\left(\frac{1}{4}+\frac{1}{8}\right)V_2-\left(\frac{1}{4}\right)V_1=3-4 \qquad 3V_2-2V_1=-8$$



Write the nodal equations for the following networks.





$$\left(\frac{1}{2}+\frac{1}{2}\right)V_1-\left(\frac{1}{2}\right)V_2=-5$$

$$2V_1-V_2=-10$$

$$\left(\frac{1}{2}+\frac{1}{9}+\frac{1}{7}+\frac{1}{2}\right)V_2-\left(\frac{1}{2}\right)V_1-\left(\frac{1}{2}\right)V_3=0$$

$$158V_2-63V_1-63V_3=0$$

$$\left(\frac{1}{2}+\frac{1}{2}+\frac{1}{4}\right)V_3-\left(\frac{1}{2}\right)V_2=5$$

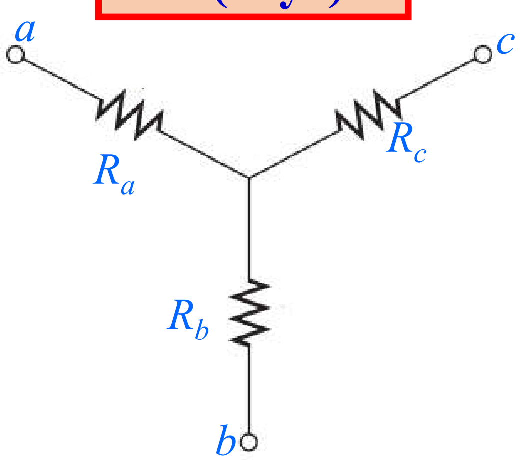
$$5V_2-2V_2=20$$

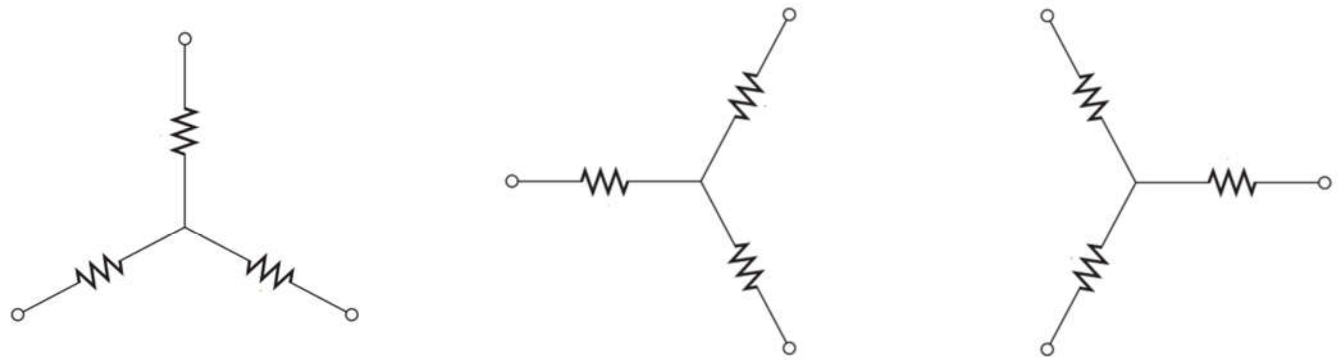
Practice Book [Ch 8] Problem: 41 ~ 44

8.12 Y–Δ (T–Π) and Δ–Y (Π–T) CONVERSIONS

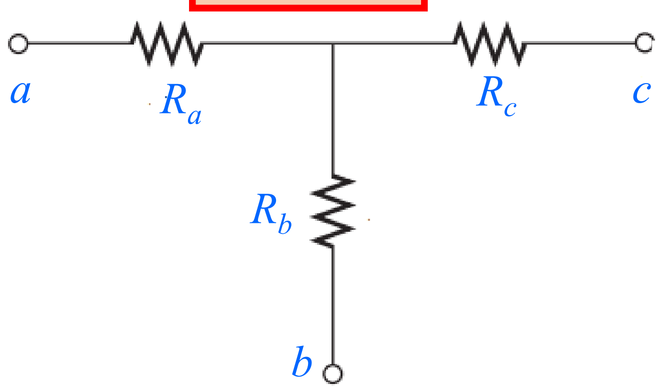
Circuit configurations are often encountered in which the resistors do **not appear to be in series or parallel**. Under these conditions, it may be necessary to convert the circuit from one form to another [*i.e*. convert from **Y/T to Δ/Π or Δ/Π to Y/T**] to solve for any unknown quantities

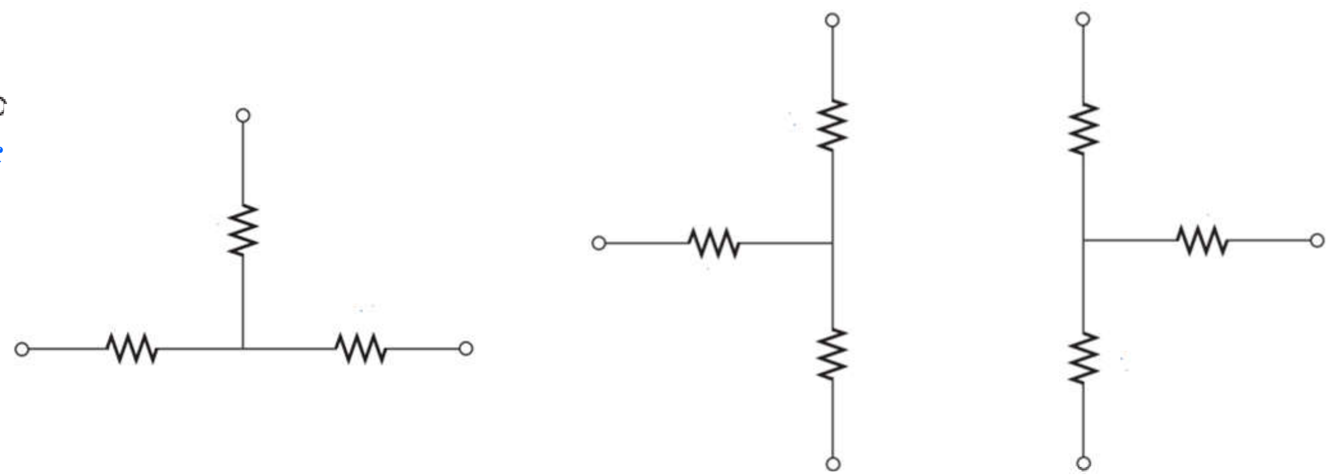
Y (Wye)



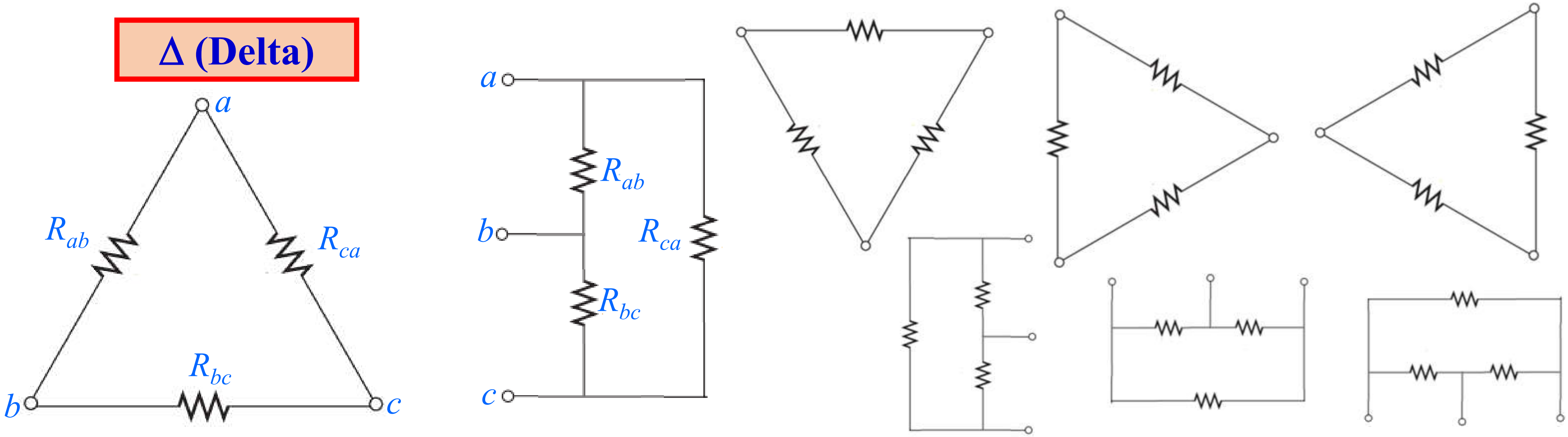


T (Tee)

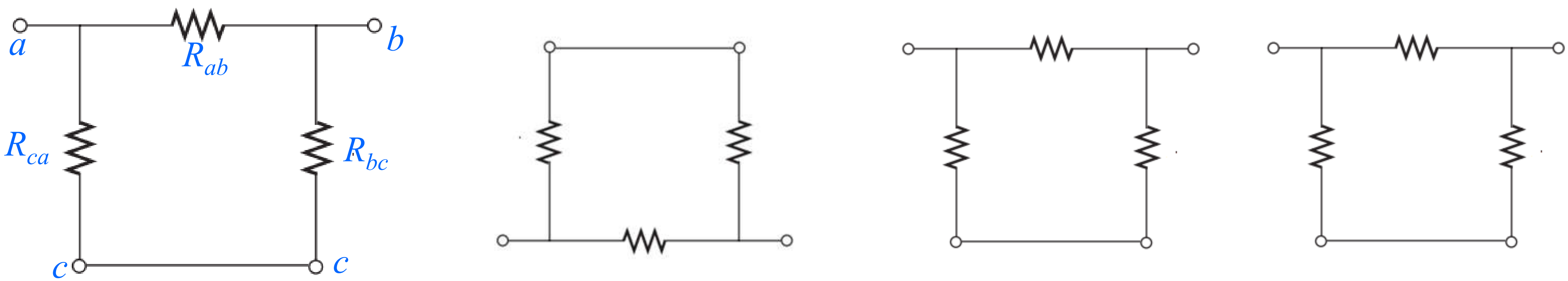




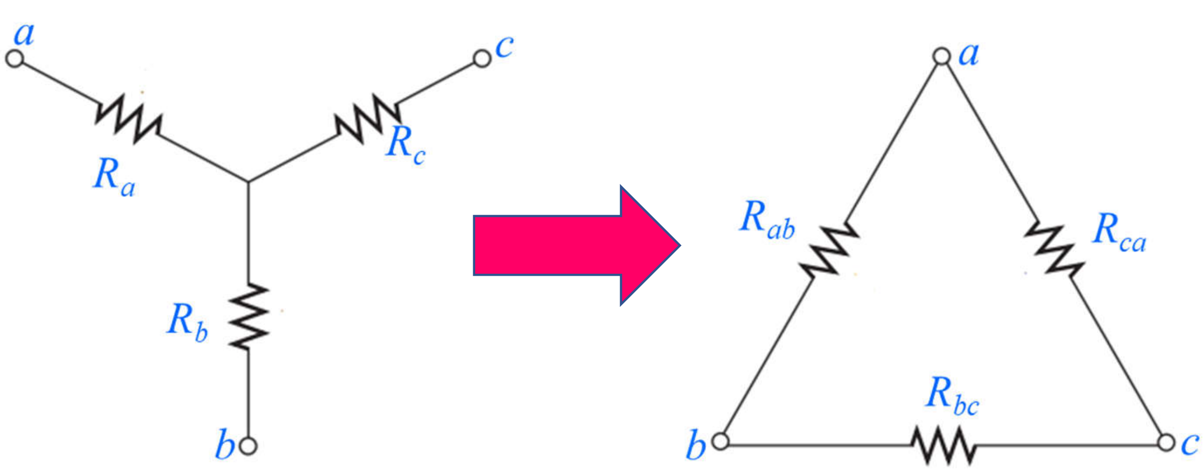
Δ (Delta)



Π (Pai)



Conversion from Y (T) to Δ (Π)



Here, R_a, R_b and R_c are known.

Calculate R_{ab}, R_{bc} and R_{ca}

$$R_{abc} = R_a R_b + R_b R_c + R_c R_a$$

$$R_{ab} = \frac{R_{abc}}{R_c} = \frac{R_a R_b + R_b R_c + R_c R_a}{R_c}$$

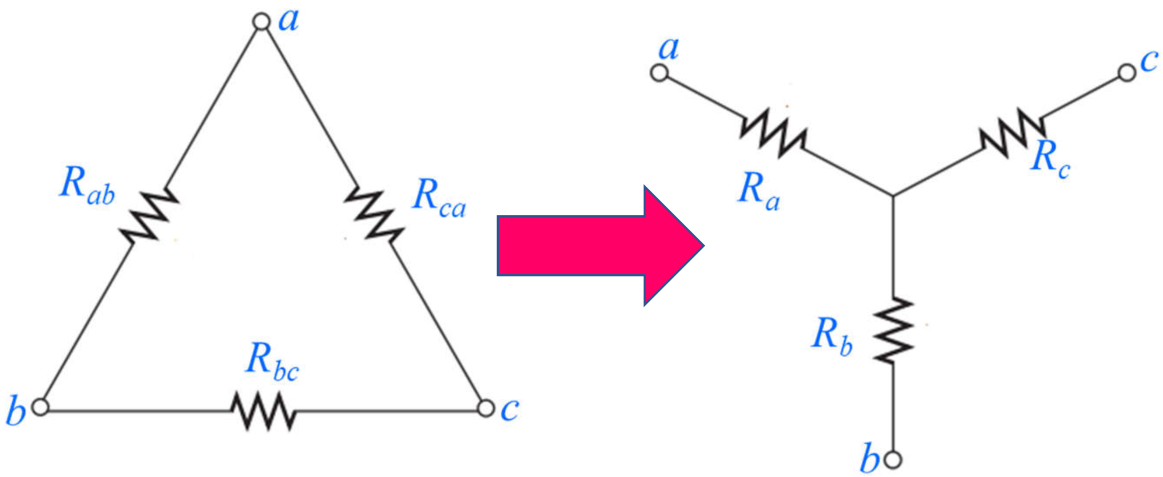
$$R_{bc} = \frac{R_{abc}}{R_a} = \frac{R_a R_b + R_b R_c + R_c R_a}{R_a}$$

$$R_{ca} = \frac{R_{abc}}{R_b} = \frac{R_a R_b + R_b R_c + R_c R_a}{R_b}$$

If $R_a = R_b = R_c = R_Y$ then $R_{ab} = R_{bc} = R_{ca} = R_\Delta = 3R_Y$

For derivation of these equations go through Eq. (8.3a) to (8.4 c)

Conversion from Δ (Π) to Y (T)



Here, R_{ab}, R_{bc} and R_{ca} are known.

Calculate R_a, R_b and R_c

$$R_{abc} = R_{ab} + R_{bc} + R_{ca}$$

$$R_a = \frac{R_{ab}R_{ca}}{R_{abc}} = \frac{R_{ab}R_{ca}}{R_{ab} + R_{bc} + R_{ca}}$$

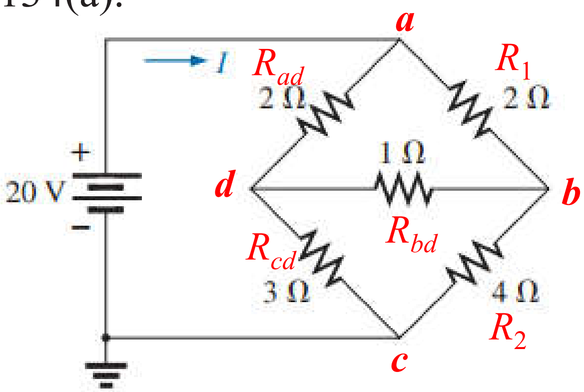
$$R_b = \frac{R_{bc}R_{ab}}{R_{abc}} = \frac{R_{bc}R_{ab}}{R_{ab} + R_{bc} + R_{ca}}$$

$$R_c = \frac{R_{ca}R_{bc}}{R_{abc}} = \frac{R_{ca}R_{bc}}{R_{ab} + R_{bc} + R_{ca}}$$

If $R_{ab} = R_{bc} = R_{ca} = R_\Delta$ then $R_{ab} = R_{bc} = R_{ca} = R_Y = \dfrac{R_\Delta}{3}$

For derivation of these equations go through Eq. (8.5a) to (8.5c)

Problem 51(a) [P. 342]. Using a Δ-Y or Y-Δ conversion, find the current I in each of the networks in Fig. 8.134(a).



Solution: First, marked four nodes (a, b, c, and d) in the circuit.

Let, R_{ad} = 2 Ω, R_{bd} = 1 Ω, R_{cd} = 3 Ω, R_1 = 2 Ω, and R_2 = 4 Ω

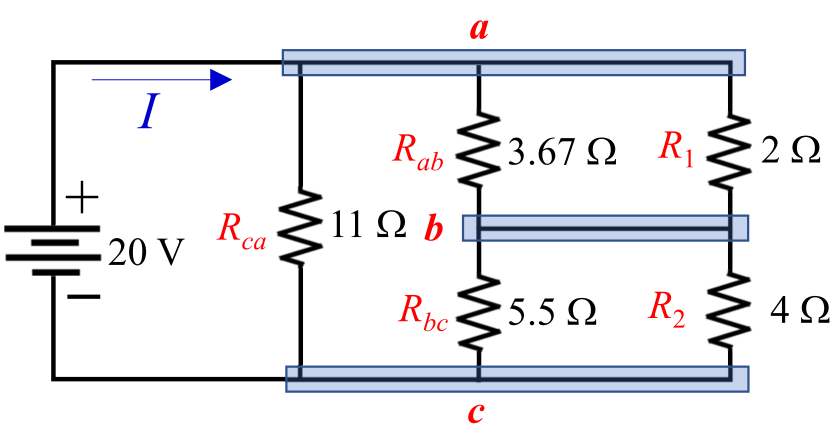
Here, R_{ad}, R_{bd}, and R_{cd} made Y connection where d is common point. So, this Y connection is going to convert Δ connection.

$$R_{abc} = R_{ad}R_{bd} + R_{bd}R_{cd} + R_{cd}R_{ad}$$
$$= (2\Omega)(1\Omega) + (1\Omega)(3\Omega) + (3\Omega)(2\Omega)$$
$$= 2\Omega + 3\Omega + 6\Omega = 11\Omega$$

$$R_{ab} = \frac{R_{abc}}{R_{cd}} = \frac{11\Omega}{3\Omega} = 3.67\Omega$$

$$R_{bc} = \frac{R_{abc}}{R_{ad}} = \frac{11\Omega}{2\Omega} = 5.5\Omega$$

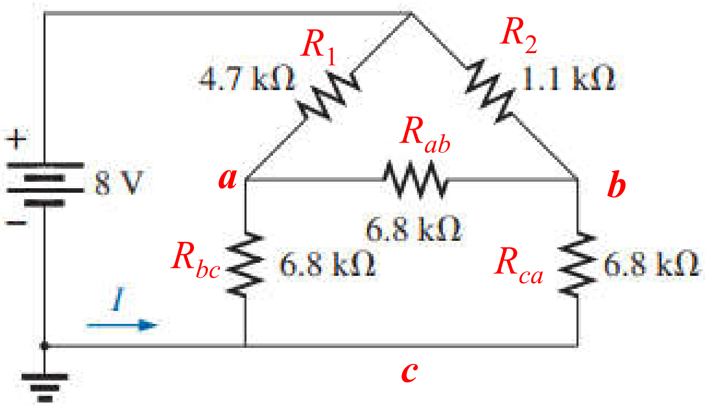
$$R_{ca} = \frac{R_{abc}}{R_{bd}} = \frac{11\Omega}{1\Omega} = 11\Omega$$



$$
\begin{aligned}
R_T &= (R_{ca}) // \{(R_{ab} // R_1) + (R_{bc} // R_2)\} \\
&= (11\Omega) // \{(3.67\Omega // 2\Omega) + (5.5\Omega // 4\Omega)\} \\
&= (11\Omega) // \{1.29\Omega + 2.32\Omega\} \\
&= (11\Omega) // \{3.61\Omega\} \\
&= 2.72\ \Omega
\end{aligned}
$$

$$
I = \frac{E}{R_T} = \frac{20\,\text{V}}{2.72\,\Omega} = \mathbf{7.35\,V}
$$

Problem 51(b) [P. 342]. Using a Δ-Y or Y-Δ conversion, find the current I in each of the networks in Fig. 8.134(b).



Solution: First, marked four nodes (a, b, and c) in the circuit.

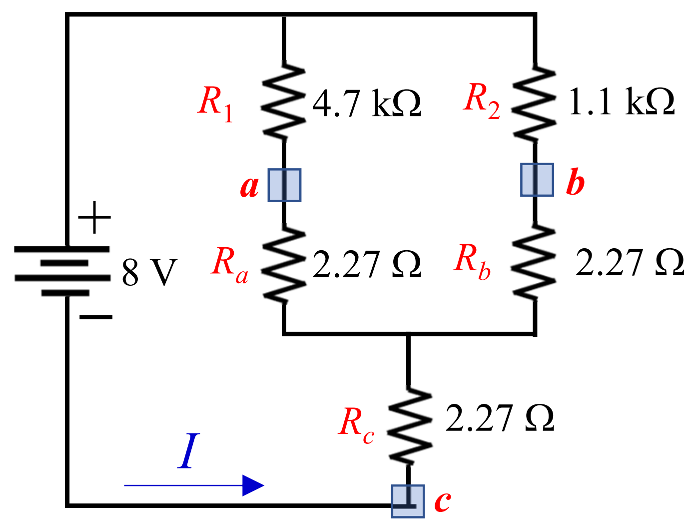
Let, R_1=4.7 kΩ, R_2=1.1 kΩ, R_{ab}=R_{bc}=R_{ca}= 6.8 kΩ

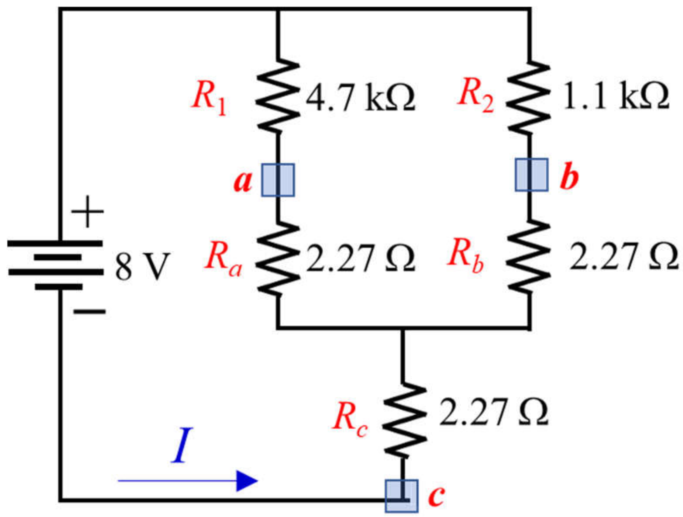
Here, R_{ab}, R_{bc}, and R_{ca} made Δ connection. So, this Δ connection is going to convert Y connection.

Here, $R_{ab} = R_{bc} = R_{ca} = R_\Delta = 6.8\ \text{k}\Omega$

so, $R_a = R_b = R_c = R_Y = \dfrac{R_\Delta}{3}$

$$= \frac{6.8\ \text{k}\Omega}{3} = 2.27\ \text{k}\Omega$$





$$
\begin{aligned}
R_T &= [(R_1 + R_a)//(R_2 + R_b)] + R_c \\
&= [(4.7\text{k}\Omega + 2.27\text{k}\Omega)//(1.1\text{k}\Omega + 2.27\text{k}\Omega)] + 2.27\text{k}\Omega \\
&= [(6.97\text{k}\Omega)//(3.37\text{k}\Omega)] + 2.27\text{k}\Omega \\
&= 2.27\text{k}\Omega + 2.27\text{k}\Omega \\
&= 4.54\text{k}\Omega
\end{aligned}
$$

$$I = \frac{E}{R_T} = \frac{8\,\text{V}}{4.54\ \text{k}\Omega} = \mathbf{1.76\ \ mA}$$

Practice Book [Ch 8] Problem: 51 ~ 56

8.11 BRIDGE NETWORKS

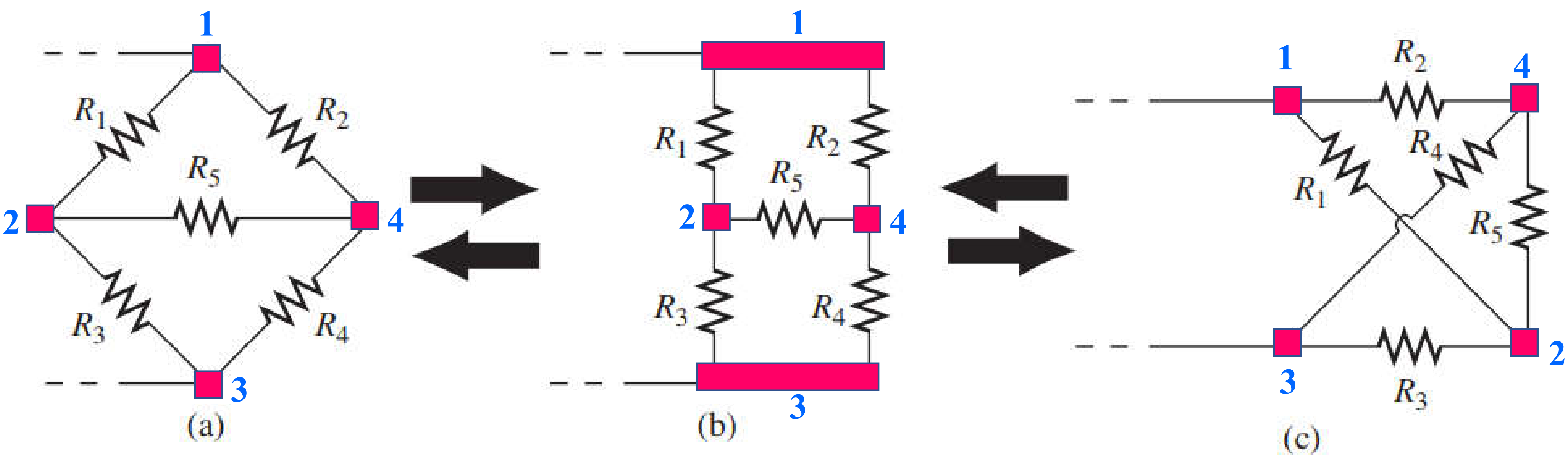
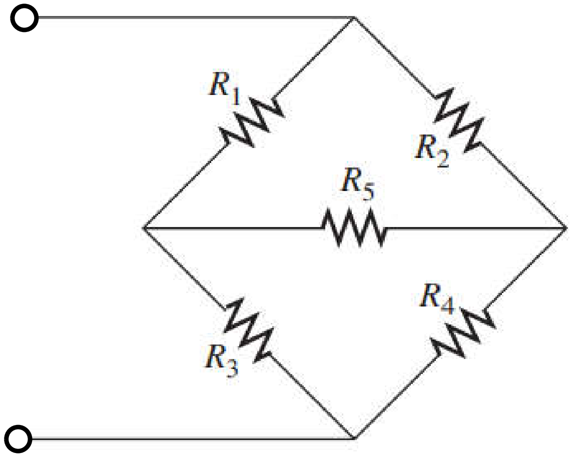


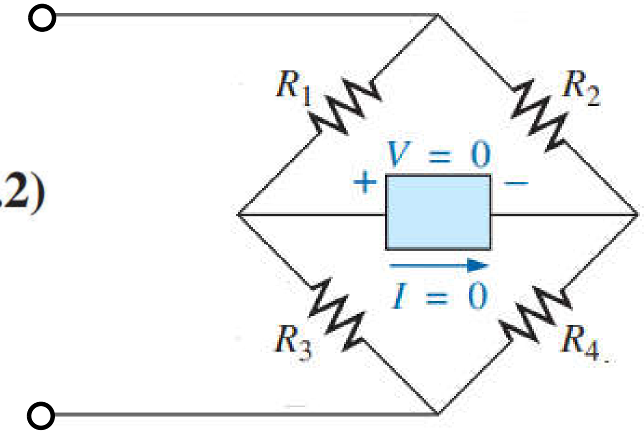
FIG. 8.69 *Various formats for a bridge network.*

Balanced Bridge Network



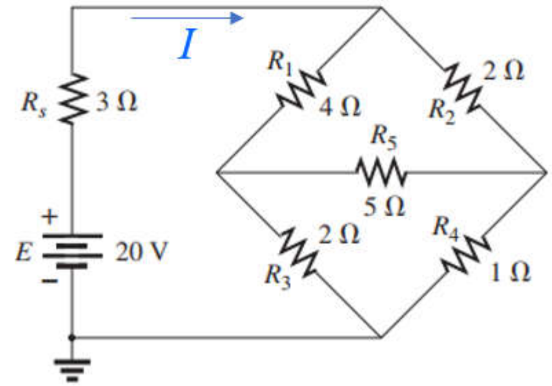
If the ratio of R_1 to R_3 is equal to that of R_2 to R_4, the bridge is balanced, and $I = 0$ A or $V = 0$ V.

$$\boxed{\frac{R_1}{R_3} = \frac{R_2}{R_4}} \qquad \textbf{(8.2)}$$

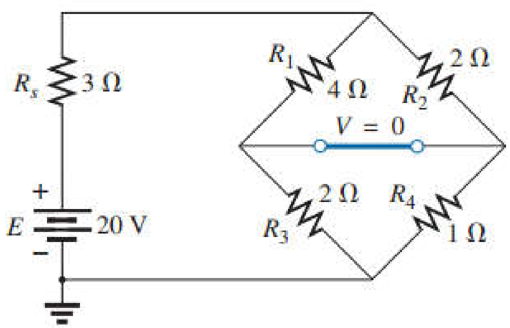


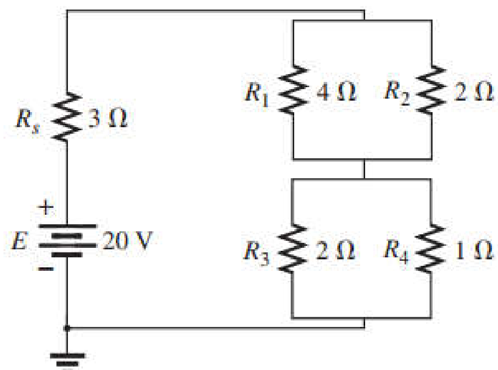
If this condition is not satisfied, to solve the bridge circuit use the other techniques.

Example 8.11.1: Find the current I for the following network.



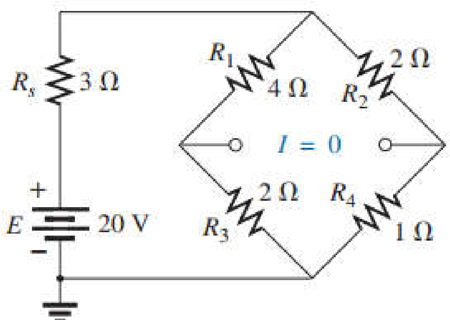
Solution: Here: $\dfrac{R_1}{R_3} = \dfrac{4\Omega}{2\Omega} = 2$ and $\dfrac{R_2}{R_4} = \dfrac{2\Omega}{1\Omega} = 2$ so $\dfrac{R_1}{R_3} = \dfrac{R_2}{R_4}$

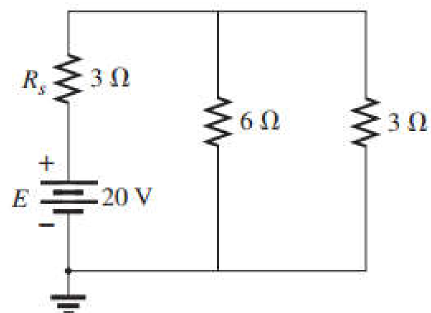




$$R_T = 3\Omega + (4\Omega // 2\Omega) + (2\Omega // 1\Omega) = 5\Omega$$

$$I = \frac{E}{R_T} = \frac{20\,\text{V}}{5\Omega} = \mathbf{4\,A}$$





$$R_T = 3\Omega + (6\Omega // 3\Omega) = 5\Omega$$

$$I = \frac{E}{R_T} = \frac{20\,\text{V}}{5\Omega} = \mathbf{4\,A}$$