Complex Variable, Laplace & Z-transformation

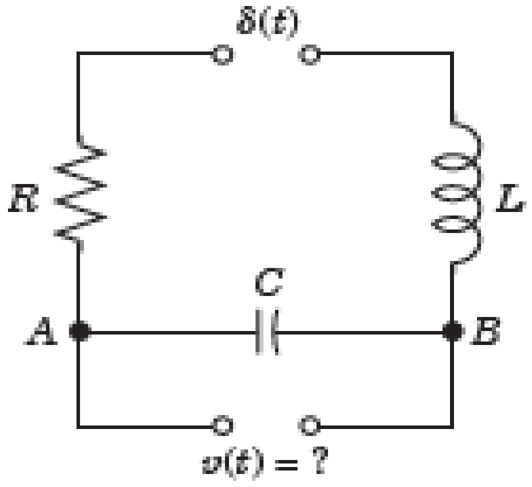
Lecture 08

This Lecture Covers-

This lecture focuses on some application problem where actually Laplace transformation have an important course of action.

Example of some Application problems

Find the output voltage response in the following figure, if $\boldsymbol{R} = \mathbf{20}\ \boldsymbol{\Omega}$, $\boldsymbol{L} = \mathbf{1}\ \boldsymbol{H}, \boldsymbol{C} = \mathbf{10^{-4}}\ \boldsymbol{F}$, the input is $\boldsymbol{\delta(t)}$, (a unit impulse at time $\boldsymbol{t} = \mathbf{0}$), and current and charge are zero at time $\boldsymbol{t} = \mathbf{0}$.



Network

Solution:

To understand what is going on, note that the network is an LC- circuit to which two wires at A and B for recording the voltage $v(t)$ on the capacitor. The current $i(t)$ and charge $q(t)$ are related by $i = q' = \frac{dq}{dt}$, we obtain the model

$$Li' + Ri + \frac{q}{C} = \delta(t)$$

$$\Rightarrow L\,q'' + Rq' + \frac{q}{C} = \delta(t)$$

$$\Rightarrow q'' + 20\,q' + 10{,}000\,q = \delta(t) \ldots\ldots\ldots\ldots (i)$$

After applying Laplace transformation in (i) we get,

$$(s^2 + 20s + 10{,}000)Q = 1$$

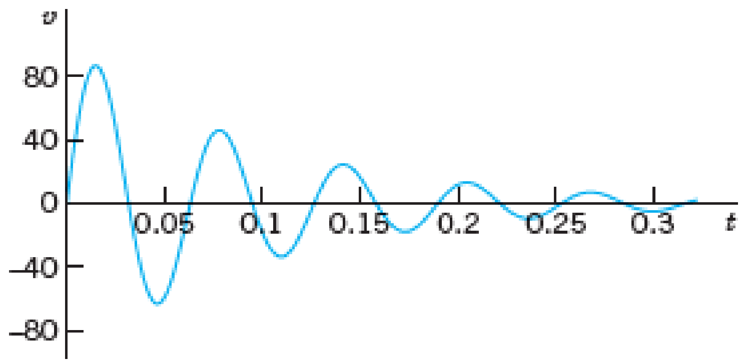
$$\Rightarrow Q = \frac{1}{(s+10)^2 + 9900} \ldots\ldots\ldots\ldots (ii)$$

Applying inverse Laplace transformation on both sides of (ii) ,

$$q = \frac{1}{99.5}\,e^{-10\,t}\sin(99.5\,t)$$

Now, $v = \frac{q}{C} = 100.5\,e^{-10\,t}\sin(99.5\,t)$.

Ans.



Voltage on the capacitor

Learning Outcomes

After finishing this lecture students can easily understand the influence of Laplace transformation on some application problem.