AIUB

SOLUTION

Sample Exercise 1:

1. $f(t) = 3t + 12$

Now, $\mathcal{L}\{(3t+12)\}$

$= \mathcal{L}\{3t\} + \mathcal{L}\{12\}$

$= 3 \cdot \frac{1!}{s} + \frac{12}{s}$

$= \frac{3}{s^2} + \frac{12}{s}$ (Ans)

2. $f(t) = e^{5t}$

Now, $\mathcal{L}\{e^{5t}\}$

$= \frac{1}{s-5}$ (Ans)

3. $f(t) = e^{-2t}$

Now, $\mathcal{L}\{e^{-2t}\}$

$= \frac{1}{s+2}$ (Ans)

4. $f(t) = (a-bt)^2$

Now, $\mathcal{L}\{(a-bt)^2\}$

$= \mathcal{L}\{a^2 - 2abt + b^2t^2\}$

$= \mathcal{L}\{a^2\} - 2ab\,\mathcal{L}\{t\} + b^2\mathcal{L}\{t^2\}$

$= \frac{a^2}{s} - 2ab\left(\frac{1!}{s^2}\right) + b^2 \cdot \frac{2!}{s^3}$

$= \frac{a^2}{s} - \frac{2ab}{s^2} + \frac{2b^2}{s^3}$

5. $f(t) = \cos \pi t$

Now, $\mathcal{L}\{\cos \pi t\}$

$= \frac{s}{s^2 + \pi^2}$ (Ans)

6. $f(t) = \cos^2 \omega t$

Now, $\mathcal{L}\{\cos^2 \omega t\}$

$= \mathcal{L}\{\frac{1}{2}(1 + \cos 2\omega t)\}$

$= \frac{1}{2}\mathcal{L}\{1\} + \frac{1}{2}\mathcal{L}\{\cos 2\omega t\}$

$= \frac{1}{2s} + \frac{s}{2(s^2 + 4\omega^2)}$

(Ans)

7. $f(t) = \sin(\omega t + \theta)$

$\mathcal{L}\{\sin(\omega t+\theta)\}$

$= \mathcal{L}\{\sin\omega t \cos\theta + \sin\theta \cos\omega t\}$

$= \cos\theta \, \mathcal{L}\{\sin\omega t\} + \sin\theta \, \mathcal{L}\{\cos\omega t\}$

$= \cos\theta \cdot \frac{\omega}{s^2+\omega^2} + \sin\theta \cdot \frac{s}{s^2+\omega^2}$

$= \frac{\omega\cos\theta}{s^2+\omega^2} + \frac{s.\sin\theta}{s^2+\omega^2}$ (Ans)

8. $f(t) = 1.5 \sin\left(3t - \frac{\pi}{2}\right)$

Now, $\mathcal{L}\{1.5 \sin(3t - \frac{\pi}{2})\}$

$= 1.5 \, \mathcal{L}\{\sin 3t \cos\frac{\pi}{2} - \sin\frac{\pi}{2} \cos 3t\}$

$= 1.5 \cos\frac{\pi}{2} \, \mathcal{L}\{\sin 3t\} - 1.5 \sin\frac{\pi}{2} \, \mathcal{L}\{\cos 3t\}$

$= 0 \times \frac{3}{s^2+9} - 1.5 \frac{s}{s^2+9}$

$= -1.5 \frac{s}{s^2+9}$

(Ans)

Lecture: 02

Exercise Set on Shifting Property.

1. $f(t) = e^{2t} \sinh 3t$

Now, $\mathcal{L}\{e^{2t} \sinh 3t\}$

$$= \frac{3}{(s-2)^2 - 9} \qquad \text{(Ans)}$$

2. $f(t) = e^{-t} \sinh 4t$

$\mathcal{L}\{e^{-t} \sinh 4t\}$

$= F(s+1)$

Now, $F(s) = \mathcal{L}\{\sinh 4t\}$

$$= \frac{4}{s^2 - 16}$$

So, $\mathcal{L}\{e^{-t} \sinh 4t\} = \frac{4}{(s+1)^2 - 16}$ (Ans)

3. $f(t) = e^{2t} \cos 3t$

$\mathcal{L}\{e^{2t} \cos 3t\}$

$= F(s-2)$

Now, $F(s) = \mathcal{L}\{\cos 3t\}$

$$= \frac{s}{s^2 + 9}$$

So, $\mathcal{L}\{e^{2t} \cos 3t\} = \frac{(s-2)}{(s-2)^2 + 9}$

(Ans)

4. $f(t) = t^{10} e^{-7t}$

$\mathcal{L}\{t^{10} e^{-7t}\}$

$= F(s+7)$

Now, $F(s) = \mathcal{L}\{t^{10}\}$

$= \dfrac{10!}{s^{11}}$

So, $\mathcal{L}\{t^{10} e^{-7t}\} = \dfrac{10!}{(s+7)^{11}}$ (Ans)

5. $f(t) = e^{5t} \cosh 6t$

$\mathcal{L}\{e^{5t} \cosh 6t\}$

$= F(s-5)$

Now, $F(s) = \dfrac{s}{s^2 - 36}$

So $\mathcal{L}\{e^{5t} \cosh 6t\} = \dfrac{(s-5)}{(s-5)^2 - 36}$

(Ans)

Exercise Set On Multiplication by t^n property.

1. $f(t) = t \sin 2t$

$$\mathcal{L}\{t \sin t\} = (-1)\frac{d}{ds}[F(s)]$$

$$= -\frac{d}{ds}\left[\mathcal{L}\{\sin 2t\}\right]$$

$$= -\frac{d}{ds}\left(\frac{2}{s^2+4}\right)$$

$$= -\left[\frac{(s^2+4)\frac{d}{ds}(2) - 2\frac{d}{ds}(s^2+4)}{(s^2+4)^2}\right]$$

$$= -\left[\frac{-4s}{(s^2+4)^2}\right]$$

$$= \frac{4s}{(s^2+4)^2}$$

(Ans)

2. $f(t) = t\cos bt$

$$\mathcal{L}\{t\cos bt\} = (-1)\frac{d}{ds}[F(s)]$$

$$= -\frac{d}{ds}[\mathcal{L}\{\cos bt\}]$$

$$= -\frac{d}{ds}\left(\frac{s}{s^2+b^2}\right)$$

$$= -\frac{(s^2+b^2)\frac{d}{ds}(s) - s.\frac{d}{ds}(s^2+b^2)}{(s^2+b^2)^2}$$

$$= -\left[\frac{s^2+b^2-2s^2}{(s^2+b^2)^2}\right]$$

$$= \frac{s^2-b^2}{(s^2+b^2)^2}$$

(Ans)

3. $f(t) = t^2 e^{-4t}$

$$\mathcal{L}\{t^2 e^{-4t}\} = (-1)^2 \frac{d^2}{ds^2}[F(s)]$$

$$= \frac{d^2}{ds^2}\left[\mathcal{L}\{e^{-4t}\}\right]$$

$$= \frac{d^2}{ds^2}\left(\frac{1}{s+4}\right)$$

$$= \frac{d}{ds}\left(\frac{-1}{(s+4)^2}\right)$$

$$= \frac{2}{(s+4)^3}$$

(Ans)

4. $f(t) = t \sinh 3t$

$$\mathcal{L}\{t \sinh 3t\} = (-1) \frac{d}{ds}[F(s)]$$

$$= -\frac{d}{ds}\left[\mathcal{L}\{\sinh 3t\}\right]$$

$$= -\frac{d}{ds}\left(\frac{3}{s^2-9}\right)$$

$$= -\left[-\frac{3 \times 2s}{(s^2-9)^2}\right]$$

$$= \frac{6s}{s^2-9}$$

(Ans)

5. $f(t) = t\cosh 2t$

$$\mathcal{L}\{t\cosh 2t\} = (-1)\frac{d}{ds}[F(s)]$$

$$= -\frac{d}{ds}[\mathcal{L}\{\cosh 2t\}]$$

$$= -\frac{d}{ds}\left(\frac{s}{s^2-4}\right)$$

$$= -\left[\frac{(s^2-4)\frac{d}{ds}(s) - s\frac{d}{ds}(s^2-4)}{(s^2-4)^2}\right]$$

$$= -\left[\frac{s^2-4-2s^2}{(s^2-4)^2}\right]$$

$$= \frac{s^2+4}{(s^2-4)^2}$$

(Ans)

Exercise Set on Laplace Transformation of Unit step function.

1. $f(t) = t.u(t-1)$

$\mathcal{L}\{t.u(t-1)\}$

$= e^{-s}\mathcal{L}\{f(t+1)\}$

$= e^{-s}\mathcal{L}\{t+1\}$

$= e^{-s}\left(\frac{1}{s^2}+\frac{1}{s}\right)$ (Ans)

2. $f(t) = (t-1)u(t-3)$

$\mathcal{L}\{(t-1)u(t-3)\}$

$= e^{-3s}\mathcal{L}\{f(t+3)\}$

$= e^{-3s}\mathcal{L}\{t+3-1\}$

$= e^{-3s}\mathcal{L}\{t+2\}$

$= e^{-3s}\left(\frac{1}{s^2}+\frac{2}{s}\right)$ (Ans)

3. $f(t) = (t+2)^2 u(t-1)$

$\mathcal{L}\{(t+2)^2 u(t-1)\}$

$= e^{-s} \mathcal{L}\{f(t+1)\}$

$= e^{-s} \mathcal{L}\{(t+1+2)^2\}$

$= e^{-s} \mathcal{L}\{t^2+6t+9\}$

$= e^{-s}\left(\frac{2}{s^3} + \frac{6}{s^2} + \frac{9}{s}\right)$ (Ans)

4. $f(t) = e^{-2t} u(t-3)$

$\mathcal{L}\{e^{-2t} . u(t-3)\}$

$= e^{-3s} \mathcal{L}\{f(t+3)\}$

$= e^{-3s} \mathcal{L}\{e^{-2(t+3)}\}$

$= e^{-3s} \mathcal{L}\{e^{-2t} . e^{-6}\}$

$= e^{-3s} e^{-6} \mathcal{L}\{e^{-2t}\}$

$= e^{-3s} e^{-6} \left(\frac{1}{s+2}\right)$

$= \frac{e^{-3s} e^{-6}}{(s+2)}$ (Ans)

5. $f(t) = 4\cos t \, u(t-\pi)$

$\mathcal{L}\{4\cos t \cdot u(t-\pi)\}$

$= e^{-\pi s} \mathcal{L}\{f(t+\pi)\}$

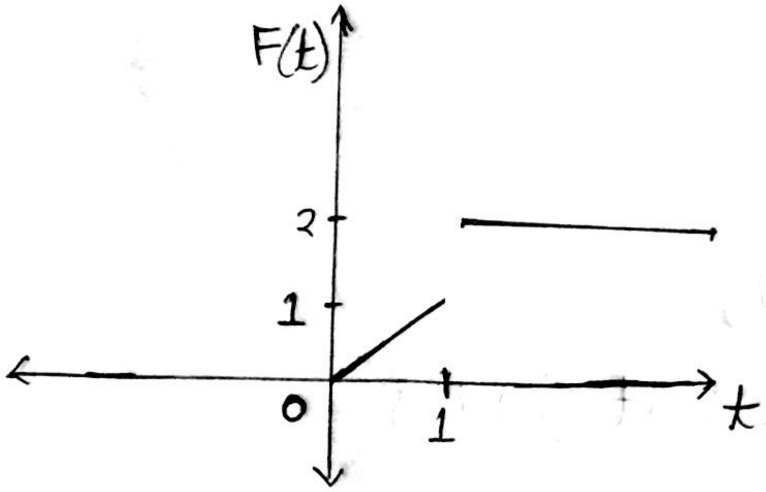
$= e^{-\pi s} \mathcal{L}\{4\cos(t+\pi)\}$

$= e^{-\pi s} \cdot 4\mathcal{L}\{\cos t \cos\pi - \sin t \sin\pi\}$

$= e^{-\pi s} \cdot 4\mathcal{L}\{-\cos t\}$

$= e^{-\pi s} \cdot \dfrac{-4s}{s^2+1} = \dfrac{-4se^{-\pi s}}{s^2+1}$ (Ans)

6. $f(t) = \begin{cases} t\ ; & 0<t<1 \\ 2\ ; & t>1 \end{cases}$

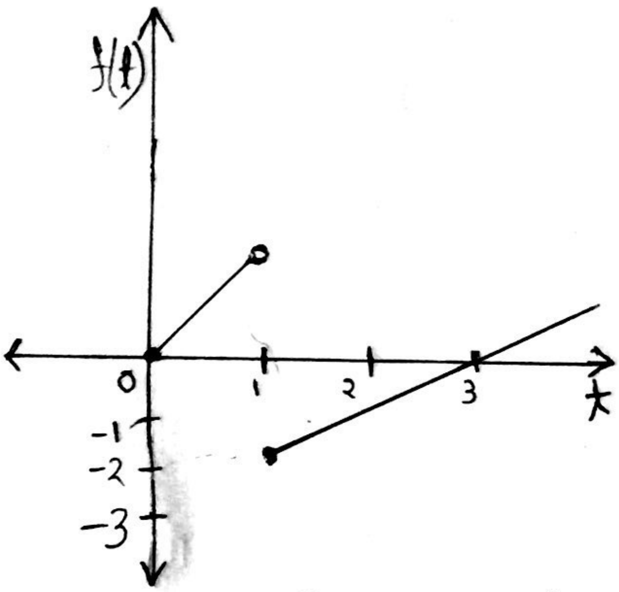


$$f(t) = 2u(t-1) + t[u(t) - u(t-1)]$$
$$= 2u(t-1) + t\,u(t) - t\,u(t-1)$$
$$= u(t-1)(2-t) + t\,u(t)$$

$$F(s) = L\{(2-t)\,u(t-1)\} + L\{t\,u(t)\}$$
$$= e^{-s} L\{f(t+1)\} + e^{0\times s} L\{f(t)\}$$
$$= e^{-s} L\{2-t-1\} + L\{t\}$$
$$= e^{-s} L\{1-t\} + L\{t\}$$
$$= \frac{e^{-s}}{s} - \frac{e^{-s}}{s^2} + \frac{1}{s^2}$$

(Ans)

7. $f(t) = \begin{cases} t^2; & 0 \le t < 1 \\ t-3; & t \geq 1 \end{cases}$



$$f(t) = t^2[u(t) - u(t-1)] + (t-3)[u(t-1)]$$
$$= t^2u(t) - t^2u(t-1) + (t-3)\,u(t-1)$$
$$= t^2u(t) + u(t-1)(t-3-t^2)$$

$$F(s) = \mathcal{L}\{t^2 u(t)\} + \mathcal{L}\{u(t-1)\cdot(t-t^2-3)\}$$
$$= e^{0\times s}\mathcal{L}\{f(t)\} + e^{-s}\mathcal{L}\{f(t+1)\}$$
$$= \mathcal{L}\{t^2\} + e^{-s}\mathcal{L}\{t+1-(t+1)^2-3\}$$
$$= \frac{2}{s^3} + e^{-s}\mathcal{L}\{t-2-t^2-2t-1\}$$
$$= \frac{2}{s^3} + e^{-s}\mathcal{L}\{-t^2-t-3\}$$
$$= \frac{2}{s^3} - e^{-s}\left(\frac{2}{s^3} + \frac{1}{s^2} + \frac{3}{s}\right)$$

(Ans)