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Course - Complex Varciable Laplace & Z-transformation

Section - [V]

Assignment (Mid-term)

SUB.:

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① Inverse Laplace transformation using partial fraction:

(a) Given $F(s) = \frac{3s+1}{(s+1)(s^2+1)}$.

Solution - $F(s)$ as $\frac{A}{s+1} + \frac{Bs+C}{s^2+1}$

$$\frac{3s+1}{(s+1)(s^2+1)} = \frac{A}{s+1} + \frac{Bs+C}{s^2+1} \text{ ——— (i)}$$

$$3s+1 = A(s^2+1) + (Bs+C)(s+1) \text{ ——— (ii)}$$

$$3s+1 = As^2 + A + Bs^2 + Bs + Cs + C \text{ ——— (iii)}$$

if $s = -1$,

$$3\times(-1)+1 = A\{(-1)^2+1\} + \{B\times(-1)\times C\}\{-1+1\}$$

$$\Rightarrow -3+1 = 2A+0$$

$$\Rightarrow -2 = 2A$$

$$\Rightarrow A = -1$$

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Comparing cofficient :-

$$0 = A + B$$

$$0 = -1 + B$$

$$B = 1$$

Then, $3 = B + C$; $1 + C$ $\therefore C = 2$

Put value of A, B, C

$$f(s) = \frac{3s+1}{(s+1)(s^2+1)} = \frac{-1}{s+1} + \frac{1 \times s + 2}{s^2+1}$$

$$= \frac{-1}{s+1} + \frac{s+2}{s^2+1}$$

(Ans)

(ii)

find $f(t) = L^{-1}\{F(s)\}$

$$= L^{-1}\left\{ -\frac{1}{s+1} + \frac{s+2}{s^2+1} \right\}$$

$$= L^{-1}\left\{ -\frac{1}{s+1} + \frac{s}{s^2+1} + 2\frac{1}{s^2+1} \right\}$$

$$= e^{-t} + \cos t + 2\sin t$$

(Ans)

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(b) Given, $f(s) = \frac{s+1}{(s+1)(s-5)}$

Solution - $f(s) = \frac{s+1}{(s+1)(s-5)} = \frac{A}{s+1} + \frac{B}{s-5}$ — (i)

$s+1 = A(s-5) + B(s+1)$ —— (ii)

if $s = 5$, $\therefore 5+1 = A(5-5) + B(5+1)$

$6 = 0 + 6B$

$B = 1$

$s = -1$; $-1+1 = A(-1-5) + B(-1+1)$

$0 = -6A + 0$

$\therefore A = 0$

Put value of A, B —

$f(s) = \frac{s+1}{(s+1)(s-5)} = \frac{0}{s+1} + \frac{1}{s-5}$

$= 0 + \frac{1}{s-5}$

$= \frac{1}{s-5}$

(Ans)

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(ii)

find $f(t) = L^{-1}\{F(s)\}$

$= L^{-1}\left\{\frac{1}{s-5}\right\}$

$= e^{5t}$

(Ans)

2

Solving the differential equation using Laplace transformation.

(a) $\ddot{y}(t) + 9y(t) = 10e^{-t}$, $y(0) = \dot{y}(0) = 0$

Where, $\ddot{y}(t) = \frac{d^2y(t)}{dt^2}$, $\dot{y}(t) = \frac{dy(t)}{dt}$

(i) Solution -

$\ddot{y}(t) + 9y(t) = 10e^{-t}$

$\Rightarrow L\{\ddot{y}(t)\} + 9L\{y(t)\} = 10\{L\{e^{-t}\}\}$

$\Rightarrow sy(s) - sy(0) - \dot{y}(0) + 9y(s) = 10\left\{\frac{1}{s+1}\right\}$

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$$\Rightarrow s^2y(s) - 0 - 0 + 9y(s) = \frac{10}{s+1}$$

$$\Rightarrow (s^2+9)\, y(s) = \frac{10}{s+1}$$

$$\Rightarrow y(s) = \frac{10}{(s+1)(s^2+9)}$$

Ans

(ii)

Solve the equation obtained in (i) for $y(s)$

Solution - $y(s) = \frac{10}{(s+1)(s^2+9)} = \frac{A}{(s+1)} + \frac{Bs+C}{(s^2+9)}$ --- (i)

$10 = A(s^2+9) + (Bs+C)(s+1)$ ——— (ii)

$\Rightarrow 10 = As^2 + 9A + Bs^2 + Bs + Cs + C$ ——— (iii)

if $s = -1$,

$10 = A\{(-1)^2+9\} + \{B\times(-1)+C\}\{-1+1\}$

$\Rightarrow 10 = 10A + 0$

$\Rightarrow A = 1$

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Comparing cofficient of s^2 from (ii)

$$\cancel{0+}\ 0 = A+B$$

$$B = -1$$

Then,

$$0 = B+C, \ -1+C \ \therefore C = 1$$

Put value of A, B, C →

$$y(s) = \frac{10}{(s+1)(s^2+9)} = \frac{1}{(s+1)} + \frac{(-1)\times s+1}{(s^2+9)}$$

$$= \frac{1}{(s+1)} + \frac{-s+1}{(s^2+9)}$$

(Ans)

(iii)

find $y(t)$ using inverse laplace transformatio of $y(s)$ in,

$$y(s) = \frac{10}{(s+1)(s^2+9)} = \frac{1}{(s+1)} + \frac{-s+1}{(s^2+9)}$$

$$L^{-1}\{(y(s)\} = L^{-1}\left\{\frac{1}{s+1} + \frac{-s+1}{s^2+9}\right\}$$

$$y(t) = L^{-1}\left\{\frac{1}{s+1} - \frac{s}{s^2+3^2} + \frac{1}{s^2+3^2}\right\}$$

$$= e^{-t} - \cos 3t + \frac{1}{3}\sin 3t.$$

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(iv) Solution - $e^{-t} - \cos 3t + \frac{1}{3}\sin 3t$

$$L\{y(t)\} = L\{e^{-t} - \cos 3t + \frac{1}{3}\sin 3t\}$$

$$Y(s) = \frac{1}{(s-(-1))} - \frac{s}{s^2+3^2} + \frac{1}{3}\cdot\frac{3}{s^2+3^2}$$

$$= \frac{1}{s+1} - \frac{s}{s^2+9} + \frac{1}{s^2+9}$$

$$= \frac{s^2+9-s(s+1)+s+1}{(s+1)(s^2+9)}$$

$$= \frac{s^2+9-s^2-s+s+1}{(s+1)(s^2+9)}$$

$$= \frac{10}{(s+1)(s^2+9)}$$

$$Y(s) = \frac{10}{(s+1)(s^2+9)}$$

(Ans)

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(b)

$\dot{y}(t) - y(t) = 8\cosh 2t$, $y(0) = 0$, $\dot{y}(t) = \frac{dy(t)}{dt}$

Write the laplace transformation.

Solution - $\dot{y}(t) - y(t) = 8\cosh 2t$

$L\{\dot{y}(t) - y(t)\} = L\{8\cosh 2t\}$

$\Rightarrow sy(s) - y(0) - y(s) = 8\frac{s}{s^2 2^2}$

$\Rightarrow sy(s) - 0 - y(s) = \frac{8s}{(s+2)(s-2)}$

$\Rightarrow (s-1)y(s) = \frac{8s}{(s+2)(s-2)}$

$\Rightarrow y(s) = \frac{8s}{(s-1)(s+2)(s-2)}$

(ii)

Solution - $y(s) = \frac{8s}{(s-1)(s+2)(s-2)} = \frac{A}{s-1} + \frac{B}{s+2} + \frac{C}{s-2}$ —— (1)

$8s = A(s+2)(s-2) + B(s-1)(s-2) + C(s-1)(s+2)$ —— (ii)

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if $s = 1$, $8 \times 1 = A(1+2)(1-2) + B(1-1)(1-2) + C(1-1)(1+2)$

$$8 = -3A$$

$$A = -\frac{8}{3}$$

if $s = -2$, $8 \times (-2) = A(-2+2)(-2-2) + B(-2-1)(-2-2)$
$+ C(-2-1)(-2+2)$

$$\Rightarrow -16 = 0 + 12B + 0$$

$$\Rightarrow B = \frac{-16}{12}$$

$$\Rightarrow B = -4/3$$

if $s = 2$, $8 \times 2 = A(2+2)(2-2) + B(2-1)(2-2) +$
$C(2-1)(2+2)$

$$16 = 0 + 0 + 4C$$

$$C = 4$$

Put the value of A, B, C

$$Y(s) = \frac{8s}{(s-1)(s+2)(s-2)} = \frac{-8/3}{s-1} + \frac{-4/3}{s+2} + \frac{4}{s-2}$$

$$= -\frac{8}{3(s-1)} - \frac{4}{3(s+2)} + \frac{4}{s-2}$$

(Ans)

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(iii)

Solution - $y(s) = \frac{8s}{(s-1)(s+2)(s+2)} - \frac{8}{3(s-1)} - \frac{4}{3(s+2)} + \frac{4}{(s-2)}$

$L^{-1}\{y(s)\} = L^{-1}\left\{-\frac{8}{3(s-1)} - \frac{4}{3(s+2)} + \frac{4}{(s-2)}\right\}$

$= -\frac{8}{3}e^{t} - \frac{4}{3}e^{-2t} + 4e^{2t}$

(Ans)

(iv)

$y(t) = -\frac{8}{3}e^{t} - \frac{4}{3}e^{-2t} + 4e^{2t}$

$L\{y(t)\} = -\frac{8}{3}L^{-1}\{e^{t}\} - \frac{4}{3}L^{-1}\{e^{-2t}\} + 4\{L^{-1}\{e^{2t}\}$

$y(s) = -\frac{8}{3}\cdot\frac{1}{(s-1)} - \frac{4}{3}\frac{1}{(s+2)} + 4\frac{1}{(s-2)}$

$= -\frac{8}{3(s-1)} - \frac{4}{3(s+2)} + \frac{4}{(s-2)}$

$= \frac{-8(s+2)(s-2) - 4(s-1)(s-2) + 4\times 3(s-1)(s+2)}{3(s-1)(s+2)(s-2)}$

$= \frac{-8s^2 + 32 - 4s^2 + 8s + 4s - 8 + 12s^2 + 24s - 12s - 24}{3(s-1)(s+2)(s-2)}$

$= \frac{24}{3(s-1)(s+2)(s-2)} = \frac{8s}{(s-1)(s+2)(s-2)}$

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Describe and sketch the locus represented by each of the following.

(i) $1 < |z+i| \le 2$

Solution - $1 < |z+i| \le 2$

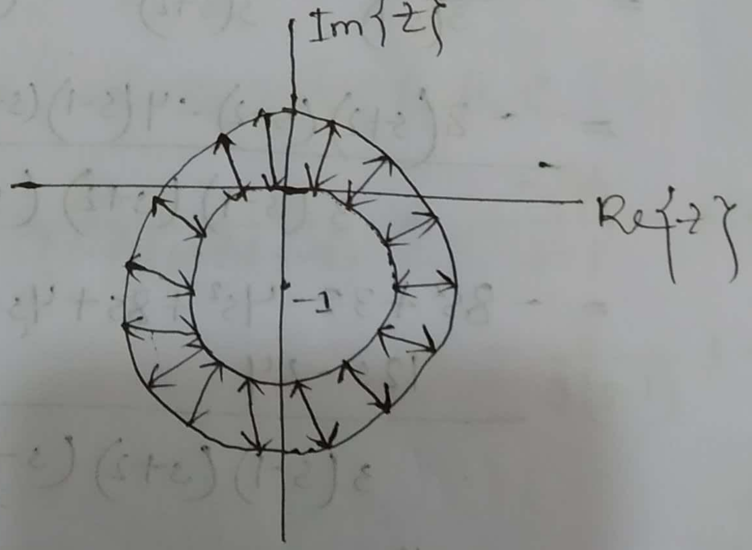
$\Rightarrow 1 < |x+iy+i| \le 2$

$\Rightarrow 1 < \sqrt{x^2+(y+1)^2} \le 2$

$\Rightarrow 1 < (x-0)^2+(y-(-1))^2 \le 2^2$

$\therefore$ centere $(0,-1)$

redius 2



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$$|z+3i| > 4$$

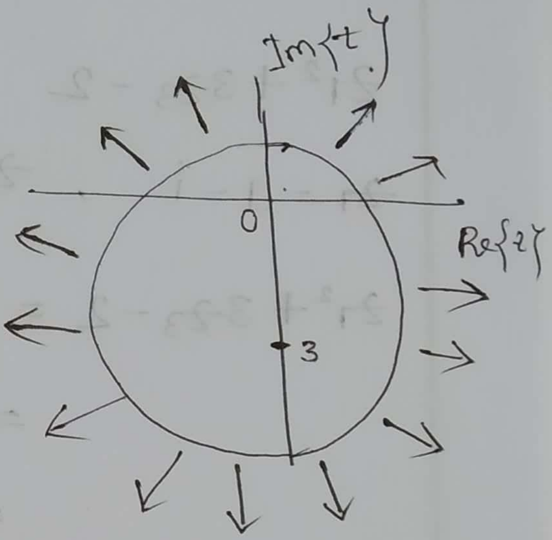
$$\Rightarrow |x+iy+3i| > 4$$

$$\Rightarrow \sqrt{x^2+(y+3)^2} > 4$$

$$\Rightarrow (x-0)^2+(y-(3)^2 > 4$$

center $(0,-3)$

redius 3



(b)

if $z_1 = 1-i$, $z_2 = -2+4i$ and $z_3 = \sqrt{3}-2i$

(i) $|2z_1 - 3z_1|$

$z_1 = 1-i$

$3z_1 = 3(1-i)$

$= 3-3i$

$z_2 = -2+4i$

$2z_2 = 2(-2+4i)$

$= -4+8i$

$2z_2 - 3z_1 = (-4-3)+i(8+3)$

$= -7+11i$

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(ii)

$z_1^2 + 3z_3 - 2$

$z_1 = 1 - i \quad , \quad z_3 = \sqrt{3} - 2i$

$$z_1^2 + 3z_3 - 2 = (1-i)^2 + 3(\sqrt{3} - 2i) - 2$$
$$= 1 - 2i + i^2 + 3\sqrt{3} - 6i - 2$$
$$= -1 + 3\sqrt{3} - 8i$$

(iii)

$\operatorname{Re}\left(\frac{z}{\bar{z}}\right)$

Solution - $z = x + iy \;,\; \bar{z} = x - iy$

$\operatorname{Re}(z) = x \quad \operatorname{Re}(\bar{z}) = x$

$$\operatorname{Re}\left(\frac{z}{\bar{z}}\right) = \frac{\operatorname{Re}(z)}{\operatorname{Re}(\bar{z})} = \frac{x}{x} = 1$$

(Ans)

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