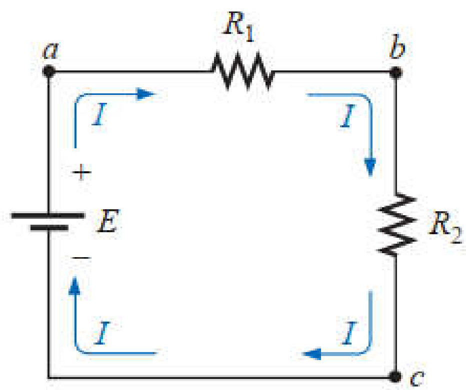
Chapter 5

Series DC Circuit

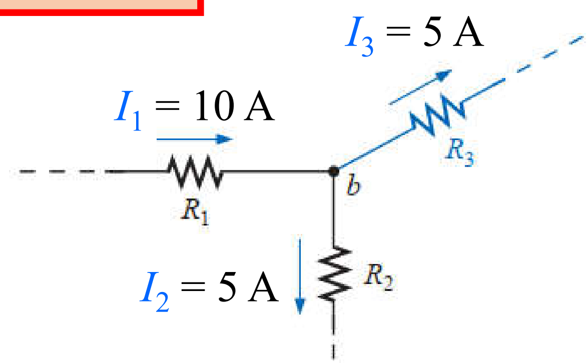
Two Elements are in Series

Two elements are in series if

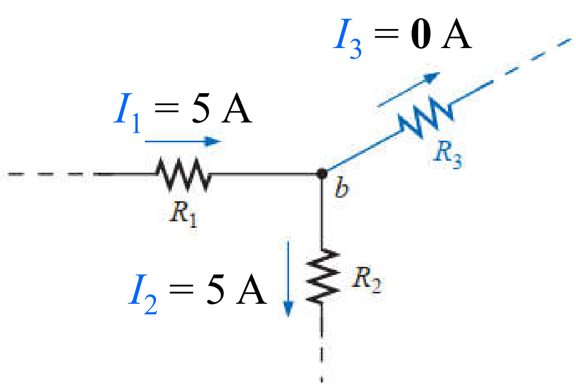
1. They have **only one terminal in common** (i.e., one lead of one is connected to only one lead of the other).
2. The common point between the two elements is **not connected to another current-carrying element**.
3. The current is the same through the two series elements.



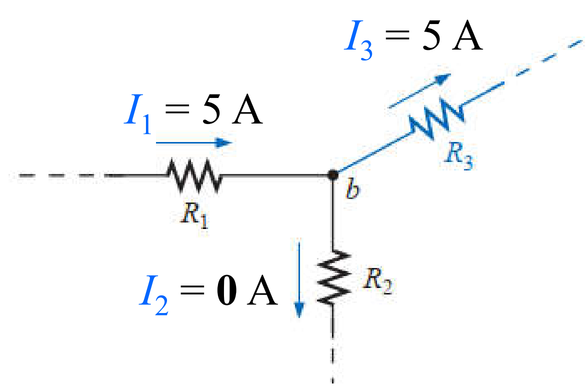
R_1 and R_2 are in series



R_1 and R_2 are not in series
R_2 and R_3 are not in series



R_1 and R_2 may be in series



R_1 and R_3 may be in series

5.2 Series Resistance

Total or Net or Effective Resistance of a Series Circuit or a Branch:

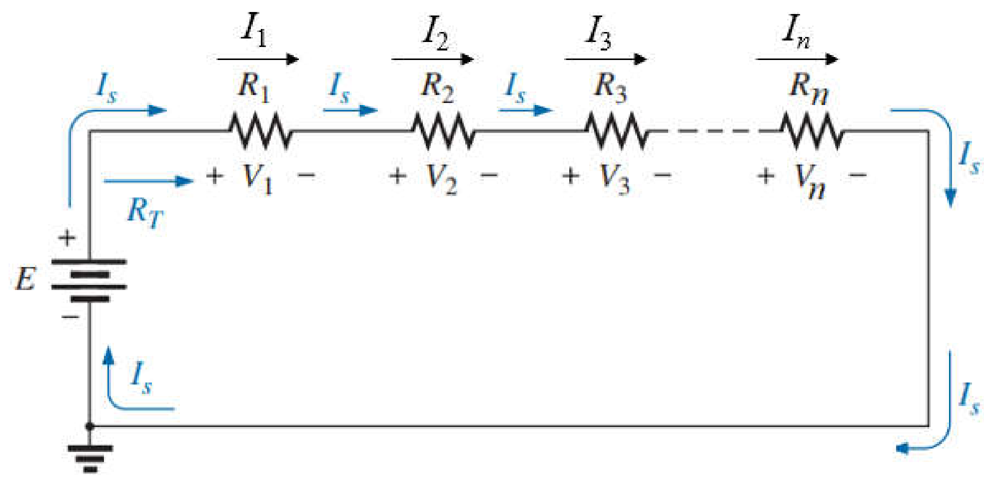
- The total resistance of a series configuration is the sum of the value of individual resistance, that is Eq. (**5.1**).
- The more resistors we add in series, the greater the resistance, no matter what their value.
- The total resistance greater than the value of all the individual resistance , that is Eq. (**5.1.1**).
- The total resistance of n resistors of the same value in series is simply ***multiply the value of one of the resistors by the number in series***; that is as Eq. (**5.2**)

$$\boxed{R_T = R_{eff} = R_1 + R_2 + R_3 + \ldots\ldots + R_n} \quad \textbf{(5.1)}$$

If $R_1 = R_2 = R_3 = \ldots\ldots = R_n = R$

$$\boxed{R_T = R_{eff} = nR} \quad \textbf{(5.2)}$$

$$\boxed{R_T > R_1;\ \ R_T > R_2; \ldots\ldots, R_T > R_N} \quad \textbf{(5.1.1)}$$



EXAMPLE 5.1 Determine the total resistance of the series connection in Fig. 5.6. Note that all the resistors appearing in this network are standard values.

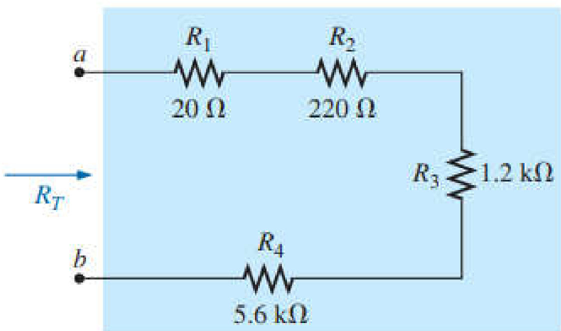


FIG. 5.6

Series connection of resistors for Example 5.1.

Solution: Note in Fig. 5.6 that even though resistor R_3 is on the vertical and resistor R_4 returns at the bottom to terminal b, all the resistors are in series since there are only two resistor leads at each connection point.

Applying Eq. (5.1):

$$R_T = R_1 + R_2 + R_3 + R_4$$
$$R_T = 20\ \Omega + 220\ \Omega + 1.2\ \text{k}\Omega + 5.6\ \text{k}\Omega$$

and $$R_T = 7040\ \Omega = \mathbf{7.04\ k\Omega}$$

EXAMPLE 5.2 Find the total resistance of the series resistors in Fig. 5.7. Again, recognize 3.3 kΩ as a standard value.

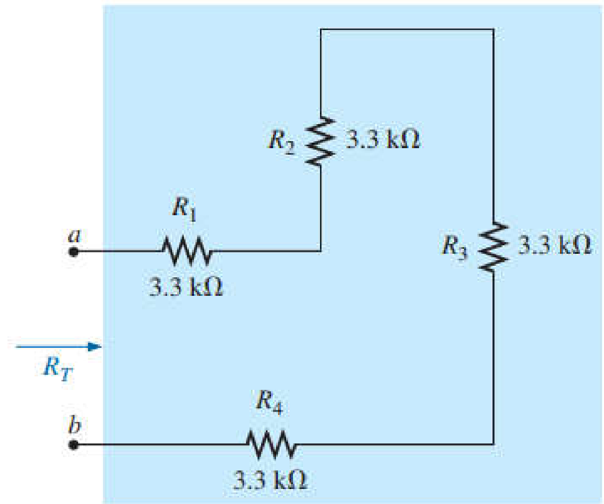


FIG. 5.7

Series connection of four resistors of the same value (Example 5.2).

Solution: Again, don't be concerned about the change in configuration. Neighboring resistors are connected only at one point, satisfying the definition of series elements.

Eq. (5.2): $$R_T = NR$$
$$= (4)(3.3\ \text{k}\Omega) = \mathbf{13.2\ k\Omega}$$

Practice Book Problem [5.2 Series Resistance] Problems: 1, 2, 5,6

Test Your Knowledge

2. Find the total resistance R_T for each configuration in Fig. 5.86. Note that only standard resistor values were used.

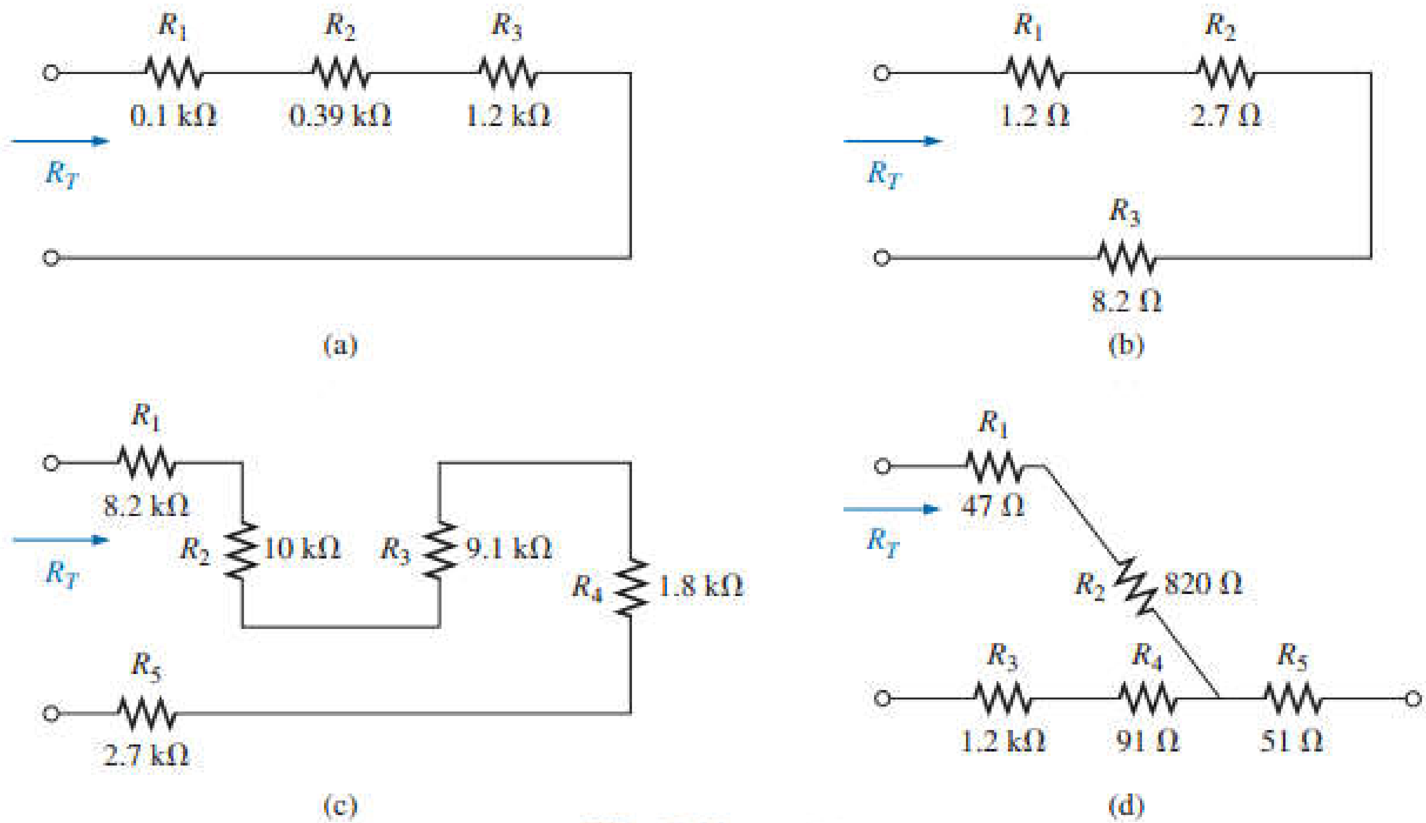
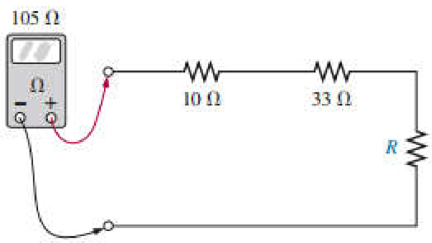
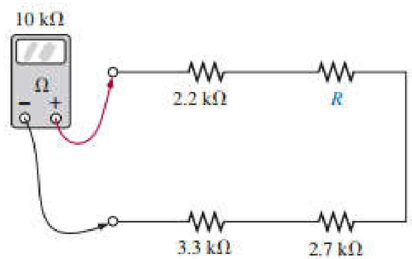


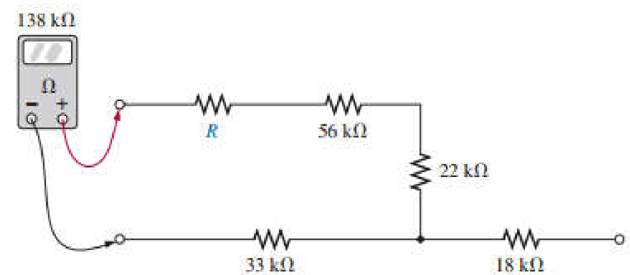
FIG. 5.86 *Problem 2.*

Test Your Knowledge

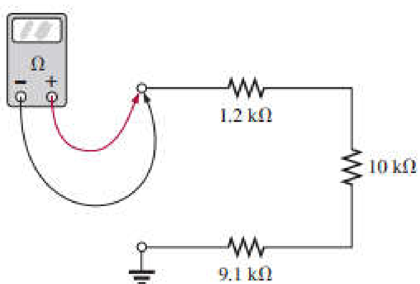
What is the value of ***R*** for the following cases of the reading of **OHM METER**?



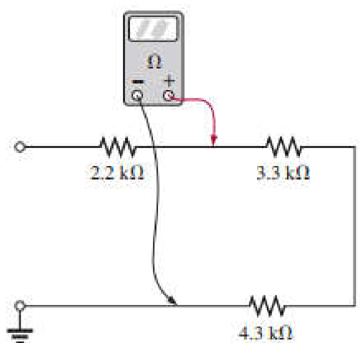




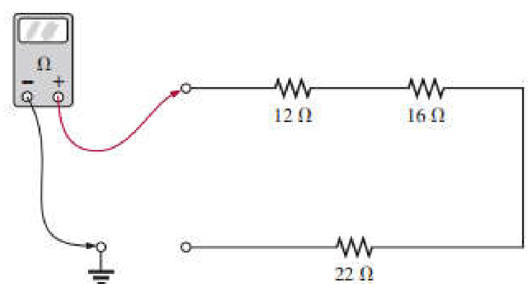
What is the reading of **OHM METER** for the following cases?



(c)



(b)



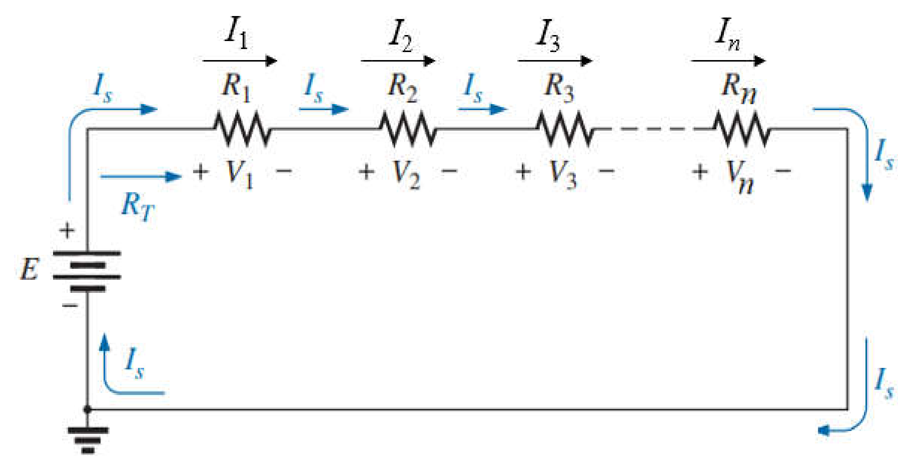
(d)

5.3 Series Circuit

A **series circuit** is one in which several resistances are connected one after the other.

Current in a series circuit: The current is the same at every point in a series circuit, that is as following equation for the following circuit.

$$I_1 = I_2 = I_3 = \ldots\ldots = I_n = I_s$$



Current Calculation of a series circuit: According ohm's law, the current of the following circuit is as follows:

$$I_s = \frac{E}{R_T} \quad \text{(A)} \quad \textbf{(5.3)}$$

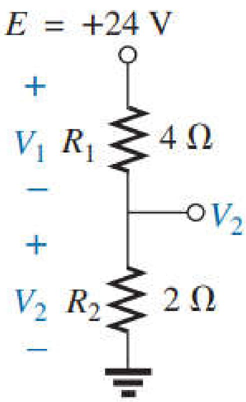
Voltage drop across individual resistance in a series circuit : According ohm's law, the of the following circuit is as follows:

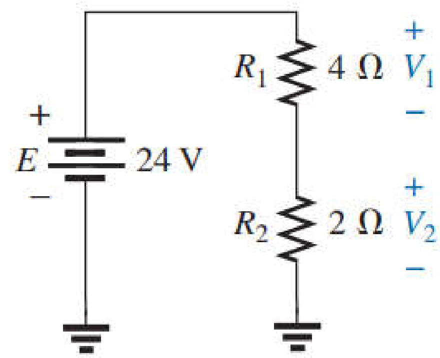
$$V_1 = I_1R_1 = I_sR_1 \qquad V_2 = I_2R_2 = I_sR_2$$
$$V_3 = I_3R_3 = I_sR_3 \;\; \ldots\ldots. \;\; V_n = I_nR_n = I_nR_2 \quad \text{(V)} \quad \textbf{(5.4)}$$

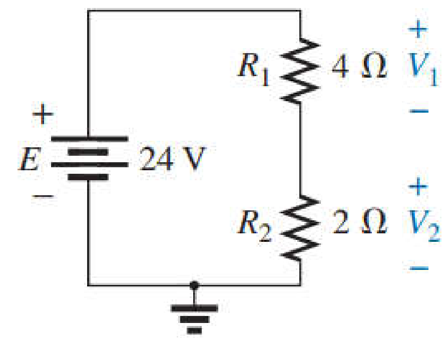
If $R_1 = R_2 = R_3 = \ldots\ldots = R_n = R$

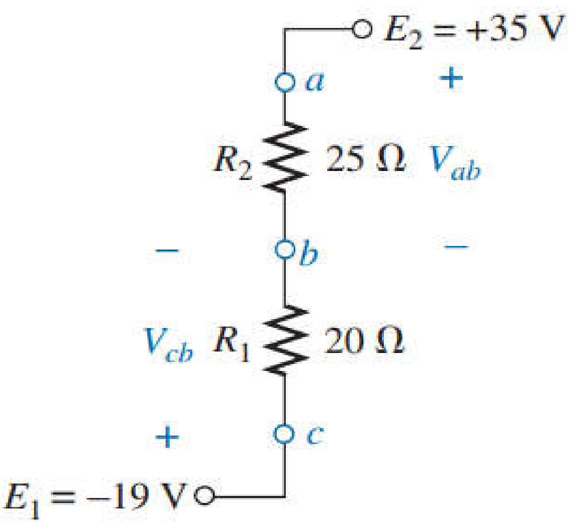
$$V_1 = V_2 = V_3 = \ldots. = V_n = \frac{E}{n}$$

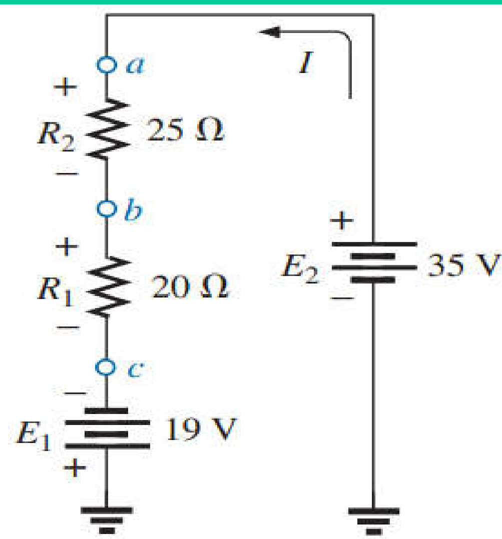
Different Ways to Sketch the Same Series Circuit

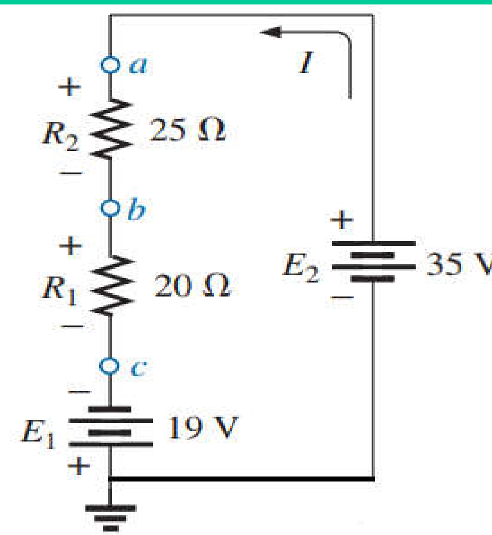












EXAMPLE 5.4 For the series circuit in Fig. 5.15:

a. Find the total resistance R_T.

b. Calculate the resulting source current I_s.

c. Determine the voltage across each resistor.

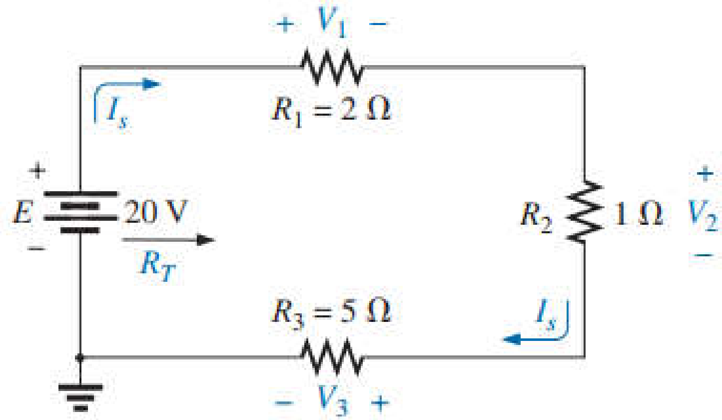


FIG. 5.15

Series circuit to be investigated in Example 5.4.

Solutions:

a. $R_T = R_1 + R_2 + R_3$

$= 2\ \Omega + 1\ \Omega + 5\ \Omega$

$R_T = \mathbf{8\ \Omega}$

b. $I_s = \dfrac{E}{R_T} = \dfrac{20\text{ V}}{8\ \Omega} = \mathbf{2.5\ A}$

c. $V_1 = I_1R_1 = I_sR_1 = (2.5\text{ A})(2\ \Omega) = \mathbf{5\ V}$

$V_2 = I_2R_2 = I_sR_2 = (2.5\text{ A})(1\ \Omega) = \mathbf{2.5\ V}$

$V_3 = I_3R_3 = I_sR_3 = (2.5\text{ A})(5\ \Omega) = \mathbf{12.5\ V}$

EXAMPLE 5.6 Given R_T and I_3, calculate R_1 and E for the circuit in Fig. 5.18.

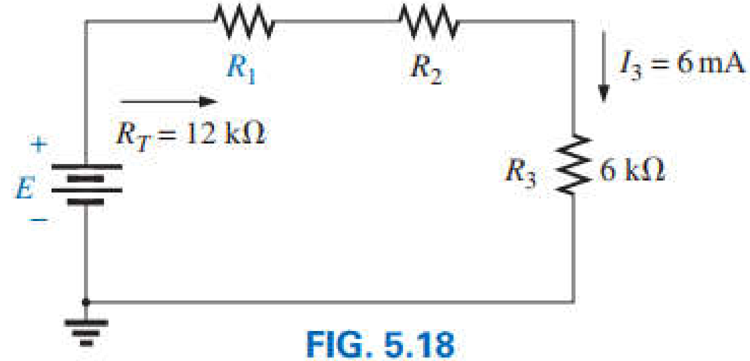


FIG. 5.18
Series circuit to be analyzed in Example 5.6.

Solution: Since we are given the total resistance, it seems natural to first write the equation for the total resistance and then insert what we know.

$$R_T = R_1 + R_2 + R_3$$

We find that there is only one unknown, and it can be determined with some simple mathematical manipulations. That is,

$$12\text{ k}\Omega = R_1 + 4\text{ k}\Omega + 6\text{ k}\Omega = R_1 + 10\text{ k}\Omega$$

and $\quad 12\text{ k}\Omega - 10\text{ k}\Omega = R_1$

so that $\quad R_1 = \mathbf{2\text{ k}\Omega}$

The dc voltage can be determined directly from Ohm's law.

$$E = I_s R_T = I_3 R_T = (6\text{ mA})(12\text{ k}\Omega) = \mathbf{72\text{ V}}$$

Practice Book Problem [5.3 Series Circuit] Problems: 7 to 11

5.7 Voltage Division in Series Circuit
Voltage Divider Rule (VDR)

Voltage drop across individual resistance in a series circuit : According ohm's law, the of the following circuit is as follows:

$$V_1 = I_s R_1 = \left(\frac{E}{R_T}\right) R_1 = \frac{R_1}{R_T} E \qquad \textbf{(5.7.1)}$$

$$V_2 = I_s R_2 = \left(\frac{E}{R_T}\right) R_2 = \frac{R_2}{R_T} E \qquad \textbf{(5.7.2)}$$

$$V_3 = I_s R_3 = \left(\frac{E}{R_T}\right) R_3 = \frac{R_3}{R_T} E \qquad \textbf{(5.7.3)}$$

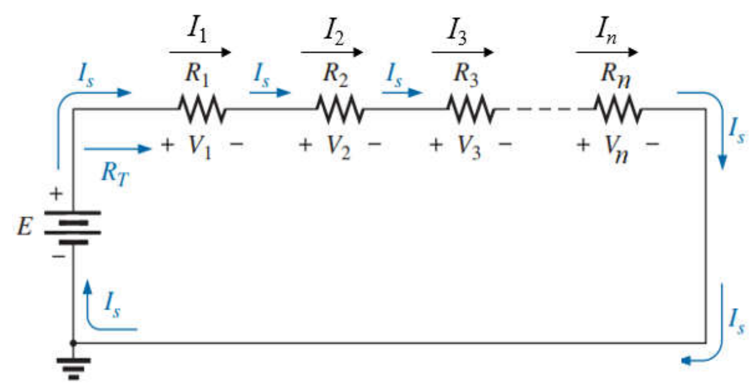
.

$$V_n = I_s R_n = \left(\frac{E}{R_T}\right) R_n = \frac{R_n}{R_T} E \qquad \textbf{(5.7.n)}$$

Based on the Eq. (5.7.1) to (5.7.n), a general equation can be written as follows:

$$\boxed{V_x = R_x \frac{E}{R_T} = \frac{R_x}{R_T} E \qquad \textbf{(5.10)}}$$

Voltage Divider Rule (VDR): The voltage across a resistor in a series circuit is equal to the value of that resistor (R_x) times the total applied voltage (E) divided by the total resistance (R_T) of the series configuration.



EXAMPLE 5.16 Using the voltage divider rule, determine voltages V_1, V_2 and V_3 for the series circuit in **Fig. 5.38**.

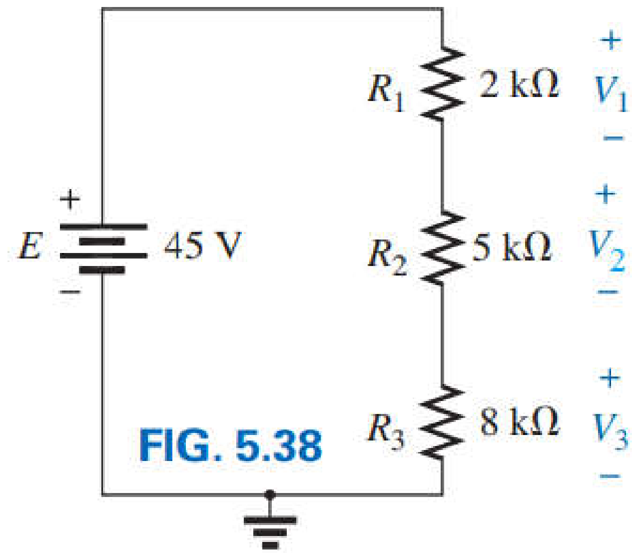


FIG. 5.38

Solution:

$$R_T = R_1 + R_2 + R_3$$
$$= 2\text{ k}\Omega + 5\text{ k}\Omega + 8\text{ k}\Omega$$
$$= 15\text{ k}\Omega$$

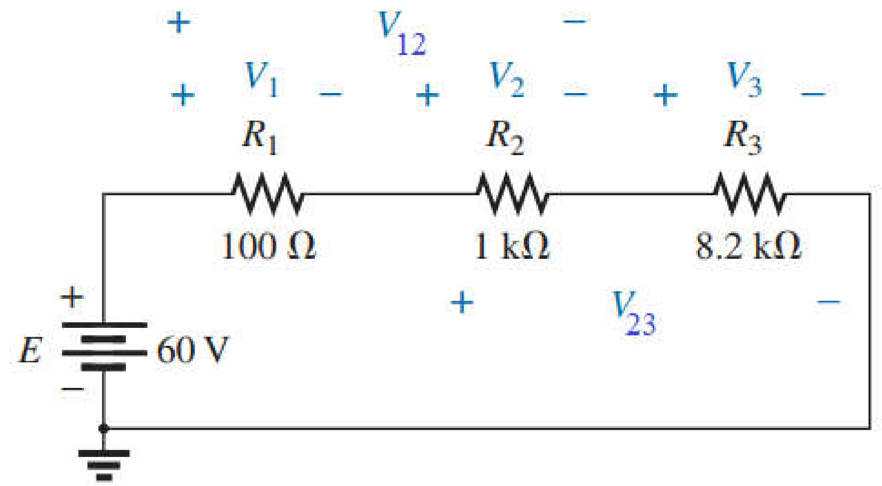
$$V_1 = R_1 \frac{E}{R_T} = 2\text{ k}\Omega\left(\frac{45\text{ V}}{15\text{ k}\Omega}\right) = \mathbf{6\ V}$$

$$V_2 = R_2 \frac{E}{R_T} = 5\text{ k}\Omega\left(\frac{45\text{ V}}{15\text{ k}\Omega}\right) = \mathbf{15\ V}$$

$$V_3 = R_3 \frac{E}{R_T} = 8\text{ k}\Omega\left(\frac{45\text{ V}}{15\text{ k}\Omega}\right) = \mathbf{24\ V}$$

Practice Book Problem [5.7 Voltage Divider Rule] Problems: 24, 25 and 26

EXAMPLE 5.7.1: Using the voltage divider rule, determine voltages V_1, V_2, V_3, V_{12} and V_{23} for the following series circuit.



Solution: $R_T = R_1 + R_2 + R_3$

$$= 100\ \Omega + 1\ \text{k}\Omega + 8.2\ \text{k}\Omega = 9.3\ \text{k}\Omega$$

$$V_1 = R_1 \frac{E}{R_T} = 100\ \Omega\left(\frac{60\ \text{V}}{9.3\ \text{k}\Omega}\right) = \mathbf{0.65\ V}$$

$$V_2 = R_2 \frac{E}{R_T} = 1\ \text{k}\Omega\left(\frac{60\ \text{V}}{9.3\ \text{k}\Omega}\right) = \mathbf{6.45\ V}$$

$$V_3 = R_3 \frac{E}{R_T} = 8.2\ \text{k}\Omega\left(\frac{60\ \text{V}}{9.3\ \text{k}\Omega}\right) = \mathbf{52.9\ V}$$

V_{12} is the voltage drop across the series combination of R_1 and R_2 and V_{23} is the voltage drop across the series combination of R_2 and R_3.

$$R_{12} = R_1 + R_2 = 100\ \Omega + 1\ \text{k}\Omega = 1.1\ \text{k}\Omega$$

$$R_{23} = R_2 + R_3 = 1\ \text{k}\Omega + 8.2\ \text{k}\Omega = 9.2\ \text{k}\Omega$$

$$V_{12} = R_{12} \frac{E}{R_T} = 1.1\ \text{k}\Omega\left(\frac{60\ \text{V}}{9.3\ \text{k}\Omega}\right) = \mathbf{7.1\ V}$$

$$V_{23} = R_{23} \frac{E}{R_T} = 9.2\ \text{k}\Omega\left(\frac{60\ \text{V}}{9.3\ \text{k}\Omega}\right) = \mathbf{59.35\ V}$$

5.6 KIRCHHOFF'S VOLTAGE LAW (KVL)

Statement:

(**1**) The algebraic sum of the potential rises and drops around a closed path (or closed loop) is zero.

OR

(**2**) The sum of the applied or supplied or rise voltage of a series dc circuit will equal the sum of the voltage drops of the circuit.

According to Statement (1):

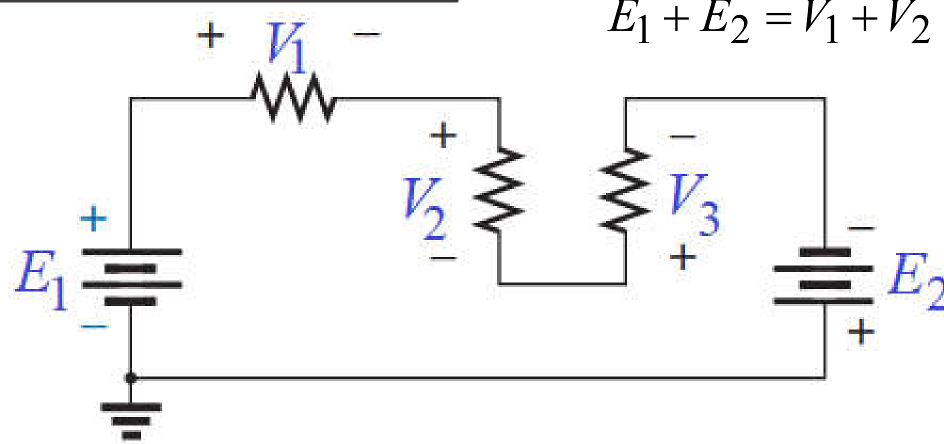
$$\boxed{\Sigma_{\circlearrowright} V = 0} \quad \text{(Kirchhoff's voltage law in symbolic form)} \quad \textbf{(5.8)}$$

$$E_1 - V_1 - V_2 - V_3 + E_2 = 0$$

According to Statement (2):

$$\boxed{\Sigma_{\circlearrowright} V_{\text{rises}} = \Sigma_{\circlearrowright} V_{\text{drops}}} \quad \textbf{(5.9)}$$

$$E_1 + E_2 = V_1 + V_2 + V_3$$



EXAMPLE 5.8 Use Kirchhoff's voltage law to determine the unknown voltage (V_1) for the circuit in Fig. 5.27.

Solution: Let considered current (I) make closed path by flowing in clockwise (CW) direction.

I is entering through **negative** terminal of E_1.

Now, let E_1 is **positive**.

I is entering through **positive** terminal of V_1.

So V_1 is **negative**.

I is entering through **positive** terminal of 4.2 V.

So 4.2 V is **negative**.

I is entering through **positive** terminal of E_2.

So E_2 is **negative**.

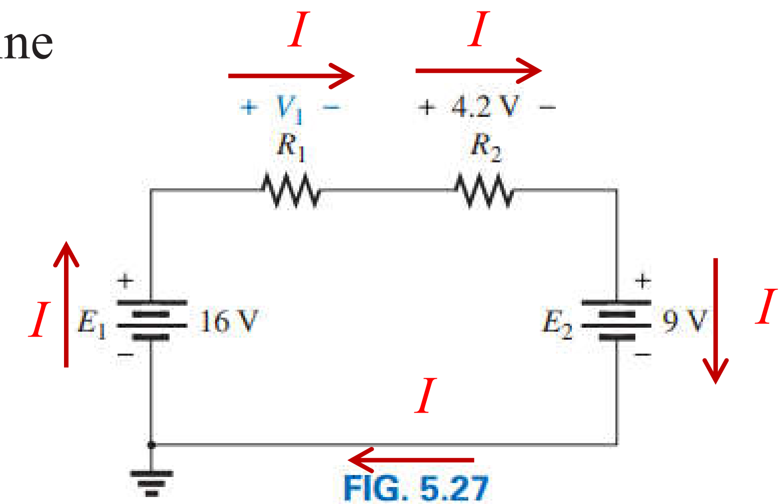


FIG. 5.27

Series circuit to be examined in Example 5.8.

According to KVL we have:

$$+E_1 - V_1 - 4.2\text{V} - E_2 = 0$$

$$V_1 = E_1 - 4.2\text{V} - E_2$$

$$V_1 = 16\text{ V} - 4.2\text{ V} - 9\text{ V}$$

$$= \mathbf{2.8\ V}$$

EXAMPLE 5.9 Determine the unknown voltage for the circuit in Fig. 5.28.

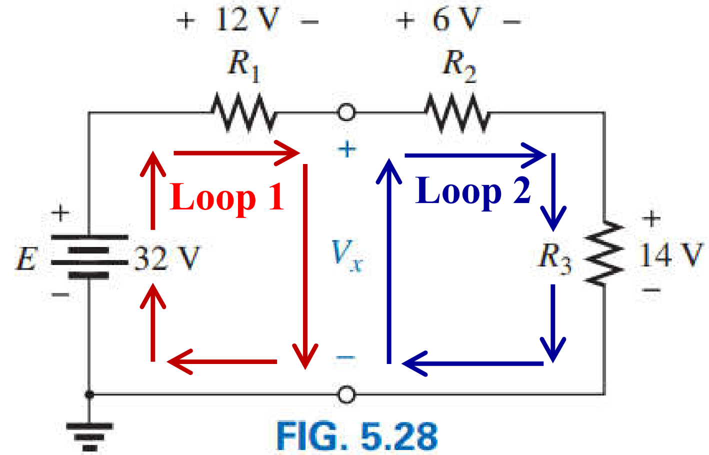


FIG. 5.28

Solution: This problem can be solved by two ways.

First way: Apply KVL around a path, including the source E and R_1 (as **Loop 1**).

$$+E_1 - 12\text{V} - V_x = 0$$

$$V_x = E_1 - 12\text{V} = 22\text{V} - 12\text{V} = \mathbf{20\ V}$$

Second way: Apply KVL around a path, including the R_2 and R_3 (as **Loop 2**).

$$+V_x - 6\text{V} - 14\text{V} = 0$$

$$V_x = 6\text{V} + 14\text{V} = \mathbf{20\ V}$$

EXAMPLE 5.6.1 Determine the unknown voltages for the following circuit.

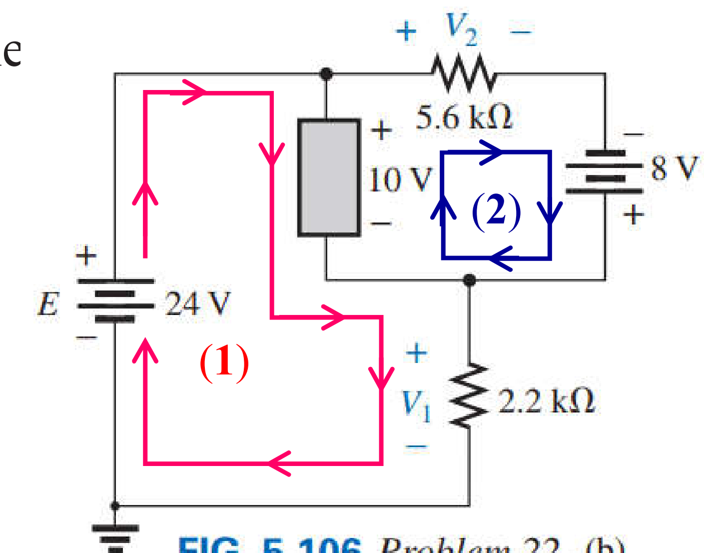


FIG. 5.106 *Problem 22.* (b)

Solution: Apply KVL in **Loop 1**:

$$+E_1 - 10\text{V} - V_1 = 0$$

$$V_1 = E_1 - 10\text{V} = 24\text{V} - 10\text{V} = \mathbf{14\ V}$$

Apply KVL in **Loop 2**:

$$+E_2 + 10\text{V} - V_2 = 0$$

$$V_1 = E_2 + 10\text{V} = 8\text{V} + 10\text{V} = \mathbf{18\ V}$$

Practice Book Problem [SECTION 5.6 Kirchhoff's Voltage Law] Problems: 20 to 23

5.4 Power Distribution in a Series Circuit

In any electrical system, the power supplied or applied or delivered will equal the power dissipated or absorbed or consumed.

$$\boxed{P_E = P_{R_1} + P_{R_2} + P_{R_3}} \quad \textbf{(5.5)}$$

$$\boxed{P_E = EI_s} \qquad \text{(watts, W)} \qquad \textbf{(5.6)}$$

$$\begin{aligned}
P_1 &= V_1 I_1 = I_1^2 R_1 = \frac{V_1^2}{R_1} = V_1 I_s = I_s^2 R_1 \\
P_2 &= V_2 I_2 = I_2^2 R_2 = \frac{V_2^2}{R_2} = V_2 I_s = I_s^2 R_2 \\
P_3 &= V_3 I_3 = I_3^2 R_3 = \frac{V_3^2}{R_3} = V_3 I_s = I_s^2 R_3
\end{aligned} \qquad \text{(W)} \quad \textbf{(5.7)}$$

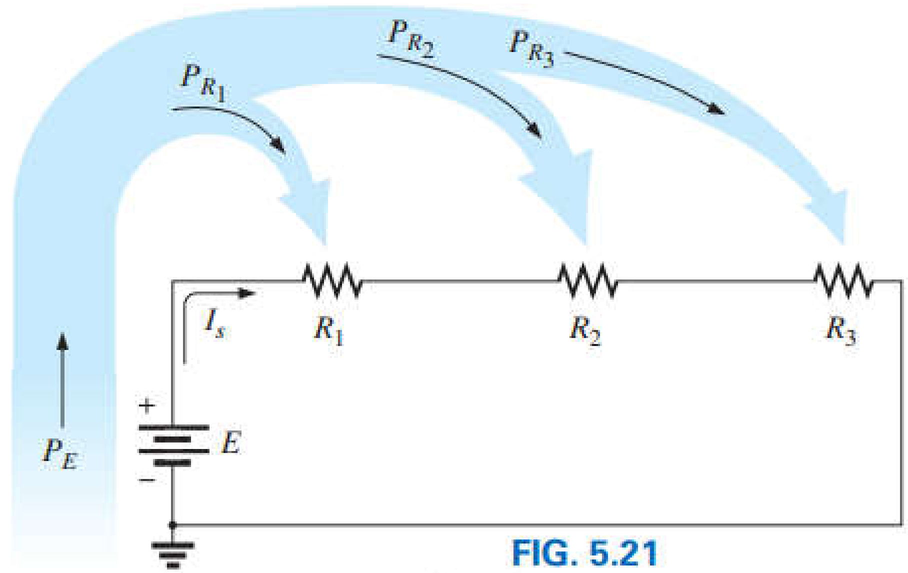


FIG. 5.21
Power distribution in a series circuit.

EXAMPLE 5.7 For the series circuit in Fig. 5.22 (all standard values):

a. Determine the total resistance R_T.

b. Calculate the current I_s.

c. Determine the voltage across each resistor.

d. Find the power supplied by the battery.

e. Verify Kirchhoff's Voltage Law (KVL).

f. Determine the power dissipated by each resistor.

g. Comment on whether the total power supplied equals the total power dissipated.

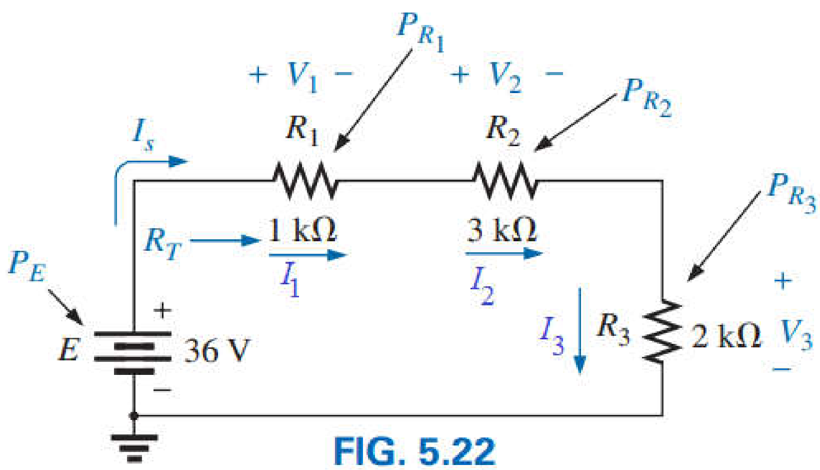


FIG. 5.22

Series circuit to be investigated in Example 5.7.

Solution:

(*a*) $R_T = R_1 + R_2 + R_3 = 1\text{ k}\Omega + 3\text{ k}\Omega + 2\text{ k}\Omega = \mathbf{6\text{ k}\Omega}$

(**b**) $I_s = I_1 = I_2 = I_3 = \dfrac{E}{R_T} = \dfrac{36\text{ V}}{6\text{ k}\Omega} = \mathbf{6\text{ mA}}$

(**c**) $V_1 = R_1\dfrac{E}{R_T} = I_1R_1 = I_sR_1 = (6\text{ mA})(1\text{ k}\Omega) = \mathbf{6\text{ V}}$

$V_2 = R_2\dfrac{E}{R_T} = I_2R_2 = I_sR_2 = (6\text{ mA})(3\text{ k}\Omega) = \mathbf{18\text{ V}}$

$V_3 = R_3\dfrac{E}{R_T} = I_3R_3 = I_sR_3 = (6\text{ mA})(2\text{ k}\Omega) = \mathbf{12\text{ V}}$

(**d**) $V_1 + V_2 + V_3 = 6\text{ V} + 18\text{ V} + 12\text{ V} = \mathbf{36\text{ V}} = E$ **[Verified]**

(**e**) $P_E = I_s^2R_T = \dfrac{E^2}{R_T} = EI_s = (36\text{ V})(6\text{ mA}) = \mathbf{216\text{ mW}}$

EXAMPLE 5.7 For the series circuit in Fig. 5.22 (all standard values):

f. Determine the power dissipated by each resistor.

g. Comment on whether the total power supplied equals the total power dissipated.

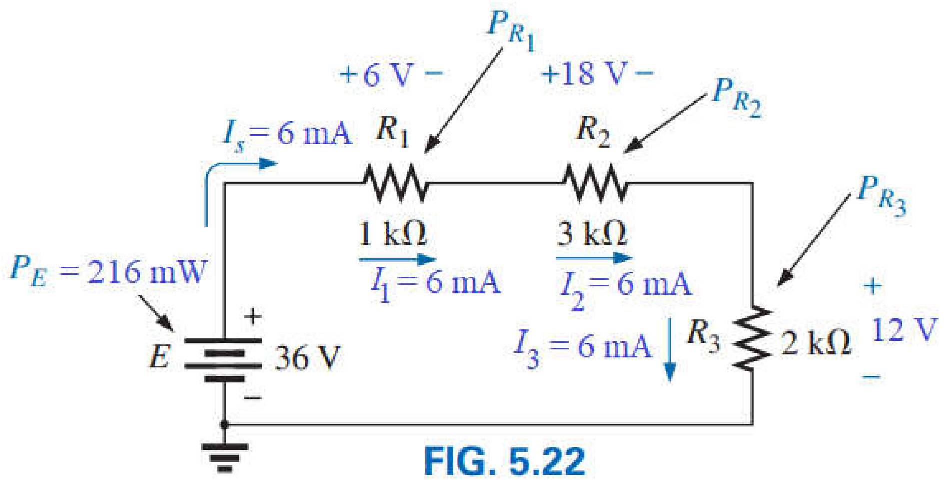


FIG. 5.22
Series circuit to be investigated in Example 5.7.

Solution:

(e) $P_{R1} = I_1^2 R_1 = I_s^2 R_1 = V_1 I_1 = \dfrac{V_1^2}{R_1} = V_1 I_s = (6\text{ V})(6\text{ mA}) = \mathbf{36\text{ mW}}$

$P_{R2} = I_s^2 R_2 = V_2 I_2 = V_2 I_s = \dfrac{V_2^2}{R_2} = I_2^2 R_2 = (6\text{ mA})^2 (3\text{ k}\Omega) = \mathbf{108\text{ mW}}$

$P_{R3} = I_3^2 R_3 = I_s^2 R_3 = V_3 I_3 = V_3 I_s = \dfrac{V_3^2}{R_3} = \dfrac{(12\text{ V})^2}{2\text{ k}\Omega} = \mathbf{72\text{ mW}}$

(f)
$$\begin{aligned} P_E &= P_{R1} + R_{R2} + R_{R3} \\ &= 36\text{ mW} + 108\text{ mW} + 72\text{ mW} \\ &= \mathbf{216\text{ mW}}\ \ \text{(Checked)} \end{aligned}$$

Practice Book Problem [5.4 Power Distribution] Problems: 12 to 16

Single-Subscript Notation of Voltage

A single-subscript notation of voltage used to provide the voltage at a point with respect to ground.

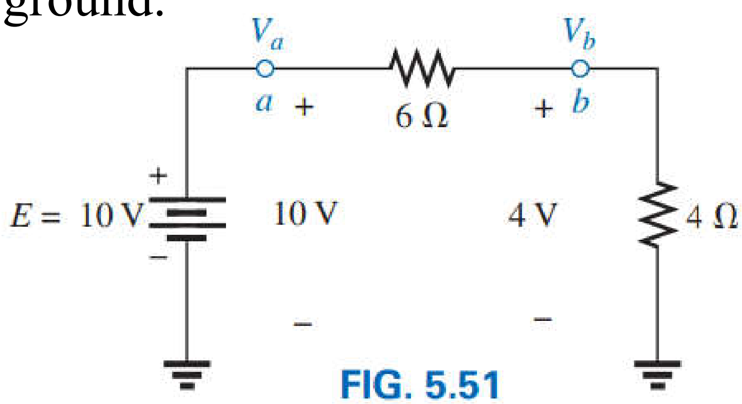


FIG. 5.51
Defining the use of single-subscript notation for voltage levels.

In Fig. 5.51:

V_a is the voltage from point ***a*** to ***ground***. In this case, it is obviously 10 V since it is right across the source voltage E. i.e. $V_a = E = 10$ V.

V_b is the voltage from point ***b*** to ***ground***. Because it is directly across the 4 Ω resistor, $V_b = 4$ V.

Here, $V_{ab} = V_a - V_b = 6$ V $\quad V_{ba} = V_b - V_a = -6$ V

EXAMPLE 5.25 Determine V_{ab}, V_{cb}, and V_c for the network in Fig. 5.59.

Solution:

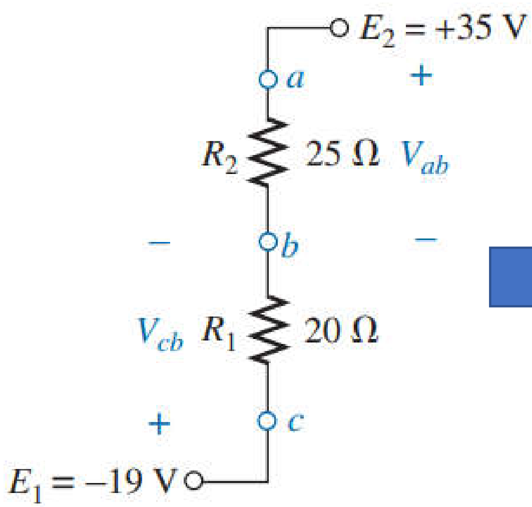
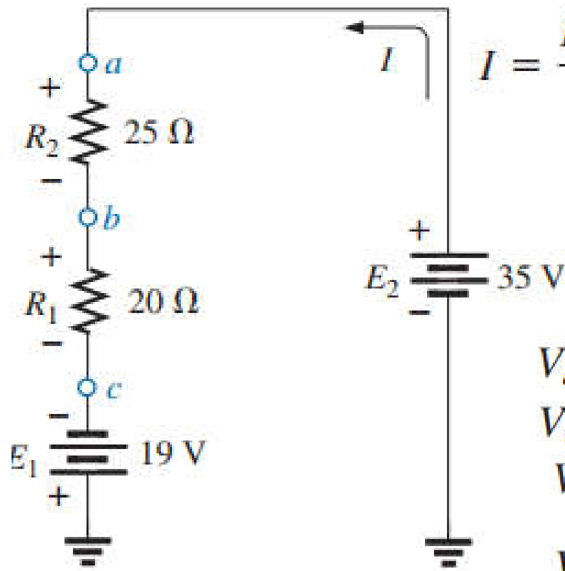


FIG. 5.59 *Example 5.25.*

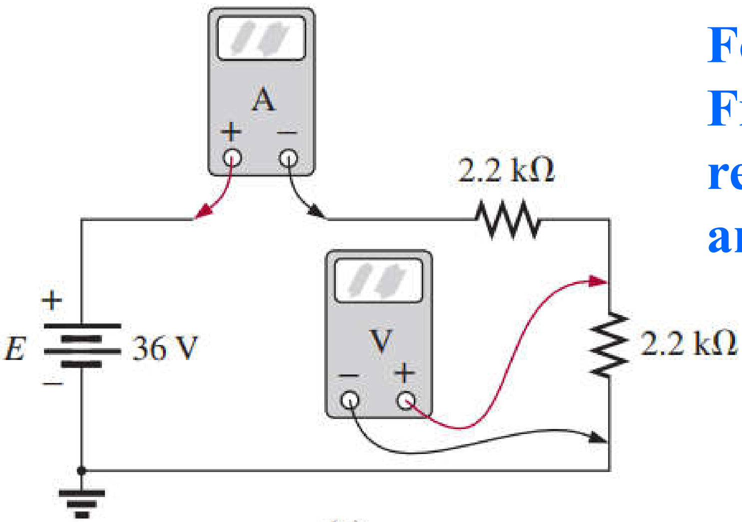


$$I = \frac{E_1 + E_2}{R_T} = \frac{19\text{ V} + 35\text{ V}}{45\ \Omega} = \frac{54\text{ V}}{45\ \Omega} = 1.2\text{ A}$$

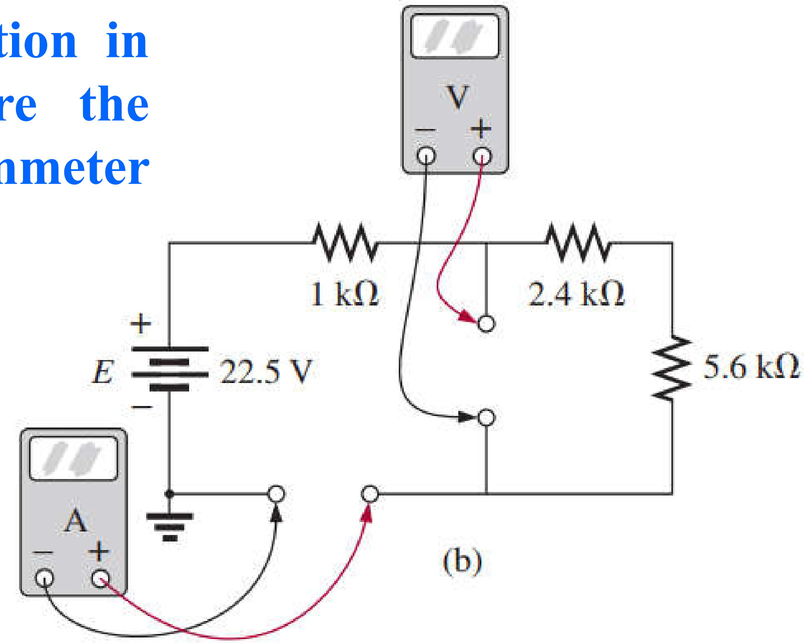
$$V_{ab} = IR_2 = (1.2\text{ A})(25\ \Omega) = \mathbf{30\ V}$$
$$V_{cb} = -IR_1 = -(1.2\text{ A})(20\ \Omega) = \mathbf{-24\ V}$$
$$V_c = E_1 = \mathbf{-19\ V}$$

$V_{ab} = \mathbf{30\ V}$ $\quad V_{cb} = \mathbf{-24\ V}$ $\quad V_c = \mathbf{-19\ V}$

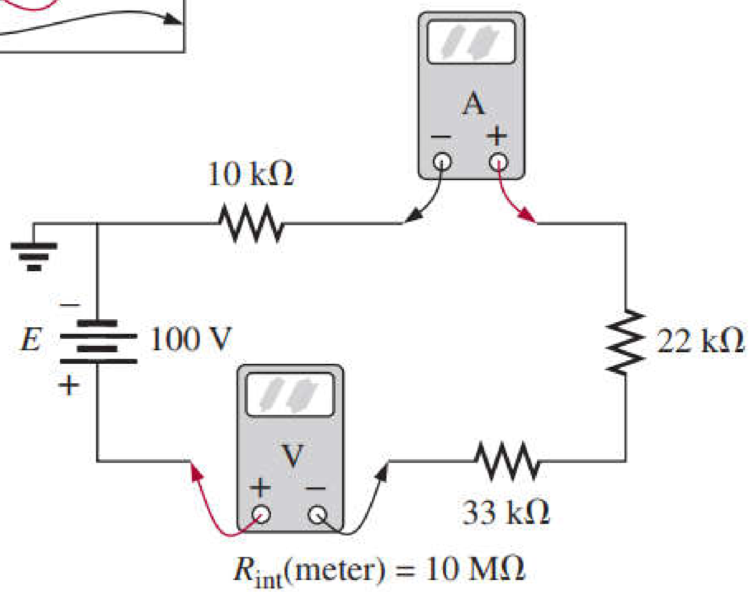
For each configuration in Fig. 5.95, what are the readings of the ammeter and the voltmeter?



(a)



(b)



(c)

VOLTAGE SOURCES IN SERIES

Voltage sources can be connected in series.

The voltage sources to be connected in series must have same current ratings through their voltage rating may be same or different.

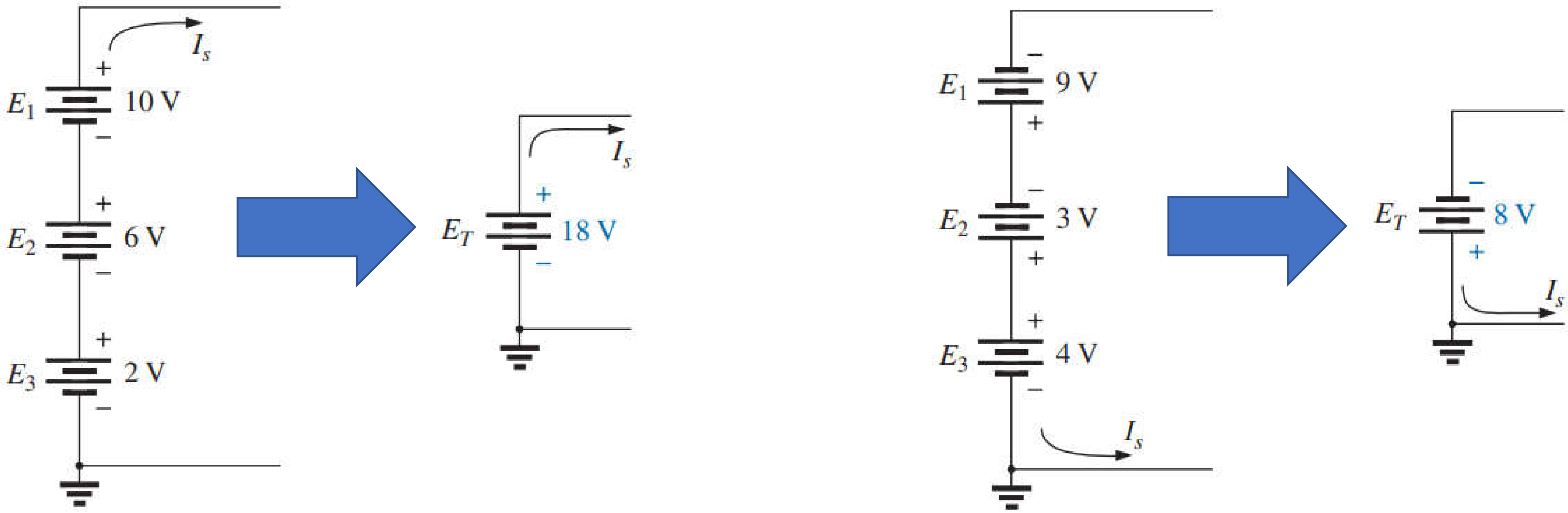
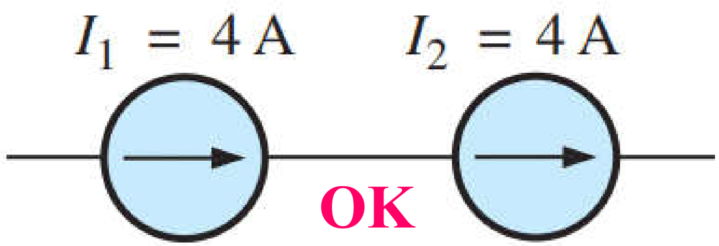


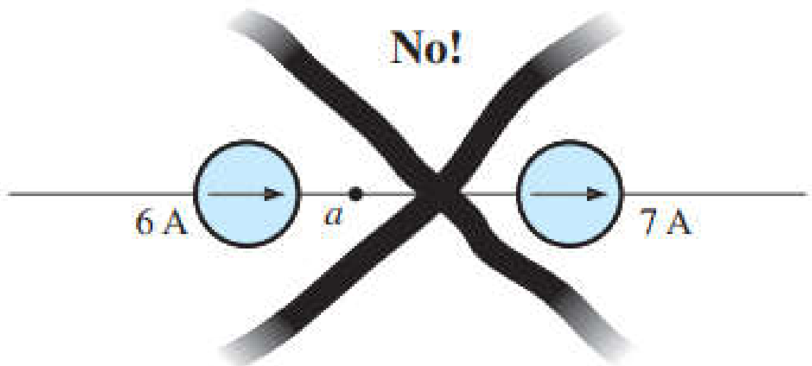
Fig. 5.3 Reducing Series Voltage Sources to a Single Source

CURRENT SOURCES IN SERIES

Current sources to be connected in series must have same current ratings through their voltage ratings may be same or different.



Current sources of different current ratings should not be connected in series.



Chapter 6

Parallel DC Circuit

Two Elements are in Parallel

Two **elements**, **branches**, or **circuits** are in parallel:

1. They have **two points in common**.
2. The **currents are not the same** through the two parallel elements.
3. The **voltage are the same across** the two parallel elements.

Figure 1: R_1 and R_2 are parallel since they have two common points a and b.

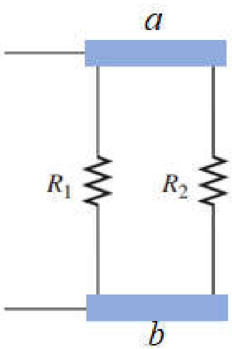


Figure 1

Figure 2: R_1 and R_2 are parallel since they have two common points a and b. R_3 is series with the parallel combination of R_1 and R_2.

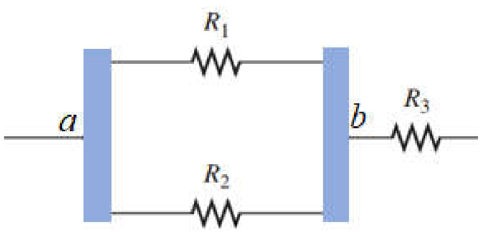


Figure 2

Figure 3: R_1 and R_2 are in series. The series combination of R_1 and R_2 are parallel with R_3

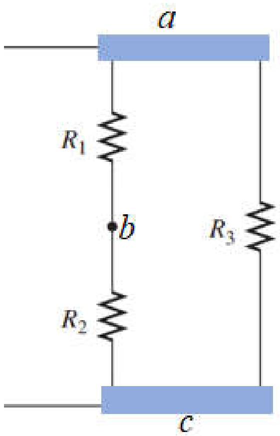
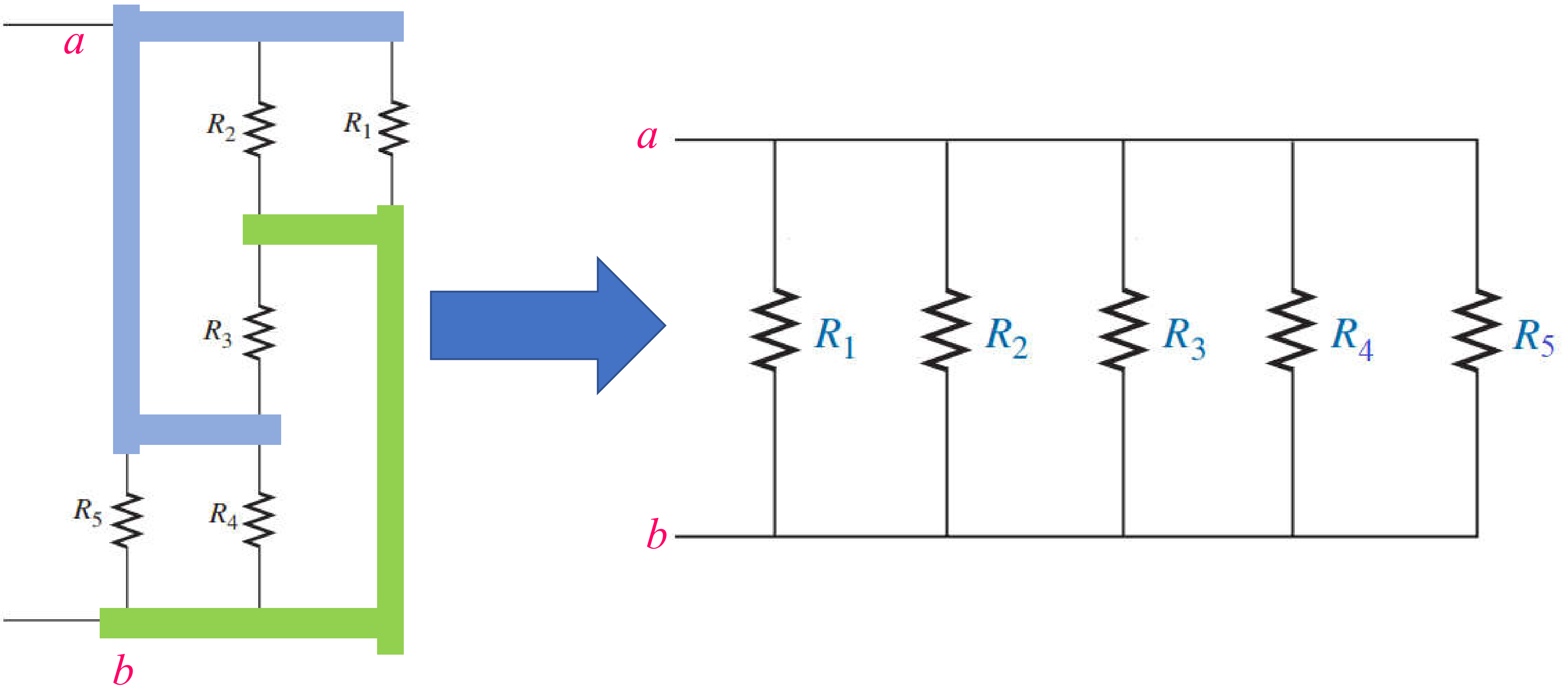


Figure 3

Identify the Combination of Connection for the Following Circuit



6.2 Parallel Resistance

- The **total or net or effective or equivalent conductance** of a parallel configuration is **the sum of the value of individual conductance**, that is Eq. (**6.2**).
- The reciprocal of the total or effective or net or equivalent resistance of a parallel circuit is equal to the sum of the reciprocal of the individual resistances, that is Eq. (**6.1**). Eq. (**6.3**) can be obtained from Eq. (6.1).

$$\boxed{G_T = G_1 + G_2 + G_3 + \cdots + G_N} \quad \text{(siemens, S)} \quad \textbf{(6.2)}$$

$$\boxed{\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_N}} \quad \textbf{(6.1)}$$

$$R_T = \frac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3} + \cdots + \dfrac{1}{R_N}} \quad \textbf{(6.3)}$$

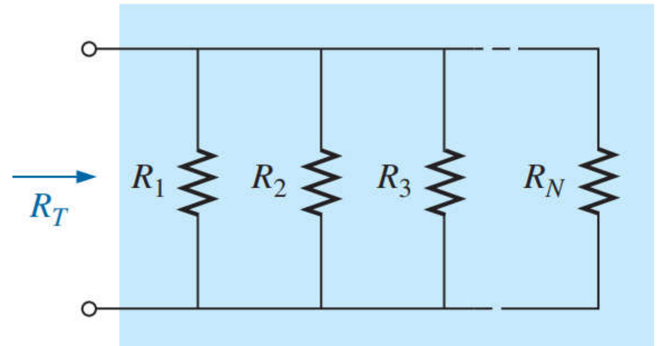


FIG. 6.3 *Parallel combination of resistors.*

- The total or equivalent resistance is the smallest of all the individual resistances. that is Eq. **(6.1.1)**.
- The total resistance of n resistors of the same value in parallel is simply ***divide the value of one of the resistors by the number of resistor in parallel***; that is as Eq. (**6.4**)

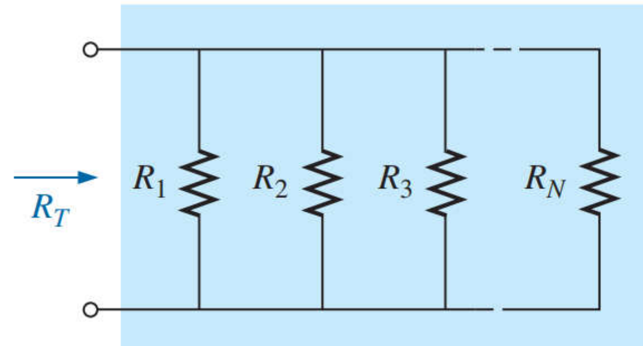


FIG. 6.3 *Parallel combination of resistors.*

$$\boxed{R_T < R_1;\ \ R_T < R_2; \ldots\ldots, R_T < R_N} \quad \textbf{(6.1.1)}$$

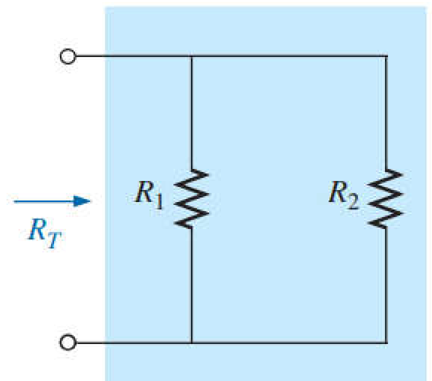
$$\text{If } R_1 = R_2 = R_3 = \ldots\ldots = R_N = R \qquad R_N = \frac{R}{N} \quad \textbf{(6.4)}$$

Special Case: Two Parallel Resistors

The total resistance of two parallel resistors is simply the product of their values divided by their sum.

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_1 + R_2}{R_1 R_2}$$

$$\therefore\ R_T = \frac{R_1 R_2}{R_1 + R_2} \quad \textbf{(6.5)}$$



EXAMPLE 6.1

a. Find the total conductance of the parallel network in Fig. 6.4.

b. Find the total resistance of the same network using (***i***) the results of part (a), (***ii***) using Eq. (6.3) and (***iii***) using Eq. (6.5).

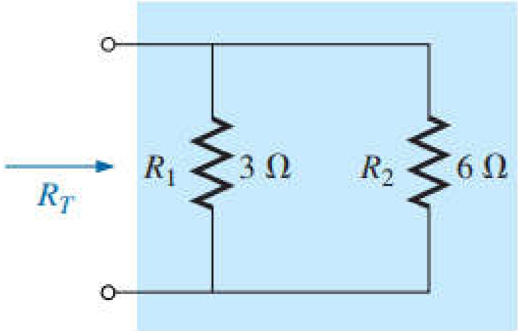


FIG. 6.4

Solution: ***a***. $G_1 = \frac{1}{R_1} = \frac{1}{3\Omega} = 0.333\text{ S},\qquad G_2 = \frac{1}{R_2} = \frac{1}{6\Omega} = 0.167\text{ S}$

$$G_T = G_1 + G_2 = 0.333\text{ S} + 0.167\text{ S} = \mathbf{0.5\ S}$$

$$\boldsymbol{b}.\ (\boldsymbol{i})\ R_T = \frac{1}{G_T} = \frac{1}{0.5\text{ S}} = \mathbf{2\ \Omega}$$

(***ii***) Applying Eq. (6.3):

$$R_T = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{1}{\frac{1}{3\Omega} + \frac{1}{6\Omega}}$$

$$= \frac{1}{0.333\text{ S} + 0.167\text{ S}} = \frac{1}{0.5\text{ S}} = \mathbf{2\ \Omega}$$

(***iii***) Applying Eq. (6.5):

$$R_T = \frac{R_1 R_2}{R_1 + R_2} = \frac{(3\Omega)(6\Omega)}{3\Omega + 6\Omega} = \frac{18\Omega}{9\Omega} = \mathbf{2\ \Omega}$$

EXAMPLE 6.3 Find the total resistance of the configuration in Fig. 6.7.

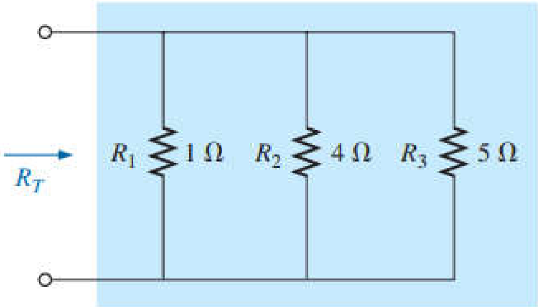


FIG. 6.7

Solution: Method 1

$$G_1 = \frac{1}{R_1} = \frac{1}{1\Omega} = 1.0\text{ S},\quad G_2 = \frac{1}{R_2} = \frac{1}{4\Omega} = 0.25\text{ S},\quad G_3 = \frac{1}{R_3} = \frac{1}{5\Omega} = 0.2\text{ S}$$

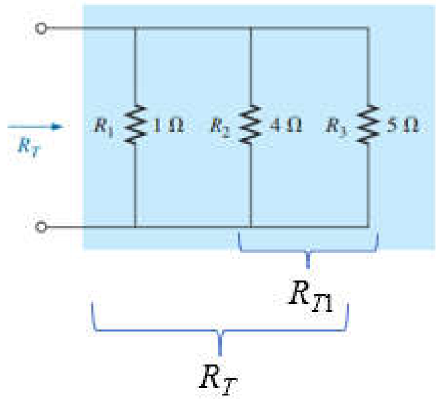
$$G_T = G_1 + G_2 + G_3 = 1.0\text{ S} + 0.25\text{ S} + 0.2\text{ S} = 1.45\text{ S} \qquad R_T = \frac{1}{G_T} = \frac{1}{1.45\text{ S}} = \mathbf{0.69\ \Omega}$$

Method 2: Applying Eq. (6.3):

$$R_T = \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}} = \frac{1}{\frac{1}{1\Omega}+\frac{1}{4\Omega}+\frac{1}{5\Omega}} = \frac{1}{1.0\text{ S}+0.25\text{ S}+0.2\text{ S}} = \frac{1}{1.45\text{ S}} \approx \mathbf{0.69\ \Omega}$$

Method 2: Applying Eq. (6.5):

$$R_{T1} = \frac{R_2 R_3}{R_2 + R_3} = \frac{(4\ \Omega)(5\ \Omega)}{4\ \Omega + 5\ \Omega} \cong 2.22\ \Omega$$



$$R_T = \frac{R_1 R_{T1}}{R_1 + R_{T1}} = \frac{(1\ \Omega)(2.22\ \Omega)}{1\ \Omega + 2.22\ \Omega} \cong \mathbf{0.69\ \Omega}$$

EXAMPLE 6.5 Find the total resistance of the parallel resistors in (***i***) Fig. 6.9 and (***ii***) Fig. 6.11.

Solution: In both figures the individual resistance value are equal.

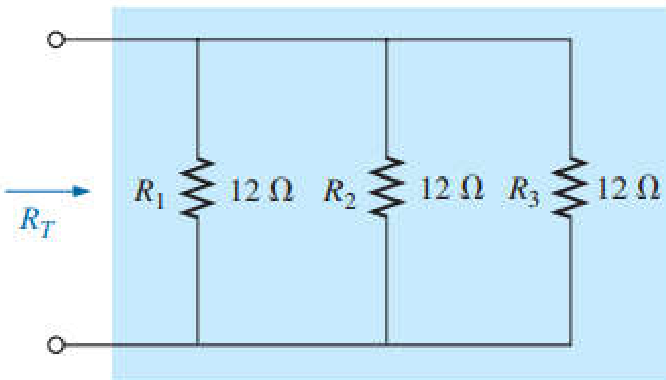


FIG. 6.9

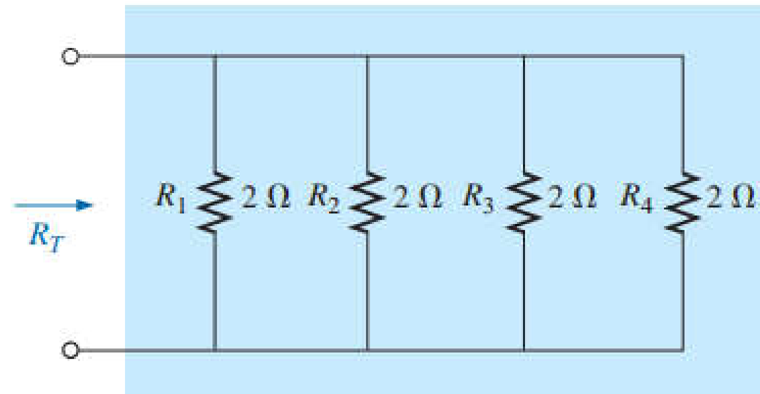


FIG. 6.11

For Fig. 6.9:

$$R_1 = R_2 = R_3 = R = 12\ \Omega$$

Total resistance: Applying Eq. (6.4):

$$R_T = \frac{R}{N} = \frac{12\ \Omega}{3} = \mathbf{4\ \Omega}$$

For Fig. 6.11:

$$R_1 = R_2 = R_3 = R_4 = R = 2\ \Omega$$

Total resistance: Applying Eq. (6.3):

$$R_T = \frac{R}{N} = \frac{2\ \Omega}{4} = \mathbf{0.5\ \Omega}$$

Problem 9 [P. 235] Determine R_1 for the network in Fig. 6.80

Practice Example 6.10 and 6.11

Solution: Here, all resistance are connected in parallel.

Let, $R_2 = 24\ \Omega$, $R_3 = 24\ \Omega$, and $R_4 = 120\ \Omega$

We know that, $$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}$$

$$\frac{1}{R_1} = \frac{1}{R_T} - \left[\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}\right]$$

$$= \frac{1}{10\ \Omega} - \left[\frac{1}{24\ \Omega} + \frac{1}{24\ \Omega} + \frac{1}{120\ \Omega}\right]$$

$$= 0.1\ \text{S} + 0.042\ \text{S} + 0.042\ \text{S} + 0.008\ \text{S} = 0.192\ \text{S}$$

$$R_1 = \frac{1}{0.192\ \text{S}} = \mathbf{5.21\ \Omega}$$

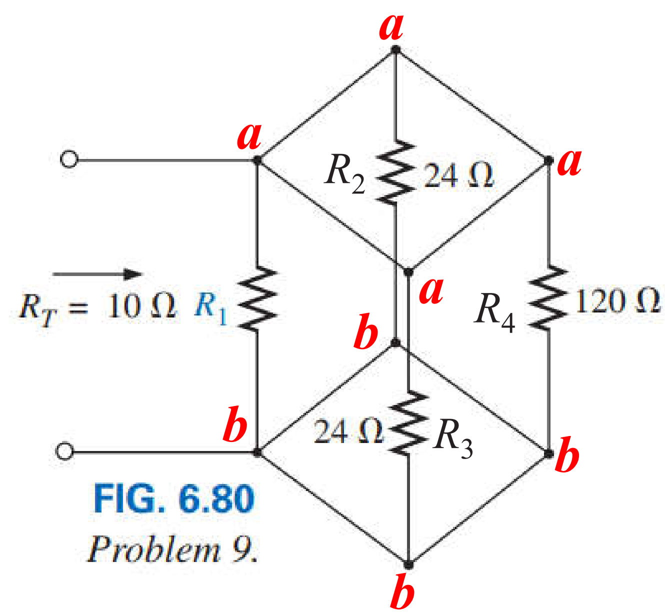
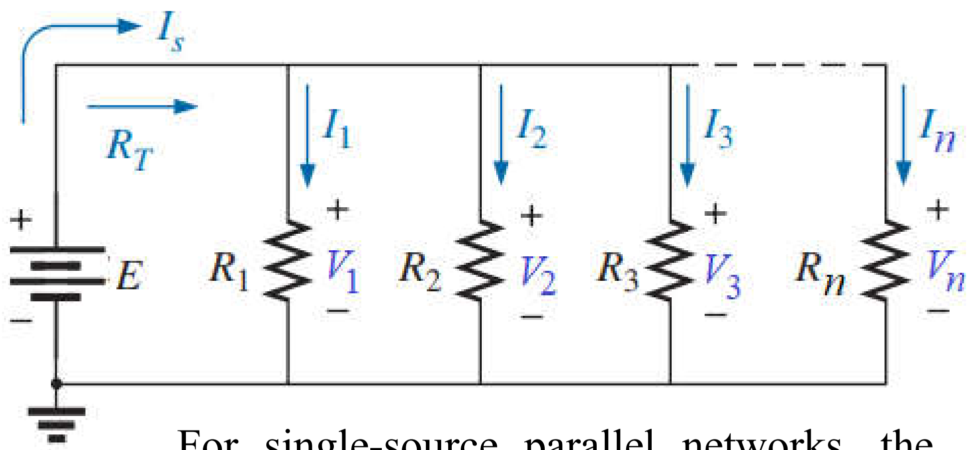


FIG. 6.80
Problem 9.

Practice Book Problem [SECTION 6.2 Parallel Resistors] Problems: 1, 3, 4, 5, 7 and 8

6.3 Parallel Circuit

A **parallel circuit** is one in which several resistances are connected across one another in such a way that one terminal of each is connected to from a junction point while the remaining ends are also joined to form another junction point.



Voltage in a parallel circuit: the voltage is always the same across parallel elements.

$$V_1 = V_2 = V_3 = \ldots\ldots = V_n = E \qquad \textbf{(6.6)}$$

Current Calculation of a Parallel circuit: According ohm's law, the current of the following circuit is as follows:

$$I_s = \frac{E}{R_T} \qquad \textbf{(6.7)}$$

$$I_1 = \frac{V_1}{R_1} = \frac{E}{R_1}; \quad I_2 = \frac{V_2}{R_2} = \frac{E}{R_2}; \quad I_n = \frac{V_n}{R_n} = \frac{E}{R_n} \qquad \textbf{(6.8)}$$

For single-source parallel networks, the source current (I_s) is always equal to the sum of the individual branch currents.

$$I_s = I_1 + I_2 + I_3 \ldots . I_n \qquad \textbf{(6.9)}$$

If $R_1 = R_2 = R_3 = \ldots\ldots = R_n = R$

$$I_1 = I_2 = I_3 = \ldots . = I_n = \frac{I_s}{n}$$

EXAMPLE 6.13 For the parallel network in Fig. 6.23.

a. Find the total resistance.

b. Calculate the source current.

c. Determine the current through each branch.

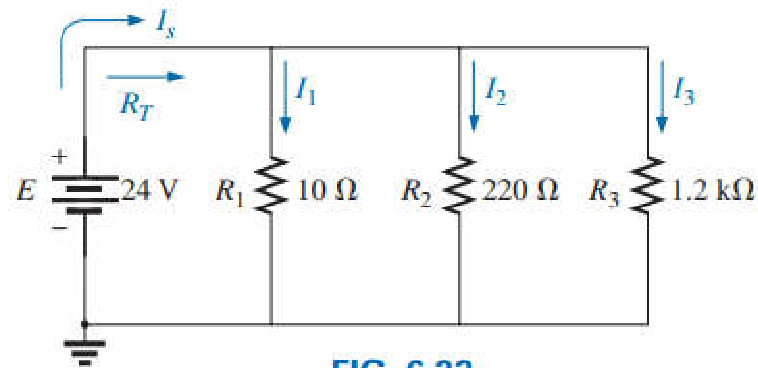


FIG. 6.23
Parallel network for Example 6.13.

Solution:

a. Applying Eq. (6.3):

$$R_T = \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}} = \frac{1}{\frac{1}{10\ \Omega}+\frac{1}{220\ \Omega}+\frac{1}{1.2\ \text{k}\Omega}}$$

$$= \frac{1}{100\times10^{-3}+4.545\times10^{-3}+0.833\times10^{-3}} = \frac{1}{105.38\times10^{-3}}$$

$$R_T = \mathbf{9.49\ \Omega}$$

b. Using Ohm's law:

$$I_s = \frac{E}{R_T} = \frac{24\ \text{V}}{9.49\ \Omega} = \mathbf{2.53\ A}$$

c. Applying Ohm's law:

$$I_1 = \frac{V_1}{R_1} = \frac{E}{R_1} = \frac{24\ \text{V}}{10\ \Omega} = \mathbf{2.4\ A}$$

$$I_2 = \frac{V_2}{R_2} = \frac{E}{R_2} = \frac{24\ \text{V}}{220\ \Omega} = \mathbf{0.11\ A}$$

$$I_3 = \frac{V_3}{R_3} = \frac{E}{R_3} = \frac{24\ \text{V}}{1.2\ \text{k}\Omega} = \mathbf{0.02\ A}$$

EXAMPLE 6.14 Given the information provided in Fig. 6.24.

a. Determine R_3.
b. Find the applied voltage E.
c. Find the source current I_s.
d. Find I_2.

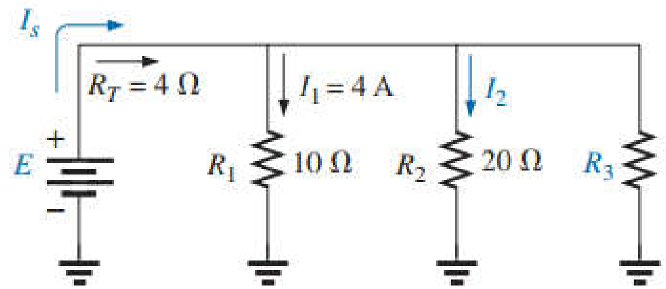


FIG. 6.24
Parallel network for Example 6.14.

Solutions:

a. Applying Eq. (6.1): $\dfrac{1}{R_T} = \dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3}$

Substituting: $\dfrac{1}{4\ \Omega} = \dfrac{1}{10\ \Omega} + \dfrac{1}{20\ \Omega} + \dfrac{1}{R_3}$

so that $0.25\ \text{S} = 0.1\ \text{S} + 0.05\ \text{S} + \dfrac{1}{R_3}$

and $0.25\ \text{S} = 0.15\ \text{S} + \dfrac{1}{R_3}$

with $\dfrac{1}{R_3} = 0.1\ \text{S}$

and $R_3 = \dfrac{1}{0.1\ \text{S}} = \mathbf{10\ \Omega}$

b. Using Ohm's law:

$$E = V_1 = I_1R_1 = (4\ \text{A})(10\ \Omega) = \mathbf{40\ V}$$

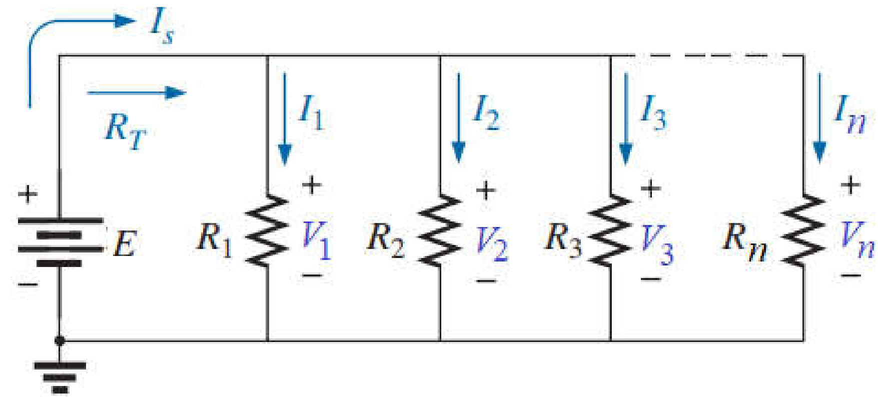
c. $$I_s = \frac{E}{R_T} = \frac{40\ \text{V}}{4\ \Omega} = \mathbf{10\ A}$$

d. Applying Ohm's law:

$$I_2 = \frac{V_2}{R_2} = \frac{E}{R_2} = \frac{40\ \text{V}}{20\ \Omega} = \mathbf{2\ A}$$

Practice Book Problem [SECTION 6.3 Parallel Circuits] Problems: 10 to 17

6.6 Current Division in Parallel Circuit
Current Divider Rule (CDR)



Current pass through individual resistance in a series circuit : According ohm's law, the of the following circuit is as follows:

$$I_1 = \frac{V_1}{R_1} = \frac{E}{R_1} = \frac{R_T}{R_1} I_s = \frac{G_1}{G_T} I_s \qquad (6.14.1)$$

$$I_2 = \frac{V_2}{R_2} = \frac{E}{R_2} = \frac{R_T}{R_2} I_s = \frac{G_2}{G_T} I_s \qquad (6.14.2)$$

. . .

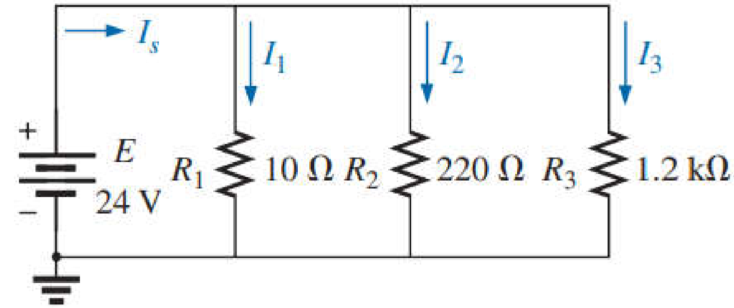
$$I_n = \frac{V_n}{R_n} = \frac{E}{R_n} = \frac{R_T}{R_n} I_s = \frac{G_n}{G_T} I_s \qquad (6.14.n)$$

Based on the Eq. (6.14.1) to (16.14.n), a general equation can be written as follows:

$$\boxed{I_x = \frac{R_T}{R_x} I_s = \frac{G_x}{G_T} I_s \qquad \mathbf{(6.8)}}$$

Current Divider Rule (CDR): The current through any branch of a parallel resistive network is equal to the total resistance of the parallel network divided by the resistor of interest and multiplied by the total current entering the parallel configuration.

EXAMPLE 6.6.1 For the following parallel network as shown in following figure. Using current divider rule, determine the current through each branch.



Solution:

$$\frac{1}{R_T}=\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}=\frac{1}{10\Omega}+\frac{1}{220\Omega}+\frac{1}{1.2\times10^3\,\Omega}$$

$$=105.38\times10^{-3}\,\mathrm{S}$$

$$R_T=\frac{1}{105.38\times10^{-3}\,\mathrm{S}}=9.49\ \Omega$$

$$I_s=\frac{E}{R_T}=\frac{24\ \mathrm{V}}{9.49\ \Omega}=2.53\ \mathrm{A}$$

Applying current divider rule (CDR) Eq. (6.8):

$$I_1=\frac{R_T}{R_1}I_s=\frac{9.49\ \Omega}{10\ \Omega}\times2.53\ \mathrm{A}=\mathbf{2.4\ A}$$

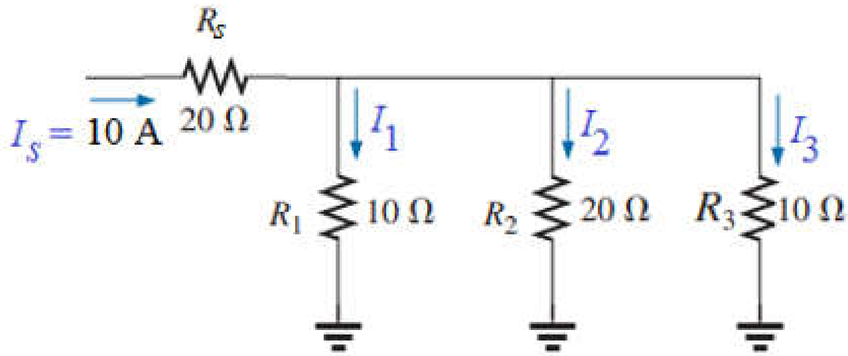
$$I_2=\frac{R_T}{R_2}I_s=\frac{9.49\ \Omega}{220\ \Omega}\times2.53\ \mathrm{A}=\mathbf{0.11\ A}$$

$$I_3=\frac{R_T}{R_3}I_s=\frac{9.49\ \Omega}{1.2\times10^3\ \Omega}\times2.53\ \mathrm{A}=\mathbf{0.02\ A}$$

EXAMPLE 6.6.2 For the following parallel network as shown in following figure. Using current divider rule, determine the current through each branch.

Solution:

$$R_T = \frac{1}{\dfrac{1}{R_1}+\dfrac{1}{R_2}+\dfrac{1}{R_3}} = \frac{1}{\dfrac{1}{10\Omega}+\dfrac{1}{20\Omega}+\dfrac{1}{10\Omega}} = 4\ \Omega$$



Applying current divider rule CDR) Eq. (6.8):

$$I_1 = \frac{R_T}{R_1} I_s = \frac{4\ \Omega}{10\ \Omega}\times 10\ \text{A} = \mathbf{4\ A}$$

$$I_2 = \frac{R_T}{R_2} I_s = \frac{4\ \Omega}{20\ \Omega}\times 10\ \text{A} = \mathbf{2\ A}$$

$$I_3 = \frac{R_T}{R_3} I_s = \frac{4\ \Omega}{10\ \Omega}\times 10\ \text{A} = \mathbf{4\ A}$$

Since R_1 and R_3 are equal, the current I_1 and I_3 must be equal.

Since R_2 is twice R_1 or R_3, the current I_2 must be one-half I_1 or I_3.

Practice Book Problem [SECTION 6.6 CDR] Problems: 29 to 34

6.5 KIRCHHOFF'S CURRENT LAW (KCL)

Statement:

(**1**) The algebraic sum of the currents entering and leaving a junction (or region) of a network is zero.

$$\sum I_{entering} - \sum I_{leaving} = 0 \qquad \textbf{(6.13.1)}$$

(**2**) The sum of the currents entering a junction (or region) of a network must equal the sum of the currents leaving the same junction (or region).

$$\sum I_{entering} = \sum I_{leaving} \qquad \textbf{(6.13)}$$

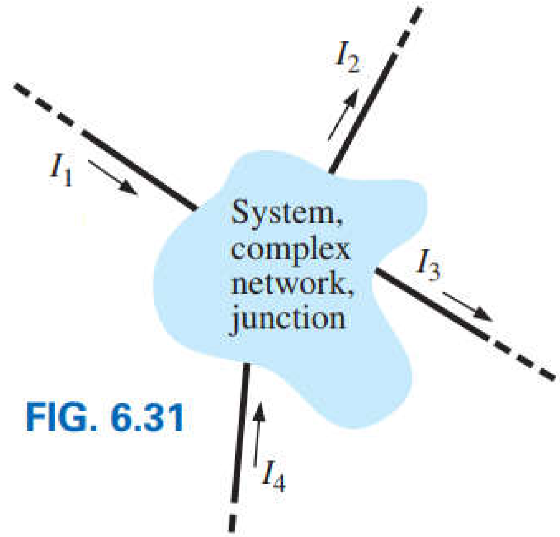


FIG. 6.31

$$I_1 + I_4 - I_2 - I_3 = 0$$

$$I_1 + I_4 = I_2 + I_3$$

EXAMPLE 6.17 Determine currents I_1, I_3, I_4, and I_5 for the network in Fig. 6.34.

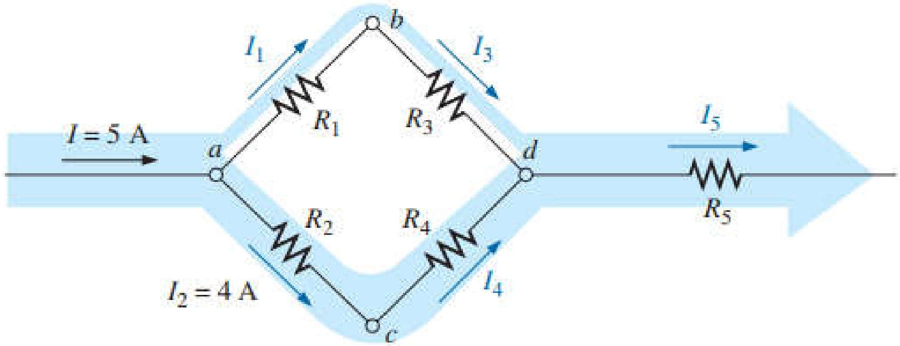


FIG. 6.34
Four-node configuration for Example 6.17.

Solution: At node *a:*

$$\Sigma I_i = \Sigma I_o$$
$$I = I_1 + I_2$$
$$5\text{ A} = I_1 + 4\text{ A}$$

and $$I_1 = 5\text{ A} - 4\text{ A} = \mathbf{1\ A}$$

At node *b:*

$$\Sigma I_i = \Sigma I_o$$
$$I_1 = I_3$$

and $$I_3 = I_1 = \mathbf{1\ A}$$

At node *c:*

$$\Sigma I_i = \Sigma I_o$$
$$I_2 = I_4$$

and $$I_4 = I_2 = \mathbf{4\ A}$$

At node *d:*

$$\Sigma I_i = \Sigma I_o$$
$$I_3 + I_4 = I_5$$
$$1\text{ A} + 4\text{ A} = I_5 = \mathbf{5\ A}$$

EXAMPLE 6.21

a. Determine currents I_1 and I_3 for the network in Fig. 6.40.
b. Find the source current I_s.

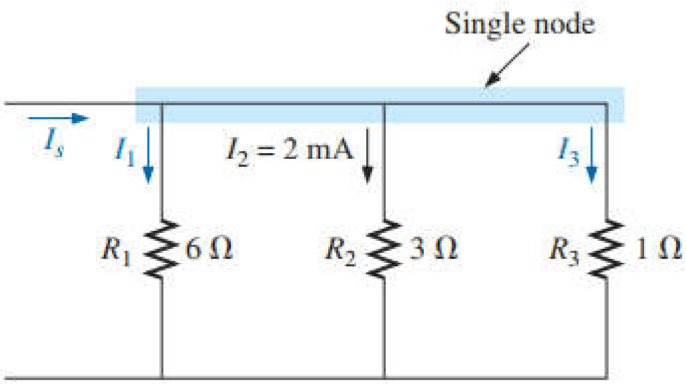


FIG. 6.40
Parallel network for Example 6.21.

Solutions:

a. Since R_1 is twice R_2, the current I_1 must be one-half I_2, and

$$I_1 = \frac{I_2}{2} = \frac{2\text{ mA}}{2} = \mathbf{1\text{ mA}}$$

Since R_2 is three times R_3, the current I_3 must be three times I_2, and

$$I_3 = 3I_2 = 3(2\text{ mA}) = \mathbf{6\text{ mA}}$$

b. Applying Kirchhoff's current law:

$$\Sigma I_i = \Sigma I_o$$
$$I_s = I_1 + I_2 + I_3$$
$$I_s = 1\text{ mA} + 2\text{ mA} + 6\text{ mA} = \mathbf{9\text{ mA}}$$

Problem 25 [P. 238] Using Kirchoff's current law, find the unknown currents for the complex configurations in Fig. 6.95(b).

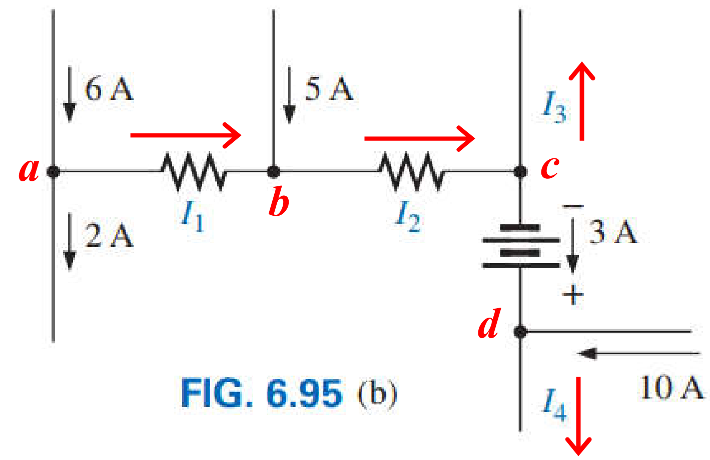


FIG. 6.95 (b)

Solution: Consider four (a, b, c, and d) nodes are here.

At node *a*: 6 A is entering and 2 A is leaving so consider I_1 is leaving.

According to KCL at node a, we have: $6\text{ A} = I_1 + 2\text{ A} = 9\text{ A}$

$$\therefore\ I_1 = 6\text{ A} - 2\text{ A} = 4\text{ A}$$

At node *b*: $I_1 = 4$ A and 5 A are entering so consider I_2 is leaving.

According to KCL at node b, we have: $I_2 = 4\text{ A} + 5\text{ A} = 9\text{ A}$

At node *c*: $I_2 = 9$ A current is entering and 3 A is leaving so consider I_3 is leaving.

According to KCL at node c, we have: $9\text{ A} = I_3 + 3\text{ A}$ $\quad\therefore\ I_3 = 9\text{ A} - 3\text{ A} = 6\text{ A}$

At node *d*: 3 A and 10 A are entering so consider I_4 is leaving.

According to KCL at node d, we have: $I_4 = 3\text{ A} + 10\text{ A} = 13\text{ A}$

Practice Book Problem [SECTION 6.5 KCL] Problems: 24 to 28

6.4 Power Distribution in a Parallel Circuit

In any electrical system, the power supplied or applied or delivered will equal the power dissipated or absorbed or consumed.

$$P_E = P_{R1} + P_{R2} + P_{R3} \qquad \textbf{(6.10)}$$

$$P_E = EI_s \;\; \text{(watt, W)} \qquad \textbf{(6.11)}$$

$$P_{R1} = V_1 I_1 = I_1^2 R_1 = \frac{V_1^2}{R_1}$$

$$P_{R2} = V_2 I_2 = I_2^2 R_2 = \frac{V_2^2}{R_2} \qquad \text{(watt, W)} \qquad \textbf{(6.12)}$$

$$P_{R3} = V_3 I_3 = I_3^2 R_3 = \frac{V_3^2}{R_3}$$

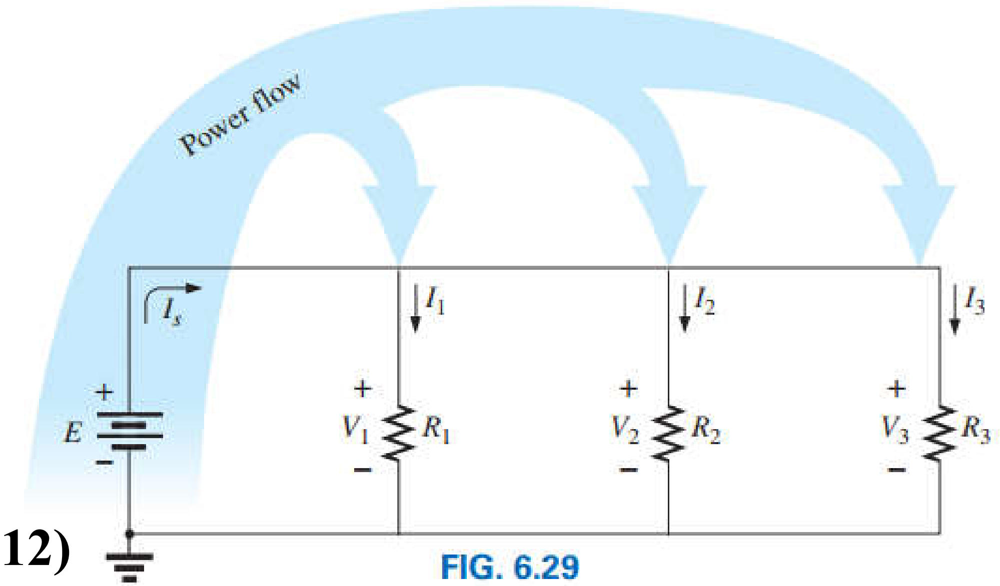


FIG. 6.29
Power flow in a dc parallel network.

EXAMPLE 6.15 For the parallel network in Fig. 6.30:

a. Determine the total resistance R_T.

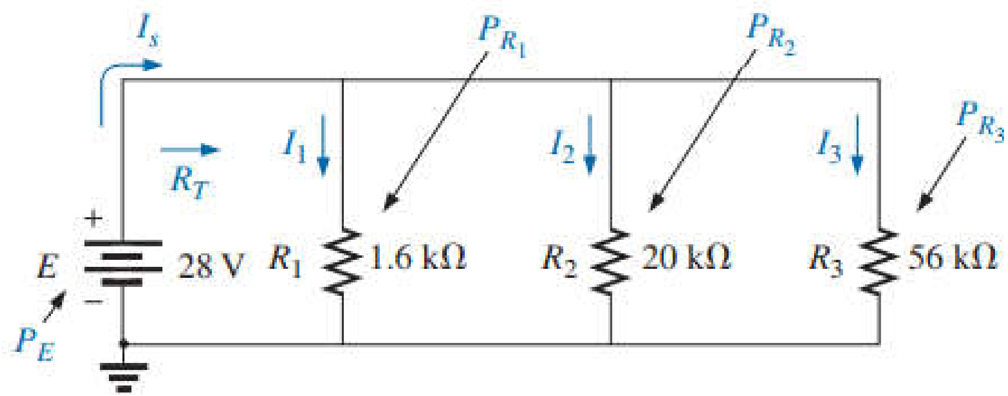
b. Find the source current (I_s) and the current (I_1, I_2, and I_3) through each resistor.

c. Verify KCL.

d. Calculate the power delivered by the source.

e. Determine the power absorbed by each parallel resistor.

f. Verify Eq. (6.10)

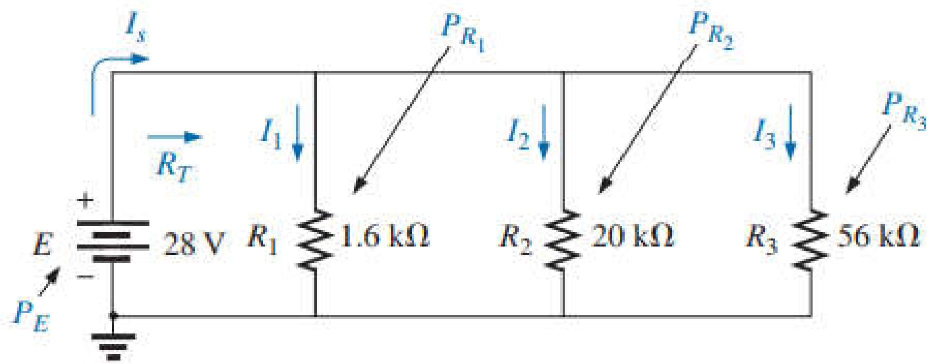


Solution: (*a*) $R_T = \dfrac{1}{\dfrac{1}{R_1}+\dfrac{1}{R_2}+\dfrac{1}{R_3}} = \dfrac{1}{\dfrac{1}{1.6\times10^3\,\Omega}+\dfrac{1}{20\times10^3\,\Omega}+\dfrac{1}{56\times10^3\,\Omega}}$

$$= \frac{1}{625\times10^{-6}\,\text{S}+50\times10^{-6}\,\text{S}+17.867\times10^{-6}\,\text{S}} = \frac{1}{692.867\times10^{-6}\,\text{S}} = \mathbf{1.44\ k\Omega}$$

b. Find the source current (I_s) and the current (I_1, I_2, and I_3) through each resistor.

$R_T = \mathbf{1.44\ k\Omega}$



(***b***) Applying Ohm's Law :

$$I_s = \frac{E}{R_T} = \frac{28\ \text{V}}{1.44 \times 10^3\ \Omega} = \mathbf{19.4\ mA}$$

$$I_1 = \frac{R_T}{R_1} I_s = \frac{E}{R_1} = \frac{26\ \text{V}}{1.6 \times 10^3\ \Omega} = \mathbf{17.5\ A}$$

$$I_2 = \frac{R_T}{R_2} I_s = \frac{E}{R_2} = \frac{26\ \text{V}}{20 \times 10^3\ \Omega} = \mathbf{1.4\ A}$$

$$I_3 = \frac{R_T}{R_3} I_s = \frac{E}{R_3} = \frac{26\ \text{V}}{56 \times 10^3\ \Omega} = \mathbf{0.5\ A}$$

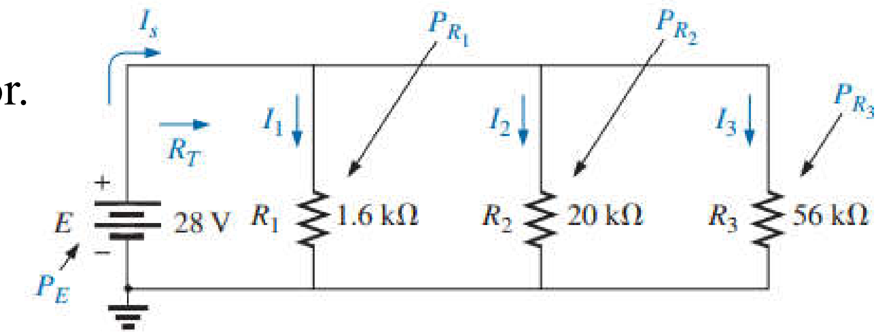
(***c***) According to KCL : $I_s = I_1 + I_2 + I_3$

$$\mathbf{19.4\ mA} = 17.5\,\text{mA} + 1.4\,\text{mA} + 0.5\,\text{mA} = \mathbf{19.4\ mA}$$

d. Calculate the power delivered by the source.

e. Determine the power absorbed by each parallel resistor.

f. Verify Eq. (6.10)



Solution: $R_T = \mathbf{1.44\ k\Omega}$ $\quad I_s = \mathbf{19.44\ mA}$

$I_1 = \mathbf{17.5\ A};\ I_2 = \mathbf{1.4\ A};\ I_3 = \mathbf{0.5\ A}$

d. Applying Eq. (6.11): $P_E = EI_s = (28\text{ V})(19.4\text{ mA}) = \mathbf{543.2\ mW}$

e. Applying each form of the power equation:

$$P_1 = V_1 I_1 = EI_1 = (28\text{ V})(17.5\text{ mA}) = \mathbf{490\ mW}$$

$$P_2 = I_2^2 R_2 = (1.4\text{ mA})^2(20\text{ k}\Omega) = \mathbf{39.2\ mW}$$

$$P_3 = \frac{V_3^2}{R_3} = \frac{E^2}{R_3} = \frac{(28\text{ V})^2}{56\text{ k}\Omega} = \mathbf{14\ mW}$$

f. $$P_E = P_{R_1} + P_{R_2} + P_{R_3}$$

$$\mathbf{543.2\ mW} = 490\text{ mW} + 39.2\text{ mW} + 14\text{ mW} = \mathbf{543.2\ mW} \quad \text{(checks)}$$

Practice Book Problem
[SECTION 6.4 Power Distribution]

Problems: 19 to 23

6.7 VOLTAGE SOURCES IN PARALLEL

Voltage sources can be placed in parallel only if:

(***i***) They have the same voltage rating.

(***ii***) Positive terminal should be connected with positive terminal and Negative terminal should be connected with negative terminal

Voltage sources to be connected in parallel must have same voltage rating through their current rating may be same or different.

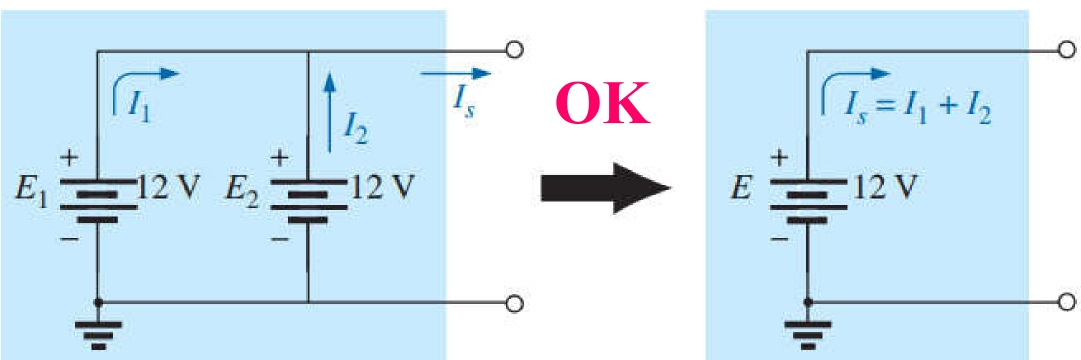


FIG. 6.47

Demonstrating the effect of placing two ideal supplies of the same voltage in parallel.

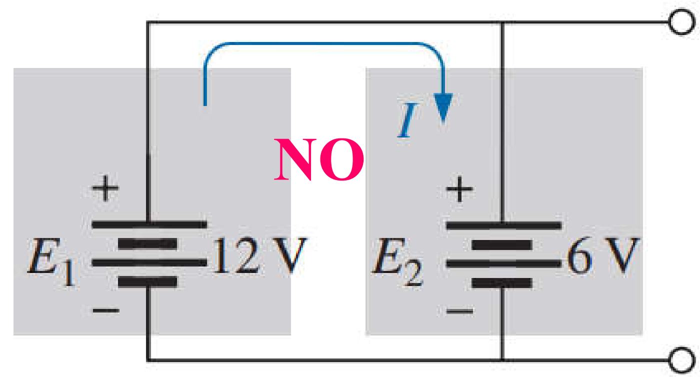


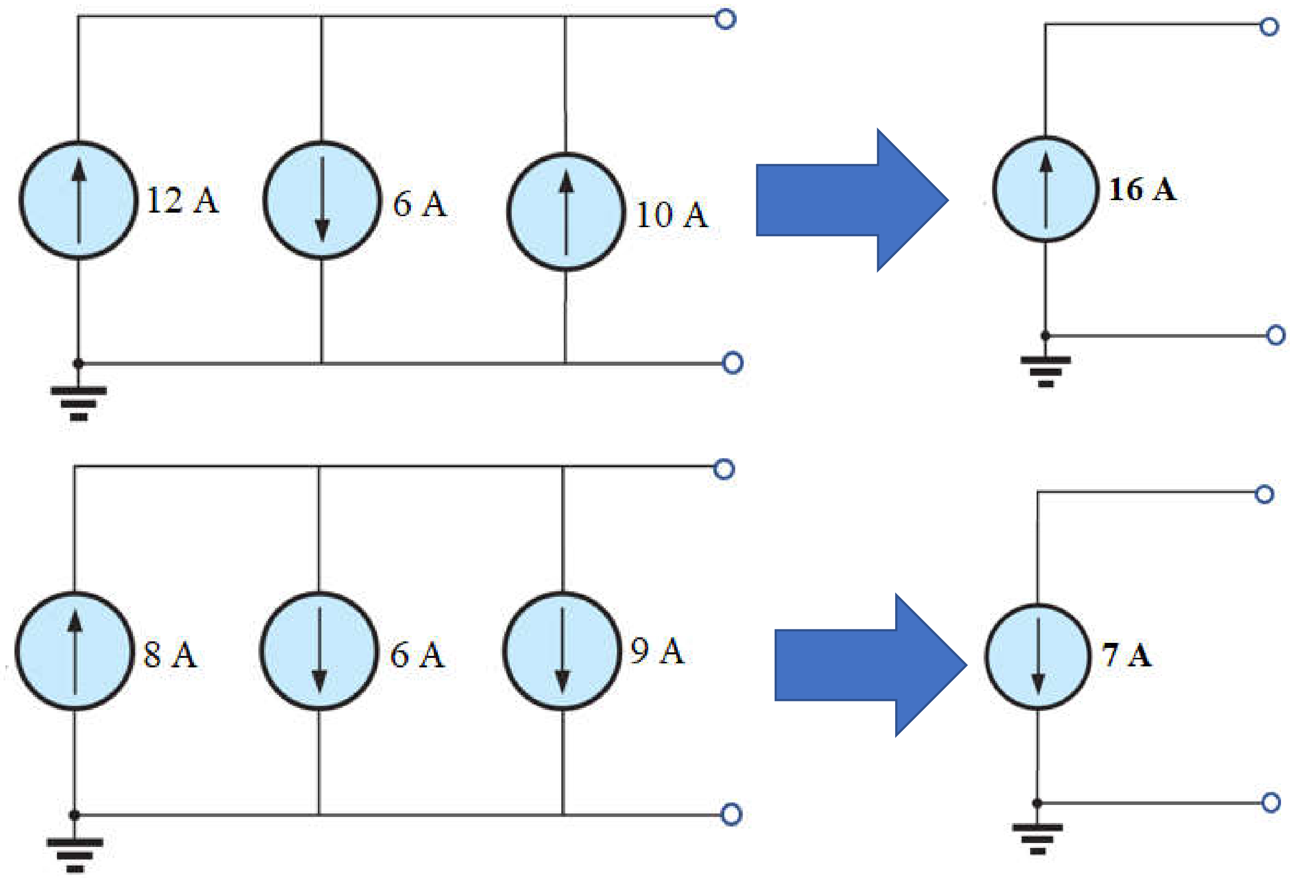
FIG. 6.48

Should not be connected

CURRENT SOURCES IN PARALLEL

Current sources of different current ratings are not connected in series.

The current sources to be connected in parallel must have same voltage rating through their current ratings may be same or different.



Reducing Parallel Current Sources to a Single Source

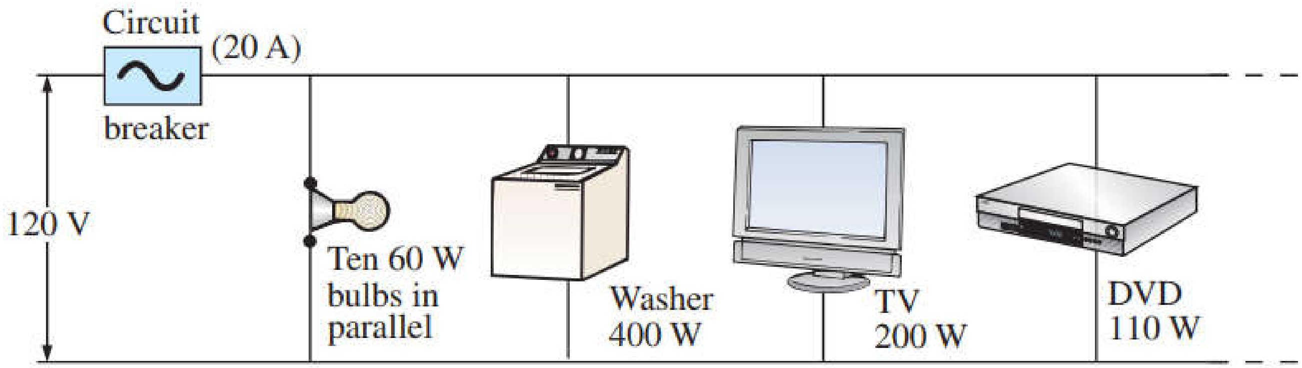
Problem 22 [P. 238] A portion of a residential service to a home is depicted in Fig. 6.92.

a. Determine the current through each parallel branch of the system.

b. Calculate the current drawn from the 120 V source. Will the 20 A breaker trip?

c. What is the total resistance of the network?

d. Determine the power delivered by the source. How does it compare to the sum of the wattage ratings appearing in Fig. 6.92?



Solution: **a**. We know that: $P = VI \quad \therefore\ I = \dfrac{P}{V}$

$$\text{For bulbs:}\quad I_b = \frac{10\times 60\ \text{W}}{120\ \text{V}} = 5\ \text{A}$$

$$\text{For washer:}\quad I_w = \frac{400\ \text{W}}{120\ \text{V}} = 3.33\ \text{A}$$

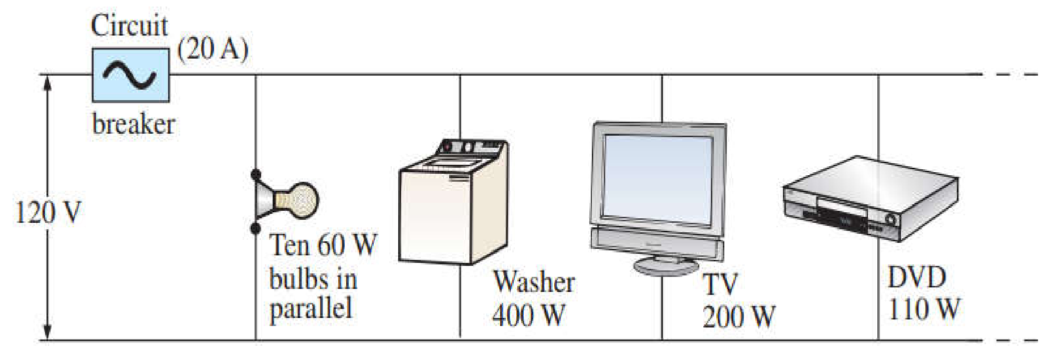
$$\text{For TV:}\quad I_{tv} = \frac{200\ \text{W}}{120\ \text{V}} = 1.667\ \text{A}$$

$$\text{For DVD:}\quad I_{dvd} = \frac{110\ \text{W}}{120\ \text{V}} = 0.917\ \text{A}$$

b. Calculate the current drawn from the 120 V source. Will the 20 A breaker trip?

c. What is the total resistance of the network?

d. Determine the power delivered by the source. How does it compare to the sum of the wattage ratings appearing in Fig. 6.92?



b. Applying KCL we have:

$$I_s = I_b + I_w + I_{tv} + I_{dvd}$$
$$= 5\ \text{A} + 3.333\ \text{A} + 1.667\ \text{A} + 0.917\ \text{A}$$
$$= 10.917\ \text{A}$$

Since source current less than 20 circuit breaker will not trip.

c. $$R_T = \frac{E}{I_s} = \frac{120\ \text{V}}{10.917\ \text{A}} = 10.99\ \Omega$$

d. $$P_E = EI_s = (120\ \text{V})(10.917\ \text{A}) = 1310\ \text{W}$$

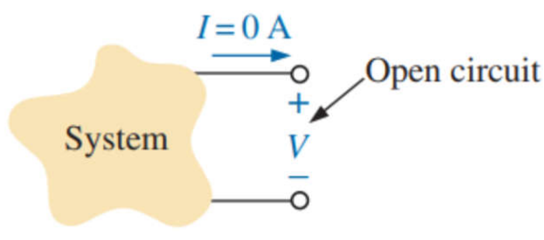
$$P_w = (10 \times 60)\,\text{W} + 400\,\text{W} + 200\,\text{W} + 110\,\text{W} = 1310\ \text{W}$$

Power delivered by the source (P_E) is **equal** to the sum of the wattage ratings (P_w).

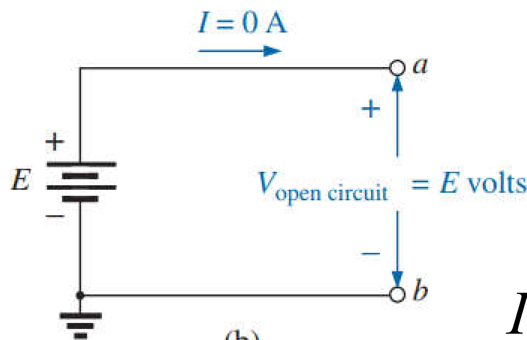
OPEN CIRCUITS
SHORT CIRCUITS

6.8 OPEN CIRCUITS

An **open circuit** is two isolated terminals not connected by any kind of element.



(a)

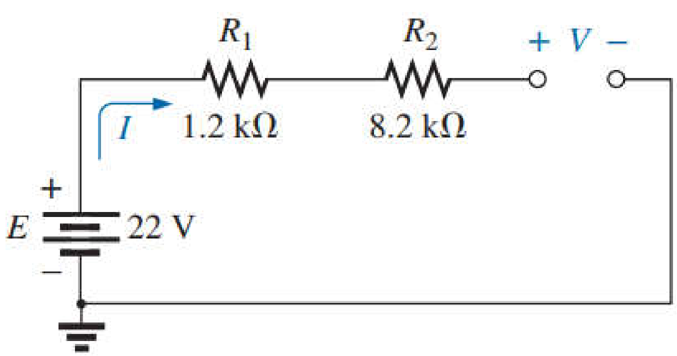


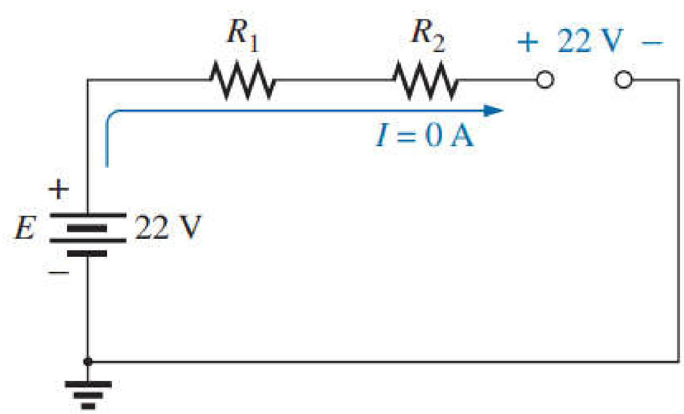
(b)

An open circuit can have a potential difference (voltage) across its terminals, but the current is always zero (0) amperes.

$I_{oc} = 0\ \text{A}$ $\qquad$ $R_{oc} = \infty$ (Infinite)

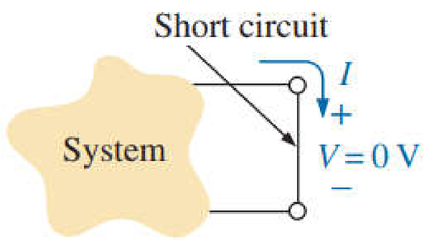
EXAMPLE 6.27.1 Determine the unknown voltage (V) and current (I) for the following network.

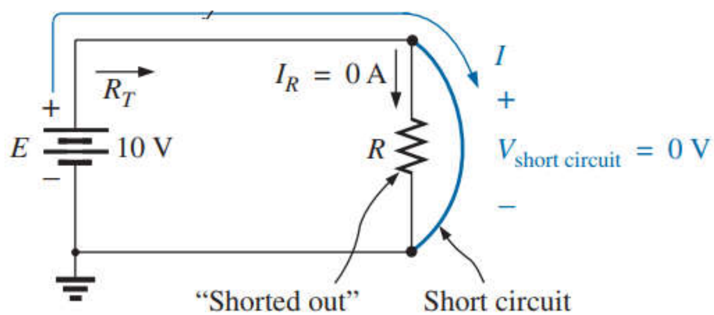




6.8 Short CIRCUITS

A **short circuit** is a very low resistance, direct connection between two terminals of a network.

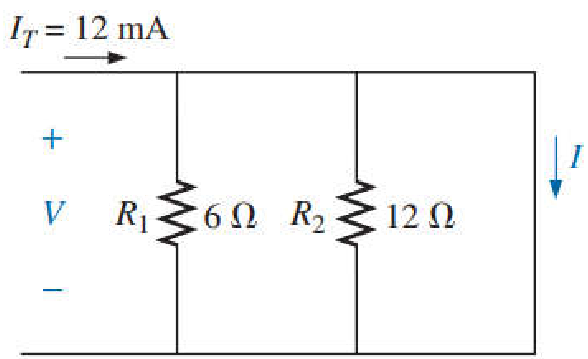


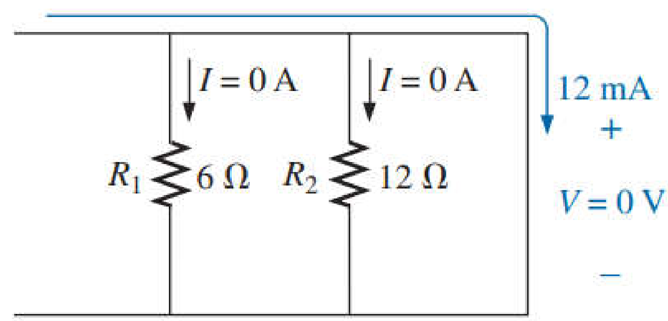


Aa short circuit can carry a current of a level determined by the external circuit, but the potential difference (voltage) across its terminals is always zero (0) volts.

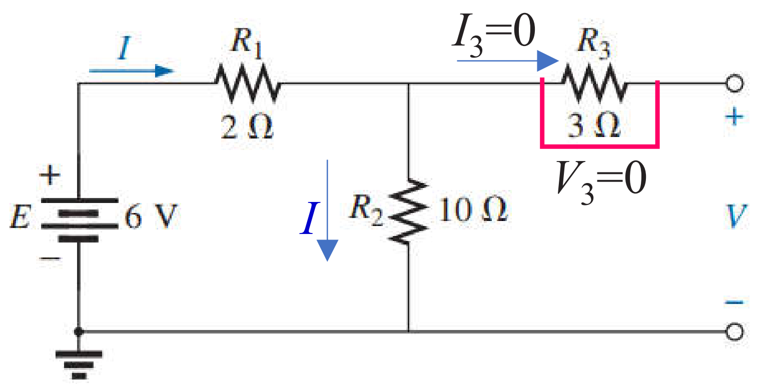
$$V_{sc} = 0\text{ A} \qquad R_{sc} = 0\text{ (Zero)}$$

EXAMPLE 6.27.2 Determine the unknown voltage (V) and current (I) for the following network.





EXAMPLE 6.28.1 Determine the unknown voltage (V) and current (I) for the following network.

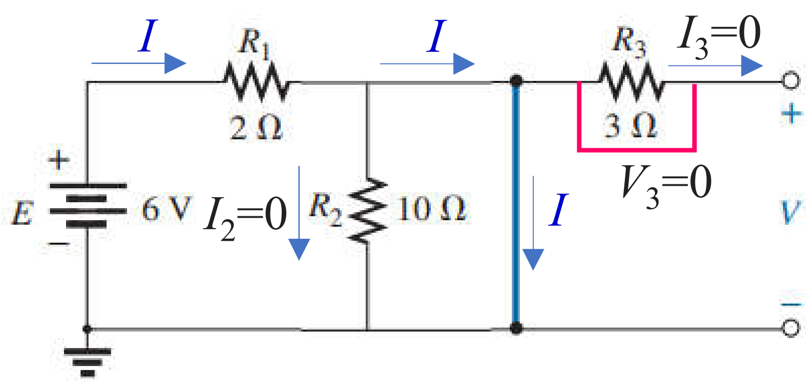


Due to the open circuit, the current flow (I_3) through the R_3 is zero. Thus, the current I flows through R_1 and R_2 and V is equal to the voltage drop across the resistance R_2.

$$V = \frac{R_2}{R_1 + R_2} E = \frac{10\ \Omega}{2\ \Omega + 10\ \Omega} \times 6\ \text{V} = \mathbf{5\ V}$$

$$I = \frac{E}{R_1 + R_2} = \frac{6\ \text{V}}{2\ \Omega + 10\ \Omega} = \mathbf{0.5\ A}$$

EXAMPLE 6.28.2 Determine the unknown voltage (V) and current (I) for the following network.



Due to the open circuit, the current flow (I_3) through the R_3 is zero and since R_2 is shorted the current flow (I_2) through the R_2 is zero. Thus, the current I flows through R_1 and short circuit and $V = \mathbf{0\ V}$.

$$I = \frac{E}{R_1} = \frac{6\ \text{V}}{2\ \Omega} = \mathbf{3\ A}$$

Problem 38 [Ch 6] A For the network in Fig. 6.108:

(**a**) Determine I_s and V_L.

(**b**) Determine I_s if R_L is shorted out.

(**c**) Determine V_L if R_L is replaced by an open circuit.

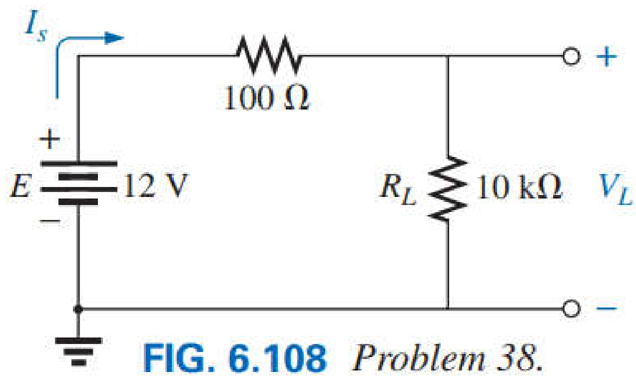
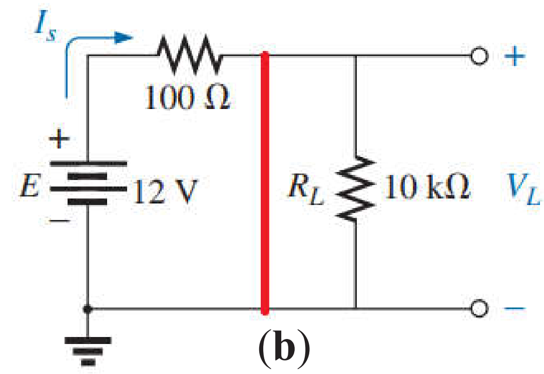


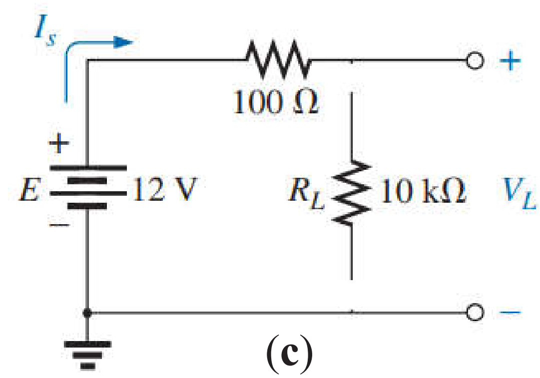
FIG. 6.108 *Problem 38.*

$$\textbf{(a)} \quad I_S = \frac{12\text{ V}}{100\ \Omega + 1\text{ k}\Omega} = \mathbf{10.91\ mA}$$

$$V_L = I_S R_L = (10.91\times10^{-3})\times(1\times10^{3}) = \mathbf{10.91\ V}$$



$$\textbf{(b)} \quad I_S = \frac{12\text{ V}}{100\ \Omega} = \mathbf{120\ mA}$$



$$\textbf{(c)} \quad V_L = E = \mathbf{12\ V}$$

Problem 39 [Ch 6] For the network in Fig. 6.109:

a. Determine the open-circuit voltage V_L.

b. If the 2.2 kΩ resistor is short circuited, what is the new value of V_L?

c. Determine V_L if the 4.7 k Ω resistor is replaced by an open circuit.

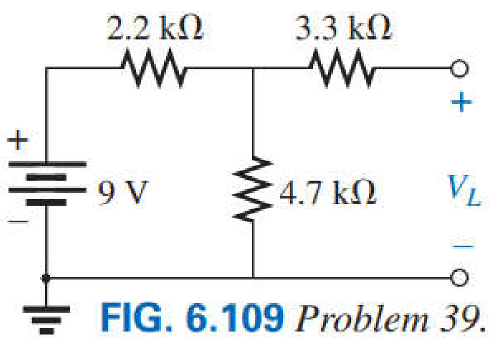
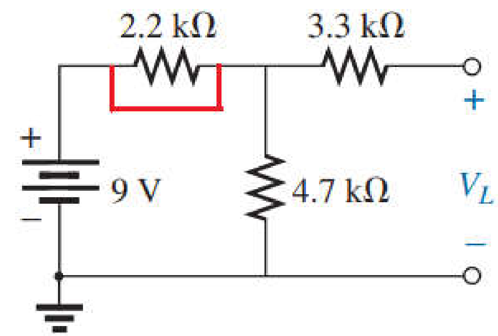
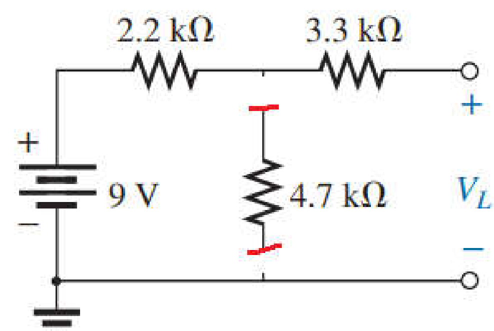


FIG. 6.109 *Problem 39.*

$$(\mathbf{a})\quad V_L = \frac{(4.7\,\text{k}\Omega)\times(9\text{ V})}{2.2\,\text{k}\Omega + 4.7\,\text{k}\Omega} = \mathbf{6.13\ V}$$



$$(\mathbf{b})\quad V_L = E = \mathbf{9\ V}$$

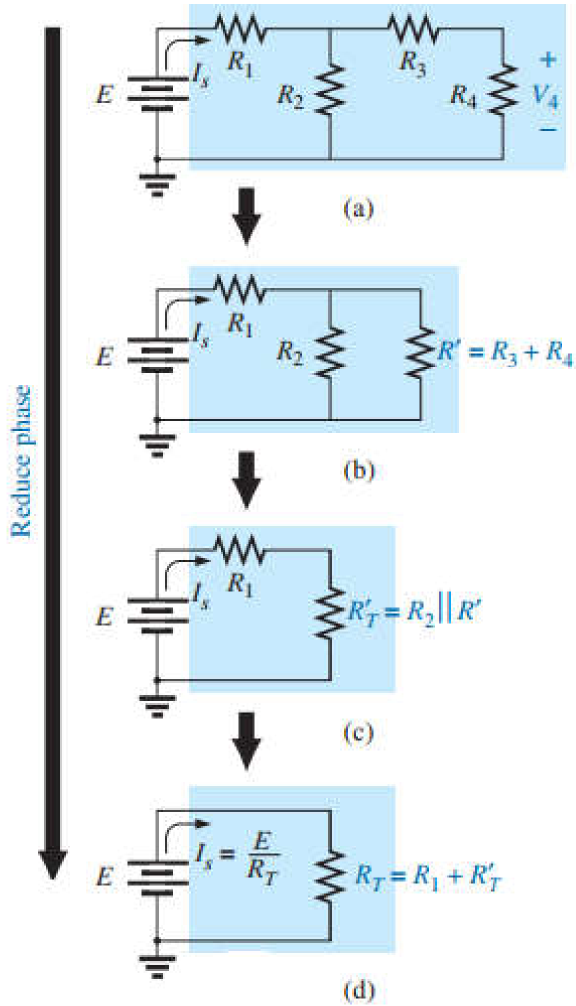


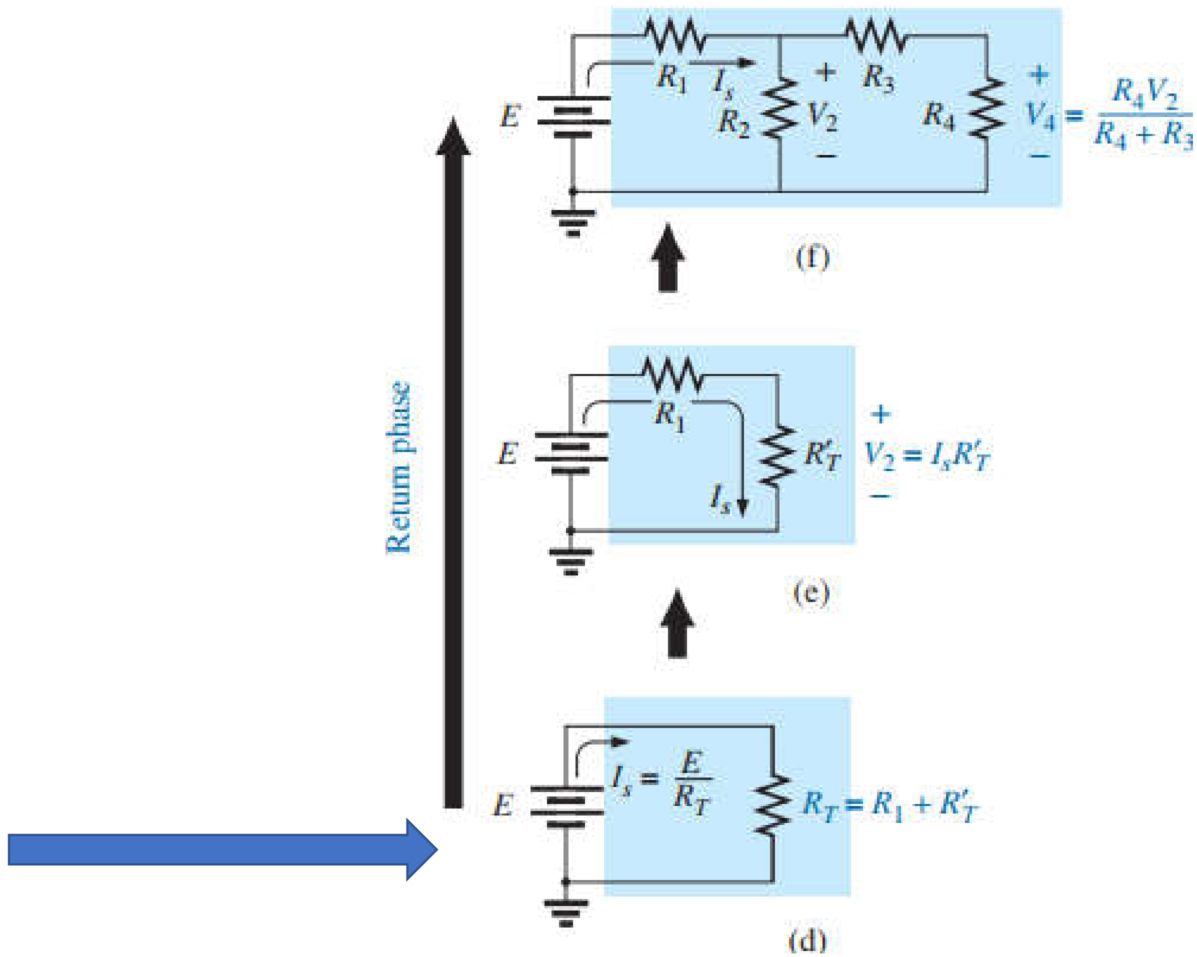
$$(\mathbf{c})\quad V_L = E = \mathbf{9\ V}$$

Chapter 7

Series-Parallel DC Circuit

7.3 REDUCE AND RETURN APPROACH





EXAMPLE 7.1 Find current I_3 for the series-parallel network in Fig. 7.3.

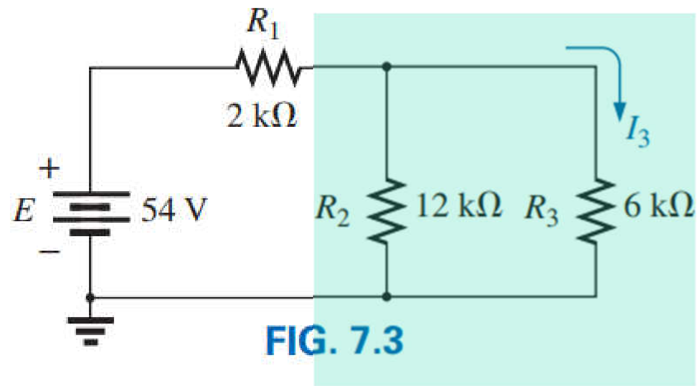


FIG. 7.3

Solution: Checking for series and parallel elements, we find that resistors R_2 and R_3 are in parallel. Their total resistance is:

$$R_4 = R_2 \| R_3 = \frac{R_2R_3}{R_2 + R_3} = \frac{(12\text{ k}\Omega)(6\text{ k}\Omega)}{12\text{ k}\Omega + 6\text{ k}\Omega} = 4\text{ k}\Omega$$

Redraw the circuit showing the calculated resistance R_4.

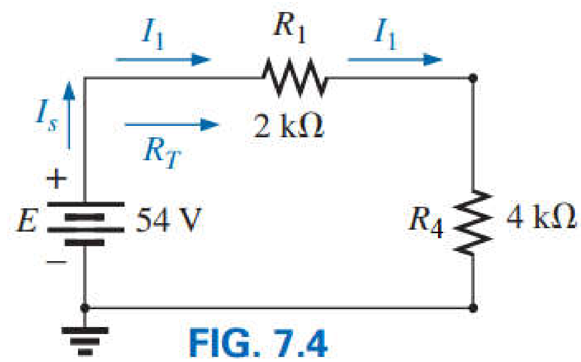


FIG. 7.4

Now, resistors R_1 and R_4 are in series, resulting in a total resistance of

$$R_T = R_1 + R_4 = 2\text{ k}\Omega + 4\text{ k}\Omega = 6\text{ k}\Omega$$

The source current is then determined using Ohm's law:

$$I_s = \frac{E}{R_T} = \frac{54\text{ V}}{6\text{ k}\Omega} = 9\text{ mA}$$

Returning to Fig. 7.3, we find the current I_3 as follows:

$$I_3 = \frac{R_4}{R_3}I_1 = \left(\frac{R_2}{R_2 + R_3}\right)I_1 = \left(\frac{12\text{ k}\Omega}{12\text{ k}\Omega + 6\text{ k}\Omega}\right)9\text{ mA} = \mathbf{6\text{ mA}}$$

EXAMPLE 7.2 For the network in Fig. 7.5:

a. Determine currents I_4 and I_s and voltage V_2.
b. Insert the meters to measure current I_4 and voltage V_2.

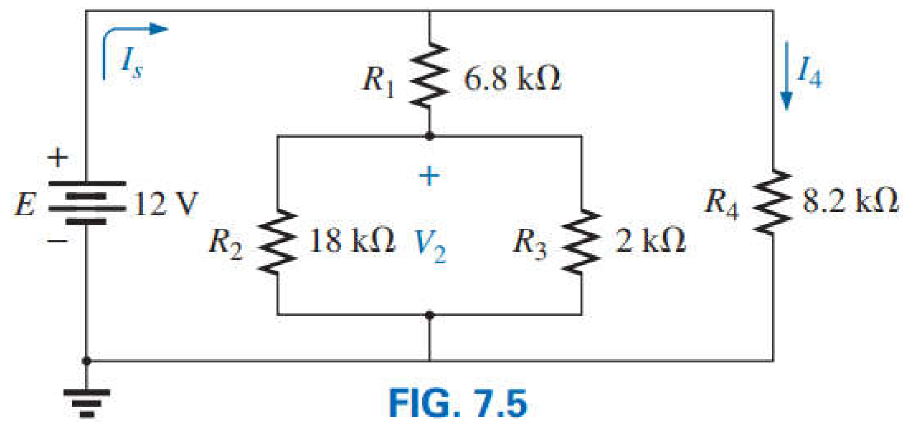


FIG. 7.5

Solution:

$$R_5 = R_2 \parallel R_3 = \frac{R_2 R_3}{R_2 + R_3} = \frac{(18\text{ k}\Omega)(2\text{ k}\Omega)}{18\text{ k}\Omega + 2\text{ k}\Omega} = 1.8\text{ k}\Omega$$

$$I_4 = \frac{V_4}{R_4} = \frac{E}{R_4} = \frac{12\text{ V}}{8.2\text{ k}\Omega} = \mathbf{1.46\text{ mA}}$$

$$V_2 = \left(\frac{R_5}{R' + R_1}\right)E = \left(\frac{1.8\text{ k}\Omega}{1.8\text{ k}\Omega + 6.8\text{ k}\Omega}\right)12\text{ V} = \mathbf{2.51\text{ V}}$$

$$I_1 = \frac{E}{R_1 + R_5} = \frac{12\text{ V}}{6.8\text{ k}\Omega + 1.8\text{ k}\Omega} = 1.40\text{ mA}$$

and $\quad I_s = I_1 + I_4 = 1.40\text{ mA} + 1.46\text{ mA} = \mathbf{2.86\text{ mA}}$

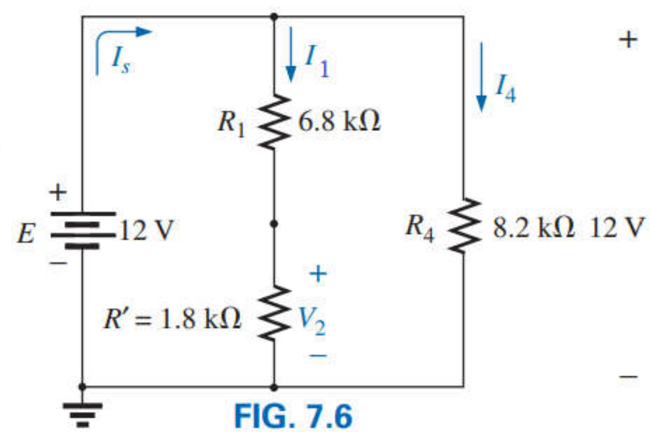


FIG. 7.6

Practice Book Problem [SECTIONS 7.2–7.5 Series Parallel Networks] Problems: 1 ~ 24

7.4 BLOCK DIAGRAM APPROACH

EXAMPLE 7.4 Determine all the currents and voltages for the series-parallel network in Fig. 7.12.

Solution:

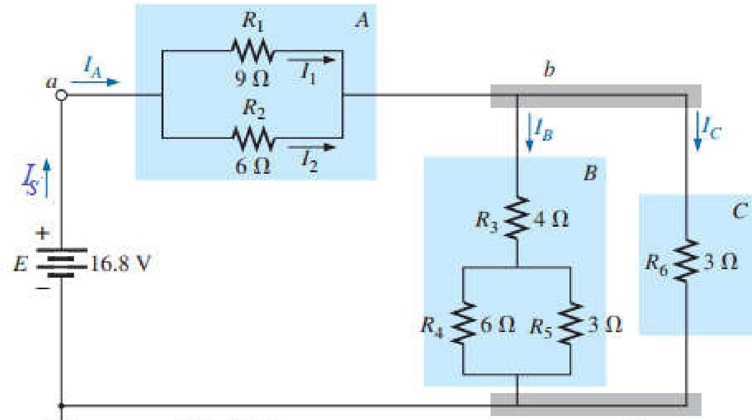


FIG. 7.12

$$R_A = \frac{R_1 R_2}{R_1 + R_2} = \frac{(9\ \Omega)\times(6\ \Omega)}{9\ \Omega + 6\ \Omega} = 3.6\ \Omega$$

$$R_B = R_3 + \frac{R_4 R_5}{R_4 + R_5} = 4\ \Omega + \frac{(3\ \Omega)\times(3\ \Omega)}{6\ \Omega + 3\ \Omega} = 4\ \Omega + 2\ \Omega = 6\ \Omega$$

$$R_C = R_6 = 3\ \Omega$$

$$R_{B//C} = \frac{R_B R_C}{R_B + R_C} = \frac{(6\ \Omega)\times(3\ \Omega)}{6\ \Omega + 3\ \Omega} = 2\ \Omega$$

$$R_T = R_A + R_{B//C} = 36\ \Omega + 2\ \Omega = 5.6\ \Omega$$

$$I_s = I_A = \frac{E}{R_T} = \frac{16.8\ \text{V}}{5.6\ \Omega} = \mathbf{3\ A}$$

$$I_1 = \frac{R_A}{R_1} I_s = \frac{R_2}{R_1 + R_2} I_s = \frac{6\ \Omega}{9\ \Omega + 6\ \Omega} \times 3\ \text{A} = \mathbf{1.2\ A}$$

$$I_2 = \frac{R_A}{R_2} I_s = \frac{R_1}{R_1 + R_2} I_s = I_s - I_1 = 3\ \text{A} - 1.2\ \text{A} = \mathbf{1.8\ A}$$

$$I_B = \frac{R_{B//C}}{R_B} I_s = \frac{R_C}{R_B + R_C} I_s = \frac{3\ \Omega}{6\ \Omega + 3\ \Omega} \times 3\ \text{A} = \mathbf{1\ A}$$

$$I_C = \frac{R_{B//C}}{R_C} I_s = \frac{R_B}{R_B + R_C} I_s = I_s - I_B = 3\ \text{A} - 1\ \text{A} = \mathbf{2\ A}$$

EXAMPLE 7.7

a. Find the voltages V_1, V_3, and V_{ab} for the network in Fig. 7.20.
b. Calculate the source current I_s.

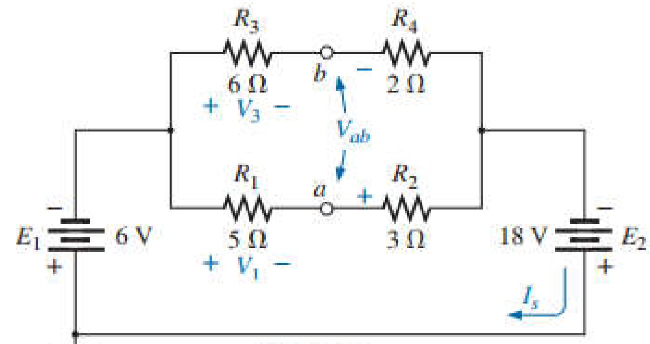


FIG. 7.20

Solution:

$$V_1 = \frac{R_1 E}{R_1 + R_2} = \frac{(5\ \Omega)(12\ \text{V})}{5\ \Omega + 3\ \Omega} = \frac{60\ \text{V}}{8} = \mathbf{7.5\ V}$$

$$V_3 = \frac{R_3 E}{R_3 + R_4} = \frac{(6\ \Omega)(12\ \text{V})}{6\ \Omega + 2\ \Omega} = \frac{72\ \text{V}}{8} = \mathbf{9\ V}$$

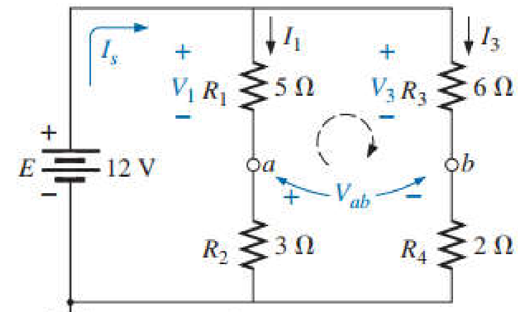


FIG. 7.21

$$+V_1 - V_3 + V_{ab} = 0$$

and $V_{ab} = V_3 - V_1 = 9\ \text{V} - 7.5\ \text{V} = \mathbf{1.5\ V}$

b. By Ohm's law, $I_1 = \dfrac{V_1}{R_1} = \dfrac{7.5\ \text{V}}{5\ \Omega} = 1.5\ \text{A}$

$$I_3 = \frac{V_3}{R_3} = \frac{9\ \text{V}}{6\ \Omega} = 1.5\ \text{A}$$

Applying Kirchhoff's current law,

$$I_s = I_1 + I_3 = 1.5\ \text{A} + 1.5\ \text{A} = \mathbf{3\ A}$$

EXAMPLE 7.8 For the network in Fig. 7.22, determine the voltages V_1 and V_2 and the current I.

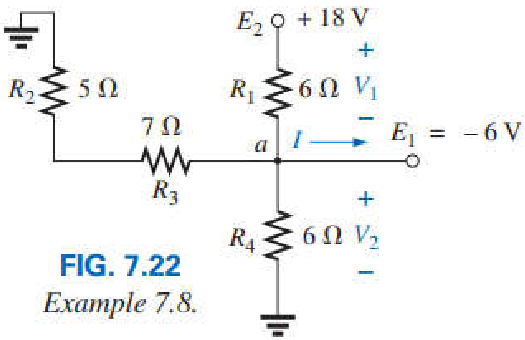


FIG. 7.22
Example 7.8.

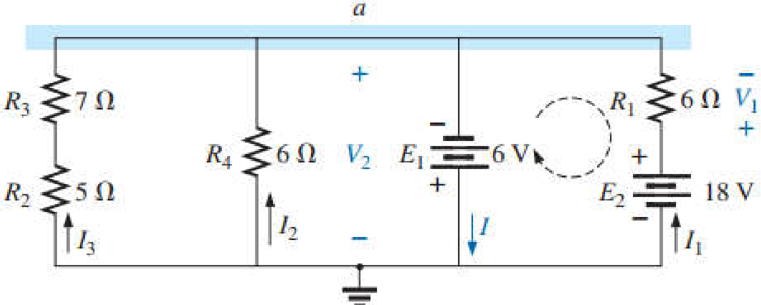


FIG. 7.23
Network in Fig. 7.22 redrawn.

Solution:

Applying Kirchhoff's voltage law:

$$V_2 = -E_1 = \mathbf{-6\ V}$$

Applying Kirchhoff's voltage law:

$$-E_1 + V_1 - E_2 = 0$$

and $\quad V_1 = E_2 + E_1 = 18\text{ V} + 6\text{ V} = \mathbf{24\ V}$

Applying Kirchhoff's current law to note a yields

$$\begin{aligned} I &= I_1 + I_2 + I_3 \\ &= \frac{V_1}{R_1} + \frac{E_1}{R_4} + \frac{E_1}{R_2 + R_3} \\ &= \frac{24\text{ V}}{6\ \Omega} + \frac{6\text{ V}}{6\ \Omega} + \frac{6\text{ V}}{12\ \Omega} \\ &= 4\text{ A} + 1\text{ A} + 0.5\text{ A} \\ I &= \mathbf{5.5\ A} \end{aligned}$$

Problem 10:

a. Find the magnitude and direction for current I, I_1, I_2, and I_3, for the network in Fig. 7.70.

b. Indicate their direction on Fig. 7.70.

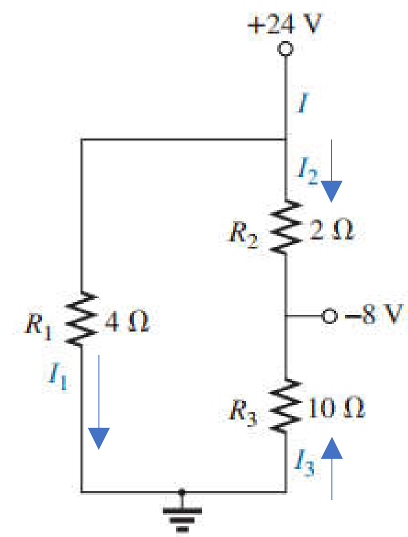


FIG. 7.70 *Problem 10.*

Solution: Voltage drop across the resistance R_1 equal to 24 V. So:

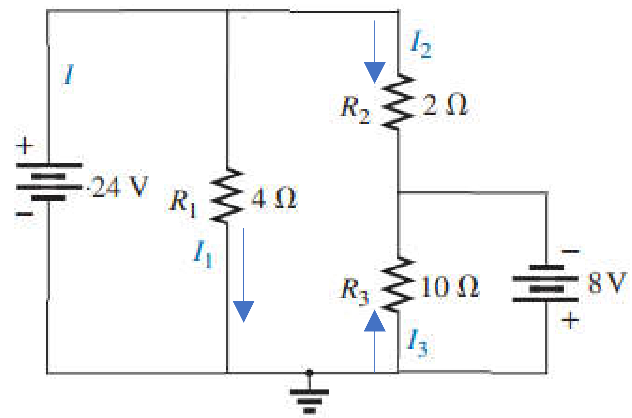
$$I_1 = \frac{24\ \text{V}}{4\ \Omega} = \mathbf{6\ A}$$

Voltage drop across the resistance R_3 equal to 8 V. So: $I_3 = \dfrac{8\ \text{V}}{10\ \Omega} = \mathbf{0.8\ A}$

Voltage drop across the resistance R_2 equal to the difference of 24 V and –8 V. So:

$$I_2 = \frac{24\ \text{V} - (-8\ \text{V})}{2\ \Omega} = \frac{32\ \text{V}}{2\ \Omega} = \mathbf{16\ A}$$

According to KCL: $I = I_1 + I_2 = 6\ \text{A} + 16\ \text{A} = 22\ \text{A}$

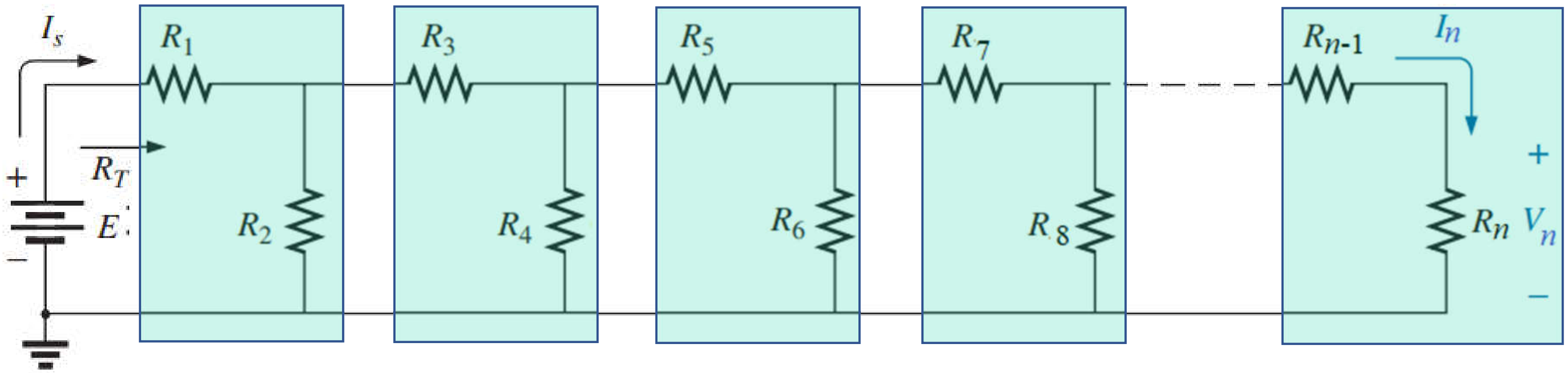


Practice Book Problem [SECTIONS 7.2–7.5 Series Parallel Networks] Problems: 1 ~ 24

7.6 LADDER NETWORKS

A ***n*-section ladder network** appears in the following Figure.

The reason for the terminology is quite obvious for the repetitive structure.

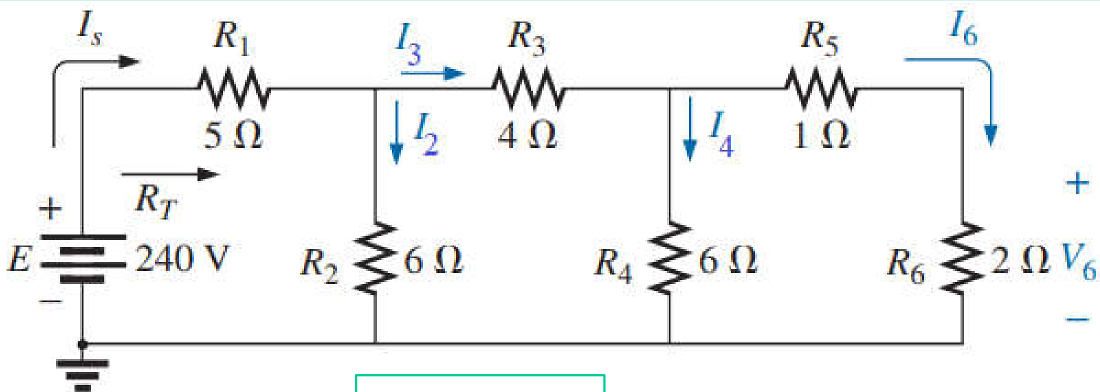


Basically, **two approaches** are used to solve networks of this type.

(**1**) Calculate the total resistance and resulting source current, and then work back through the ladder until the desired current or voltage is obtained [**Reduce and Return Approach**].

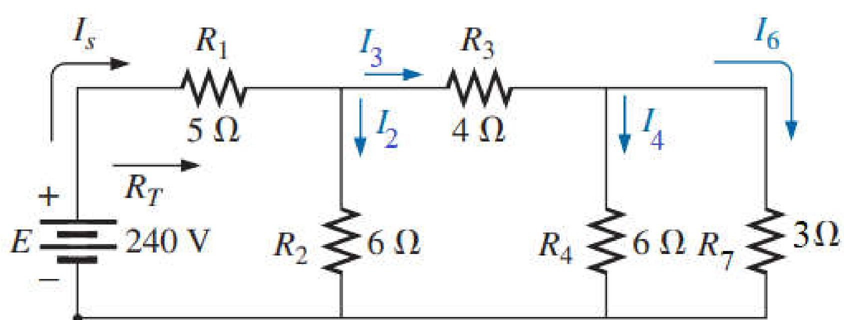
(**2**) Assign a letter symbol to the last branch current (I_n) and voltage (V_n) and work back through the network to the source, maintaining this assigned current or other current of interest. The desired current can then be found directly.

EXAMPLE FIG. 7.3 Determine the unknown currents I_s, I_2, I_3, I_4, and I_6 and voltage (V_6) for the following network.



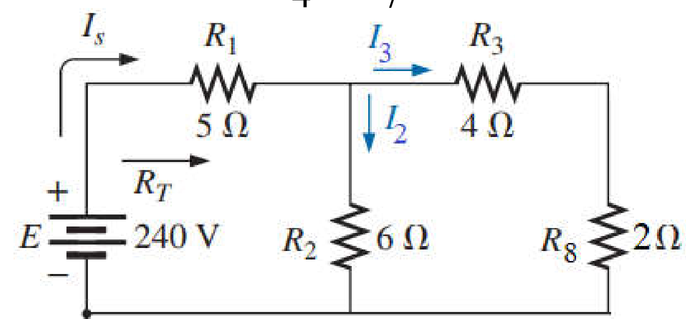
Method 1

$$R_7 = R_5 + R_6 = 3\ \Omega$$



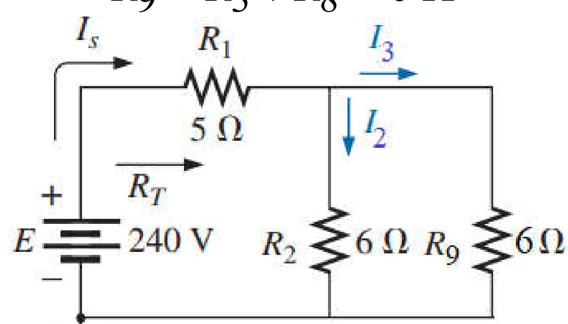
(*a*)

$$R_8 = \frac{R_4 R_7}{R_4 + R_7} = 2\ \Omega$$



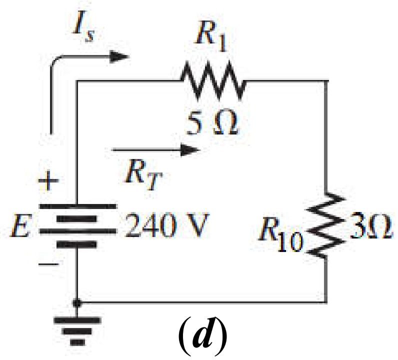
(*b*)

$$R_9 = R_3 + R_8 = 6\ \Omega$$



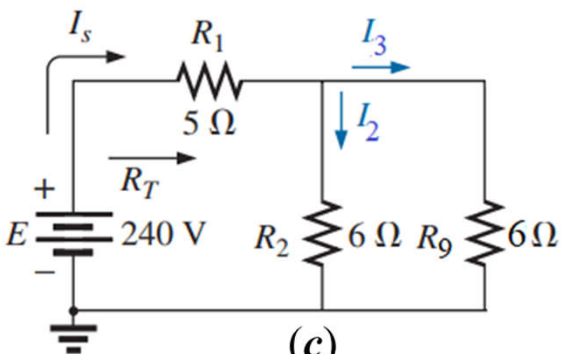
(*c*)

$$R_{10} = \frac{R_2 R_9}{R_2 + R_9} = 3\ \Omega$$



(*d*)

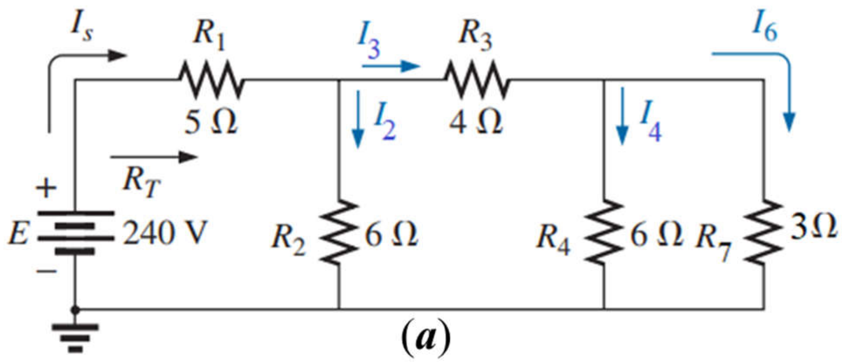
$$I_s = \frac{E}{R_1 + R_{10}} = \frac{240\,\text{V}}{5\ \Omega + 3\ \Omega} = 30\,\text{A}$$



(*c*)

From Fig. (c), we have:

$$I_2 = I_3 = \frac{I_s}{2} = 15\,\text{A}$$



(*a*)

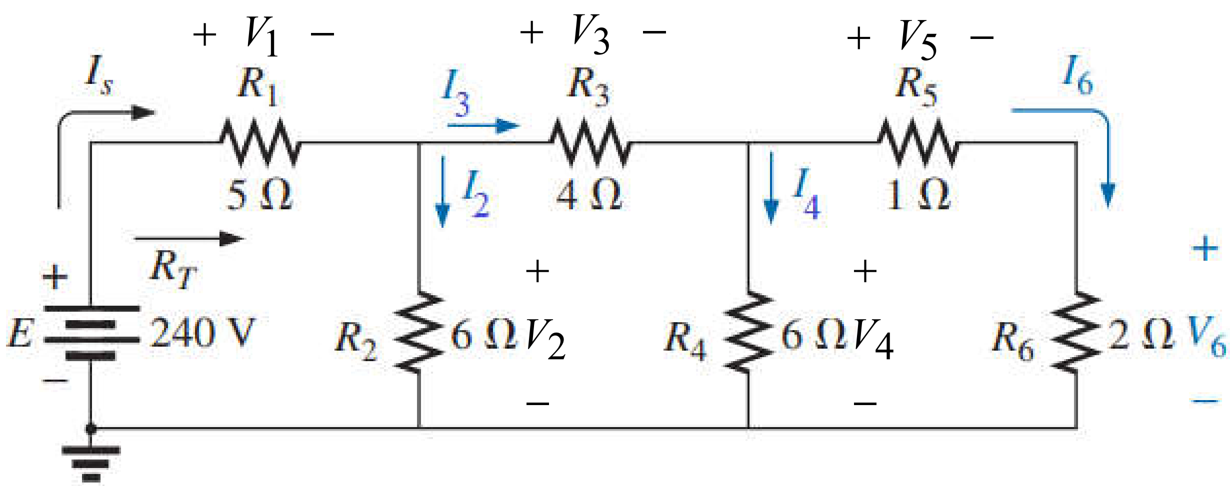
From Fig. (a), we have:

$$I_4 = \frac{R_7}{R_4 + R_7} I_3 = 5\,\text{A}$$

$$I_6 = \frac{R_4}{R_4 + R_7} I_3 = 10\,\text{A}$$

$$V_6 = I_6 R_6 = 10\,\text{A} \times 2\,\Omega = \mathbf{20\,V}$$

Method 2



The assigned notation for the current through the final branch is I_6:

$$I_6 = \frac{V_5 + V_6}{R_5 + R_6} = \frac{V_4}{R_5 + R_6} = \frac{V_4}{3\ \Omega} \qquad \therefore V_4 = (3\ \Omega)I_6$$

$$I_4 = \frac{V_4}{R_4} = \frac{V_4}{6\ \Omega} = \frac{(3\Omega)I_6}{6\ \Omega} = 0.5I_6$$

$$I_3 = I_4 + I_6 = 0.5I_6 + I_6 = 1.5I_6$$

$$V_3 = I_3R_3 = (1.5I_6)(4\Omega) = (6\Omega)I_6$$

Practice Problem
[SECTIONS 7.6]
Problems: 25 ~ 28

$$V_2 = V_3 + V_4 = (6\Omega)I_6 + (3\Omega)I_6 = (9\Omega)I_6$$

$$I_2 = \frac{V_2}{R_2} = \frac{(9\Omega)I_6}{6\ \Omega} = 1.5I_6$$

$$I_s = I_2 + I_3 = 1.5I_6 + 1.5I_6 = 3I_6$$

$$V_1 = I_sR_1 = 3I_6(5\Omega) = (15\Omega)I_6$$

$$E = V_1 + V_2 = (15\Omega)I_6 + (9\Omega)I_6 = (24\Omega)I_6$$

$$I_6 = \frac{E}{24\Omega} = \frac{240\text{V}}{24\Omega} = \mathbf{10\ A}$$

$$V_6 = I_6R_6 = 10\text{ A} \times 2\Omega = \mathbf{20\,V}$$

$$I_4 = 0.5I_6 = 0.5 \times 10\text{ A} = \mathbf{5\ A}$$

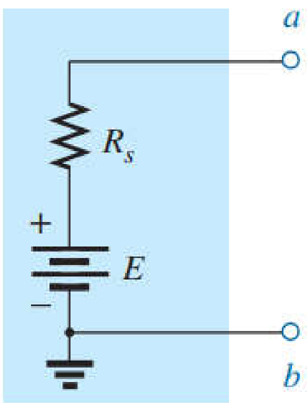
$$I_3 = 1.5I_6 = 1.5 \times 10\text{ A} = \mathbf{15\ A}$$

$$I_2 = 1.5I_6 = 1.5 \times 10\text{ A} = \mathbf{15\ A}$$

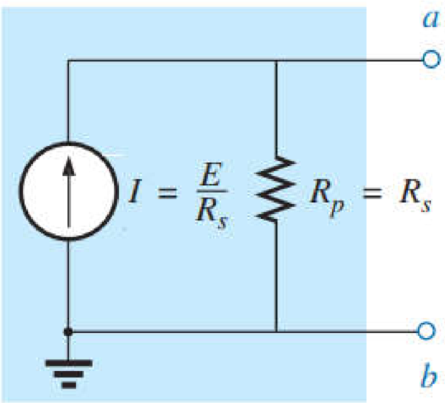
$$I_s = 3I_6 = 3 \times 10\text{ A} = \mathbf{30\ A}$$

8.3 SOURCE Transformation

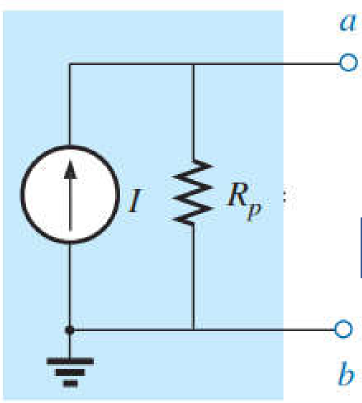
Voltage Source Convert to Current Source



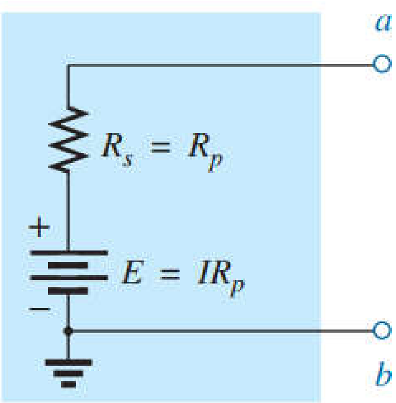
$$I = \frac{E}{R_s} \qquad R_p = R_s$$



Current Source Convert to Voltage Source



$$E = IR_p \qquad R_s = R_p$$



13. Convert the voltage sources in Fig. 8.114 to current sources.

a. Find the voltage V_{ab} and the polarity of points a and b.

b. Find the magnitude and direction of the current I_3.

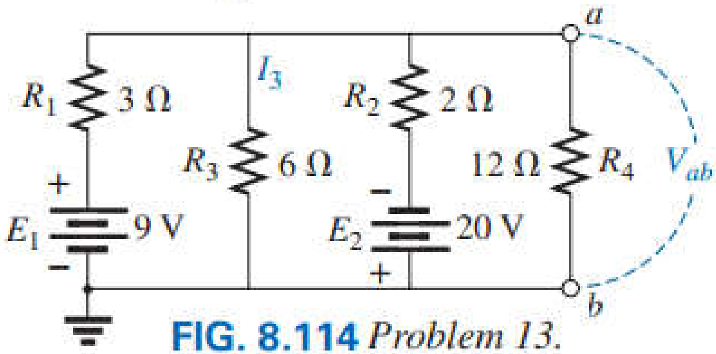
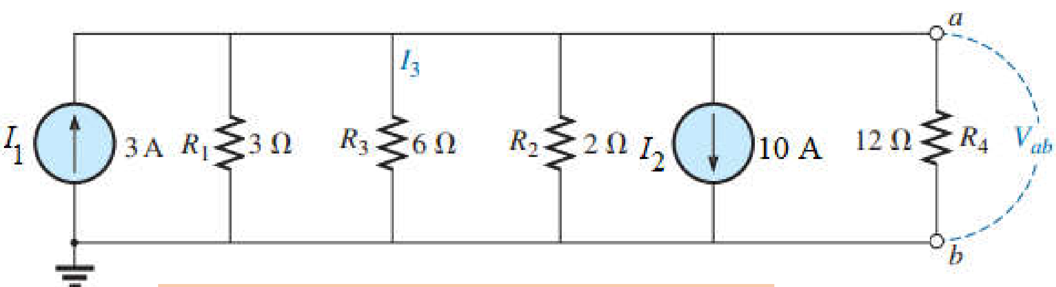
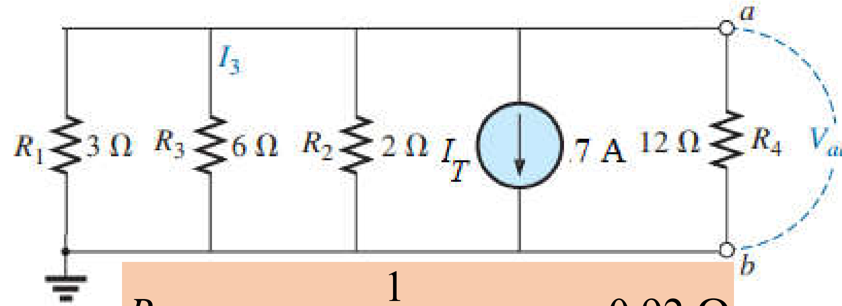


FIG. 8.114 *Problem 13.*

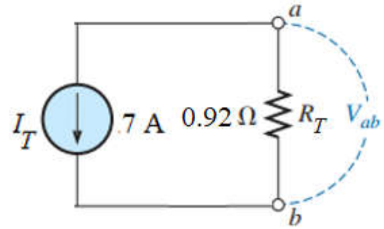
$$I_1 = \frac{E_1}{R_1} = \frac{9\ \text{V}}{3\ \Omega} = 3\ \text{A} \qquad I_2 = \frac{E_2}{R_2} = \frac{20\ \text{V}}{2\ \Omega} = 10\ \text{A}$$



$$I_T \downarrow = I_2 - I_1 = 10\ \text{A} - 3\ \text{A} = 7\ \text{A}$$



$$R_T = \frac{1}{\frac{1}{R_1} + \frac{1}{R_6} + \frac{1}{R_2} + \frac{1}{R_4}} = 0.92\ \Omega$$



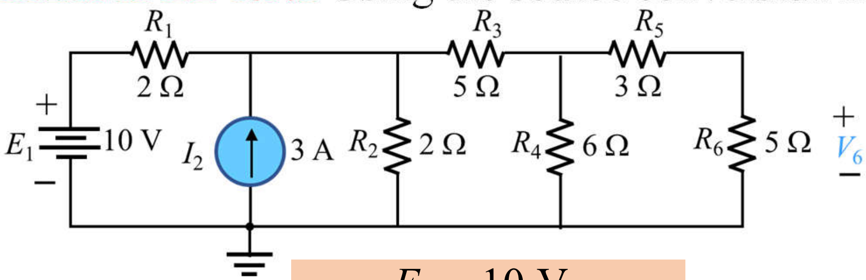
$$V_{ab} = -I_T R_T = \mathbf{-6.44\ V}$$

$$I_3 \uparrow = \frac{-V_{ab}}{R_3} = \frac{6.44\ \text{V}}{6\ \Omega} = \mathbf{1.07\ A}$$

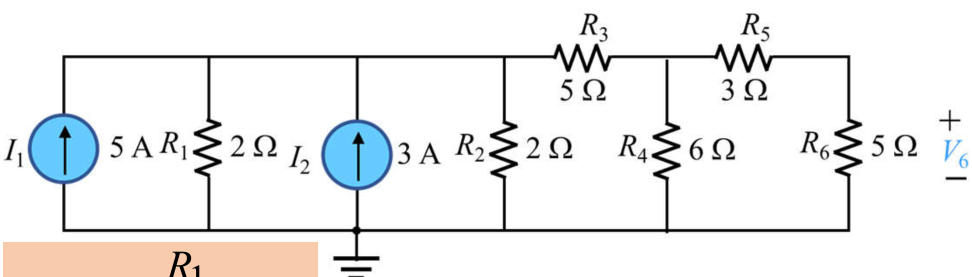
Practice Book [Ch 8]

Problem: 7 ~ 10 and 14

EXAMPLE 8.3.1 Using the source conversion find the value of voltage V_6.

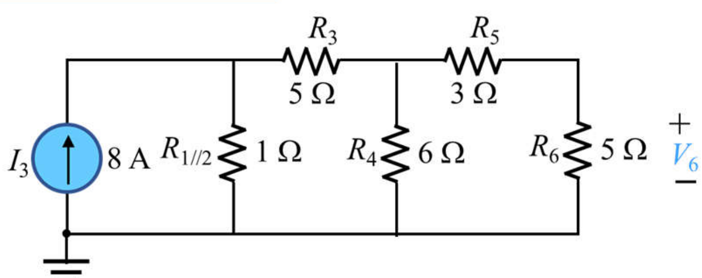


$$I_1 = \frac{E_1}{R_1} = \frac{10\ \text{V}}{2\ \Omega} = 5\ \text{A}$$

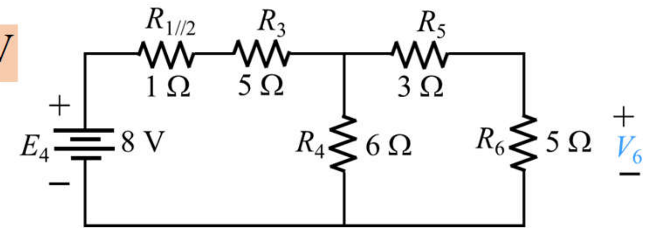


$$R_{1//2} = \frac{R_1}{2} = 1\ \Omega$$

$$I_3 = I_1 + I_2 = 8\ \text{A}$$

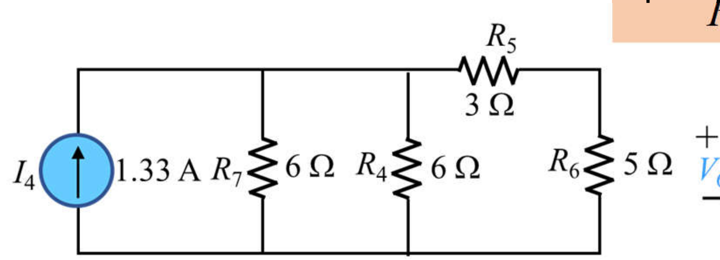


$$E_4 = I_3 R_{1//2} = 8\ \text{V}$$



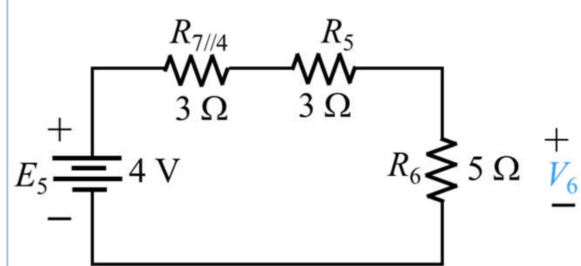
$$R_7 = R_{1//2} + R_3 = 6\ \Omega$$

$$I_4 = \frac{E_4}{R_7} = 1.33\ \text{A}$$



$$R_{7//4} = \frac{R_7}{2} = 3\ \Omega$$

$$E_5 = I_4 R_{7//4} = 4\ \text{V}$$



$$V_6 = \frac{R_6}{R_{7//4} + R_5 + R_6} E_5$$
$$= 1.1\ \text{V}$$

More Examples Related to Series-Parallel Circuits

Problem 12 [Ch 7]: For the network in Fig. 7.72:

a. Determine the current I_1.

b. Calculate the currents I_2 and I_3.

c. Determine the voltage levels V_a and V_b.

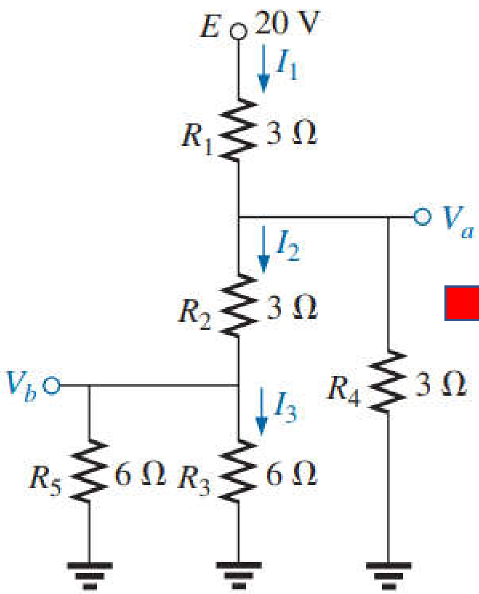
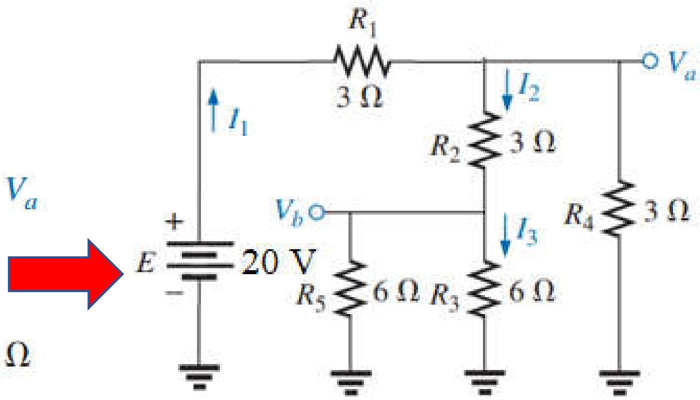


FIG. 7.72
Problem 12.



$$R_b = R_5 \,//\, R_6 = \frac{6\ \Omega}{2} = 3\ \Omega$$

$$R_{b2} = R_b + R_2 = 3\ \Omega + 3\ \Omega = 6\ \Omega$$

$$R_a = \frac{R_{b2}R_4}{R_{b2} + R_4} = 2\ \Omega$$

$$R_T = R_1 + R_a = 3\ \Omega + 2\ \Omega = 5\ \Omega$$

$$I_1 = \frac{E}{R_T} = \frac{20\ \text{V}}{5\ \Omega} = \mathbf{4\ A}$$

$$V_a = \frac{R_a}{R_T}E = \frac{2\ \Omega}{5\ \Omega} \times 20\ \text{V} = \mathbf{8\ V}$$

$$I_2 = \frac{V_a}{R_{b2}} = \frac{8\ \text{V}}{6\ \Omega} = \mathbf{1.333\ A}$$

$$V_b = I_2 R_b = 1.333\ \text{A} \times 3\ \Omega \cong \mathbf{4\ V}$$

$$I_3 = \frac{V_b}{R_3} = \frac{I_2}{2} = \frac{1.333\ \text{A}}{2} = \mathbf{0.667\ A}$$

Problem 10 [Ch 7] Determine the unknown voltage (V) and current (I) for the following network.

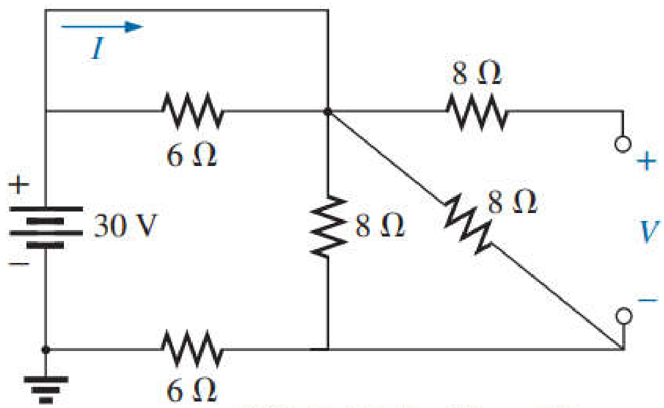
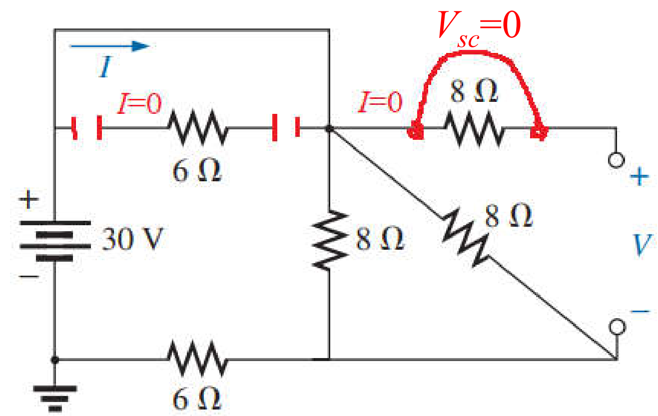
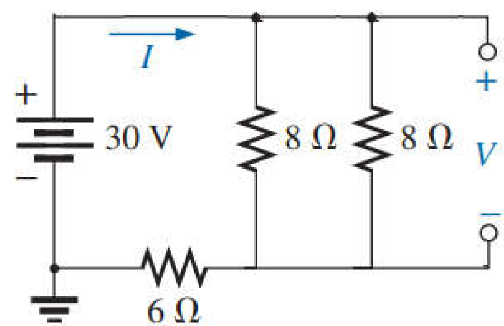
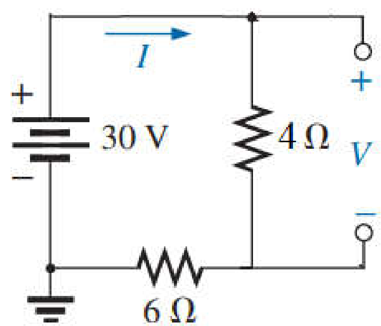


FIG. 7.78 *Problem 18.*





$$R_{8//8} = \frac{8\ \Omega}{2} = 4\ \Omega$$



$$I = \frac{30\ \text{V}}{4\ \Omega + 6\ \Omega} = \mathbf{3\ A}$$

$$V = 3\ \text{A} \times 4\ \Omega = \mathbf{12\ V}$$

Problem 26 For the ladder network in Fig. 7.86: ***a***. Determine R_T. ***b***. Calculate I. ***c***. Find I_8. d. Power consumed by R_6 resistance. e. Power delivered by the 2 V supply.

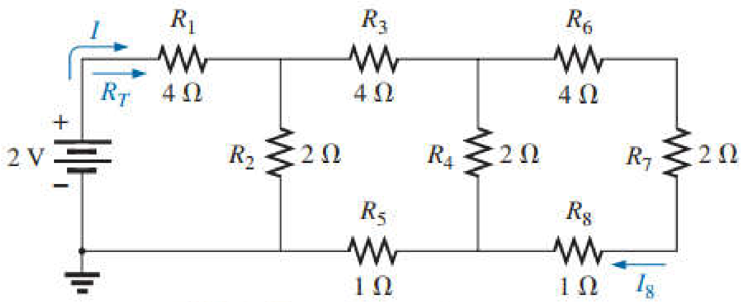
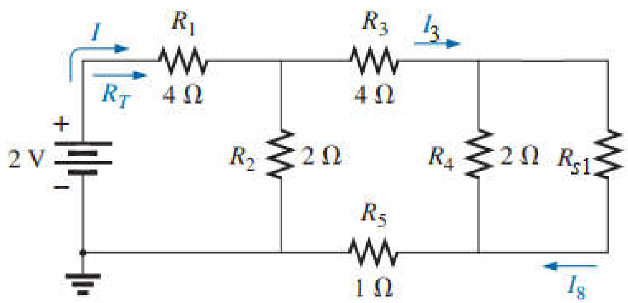
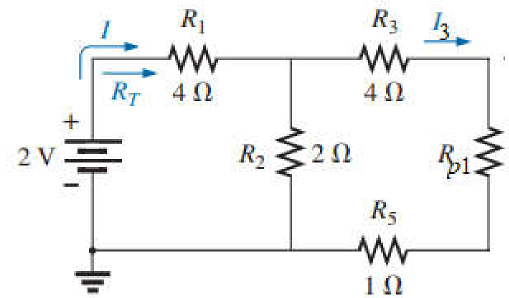


FIG. 7.86 *Problem 26.*

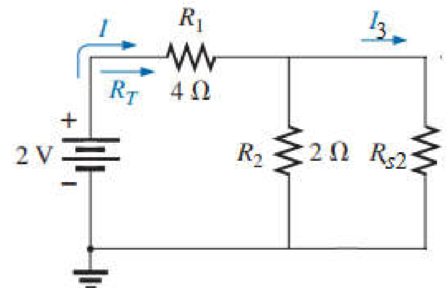
$$R_{s1} = R_6 + R_7 + R_8 = 7\ \Omega$$



$$R_{p1} = \frac{R_{s1}R_4}{R_{s1} + R_4} = 1.56\ \Omega$$



$$R_{s2} = R_3 + R_{p1} + R_5 = 6.56\ \Omega$$



$$R_{p2} = \frac{R_{s2}R_2}{R_{s2} + R_2} = 1.53\ \Omega$$

$$R_T = R_{p2} + R_1 = \mathbf{5.53\ \Omega}$$

$$I = \frac{E}{R_T} = \frac{20\,\text{V}}{5.53\ \Omega} = \mathbf{361.66\,mA}$$

$$I_3 = \frac{R_2}{R_2 + R_{s2}} I = \mathbf{84.50\,mA}$$

$$I_8 = \frac{R_4}{R_4 + R_{s1}} I_3 = \mathbf{18.78\,mA}$$

$$P_6 = I_8^2 R_6 = (18.78 \times 10^{-3})^2 \times 4\Omega$$
$$= \mathbf{1.41\,mW}$$

$$P_E = EI = (2\,\text{V}) \times (361.66 \times 10^{-3})$$
$$= \mathbf{723.32\,mW}$$

Problem 28. [Ch. 7] For the multiple ladder configuration in Fig. 7.88:

a. Determine I. **b.** Calculate I_4. **c.** Find I_6. **d.** Find I_{10}.

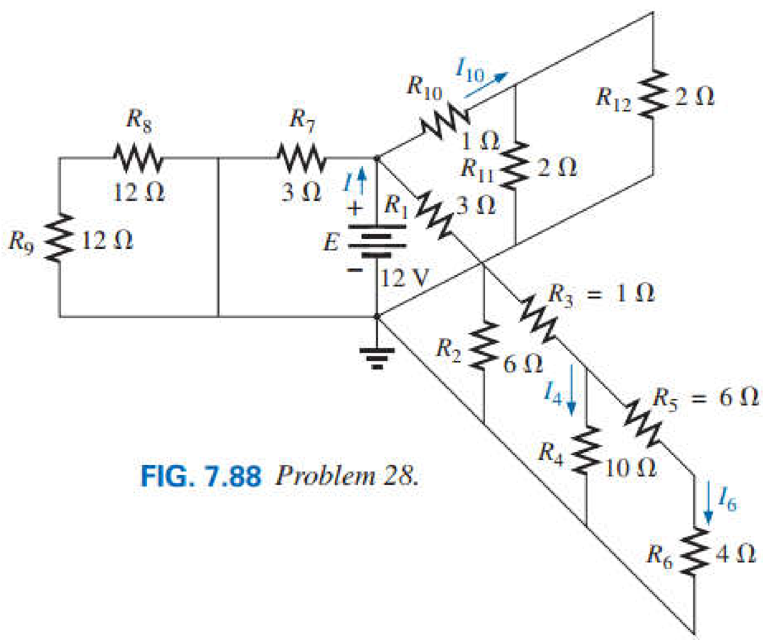


FIG. 7.88 *Problem 28.*

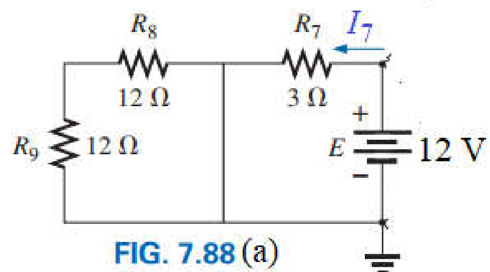


FIG. 7.88 (a)

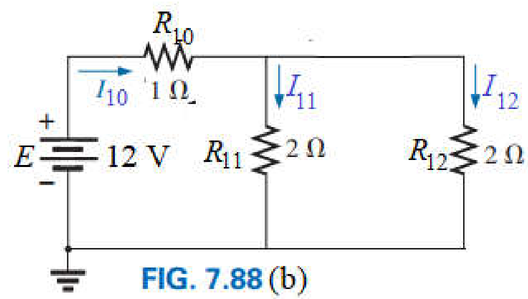


FIG. 7.88 (b)

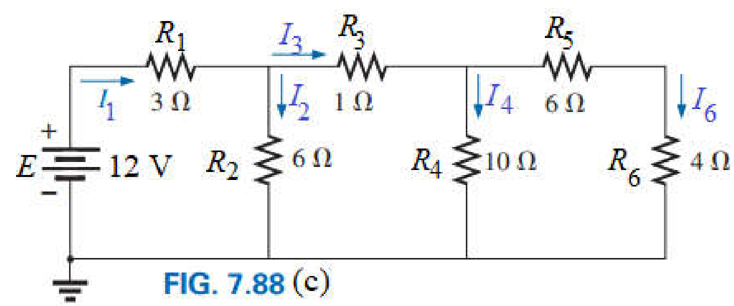


FIG. 7.88 (c)