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AIUB COURSE SOLUTION-ACS

Complex Variables, Laplace & Z-Transfrom

MATH – 3

Chapter -6.1

(((((FULL)))))

Fall 2018-19

Solved By

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AIUB COURSE SOLUTION-ACS

Sample Exercise (Short)

1.Find and sketch the path and its orientation given by:

(i) $z(t) = (1+4i)t\ (1 \le t \le 4)$

(iii) $z(t) = 5e^{it}\ (0 \le t \le \pi)$

(ii) $z(t) = (2-i)t\ (-2 \le t \le 2)$

(iv) $z(t) = 1 + i + 2e^{it} (0 \le t \le 2\pi)$

(v) $z(t) = 2\sin(t) + i\,3\cos(t)\ (0 \le t \le 2\pi)$

(vi) $z(t) = 4\cos(t) + i(2+\sin(t))(0 \le t$

(vii) $z(t) = \cosh(t) + i\sinh(t)(0 \le t \le 4)$.

2. Sketch and represent them parametrically:

(i) Line segment from $1+3i$ to $2-3i$

(ii) unit circle (clockwise)

(iii) $|z-4i| = 4$ (counter clockwise)

(iv) $|z-2+i| = 2$ (counter clockwise)

3. Sketch the path C from $z=0$ to $z=2i$ and hence evaluate $\int_C \operatorname{Im} z\,dz$.

4. Sketch the path C from $z=0$ to $z=4$and hence evaluate $\int_C \operatorname{Re} z^2\,dz$.

5. Sketch the path $C$from $z=1$ to $z=3$and hence evaluate $\int_C 2z\,dz$.

6. Sketch the path $C$from $z=2$ to $z=2+i$and henceevaluate $\int_C \bar{z}\,dz$.

7. Sketch the path C, whichis the unit circle $|z|=1$ and hence evaluate $\int_C (z+z^{-1})dz$.

Exercise Set: 6.1

Sample Exercise (Short)

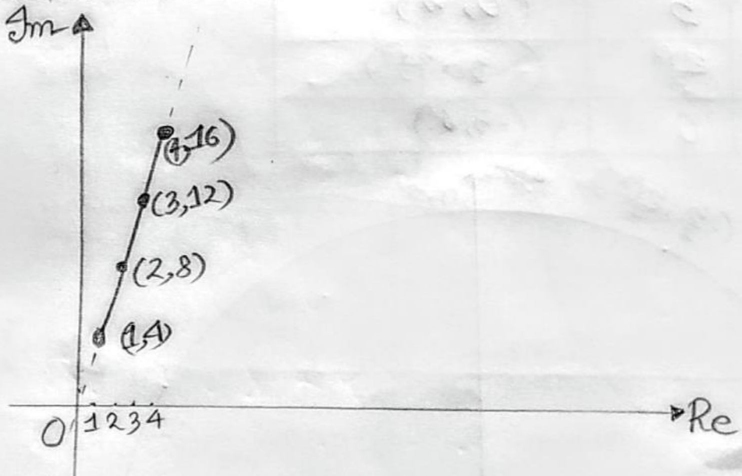
(i) $z(t) = (1+4i)t \quad (1 \le t \le 4)$

$x(t) + iy(t) = t + i4t$

Comparing real and imaginary part, we get

$x(t) = t, \; y(t) = 4t \quad (1 \le t \le 4)$

t	x	y	(x,y)
1	1	4	(1,4)
2	2	8	(2,8)
3	3	12	(3,12)
4	4	16	(4,16)



So, $z(t) = (1+4i)t \; (1 \le t \le 4)$ represents the line segment from $(1,4)$ to $(4,16)$ in complex plane.

1 (ii) $z(t) = (2-i)t \cdot \quad (-2 \le t \le 2$

$z(t) = 2t - it$

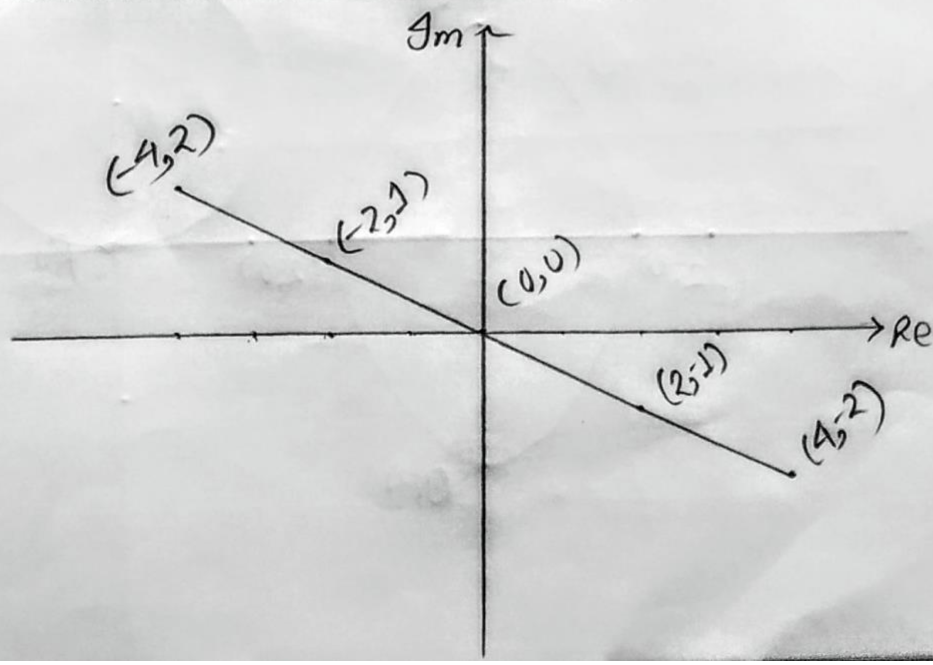
$x(t) + iy(t) = 2t + i(-t)$

$x(t) = 2t$

$y(t) = -t$

$(-2 \le t \le 2)$

t	x	y	x, y
-2	-4	2	$(-4,2)$
-1	-2	1	$(-2,1)$
0	0	0	$(0,0)$
1	2	-1	$(2,-1)$
2	4	-2	$(4,-2)$



(iii) $z(t) = 5e^{it} \; (0 \le t \le \pi)$

$x(t) + iy(t) = 5\cos(t) + i5\sin(t)$

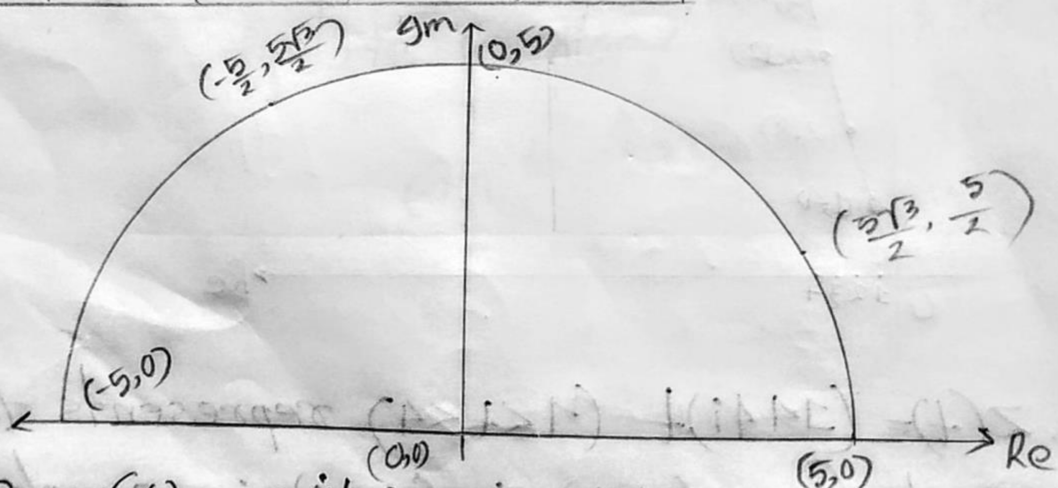
Comparing real and imaginary part, we get

$x(t) = 5\cos(t)$

$y(t) = 5\sin(t)$

$(0 \le t \le \pi)$

t	x	y	(x,y)
0	5	0	$(5,0)$
$\frac{\pi}{6}$	$\frac{5\sqrt{3}}{2}$	$\frac{5}{2}$	$\left(\frac{5\sqrt{3}}{2}, \frac{5}{2}\right)$
$\frac{\pi}{2}$	0	5	$(0,5)$
$\frac{2\pi}{3}$	$-\frac{5}{2}$	$\frac{5\sqrt{3}}{2}$	$\left(-\frac{5}{2}, \frac{5\sqrt{3}}{2}\right)$
π	-5	0	$(-5,0)$



So $z(t) = 5e^{it} \; (0 \le t \le \pi)$ represents upper semicircle of radius 2 with center $(0,0)$.

(iv) $z(t) = 1 + i + 2e^{it} \quad (0 \leq t \leq 2\pi)$

$x(t) + iy(t) = 1 + i + 2\cos(t) + i\,2\sin(t)$

Comparing real and imaginary part, we get

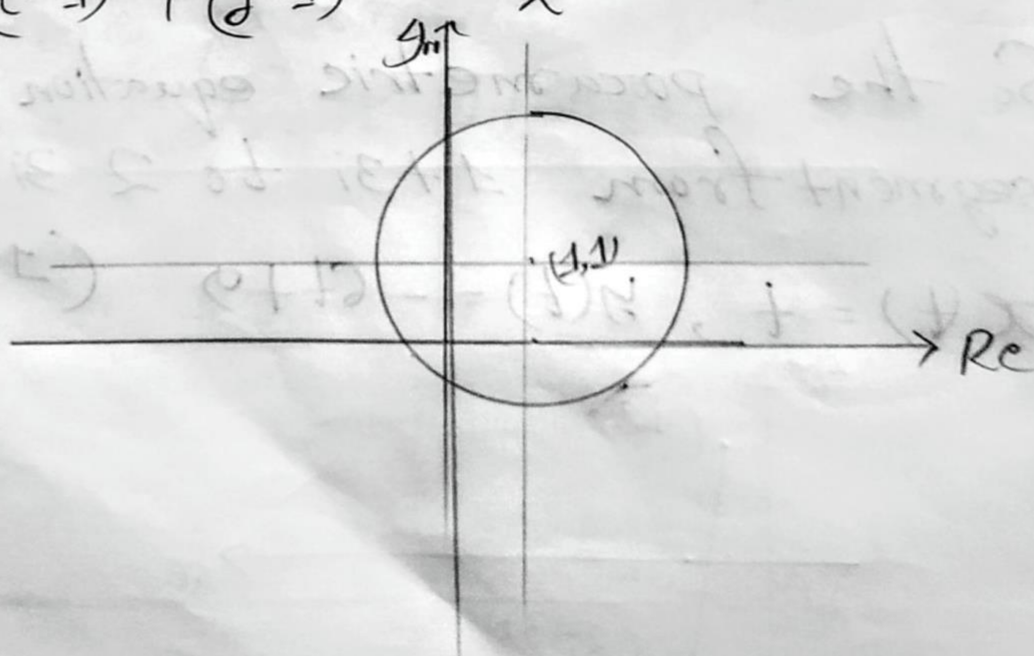
$x(t) = 1 + 2\cos(t) \Rightarrow (x-1) = 2\cos(t)$ — (i)

$y(t) = 1 + 2\sin(t) \Rightarrow (y-1) = 2\sin(t)$ — (ii)

Squaring and subtracting i & ii, we get

$(x-1)^2 + (y-1)^2 = 2^2(\cos^2(t) + \sin^2(t))$

$(x-1)^2 + (y-1)^2 = 2^2$



(v) $z(t) = 2\sin(t) + i3\cos(t) \quad (0 \le t \le 2\pi)$

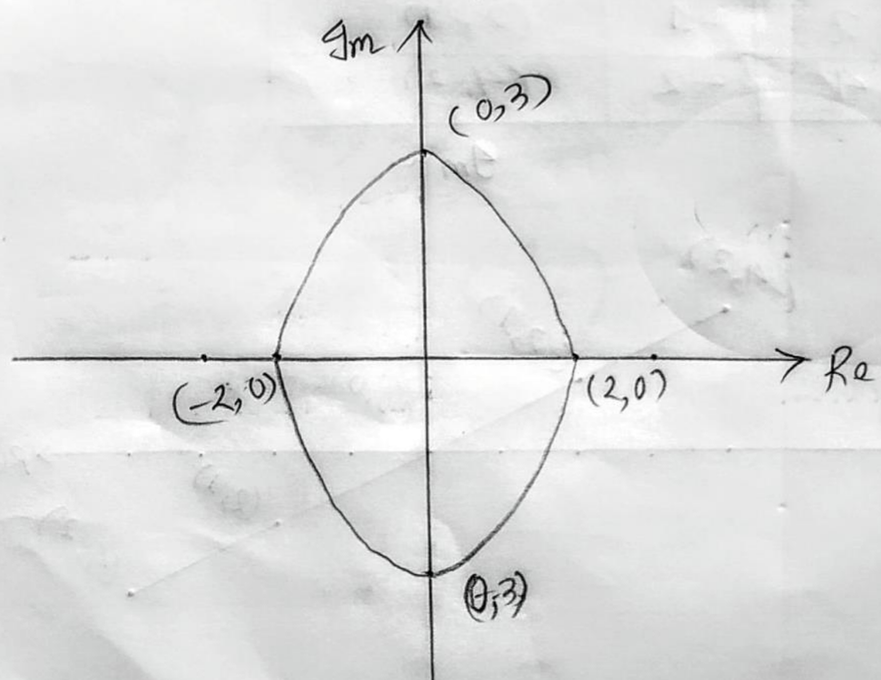
$x(t) + iy(t) = 2\sin(t) + i3\cos(t)$

$x(t) = 2\sin(t)$

$y(t) = 3\cos(t)$

$(0 \le t \le 2\pi)$

t	x	y	(x, y)
0	0	3	(0,3)
$\frac{\pi}{2}$	2	0	(2,0)
π	0	3	(0, 3)
$\frac{3\pi}{2}$	−2	0	(−2,0
2π	0	3	(0,3)



(vi) $z(t) = 4\cos(t) + i(2+\sin(t)) \quad (0 \leq t \leq 2\pi)$

$x(t) + iy(t) = 4\cos(t) + i(2+\sin(t))$

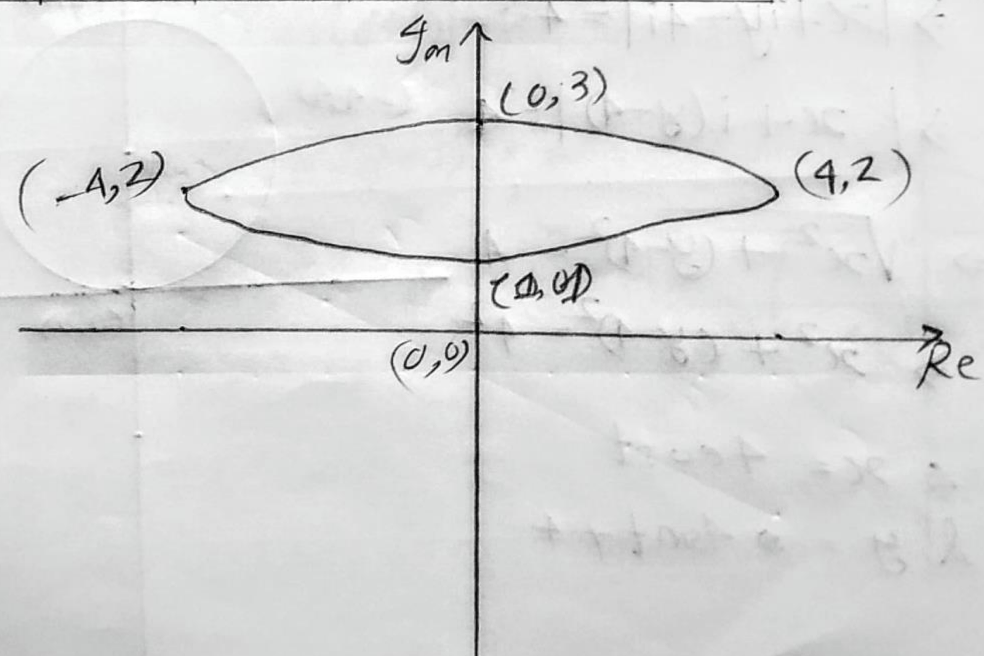
$x(t) = 4\cos(t) \quad \text{--- (i)}$

$y(t) = 2+\sin(t)$

$y(t) - 2 = \sin(t) \quad \text{--- (ii)}$

$(0 \leq t \leq 2\pi)$

t	x	y	(x,y)
0	4	2	$(4,2)$
$\frac{\pi}{2}$	0	3	$(0,3)$
π	-4	2	$(-4,2)$
$\frac{3\pi}{2}$	0	1	$(0,1)$
2π	4	2	$(4,2)$



(vii) $z(t) = \cosh(t) + i\sinh(t) \quad (0 \leq t \leq 4)$

$x(t) + iy(t) = \cosh(t) + i\sinh(t)$

Comparing real and im part, we get

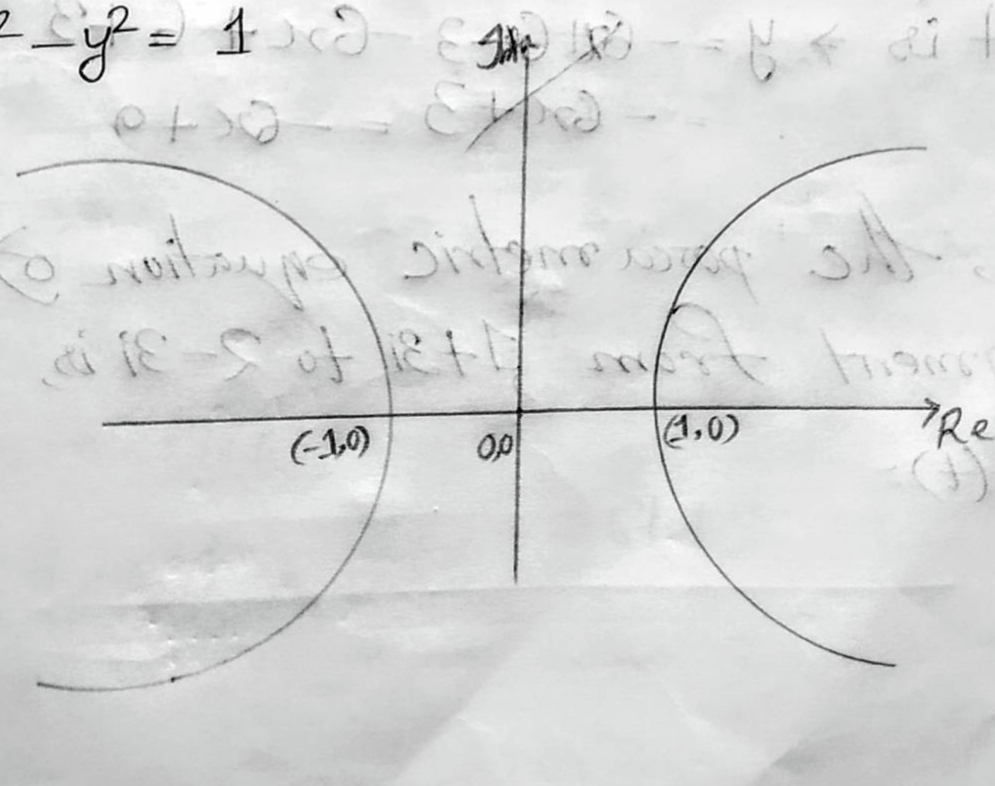
$x(t) = \cosh(t)$ ... (i)

$y(t) = \sinh(t)$ ... (ii)

$(0 \leq t \leq 4)$

By squaring and subtracting i & ii, we get

$x^2 - y^2 = 1$



2.

(i) Line segment from $1+3i$ to $2-3i$

The equation of straight line passing through the points $(1,3)$ to $(2,-3)$ is

$$\therefore y-3 = \frac{-3-3}{2-1}(x-1) \quad \left[y-y_1 = \frac{y_2-y_1}{x_2-x_1}(x-x_1) \right]$$

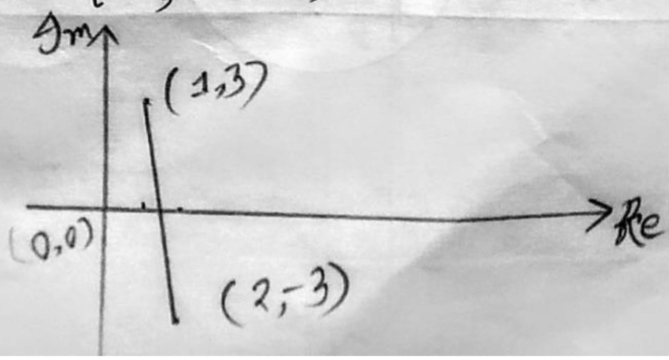
$$\Rightarrow y-3 = -6x+6$$

$$\Rightarrow y = -6x+9$$

Let, $x=t$ then $y=-6t+9$ where t varies from $t=1$ to $t=2$

So the parametric equation of line segment from $1+3i$ to $2-3i$

$$x(t) = t, \quad y(t) = -6t+9 \quad (1 \le t \le 2)$$



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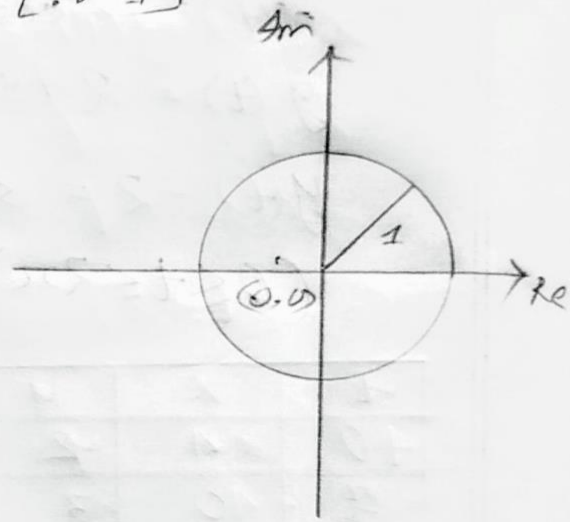
2 (ii) Unit circle equation (clockwise)

$x^2+y^2=1$

Let $x = r\cos t = \cos t$ [$\because r=1$]

$y = r\sin t = \sin t$

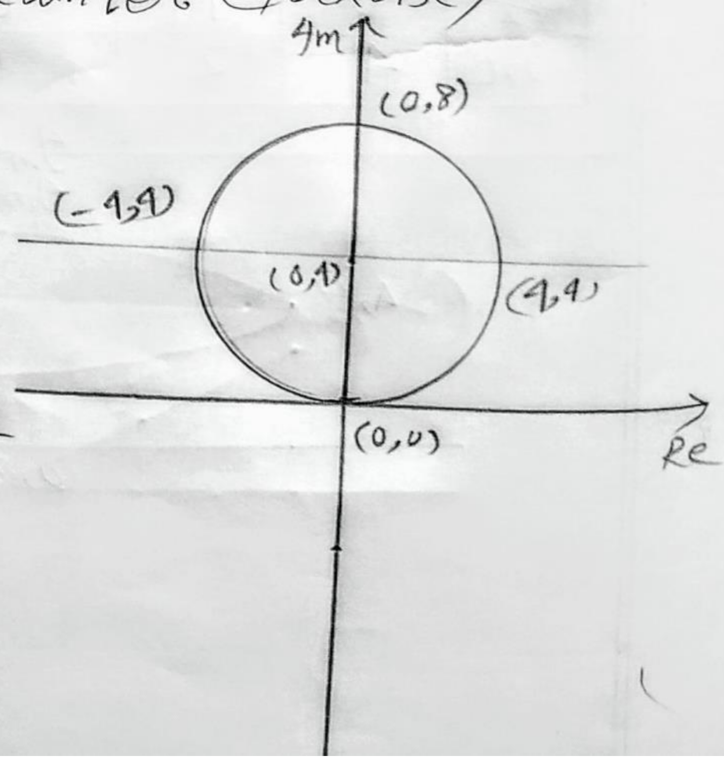


$\therefore z = \cos t + i\sin t$

$\Rightarrow z = e^{it}$

$\Rightarrow |z| = 1$

(iii) $|z-4i| = 4$ (Counter Clockwise)



$\Rightarrow |x+iy-4i| = 4$

$\Rightarrow |x+i(y-4)| = 4$

$\Rightarrow \sqrt{x^2+(y-4)^2} = 4$

$\Rightarrow x^2+(y-4)^2 = 4^2$

$\therefore x = 4\cos t$

& $y = 4\sin t + 4$

$\therefore z = x + iy$

$= 4\cos t + i(4\sin t + 4)$

$= 4\cos t + i4\sin t + 4i$

$= 4e^{it} + 4i$

$= 4(e^{it} + i)$

(iv) $|z - 2 + i| = 2$ (counter clockwise)

$\Rightarrow |x + iy - 2 + i| = 2$

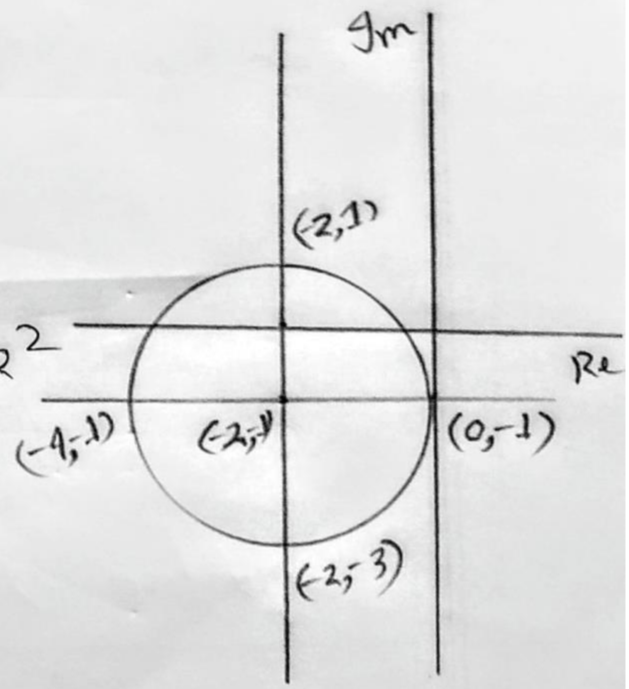
$\Rightarrow |(x-2) + i(y+1)| = 2$

$\Rightarrow \sqrt{(x-2)^2 + (y+1)^2} = 2$

$\Rightarrow (x-2)^2 + (y-(-1))^2 = 2^2$

$\therefore x = 2 + 2\cos t$

$\&\ y = -1 + 2\sin t$



$\therefore z = x + iy$

$= 2 + 2\cos t + i(-1 + 2\sin t$

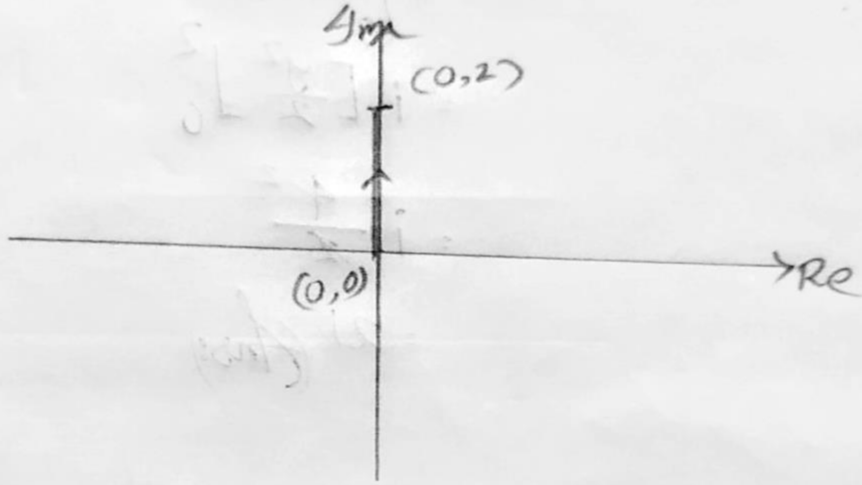
$= 2 + 2\cos t + i2\sin t - i$

$= 2 - i + 2e^{it}$

Ans:

3. $C: z=0$ to $z=2i$ hence $\int_C \operatorname{Im} z\, dz$

$(0,0)$ $(0,2)$



equation of $C \Rightarrow x=0$

Now, $\operatorname{Im}\{z\} = \operatorname{Im}\{x+iy\}$

$= y$

again $z = x+iy$

$= 0+iy$

$\therefore \frac{dz}{dy} = i$

$\therefore dz = i\,dy$

$$\therefore \int_C \mathrm{Im}\, z\, dz = \int_0^2 yi\,dy$$

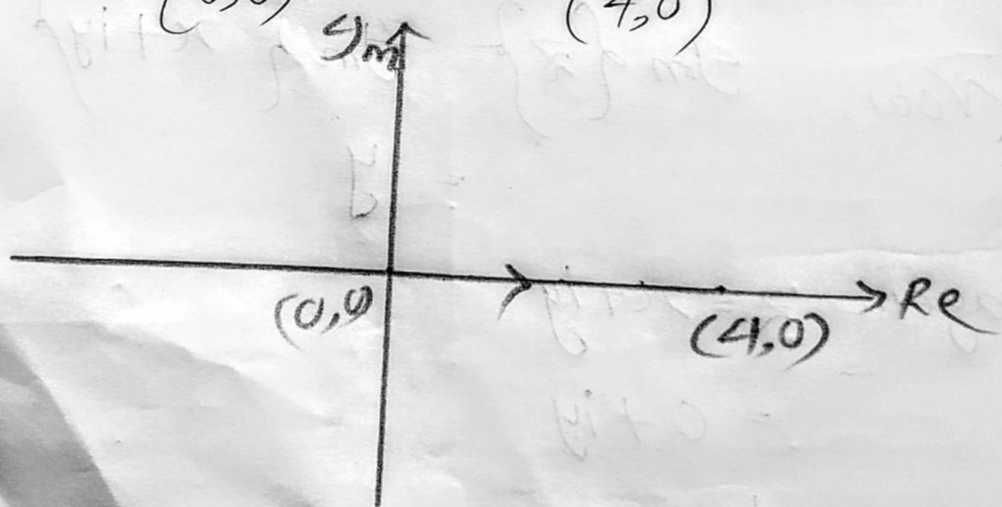
$$= i\int_0^2 y\,dy$$

$$= i\left[\frac{y^2}{2}\right]_0^2$$

$$= i\,\frac{\cancel{4}^{\,2}}{\cancel{2}}$$

$$= 2i \text{ (Ans:)}$$

4. C : $z=0$ to $z=4$ hence $\int_C \mathrm{Re}\, z^2 dz$
$(0,0)$ $(4,0)$



equation of $C \Rightarrow y=0$

$\therefore \ Re\{z^2\} = Re\{(x+iy)^2\}$

$= Re\{x^2 + 2xiy - y^2\}$

$= x^2 - y^2$

$= x^2 - 0^2$

$= x^2$

$\therefore \ Re\{z^2\} = x^2$

again, $z = (x+iy)$

$z = x$

$\therefore dz = dx$

$\therefore \int_C Re\{z^2\}\, dz = \int_0^4 x^2\, dx$

$= \left[\frac{x^3}{3}\right]_0^4$

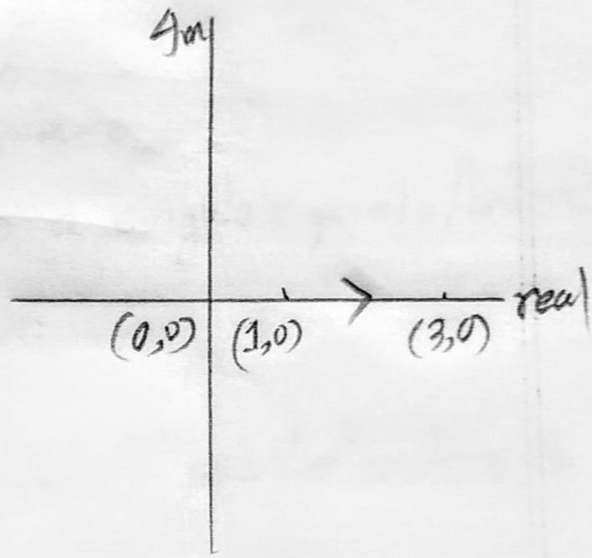
$= \frac{64}{3}$

(Ans:)

5. Given $z=1$ to $z=3$

Along C: $y=0$

$$z = x \qquad 1 \leq x \leq 3$$



$$\Rightarrow \frac{dz}{dx} = 1$$

$$\Rightarrow dz = dx$$

$$\int_C 2z\,dz = 2\int_1^3 x\,dx$$

$$= 2\left[\frac{x^2}{2}\right]_1^3$$

$$= 2\left(\frac{9}{2} - \frac{1}{2}\right)$$

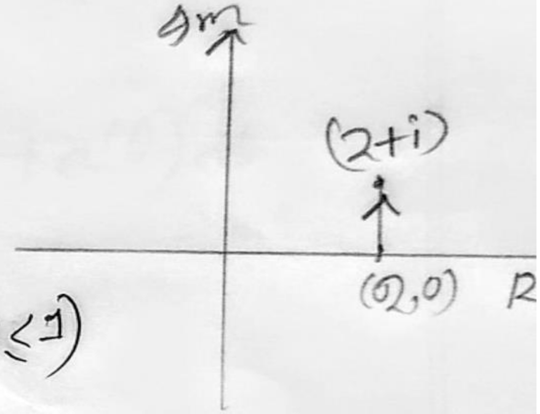
$$= 2\,\frac{\cancel{8}^{\,4}}{\cancel{2}}$$

$$= 2 \times 4$$

$$= 8 \text{ (Ans:)}$$

6. $z=2$ to $z=2+i$ hence Evaluate $\int_c \bar{z}\,dz$

Given, $z=2$ to $z=2+i$



Along c: $x=2$

$$z = 2+iy$$

$$dz = i\,dy \quad (0 \leq y \leq 1)$$

$$\int \bar{z}\,dz = \int_0^1 i(2-iy)\,dy$$

$$= i\int_0^1 2\,dy - i\int_0^1 iy\,dy$$

$$= i\left[2y\right]_0^1 - i^2\left[\frac{y^2}{2}\right]_0^1$$

$$= 2i - \frac{1}{2}i^2$$

(Ans.)

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7.

Along $C: |z| = 1$

$$\Rightarrow z = e^{i\theta} \quad dz = ie^{i\theta}\, d\theta \;(0 \leq \theta \leq 2\pi)$$

Evaluate $\int_C (z + z^{-1})\, dz$

$$= \int_0^{2\pi} ie^{i\theta}(e^{i\theta} + e^{-i\theta})\, d\theta$$

$$= i\int_0^{2\pi} (e^{2i\theta} + 1)\, d\theta$$

$$= i\left[\frac{1}{2i} e^{i2\theta} + \theta\right]_0^{2\pi}$$

$$= i\left[\frac{e^{i4\pi}}{2i} + 2\pi - \frac{1}{2i}\right]$$

$$= i\left(\frac{1}{2i}(\cos 4\pi + i\sin 4\pi) + 2\pi - \frac{1}{2i}\right)$$

$$= i\left(\cancel{\frac{1}{2i}} + 2\pi - \cancel{\frac{1}{2i}}\right)$$

$$= 2\pi i \quad \text{(Ans:)}$$

Sample Exercise

Sketch the corresponding paths and hence evaluate the followings:

8. $\int_C e^{2z} dz$, C is the shortest path from i to $2i$.

9. $\int_C \bar{z}\, dz$, C from 0 to $2+4i$ along the parabola $y = x^2$.

10. $\int_C \operatorname{Re} z^2 dz$, C is the boundary of the square with vertices 0, i, 1 + i, 1 clockwise.

11. $\int_C \operatorname{Im} z^2 dz$, C is the patharound the triangle with vertices $0, 2i, 1+2i$ (counter clockwise)

12. $\int_C \frac{1}{z-3} dz$, C is a circle $|z-3| = R$.

13. $\int_C \left(\frac{1}{z+i} - \frac{2}{(z+i)^2}\right) dz$, C is the circle $|z+i| = 2$, clockwise.

14. $\int_C \operatorname{Re} z\, dz$, C isthe shortest path from 0 to $1-3i$ along $z(t) = t - i3t, 0 \le t \le 1$.

15. $\int_C f(z) dz$, where $f(z) = 2x + y - 2xi$

along C

(a) is shortest path from $z = 0$ to $z = i + 1$.

(b) consists of two line segments, one from $z = 0$ to $z = i$ and other from $z = i$ to $z = i + 1$.

16. $\int_C \sinh \pi z\ dz$, C from i along the y-axis to 0.

17. $\int_C \ln(z) dz$, C is the shortest path from 1 to 2.

18. $\int_C \operatorname{Re} z dz$, along C where

(a) C isthe shortest path from 0 to $1+2i$.

(b) C is consisting of $C1$ and $C2$.

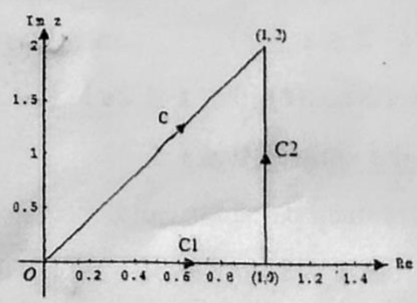
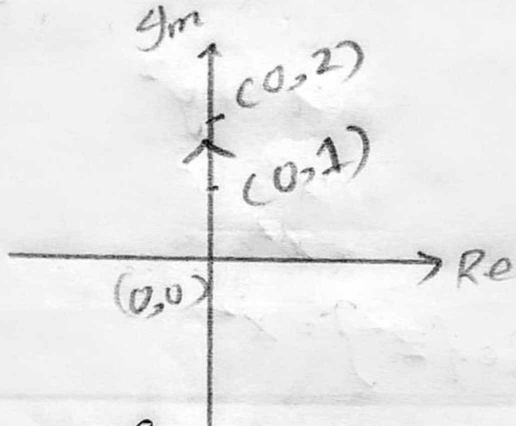


Fig: 8(i)

Cauchy Goursat Theorem (CCT)

8. $\int_c e^{2z} dz$, c : i to $2i$
$(0,1)$ to $(0,2)$



Equation ~~of~~ c : $x = 0$

Now $e^{2z} = e^{2(x+iy)}$

$= e^{2iy}$

and $z = x+iy$

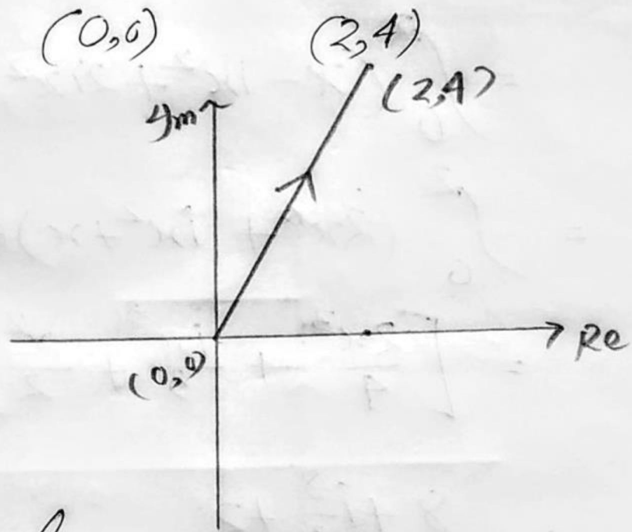
$z = iy$

$\therefore dz = i\,dy$

$\therefore \int_c e^{2z} dz = \int_1^2 e^{2iy}\, i\,dy$

$= i\left[\frac{e^{2iy}}{2i}\right]_1^2 = \frac{1}{2} e^{4i} - \frac{e^{2i}}{2}$

9. $\int_C \bar{z}\,dz$ C: 0 to $2+4i$, along $y = x^2$ [Parabola]



Equation of c : $y = x^2$

Now, $\bar{z} = x - iy$

$\therefore \bar{z} = x - ix^2$

& $\frac{dz}{dx} = 1 - 2ix$

$dz = dx - 2ix\,dx$

$= 1 -$

$z = x + iy$

$z = x + ix^2$

$\frac{dz}{dx} = 1 + 2ix$

$dz = (1 + 2ix)\,dx$

$$\therefore \int_C \bar{z}\, dz = \int_0^2 (x - ix^2)(1 + 2ix)\,dx$$

$$= \int_0^2 (x - ix^2 + 2ix^2 + 2x^3)\,dx$$

$$= \int_0^2 (2x^3 + ix^2 + x)\,dx$$

$$= \left[\frac{2x^3}{4} + \frac{ix^3}{3} + \frac{x^2}{2}\right]_0^2$$

$$= 8 + i\frac{8}{3} + 2$$

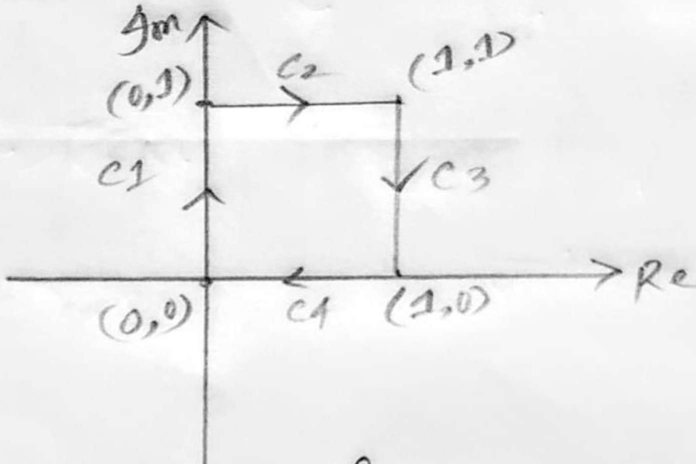
$$= 10 + \frac{8}{3}i \text{ (Ans:)}$$

10.

$\int_C \mathrm{Re}\, z^2 dz$, c: boundary of square with vertices

0, i, 1+i, 1

(0,0), (0,1), (1,1), (1,0)



For C_1: equation of C_1 is: $x = 0$

Now $\mathrm{Re}\{z^2\} = \mathrm{Re}\{(x+iy)^2\}$

$= \mathrm{Re}\{x^2 - y^2 + 2ixy\}$

$= x^2 - y^2$

$= 0^2 - y^2$

$= -y^2$

And $z = x + iy$

$= iy$

$dz = i\,dy$

$\therefore \int_{C_1} Re\{z^2\}dz = \int_0^1 -y^2 i\,dy$

$= -i\left[\frac{y^3}{3}\right]_0^1$

$= \frac{-i}{3}$

For c_2 : equation of c_2 is $y = 1$

Now, $Re\{z^2\} = x^2 - y^2$

$= x^2 - 1$

And $z = x + iy$

$z = x + i$

$dz = dx$

$\therefore \int_{C_2} Re\{z^2\}dz = \int_0^1 (x^2 - 1)\,dx$

$= \left[\frac{x^3}{3} - x\right]_0^1$

$= \frac{1}{3} - 1$

$= \frac{-2}{3}$

For C_3 equation of C_3 : $x=1$

Now, $Re\{z^2\} = x^2 - y^2$

$= 1 - y^2$

And, $z = x + iy$ $\quad dz = i\,dy$

$= 1 + iy$

$\therefore \int_{C_3} Re\{z^2\}\,dz = \int_1^0 (1-y^2)\,i\,dy$

$= -\frac{2i}{3}$

For C_4 equation C_4 is $y=0$

$\therefore Re\{z^2\} = x^2$ & $z = x + iy$

$z = x$

$\therefore dz = dx$

$\therefore \int_{C_4} Re\{z^2\}\,dz = \int_1^0 x^2\,dx$

$= \left[\frac{x^3}{3}\right]_1^0$

$= -\frac{1}{3}$

$\int_C Re\{z^2\}\,dz = -\frac{i}{3} - \frac{2i}{3} - \frac{1}{3} = -1 - i$

11.

Along C_1 : $x=0$ $\quad z = iy$

$dz = idy$

$$\therefore \int_{C_1} Im(z^2)dz = \int_0^2 2xiy\,d\cancel{x}y$$

$$= \int_0^2 2iy \times 0 \times i\,dy$$

$$= 0$$

Along C_2 :

$(x_1, y_1) = (0,2)$
$(x_2, y_2) = (1,2)$

$$\frac{y-y_1}{y_1-y_2} = \frac{x-x_1}{x_1-x_2}$$

$$\Rightarrow \frac{y-2}{\cancel{2}-\cancel{2}} = \frac{x-0}{0-1}$$

$$\Rightarrow \frac{y-2}{0} = \frac{x-0}{-1}$$

$$\Rightarrow 0 = \frac{x}{-1}$$

$$\therefore x = 0$$

$z = x + iy$

$z = iy$

$dz = idy$

$$\int_{c_2} Im\,(z^2)\,dz = \int_1^1 2xyxi\,dy$$
$$= 0$$

Aloge c_3: $\frac{y-y_1}{y_1-y_2} = \frac{x-x_1}{x_1-x_2}$

$(x_1, y_1) = (1, 2)$
$(x_2, y_2) = (0, 0)$

$$\Rightarrow \frac{y-2}{2-0} = \frac{x-1}{1-0}$$

$$\Rightarrow y-2 = 2x-1$$
$$\Rightarrow y = 2x+1$$

$$z = x+iy$$
$$= x + 2xi + i$$

$$dz = (1+2i)\,dx$$

$$\therefore \int_{c_3} Im\,(z^2)\,dz = \int_1^0 2xy\,(1+2i)\,dx$$

$$= \int_1^0 2xy + 4xyi\,dx$$

$$= \cancel{\int 2y \left\{ \left[\frac{x^2}{2}\right]_1^0 + i\left[\frac{x^2}{2}\right]_1^0 \right\}}$$

$$\cancel{= -2y\frac{1}{2} - i2y\frac{1}{2}}$$

$$\cancel{= -y - iy}$$

$$\cancel{\therefore \int_C Im(z^2)\,dz = 0 + 0 - y - iy}$$

$$\int_1^0 (4x^2+2x)(1+2i)\,dx$$

$$= \int_1^0 (4x^2 + 8ix^2 + 2x + 4ix)\,dy$$

$$= 2\left\{\left[\frac{2x^3}{3}\right]_1^0 + \left[\frac{8ix^3}{3}\right]_1^0 + \left[\frac{2x^2}{2}\right]_1^0 + \left[\frac{4ix^2}{2}\right]_1^0\right\}$$

$$= -2\frac{2}{3} + \left(-\frac{8i}{3}\right) + (-1) + \left(-\frac{4i}{2}\right)$$

$$= -\frac{4}{3} - \frac{8i}{3} - 1 - 2i$$

$$= -\frac{7}{3} - \frac{14}{3}i$$

$$\therefore \int_C Im(z^2)\,dz = 0 + 0 - \frac{7}{3} - \frac{14}{3}i$$

12.

$$\int_C \frac{1}{z-3}\,dz \qquad C: |z-3| = R$$

$$\Rightarrow z-3 = Re^{i\theta}$$

$$\therefore \int_0^{2\pi} \frac{1}{Re^{i\theta}} Rie^{i\theta} d\theta \qquad \Rightarrow dz = Rie^{i\theta} d\theta$$

$$= \int_0^{2\pi} i\,d\theta$$

$$= i\,[\theta]_0^{2\pi}$$

$$= i.2\pi \text{ (Ans:)}$$

13. $$\int_C \left(\frac{1}{z+i} - \frac{2}{(z+i)^2}\right) dz \qquad C: |z+i| = 2$$

Equation of $C: |z+i| = 2$

$$z+i = 2e^{i\theta}$$

and $z = re^{i\theta}$

$$z = 2e^{i\theta}$$

$$dz = 2ie^{i\theta} d\theta.$$

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$$\int_C \left(\frac{1}{(z+i)} - \frac{2}{(z+i)^2} \right) dz$$

$$= \int_{2\pi}^{0} \left(\frac{1}{2e^{i\theta}} - \frac{2}{(2e^{i\theta})^2} \right) 2ie^{i\theta} d\theta$$

$$= \int_{2\pi}^{0} \left(i - \frac{i}{e^{i\theta}} \right) d\theta$$

$$= \left[i\theta - \frac{ie^{-i\theta}}{-i} \right]_{2\pi}^{0}$$

$$= 1 - 2\pi i - e^{-2\pi i}$$

$$\left[\begin{array}{l} e^{2\pi i} = e^{4\pi i} = 1 \\ e^{\pi i} = e^{3\pi i} = -1 \end{array} \right]$$

$$= -2\pi i$$

(Ans:)

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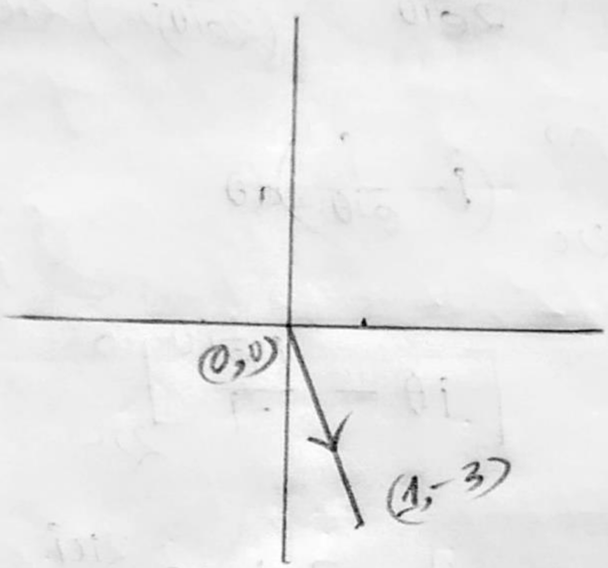
14. $\int_C Re\{z\}\,dz$ C: 0 to $1-3i$ along $z(t)=t-i3t$

$0 \le t \le 1$



equation of C: $z = t-3it$

$\Rightarrow x+iy = t-3it$

$\therefore x=t \;; y=-3t$

$Re\{z\} = Re\{x+iy\}$

$= x$

$= t$

And, $z = x + iy$

$= t - 3it$

$dz = dt - 3i\,dt$

$dz = (1-3i)\,dt$

$\therefore \int_C Re\{z\}\,dz = \int_0^1 t\,(1+3i)\,dt$

$= (1-3i)\left[\frac{t^2}{2}\right]_0^1$

$= \frac{1-3i}{2}$

15. $\int_C f(z)\,dz$ i) $f(z) = 2x + y - 2xi$

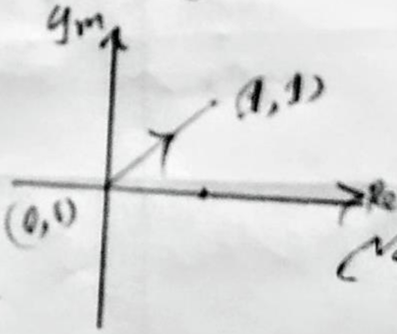
(a) C is shortest path form $z=0$ to $z=1+i$

equation of C : $y = x$

formula

$y - 0 = \frac{1-0}{1-0}(x-0)$

$\therefore y = x$



$z = x + iy$

$\frac{dz}{dx} = \frac{d}{dx}(x + ix)$

$dz = (1+i)\,dx$

Now $f(z) = 2x + y - 2xi$

$= 2x + x - 2xi$

$= 3x - 2xi$

$$\therefore \int_C f(z)\,dz = \int_0^1 (3x - 2xi)(1+i)\,dx$$

$$= (1+i)\left[\frac{3x^2}{2} - ix^2\right]_0^1$$

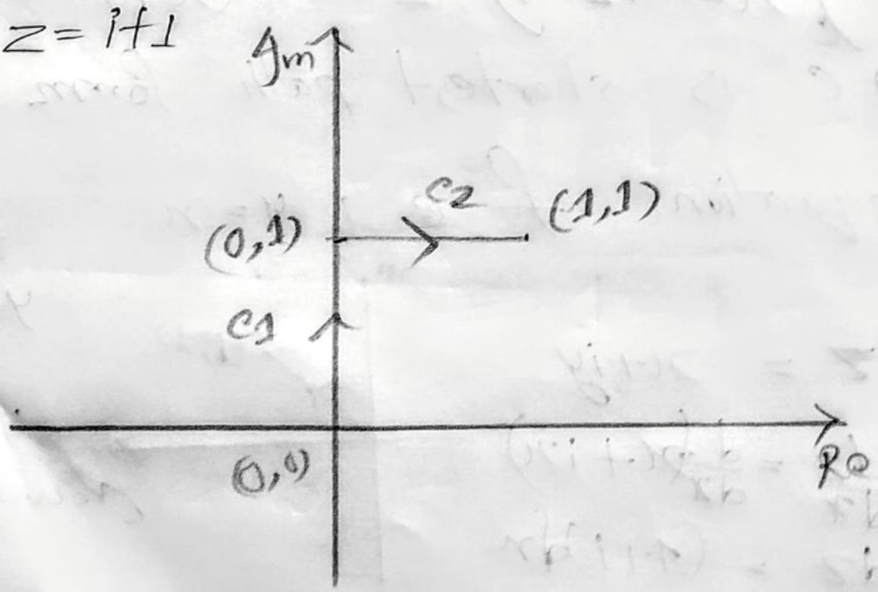
$$= (1+i)\left(\frac{3}{2} - i\right)$$

$$= \frac{5}{2} + \frac{i}{2}$$

(Ans.)

15(b) $\int_C f(z)\,dz$ where $f(z) = 2x + y - 2xi$

consists of two line segments one from $z = 0$ to $z = i$ and other from $z = i$ to $z = i+1$



$\int_C f(z)dz$ if $f(z) = 2x + y - 2xi$

for c_1 = equation of c_1 is $x=0$

$$f(z) = 2x + y - 2ix\cancel{y}$$
$$= 0 + y - 0$$
$$= y$$

and

$$z = x + iy$$
$$= iy$$
$$dz = idy$$

$$\int_{c_1} f(z)dz$$
$$= \int_0^1 y\, idy$$
$$= i\left[\frac{y^2}{2}\right]_0^1$$
$$= i\,\frac{1}{2}$$
$$= \frac{i}{2}$$

for C_2

equation of c_2 : $y = 1$

$f(z) = 2x + y - 2xi$

$= 2x + 1 - 2xi$

And $z = x + iy$

$z = x + i$

$dz = dx$

$\therefore \int_{C_2} f(z)dz = \int_0^1 (2x - 1 - 2xi)\, dx$

$= \left[\frac{2x^2}{2}\right]_0^1 - [x]_0^1 - \left[\frac{2ix^2}{2}\right]_0^1$

$= 1 - 1 - 1$

$= -1$

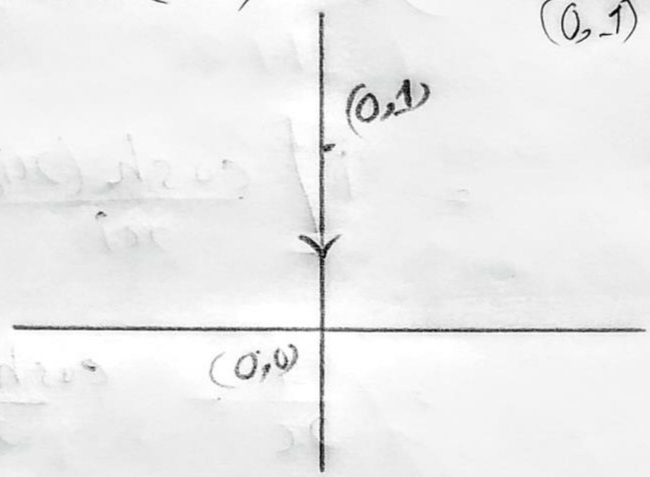
$\therefore \int_C f(z)dz = \frac{i}{2} - 1$

(Ans)

16.

$$\int_C \sinh(\pi z)\,dz \qquad c: i \text{ to } 0i$$

$(0,1) \qquad (0,0)$



equation of c: $x=0$

$$\sinh(\pi z) = \sinh\{\pi(x+iy)\} = \sinh i\pi y$$

& $z = x+iy$

$= iy$

$dz = i\,dy$

$$\therefore \int_C \sinh(\pi z)dz = \int_1^0 \sinh(\pi i y)\, i\, dy$$

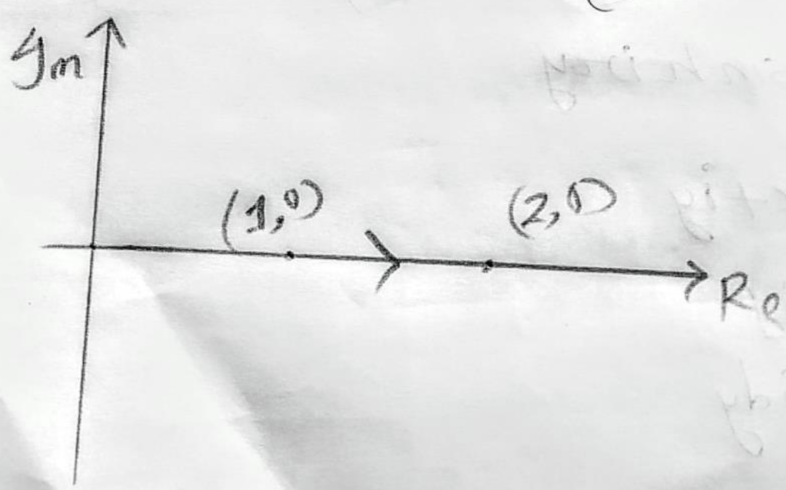
$$= i\left[\frac{\cosh(\pi i y)}{\pi i}\right]_1^0$$

$$= \frac{1}{\pi} - \frac{\cosh \pi i}{\pi}$$

(Ans)

17. $\int_C Im(z)\, dz \qquad C: 1 \text{ to } 2$

$(1,0) \qquad (2,0)$



equation c is : $y = 0$

$\Rightarrow \ln(z) = \ln(x+iy)$

$= \ln x$

$z = x+iy$

$= x+0$

$= x$

$dz = dx$

let, $\ln x = u$

$\therefore \int_c \ln(z)\,dz = \int_1^2 \ln x\,dx$

x	2	1
u	.69	0

$= \cancel{\left[x\ln(x) - x\right]_1^2}$

$= \int_0^{.69} u\,du$

$= \left[\frac{u^2}{2}\right]_0^{.69}$

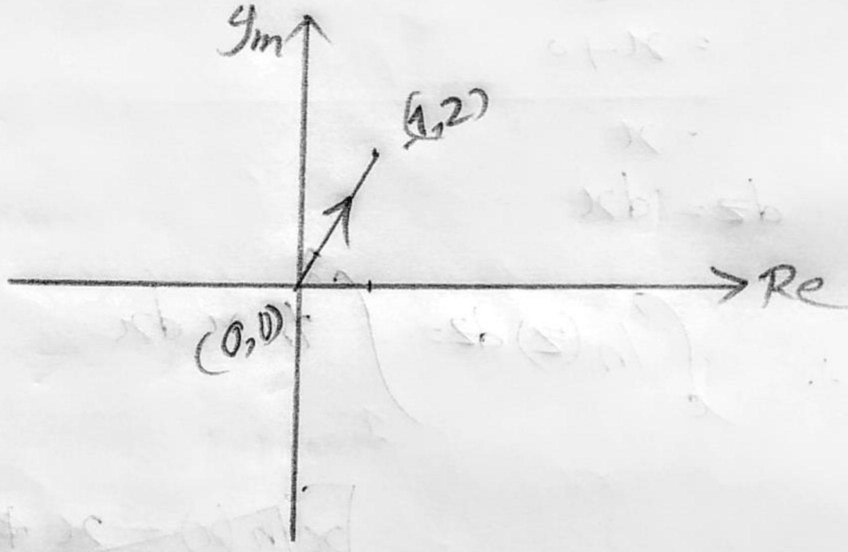
$= \frac{(.69)^2}{2}$

$= .24$ (Ans.)

18. $\int_c Re\, z\, dz$

(a) c: 0 to $1+2i$

$(0,0)$ $(1,2)$



equation to c is:

$(x_1, y_1) = (0,0)$

$(x_2, y_2) = (1,2)$

$$\frac{y-y_1}{y_1-y_2} = \frac{x-x_1}{x_1-x_2}$$

$$\Rightarrow \frac{y-0}{0-2} = \frac{x-0}{0-1}$$

$$\Rightarrow \frac{y}{-2} = \frac{x}{-1}$$

$$\Rightarrow y = 2x$$

$Re\{z\} = x$

$z = x + iy$

$= x + i\,2x$

$dz = (1+2i)\,dx$

$\therefore \int_C Re\,z\,dz = \int_0^1 x(1+2i)\,dx$

$= \int_0^1 (x + 2ix)\,dx$

$= \left[\frac{x^2}{2} + \frac{2ix^2}{2}\right]_0^1$

$= \frac{1}{2} + i$

(Ans.)

18. $\int_C Re\,z\,dz$

for C_1

equation of C: $y = 0$

$Re\{z\} = x$

& $z = x + iy$

$= x$

$dz = dx$

$$\int_{C_1} Re\, z\, dz = \int_0^1 x\, dx$$
$$= \left[\frac{x^2}{2}\right]_0^1$$
$$= \frac{1}{2}$$

for C_2

equation of C_2: $x = 1$

$Re\{z\} = x$
$= 1$

$\therefore z = x + iy$
$= 1 + iy$

$dz = i\,dy$

$$\therefore \int_{C_2} Re\, z\, dz = \int_0^2 1\, i\,dy$$
$$= i\,[1y]_0^2$$
$$= i\,2$$
$$= 2i$$

$$\therefore \int_C Re(z)\, dz = \frac{1}{2} + 2i \quad \text{(Ans)}$$

Thank You