Assignment MIDterm

Complex Variable, Laplace and Z-transformations

1. Inverse Laplace transformation using **partial fraction**:

(a) Given $F(s) = \frac{3s+1}{(s+1)(s^2+1)}$

(i) decompose $F(s)$ as $\frac{A}{(s+1)} + \frac{Bs+C}{(s^2+1)}$

(ii) find $f(t) = \mathcal{L}^{-1}\{F(s)\}$.

(b) Given $F(s) = \frac{s+1}{(s+1)(s-5)}$

(i) decompose $F(s)$ as $\frac{A}{(s+1)} + \frac{B}{(s-5)}$

find $f(t) = \mathcal{L}^{-1}\{F(s)\}$.

2. Solving the differential equations using **Laplace transformation**:

(a) $\ddot{y}(t) + 9y(t) = 10e^{-t}$, $y(0) = \dot{y}(0) = 0$, where $\ddot{y}(t) \equiv \frac{d^2y(t)}{dt^2}$, $\dot{y}(t) \equiv \frac{dy(t)}{dt}$.

(b) $\dot{y}(t) - y(t) = 8\cosh 2t$, $y(0) = 0$, where $\dot{y}(t) \equiv \frac{dy(t)}{dt}$.

(i) write Laplace transformation of both sides of the above differential equation,

(ii) solve the equation obtained in (i) for $Y(s)$ and,

(i) find $y(t)$, using **inverse Laplace transformation** of $Y(s)$ in (ii),

(ii) justify your answers.

3. **Complex variable:**

(a) Describe and sketch the locus represented by each of the followings:

(i) $1 < |z+i| \le 2$ and, (ii) $|z+3i| > 4$.

(b) If $z_1 = 1-i$, $z_2 = -2+4i$ and $z_3 = \sqrt{3} - 2i$, evaluate each of the following

(i) $|2z_2 - 3z_1|$, (ii) $z_1^2 + 3z_3 - 2$, (iii) $\mathrm{Re}\left(\frac{z}{\bar{z}}\right)$.