

Lecture 02

Complex Variable, Laplace & Z- transformation

This Lecture Covers -

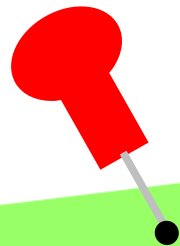
1. Laplace Transformation Using First Shifting Property.
2. Some Examples & Exercises on First Shifting Property.
3. Laplace Transformation Using Multiplication by t^n Property.
4. Some Examples & Exercises on Multiplication by t^n Property.



First Shifting or Translation Property

If $\mathcal{L}\{f(t)\} = F(s)$, then

$$\mathcal{L}\{e^{at} f(t)\} = F(s - a)$$



Property of Multiplication by t^n

If $\mathcal{L}\{f(t)\} = F(s)$ then

$$\mathcal{L}\{t^n f(t)\} = (-1)^n \frac{d^n}{ds^n} [F(s)]$$

Examples on Shifting Property

Example 1:

$$\begin{aligned}\mathcal{L}\{e^{2t} \sin t\} \\ = F(s - 2)\end{aligned}$$

Now,

$$\begin{aligned}F(s) &= \mathcal{L}\{\sin t\} \\ &= \frac{1}{s^2 + 1}\end{aligned}$$

$$\text{So, } \mathcal{L}\{e^{2t} \sin t\} = \frac{1}{(s-2)^2 + 1}.$$

Ans.

Example 2:

$$\begin{aligned}\mathcal{L}\{e^{-t} \cos 2t\} \\ = F(s + 1)\end{aligned}$$

Now,

$$\begin{aligned}F(s) &= \mathcal{L}\{\cos 2t\} \\ &= \frac{s}{s^2 + 4}\end{aligned}$$

$$\text{So, } \mathcal{L}\{e^{-t} \cos 2t\} = \frac{(s+1)}{(s+1)^2 + 4}.$$

Ans.

Examples on Shifting Property

Example 3:

$$\begin{aligned}\mathcal{L}\{e^t \cosh 3t\} \\ = F(s - 1)\end{aligned}$$

Now,

$$\begin{aligned}F(s) &= \mathcal{L}\{\cosh 3t\} \\ &= \frac{s}{s^2 - 9}\end{aligned}$$

$$\begin{aligned}\text{So, } \mathcal{L}\{e^t \cosh 3t\} &= \\ \frac{s-1}{(s-1)^2 - 9}.\end{aligned}$$

Ans.

Example 4:

$$\begin{aligned}\mathcal{L}\{e^{-3t} t^8\} \\ = F(s + 3)\end{aligned}$$

Now,

$$\begin{aligned}F(s) &= \mathcal{L}\{t^8\} \\ &= \frac{8!}{s^9}\end{aligned}$$

$$\text{So, } \mathcal{L}\{e^{-t} \cos 2t\} = \frac{8!}{(s+3)^9}.$$

Ans.

Exercise Set on Shifting Property

Find Laplace Transformation of the following function:

1. $f(t) = e^{2t} \sinh 3t,$

2. $f(t) = e^{-t} \sinh 4t,$

3. $f(t) = e^{2t} \cos 3t,$

4. $f(t) = t^{10} e^{-7t},$

5. $f(t) = e^{5t} \cosh 6t.$

Examples on Multiplication by t^n

Example 1 :

$$\begin{aligned}\mathcal{L}\{t \cos t\} &= (-1)^1 \frac{d}{ds} [F(s)] \\&= -\frac{d}{ds} [\mathcal{L}\{\cos t\}] \\&= -\frac{d}{ds} \left[\frac{s}{s^2 + 1} \right] \\&= -\frac{(s^2 + 1) \frac{d}{ds} (s) - s \frac{d}{ds} (s^2 + 1)}{(s^2 + 1)^2} \\&= -\frac{(s^2 + 1) - s \times 2s}{(s^2 + 1)^2} \\&= -\frac{s^2 + 1 - 2s^2}{(s^2 + 1)^2} \\&= \frac{s^2 - 1}{(s^2 + 1)^2}.\end{aligned}$$

Ans.

Property

Example 2 :

$$\begin{aligned}\mathcal{L}\{t^2 e^{3t}\} &= (-1)^2 \frac{d^2}{ds^2} [F(s)] \\&= \frac{d^2}{ds^2} [\mathcal{L}\{e^{3t}\}] \\&= \frac{d^2}{ds^2} \left[\frac{1}{s - 3} \right] \\&= \frac{d}{ds} \left[\frac{(s - 3) \frac{d}{ds} (1) - 1 \frac{d}{ds} (s - 3)}{(s - 3)^2} \right] \\&= \frac{d}{ds} \left[\frac{-1}{(s - 3)^2} \right] \\&= \frac{(-1)(-2)(s - 3)^{-3}}{2} \\&= \frac{2}{(s - 3)^3}.\end{aligned}$$

Ans.

Exercise Set on Multiplication by t^n

Property

Find Laplace Transformation of the following functions:

1. $f(t) = t \sin 2t,$

2. $f(t) = t \cos bt,$

3. $f(t) = t^2 e^{-4t},$

4. $f(t) = t \sinh 3t,$

5. $f(t) = t \cosh 2t.$

Learning Outcomes

After completing this lecture you will learn about find Laplace Transformation using two properties named as first shifting or translation and another one is multiplication by t^n property.

Sample MCQ

1. $\mathcal{L}\{t^{10} e^{-5t}\} = ?$

(a) $\frac{11!}{(s+5)^{10}}$

(b) $\frac{10!}{(s+5)^{11}}$

(c) $\frac{10}{(s+5)^{11}}$

(d) $\frac{10!}{(s+5)^{10}}$

2. $\mathcal{L}\{t \sin t\} = ?$

(a) $\frac{2s}{(s^2+1)^2}$

(b) $\frac{s}{(s^2+1)^2}$

(c) $\frac{4s}{(s^2+1)^2}$

(d) $\frac{4s}{(s^2-1)^2}$

3. $\mathcal{L}\{e^{3t} \cosh 5t\} = ?$

(a) $\frac{s}{s^2+25}$

(b) $\frac{s+3}{(s+3)^2+25}$

(c) $\frac{s+3}{(s+3)^2-25}$

(d) $\frac{s-3}{(s-3)^2-25}$

Sample MCQ

4. $\mathcal{L}\{e^{2t} \sinh t\} = ?$

(a) $\frac{2}{(s-1)^2-1}$

(b) $\frac{1}{(s-2)^2-1}$

(c) $\frac{1}{(s+2)^2-1}$

(d) none

5. $\mathcal{L}\{t \cos t\} = ?$

(a) $\frac{2s}{(s^2+1)^2}$

(b) $\frac{s^2-1}{(s^2+1)^2}$

(c) $\frac{4s}{(s^2+1)^2}$

(d) $\frac{4s}{(s^2-1)^2}$