

# Chapter 9

## Network Theorems



## 9.2 SUPERPOSITION THEOREM

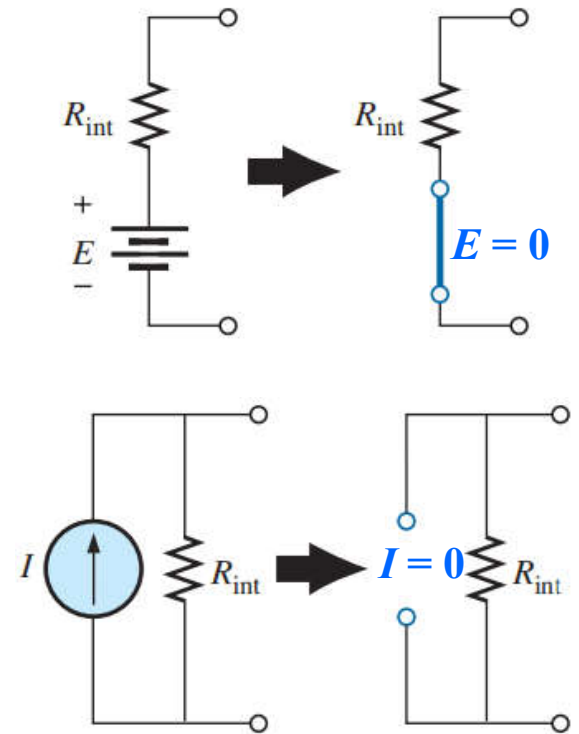


## Statement of Superposition Theorem

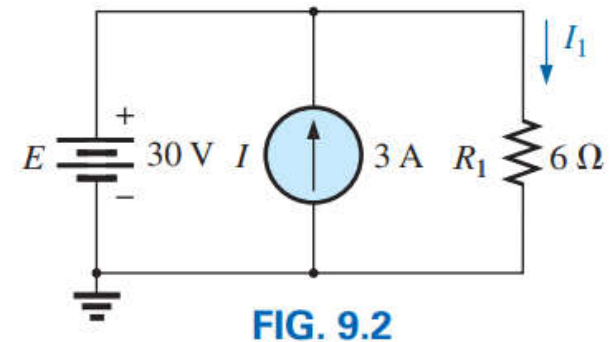
The current through, or voltage across, an element in a linear bilateral network is equal to the algebraic sum of the currents or voltages produced independently by each source.

### Steps to Apply Superposition Theorem

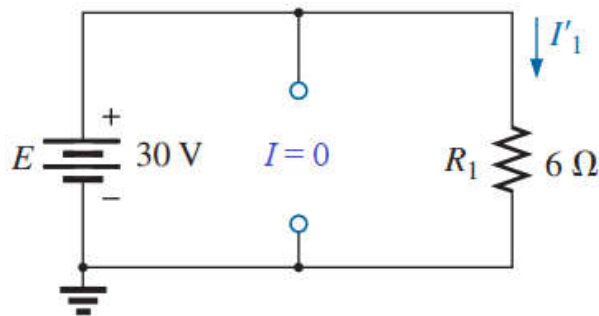
- Step 1:** Select a single source acting alone. **Short the other voltage sources to make voltage is zero** and **open the current sources to make current is zero**, if internal resistance/impedances are not known. If known, replace them by their internal impedances.
- Step 2:** Find the current through or the voltage across the required element, due to the source under consideration, using a suitable simplification technique.
- Step 3:** Repeat the above two steps for all the sources.
- Step 4:** Add all the individual effects produced by individual sources, to obtain the total current in or voltage across the element.



**EXAMPLE 9.1** Using the superposition theorem, determine current  $I_1$  for the network in Fig. 9.2.

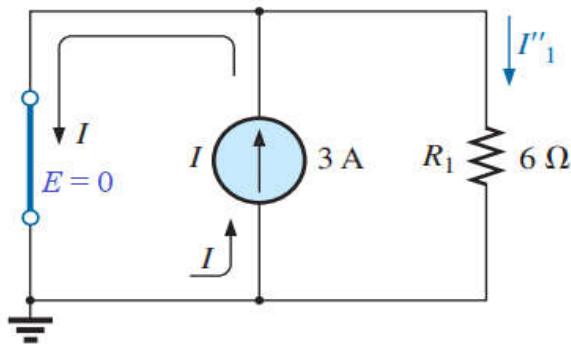


**Solution:** Consider  $E$  then and  $I = 0$  A (**open**).



$$I'_1 = \frac{E}{R_1} = \frac{30 \text{ V}}{6 \Omega} = 5 \text{ A}$$

Consider  $I$  then and  $E = 0$  V (**shorted**).

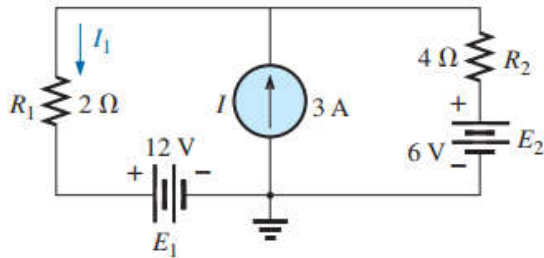


$$I''_1 = \frac{R_{sc} I}{R_{sc} + R_1} = \frac{(0 \Omega) I}{0 \Omega + 6 \Omega} = 0 \text{ A}$$

According to Superposition Theorem:

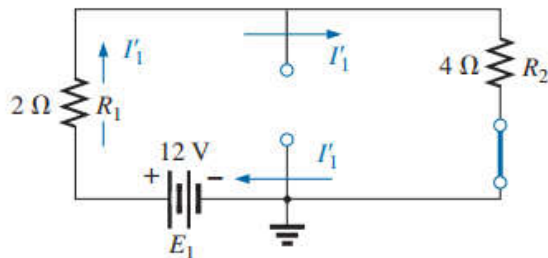
$$I_1 = I'_1 + I''_1 = 5 \text{ A} + 0 \text{ A} = \mathbf{5 \text{ A}}$$

**EXAMPLE 9.5** Find the current through the  $2\ \Omega$  resistor of the network in Fig. 9.18. The presence of three sources results in three different networks to be analyzed.



**FIG. 9.18** Example 9.5.

**Solution:** Considering the effect of the 12 V source (Fig. 9.19):

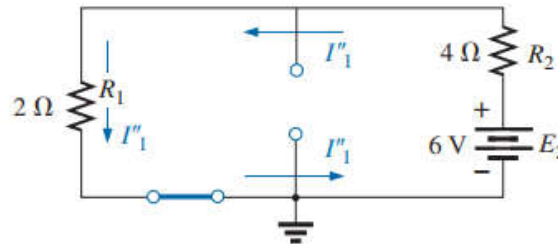


**FIG. 9.19**

The effect of  $E_1$  on the current  $I$ .

$$\begin{aligned} I'_1 &= \frac{E_1}{R_1 + R_2} \\ &= \frac{12\text{ V}}{2\ \Omega + 4\ \Omega} \\ &= \frac{12\text{ V}}{6\ \Omega} = 2\text{ A} \end{aligned}$$

Considering the effect of the 6 V source (Fig. 9.20):

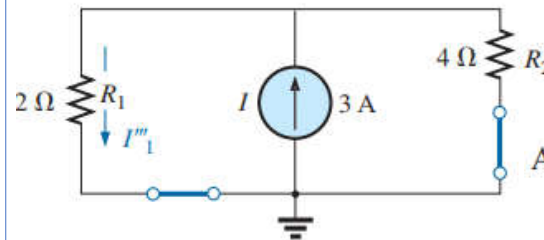


**FIG. 9.20**

The effect of  $E_2$  on the current  $I_1$ .

$$\begin{aligned} I''_1 &= \frac{E_2}{R_1 + R_2} \\ &= \frac{6\text{ V}}{2\ \Omega + 4\ \Omega} \\ &= \frac{6\text{ V}}{6\ \Omega} = 1\text{ A} \end{aligned}$$

Considering the effect of the 3 A source (Fig. 9.21):

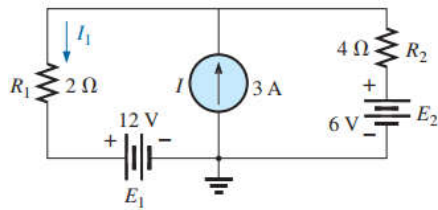


**FIG. 9.21**

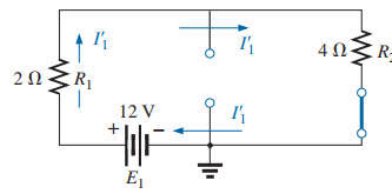
The effect of  $I$  on the current  $I_1$ .

Applying the current divider rule,

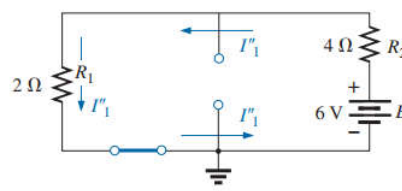
$$\begin{aligned} I'''_1 &= \frac{R_2 I}{R_1 + R_2} \\ &= \frac{(4\ \Omega)(3\text{ A})}{2\ \Omega + 4\ \Omega} \\ &= \frac{12\text{ A}}{6} = 2\text{ A} \end{aligned}$$



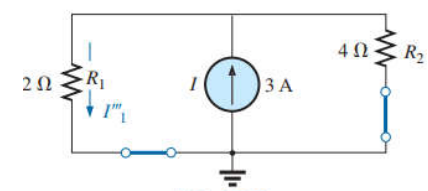
**FIG. 9.18** Example 9.5.



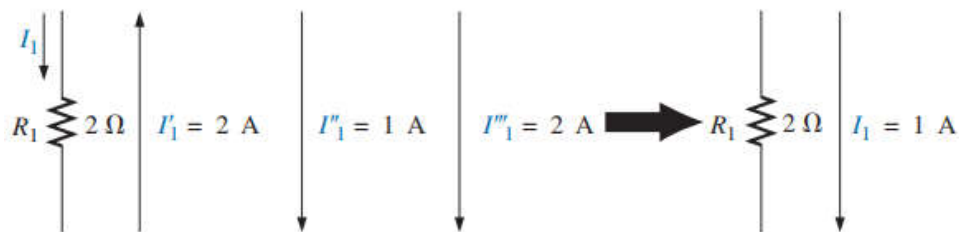
**FIG. 9.19**  
The effect of  $E_1$  on the current  $I$ .



**FIG. 9.20**  
The effect of  $E_2$  on the current  $I_1$ .



**FIG. 9.21**  
The effect of  $I$  on the current  $I_1$ .

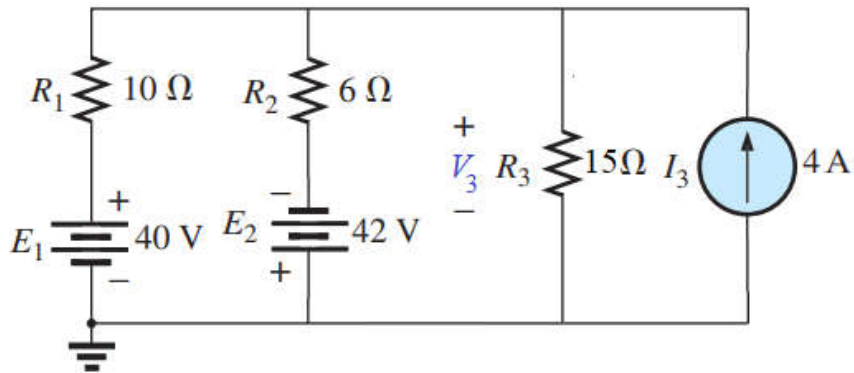


**FIG. 9.22** The resultant current  $I_1$ .

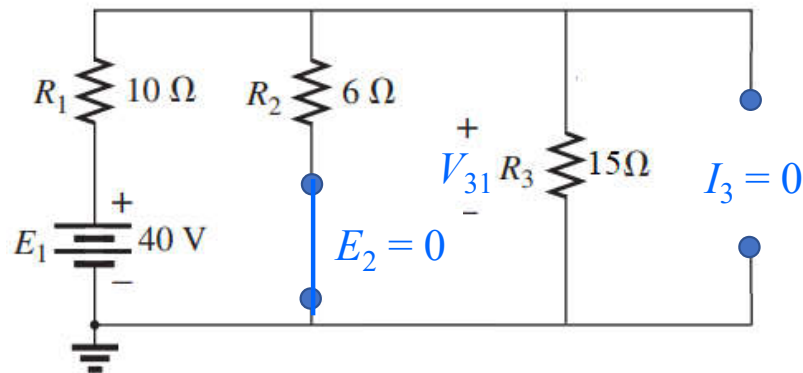
The total current through the  $2\ \Omega$  resistor appears in Fig. 9.22 and

$$\begin{aligned}
 I_1 &= \overbrace{I''_1 + I'''_1}^{\text{Same direction as } I_1 \text{ in Fig. 9.18}} - \overbrace{I'_1}^{\text{Opposite direction to } I_1 \text{ in Fig. 9.18}} \\
 &= 1\ \text{A} + 2\ \text{A} - 2\ \text{A} = 1\ \text{A}
 \end{aligned}$$

**Example 9.2.1** Using superposition, find the voltage  $V_3$  for the following network.



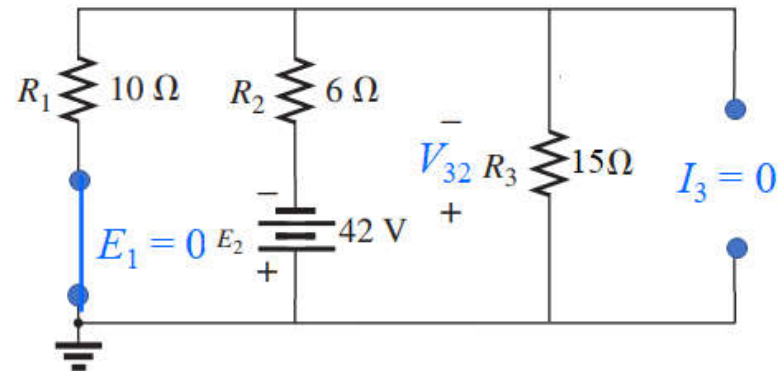
**Solution:** Consider  $E_1$  then  $E_2 = 0$  V (**shorted**) and  $I_3 = 0$  A (**open**).



$$R_{p1} = \frac{R_2 R_3}{R_2 + R_3} = 4.29 \, \Omega$$

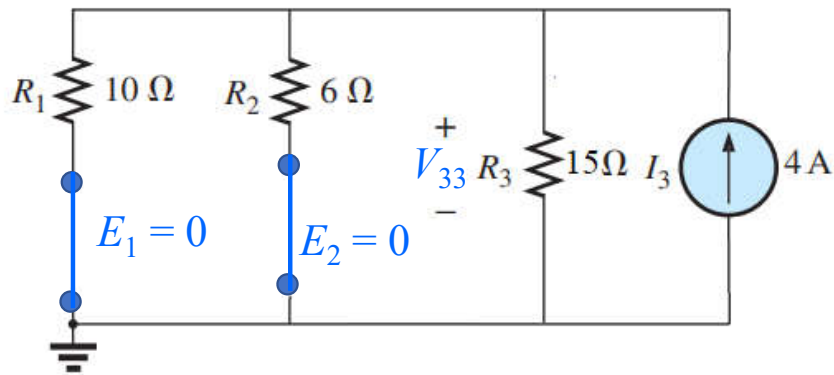
$$V_{31} = \frac{R_{p1}}{R_1 + R_{p1}} E_1 = 12 \, \text{V}$$

Consider  $E_2$  then  $E_1 = 0$  V (**shorted**) and  $I_3 = 0$  A (**open**).



$$R_{p2} = \frac{R_1 R_3}{R_1 + R_3} = 6 \, \Omega \quad V_{32} = \frac{R_{p2}}{R_2 + R_{p2}} E_2 = 21 \, \text{V}$$

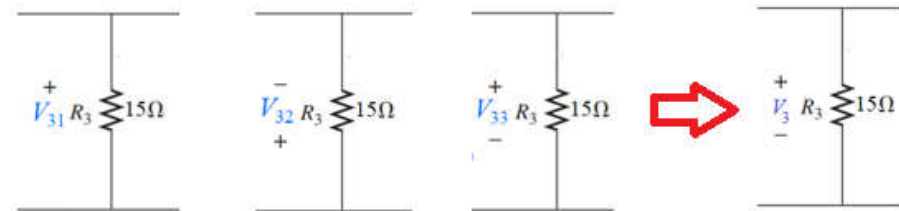
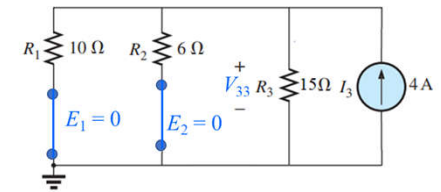
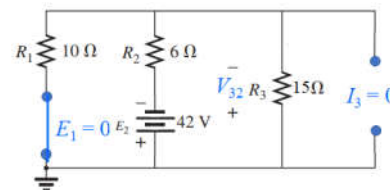
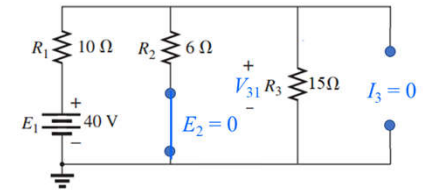
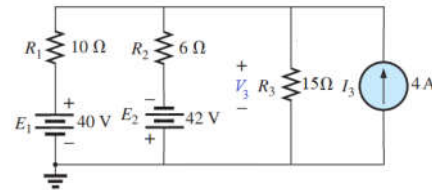
Consider  $I_3$  then  $E_1 = 0$  V (**shorted**) and  $E_2 = 0$  V (**shorted**).



$$G_{p3} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = 0.333 \text{ S}$$

$$R_{p3} = \frac{1}{G_{p3}} = 3 \text{ } \Omega$$

$$V_{33} = R_{p3} I_3 = 12 \text{ V}$$



According to Superposition Theorem:

$$V_3 = V_{31} - V_{32} + V_{33} = 12 \text{ V} - 21 \text{ V} + 12 \text{ V} = -21 \text{ V}$$

**Practice Book [Ch 9] Problem: 1 ~ 6**

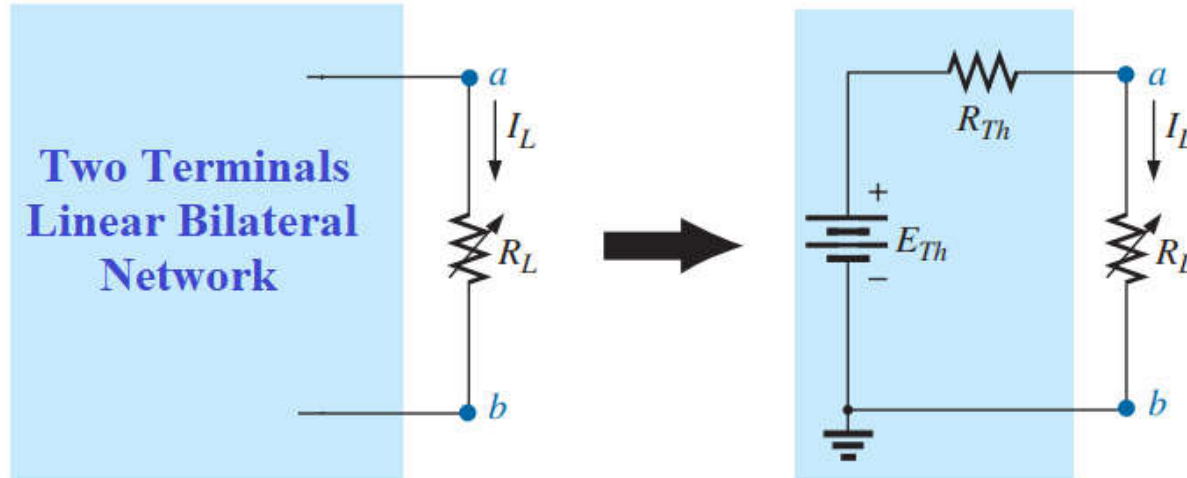


## 9.3 THÉVENIN'S THEOREM



## Statement of Thevenin's Theorem

Any two-terminal, linear bilateral network can be replaced by an equivalent circuit consisting of a **voltage source** and a **series resistance**, as shown in the following figure.



$R_{Th}$ : Thevenin's equivalent resistance

$E_{Th}$ : Thevenin's equivalent voltage

## Steps to Apply Thevenin's Theorem

- Step 1:** Remove that portion of the network where the Thévenin equivalent circuit is found.
- Step 2:** Mark the terminals of the remaining two-terminal network. (The importance of this step will become obvious as we progress through some complex networks.)
- Step 3:** Calculate  $R_{Th}/Z_{Th}$  by first setting all sources to zero (voltage sources are replaced by short circuits, and current sources by open circuits) and then finding the resultant resistance/impedance between the two marked terminals. (If the internal resistance/impedance of the voltage and/or current sources is included in the original network, it must remain when the sources are set to zero.)
- Step 4:** Calculate  $E_{Th}$  by first returning all sources to their original position and finding the open-circuit voltage between the marked terminals. (This step is invariably the one that causes most confusion and errors. In all cases, keep in mind that it is the open circuit potential between the two terminals marked in step 2.)
- Step 5:** Draw the Thévenin equivalent circuit with the portion of the circuit previously removed replaced between the terminals of the equivalent circuit.
- Step 6:** Do the remaining required calculation

**Example 9.6** Find the Thévenin equivalent circuit for the network in the shaded area of the network in Fig. 9.26. Then find the current through  $R_L$  for values of  $2\ \Omega$ ,  $10\ \Omega$ , and  $100\ \Omega$ .

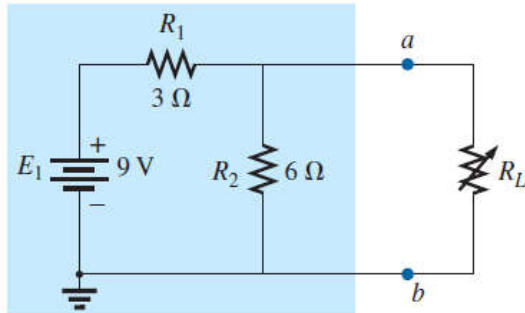
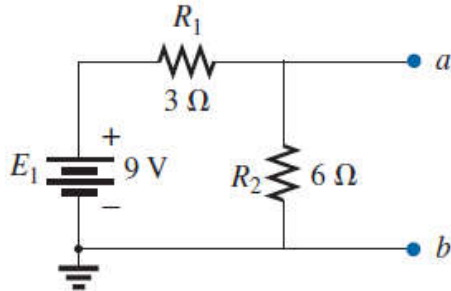


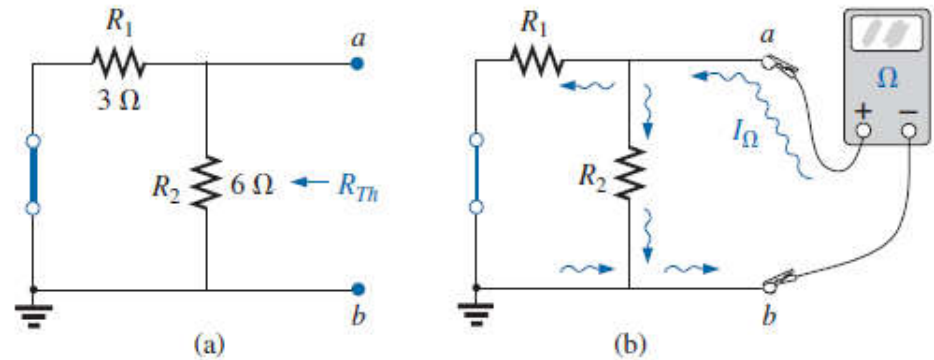
FIG. 9.26 Example 9.6.

**Step 1:** Remove that portion of the network where the Thévenin equivalent circuit is found.

**Step 2:** Mark the terminals (such as  $a$  and  $b$ ) of the remaining two-terminal network.

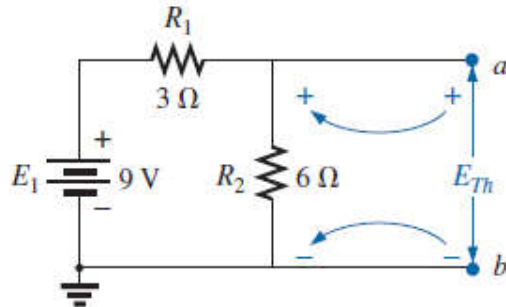


**Step 3:** Calculate  $R_{Th}$  by first setting all sources to zero (voltage sources are replaced by short circuits, and current sources by open circuits) and then finding the resultant resistance between the two marked terminals ( $a$  and  $b$ ).



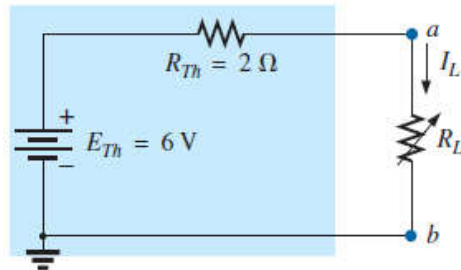
$$R_{Th} = R_1 \parallel R_2 = \frac{(3\ \Omega)(6\ \Omega)}{3\ \Omega + 6\ \Omega} = 2\ \Omega$$

**Step 4: Calculate  $E_{Th}$**  by first returning all sources to their original position and finding the open-circuit voltage between the marked terminals (*a* and *b*).



$$E_{Th} = \frac{R_2 E_1}{R_2 + R_1} = \frac{(6 \Omega)(9 \text{ V})}{6 \Omega + 3 \Omega} = \frac{54 \text{ V}}{9} = 6 \text{ V}$$

**Step 5: Draw the Thévenin equivalent circuit** with the portion of the circuit previously removed replaced between the terminals of the equivalent circuit.



$$I_L = \frac{E_{Th}}{R_{Th} + R_L}$$

$$R_L = 2 \Omega:$$

$$I_L = \frac{6 \text{ V}}{2 \Omega + 2 \Omega} = 1.5 \text{ A}$$

$$R_L = 10 \Omega:$$

$$I_L = \frac{6 \text{ V}}{2 \Omega + 10 \Omega} = 0.5 \text{ A}$$

$$R_L = 100 \Omega:$$

$$I_L = \frac{6 \text{ V}}{2 \Omega + 100 \Omega} = 0.06 \text{ A}$$

**Example 9.8** Using Thevenin's Theorem calculate the current passing through the resistor  $R_4$ .

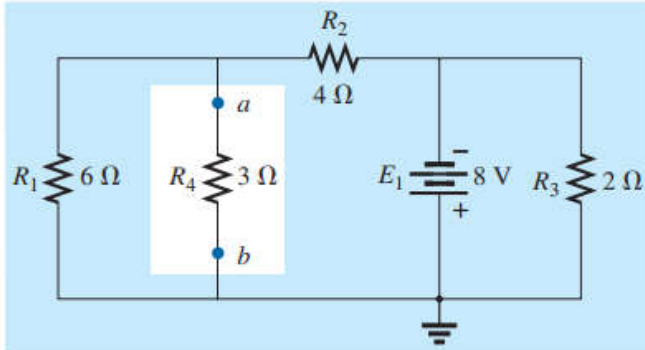
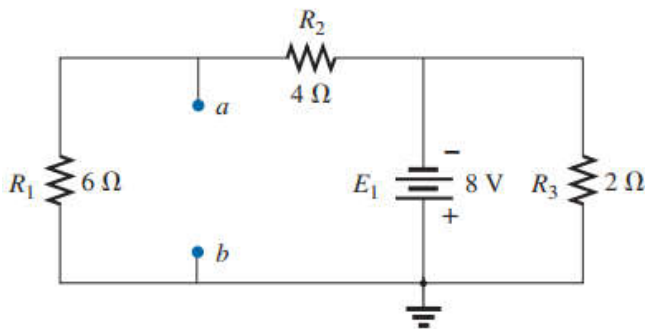


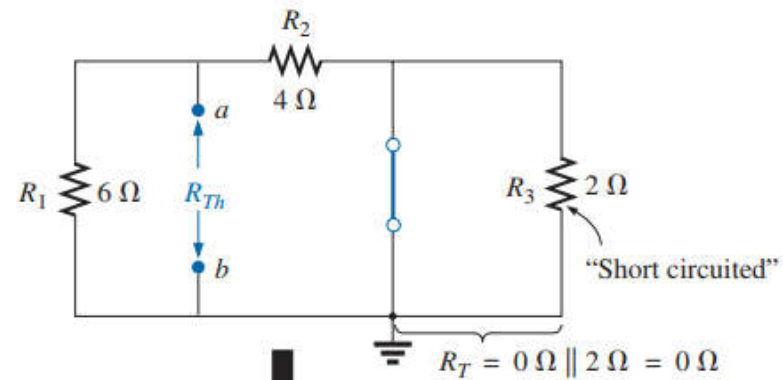
FIG. 9.37 Example 9.8.

**Step 1:** Remove that portion of the network where the Thévenin equivalent circuit is found.

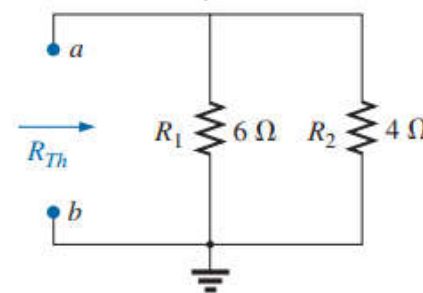
**Step 2:** Mark the terminals (such as  $a$  and  $b$ ) of the remaining two-terminal network.



**Step 3:** Calculate  $R_{Th}$  by first setting all sources to zero (voltage sources are replaced by short circuits, and current sources by open circuits) and then finding the resultant resistance between the two marked terminals ( $a$  and  $b$ ).

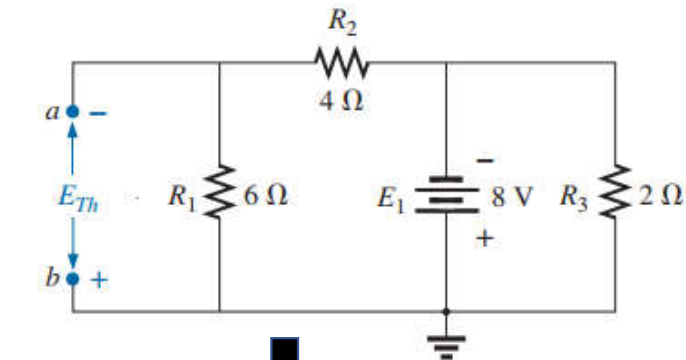


Circuit redrawn:

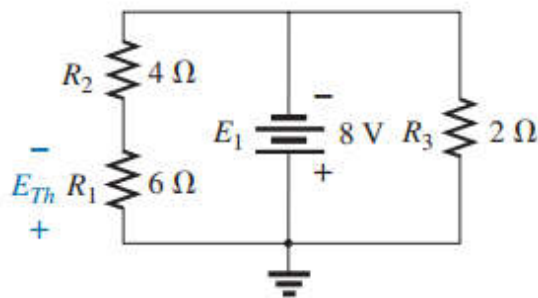


$$\begin{aligned}
 R_{Th} &= R_1 \parallel R_2 \\
 &= \frac{(6 \, \Omega)(4 \, \Omega)}{6 \, \Omega + 4 \, \Omega} \\
 &= \frac{24 \, \Omega}{10} = 2.4 \, \Omega
 \end{aligned}$$

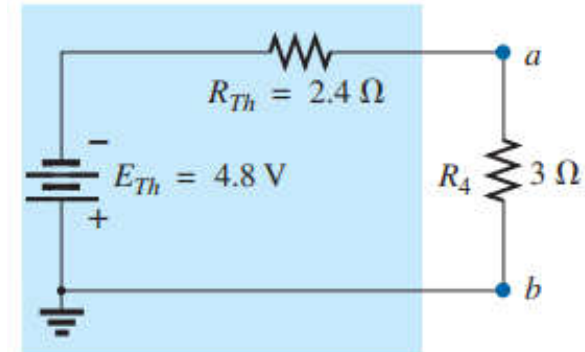
**Step 4: Calculate  $E_{Th}$**  by first returning all sources to their original position and finding the open-circuit voltage between the marked terminals ( $a$  and  $b$ ).



$$\begin{aligned}
 E_{Th} &= \frac{R_1 E_1}{R_1 + R_2} \\
 &= \frac{(6 \Omega)(8 \text{ V})}{6 \Omega + 4 \Omega} \\
 &= \frac{48 \text{ V}}{10} = 4.8 \text{ V}
 \end{aligned}$$



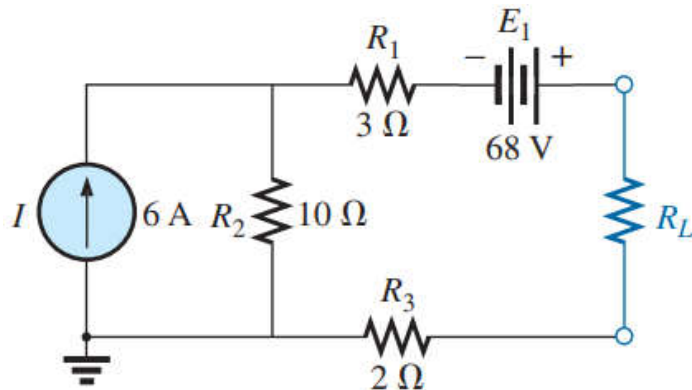
**Step 5: Draw the Thévenin equivalent circuit** with the portion of the circuit previously removed replaced between the terminals of the equivalent circuit.



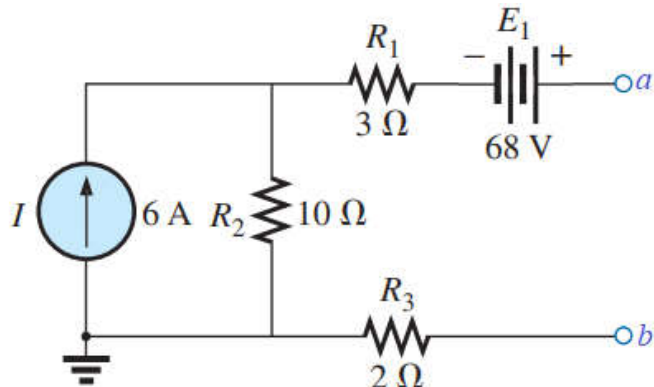
**Step 6: Calculate the current passing through the resistor  $R_4$ .**

$$I_{R4} = \frac{E_{Th}}{R_{Th} + R_4} = \frac{4.8 \text{ V}}{2.4 \Omega + 3 \Omega} = 0.89 \text{ A}$$

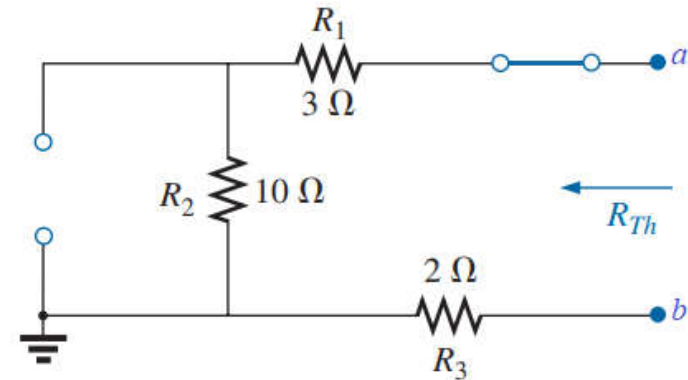
**Example 9.3.1:** Find the Thévenin equivalent circuit for the portions of the following network to points  $a$  and  $b$ .



**Step 1 and Step 2:**

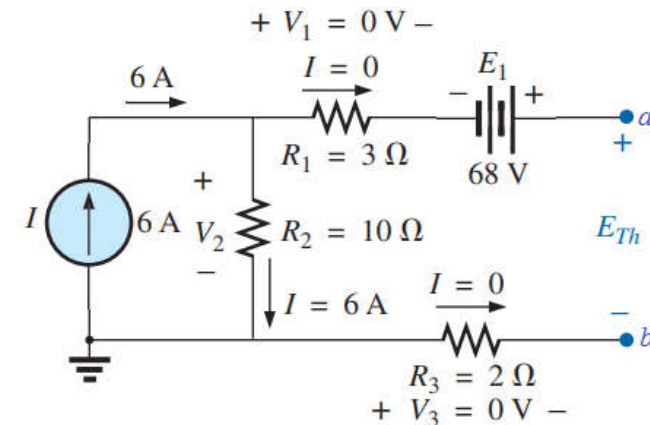


**Step 3:  $R_{Th}$  calculation**

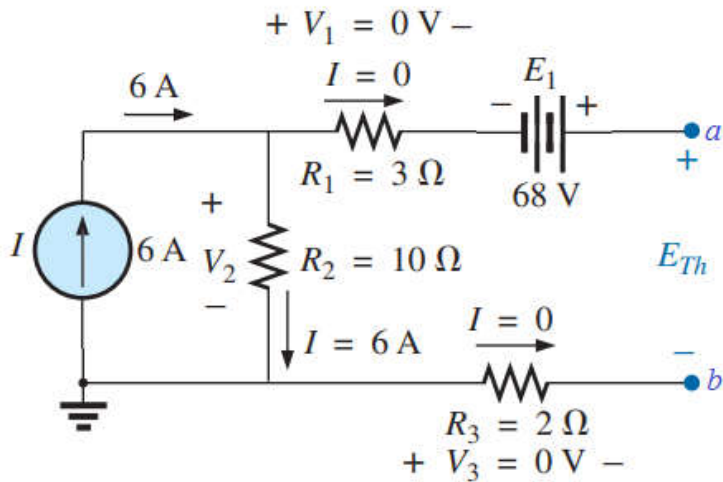


$$R_{Th} = R_1 + R_2 + R_3 = 3\ \Omega + 10\ \Omega + 2\ \Omega = 15\ \Omega$$

**Step 4:  $E_{Th}$  calculation**







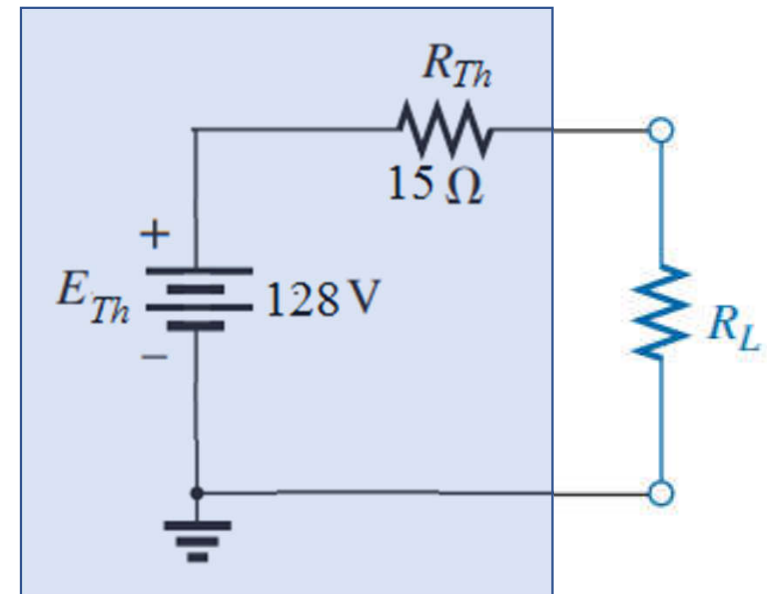
$$V_1 = V_3 = 0 \text{ V}$$

$$V_2 = I_2 R_2 = IR_2 \\ = (6 \text{ A})(10 \Omega) = 60 \text{ V}$$

Applying Kirchhoff's voltage law:

$$E_{Th} = V_2 + E_1 = 60 \text{ V} + 68 \text{ V} = 128 \text{ V}$$

**Step 5: Draw the Thévenin equivalent circuit**



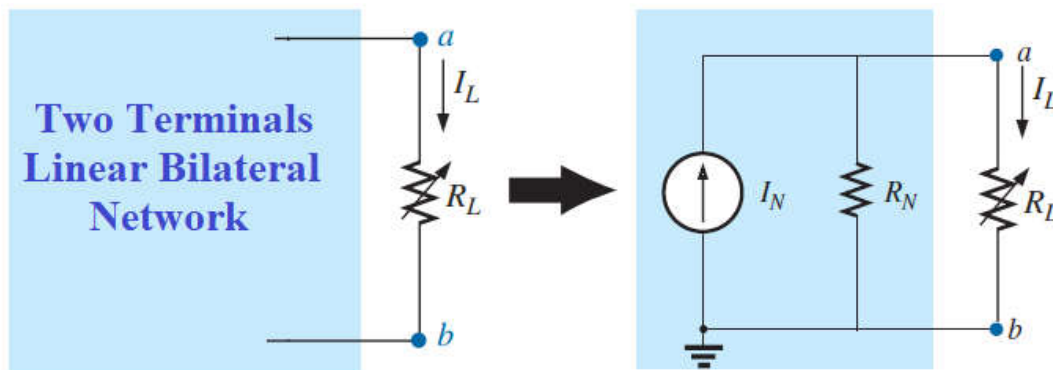
**Practice Book [Ch 9] Problem: 7 ~ 15**

## 9.4 NORTON'S THEOREM



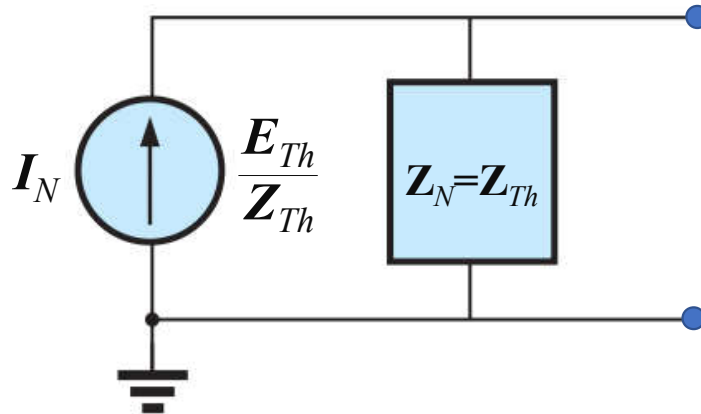
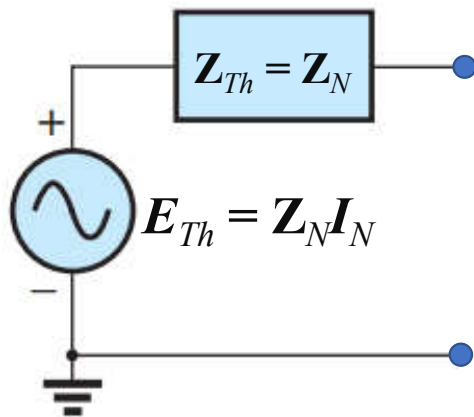
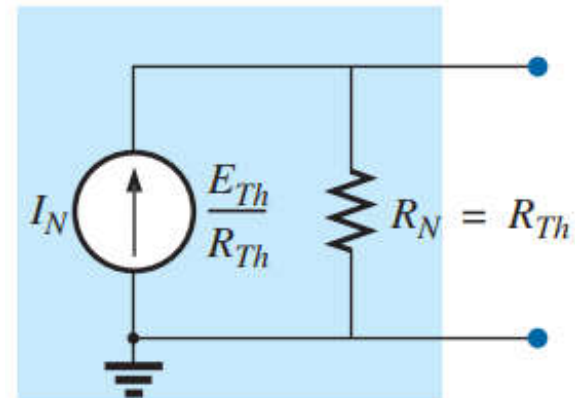
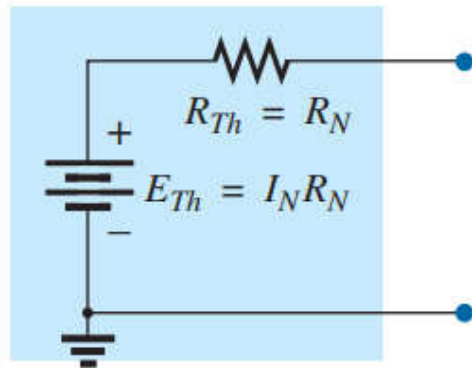
## Statement of Norton's Theorem

Any two-terminal, linear bilateral network can be replaced by an equivalent circuit consisting of a **current source** and a **parallel resistance**, as shown in the following figure.



$R_N$ : Norton's equivalent resistance  
 $I_N$ : Norton's equivalent current

## Conversion Between Thevenin and Norton's Equivalent Circuit



## Steps to Apply Norton's Theorem

- Step 1:** Remove that portion of the network where the Thévenin equivalent circuit is found.
- Step 2:** Mark the terminals of the remaining two-terminal network.
- Step 3:** Calculate  $R_N/Z_N$  by first setting all sources to zero (voltage sources are replaced by short circuits, and current sources by open circuits) and then finding the resultant resistance/impedance between the two marked terminals. (If the internal resistance/impedance of the voltage and/or current sources is included in the original network, it must remain when the sources are set to zero.)
- Step 4:** Calculate  $I_N$  by first returning all sources to their original position and then finding the short-circuit current between the marked terminals. It is the same current that would be measured by an ammeter placed between the marked terminals.
- Step 5:** Draw the Norton's equivalent circuit with the portion of the circuit previously removed replaced between the terminals of the equivalent circuit.
- Step 6:** Do the remaining required calculation



**Example 9.11** [P364] Find the Norton equivalent circuit for the network in the shaded area in Fig. 9.61.

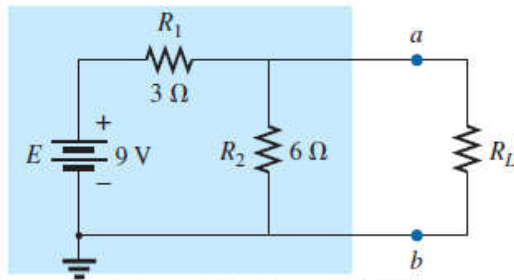
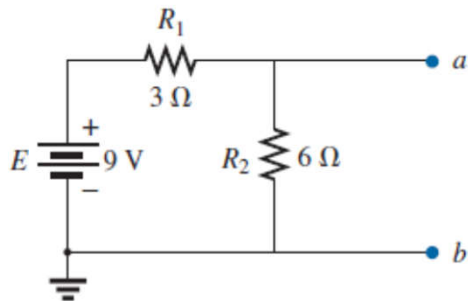


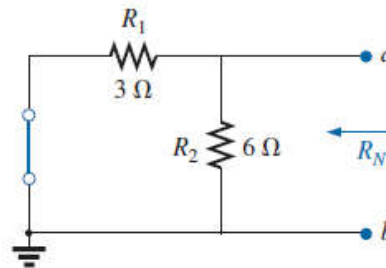
FIG. 9.61 Example 9.11.

**Step 1:** Remove that portion of the network where the Norton equivalent circuit is found.

**Step 2:** Mark the terminals (such as *a* and *b*).



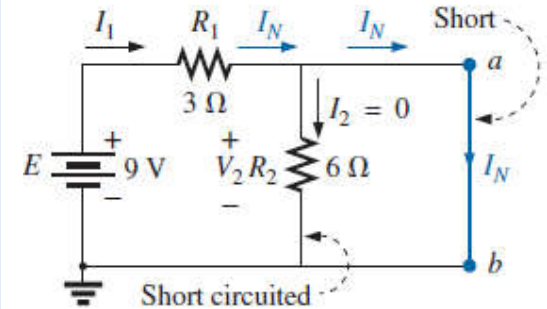
**Step 3:** Calculate  $R_N$  by first setting all sources to zero and then finding the resultant resistance between the two marked terminals (*a* and *b*).



$$R_N = R_1 \parallel R_2 = 3 \Omega \parallel 6 \Omega$$

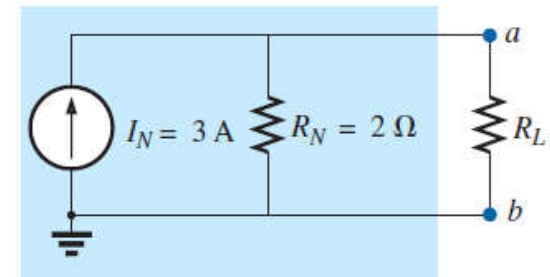
$$= \frac{(3 \Omega)(6 \Omega)}{3 \Omega + 6 \Omega} = \frac{18 \Omega}{9} = 2 \Omega$$

**Step 4:** Calculate  $I_N$  by first returning all sources to their original position then finding the short-circuit current between the marked terminals. (*a* and *b*).



$$I_N = \frac{E}{R_1} = \frac{9 \text{ V}}{3 \Omega} = 3 \text{ A}$$

**Step 5:** Draw the Norton equivalent circuit with the portion of the circuit previously removed replaced between the terminals of the equivalent circuit.



**EXAMPLE 9.13** (Two sources) Find the Norton equivalent circuit for the portion of the network to the left of  $a-b$  in Fig. 9.72.

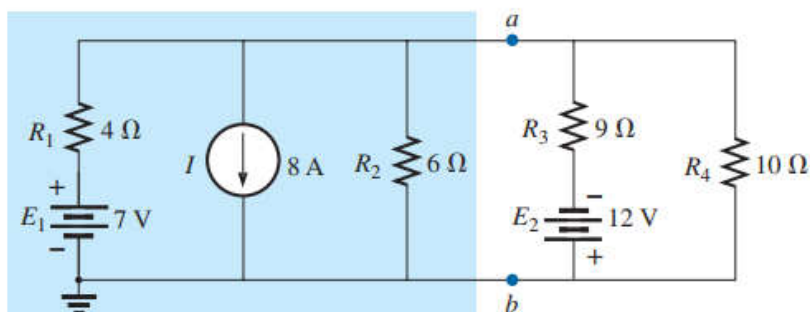
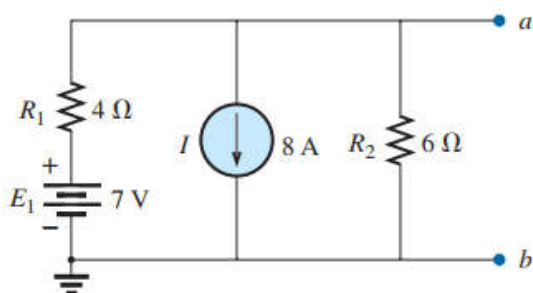
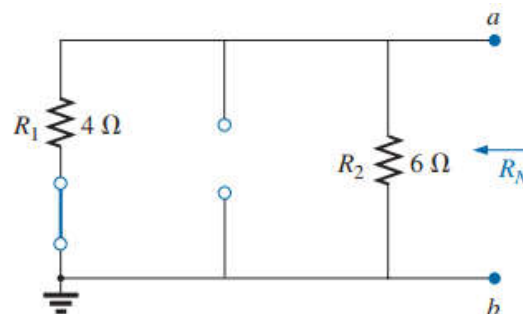


FIG. 9.72 Example 9.13.

**Step 1 and Step 2:**

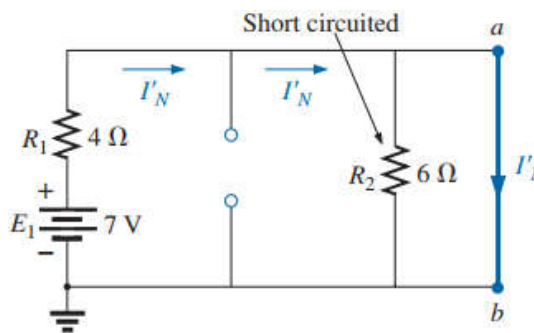


**Step 3: Calculate  $R_N$ :**



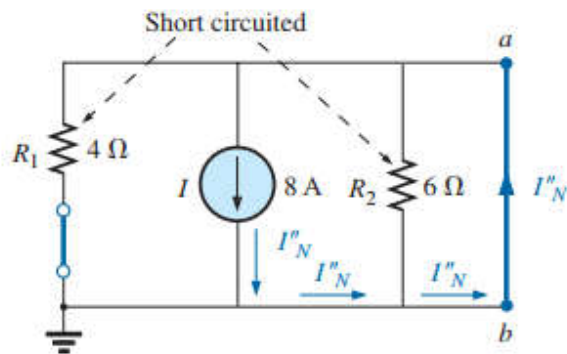
$$\begin{aligned} R_N &= R_1 \parallel R_2 = 4 \Omega \parallel 6 \Omega \\ &= \frac{(4 \Omega)(6 \Omega)}{4 \Omega + 6 \Omega} = \frac{24 \Omega}{10} \\ &= 2.4 \Omega \end{aligned}$$

**Step 4: Calculate  $I_N$ :** Since there are two sources Superposition theorem has to be applied. Consider 7 V:



$$I'_N = \frac{E_1}{R_1} = \frac{7 \text{ V}}{4 \Omega} = 1.75 \text{ A}$$

Consider 8 A:

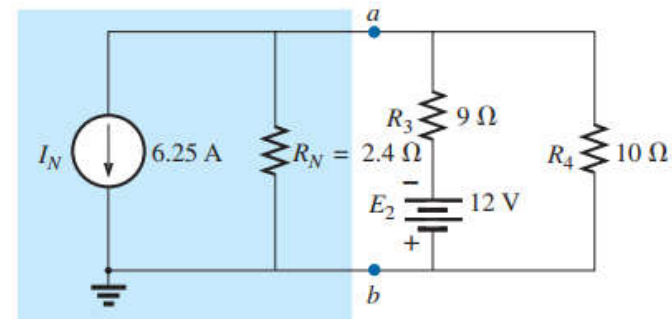


$$I''_N = I = 8 \text{ A}$$

According to Superposition Theorem

$$\begin{aligned} I_N &= I''_N - I'_N \\ &= 8 \text{ A} - 1.75 \text{ A} \\ &= \mathbf{6.25 \text{ A}} \end{aligned}$$

Step 5: Draw the Norton equivalent circuit:





**Example 9.12** [P365] Using the Norton Theorem to terminal  $a$  and  $b$  of Fig. 9.67, find the value of current which is passing through  $9\ \Omega$  resistor.

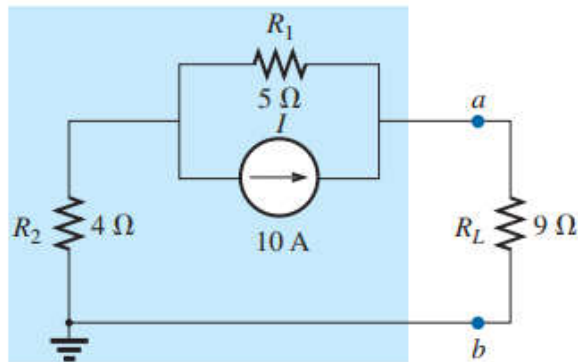
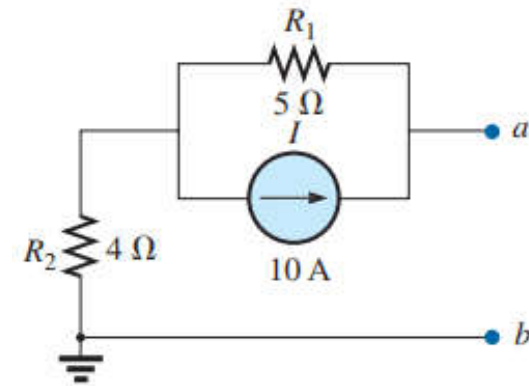


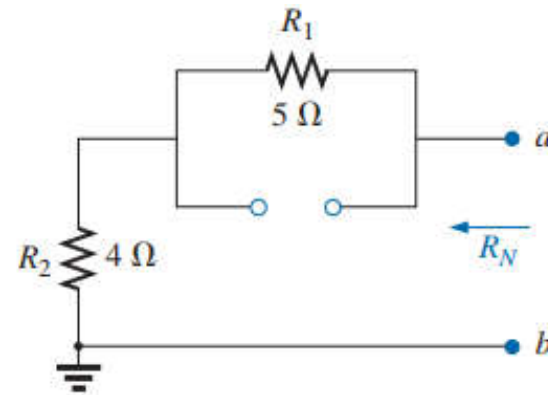
FIG. 9.67 Example 9.12.

**Step 1:** Remove that portion of the network where the Norton equivalent circuit is found.

**Step 2:** Mark the terminals (such as  $a$  and  $b$ ).

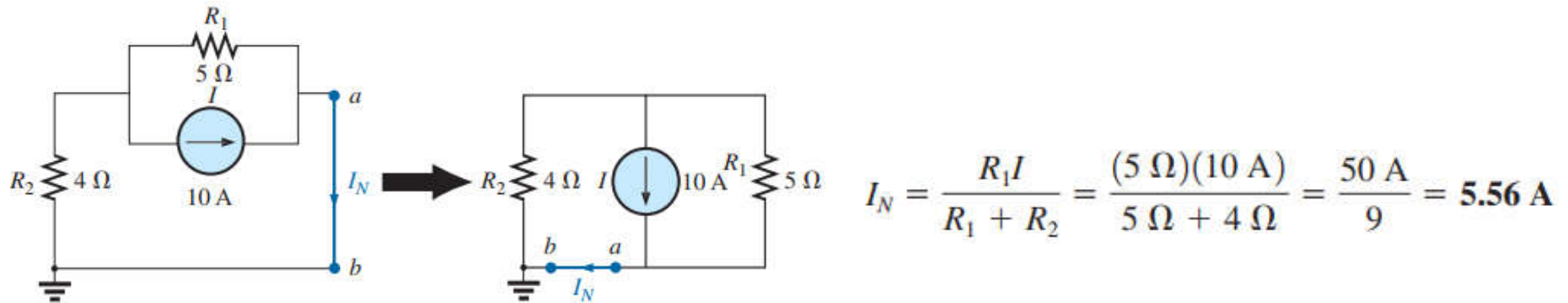


**Step 3:** Calculate  $R_N$  by first setting all sources to zero and then finding the resultant resistance between the two marked terminals ( $a$  and  $b$ ).

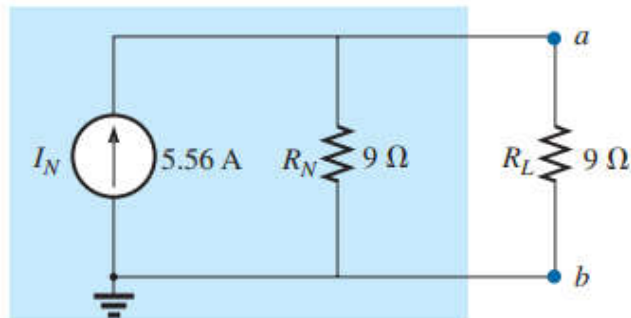


$$R_N = R_1 + R_2 = 5\ \Omega + 4\ \Omega = 9\ \Omega$$

**Step 4: Calculate  $I_N$**  by first returning all sources to their original position then finding the short-circuit current between the marked terminals. (*a* and *b*).

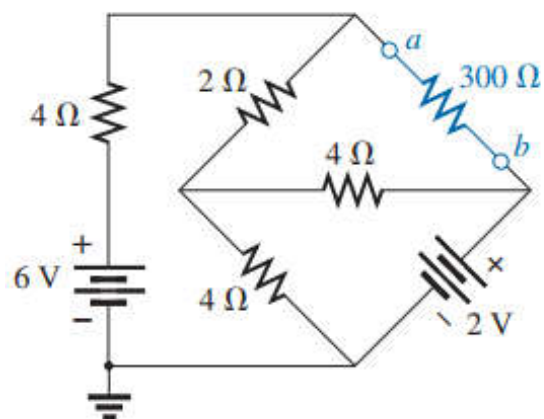


**Step 5: Draw the Norton equivalent circuit** with the portion of the circuit previously removed replaced between the terminals of the equivalent circuit.

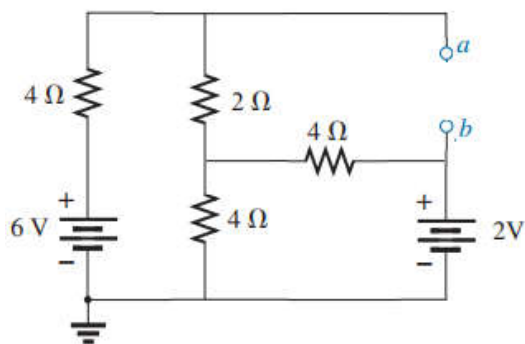


$$I_{RL} = \frac{R_N}{R_N + R_L} I_N = 2.78\ \text{A}$$

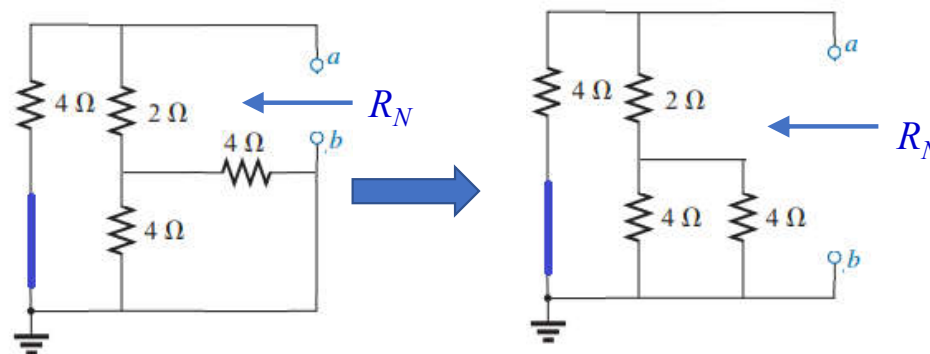
**Problem 23** [P392] Find the Norton equivalent circuit for the portions of the networks in Fig. 9.136(b) external to branch  $a$ - $b$ .



**Step 1 and Step 2:**

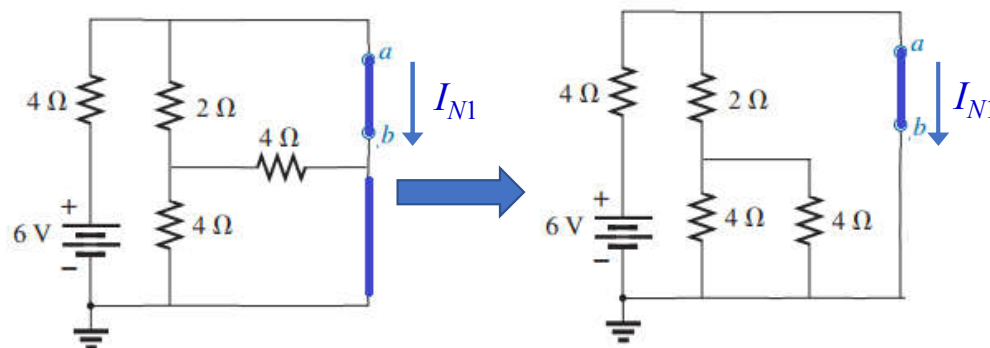


**Step 3: Calculate  $R_N$ :**



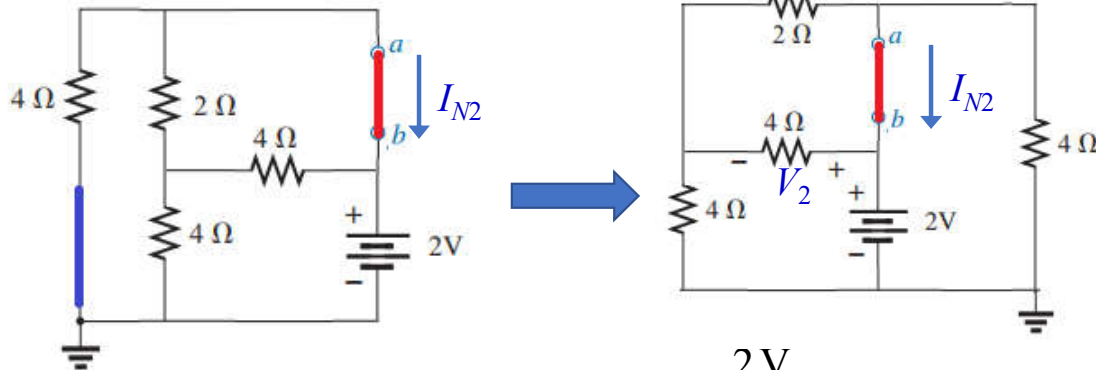
$$R_N = 4\Omega // [2\Omega + 4\Omega // 4\Omega] = 2\Omega$$

**Step 4: Calculate  $I_N$ :** Since there are two sources Superposition theorem has to be applied. Consider 6 V:



$$I_{N1} = \frac{6V}{4\Omega} = 1.5 A$$

Consider 2 V:



$$I_4 = \frac{2V}{4\Omega} = 0.5 \text{ A}$$

$$V_2 = \frac{(2\Omega // 4\Omega)}{(2\Omega // 4\Omega) + 4\Omega} 2V = 0.5 \text{ V}$$

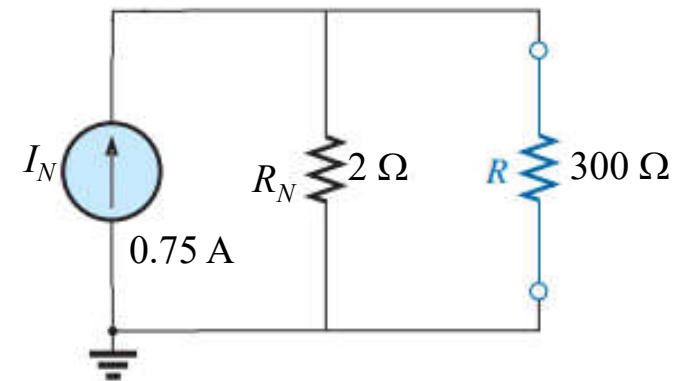
$$I_2 = \frac{0.5 \text{ V}}{2\Omega} = 0.25 \text{ A}$$

$$I_{N2} = -I_4 - I_2 = -0.75 \text{ A}$$

According to Superposition Theorem

$$I_N = I_{N1} + I_{N2} = \mathbf{0.75 \text{ A}}$$

**Step 5: Draw the Norton equivalent circuit:**



**Practice Book [Ch 9] Problem: 18 ~ 23**

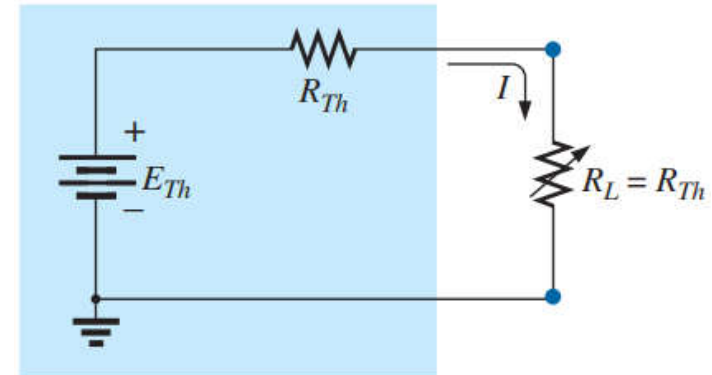
## 9.5 MAXIMUM POWER TRANSFER THEOREM



## Statement of Maximum Power Transfer Theorem

A load will receive/consume/absorb maximum power from a network when *its resistance is exactly equal to the Thévenin resistance* of the network applied to the load. That is:

$$R_L = R_{Th} \quad (9.2)$$



When  $R_L = R_{Th}$ , the maximum power delivered to the load can be determined by first finding the current:

$$I_L = \frac{E_{Th}}{R_{Th} + R_L} = \frac{E_{Th}}{R_{Th} + R_{Th}} = \frac{E_{Th}}{2R_{Th}}$$

Power can be calculated as follows:

$$P_L = I_L^2 R_L = \left( \frac{E_{Th}}{2R_{Th}} \right)^2 (R_{Th}) = \frac{E_{Th}^2 R_{Th}}{4R_{Th}^2}$$

$$P_{L_{max}} = \frac{E_{Th}^2}{4R_{Th}} \quad (9.3)$$

For the Norton equivalent circuit in Fig. 9.84, maximum power will be delivered to the load when

$$R_L = R_N \quad (9.5)$$

Power can be calculated as follows:

$$P_{L_{\max}} = \frac{I_N^2 R_N}{4} \quad (\text{W}) \quad (9.6)$$

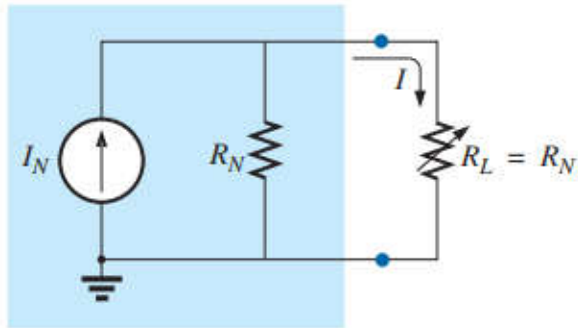
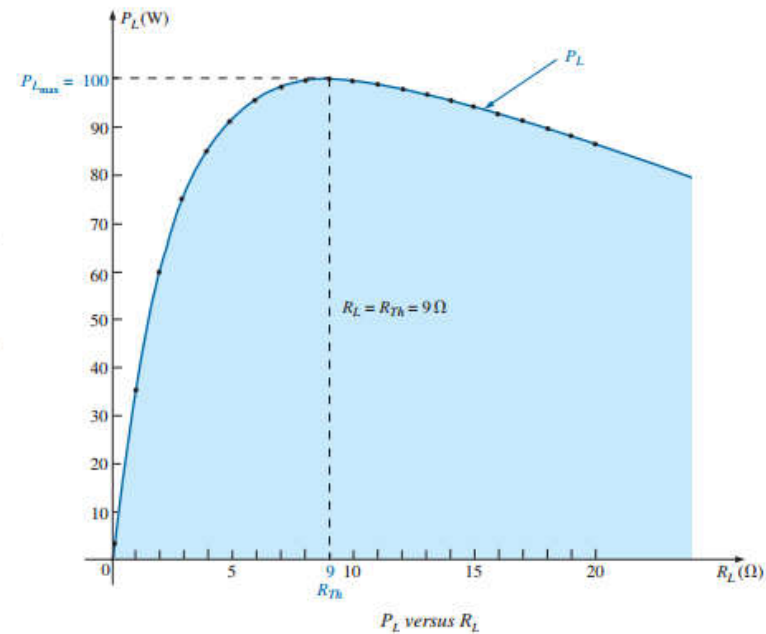
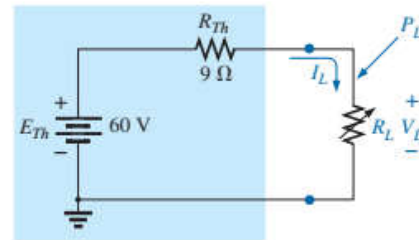


FIG. 9.84



**Example 9.5.1** For the following network, find the value of  $R_L$  for maximum power to  $RL$  and determine the maximum power to  $R_L$ .

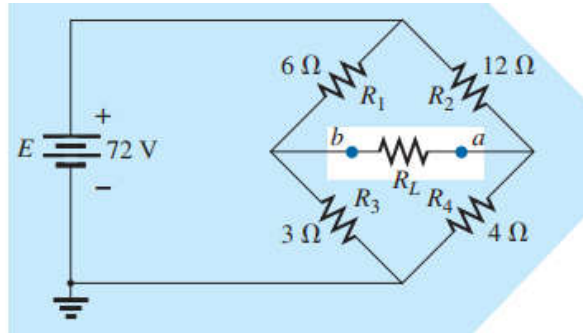
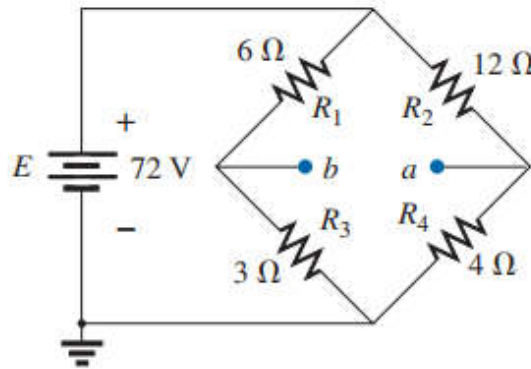


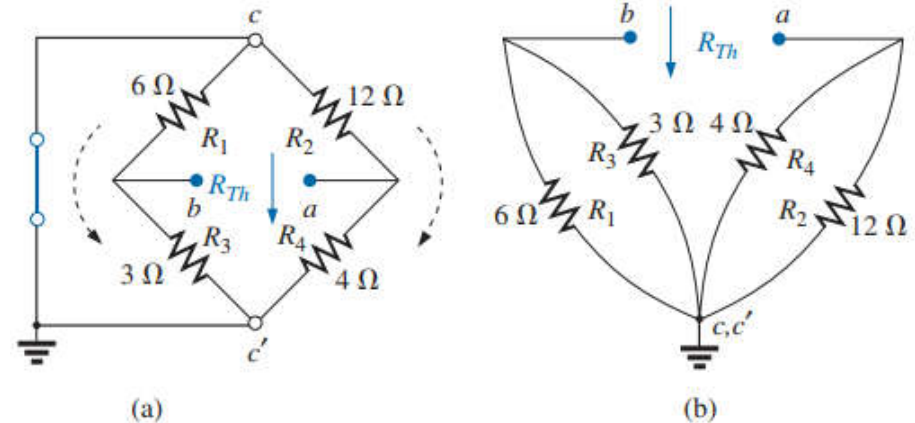
FIG. 9.43 Example 9.9.

**Step 1:** Remove that portion of the network where the Thévenin equivalent circuit is found.

**Step 2:** Mark the terminals (such as  $a$  and  $b$ ) of the remaining two-terminal network.



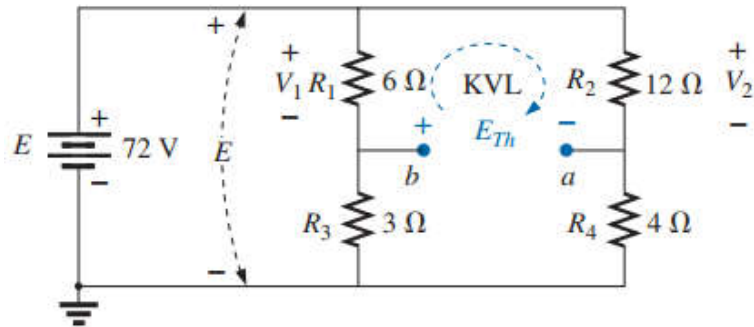
**Step 3:** Calculate  $R_{Th}$  by first setting all sources to zero (voltage sources are replaced by short circuits, and current sources by open circuits) and then finding the resultant resistance between the two marked terminals ( $a$  and  $b$ ).



$$\begin{aligned} R_{Th} &= R_{a-b} = R_1 \parallel R_3 + R_2 \parallel R_4 \\ &= 6\Omega \parallel 3\Omega + 4\Omega \parallel 12\Omega \\ &= 2\Omega + 3\Omega = 5\Omega \end{aligned}$$



**Step 4: Calculate  $E_{Th}$**  by first returning all sources to their original position and finding the open-circuit voltage between the marked terminals (*a* and *b*).

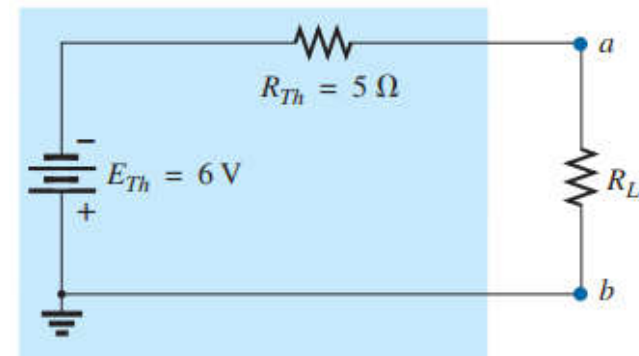


$$V_1 = \frac{R_1 E}{R_1 + R_3} = \frac{(6\ \Omega)(72\ \text{V})}{6\ \Omega + 3\ \Omega} = \frac{432\ \text{V}}{9} = 48\ \text{V}$$

$$V_2 = \frac{R_2 E}{R_2 + R_4} = \frac{(12\ \Omega)(72\ \text{V})}{12\ \Omega + 4\ \Omega} = \frac{864\ \text{V}}{16} = 54\ \text{V}$$

$$E_{Th} = V_2 - V_1 = 54\ \text{V} - 48\ \text{V} = 6\ \text{V}$$

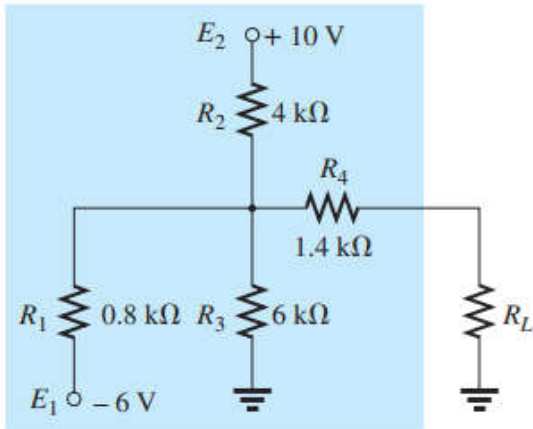
**Step 5: Draw the Thévenin equivalent circuit** with the portion of the circuit previously removed replaced between the terminals of the equivalent circuit.



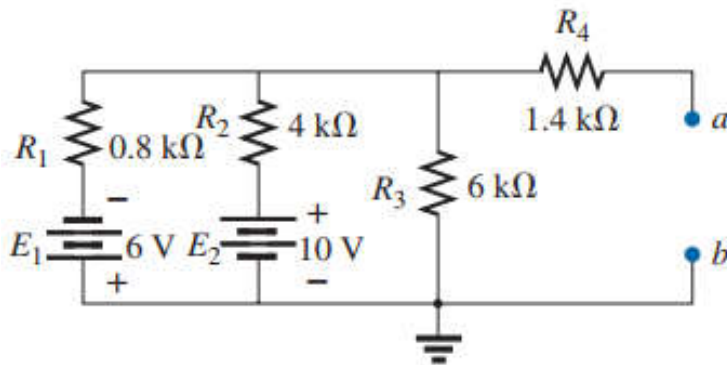
$$R_L = R_{Th} = 5\ \Omega$$

$$P_{\max} = \frac{E_{Th}^2}{4R_{Th}} = \frac{(6\text{V})^2}{4 \times 5\ \Omega} = 1.8\ \text{W}$$

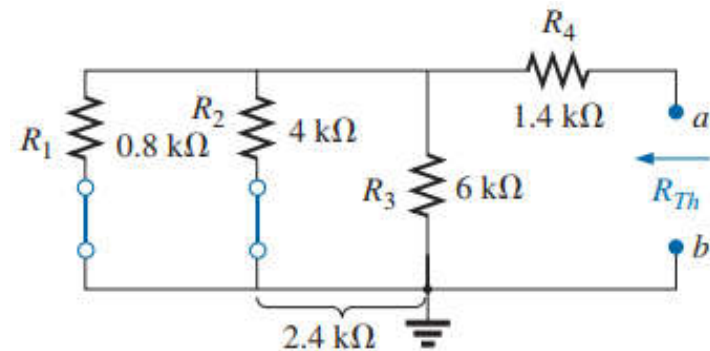
**Example 9.5.2** For the following network, find the value of  $R_L$  for maximum power to  $R_L$  and determine the maximum power to  $R_L$ .



**Step 1 and Step 2:** for Thévenin equivalent circuit



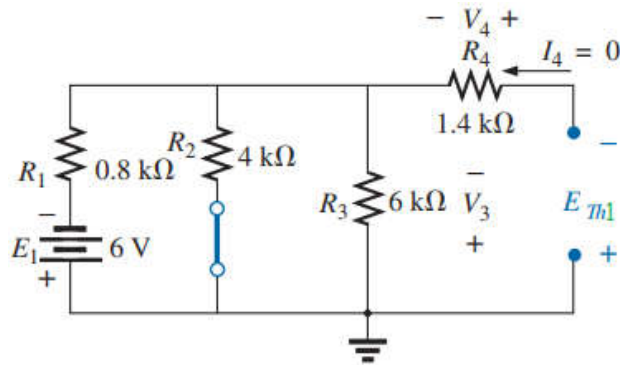
**Step 3:** Calculate  $R_{Th}$



$$\begin{aligned}
 R_{Th} &= R_4 + R_1 \parallel R_2 \parallel R_3 \\
 &= 1.4 \text{ k}\Omega + 0.8 \text{ k}\Omega \parallel 4 \text{ k}\Omega \parallel 6 \text{ k}\Omega \\
 &= 1.4 \text{ k}\Omega + 0.8 \text{ k}\Omega \parallel 2.4 \text{ k}\Omega \\
 &= 1.4 \text{ k}\Omega + 0.6 \text{ k}\Omega \\
 &= 2 \text{ k}\Omega
 \end{aligned}$$

#### Step 4: Calculate $E_{Th}$

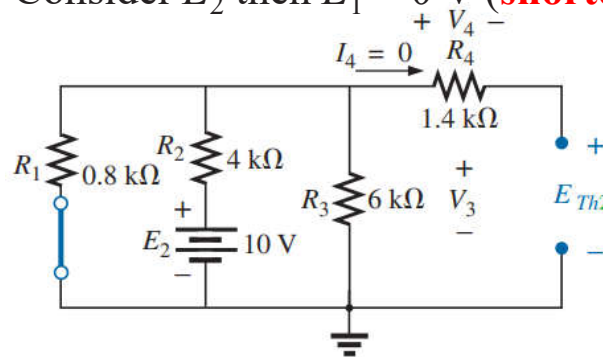
Since there are two sources Superposition theorem has to be applied. Consider  $E_1$  then  $E_2 = 0$  V (**shorted**).



$$R_{p1} = R_2 // R_3 = 2.4 \text{ k}\Omega$$

$$E_{Th1} = \frac{R_{p1}}{R_{p1} + R_1} E_1 = 4.5 \text{ V}$$

Consider  $E_2$  then  $E_1 = 0$  V (**shorted**).



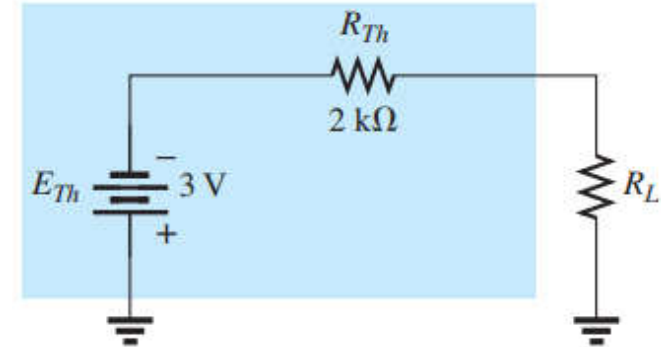
$$R_{p2} = R_1 // R_3 = 0.706 \text{ k}\Omega$$

$$E_{Th2} = \frac{R_{p2}}{R_{p2} + R_4} E_2 = 1.5 \text{ V}$$

Since  $E_{Th1}$  and  $E_{Th2}$  have opposite polarities,

$$E_{Th} = E_{Th1} - E_{Th2} = 4.5 \text{ V} - 1.5 \text{ V} = 3 \text{ V} \quad (\text{polarity of } E'_{Th})$$

#### Step 5: Draw the Thévenin equivalent circuit



$$R_L = R_{Th} = 2 \text{ k}\Omega$$

$$P_{\max} = \frac{E_{Th}^2}{4R_{Th}} = \frac{(3\text{V})^2}{4 \times (2 \times 10^3 \Omega)} = 1.125 \text{ mW}$$

### Example 9.5.2

(a) Find the Norton equivalent circuit for the network external to the resistor  $R$  for the network in Fig. 9.127.

(b) find the value of  $R$  for maximum power to  $R$  and determine the maximum power to  $R$ .

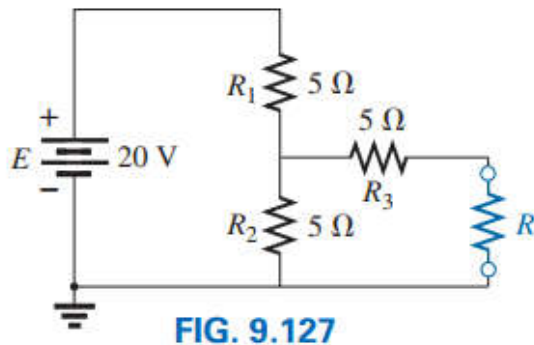
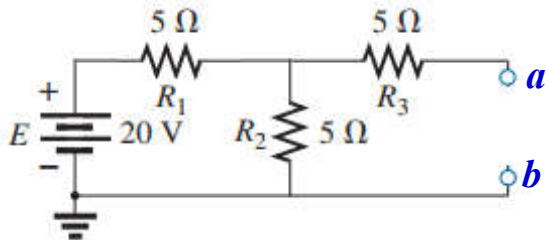
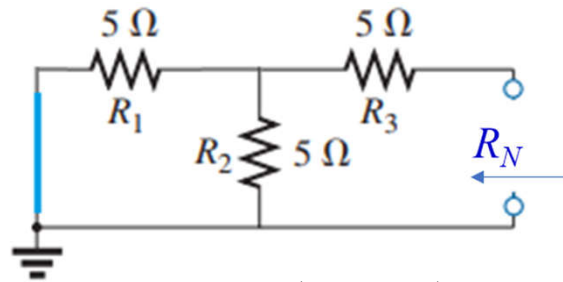


FIG. 9.127

**Step 1:** and **Step 2** for the Norton equivalent circuit.

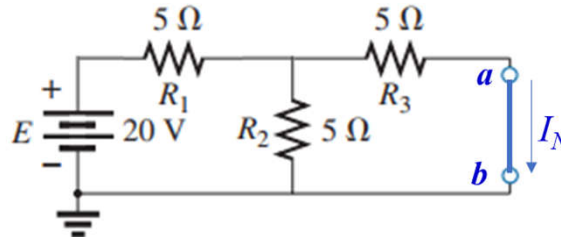


**Step 3: Calculate  $R_N$**



$$R_N = R_3 + (R_1 \parallel R_2) = 7.5 \Omega$$

**Step 4: Calculate  $I_N$**

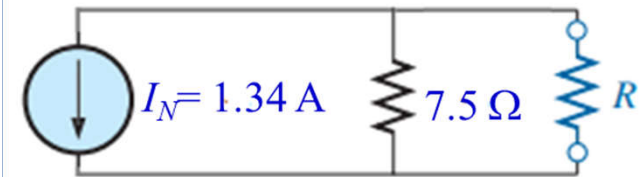


$$R_T = R_1 + (R_2 \parallel R_3) = 7.5 \Omega$$

$$I_T = \frac{E}{R_T} = 2.67 \text{ A}$$

$$I_N = \frac{I_T}{2} = 1.34 \text{ A}$$

**Step 5: Draw the Norton equivalent circuit:**



$$R_L = R_N = 7.5 \Omega$$

$$\begin{aligned} P_{\max} &= \frac{I_N^2 R_N}{4} \\ &= \frac{(1.34 \text{ A})^2 \times 7.5 \Omega}{4} \\ &= 3.367 \text{ W} \end{aligned}$$

**Practice Book [Ch 9]**  
**Problem: 18 ~ 23**