ASSIGNMENT

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SECTION - [V]

COURSE - Complex Variable Laplace & Z- transformation

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(1)

Given,

$\int_C \text{Im}\, z^2 dz$, vertices 0, 2i, 1+2i

Here,

$z = 0$

$\Rightarrow x+iy = 0 + i \cdot 0$

$\Rightarrow (x,y) = (0,0)$

$z = 2i$

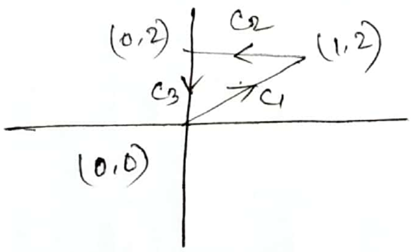
$x+iy = 0+2i$

$\Rightarrow (x,y) = (0,2)$

$z = 1+2i$

$\Rightarrow (x+iy) = (1+2i)$

$\Rightarrow (x,y) = (1,2)$



Here,

Path C_1 -

(0,0) , (1,2)

Slope,

$y - 0 = \frac{2-0}{1-0}(x-0)$

$\Rightarrow y = 2x$

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$$f(z) = \operatorname{Im} z^2$$
$$= \operatorname{Im}(x+iy)^2$$
$$= \operatorname{Im}\{x^2 - y^2 + i\cdot 2xy\}$$
$$= \operatorname{Im}\{x^2 - (2x)^2 + i\cdot 2x\cdot 2x\}$$
$$= \operatorname{Im}\{x^2 - 4x^2 + i\cdot 4x^2\}$$
$$= \operatorname{Im}\{-3x^2 + i\cdot 4x^2\}$$
$$= 4x^2$$

and,

$$z = x+iy$$
$$\Rightarrow \frac{dz}{dx} = \frac{d}{dx}(x+iy)$$
$$\Rightarrow \frac{dz}{dx} = \frac{d}{dx}(x+i2x)$$
$$\Rightarrow dz = (1+2i)dx$$

So,

$$\int_{C_3} \operatorname{Im} z^2 dz = \int_0^{\cancel{0}\,1} 4x^2(1+2i)dx$$
$$= 4(1+2i)\int_0^1 x^2 dx$$
$$= 4(1+2i)\left[\frac{x^3}{3}\right]_0^1$$
$$= 4(1+2i)\frac{1}{3}$$

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$$= \frac{4(1+2i)}{3}$$

Path C_2 -

(1,2) , (0,2)

Slope,

$$y-2 = \frac{2-2}{0-1}(x-1)$$

$$\Rightarrow y-2 = 0$$

$$\Rightarrow y = 2$$

$$\therefore f(z) = \operatorname{Im} z^2$$

$$= \operatorname{Im}(x+iy)^2$$

$$= \operatorname{Im}\{x^2-y^2+2xy\cdot i\}$$

$$= 2xy \quad [\text{imagize}]$$

$$= 2x\cdot 2 \quad [y=2]$$

$$= 4x$$

and,

$$z = x+iy$$

$$\Rightarrow \frac{dz}{dx} = \frac{d}{dx}(x+iy)$$

$$\Rightarrow \frac{dz}{dx} = 1 \quad [y=2]$$

$$\Rightarrow dz = dx$$

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So,

$$\int_{C_2} \operatorname{Im} z^2 \, dz = \int_0^1 4x \, dx$$

$$= 4\left[\frac{x^2}{2}\right]_0^1$$

$$= 4 \cdot \frac{1}{2}$$

$$= 2$$

Again,

Path C_3:

$$x = 0$$

$$f(z) = \operatorname{Im} z^2$$

$$= \operatorname{Im}(x+iy)^2$$

$$= -y^2$$

and,

$$z = x+iy$$

$$\frac{dz}{dy} = \frac{d}{dy}(x+iy)$$

$$\Rightarrow \frac{dz}{dy} = \frac{d}{dy}(i \cdot y) \quad [x=0]$$

$$\Rightarrow dz = i \cdot dy$$

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$$\int_{C_3} \operatorname{Im} z^2 \, dz = \int_0^2 -y^2 i \, dy$$

$$= -i \int_0^2 y^2 \, dy$$

$$= -i \left[\frac{y^3}{3}\right]_0^2$$

$$= -i \, 8/3$$

So,

$$\int_C \operatorname{Im} z^2 \, dz = \frac{4}{3}(1+2i) + 2 - \frac{8i}{3}$$

$$= \frac{4(1+2i) + 6 - 8i}{3}$$

$$= \frac{4+8i+6-8i}{3}$$

$$= \frac{10}{3} \quad \text{(Ans)}$$

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2(a) c is shortest path from $z=0$ to $z=i+1$

Ans- Here, C is the line segment from $z=0 (0,0)$ to $z=i+1$ $(1,1)$

$$\frac{y-y_1}{y_1-y_2} = \frac{x-x_1}{x_1-x_2}$$

$$\Rightarrow \frac{y-0}{0-1} = \frac{x-0}{0-1}$$

$$\Rightarrow -y = -x$$

$$\Rightarrow y = x$$

let, $x=t$

$y=t$

$z = x+iy = t+it$ where, $t=0$ to $t=1$

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2 (b) Consists of two line segments one form $z=0$ to $z=2i$ and other from $z=i$ to $z=i+1$

Ans - Here,

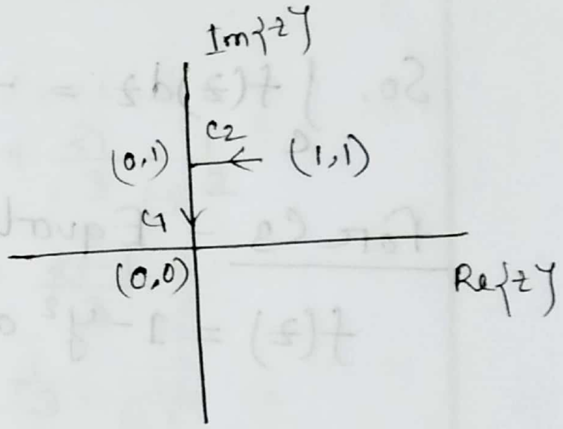
$$f(z) = 2x+y-2xi$$

for C_1 : Equation of path is $x=0$

$$f(z) = y$$

and $z = x+iy = iy$

$$dz = i\,dy$$



$$\int_{C_1} f(z)\,dz = \int_1^0 y\,i\,dy$$

$$= i\int_1^0 y\,dy$$

$$= i\left[\frac{y^2}{2}\right]_1^0$$

$$= \frac{-i}{2}$$

for C_2 - Equation of path is $y=1$

$$f(z) = 2x+1-2xi$$

and $z = x+iy = x+i$

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$$\Rightarrow dz = dx$$

$$\int_{C_2} f(z)\,dz = \int_1^0 (2x-1-2xi)\,dx$$

$$= [x^2]_1^0 - [x]_1^0 - i[x^2]_1^0$$

$$= -1 - 1 - i$$

$$= -2 - i$$

So, $\int_C f(z)\,dz = -\frac{i}{2} - 2 - i = -2 - \frac{3i}{2}$

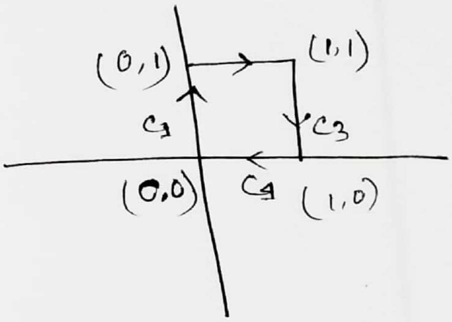
3. $\int_C \text{Re}\, z^2 dz$, C is the boundary of the square with verctices 0, i, 1+i, 1 clockwise.

Ans - Here,

$$f(z) = \text{Re}\{z^2\} = \text{Re}\{(x+iy)^2\} = \text{Re}\{x^2+2ixy-y^2\}$$
$$= x^2 - y^2$$

for C_1 - Equation of the path is $x=0$

$$f(z) = -y^2 \text{ and } z = x+iy = iy$$
$$\Rightarrow dz = idy$$



$$\int_{C_1} \text{Re}\, z^2 dz = \int_0^1 y^2 i dy$$
$$= -i\left[\frac{y^3}{3}\right]_0^1$$
$$= -\frac{i}{3}$$

for C_2 - Equation of the path is $y=1$

$$f(z) = x^2 - 1 \quad z = x+iy = x+i$$
$$dz = dx$$

$$\int_{C_2} \text{Re}\, z^2 dz = \int_0^1 (x^2-1)\, dx$$
$$= \left[\frac{x^3}{3}\right]_0^1 - [x]_0^1$$
$$= \frac{1}{3} - 1 = -2/3$$

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For C_4 - Equation of the path is $y=0$

$$f(z) = x^2 \quad ; \quad z = x+iy = x$$

$$dz = dx$$

$$\int_{C_4} \operatorname{Re} z^2 dz = \int_1^0 x\,dx$$

$$= \left[\frac{x^2}{2}\right]_1^0$$

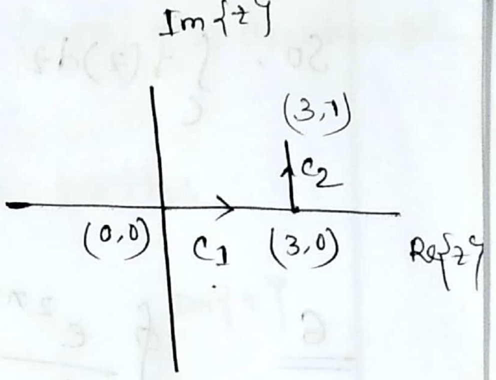
$$= -\frac{1}{2}$$

So, $\int_C \operatorname{Re} z^2 dz = -\frac{i}{3} - \frac{2}{3} + \frac{2i}{3} - \frac{1}{2}$

$$= \frac{-4-3}{6} + \frac{2i-i}{3}$$

$$= -\frac{7}{6} + \frac{i}{3}$$

4 Here,



$$f(z) = \bar{z} = x - iy$$

for C_1 Equation of the path

$$y = 0$$

$$f(z) = x, \quad z = x + iy = x$$

$$dz = dx$$

$$\int_{C_1} f(z)\,dz = \int_0^3 x\,dx$$

$$= \left[\frac{x^2}{2}\right]_0^3$$

$$= 9/2$$

for C_2 Equation of the path is

$$x = 3$$

$$f(z) = 3 - iy, \quad z = x + iy = 3 + iy$$

$$dz = i\,dy$$

$$\int_{C_2} f(z)\,dz = \int_0^1 (3 - iy)\, i\,dy$$

$$= i\left[3y - \frac{iy^2}{2}\right]_0^1$$

$$= i(3 - 1/2)$$

$$= 3i + 1/2$$

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So, $\int_C f(z)dz = 9/2 + 3i + 1/2$

$= 3i + 5$

6 $\oint_C \frac{e^{2\pi z}}{(z+i)^2} dz \qquad C\,|z+i| = 2$

for singular points $(z+i)^2 = 0$

$\Rightarrow z+i = 0$

$\Rightarrow z = -i$

Singular point $z = -i$ is a pole of order 2

$$\text{Res}(z = -i) = \lim_{z \to i} \frac{1}{(2-1)!} \frac{d}{dz}\left[\frac{e^{2\pi z}}{(z+i)^2}(z+i)^2\right]$$

$$= \lim_{z \to -i} \frac{d}{dz}\left[e^{2\pi z}\right]$$

$$= \lim_{z \to -i} 2\pi e^{2\pi z}$$

$$= 2\pi e^{i\pi}$$

$$= 2\pi(\cos\pi + i\sin\pi)$$

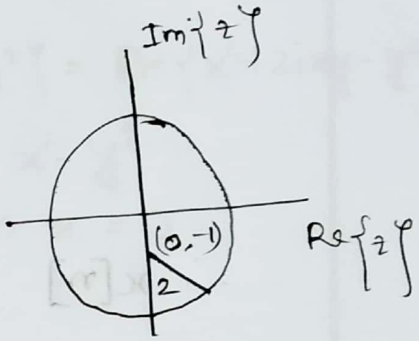
$$= -2\pi$$

5 $\int_C \left(\frac{1}{z+i} - \frac{2}{(z+i)^2} \right) dz$ $\quad |z+i| = 2$ clockwise

Ans- Here equation of the paths,

$$|z+i| = 2$$

$$\Rightarrow z+i = 2e^{i\theta}$$



$$f(z) = \left(\frac{1}{z+i} - \frac{2}{(z+i)^2} \right)$$

$$= \frac{1}{2e^{i\theta}} - \frac{2}{4e^{i2\theta}}$$

and $z = \cancel{rei}\; re^{i\theta} = 2e^{i\theta}$

$$\Rightarrow dz = 2ie^{i\theta} d\theta \qquad [r=2]$$

$$\int_C f(z)dz = \int_{2\pi}^{0} \left(\frac{1}{2e^{i\theta}} - \frac{2}{4e^{i2\theta}} \right) 2ie^{i\theta} d\theta$$

$$= i \int_{2\pi}^{0} \left(1 - \frac{1}{e^{i\theta}} \right) d\theta$$

$$= i \left[\theta - \frac{e^{-i\theta}}{(-i)} \right]_{2\pi}^{0}$$

$$= i \left[-2\pi + \frac{1}{i} - \frac{e^{-i2\pi}}{i} \right]$$

$$= -2\pi i$$

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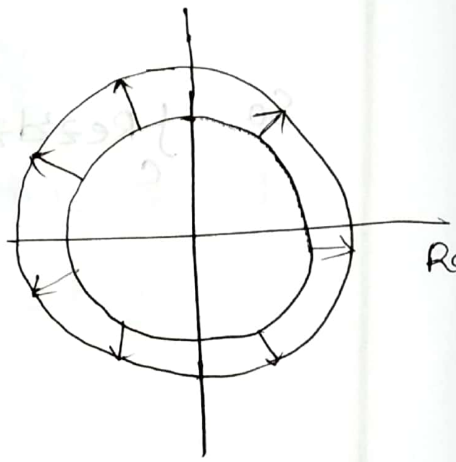
7. $f(z) = \frac{z}{(z-3)(4-z)}$ at $3<|z|<4$

Here, $f(z) = \frac{3z}{(z-3)(4-z)}$

$= \frac{9}{z-3} + \frac{12}{4-z}$

(using parctial function)



Now given that,

$\|z\| > 3$	$\|z\| < 4$
$\frac{\|z\|}{3} > 1$	$\frac{\|z\|}{4} < 1$
$\Rightarrow \frac{3}{\|z\|} < 1$	

$\therefore f(z) = \frac{9}{z(1-\frac{3}{z})} + \frac{12}{4(1-\frac{z}{4})}$

$= \frac{9}{z}\left(1-\frac{3}{z}\right)^{-1} + 3\left(1-\frac{z}{4}\right)^{-1}$

$= \frac{9}{z}\left(1+\frac{3}{z}+\frac{9}{z^2}+\cdots\right) + 3\left(1+\frac{3}{4}+\frac{z^2}{16}+\cdots\right)$

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8 $$X(z) = \frac{2}{(1-z^{-1})(1-0.5z^{-1})} \quad |z|>1$$

$$= \frac{2}{(1-z^{-1})(1-\frac{1}{2}z^{-1})}$$

$$= \frac{4}{1-z^{-1}} - \frac{2}{1-\frac{1}{2}z^{-1}}$$

$$\therefore x[n] = 4\,u[n] - 2\left(\frac{1}{2}\right)^n u[n]$$