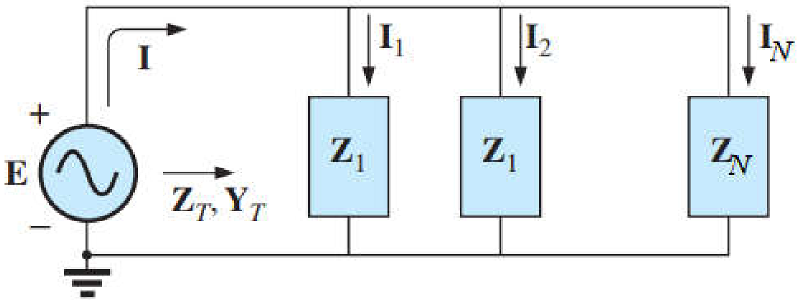
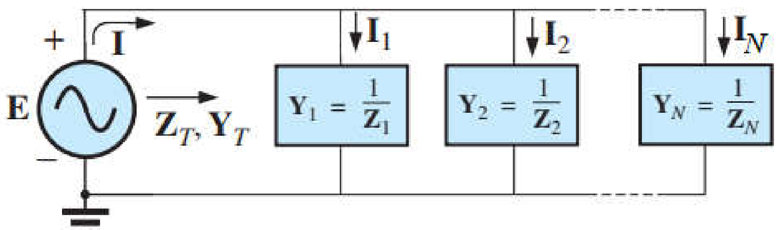
Chapter 15

Parallel Circuits

Parallel Configuration



$$Y_1 = \frac{1}{Z_1}; \quad Y_2 = \frac{1}{Z_2}; \ldots\ldots \quad Y_N = \frac{1}{Z_N}$$



The **total admittance** of a parallel configuration is the sum of the individual admittances:

$$Y_T = Y_1 + Y_2 + \ldots\ldots + Y_N \qquad \textbf{(15.16)}$$

The **total impedance** of a parallel configuration can be calculated as follows:

$$Z_T = \frac{1}{Y_T}$$

$$\frac{1}{Z_T} = \frac{1}{Z_1} + \frac{1}{Z_2} + \ldots\ldots + \frac{1}{Z_N} \qquad \textbf{(15.17)}$$

$$Z_T = \frac{1}{\frac{1}{Z_1} + \frac{1}{Z_2} + \ldots\ldots + \frac{1}{Z_N}} \qquad \textbf{(15.18)}$$

For **two impedance in parallel** :

$$Z_T = \frac{Z_1 Z_2}{Z_1 + Z_2} \qquad \textbf{(15.19)}$$

Current

$$I = \frac{E}{Z_T} = EY_T$$

$$I_1 = \frac{E}{Z_1} = EY_1$$

$$I_2 = \frac{E}{Z_2} = EY_2$$

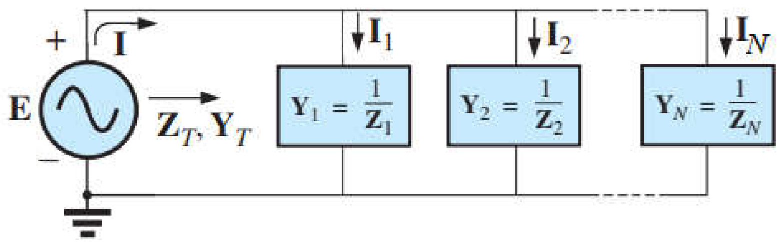
$$I_N = \frac{E}{Z_N} = EY_N$$

If $Z_1 = Z_2 = \ldots. = Z_N = Z_p$

$$Y_1 = Y_2 = \ldots. = Y_N = Y_p$$

$$Y_T = N \times Y_p \qquad Z_T = \frac{Z_p}{N}$$

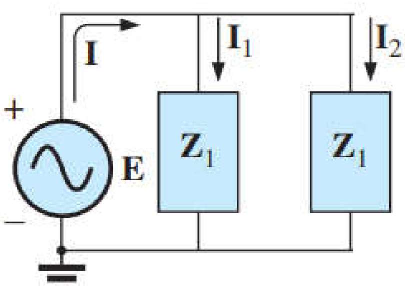
$$I_1 = I_2 = .. = I_N = \frac{I}{N}$$



Current Divider Rule (CDR)

The current flows through an admittance in a parallel circuit is equal to the value of that admittance (Y_x) times the total current (I) divided by the total admittance (Y_T) of the parallel configuration.

$$I_x = \frac{Y_x}{Y_T} I = \frac{Z_T}{Z_x} I$$



$$I_1 = \frac{Z_2}{Z_1 + Z_2} I$$

$$I_2 = \frac{Z_1}{Z_1 + Z_2} I$$

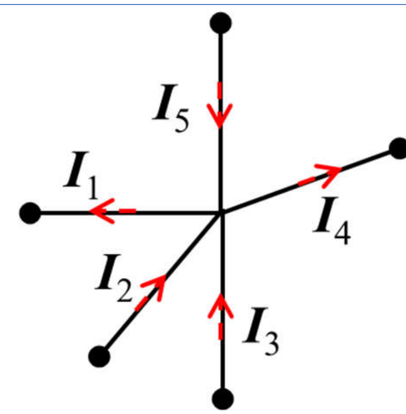
Kirchhoff's Current Law (KCL)

(**1**) The algebraic sum of the currents entering and leaving a junction (or region) of a network is zero.

$$\sum I_{entering} - \sum I_{leaving} = 0$$

(**2**) The sum of the currents entering a junction (or region) of a network must equal the sum of the currents leaving the same junction (or region).

$$\sum I_{entering} = \sum I_{leaving}$$

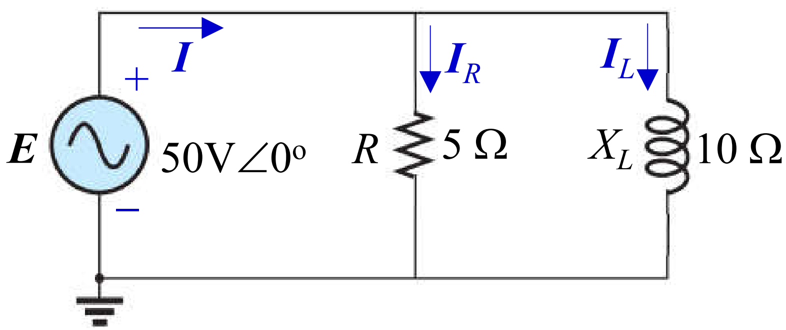


(**1**) $(I_2 + I_3 + I_5) - (I_1 + I_4) = 0$

(**2**) $I_2 + I_3 + I_5 = I_1 + I_4$

R-*L* Parallel Circuit

R-L Parallel Circuit



Admittance

$$Z_R = 5\Omega\angle 0° = 5\ \Omega \qquad Z_L = 10\Omega\angle 90° = j10\ \Omega$$

$$Y_R = \frac{1}{Z_R} = \frac{1}{5\Omega\angle 0°} = 0.2\text{S}\angle 0° = 0.2\ \text{S}$$

$$Y_L = \frac{1}{Z_L} = \frac{1}{10\Omega\angle 90°} = 0.1\text{S}\angle -90° = -j0.1\ \text{S}$$

$$Y_T = Y_R + Y_L = 0.2\ \text{S} - j0.1\ \text{S} = 0.224\text{S}\angle -26.57°$$

Admittance Diagram

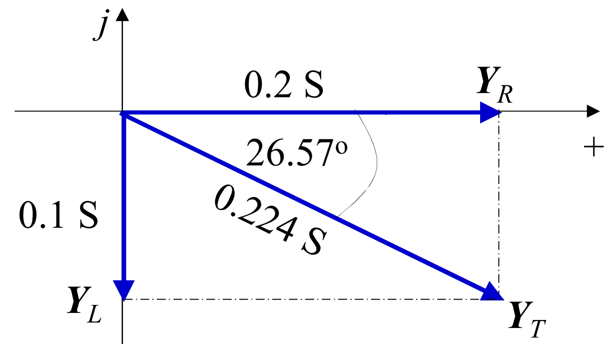


Fig. Admittance diagram

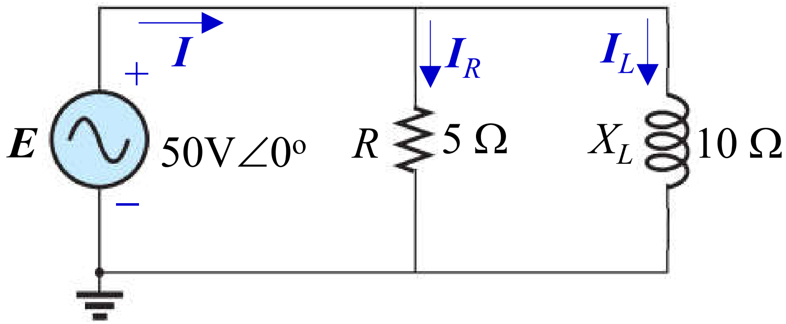
Impedance

$$Z_T = \frac{1}{Y_T} = \frac{1}{0.224\text{S}\angle -26.57°} = 4.46\Omega\angle 26.57° \cong 4 + j2\Omega$$

$$Z_T = \frac{1}{\frac{1}{Z_R} + \frac{1}{Z_L}} = \frac{1}{\frac{1}{5\Omega} + \frac{1}{j10\Omega}} = 4 + j2\Omega = 4.47\Omega\angle 26.57$$

$$Z_T = \frac{Z_R Z_L}{Z_R + Z_L} = \frac{(5\Omega)(j10\Omega)}{5\Omega + j10\Omega} = 4 + j2\ \Omega = 4.47\Omega\angle 26.57°$$

R-L Parallel Circuit



Current

$$\boldsymbol{I} = \frac{\boldsymbol{E}}{\boldsymbol{Z}_T} = \boldsymbol{E}\boldsymbol{Y}_T = \frac{50\text{V}\angle 0^\circ}{4.47\Omega\angle 26.57^\circ}$$
$$= 11.18\text{A}\angle -26.57^\circ$$

$$\boldsymbol{I}_R = \frac{\boldsymbol{E}}{\boldsymbol{Z}_R} = \boldsymbol{E}\boldsymbol{Y}_R = \frac{50\text{V}\angle 0^\circ}{5\Omega\angle 0^\circ} = 10\text{A}\angle 0^\circ$$

$$\boldsymbol{I}_L = \frac{\boldsymbol{E}}{\boldsymbol{Z}_L} = \boldsymbol{E}\boldsymbol{Y}_L = \frac{50\text{V}\angle 0^\circ}{10\Omega\angle 90^\circ} = 5\text{A}\angle -90^\circ$$

KCL:

$$\boldsymbol{I}_R + \boldsymbol{I}_L = 10\text{A}\angle 0^\circ + 5\text{A}\angle -90^\circ = 11.18\text{A}\angle -26.57^\circ = \boldsymbol{I}$$

Phasor Diagram

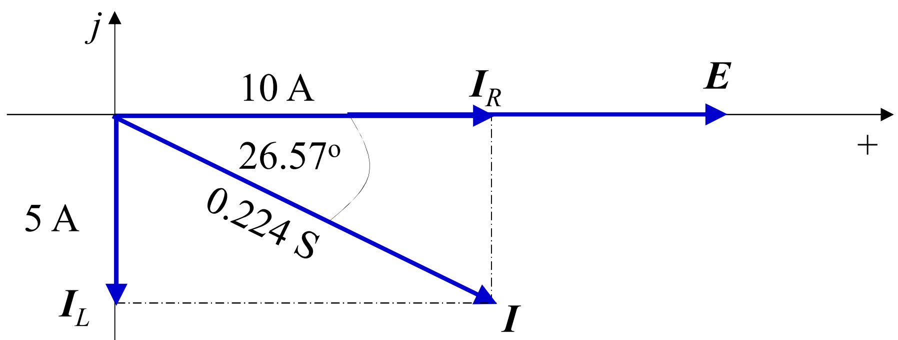
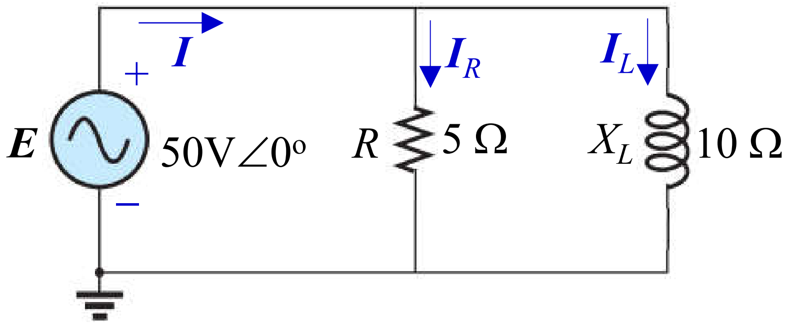


Fig. Phasor diagram

Practice Solution of Fig. 15.68 [Ch. 15], Problems 28 and 30

R-L Parallel Circuit



Power Factor and Reactive Factor

$pf = (G/Y_T) = \cos\theta_z = \cos(26.57^o) = \mathbf{0.894\ lagging}$

$rf = (B_L/Y_T) = \sin\theta_z = \sin(26.57^o) = \mathbf{0.447}$

Power [Total watts]

$P_E = EI\cos\theta_z = 50\times11.18\cos(26.57^o) = \mathbf{500.19\ W}$

$P_R = I_R^2R = (E^2/R) = (50V)^2/5\Omega = \mathbf{500\ W}$

Reactive Power [volt-ampere reactive]

$Q_E = EI\sin\theta_z = 50\times11.18\sin(26.57^o) = \mathbf{250.1\ Var}$

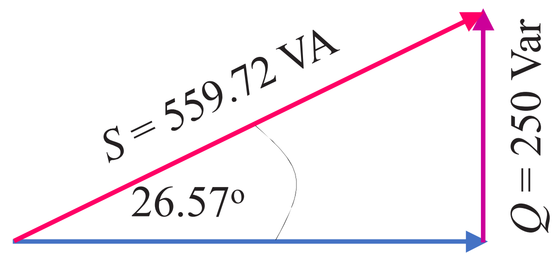
$Q_L = I_L^2X_L = (E^2/X_L) = (50V)^2/10\Omega = \mathbf{250\ Var}$

Apparent Power [volt-ampere]

$S_E = EI = 50\times11.18 = \mathbf{559.5\ VA}$

$S_Z = I^2Z = (E^2/Z) = (50V)^2/4.47\Omega = \mathbf{559.72\ VA}$

Power Triangle



Instantaneous Equation

$p(t) = 500(1-\cos2\omega t) + 250\sin2\omega t$ W

$e(t) = \left(\sqrt{2}\times50\right)\sin\omega t$ V

$i(t) = \left(\sqrt{2}\times11.18\right)\sin\left(\omega t - 26.57^\circ\right)$ A

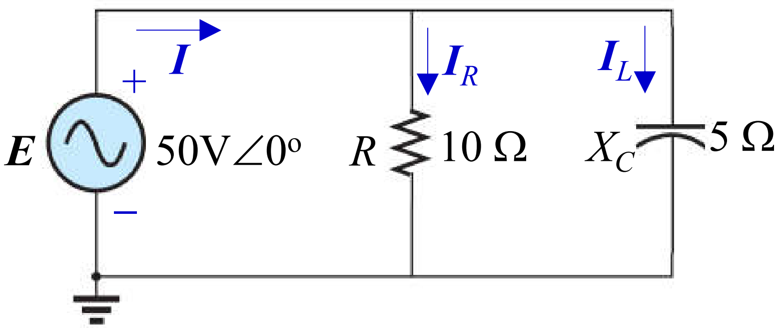
$i_R(t) = \left(\sqrt{2}\times10\right)\sin\omega t$ A

$i_L(t) = \left(\sqrt{2}\times5\right)\sin\left(\omega t - 90^\circ\right)$ A

Practice Solution of Fig. 15.68 [Ch. 15], Problems 28 and 30

R-*C* Parallel Circuit

R-C Parallel Circuit



Admittance

$$Z_R = 10\Omega\angle 0° = 10\ \Omega$$

$$Z_C = 5\Omega\angle -90° = -j5\ \Omega$$

$$Y_R = \frac{1}{Z_R} = \frac{1}{10\Omega\angle 0°} = 0.1\text{S}\angle 0° = 0.1\ \text{S}$$

$$Y_C = \frac{1}{Z_C} = \frac{1}{5\Omega\angle -90°} = 0.2\text{S}\angle 90° = j0.2\ \ \text{S}$$

$$Y_T = Y_R + Y_C = 0.1\ \text{S} + j0.2\ \text{S}$$

$$= 0.224\text{S}\angle 63.43°$$

Admittance Diagram

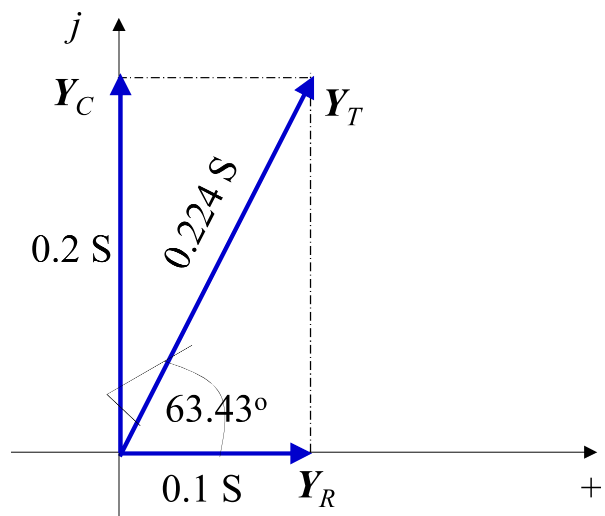


Fig. Admittance diagram

Impedance

$$Z_T = \frac{1}{Y_T} = \frac{1}{0.224\text{S}\angle 63.43°}$$

$$= 4.46\Omega\angle -63.43°$$

$$\cong 2 - j4\Omega$$

$$Z_T = \frac{Z_R Z_C}{Z_R + Z_C}$$

$$= \frac{(10\Omega)(j5\Omega)}{10\Omega -\ j5\Omega}$$

$$= 2 - j4\Omega$$

$$= 4.472\Omega\angle -63.43°$$

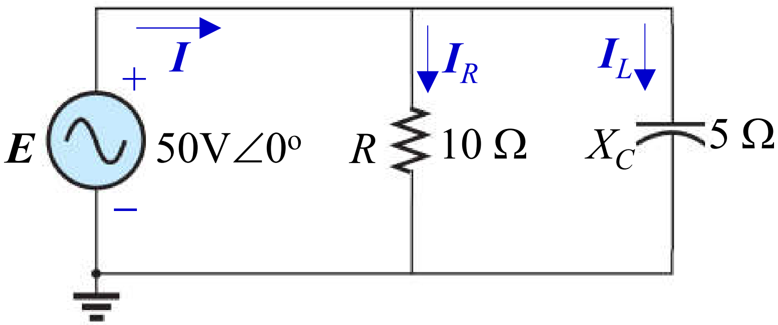
$$Z_T = \frac{1}{\frac{1}{Z_R} + \frac{1}{Z_C}}$$

$$= \frac{1}{\frac{1}{10\Omega} + \frac{1}{-j5\Omega}}$$

$$= 2 - j4\Omega$$

$$= 4.472\Omega\angle -63.43°$$

R-C Parallel Circuit



Current

$$\boldsymbol{I} = \frac{\boldsymbol{E}}{\boldsymbol{Z}_T} = \boldsymbol{E}\boldsymbol{Y}_T = \frac{50\text{V}\angle 0°}{4.47\Omega\angle -63.43°}$$

$$= 11.18\text{A}\angle 63.43°$$

$$\boldsymbol{I}_R = \frac{\boldsymbol{E}}{\boldsymbol{Z}_R} = \boldsymbol{E}\boldsymbol{Y}_R = \frac{50\text{V}\angle 0°}{10\Omega\angle 0°} = 5\text{A}\angle 0°$$

$$\boldsymbol{I}_C = \frac{\boldsymbol{E}}{\boldsymbol{Z}_C} = \boldsymbol{E}\boldsymbol{Y}_C = \frac{50\text{V}\angle 0°}{5\Omega\angle -90°} = 10\text{A}\angle 90°$$

KCL:

$$\boldsymbol{I}_R + \boldsymbol{I}_C = 5\text{A}\angle 0° + 10\text{A}\angle 90° = 11.18\text{A}\angle 63.43° = \boldsymbol{I}$$

Phasor Diagram

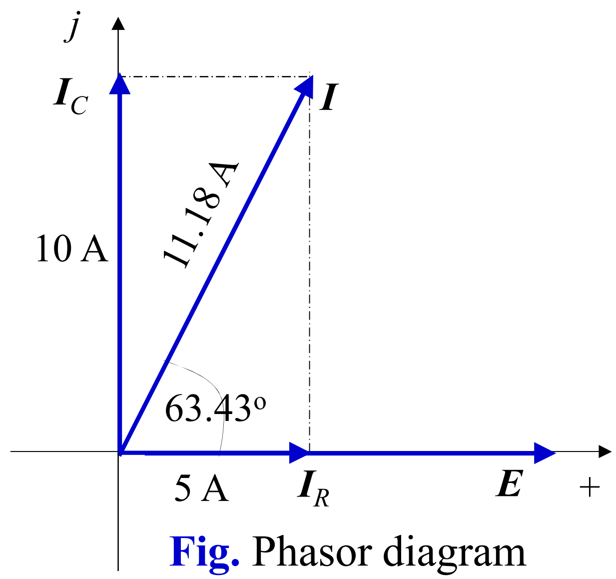
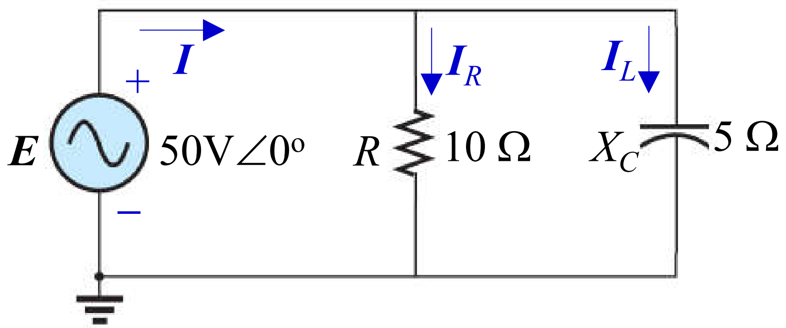


Fig. Phasor diagram

Practice Solution of Fig. 15.72 [Ch. 15], Problem 29

R-C Parallel Circuit



Power Factor and Reactive Factor

$pf = (G/Y_T) = \cos\theta_z = \cos(-63.43^o) = \mathbf{0.447\ leading}$

$rf = (B_L/Y_T) = \sin\theta_z = \sin(-63.43^o) = \mathbf{-\ 0.894}$

Power [Total watts]

$P_E = EI\cos\theta_z = 50\times 11.18\cos(-63.43^o) = \mathbf{250.1\ W}$

$P_R = I_R{}^2R = (E^2/R) = (50V)^2/10\Omega = \mathbf{250\ W}$

Reactive Power [volt-ampere reactive]

$Q_E = EI\sin\theta_z = 50\times 11.18\sin(-63.43^o) = \mathbf{-\ 500.19\ Var}$

$Q_C = -\ I_C{}^2X_C = -(E^2/X_C) = -(50V)^2/5\Omega = \mathbf{-\ 500\ Var}$

Apparent Power [volt-ampere]

$S_E = EI$

$= 50\times 11.18\ \text{VA}$

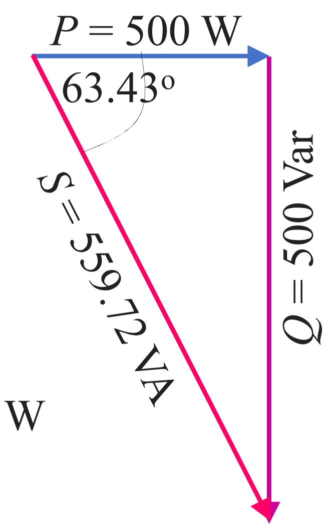
$\mathbf{= 559.5\ VA}$

$S_Z = I^2Z = (E^2/Z)$

$=(50V)^2/4.47\Omega$

$= \mathbf{559.72\ VA}$

Power Triangle



Instantaneous Equation

$p(t) = 250(1 - \cos 2\omega t) - 500\sin 2\omega t$ W

$e(t) = \left(\sqrt{2}\times 50\right)\sin\omega t$ V

$i(t) = \left(\sqrt{2}\times 11.18\right)\sin\left(\omega t + 63.43^\circ\right)$ A

$i_R(t) = \left(\sqrt{2}\times 5\right)\sin\omega t$ A

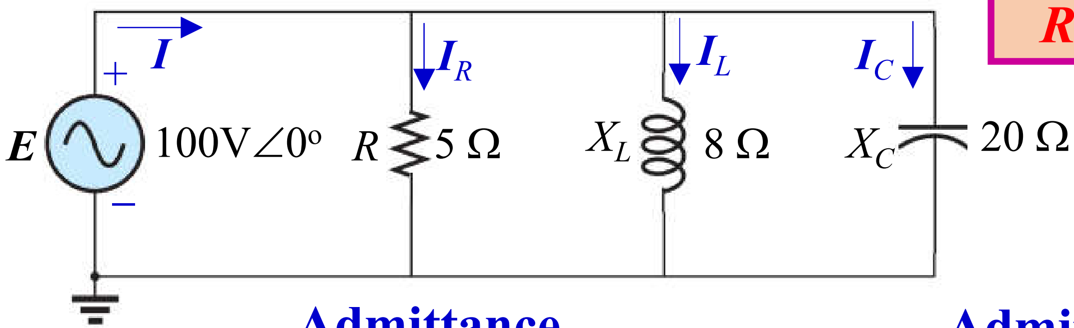
$i_C(t) = \left(\sqrt{2}\times 10\right)\sin\left(\omega t + 90^\circ\right)$ A

Practice Solution of Fig. 15.72 [Ch. 15], Problem 29

R-L-C Parallel Circuit

Example 1

R-L-C Parallel Circuit 1



Admittance

$\boldsymbol{Z}_R = 5\Omega\angle 0° = 5\ \Omega \qquad \boldsymbol{Z}_L = 8\Omega\angle 90° = j8\ \Omega$

$\boldsymbol{Z}_C = 20\Omega\angle -90° = -j20\ \Omega$

$$\boldsymbol{Y}_R = \frac{1}{\boldsymbol{Z}_R} = \frac{1}{5\Omega\angle 0°} = 0.2\text{S}\angle 0° = 0.2\ \text{S}$$

$$\boldsymbol{Y}_L = \frac{1}{\boldsymbol{Z}_L} = \frac{1}{8\Omega\angle 90°} = 0.125\text{S}\angle -90° = -j0.125\ \text{S}$$

$$\boldsymbol{Y}_C = \frac{1}{\boldsymbol{Z}_C} = \frac{1}{20\Omega\angle -90°} = 0.05\text{S}\angle 90° = j0.05\ \text{S}$$

$$\boldsymbol{Y}_T = \boldsymbol{Y}_R + \boldsymbol{Y}_L + \boldsymbol{Y}_C = 0.2\ \text{S} - j0.125\ \text{S} + j0.05\ \text{S}$$
$$= 0.2\ \text{S} - j0.075\ \text{S} = 0.214\text{S}\angle -20.56°$$

Admittance Diagram

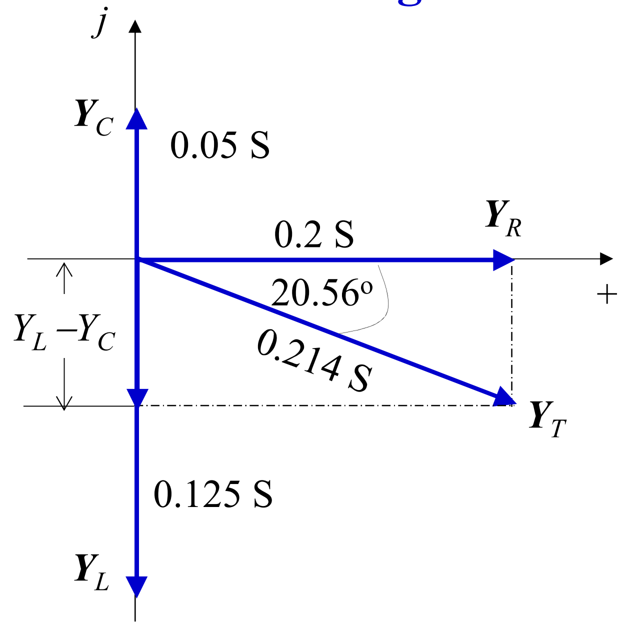
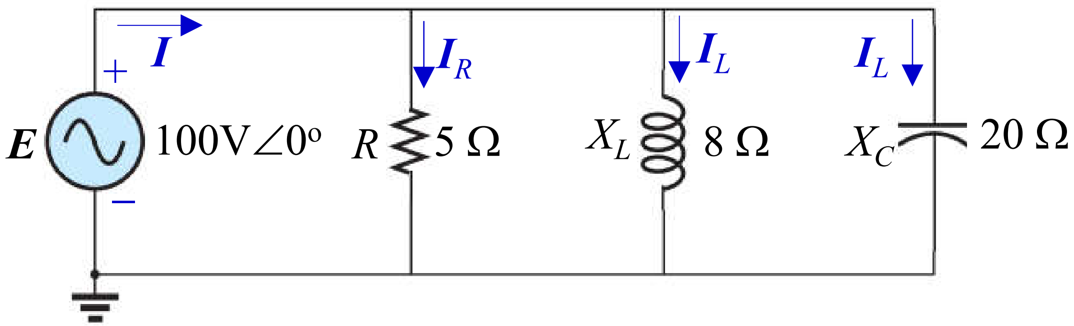


Fig. Admittance diagram

Impedance

$$\boldsymbol{Z}_T = \frac{1}{\dfrac{1}{\boldsymbol{Z}_R} + \dfrac{1}{\boldsymbol{Z}_L} + \dfrac{1}{\boldsymbol{Z}_C}}$$
$$= \frac{1}{\dfrac{1}{5\Omega} + \dfrac{1}{j8\Omega} + \dfrac{1}{-j20\Omega}}$$
$$= 4.38 + j1.64\Omega$$
$$= 4.68\Omega\angle 20.56°$$



Current

$$\boldsymbol{I} = \frac{\boldsymbol{E}}{\boldsymbol{Z}_T} = \boldsymbol{E}\boldsymbol{Y}_T = \frac{100\text{V}\angle 0°}{4.68\Omega\angle 20.56°} = 21.37\text{A}\angle -20.56°$$

$$\boldsymbol{I}_R = \frac{\boldsymbol{E}}{\boldsymbol{Z}_R} = \boldsymbol{E}\boldsymbol{Y}_R = \frac{100\text{V}\angle 0°}{5\Omega\angle 0°} = 20\text{A}\angle 0°$$

$$\boldsymbol{I}_L = \frac{\boldsymbol{E}}{\boldsymbol{Z}_L} = \boldsymbol{E}\boldsymbol{Y}_L = \frac{100\text{V}\angle 0°}{8\Omega\angle 90°} = 12.5\text{A}\angle -90°$$

$$\boldsymbol{I}_C = \frac{\boldsymbol{E}}{\boldsymbol{Z}_C} = \boldsymbol{E}\boldsymbol{Y}_C = \frac{100\text{V}\angle 0°}{20\Omega\angle -90°} = 5\text{A}\angle 90°$$

KCL:

$$\boldsymbol{I}_R + \boldsymbol{I}_L + \boldsymbol{I}_C = 20\text{A}\angle 0° + 12.5\text{A}\angle -90° + 5\text{A}\angle 90°$$
$$= 21.37\text{A}\angle -20.56° = \boldsymbol{I}$$

Phasor Diagram

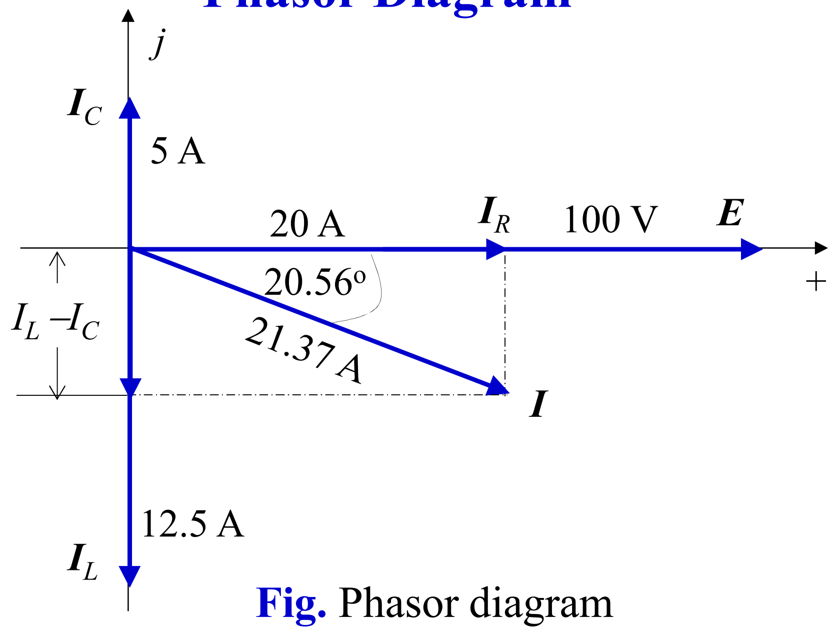
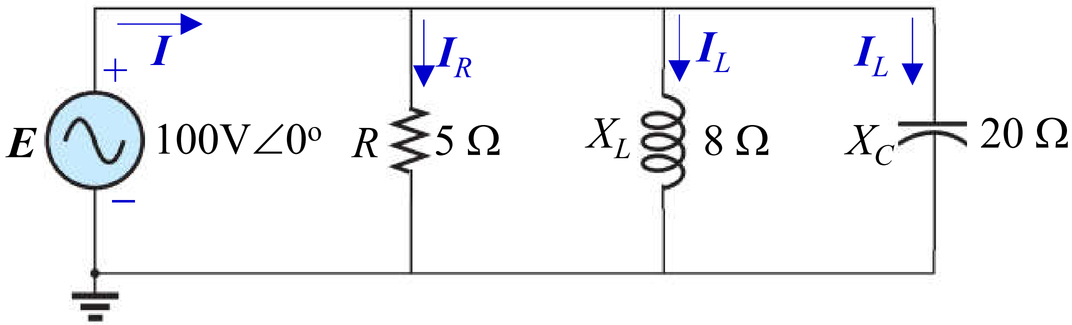


Fig. Phasor diagram

Practice Solution of Fig. 15.77 [Ch. 15], Problem 31 to 32



Apparent Power [volt-ampere]

$S_E = EI = 100\times21.37 = \mathbf{2137\ VA}$
$S_Z = I^2Z = (E^2/Z) = (100V)^2/4.68\Omega = \mathbf{2137.75\ VA}$

Power Triangle

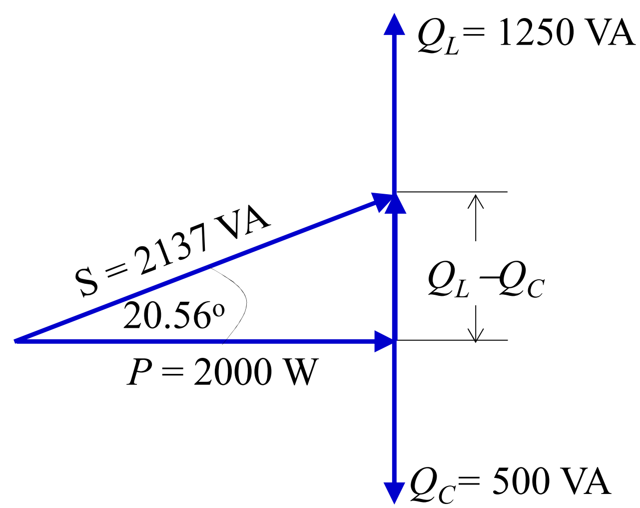


Fig. Admittance diagram

Power Factor and Reactive Factor

$pf = (G/Y_T) = \cos\theta_z = \cos(20.56^o) = \mathbf{0.936\ lagging}$
$rf = (B/Y_T) = \sin\theta_z = \sin(20.56^o) = \mathbf{0.351}$

Power [Total watts]

$P_E = EI\cos\theta_z = 100\times21.37\cos(20.56^o) = \mathbf{2000.23\ W}$
$P_R = I_R^2R = (E^2/R) = (100V)^2/5\Omega = \mathbf{2000\ W}$

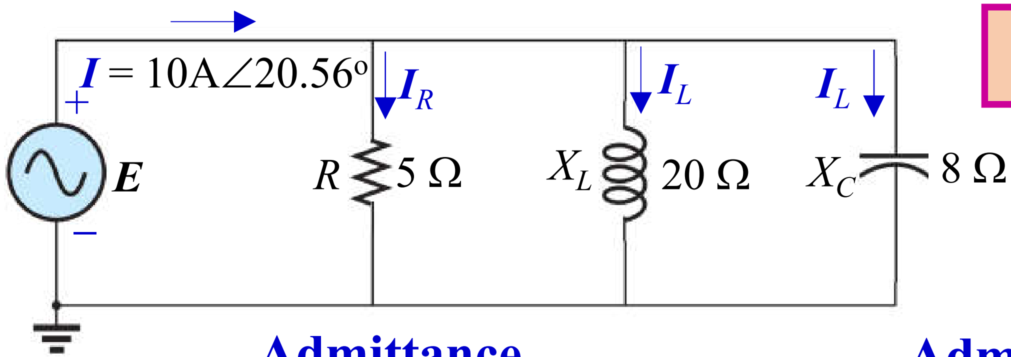
Reactive Power [volt-ampere reactive]

$Q_E = EI\sin\theta_z = 100\times21.37\sin(20.56^o) = \mathbf{750.09\ Var}$
$Q_L = I_L^2X_L = (E^2/X_L) = (100V)^2/8\Omega = \mathbf{1250\ Var}$
$Q_C = -I_C^2X_C = -(E^2/X_C) = -(100V)^2/20\Omega = \mathbf{-500\ Var}$
$Q = Q_L + Q_C = \mathbf{750\ Var}$

Practice Solution of Fig. 15.77 [Ch. 15], Problem 31 to 32

R-L-C Parallel Circuit
Example 2

R-L-C Parallel Circuit 2



Admittance

$$\boldsymbol{Z}_R = 5\Omega\angle 0° = 5\ \Omega \qquad \boldsymbol{Z}_L = 20\Omega\angle 90° = j20\ \Omega$$

$$\boldsymbol{Z}_C = 8\Omega\angle -90° = -j8\ \Omega$$

$$\boldsymbol{Y}_R = \frac{1}{\boldsymbol{Z}_R} = \frac{1}{5\Omega\angle 0°} = 0.2\text{S}\angle 0° = 0.2\ \text{S}$$

$$\boldsymbol{Y}_L = \frac{1}{\boldsymbol{Z}_L} = \frac{1}{20\Omega\angle 90°} = 0.05\text{S}\angle -90° = -j0.05\ \text{S}$$

$$\boldsymbol{Y}_C = \frac{1}{\boldsymbol{Z}_C} = \frac{1}{8\Omega\angle -90°} = 0.125\text{S}\angle 90° = j0.125\ \text{S}$$

$$\boldsymbol{Y}_T = \boldsymbol{Y}_R + \boldsymbol{Y}_L + \boldsymbol{Y}_C = 0.2\ \text{S} - j0.05\ \text{S} + j0.125\ \text{S}$$
$$= 0.2\ \text{S} + j0.075\ \text{S} = 0.214\text{S}\angle 20.56°$$

Admittance Diagram

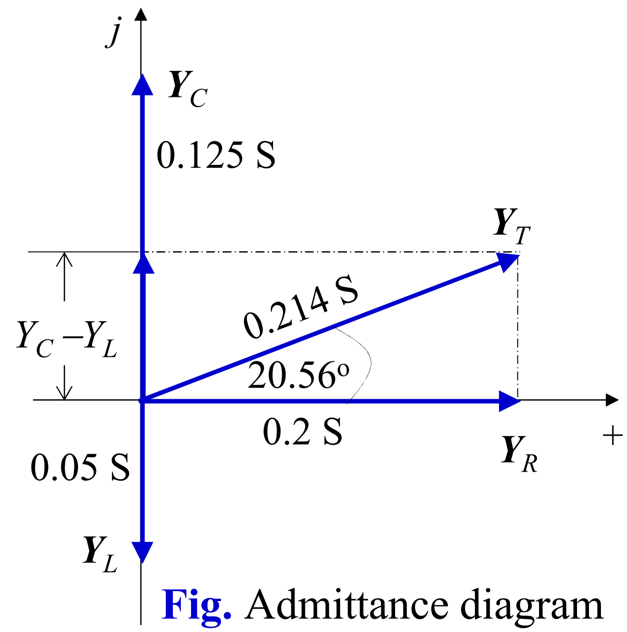


Fig. Admittance diagram

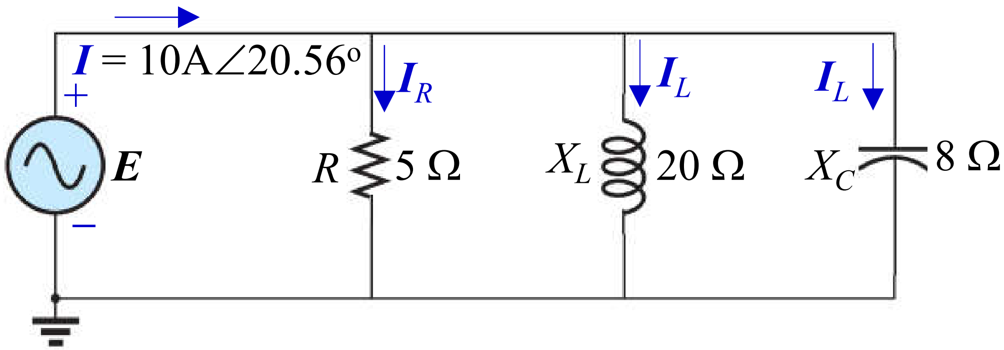
Impedance

$$\boldsymbol{Z}_T = \frac{1}{\dfrac{1}{\boldsymbol{Z}_R} + \dfrac{1}{\boldsymbol{Z}_L} + \dfrac{1}{\boldsymbol{Z}_C}}$$

$$= \frac{1}{\dfrac{1}{5\Omega} + \dfrac{1}{-j8\Omega} + \dfrac{1}{j20\Omega}}$$

$$= 4.38 - j1.64\Omega$$

$$= 4.68\Omega\angle -20.56°$$



Current

$$E = IZ_T = \frac{I}{Y_T} = \frac{10A\angle 20.56^\circ}{0.214S\angle -20.56^\circ} = 46.73V\angle 0^\circ$$

$$I_R = \frac{E}{Z_R} = EY_R = \frac{46.73V\angle 0^\circ}{5\Omega\angle 0^\circ} = 9.35A\angle 0^\circ$$

$$I_L = \frac{E}{Z_L} = EY_L = \frac{46.73V\angle 0^\circ}{20\Omega\angle 90^\circ} = 2.34A\angle -90^\circ$$

$$I_C = \frac{E}{Z_C} = EY_C = \frac{46.73V\angle 0^\circ}{8\Omega\angle -90^\circ} = 5.84A\angle 90^\circ$$

KCL:

$$I_R + I_L + I_C = 9.35A\angle 0^\circ + 2.34A\angle -90^\circ + 5.84A\angle 90^\circ$$
$$= 10A\angle 20.56^\circ = I$$

Phasor Diagram

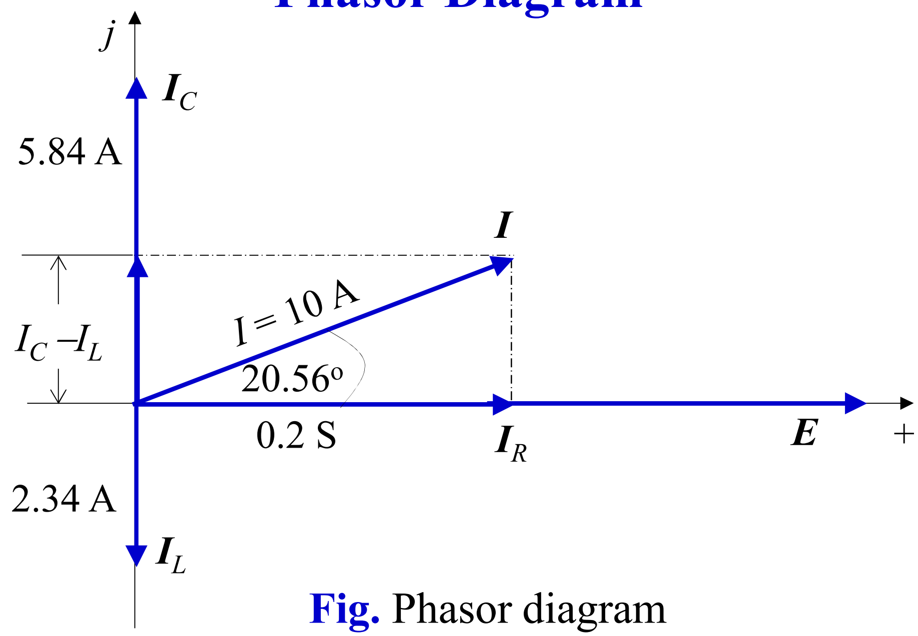
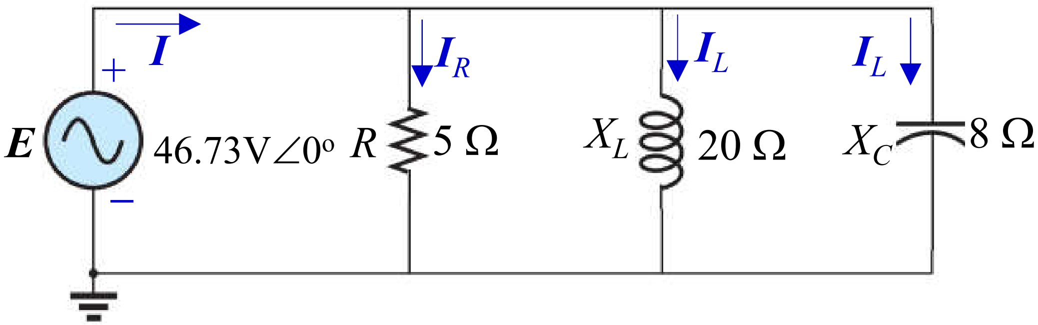


Fig. Phasor diagram



Power Factor and Reactive Factor

$pf = (G/Y_T) = \cos\theta_z = \cos(-20.56°)$ **= 0.351 leading**

$rf = (B/Y_T) = \sin\theta_z = \sin(-20.56°)$ **= – 0.936**

Power [Total watts]

$P_E = EI\cos\theta_z = 46.73\times10\cos(-20.56°)$ **= 437.39 W**

$P_R = I_R^2R = (E^2/R) = (46.73\text{ V})^2/5\Omega$ **= 437.11 W**

Reactive Power [volt-ampere reactive]

$Q_E = EI\sin\theta_z = 46.73\times21.37\sin(-20.56°)$ **= – 164.02 Var**

$Q_L = I_L^2X_L = (E^2/X_L) = (46.73\text{V})^2/20\Omega$ **= 109.51 Var**

$Q_C = -I_C^2X_C = -(E^2/X_C) = -(46.73\text{V})^2/8\Omega$ **= – 272.84 Var**

$Q = Q_L + Q_C$ **= – 163.33 Var**

Apparent Power [volt-ampere]

$S_E = EI = 46.73\times10$ **= 467.3 VA**

$S_Z = I^2Z = (E^2/Z) = (46.73\text{V})^2/4.68\Omega$ **= 468 VA**

Power Triangle

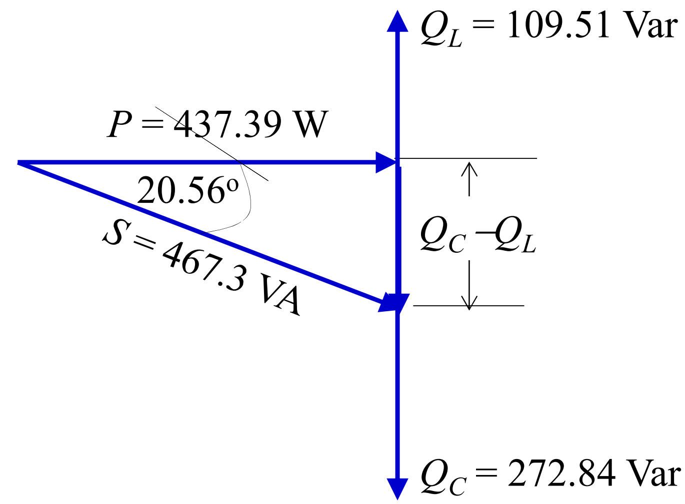


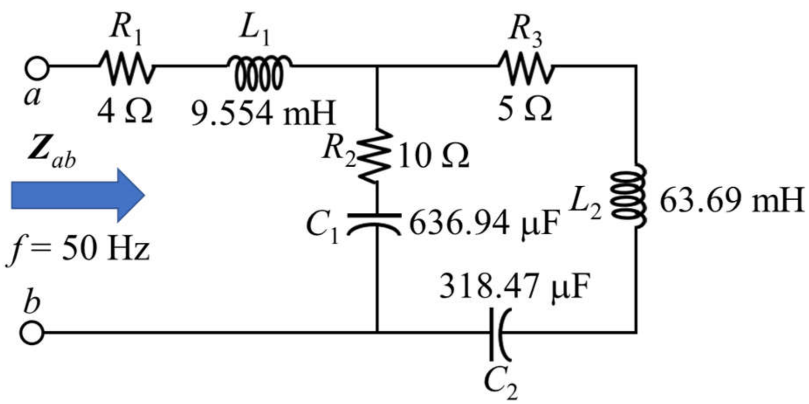
Fig. Phasor diagram

Practice Solution of Fig. 15.77 [Ch. 15], Problem 31 to 32

Chapter 16

Series-Parallel Circuits

EXAMPLE: Calculate the impedance at terminals *a* and *b* for the following electrical network.



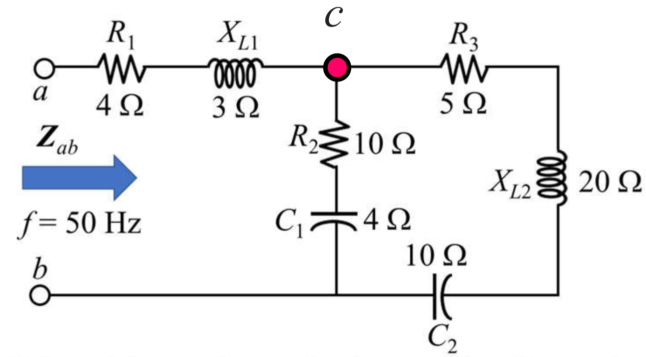
Solution: (1) Calculate all reactance if needed.

$$X_{L1} = 2\pi f L_1 = 2\pi \times 50 \times \left(9.554 \times 10^{-3}\right) = 3\ \Omega$$

$$X_{L2} = 2\pi f L_2 = 2\pi \times 50 \times \left(63.69 \times 10^{-3}\right) = 20\ \Omega$$

$$X_{C1} = \frac{1}{2\pi f C_1} = \frac{1}{2\pi \times 50 \times \left(636.94 \times 10^{-6}\right)} = 5\ \Omega$$

$$X_{C2} = \frac{1}{2\pi f C_2} = \frac{1}{2\pi \times 50 \times \left(318.47 \times 10^{-6}\right)} = 10\ \Omega$$



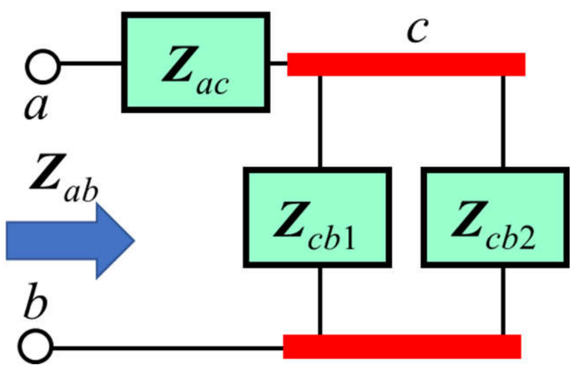
(2) Identify and mark the nodes/junctions.

There are two branches are connected between terminals *c* and *b*.

Write the impedances in different branches.

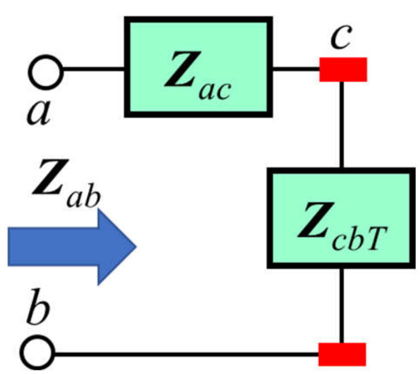
$$\boldsymbol{Z}_{ac} = 4 + j3\ \Omega$$
$$\boldsymbol{Z}_{cb1} = 10 - j5\ \Omega$$
$$\boldsymbol{Z}_{cb2} = 5 + j20 - j10 = 5 + j10\ \Omega$$

Redraw the circuit showing the impedance.



$Z_{ac} = 4 + j3\ \Omega$
$Z_{cb1} = 10 - j5\ \Omega$
$Z_{cb2} = 5 + j20 - j10 = 5 + j10\ \Omega$

$$Z_{cbT} = \frac{Z_{cb1}Z_{cb2}}{Z_{cb1} + Z_{cb2}} = \frac{(10 - j5)(5 + j10)}{(10 - j5) + (5 + j10)} = \frac{100 + j75}{15 + j5} = 7.5 + j2.5\ \Omega$$



$$Z_{ab} = Z_{ac} + Z_{cbT} = 4 + j3 + 7.5 + j2.5 = 11.5 + j5.5\ \Omega$$

EXAMPLE 16.1: For the network in Fig. 16.1:

(***a***) Calculate $\boldsymbol{Z}_T$. (***b***) Determine $\boldsymbol{I}_s$.

(***c***) Calculate $\boldsymbol{V}_R$, $\boldsymbol{V}_C$ and $\boldsymbol{V}_L$. (***d***) Find the $\boldsymbol{I}_C$ and $\boldsymbol{I}_L$.

(***e***) Compute the power delivered.

(***e***) Find power factor (F_p) of the network.

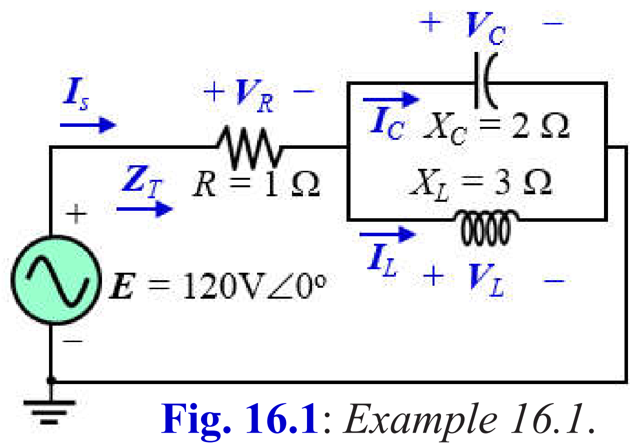


Fig. 16.1: *Example 16.1.*

Solution: (a) Let, $\boldsymbol{Z}_1 = 1\ \Omega = 1\Omega\angle 0^\circ$; $\boldsymbol{Z}_2 = -j2\ \Omega = 2\Omega\angle -90^\circ$; $\boldsymbol{Z}_3 = j3\ \Omega = 3\Omega\angle 90^\circ$;

Fig. 16.1(a) shows the redrawing circuit of Fig. 16.1.

$$\boldsymbol{Z}_4 = \frac{\boldsymbol{Z}_2\boldsymbol{Z}_3}{\boldsymbol{Z}_2+\boldsymbol{Z}_3} = \frac{(-j2)(j3)}{(-j2)+(j3)} = -j6\ \Omega = 6\Omega\angle -90^\circ$$

Fig. 16.1(b) shows the redrawing circuit of Fig. 16.1(a).

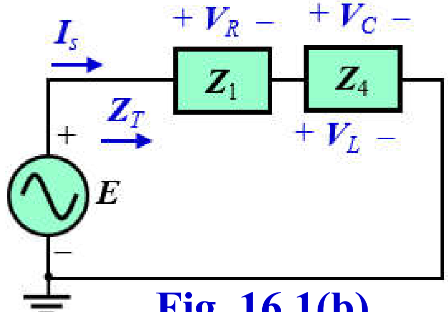


Fig. 16.1(b)

$$\boldsymbol{Z}_T = \boldsymbol{Z}_1 + \boldsymbol{Z}_4 = 1 - j6\,\Omega$$
$$= \mathbf{6.08\Omega\angle -80.54^\circ}$$

$$(\boldsymbol{b})\ \boldsymbol{I}_s = \frac{\boldsymbol{E}}{\boldsymbol{Z}_T} = \frac{120\text{V}\angle 0^\circ}{6.08\Omega\angle -80.54^\circ}$$
$$= 19.74\text{A}\angle 80.54^\circ$$

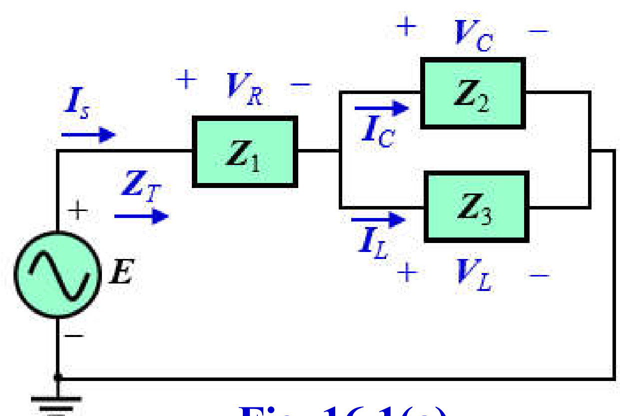


Fig. 16.1(a)

(**c**) Referring to Fig. 16.1(b), we have.

$$\boldsymbol{V}_R = \boldsymbol{I}_s\boldsymbol{Z}_1 = (19.74\text{A}\angle 80.54°)(1\Omega\angle 0°) = \mathbf{19.74}V\angle\mathbf{80.54}°$$

$$\boldsymbol{V}_C = \boldsymbol{V}_L = \boldsymbol{I}_s\boldsymbol{Z}_4 = (19.74\text{A}\angle 80.54°)(6\Omega\angle -90°) = \mathbf{118.44}\text{V}\angle -\mathbf{9.46}°$$

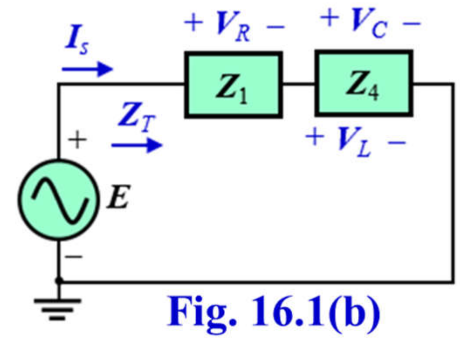


Fig. 16.1(b)

(**d**) Referring to Fig. 16.1(b), we have.

$$\boldsymbol{I}_C = \frac{\boldsymbol{V}_C}{\boldsymbol{Z}_C} = \frac{118.44\text{V}\angle -9.46°}{2\Omega\angle -90°} = \mathbf{59.22A}\angle\mathbf{80.54}°$$

$$\boldsymbol{I}_L = \frac{\boldsymbol{V}_L}{\boldsymbol{Z}_L} = \frac{118.44\text{V}\angle -9.46°}{3\Omega\angle 90°} = \mathbf{39.48A}\angle -\mathbf{99.46}°$$

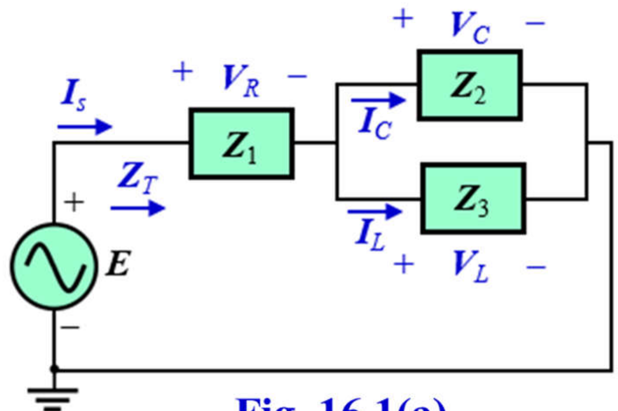


Fig. 16.1(a)

$(\boldsymbol{e})\ P_{del} = I_s^2 R = (19.74)^2\,(1\Omega) = \mathbf{389.67\ W}$

$(\boldsymbol{f})\ pf = F_p = cos\theta = \cos(80.54°) = \mathbf{0.164\ Leading}$

EXAMPLE 16.3: For the network in Fig. 16.5:

(*a*) Calculate the total impedance $\boldsymbol{Z}_T$ and the current $\boldsymbol{I}_s$.

(*b*) Calculate the voltage $\boldsymbol{V}_C$ using the voltage divider rule.

(*c*) Calculate the currents $\boldsymbol{I}_1$ and $\boldsymbol{I}_2$ using the current divider rule.

(*d*) Calculate the power consumption by *R*, the reactive power consumption by *L* and the reactive power supplied by *C*.

(*e*) Calculate the apparent power, the power and the reactive power delivered by source.

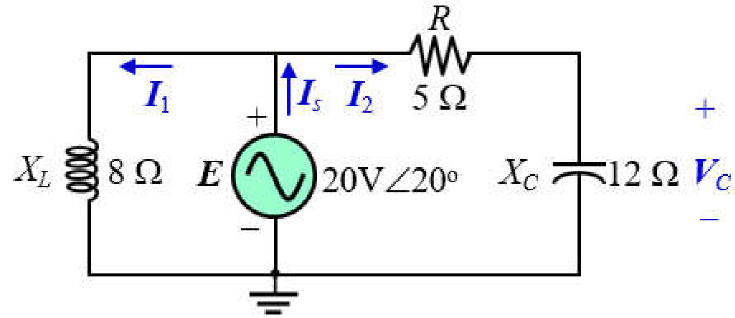


Fig. 16.5: *Example 16.3.*

Solution: (a) Let, $\boldsymbol{Z}_1 = 5\ \Omega = 5\Omega\angle 0^o$;

$\boldsymbol{Z}_2 = -j12\ \Omega = 12\Omega\angle -90^o$;

$\boldsymbol{Z}_3 = j8\ \Omega = 8\Omega\angle 90^o$;

Fig. 16.5(a) shows the redrawing circuit of Fig. 16.5.

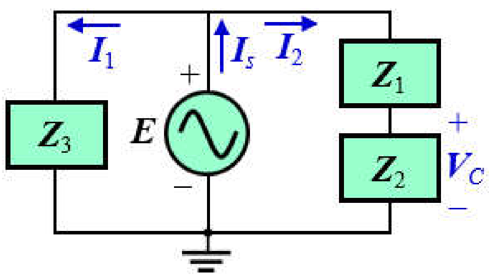


Fig. 16.5(a)

$\boldsymbol{Z}_4 = \boldsymbol{Z}_1 + \boldsymbol{Z}_2 = 5 - j12\ \Omega = 13\Omega\angle -67.38^o$

Fig. 16.5(b) shows the redrawing circuit of Fig. 16.5(a).

$$\boldsymbol{Z}_T = \frac{\boldsymbol{Z}_3\boldsymbol{Z}_4}{\boldsymbol{Z}_3+\boldsymbol{Z}_4} = \frac{(j8)(5-j3)}{(j8)+(5-j3)}$$
$$= 7.8 + j14.24\ \Omega$$
$$= 16.24\Omega\angle 61.29^o$$

$$\boldsymbol{I}_s = \frac{\boldsymbol{E}}{\boldsymbol{Z}_T} = \frac{20\text{V}\angle 20°}{16.24\Omega\angle 61.29°}$$
$$= 1.23\text{A}\angle 40.29°$$

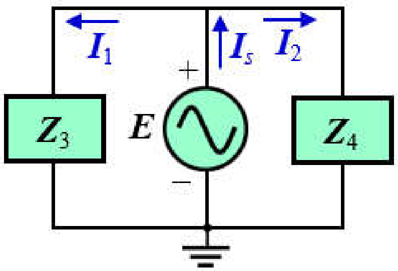


Fig. 16.5(b)

(**b**) Calculate the voltage $\boldsymbol{V}_C$ using the voltage divider rule.

Referring to Fig. 16.5(a), we have.

$$\boldsymbol{V}_C = \frac{\boldsymbol{Z}_2\boldsymbol{E}}{\boldsymbol{Z}_1+\boldsymbol{Z}_2} = \frac{(12\Omega\angle-90°)(20\text{V}\angle 20°)}{5-j12}$$
$$= 18.46\text{V}\angle-2.62°$$

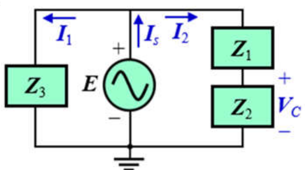


Fig. 16.5(a)

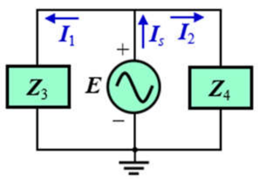


Fig. 16.5(b)

(**c**) Calculate the currents $\boldsymbol{I}_1$ and $\boldsymbol{I}_2$ using the current divider rule.

Referring to Fig. 16.5(b), we have.

$$\boldsymbol{I}_1 = \frac{\boldsymbol{Z}_4\boldsymbol{I}_s}{\boldsymbol{Z}_3+\boldsymbol{Z}_4} = \frac{(5-j12\ \Omega)(1.23\text{A}\angle 40.29°)}{(j8)+(5-j12)}$$
$$= 0.87-j2.35\text{ A} = 2.51\text{A}\angle-68.68°$$

$$\boldsymbol{I}_2 = \frac{\boldsymbol{Z}_3\boldsymbol{I}_s}{\boldsymbol{Z}_3+\boldsymbol{Z}_4} = \frac{(j8\ \Omega)(1.23\text{A}\angle 40.29°)}{(j8)+(5-j12)}$$
$$= 0.06+j1.54\text{ A} = 1.54\text{A}\angle 87.77°$$

(**d**) Calculate the power consumption by R, the reactive power consumption by L and the reactive power supplied by C.

$$P_R = I_2^2R = (1.54\text{A})^2\times(5\Omega) = 11.86\text{ W}$$

$$Q_L = I_1^2X_L = (2.51\text{A})^2\times(8\Omega) = 50.4\text{ VAR}$$

$$Q_C = -I_2^2X_C = -(1.54\text{A})^2\times(12\Omega) = -28.46\text{ VAR}$$

(**e**) Calculate the apparent power, the power and the reactive power delivered by source.

$$S = EI_s = (20\text{V})\times(1.23\text{A}) = 24.6\text{ VA}$$

$$P = EI_s cos\theta = EI_s\cos\theta_z$$
$$= (20\text{V})\times(1.23\text{A})\cos(61.29°)$$
$$= 11.82\text{ W}$$

$$Q = EI_s sin\theta = EI_s\sin\theta_z$$
$$= (20\text{V})\times(1.23\text{A})\sin(61.29°)$$
$$= 21.58\text{ VAR}$$

EXAMPLE 16.6 For the network in Fig. 16.12:

a. Determine the current $\mathbf{I}$.

b. Find the voltage $\mathbf{V}$.

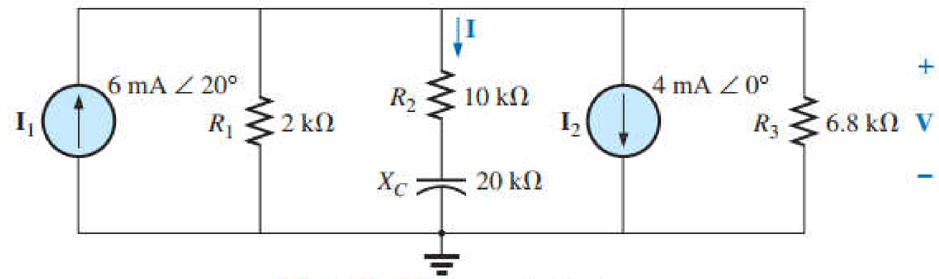


FIG. 16.12 *Example 16.6.*

Solutions:

a. The rules for parallel current sources are the same for dc and ac networks. That is, the equivalent current source is their sum or difference (as phasors). Therefore,

$$\begin{aligned}\mathbf{I}_T &= 6\text{ mA}\angle 20^\circ - 4\text{ mA}\angle 0^\circ\\ &= 5.638\text{ mA} + j\,2.052\text{ mA} - 4\text{ mA}\\ &= 1.638\text{ mA} + j\,2.052\text{ mA}\\ &= 2.626\text{ mA}\angle 51.402^\circ\end{aligned}$$

Redrawing the network using block impedances results in the network in Fig. 16.13 where

$$\mathbf{Z}_1 = 2\text{ k}\Omega\angle 0^\circ \parallel 6.8\text{ k}\Omega\angle 0^\circ = 1.545\text{ k}\Omega\angle 0^\circ$$

and $$\mathbf{Z}_2 = 10\text{ k}\Omega - j\,20\text{ k}\Omega = 22.361\text{ k}\Omega\angle -63.435^\circ$$

Note that $\mathbf{I}$ and $\mathbf{V}$ are still defined in Fig. 16.13.

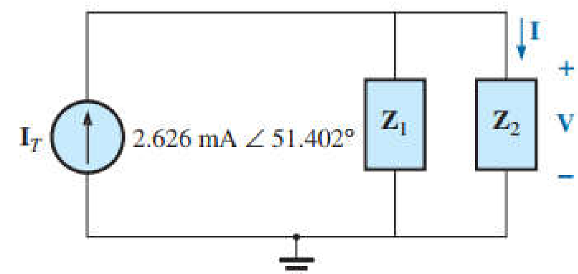


FIG. 16.13
Network in Fig. 16.12 following the assignment of the subscripted impedances.

Calculate the currents ***I***.

Current divider rule:

$$\mathbf{I} = \frac{\mathbf{Z}_1\mathbf{I}_T}{\mathbf{Z}_1 + \mathbf{Z}_2} = \frac{(1.545\text{ k}\Omega \angle 0°)(2.626\text{ mA} \angle 51.402°)}{1.545\text{ k}\Omega + 10\text{ k}\Omega - j\,20\text{ k}\Omega}$$

$$= \frac{4.057\text{ A} \angle 51.402°}{11.545 \times 10^3 - j\,20 \times 10^3} = \frac{4.057\text{ A} \angle 51.402°}{23.093 \times 10^3 \angle -60.004°}$$

$$= \mathbf{0.18\ mA \angle 111.41°}$$

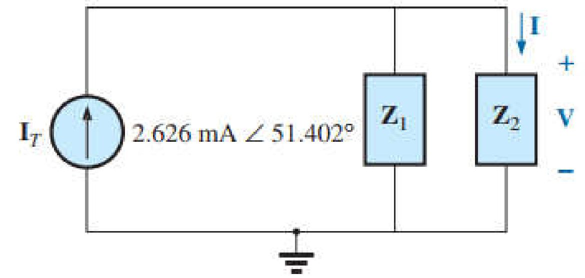


FIG. 16.13
Network in Fig. 16.12 following the assignment of the subscripted impedances.

(***b***) Calculate the voltage ***V***.

$$\mathbf{V} = \mathbf{IZ}_2$$

$$= (0.176\text{ mA} \angle 111.406°)(22.36\text{ k}\Omega \angle -63.435°)$$

$$= \mathbf{3.94\ V \angle 47.97°}$$

EXAMPLE 16.7 For the network in Fig. 16.14:

a. Compute **I**.
b. Find $\mathbf{I}_1$, $\mathbf{I}_2$, and $\mathbf{I}_3$.
c. Verify Kirchhoff's current law by showing that

$$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3$$

d. Find the total impedance of the circuit.

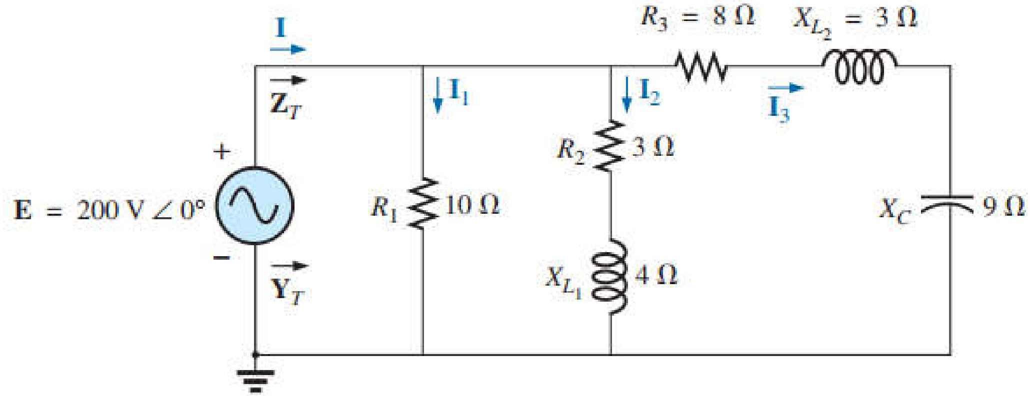


FIG. 16.14
Example 16.7.

Solutions:

a. $\mathbf{Z}_1 = R_1 = 10\ \Omega\ \angle 0°$

$\mathbf{Z}_2 = R_2 + j X_{L_1} = 3\ \Omega + j\,4\ \Omega$

$\mathbf{Z}_3 = R_3 + j X_{L_2} - j X_C = 8\ \Omega + j\,3\ \Omega - j\,9\ \Omega = 8\ \Omega - j\,6\ \Omega$

Redrawing the circuit as in Fig. 16.15 reveals a strictly parallel network where

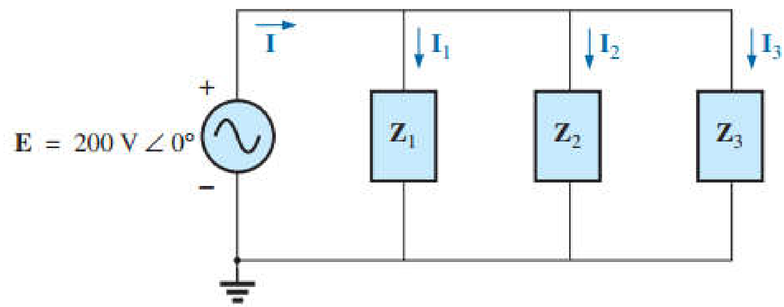


FIG. 16.15 *Network in Fig. 16.14 following the assignment of the subscripted impedances.*

The total admittance is

$$\mathbf{Y}_T = \mathbf{Y}_1 + \mathbf{Y}_2 + \mathbf{Y}_3$$

$$= \frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2} + \frac{1}{\mathbf{Z}_3} = \frac{1}{10\ \Omega} + \frac{1}{3\ \Omega + j\,4\ \Omega} + \frac{1}{8\ \Omega - j\,6\ \Omega}$$

$$= 0.1\ \text{S} + \frac{1}{5\ \Omega\ \angle 53.13°} + \frac{1}{10\ \Omega\ \angle -36.87°}$$

$$= 0.3\ \text{S} - j\,0.1\ \text{S} = 0.316\ \text{S}\ \angle -18.435°$$

The current $\mathbf{I}$:

$$\mathbf{I} = \mathbf{EY}_T = (200\text{ V}\angle 0°)(0.326\text{ S}\angle -18.435°)$$
$$= \mathbf{63.2\text{ A}\angle -18.44°}$$

(*b*) Find the currents I_1, I_2 and I_3.

b. Since the voltage is the same across parallel branches,

$$\mathbf{I}_1 = \frac{\mathbf{E}}{\mathbf{Z}_1} = \frac{200\text{ V}\angle 0°}{10\ \Omega\angle 0°} = \mathbf{20\text{ A}\angle 0°}$$

$$\mathbf{I}_2 = \frac{\mathbf{E}}{\mathbf{Z}_2} = \frac{200\text{ V}\angle 0°}{5\ \Omega\angle 53.13°} = \mathbf{40\text{ A}\angle -53.13°}$$

$$\mathbf{I}_3 = \frac{\mathbf{E}}{\mathbf{Z}_3} = \frac{200\text{ V}\angle 0°}{10\ \Omega\angle -36.87°} = \mathbf{20\text{ A}\angle +36.87°}$$

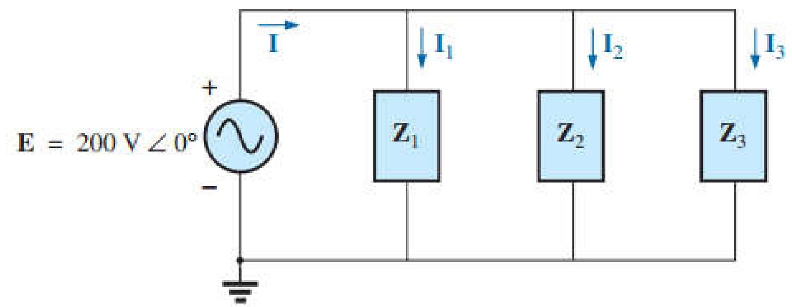


FIG. 16.15 *Network in Fig. 16.14 following the assignment of the subscripted impedances.*

(*c*) Verify KCL:

c. $$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3$$
$$60 - j\,20 = 20\angle 0° + 40\angle -53.13° + 20\angle +36.87°$$
$$= (20 + j\,0) + (24 - j\,32) + (16 + j\,12)$$
$$60 - j\,20 = 60 - j\,20 \quad \text{(checks)}$$

(*d*) Find the total impedance of the circuit.

d. $$\mathbf{Z}_T = \frac{1}{\mathbf{Y}_T} = \frac{1}{0.316\text{ S}\angle -18.435°}$$
$$= \mathbf{3.17\ \Omega\angle 18.44°}$$

EXAMPLE 16.8 For the network in Fig. 16.18:

a. Calculate the total impedance $\mathbf{Z}_T$.
b. Compute $\mathbf{I}$.
c. Find the total power factor.
d. Calculate $\mathbf{I}_1$ and $\mathbf{I}_2$.
e. Find the average power delivered to the circuit.

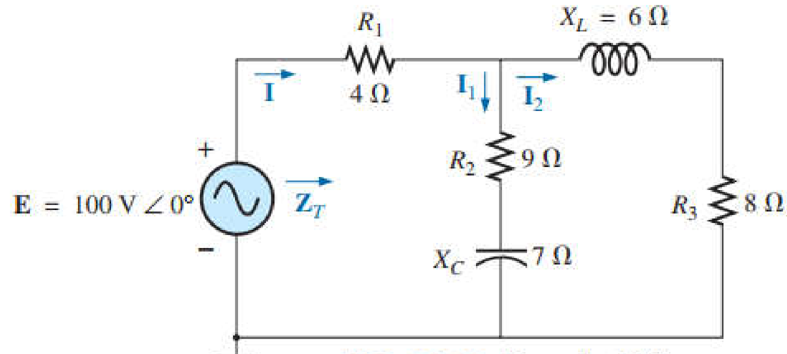


FIG. 16.18 *Example 16.8.*

Solutions:

a. $\mathbf{Z}_1 = R_1 = 4\ \Omega \angle 0°$
$\mathbf{Z}_2 = R_2 - j X_C = 9\ \Omega - j\,7\ \Omega = 11.40\ \Omega \angle -37.87°$
$\mathbf{Z}_3 = R_3 + j X_L = 8\ \Omega + j\,6\ \Omega = 10\ \Omega \angle +36.87°$

Redrawing the circuit as in Fig. 16.19, we have

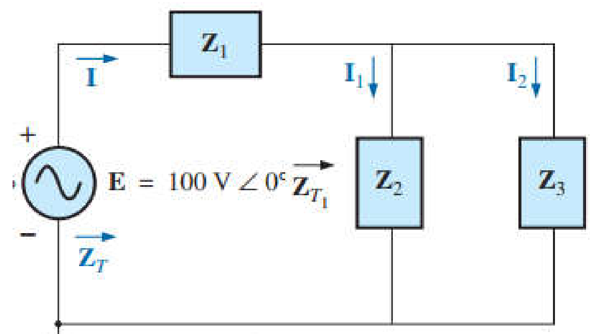


FIG. 16.19

$$\mathbf{Z}_T = \mathbf{Z}_1 + \mathbf{Z}_{T_1} = \mathbf{Z}_1 + \frac{\mathbf{Z}_2\mathbf{Z}_3}{\mathbf{Z}_2 + \mathbf{Z}_3}$$

$$= 4\ \Omega + \frac{(11.4\ \Omega \angle -37.87°)(10\ \Omega \angle 36.87°)}{(9\ \Omega - j\,7\ \Omega) + (8\ \Omega + j\,6\ \Omega)}$$

$$= 4\ \Omega + 6.68\ \Omega + j\,0.28\ \Omega = 10.68\ \Omega + j\,0.28\ \Omega$$

$$\mathbf{Z}_T = \mathbf{10.68\ \Omega \angle 1.5°}$$

b. $\mathbf{I} = \dfrac{\mathbf{E}}{\mathbf{Z}_T} = \dfrac{100\text{ V} \angle 0°}{10.684\ \Omega \angle 1.5°} = \mathbf{9.36\ A \angle -1.5°}$

c. $F_p = \cos\theta_T = \dfrac{R}{Z_T} = \dfrac{10.68\ \Omega}{10.684\ \Omega} \cong \mathbf{1}$

d. Current divider rule:

$$\mathbf{I}_2 = \frac{\mathbf{Z}_2\mathbf{I}}{\mathbf{Z}_2 + \mathbf{Z}_3} = \frac{(11.40\ \Omega \angle -37.87°)(9.36\text{ A} \angle -1.5°)}{(9\ \Omega - j\,7\ \Omega) + (8\ \Omega + j\,6\ \Omega)}$$
$$= \frac{106.7\text{ A} \angle -39.37°}{17 - j\,1} = \frac{106.7\text{ A} \angle -39.37°}{17.03 \angle -3.37°}$$
$$\mathbf{I}_2 = \mathbf{6.27\ A \angle -36°}$$

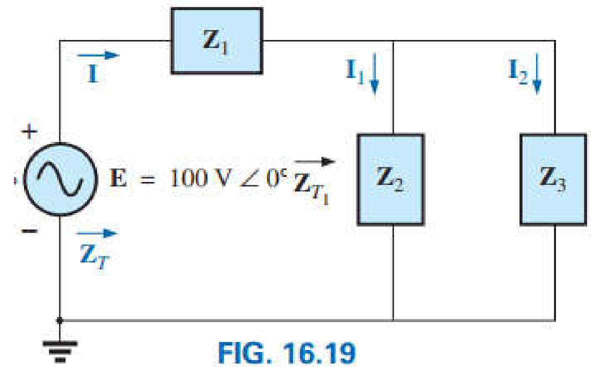


FIG. 16.19

Applying Kirchhoff's current law (rather than another application of the current divider rule) yields

$$\mathbf{I}_1 = \mathbf{I} - \mathbf{I}_2$$

or

$$\mathbf{I} = \mathbf{I}_1 - \mathbf{I}_2$$
$$= (9.36\text{ A} \angle -1.5°) - (6.27\text{ A} \angle -36°)$$
$$= (9.36\text{ A} - j\,0.25\text{ A}) - (5.07\text{ A} - j\,3.69\text{ A})$$
$$\mathbf{I}_1 = 4.29\text{ A} + j\,3.44\text{ A} = \mathbf{5.5\ A \angle 38.72°}$$

e. $P_T = EI\cos\theta_T$
$$= (100\text{ V})(9.36\text{ A})\cos 1.5°$$
$$= (936)(0.99966)$$
$$P_T = \mathbf{935.68\ W}$$

*10. For the network in Fig. 16.48:

a. Find the total impedance $\mathbf{Z}_T$ and the admittance $\mathbf{Y}_T$.
b. Find the source current $\mathbf{I}_s$ in phasor form.
c. Find the currents $\mathbf{I}_1$ and $\mathbf{I}_2$ in phasor form.
d. Find the voltages $\mathbf{V}_1$ and $\mathbf{V}_{ab}$ in phasor form.
e. Find the average power delivered to the network.
f. Find the power factor of the network, and indicate whether it is leading or lagging.

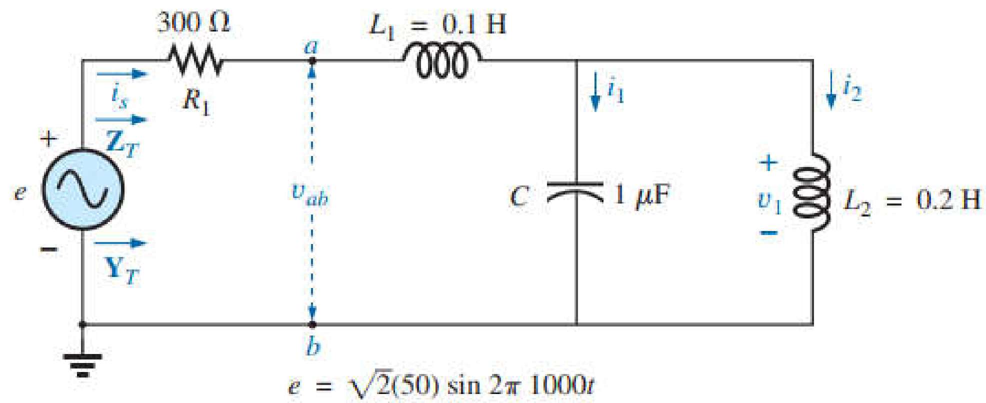


FIG. 16.48 *Problem 10.*

Solution: Here, E = 50 V, f =1000 Hz and ω =2π×1000 =6280 rad/s $\quad \boldsymbol{E} = 50\text{V}\angle 0°$

$$X_{L1} = \omega L_1 = (6280\ \text{rad/s})(0.1\ \text{H}) = 628\ \Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{(6280\ \text{rad/s})(1 \times 10^{-6}\ \text{F})} = 159.24\ \Omega$$

$$X_{L2} = \omega L_2 = (6280\ \text{rad/s})(0.2\ \text{H}) = 1256\ \Omega$$

Let,

$$\boldsymbol{Z}_1 = 300\ \Omega = 300\Omega\angle 0°$$

$$\boldsymbol{Z}_2 = j628\ \Omega = 628\Omega\angle 90°$$

$$\boldsymbol{Z}_3 = -j159.24\ \Omega = 159.24\Omega\angle -90°$$

$$\boldsymbol{Z}_4 = j1256\ \Omega = 1256\Omega\angle 90°$$

Fig. 16.48(a) shows the redrawing circuit of Fig. 16.48.

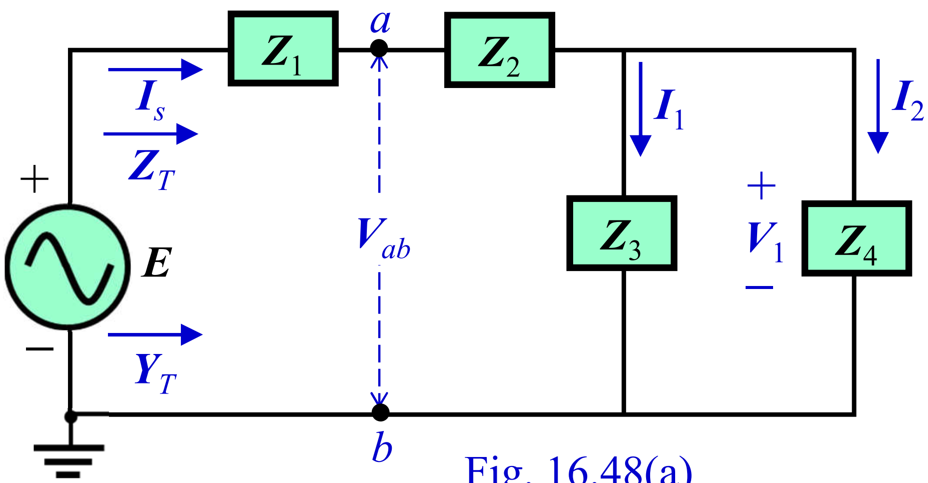


Fig. 16.48(a)

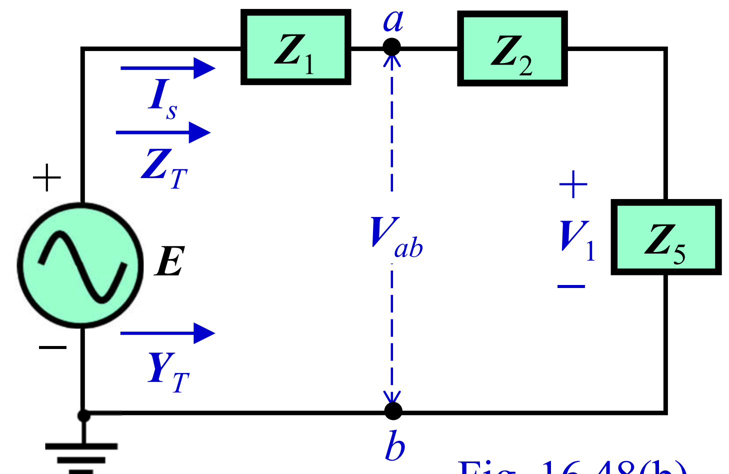


Fig. 16.48(b)

$$Z_5 = \frac{Z_3 Z_4}{Z_3 + Z_4} = \frac{(-j159.24\ \Omega)(j1256\ \Omega)}{(-j159.24\ \Omega) + (j1256\ \Omega)}$$
$$= -j182.36\ \Omega = 182.36\Omega\angle -90°$$

Fig. 16.48(b) shows the redrawing circuit of Fig. 16.48(a).

$$Z_6 = Z_2 + Z_5 = j445.64\ \Omega = 445.64\Omega\angle 90°$$

Fig. 16.48(c) shows the redrawing circuit of Fig. 16.48(b).

$$Z_T = Z_1 + Z_6 = 300 + j445.64\ \Omega = 537.21\Omega\angle 56.05°$$

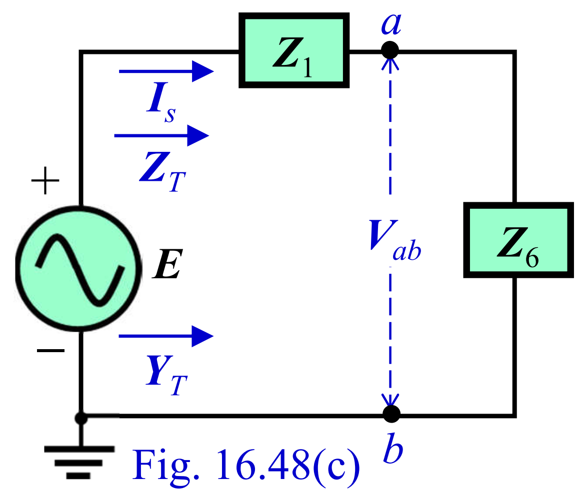


Fig. 16.48(c)

Fig. 16.48(d) shows the redrawing circuit of Fig. 16.48(c).

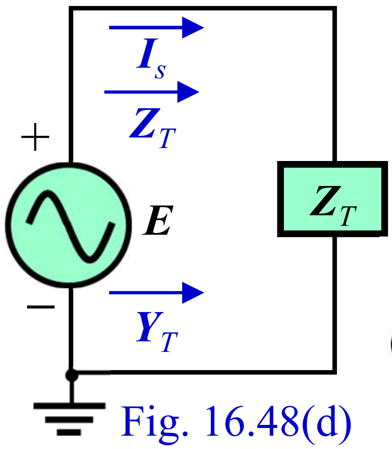


Fig. 16.48(d)

$$Y_T = \frac{1}{Z_T} = \frac{1}{537.21\Omega\angle 56.0°}$$
$$= \mathbf{1.861mS\angle - 56.05°}$$

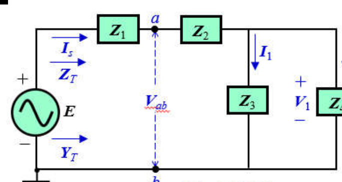


Fig. 16.48(a)

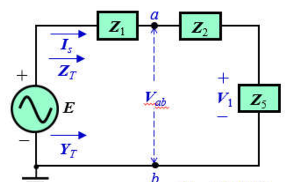


Fig. 16.48(b)

(*b*) $I_s = \frac{E}{Z_T} = \frac{50V\angle 0°}{537.21\Omega\angle 56.05°} = \mathbf{93.07mA\angle - 56.05°}$

(*c*) Referring to Fig. 16.48(a) and Fig. 16.48(b), I_1 and I_2 are:

$$I_1 = \frac{Z_5}{Z_3} I_s = \mathbf{106.58mA\angle - 56.05°}$$

$$I_2 = \frac{Z_5}{Z_4} I_s = \mathbf{13.52mA\angle 123.96°}$$

(*d*) Referring to Fig. 16.48(b) and Fig. 16.48(c), V_{ab} and V_1 are:

$$V_{ab} = Z_6 I_s = \mathbf{41.48V\angle 33.95°}$$

$$V_1 = Z_5 I_s = \mathbf{16.98V\angle 213.95°}$$

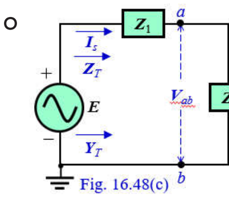


Fig. 16.48(c)

(*e*) Average power:

$$P = EI_s \cos\theta_T$$
$$= 50 \times (93.07\text{mA})\cos(56.05°) = \mathbf{2.6\ W}$$

(*f*) Power Factor:

$$F_p = \cos\theta_T = \mathbf{0.558\ Lagging}$$

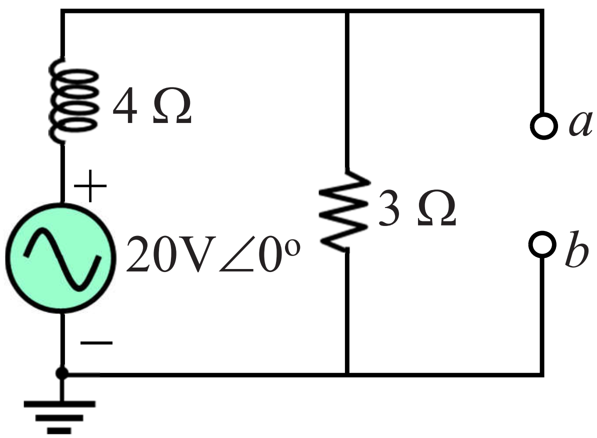
Practice Book Remaining Examples

And

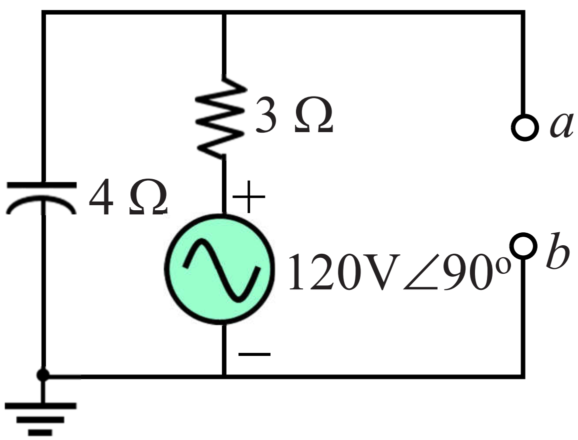
All Problems of Chapter 16

Example Related to Open Circuit and Short Circuit

Example: For the following circuits, find voltage drop across the terminals *a* and *b*.

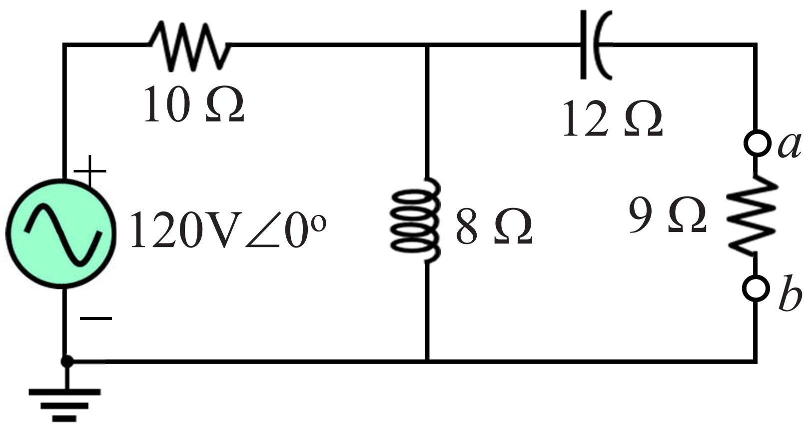


$$V_{ab} = \frac{(3\Omega)}{(3\Omega)+(j4\ \Omega)}(20\text{V}\angle 0°)$$
$$= 7.2 - j9.6\ \text{V}$$
$$= 12\text{V}\angle -53.13°$$

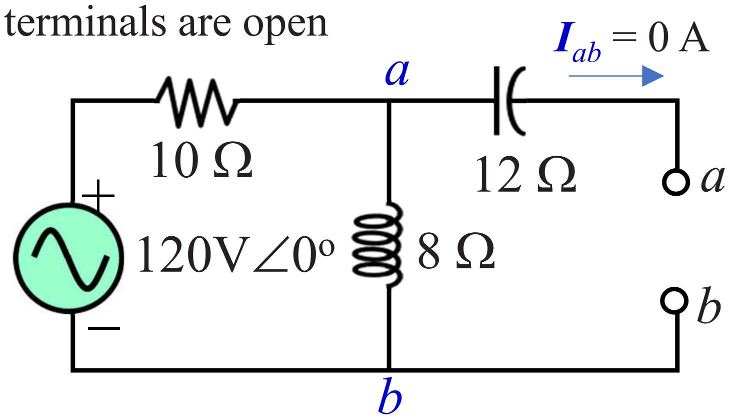


$$V_{ab} = \frac{(-j4\Omega)}{(3\Omega)+(j4\ \Omega)}(120\text{V}\angle 90°)$$
$$= 57.6 + j76.8\ \text{V}$$
$$= 96\text{V}\angle 53.13°$$

Example: Find the voltage drop across the terminals *a* and *b*. when the terminals are open.



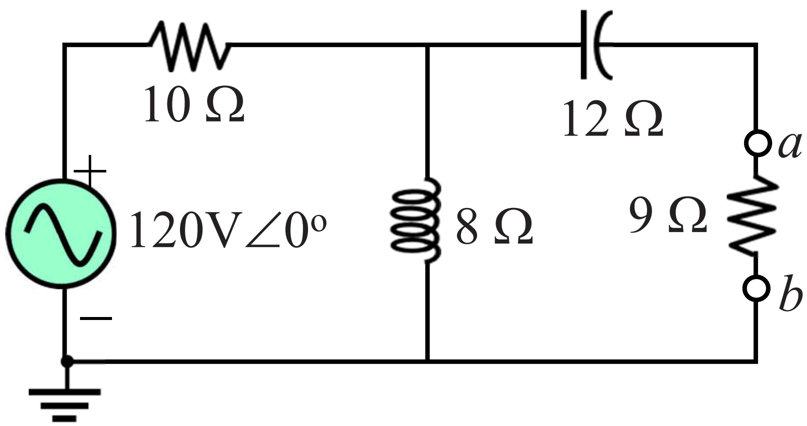
Solution: Redraw the circuit by considering *a* and *b* terminals are open



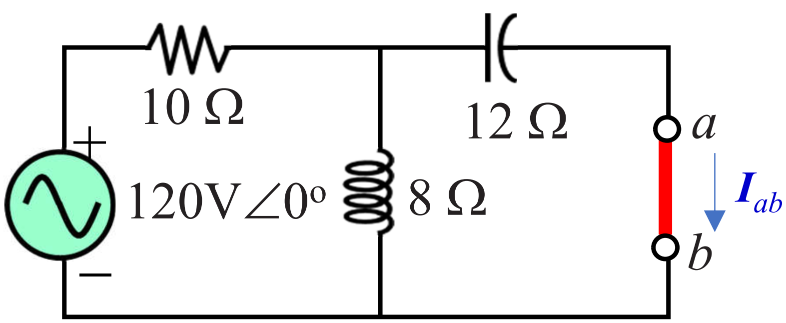
Due to the open circuit no current flows through the 12 Ω capacitive reactance, so the voltage drop across the *a* and *b* terminals is equal to the voltage drop across the 8 Ω inductive reactance.

$$\begin{aligned} V_{ab} &= \frac{(j8\Omega)}{(10\Omega)+(j8\ \Omega)}(120\text{V}\angle 0^\circ) \\ &= 46.83 + j58.54 \text{ V} \\ &= 74.97\text{V}\angle 51.34^\circ \end{aligned}$$

Example: Find the current passing through the terminals *a* and *b*. when the terminals are shorted.



Solution: (***a***) Redraw the circuit by considering *a* and *b* terminals are open

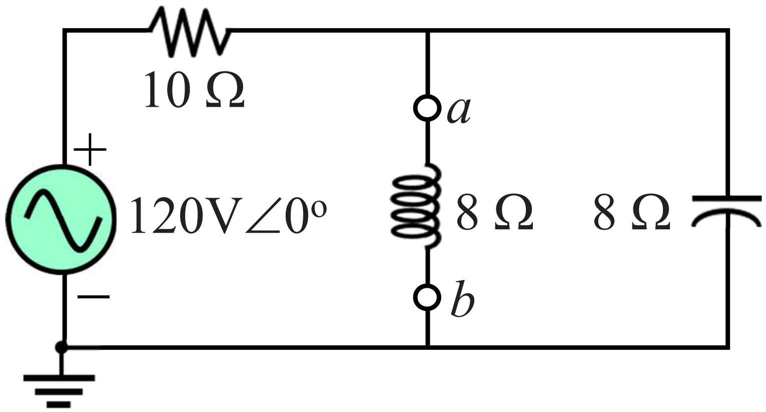


$$\boldsymbol{Z}_p = \frac{(j8\,\Omega)(-j12\,\Omega)}{(j8\,\Omega)+(-j12\,\Omega)}$$
$$= 46.83 + j58.54\ \Omega$$
$$= 74.97\Omega\angle 51.34^\circ$$

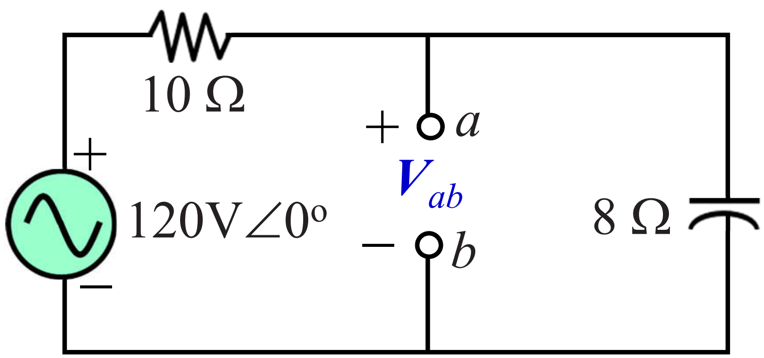
$$\boldsymbol{V}_p = \frac{(10)(\boldsymbol{Z}_p)}{(10)+(\boldsymbol{Z}_p)}\boldsymbol{E}$$
$$= \frac{(10)(46.83+j58.54)}{(10)+(46.83+j58.54)}(120\text{V}\angle 0^\circ)$$
$$= 102.25 + j42.6\ \text{V}$$
$$= 110.77\text{V}\angle 22.62^\circ$$

$$\boldsymbol{I}_{ab} = \frac{\boldsymbol{V}_p}{(-j12\ \Omega)} = \frac{(102.25 + j42.6\ \text{V})}{(-j12\ \Omega)}$$
$$= -3.55 + j8.52\ \text{A}$$
$$= 9.23\text{A}\angle 112.62^\circ$$

Example: Find the voltage drop across the terminals *a* and *b*. when the terminals are open.



Solution: Redraw the circuit by considering *a* and *b* terminals are open

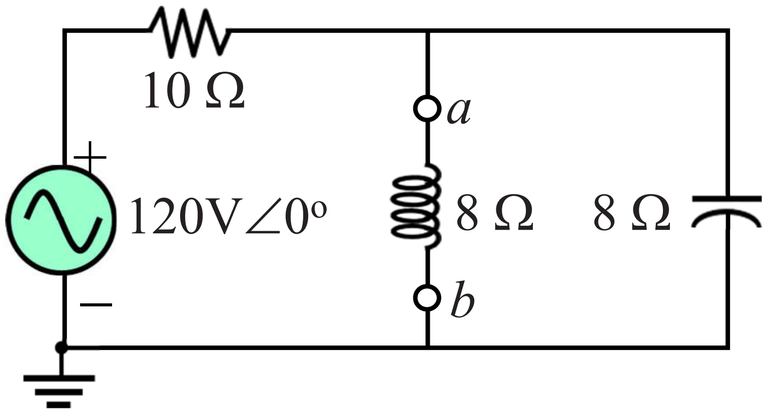


$$V_{ab} = \frac{(10\Omega)(-j8\Omega)}{(10\Omega)(-j8\Omega)}(120\text{V}\angle 0^\circ)$$

$$= 46.83 - j58.54 \text{ V}$$

$$= 74.97\text{V}\angle -51.34^\circ$$

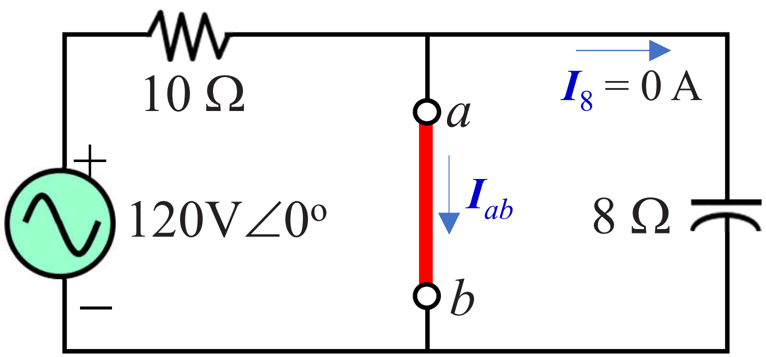
Example: Find the current passing through the terminals *a* and *b*. when the terminals are shorted.



Due to the short circuit no current flows through the 8 Ω capacitive reactance, so the current passes through the *a* and *b* terminals can be calculated as:

$$I_{ab} = \frac{E}{10\ \Omega}$$
$$= \frac{120\text{V}\angle 0^\circ}{10\ \Omega}$$
$$= 12\text{A}\angle 0^\circ$$

Solution: (***a***) Redraw the circuit by considering *a* and *b* terminals are open



MAXIMUM POWER TRANSFER THEOREM [AC]

Maximum Power Transfer or Impedance Matching Theorem

Statement: Maximum power will be delivered to a load when the load impedance is the complex conjugate of the Thévenin impedance across its terminals.

If, $\boldsymbol{Z}_L = R \pm jX$ and $\boldsymbol{Z}_{Th} = R_{Th} \pm jX_{Th}$

Then, according to maximum power transfer theorem:

$$\boldsymbol{Z}_L = R \pm jX = \left(\boldsymbol{Z}_{Th}\right)^* = R_{Th} \mp jX_{Th}$$

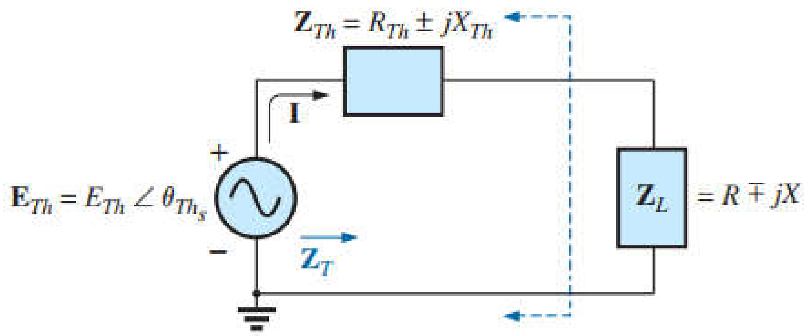


FIG. 18.82
Conditions for maximum power transfer to $\mathbf{Z}_L$.

$$\boxed{Z_L = Z_{Th} \text{ and } \theta_L = -\theta_{Th_Z}} \qquad \textbf{(18.16)}$$

$$\boxed{R_L = R_{Th} \quad \text{and} \quad \pm j\,X_{\text{load}} = \mp j\,X_{Th}} \qquad \textbf{(18.17)}$$

$$\boxed{\mathbf{Z}_T = 2R} \qquad \textbf{(18.18)}$$

$$\boxed{F_p = 1} \quad \text{(maximum power transfer)} \qquad \textbf{(18.19)}$$

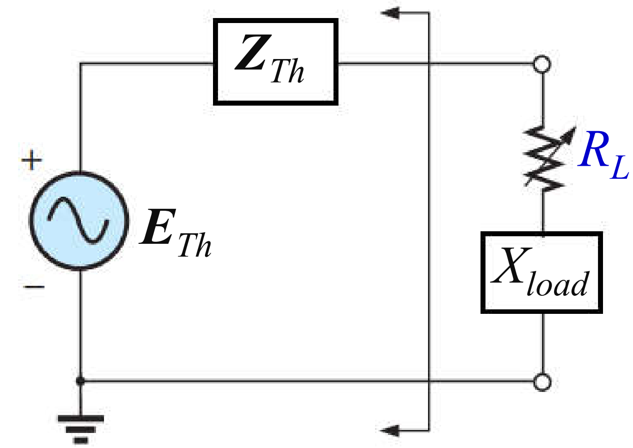
$$\boxed{P_{\max} = \frac{E_{Th}^2}{4R}} \qquad \textbf{(18.20)}$$

Special Situation: If the load resistance is adjustable but the magnitude of the load reactance cannot be set equal to the magnitude of the Thévenin reactance, then the maximum power that can be delivered to the load will occur when the load reactance is made as close to the Thévenin reactance as possible, and the load resistance is set to the following value:

$$R_L = \sqrt{R_{Th}^2 + (X_{Th} + X_{\text{load}})^2} \tag{18.21}$$

$$P = E_{Th}^2/4R_{\text{av}} \tag{18.22}$$

$$R_{\text{av}} = \frac{R_{Th} + R_L}{2} \tag{18.23}$$



EXAMPLE 18.19 Find the load impedance in Fig. 18.83 for maximum power to the load, and find the maximum power.

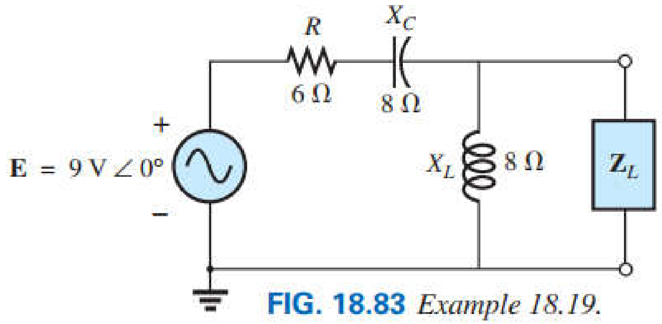
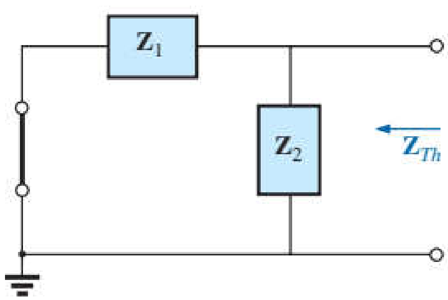


FIG. 18.83 *Example 18.19.*

Solution: $Z_1 = R - jX_C = 6\ \Omega - j\,8\ \Omega = 10\ \Omega\ \angle -53.13^\circ$

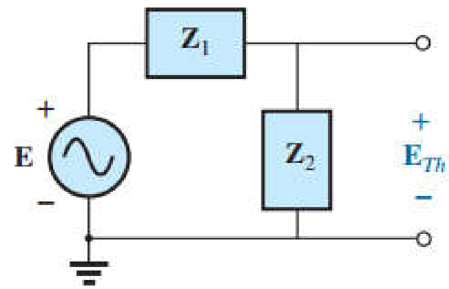
$Z_2 = +jX_L = j\,8\ \Omega$

Determine $\mathbf{Z}_{Th}$ [Fig. 18.84(a)]:



$$\mathbf{Z}_{Th} = \frac{\mathbf{Z}_1\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2} = \frac{(10\ \Omega\ \angle -53.13^\circ)(8\ \Omega\ \angle 90^\circ)}{6\ \Omega - j\,8\ \Omega + j\,8\ \Omega} = 13.33\ \Omega\ \angle 36.87^\circ = 10.66\ \Omega + j\,8\ \Omega$$

To find the maximum power, we must first find $\mathbf{E}_{Th}$ [Fig. 18.84(b)], as follows:



$$\mathbf{E}_{Th} = \frac{\mathbf{Z}_2\mathbf{E}}{\mathbf{Z}_2 + \mathbf{Z}_1} = \frac{(8\ \Omega\ \angle 90^\circ)(9\ \text{V}\ \angle 0^\circ)}{j\,8\ \Omega + 6\ \Omega - j\,8\ \Omega} = \frac{72\ \text{V}\ \angle 90^\circ}{6\ \angle 0^\circ} = 12\ \text{V}\ \angle 90^\circ$$

According to maximum power transfer theorem:

$$\mathbf{Z}_L = 13.3\ \Omega\ \angle -36.87^\circ = \mathbf{10.66\ \Omega - j\,8\ \Omega}$$

Maximum power received by load:

$$P_{\max} = \frac{E_{Th}^2}{4R} = \frac{(12\ \text{V})^2}{4(10.66\ \Omega)} = \frac{144}{42.64} = \mathbf{3.38\ W}$$

EXAMPLE 18.21 For the network in Fig. 18.90:

a. Determine the value of R_L for maximum power to the load if the load reactance is fixed at 4 Ω.
b. Find the power delivered to the load under the conditions of part (a).
c. Find the maximum power to the load if the load reactance is made adjustable to any value, and compare the result to part (b) above.

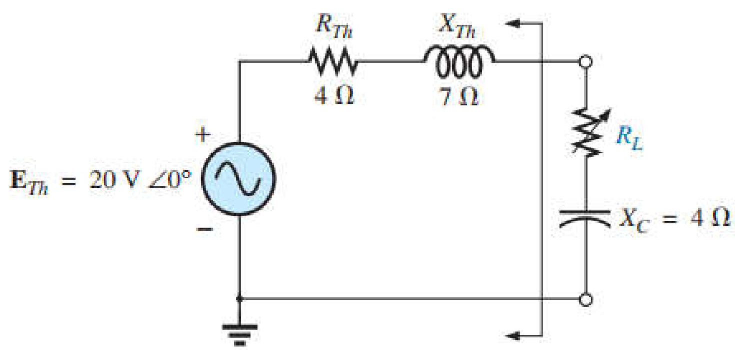


FIG. 18.90 *Example 18.21.*

Solutions:

a. Eq. (18.21):
$$R_L = \sqrt{R_{Th}^2 + (X_{Th} + X_{\text{load}})^2}$$
$$= \sqrt{(4\ \Omega)^2 + (7\ \Omega - 4\ \Omega)^2}$$
$$= \sqrt{16 + 9} = \sqrt{25}$$
$$R_L = \mathbf{5\ \Omega}$$

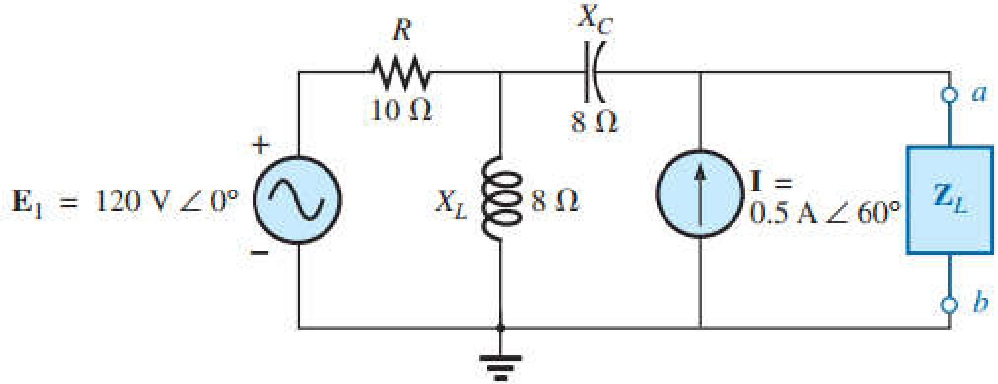
b. Eq. (18.23):
$$R_{\text{av}} = \frac{R_{Th} + R_L}{2} = \frac{4\ \Omega + 5\ \Omega}{2}$$
$$= \mathbf{4.5\ \Omega}$$

Eq. (18.22):
$$P = \frac{E_{Th}^2}{4R_{\text{av}}}$$
$$= \frac{(20\ \text{V})^2}{4(4.5\ \Omega)} = \frac{400}{18}\ \text{W}$$
$$\cong \mathbf{22.22\ W}$$

c. For $\mathbf{Z}_L = 4\ \Omega - j\,7\ \Omega$,
$$P_{\max} = \frac{E_{Th}^2}{4R_{Th}} = \frac{(20\ \text{V})^2}{4(4\ \Omega)}$$
$$= \mathbf{25\ W}$$

exceeding the result of part (b) by 2.78 W.

PROBLEM: Find the load impedance $\boldsymbol{Z}_L$ for the networks in following Figure for maximum power to the load, and find the maximum power to the load.



Solution: $\boldsymbol{Z}_1 = R\ \Omega = 10\ \Omega$; $\boldsymbol{Z}_2 = jX_L\ \Omega = j8\ \Omega$; $\boldsymbol{Z}_3 = -jX_C\ \Omega = -j8\ \Omega$;

Step 1 and **Step 2**:

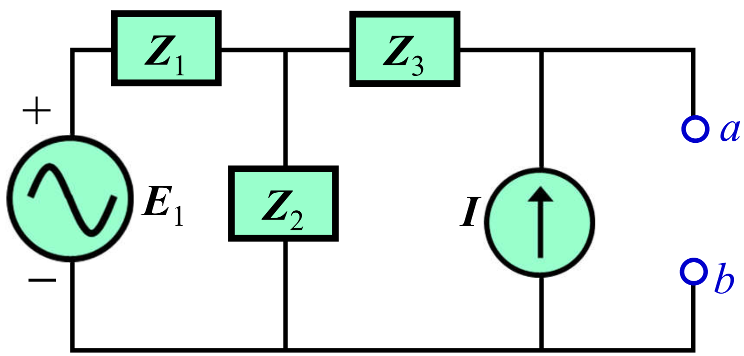


Fig. (a)

Step 3: $\boldsymbol{Z}_{Th}$ calculation

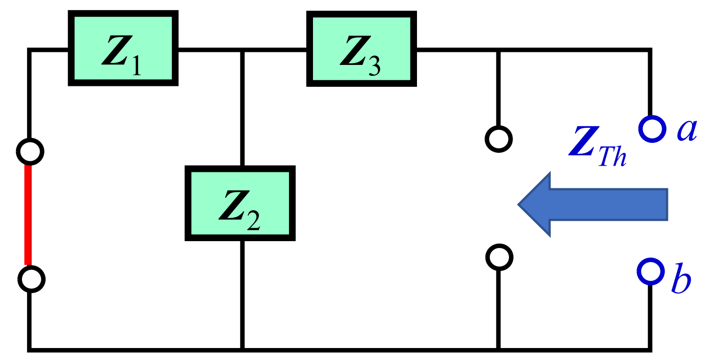


Fig. (b)

$$\boldsymbol{Z}_{Th} = \boldsymbol{Z}_3 + \frac{\boldsymbol{Z}_1\boldsymbol{Z}_2}{\boldsymbol{Z}_1 + \boldsymbol{Z}_2} = 3.9 - j3.12\ \ \Omega = 5\Omega\angle -38.66°$$

Step 4: E_{Th} calculation

Since there are two sources, Thevenin's voltage can be calculated by using Superposition Theorem.

Considering E_1:

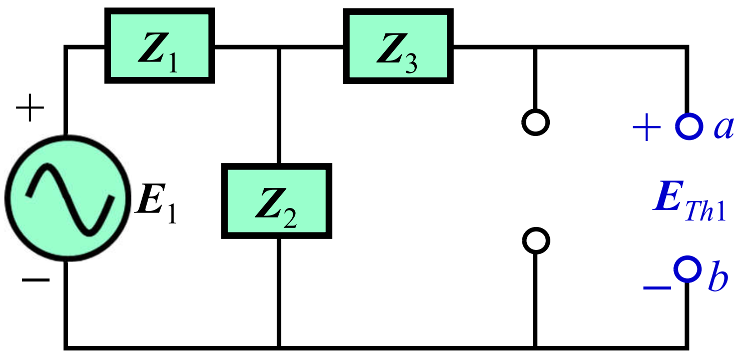


Fig. (c)

$$E_{Th1} = \frac{Z_2 E_1}{Z_1 + Z_2} = 46.83 + j58.54\ \Omega$$
$$= 74.96\Omega\angle 51.34°$$

Considering I:

$$Z_{T2} = Z_3 + \frac{Z_1 Z_2}{Z_1 + Z_2}$$
$$= 3.9 - j3.12\ \Omega$$
$$= 5\Omega\angle -38.66°$$

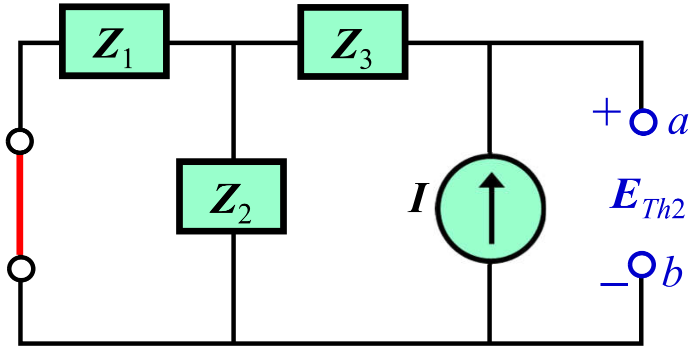
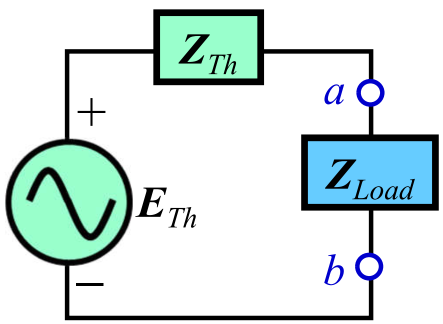


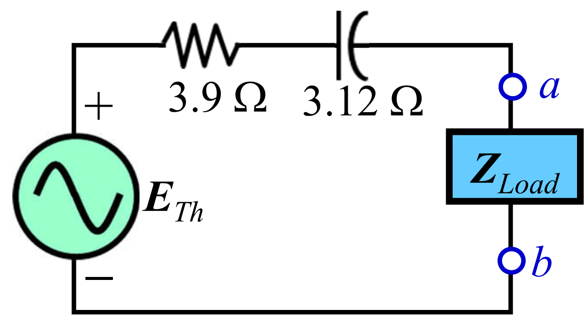
Fig. (d)

$$E_{Th2} = Z_{T2} I = 2.32 + j0.897\ \text{V} = 2.49\text{V}\angle 21.16°$$

According Superposition Theorem:

$$E_{Th} = E_{Th1} + E_{Th2} = 49.15 + j59.43\ \text{V} = 77.12\text{V}\angle 50.41°$$





According to maximum power transfer theorem:

$$\boldsymbol{Z}_L = (\boldsymbol{Z}_{Th})^* = (3.9 - j3.12)^* = 3.9 + j3.12\ \Omega$$

Maximum power received by load:

$$P_{\max} = \frac{E_{Th}^2}{4R_{Th}}$$

$$= \frac{(77.12\text{V})^2}{4 \times 3.9\Omega}$$

$$= \mathbf{381.25\ W}$$

Practice Book Remaining Examples
And
Problem 39, 40, 45 and 46 [Ch. 18]

Chapter 24

Poly-phase System

Poly-Phase Generator

An ac generator designed to **develop a single sinusoidal voltage** for each rotation of the shaft (rotor) is referred to as a **single-phase ac generator**.
If the number of coils on the rotor is increased in a specified manner, the result is a **polyphase ac generator**, which **develops more than one ac phase voltage** per rotation of the rotor.

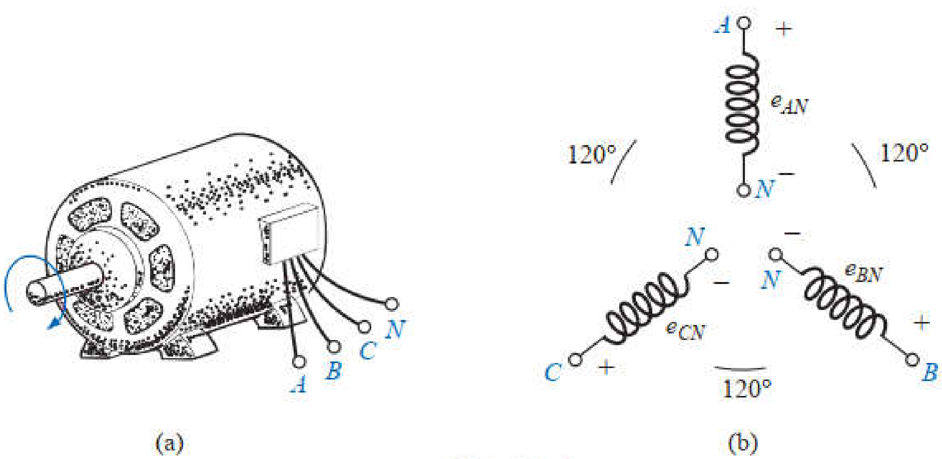


FIG. 22.1
(a) Three-phase generator; (b) induced voltages of a three-phase generator.

For a balanced three phase source, the peak value of voltage $e_{AN}(t)$, $e_{BN}(t)$, and $e_{CN}(t)$ are equal and the phase displacement from each other is 120°.

$$e_{AN}(t) = E_m \sin \omega t$$

$$e_{BN}(t) = E_m \sin(\omega t - 120°)$$

$$e_{CN}(t) = E_m \sin(\omega t - 240°) = E_m \sin(\omega t + 120°)$$

Let, the rms value of voltages $e_{AN}(t)$, $e_{BN}(t)$, and $e_{CN}(t)$ is E_p then these voltage are in phasor form as folows:

$$\boldsymbol{E}_{AN} = E_p\angle 0°$$

$$\boldsymbol{E}_{BN} = E_p\angle -120°$$

$$\boldsymbol{E}_{CN} = E_p\angle -240° = E_p\angle 120°$$

where, $E_p = \frac{1}{\sqrt{2}} E_m = 0.707 E_m$

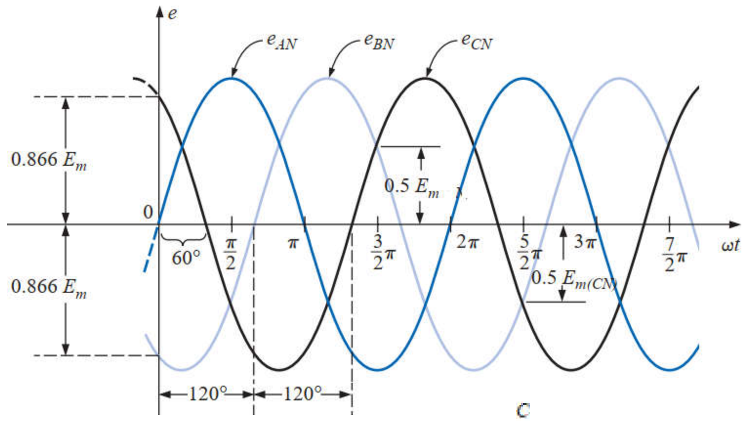
In a balanced system, **at any instant of time, the algebraic sum of the three phase voltages of a three-phase generator is *zero***. That means:

$$e_{AN}(t) + e_{BN}(t) + e_{CN}(t) = 0 \qquad \boldsymbol{E}_{AN} + \boldsymbol{E}_{CN} + \boldsymbol{E}_{CN} = 0$$

Phase Sequence or Phase Order

There are two phase sequence or phase orders:

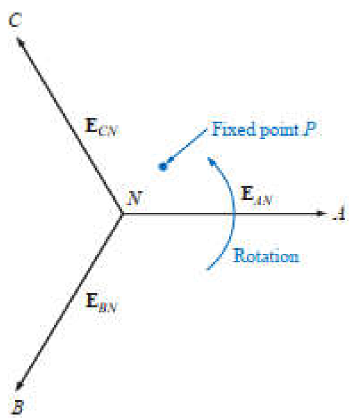
(***a***) ***ABC*–sequence** [*B* lags *A* by 120° and *C* lags *B* by 120° that means *C* lags *A* by 240°]



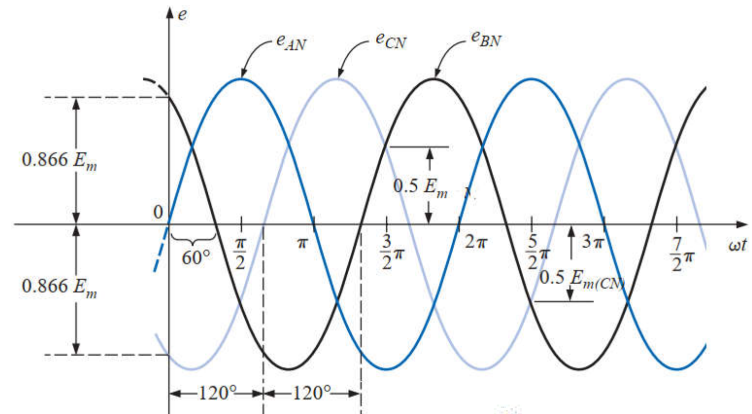
$$\boldsymbol{E}_{AN} = E_p \angle 0°$$

$$\boldsymbol{E}_{BN} = E_p \angle -120°$$

$$\boldsymbol{E}_{CN} = E_p \angle -240° = E_p \angle 120°$$



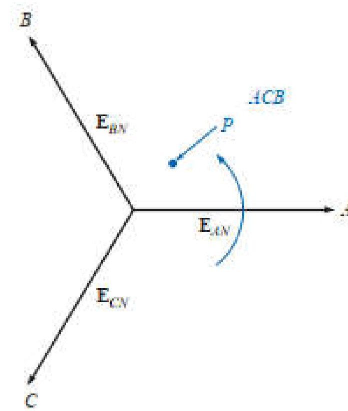
(***b***) ***ACB*–sequence** [*C* lags *A* by 120° and *B* lags *C* by 120° that means *B* lags *A* by 240°]



$$\boldsymbol{E}_{AN} = E_p \angle 0°$$

$$\boldsymbol{E}_{CN} = E_p \angle -120°$$

$$\boldsymbol{E}_{BN} = E_p \angle -240° = E_p \angle 120°$$

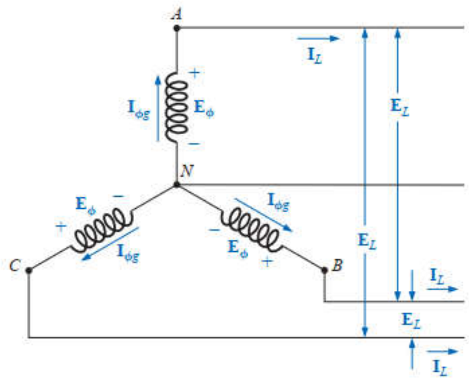


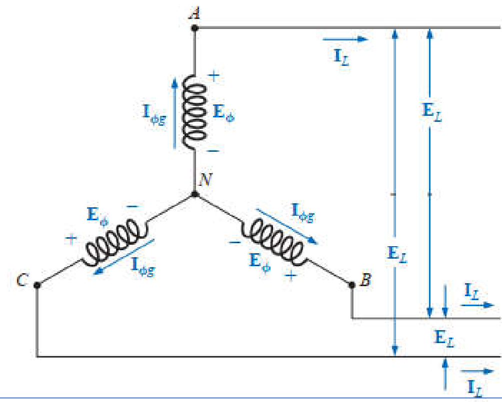
Connection of Three-Phase System

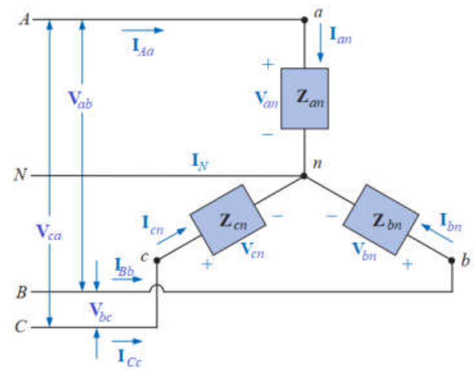
Three phase system can be connected two different ways:

(*a*) **Star** or **Y** (Wye) or **T** (Tee) connection

Balanced source voltages are equal in magnitude and are out of phase with each other by 120°.



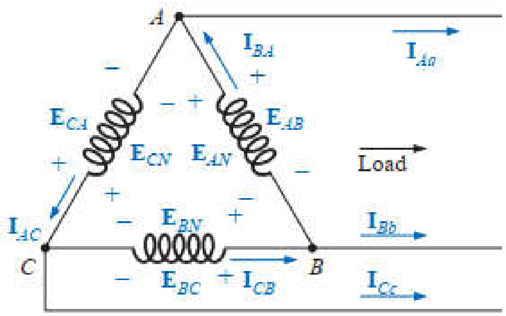


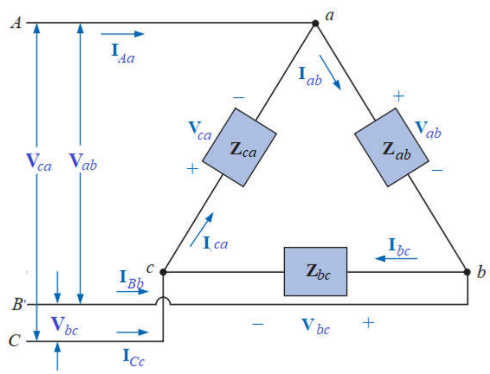


$$Z_{an} = Z_{bn} = Z_{cn} = Z_Y$$

$$Z_Y = Z\angle\theta_z = R \pm jX$$

(*b*) **Mesh** or Δ (delta) or Π (pai) connection



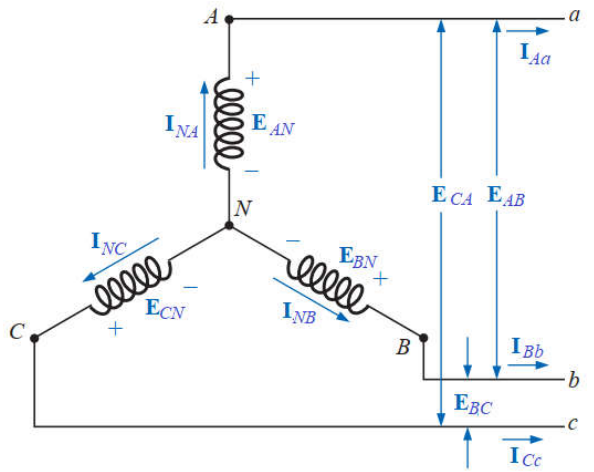


$$Z_{ab} = Z_{bc} = Z_{ca} = Z_\Delta$$

$$Z_\Delta = Z\angle\theta_z = R \pm jX$$

A balanced load is one which the phase impedance are equal in magnitude and in phase (also, equal in real part and equal in imaginary part).

Star or Y (Wye) or T (Tee) connection



Phase Voltages: $\mathbf{E}_{AN}$, $\mathbf{E}_{BN}$ and $\mathbf{E}_{CN}$

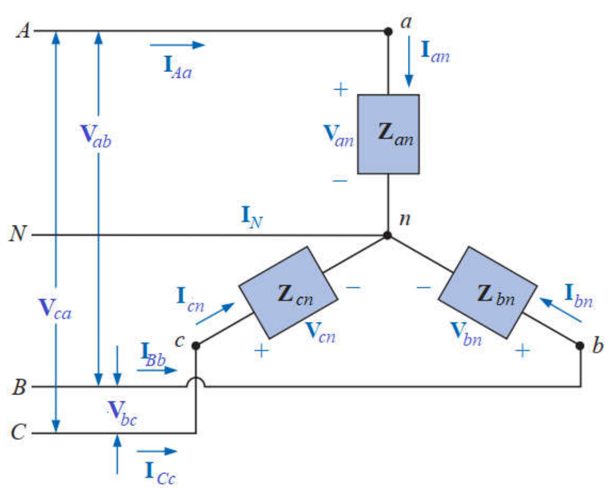
Line Voltages: $\mathbf{E}_{AB}$, $\mathbf{E}_{BC}$ and $\mathbf{E}_{CA}$

Phase Currents: $\mathbf{I}_{NA}$, $\mathbf{I}_{NB}$ and $\mathbf{I}_{NC}$

Line Currents: $\mathbf{I}_{Aa}$, $\mathbf{I}_{Bb}$ and $\mathbf{I}_{Cc}$

Line Currents = Phase Currents

$\mathbf{I}_{Aa} = \mathbf{I}_{NA}$; $\mathbf{I}_{Bb} = \mathbf{I}_{NB}$; and $\mathbf{I}_{Cc} = \mathbf{I}_{NC}$



Phase Voltages: $\mathbf{V}_{an}$, $\mathbf{V}_{bn}$ and $\mathbf{V}_{cn}$

Line Voltages: $\mathbf{V}_{ab}$, $\mathbf{V}_{bc}$ and $\mathbf{V}_{ca}$

Phase Currents: $\mathbf{I}_{an}$, $\mathbf{I}_{bn}$ and $\mathbf{I}_{cn}$

Line Currents: $\mathbf{I}_{Aa}$, $\mathbf{I}_{Bb}$ and $\mathbf{I}_{Cc}$

Line Currents = Phase Currents

$\mathbf{I}_{Aa} = \mathbf{I}_{an}$; $\mathbf{I}_{Bb} = \mathbf{I}_{bn}$; and $\mathbf{I}_{Cc} = \mathbf{I}_{cn}$

Line Voltage ≠ Phase Voltage

$$\boldsymbol{E}_{AB} = \boldsymbol{E}_{AN} - \boldsymbol{E}_{BN}$$
$$\boldsymbol{E}_{BC} = \boldsymbol{E}_{BN} - \boldsymbol{E}_{CN}$$
$$\boldsymbol{E}_{CA} = \boldsymbol{E}_{CN} - \boldsymbol{E}_{AN}$$

$$\boldsymbol{V}_{ab} = \boldsymbol{V}_{an} - \boldsymbol{V}_{bn}$$
$$\boldsymbol{V}_{bc} = \boldsymbol{V}_{bn} - \boldsymbol{V}_{cn}$$
$$\boldsymbol{V}_{ca} = \boldsymbol{V}_{cn} - \boldsymbol{V}_{an}$$

Let,

V_P and E_P: RMS value of phase voltage

V_L and E_L: RMS value of line voltage

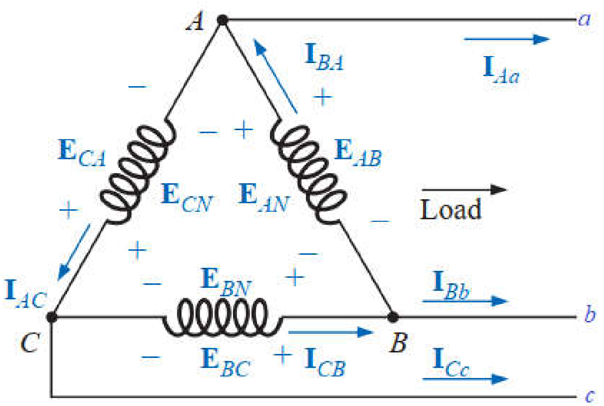
I_P: RMS value of phase current

I_L: RMS value of line current

$$E_L = \sqrt{3}E_P \qquad V_L = \sqrt{3}V_P$$

$$I_L = I_P$$

Mesh or Δ (delta) or Π (pai) connection



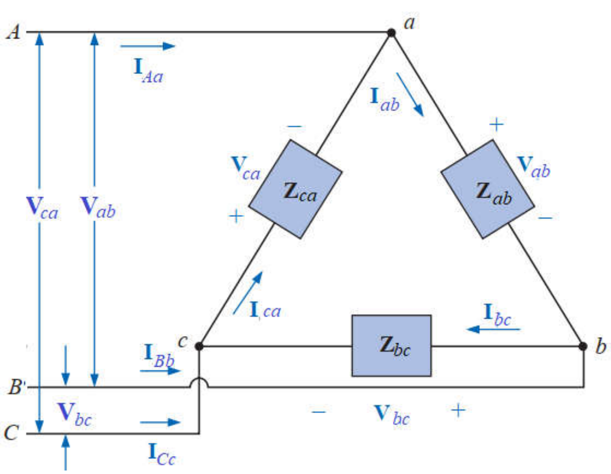
Phase Voltages: $\mathbf{E}_{AB}$, $\mathbf{E}_{BC}$ and $\mathbf{E}_{CA}$

Line Voltages: $\mathbf{E}_{AB}$, $\mathbf{E}_{BC}$ and $\mathbf{E}_{CA}$

Phase Currents: $\mathbf{I}_{BA}$, $\mathbf{I}_{AC}$ and $\mathbf{I}_{CB}$

Line Currents: $\mathbf{I}_{Aa}$, $\mathbf{I}_{Bb}$ and $\mathbf{I}_{Cc}$

Line Voltage = Phase Voltage



Phase Voltages: $\mathbf{V}_{ab}$, $\mathbf{V}_{bc}$ and $\mathbf{V}_{ca}$

Line Voltages: $\mathbf{V}_{ab}$, $\mathbf{V}_{bc}$ and $\mathbf{V}_{ca}$

Phase Currents: $\mathbf{I}_{ab}$, $\mathbf{I}_{bc}$ and $\mathbf{I}_{ca}$

Line Currents: $\mathbf{I}_{Aa}$, $\mathbf{I}_{Bb}$ and $\mathbf{I}_{Cc}$

Line Voltage = Phase Voltage

Line Current ≠ Phase Current

$$\boldsymbol{I}_{Aa} = \boldsymbol{I}_{BA} - \boldsymbol{I}_{AC}$$
$$\boldsymbol{I}_{Bb} = \boldsymbol{I}_{CB} - \boldsymbol{I}_{BA}$$
$$\boldsymbol{I}_{Cc} = \boldsymbol{I}_{AC} - \boldsymbol{I}_{CB}$$

$$\boldsymbol{I}_{Aa} = \boldsymbol{I}_{ab} - \boldsymbol{I}_{ca}$$
$$\boldsymbol{I}_{Bb} = \boldsymbol{I}_{bc} - \boldsymbol{I}_{ab}$$
$$\boldsymbol{I}_{Cc} = \boldsymbol{I}_{ca} - \boldsymbol{I}_{bc}$$

Let,

V_P and E_P: RMS value of phase voltage
V_L and E_L: RMS value of line voltage
I_P: RMS value of phase current
I_L: RMS value of line current

$$I_L = \sqrt{3} I_P$$

$$E_L = E_P \qquad V_L = V_P$$

Power Calculation

Instantaneous Power Equation: $p(t) = \frac{3}{2} E_m I_m \cos\theta = 3E_p I_p \cos\theta$ [W]

V_m and E_m: Peak value of phase voltage
I_m : Peak value of phase current
V_P and E_P: RMS value of phase voltage
V_L and E_L: RMS value of line voltage
I_P: RMS value of phase current
I_L: RMS value of line current

For Y – Connection :

$$\theta = \theta_z = \theta_{e(an)} - \theta_{i(an)} = \theta_{e(bn)} - \theta_{i(bn)} = \theta_{e(cn)} - \theta_{i(cn)}$$

For Δ – Connection :

$$\theta = \theta_z = \theta_{e(ab)} - \theta_{i(ab)} = \theta_{e(bc)} - \theta_{i(bc)} = \theta_{e(ca)} - \theta_{i(ca)}$$

Source Side

$pf = \cos\theta$ $\qquad rf = \sin\theta$

$$S = 3E_P I_P = \sqrt{3} E_L I_L$$

$$P = 3E_P I_P \cos\theta = \sqrt{3} E_L I_L \cos\theta = S\cos\theta$$

$$Q = 3E_P I_P \sin\theta = \sqrt{3} E_L I_L \sin\theta = S\sin\theta$$

Load Side

$S = 3I_P^2 Z$ $\qquad P = 3I_P^2 R$ $\qquad Q_L = 3I_P^2 X_L$

$Q_C = -3I_P^2 X_C$ $\qquad Q = Q_L + Q_C$

$pf = \frac{P}{S}$ $\qquad rf = \frac{Q}{S}$

Example: The line voltage and line current of a three-phase system are 440 V and 40 A. Calculate the phase voltage and phase current for (***i***) star-connection, and (***ii***) mesh-connection.

Solution: Given: V_L =440 V, I_L = 40 A

For Star connection $I_P = I_L = 40 \text{ A}$

$$V_P = \frac{V_L}{\sqrt{3}} = 254 \text{ V}$$

For Mesh connection $V_P = V_L = 440 \text{ V}$

$$I_P = \frac{I_L}{\sqrt{3}} = 23.1 \text{ A}$$

Example 4.1.6: The phase voltage and phase current of a three-phase system are 400 V and 20 A. Calculate the line voltage and line current for (***i***) star or Wye-connection, and (***ii***) mesh or Delta-connection.

Solution: Given: V_P =400 V, I_P = 20 A, n=3

Star or Wye connection

$$I_L = I_P = 20 \text{ A}$$

$$V_L = \sqrt{3}V_P = \sqrt{3} \times 400 = 692.82\ V$$

Mesh or Delta connection

$$V_L = V_P = 400 \text{ V}$$

$$I_L = \sqrt{3}I_P = \sqrt{3} \times 20 = 34.64 \text{ A}$$

Example: A 500 volts three-phase supply is connected with a Δ-connected load having R = 18 Ω and X_C = 24 Ω in series in per phase. Calculate (***i***) the real power, (***ii***) the reactive power, (***iii***) the apparent, (***iv***) the power factor and (***v***) the reactive factor.

Solution: Given, $V_L = 500 \text{ V} \qquad \boldsymbol{Z}_\Delta = \boldsymbol{Z}_{ab} = \boldsymbol{Z}_{bc} = \boldsymbol{Z}_{ca} = 18 - j24 = 30\angle -53.13° \quad \Omega$

$$Z_p = 30 \ \Omega \qquad \theta = \theta_z = -53.13°$$

$$V_p = V_L = 500 \text{ V} \qquad I_p = \frac{V_p}{Z_p} = \frac{500}{30} = 16.67 \text{ A} \qquad I_L = \sqrt{3} I_p = \sqrt{3} \times 16.67 = 28.87 \text{ A}$$

$$P = 3V_p I_p \cos\theta = \sqrt{3} V_L I_L \cos\theta = \sqrt{3} \times 500 \times 28.87 \times \cos(-53.13°) = 15001.33 \text{ W}$$

$$Q = 3V_p I_p \sin\theta = \sqrt{3} V_L I_L \sin\theta = \sqrt{3} \times 500 \times 28.87 \times \sin(-53.13°) = -20001.7 \text{ Var}$$

$$S = 3V_p I_p = \sqrt{3} V_L I_L = \sqrt{3} \times 500 \times 28.87 = 25002.15 \text{ VA}$$

$$pf = \cos\theta = \cos(-53.13°) = 0.6 \qquad rf = \sin\theta = \sin(-53.13°) = -0.8$$

Example: A three-phase Y-connected motor draws 5.6 kW at a power factor of 0.8 lagging when the line voltage is 220 V. Determine (***i***) the line current and (***ii***) the impedance of the motor.

Solution: Given, P = 5.6 kW = 5600 W; $pf = \cos\theta_z = \cos\theta = 0.8$ lagging, V_L = 220 V

Since $P = \sqrt{3}V_L I_L \cos\theta = 5600 \text{ W}$

$$I_L = \frac{P}{\sqrt{3}V_L \cos\theta} = \frac{5600}{\sqrt{3}\times 220\times 0.8} = 18.37 \text{ A}$$

For Y-connection:

$$I_p = I_L = 18.37 \text{ A} \qquad V_L = \sqrt{3}V_p \qquad V_p = \frac{1}{\sqrt{3}}V_L = \frac{1}{\sqrt{3}}\times 220 = 127.02 \text{ V}$$

The magnitude of impedance is: $Z_p = \dfrac{V_p}{I_p} = \dfrac{127.02}{18.37} = 6.91 \ \Omega$

Since power factor is lagging, we have $\theta_z = \theta = \cos(pf) = \cos(0.8) = 36.87°$

The impedance is: $\boldsymbol{Z} = Z_p \angle\theta_z = 6.91\Omega\angle 36.87° = 5.53 + j4.15 \ \Omega$