Learning Outcomes

Exercises Using First Shifting Property

Examples Using First Shifting Property

First Translation Property & Example

Exercise Using Direct Formula

Examples Using Direct Formula

Important Formulae

This Lecture Covers

Inverse Laplace Transformation

Lecture 04

Complex Variable, Laplace & Z-Transformation

Learning Outcomes

Exercises Using First Shifting Property

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Examples Using Direct Formula

Important Formulae

This Lecture Covers

1. Formula of Inverse Laplace Transformation.

2. Examples & Exercise of Inverse Laplace Transformation Using Direct Formula.

3. First Shifting Property of Inverse Laplace Transformation.

4. Examples & Exercises of Inverse Laplace Transformation Using First Shifting Property.

Inverse Laplace Transformation

1. $\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} = 1,$

2. $\mathcal{L}^{-1}\left\{\frac{1}{s^{n+1}}\right\} = \frac{t^n}{n!},$

3. $\mathcal{L}^{-1}\left\{\frac{1}{s-a}\right\} = e^{at},$

4. $\mathcal{L}^{-1}\left\{\frac{s}{s^2+a^2}\right\} = \cos at,$

5. $\mathcal{L}^{-1}\left\{\frac{a}{s^2+a^2}\right\} = \sin at,$

6. $\mathcal{L}^{-1}\left\{\frac{s}{s^2-a^2}\right\} = \cosh at,$

7. $\mathcal{L}^{-1}\left\{\frac{a}{s^2-a^2}\right\} = \sinh at.$

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Examples Using Direct Formula

1. $\mathcal{L}^{-1}\left\{\frac{s^2+1}{s^3}\right\}$

$= \mathcal{L}^{-1}\left\{\frac{1}{s}+\frac{1}{s^3}\right\} = 1+\frac{t^2}{2!} = 1+\frac{t^2}{2}.$

2. $\mathcal{L}^{-1}\left\{\frac{1}{2s-5}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{2(s-\frac{5}{2})}\right\} = \frac{1}{2}e^{\frac{5}{2}t}$

3. $\mathcal{L}^{-1}\left\{\frac{2s}{s^2-9}\right\} = 2\mathcal{L}^{-1}\left\{\frac{s}{s^2-3^2}\right\} = 2\cosh 3t$

4. $\mathcal{L}^{-1}\left\{\frac{5}{s}-\frac{3s}{s^2+16}+\frac{2}{s^2+4}\right\}$

$= 5\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} - 3\mathcal{L}^{-1}\left\{\frac{s}{s^2+16}\right\} + \mathcal{L}^{-1}\left\{\frac{2}{s^2+2^2}\right\}$

$= 5 - 3\cos 4t + \sin 2t.$

Important Formulae

This Lecture Covers

Inverse Laplace Transformation

Find The Inverse Laplace Transformation of the following functions:

1. $F(s) = \frac{1}{s-5}$,
2. $F(s) = \frac{1}{s^5}$,
3. $F(s) = \frac{s^3-5s^2+6}{s^4}$,
4. $F(s) = \frac{2+4s}{s^2+25}$,
5. $F(s) = \frac{3}{s^2+4}$,
6. $F(s) = \frac{3}{s^2-4}$.

First translation property

If $\mathcal{L}^{-1}\{F(s)\} = f(t)$ then

$$\mathcal{L}^{-1}\{F(s-a)\} = e^{at}\mathcal{L}^{-1}\{F(s)\}.$$

Example: 01

$$\mathcal{L}^{-1}\left\{\frac{10}{(s+3)^4}\right\}$$
$$= 10\mathcal{L}^{-1}\left\{\frac{1}{(s+3)^4}\right\}$$
$$= 10e^{-3t}\mathcal{L}^{-1}\left\{\frac{1}{s^4}\right\}$$
$$= 10e^{-3t}\frac{t^3}{3!} = \frac{10}{6}e^{-3t}t^3.$$

Example: 02

$$\mathcal{L}^{-1}\left\{\frac{1}{(s-2)^2+1}\right\}$$
$$= e^{2t}\mathcal{L}^{-1}\left\{\frac{1}{s^2+1}\right\}$$
$$= e^{2t}\sin t\,.$$

Example 3.

$$\mathcal{L}^{-1}\left\{\frac{2s+1}{s^2+4s+13}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{2s+1}{s^2+2.s.2+4+9}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{2(s+2)-3}{(s+2)^2+9}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{2(s+2)}{(s+2)^2+3^2}-\frac{3}{(s+2)^2+3^2}\right\}$$

$$= 2e^{-2t}\cos 3t - e^{-2t}\sin 3t.$$

Learning Outcomes

Exercises Using First Shifting Property

Find Inverse Laplace of the following functions:

1. $F(s) = \frac{1}{(s-3)^4}$,

2. $F(s) = \frac{3}{(s+2)^2+9}$,

3. $F(s) = \frac{s-2}{(s-2)^2-16}$,

4. $F(s) = \frac{s}{s^2+4s-9}$,

5. $F(s) = \frac{5s-7}{s^2-6s+25}$,

6. $F(s) = \frac{s}{s^2-6s+10}$.

Examples Using First Shifting Property

First Translation Property & Example

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Examples Using Direct Formula

Important Formulae

This Lecture Covers

Inverse Laplace Transformation

Learning Outcomes

After completing this chapter you can easily evaluate the inverse Laplace transformation of function using direct formula & also using property.

This Lecture Covers

- Inverse Laplace Transformation
- Important Formulae
- Examples Using Direct Formula
- Exercise Using Direct Formula
- First Translation Property & Example
- Examples Using First Shifting Property
- Exercises Using First Shifting Property

THE END


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Sample MCQ

1. $\mathcal{L}^{-1}\left\{\frac{s^2+1}{s^3}\right\}=?$

(a) $1+\frac{t}{2}$ (b) $1+\frac{t^2}{2}$ (c) $1-\frac{t^2}{2}$ (d) $\frac{t^2}{2}$

2. $\mathcal{L}^{-1}\left\{\frac{4}{s-2}-\frac{s}{s^2-16}+\frac{4}{s^2-4}\right\}=?$

(a) $e^{2t}-\cosh 4t+2\sinh 2t$ (b) $4e^{2t}+\cosh 4t+2\sinh 2t$

(c) $4e^{2t}-\cosh 4t+2\sinh 2t$ (d) $4e^{2t}-\cosh 4t$

3. $\mathcal{L}^{-1}\left\{\frac{s}{s^2+4s+13}\right\}=?$

(a) $e^{-2t}\cos 3t-\frac{2}{3}e^{-2t}\sin 3t$ (b) $e^{-2t}\cos 3t-\frac{2}{3}e^{-2t}\sin 3t$

(c) $e^{-2t}\cos 3t-\frac{2}{3}e^{-2t}\sin 3t$ (d) $e^{-2t}\cos 3t-\frac{2}{3}e^{-2t}\sin 3t$

4. $\mathcal{L}^{-1}\left\{\frac{s-2}{(s-2)^2-16}\right\}=?$

(a) $\frac{e^{2t}}{2}+\frac{e^{6t}}{2}$ (b) $\frac{e^{-2t}}{2}+\frac{e^{-6t}}{2}$ (c) $\frac{e^{-2t}}{2}+\frac{e^{4t}}{2}$ (d) $\frac{e^{-2t}}{2}+\frac{e^{6t}}{2}$