# Complex Variable, Laplace & Z-transformation

## Lecture 02

# This Lecture Covers -

1. Laplace Transformation Using First Shifting Property.
2. Some Examples & Exercises on First Shifting Property.
3. Laplace Transformation Using Multiplication by $t^n$ Property.
4. Some Examples & Exercises on Multiplication by $t^n$ Property.

## First Shifting or Translation Property

If $L\{f(t)\} = F(s)$, then

$$\mathcal{L}\{e^{at} f(t)\} = F(s-a)$$

## Property of Multiplication by $t^n$

If $\mathcal{L}\{f(t)\} = F(s)$ then

$$\mathcal{L}\{t^n f(t)\} = (-1)^n \frac{d^n}{ds^n}[F(s)]$$

# Examples on Shifting Property

## Example 1:

$$\mathcal{L}\{e^{2t}\sin t\} = F(s-2)$$

Now,

$$F(s) = \mathcal{L}\{\sin t\} = \frac{1}{s^2+1}$$

So, $\mathcal{L}\{e^{2t}\sin t\} = \frac{1}{(s-2)^2+1}$.

Ans.

## Example 2:

$$\mathcal{L}\{e^{-t}\cos 2t\} = F(s+1)$$

Now,

$$F(s) = \mathcal{L}\{\cos 2t\} = \frac{s}{s^2+4}$$

So, $\mathcal{L}\{e^{-t}\cos 2t\} = \frac{(s+1)}{(s+1)^2+4}$.

Ans.

# Examples on Shifting Property

## Example 3:

$$\mathcal{L}\{e^t \cosh 3t\} = F(s-1)$$

Now,

$$F(s) = \mathcal{L}\{\cosh 3t\} = \frac{s}{s^2-9}$$

So, $\mathcal{L}\{e^t \cosh 3t\} = \frac{s-1}{(s-1)^2-9}$.

Ans.

## Example 4:

$$\mathcal{L}\{e^{-3t}\, t^8\} = F(s+3)$$

Now,

$$F(s) = \mathcal{L}\{t^8\} = \frac{8!}{s^9}$$

So, $\mathcal{L}\{e^{-3t}\, t^8\} = \frac{8!}{(s+3)^9}$.

Ans.

# Exercise Set on Shifting Property

Find Laplace Transformation of the following function:

1. $f(t) = e^{2t} \sinh 3t,$

2. $f(t) = e^{-t} \sinh 4t\,,$

3. $f(t) = e^{2t} \cos 3t,$

4. $f(t) = t^{10}\, e^{-7t},$

5. $f(t) = e^{5t} \cosh 6t.$

# Examples on Multiplication by $t^n$ Property

## Example 1 :

$$\mathcal{L}\{t\cos t\} = (-1)^1 \frac{d}{ds}[F(s)]$$

$$= -\frac{d}{ds}\left[\mathcal{L}\{\cos t\}\right]$$

$$= -\frac{d}{ds}\left[\frac{s}{s^2+1}\right]$$

$$= -\frac{(s^2+1)\frac{d}{ds}(s) - s\frac{d}{ds}(s^2+1)}{(s^2+1)^2}$$

$$= -\frac{(s^2+1) - s\times 2s}{(s^2+1)^2}$$

$$= -\frac{s^2+1-2s^2}{(s^2+1)^2}$$

$$= \frac{s^2-1}{(s^2+1)^2}.$$

Ans.

## Example 2 :

$$\mathcal{L}\{t^2 e^{3t}\} = (-1)^2 \frac{d^2}{ds^2}[F(s)]$$

$$= \frac{d^2}{ds^2}\left[\mathcal{L}\{e^{3t}\}\right]$$

$$= \frac{d^2}{ds^2}\left[\frac{1}{s-3}\right]$$

$$= \frac{d}{ds}\left[\frac{(s-3)\frac{d}{ds}(1) - 1\frac{d}{ds}(s-3)}{(s-3)^2}\right]$$

$$= \frac{d}{ds}\left[\frac{-1}{(s-3)^2}\right]$$

$$= (-1)(-2)(s-3)^{-3}$$

$$= \frac{2}{(s-3)^3}.$$

Ans.

# Exercise Set on Multiplication by $t^n$ Property

Find Laplace Transformation of the following functions:

1. $f(t) = t \sin 2t$ ,

2. $f(t) = t \cos bt$ ,

3. $f(t) = t^2 \, e^{-4t}$,

4. $f(t) = t \sinh 3t$,

5. $f(t) = t \cosh 2t$.

# Learning Outcomes

After completing this lecture you will learn about find Laplace Transformation using two properties named as first shifting or translation and another one is multiplication by $t^n$ property.

# Sample MCQ

1. $\mathcal{L}\{t^{10}\, e^{-5t}\} = ?$

(a) $\frac{11\,!}{(s+5)^{10}}$ (b) $\frac{10\,!}{(s+5)^{11}}$ (c) $\frac{10}{(s+5)^{11}}$ (d) $\frac{10\,!}{(s+5)^{10}}$

2. $\mathcal{L}\{t \sin t\} = ?$

(a) $\frac{2s}{(s^2+1)^2}$ (b) $\frac{s}{(s^2+1)^2}$ (c) $\frac{4s}{(s^2+1)^2}$ (d) $\frac{4s}{(s^2-1)^2}$

3. $\mathcal{L}\{e^{3t} \cosh 5t\} = ?$

(a) $\frac{s}{s^2+25}$ (b) $\frac{s+3}{(s+3)^2+25}$ (c) $\frac{s+3}{(s+3)^2-25}$ (d) $\frac{s-3}{(s-3)^2-25}$

# Sample MCQ

4. $\mathcal{L}\{e^{2t}\sinh t\} = ?$

(a) $\frac{2}{(s-1)^2-1}$ (b) $\frac{1}{(s-2)^2-1}$ (c) $\frac{1}{(s+2)^2-1}$ (d) none

5. $\mathcal{L}\{t\cos t\} = ?$

(a) $\frac{2s}{(s^2+1)^2}$ (b) $\frac{s^2-1}{(s^2+1)^2}$ (c) $\frac{4s}{(s^2+1)^2}$ (d) $\frac{4s}{(s^2-1)^2}$