Lecture -1
Logic Gates & Boolean Algebra

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Logic Gates

- Logic Gates are the basic building blocks of any digital system.
- A logic gate can have one or more than one input but only one output.
- The relationship between the input/s and the output is based on a certain logic.
- The gates are named based on the logic.

Basic Logic Gates

- NOT gate
- AND gate
- OR gate

Universal Logic Gates

- NAND gate
- NOR gate

Exclusive Logic Gates

- XOR gate
- XNOR gate

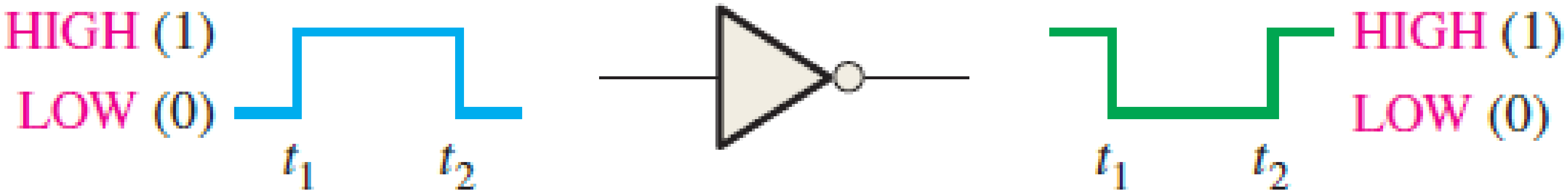
NOT Gate (Inverter)

- The output an inverter (NOT gate) is the opposite of its input.

Inverter truth table.

Input	Output
LOW (0)	HIGH (1)
HIGH (1)	LOW (0)

$x = \overline{x}$

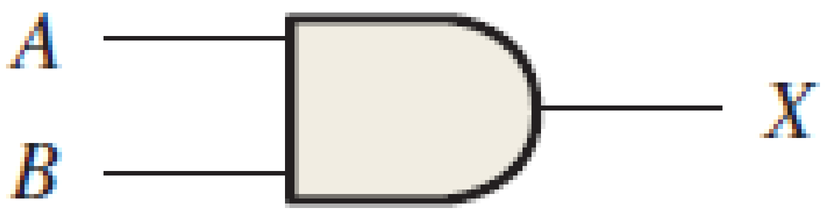


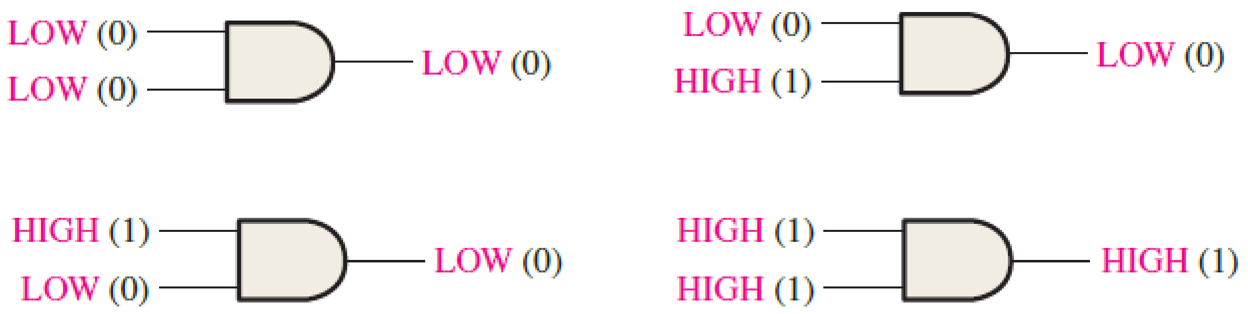
Input pulse

Output pulse

AND Gate

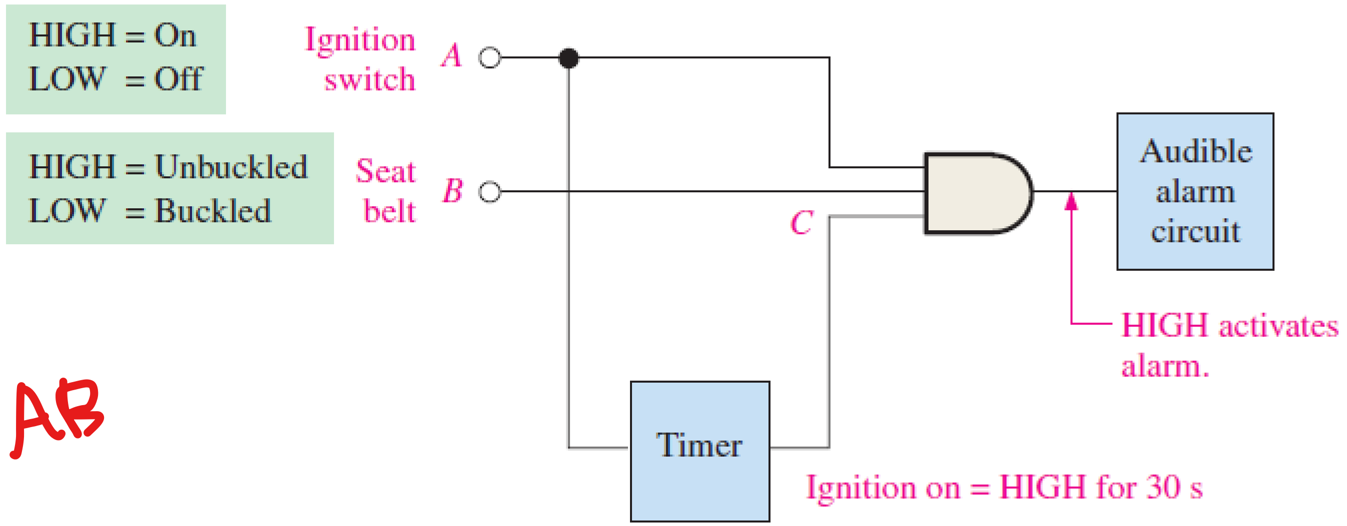
- The output an AND gate is HIGH only when both the inputs are HIGH.





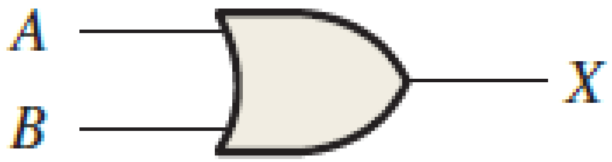
Inputs		Output
A	*B*	*X*
0	0	0
0	1	0
1	0	0
1	1	1

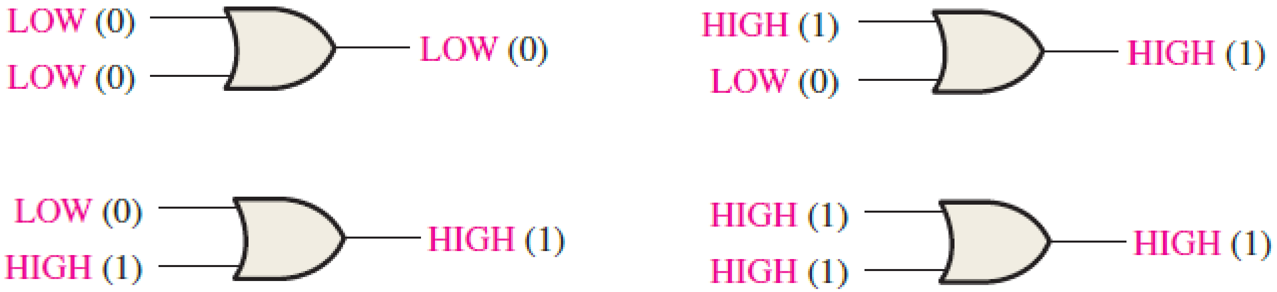
$x = AB$



OR Gate

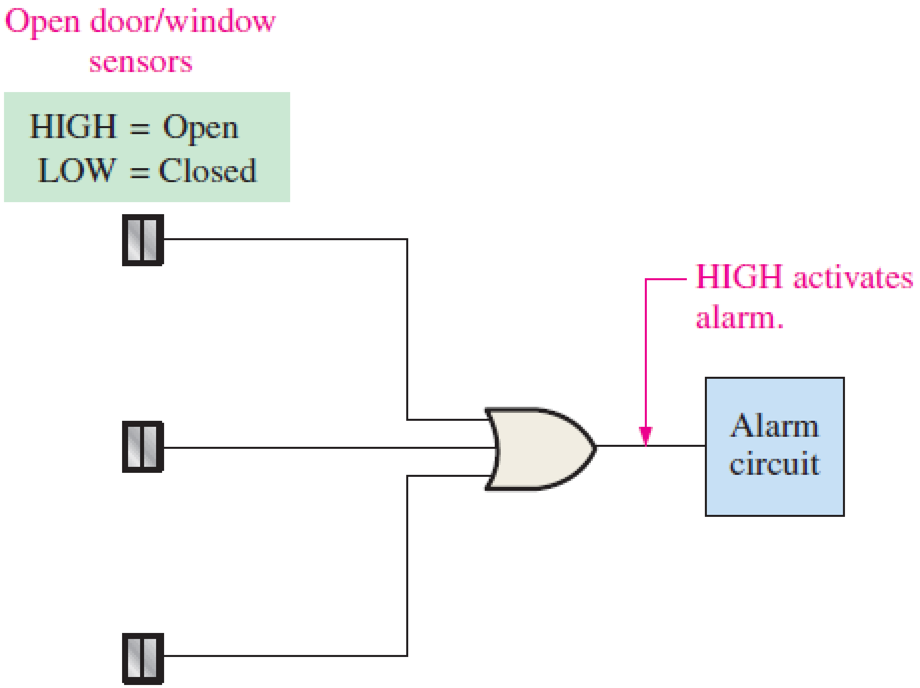
- The output an OR gate is HIGH when anyone or both the inputs are HIGH.





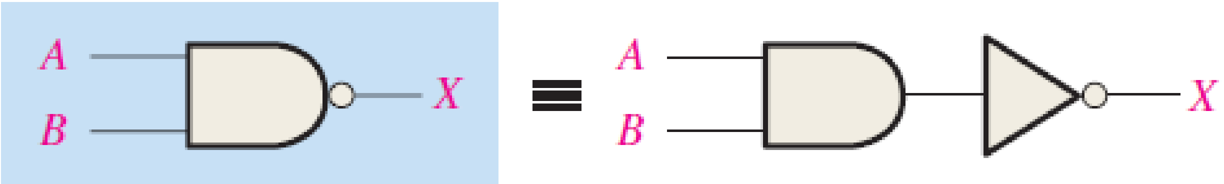
Inputs		Output
A	*B*	*X*
0	0	0
0	1	1
1	0	1
1	1	1

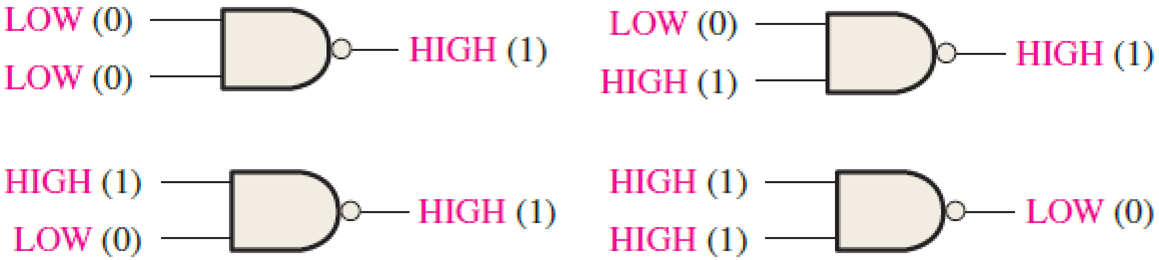
$X = A + B$

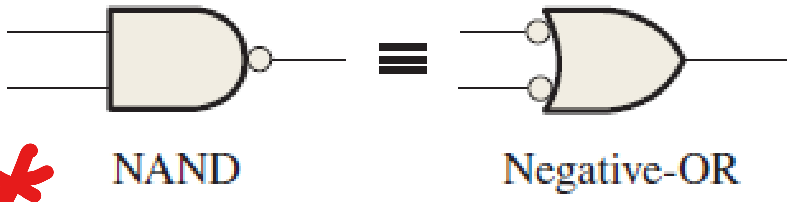


NAND Gate

- The output of a NAND gate is HIGH whenever one or more inputs are LOW.

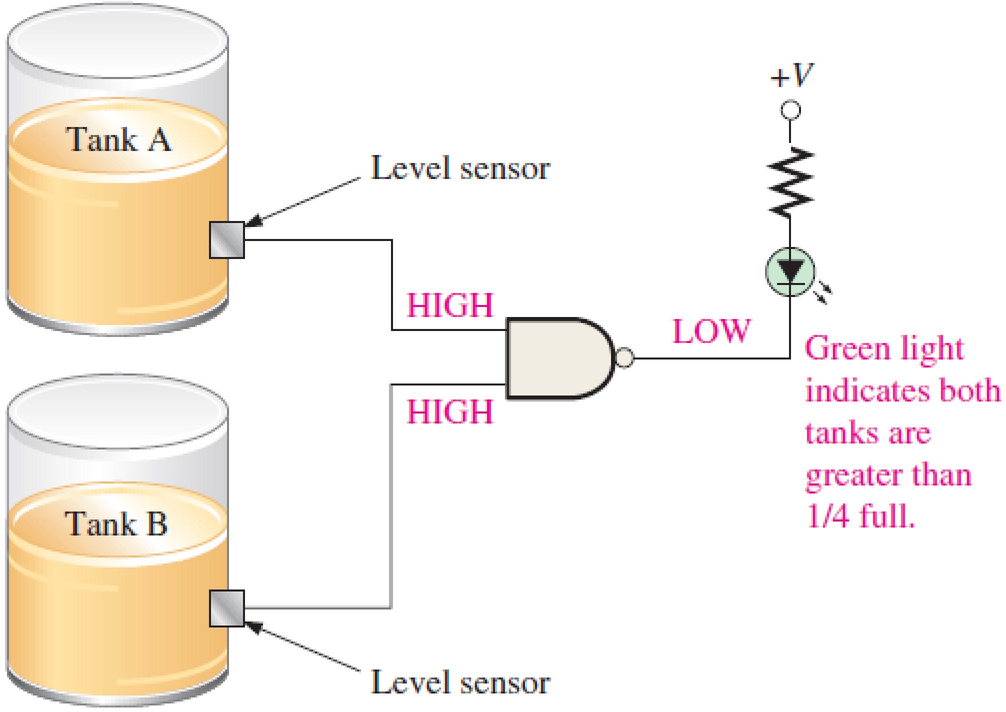






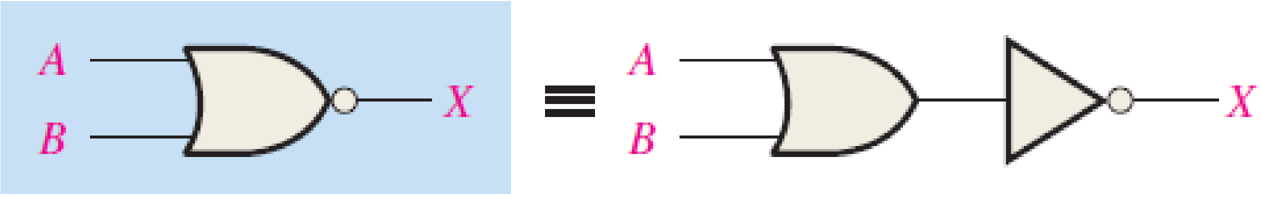
Inputs		Output
A	*B*	*X*
0	0	1
0	1	1
1	0	1
1	1	0

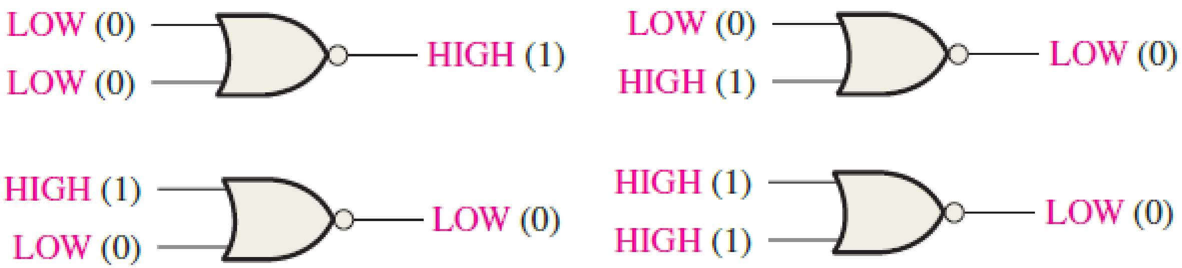
$x = \overline{AB}$



NOR Gate

- The output of a NOR gate is LOW whenever one or more inputs are HIGH.

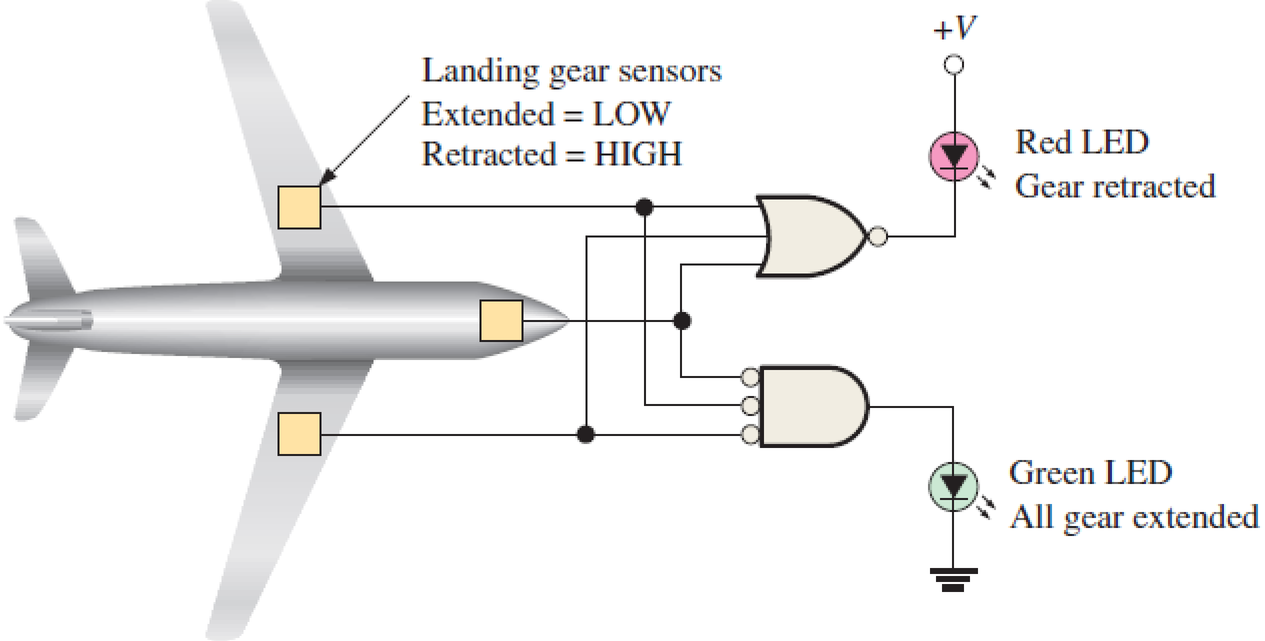




NOR Negative-AND

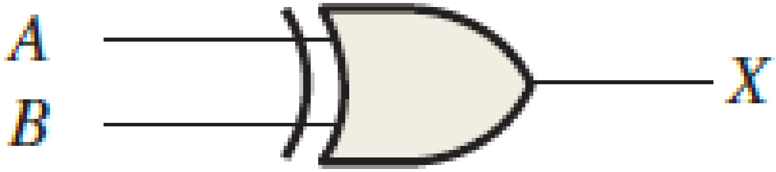
Inputs		Output
A	*B*	*X*
0	0	1
0	1	0
1	0	0
1	1	0

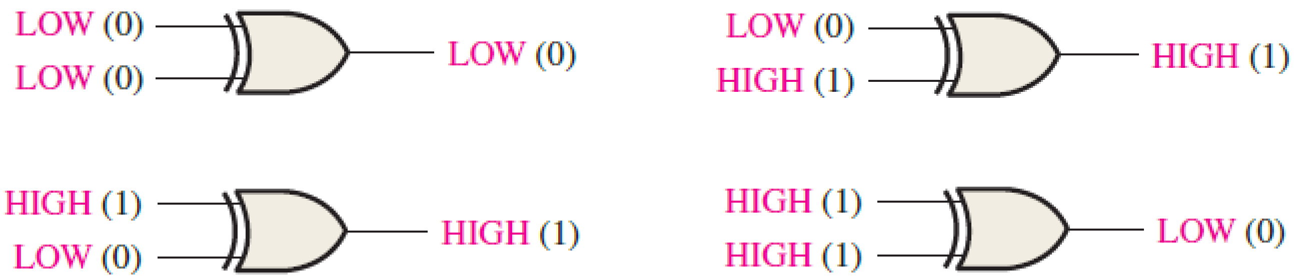
$X = \overline{A+B}$



XOR Gate

- The output of a XOR gate is HIGH whenever the two inputs are different.



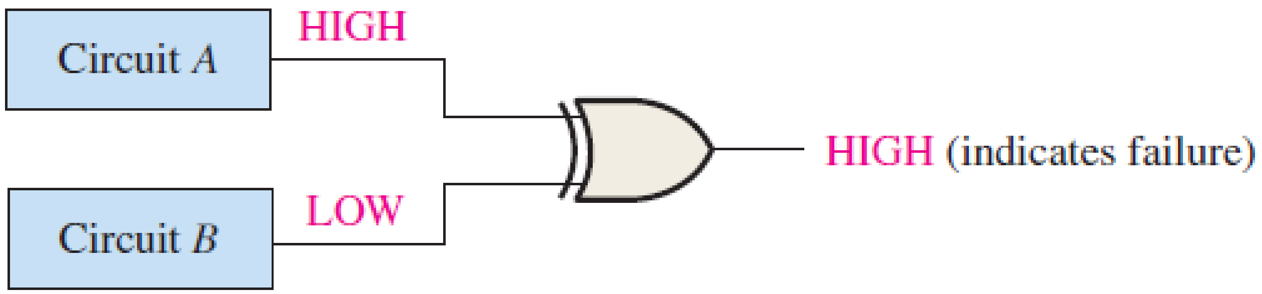


Truth table for an exclusive-OR gate.

Inputs		Output
A	*B*	*X*
0	0	0
0	1	1
1	0	1
1	1	0

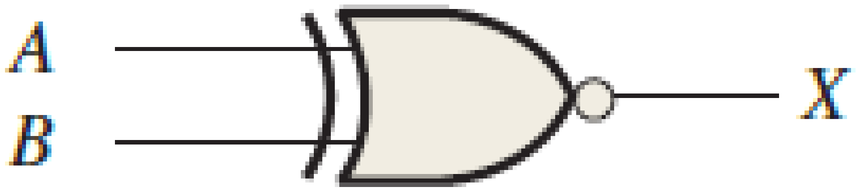
$X = A \oplus B$

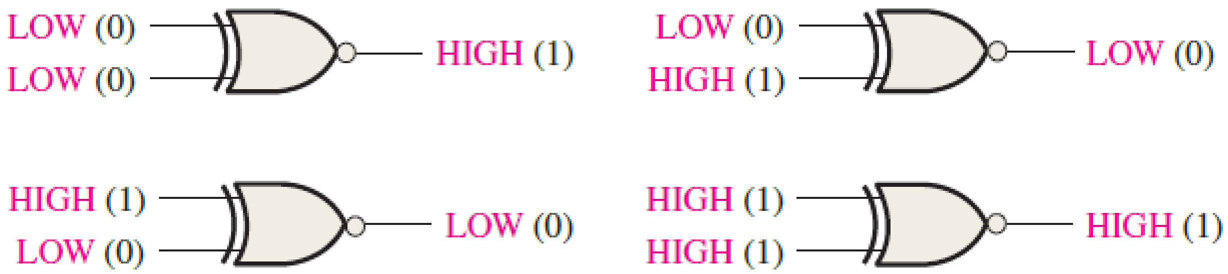
Two circuits are supposed to work parallelly in a process. If any of the circuit fails an indicator is activated.



XNOR Gate

- The output of a XNOR gate is gate is HIGH whenever the two inputs are same.



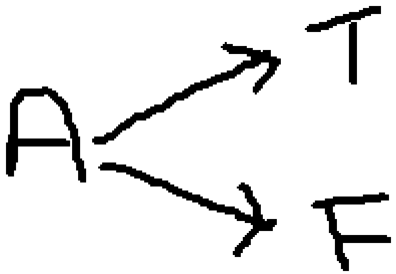


Inputs		Output
A	*B*	*X*
0	0	1
0	1	0
1	0	0
1	1	1

$$x = \overline{A \oplus B}$$

Boolean Algebra

- Boolean Variable: These are variables which can either take the value 1 or 0.



- Boolean Logic Expression: A Boolean logic expression is an expression constituted of only Boolean variables. The output of a Boolean logic expression is a Boolean value i.e. either True/False.

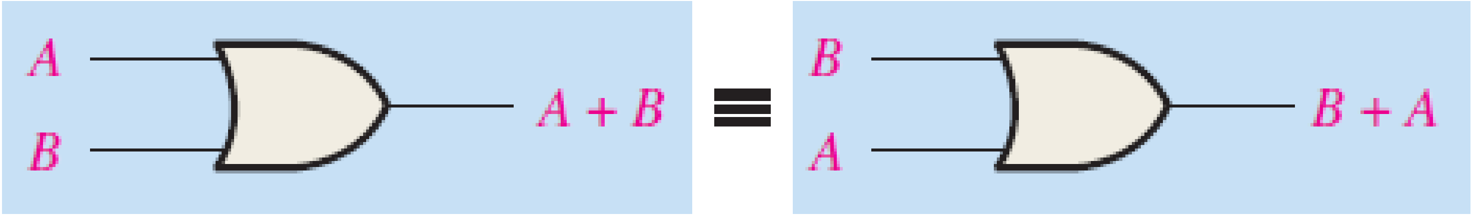
$$AB + A(B + C) + B(B + C)$$

- Boolean Algebra: It is the mathematics of digital logic. Usually Boolean algebra is used to simplify Boolean expressions or Boolean Function.

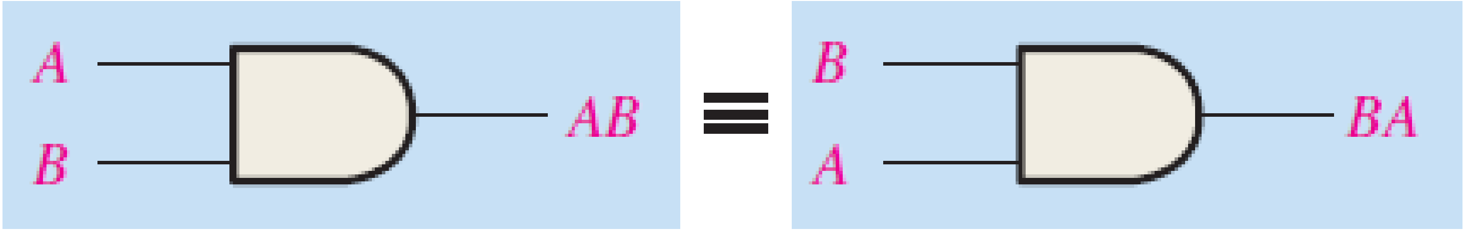
Laws of Boolean Algebra

Commutative Law:

- The commutative law for addition can be written as **A+B=B+A**



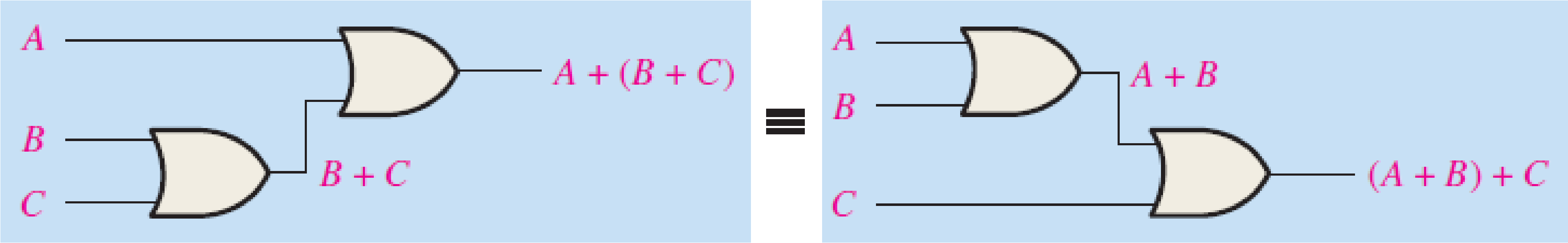
- The commutative law for multiplication can be written as **AB=BA**



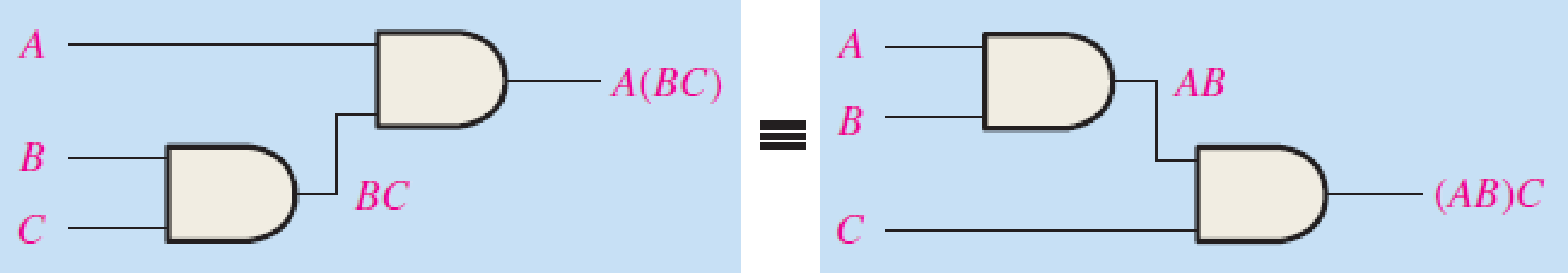
Laws of Boolean Algebra

Associative Law:

- The associative law of addition for three variables is written as **A+(B+C)=(A+B)+C**



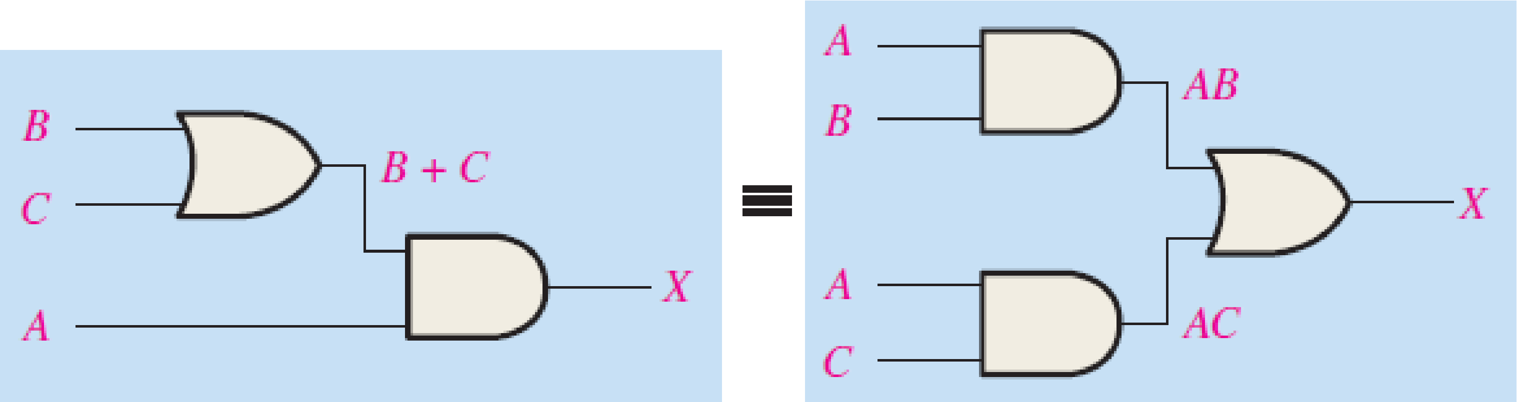
- The associative law of multiplication for three variables is written as **A(BC)=(AB)C**



Laws of Boolean Algebra

Distributive Law:

- The distributive law for three variables is written as $\mathbf{A(B+C)=AB+AC}$



$X = A(B + C)$ $\qquad$ $X = AB + AC$

Rules of Boolean Algebra

Basic rules of Boolean algebra.

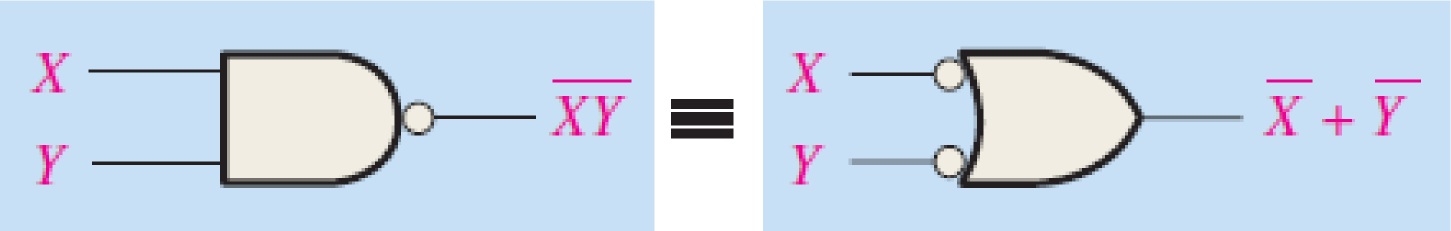
1. $A + 0 = A$	7. $A \cdot A = A$
2. $A + 1 = 1$	8. $A \cdot \overline{A} = 0$
3. $A \cdot 0 = 0$	9. $\overline{\overline{A}} = A$
4. $A \cdot 1 = A$	10. $A + AB = A$
5. $A + A = A$	11. $A + \overline{A}B = A + B$
6. $A + \overline{A} = 1$	12. $(A + B)(A + C) = A + BC$

De Morgan's Theorem

The first theorem is stated as follows:

- The complement of a product of variables is equal to the sum of the complements of complements of the variable.
- The formula of this theorem for two variables is written as:

$$\overline{\mathbf{XY}} = \overline{\mathbf{X}} + \overline{\mathbf{Y}}$$



NAND Negative-OR

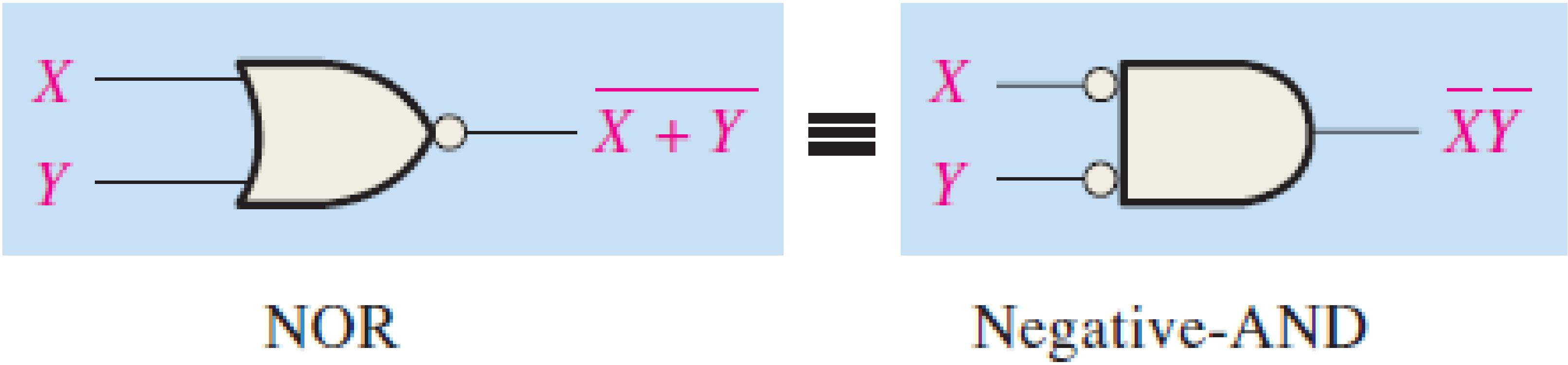
Inputs		Output	
X	Y	$\overline{XY}$	$\overline{X} + \overline{Y}$
0	0	1	1
0	1	1	1
1	0	1	1
1	1	0	0

De Morgan's Theorem

The second theorem is stated as follows:

- The complement of a sum of variables is equal to the product of the complements of the variables.
- The formula of this theorem for two variables is written as:

$$\overline{\mathbf{X+Y}} = \overline{\mathbf{X}}.\overline{\mathbf{Y}}$$



Inputs		Output	
X	Y	$\overline{X+Y}$	$\overline{X}\,\overline{Y}$
0	0	1	1
0	1	0	0
1	0	0	0
1	1	0	0

Application of De Morgan's Theorem

Apply DeMorgan's theorems to the expressions $\overline{XYZ}$ and $\overline{X + Y + Z}$.

Solution

$$\overline{XYZ} = \overline{X} + \overline{Y} + \overline{Z}$$

$$\overline{X + Y + Z} = \overline{X}\,\overline{Y}\,\overline{Z}$$

Apply DeMorgan's theorems to the expressions $\overline{WXYZ}$ and $\overline{W + X + Y + Z}$.

Solution

$$\overline{WXYZ} = \overline{W} + \overline{X} + \overline{Y} + \overline{Z}$$

$$\overline{W + X + Y + Z} = \overline{W}\,\overline{X}\,\overline{Y}\,\overline{Z}$$

Apply DeMorgan's theorems to each expression:

(a) $\overline{(\overline{A + B}) + \overline{C}}$

(b) $\overline{(\overline{A} + B) + CD}$

(c) $\overline{(A + B)\overline{C}\,\overline{D} + E + \overline{F}}$

Solution

(a) $\overline{(\overline{A + B}) + \overline{C}} = (\overline{\overline{A + B}})\overline{\overline{C}} = (A + B)C$

(b) $\overline{(\overline{A} + B) + CD} = (\overline{\overline{A} + B})\overline{CD} = (\overline{\overline{A}}\,\overline{B})(\overline{C} + \overline{D}) = A\overline{B}(\overline{C} + \overline{D})$

(c) $\overline{(A + B)\overline{C}\,\overline{D} + E + \overline{F}} = ((\overline{(A + B)\overline{C}\,\overline{D}})(\overline{E + \overline{F}}) = (\overline{A}\,\overline{B} + C + D)\overline{E}F$

Application of De Morgan's Theorem

Apply DeMorgan's theorem to the expression $\overline{\overline{X} + \overline{Y} + \overline{Z}}$.

Apply DeMorgan's theorem to the expression $\overline{\overline{W}\,\overline{X}\,\overline{Y}\,\overline{Z}}$.

Apply DeMorgan's theorems to each of the following expressions:

(a) $\overline{(A + B + C)D}$

(b) $\overline{ABC + DEF}$

(c) $\overline{A\overline{B} + \overline{C}D + EF}$

The Boolean expression for an exclusive-OR gate is $A\overline{B} + \overline{A}B$. With this as a starting point, use DeMorgan's theorems and any other rules or laws that are applicable to develop an expression for the exclusive-NOR gate.

Starting with the expression for a 4-input NAND gate, use DeMorgan's theorems to develop an expression for a 4-input negative-OR gate.

Apply DeMorgan's theorems to the following expressions:

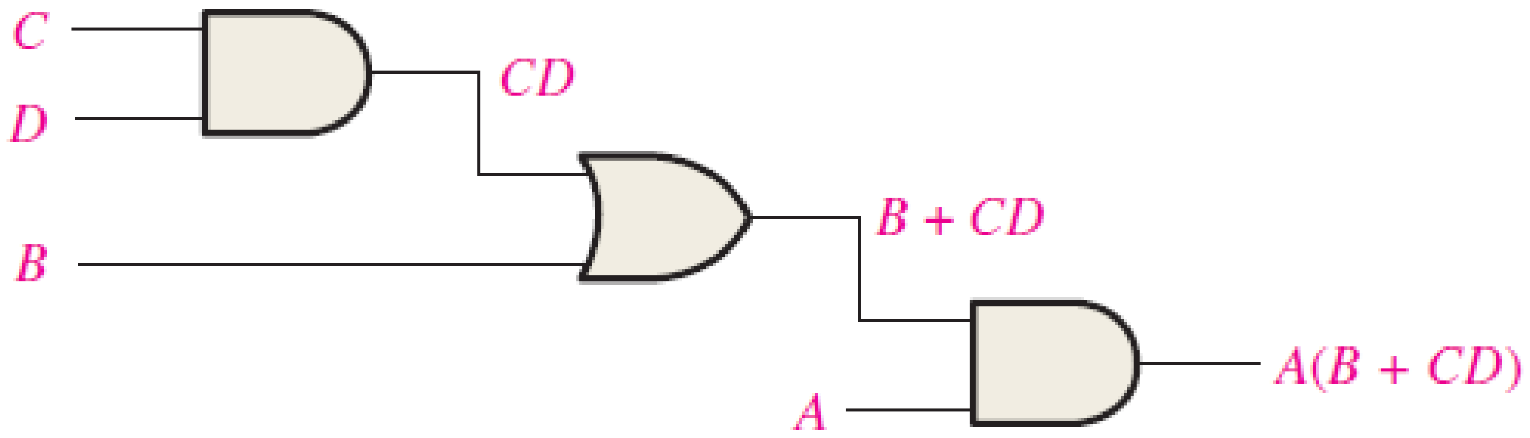
(a) $\overline{ABC} + \overline{(\overline{D} + E)}$ (b) $\overline{(A + B)C}$ (c) $\overline{A + B + C} + \overline{\overline{D}E}$

Boolean Analysis of Logic Circuit

Boolean algebra provides a concise way to express the operation of a logic circuit formed by a combination of logic gates so that the output can be determined for various combinations of input values.

- **A logic circuit can be expressed by Boolean expression and Boolean expression can be implemented by a logic circuit.**
- **The following Boolean expression can be implemented by the logic circuit below:**

$$\mathbf{A(B + CD)}$$



Constructing a Truth-table from a Boolean Expression

- Once we have the Boolean expression describing a process or a logical circuit, a truth-table to show the operation for all possible combination can be constructed.
- First, we need to determine the number of inputs in the expression.
- Then, we need to note down all possible combination of the inputs.
- Lastly, we will evaluation the expression for all possible combination.

$$\mathbf{A(B + CD)}$$

Inputs				Output
A	B	C	D	$A(B + CD)$
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

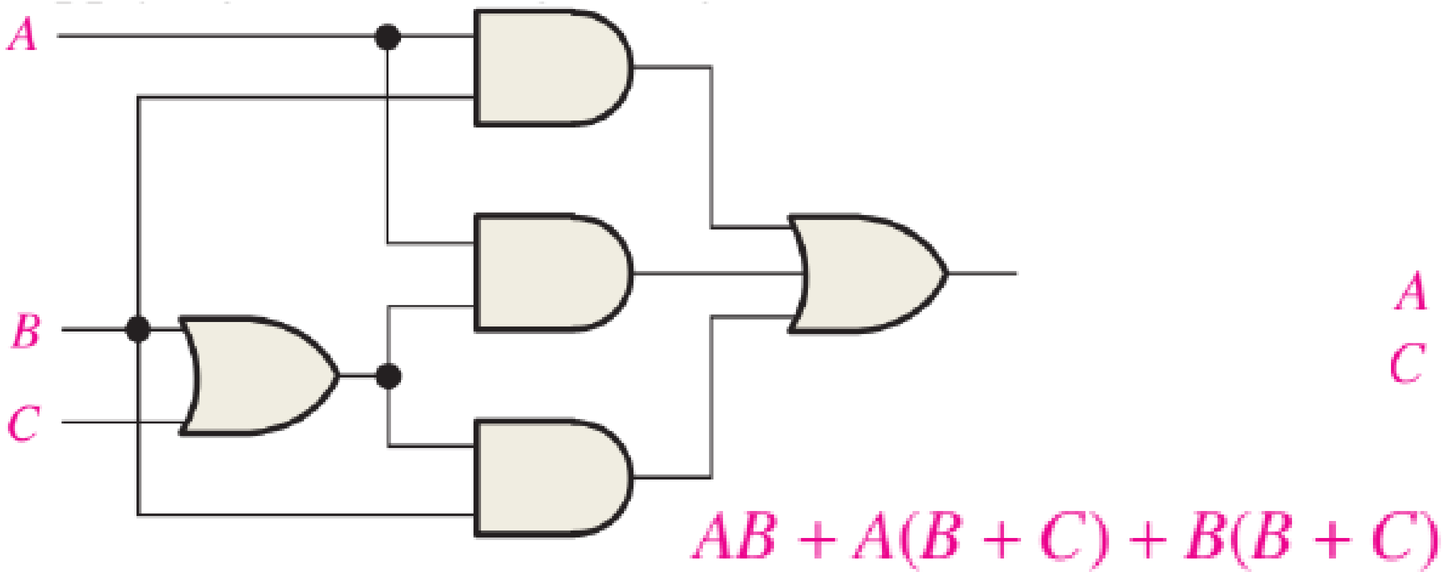
Logic Simplification

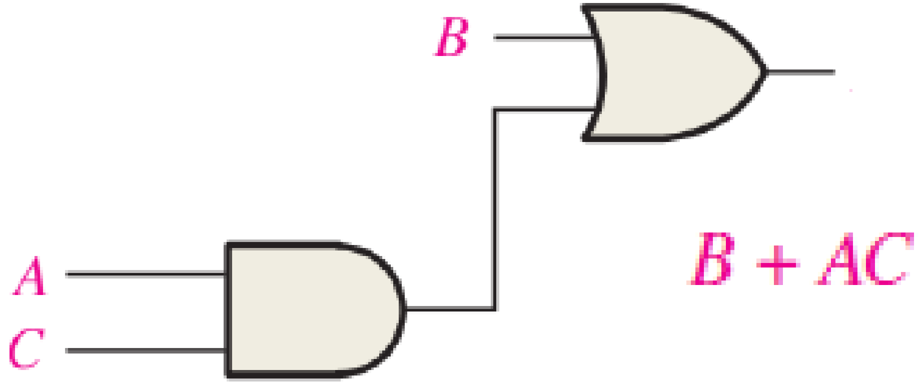
- We know that a Boolean expression can be implemented by a logical circuit.
- A large Boolean expression can often be simplified to a simpler and shorter expression.
- This is done by applying the laws and rules of Boolean algebra.
- Simplifying makes implementation simpler and thus requires lesser number of gates.
- Boolean algebra can be used to simplify the following expression:

$$\mathbf{AB + A(B + C) + B(B + C)}$$

- The simplified expression is:

$$\mathbf{B + AC}$$





Logic Simplification

Simplify the Boolean expression $A\overline{B} + A(\overline{B + C}) + B(\overline{B + C})$.

Simplify the following Boolean expression:

$$[A\overline{B}(C + BD) + \overline{A}\,\overline{B}]C$$

Simplify the following Boolean expression:

$$\overline{A}BC + A\overline{B}\,\overline{C} + \overline{A}\,\overline{B}\,\overline{C} + A\overline{B}C + ABC$$

Simplify the following Boolean expression:

$$\overline{AB + AC} + \overline{A}\,\overline{B}C$$

Simplify the Boolean expression $\overline{AB} + \overline{AC} + \overline{A}\,\overline{B}\,\overline{C}$.

***Applying rules of Boolean algebra and DeMorgan's Theorem show that:

i) $\mathbf{MN + \overline{MO} + M\overline{N}O(MN + O) = 1}$

ii) $\mathbf{\overline{A}B\overline{C} + \overline{A}BC + AB\overline{C} + ABC = B}$

1. Simplify the following Boolean expressions:

   (a) $A + AB + A\overline{B}C$ (b) $(\overline{A} + B)C + ABC$ (c) $A\overline{B}C(BD + CDE) + A\overline{C}$

2. Implement each expression in Question 1 as originally stated with the appropriate logic gates. Then implement the simplified expression, and compare the number of gates.

References

1. Thomas L. Floyd, “Digital Fundamentals” 11th edition, Prentice Hall – Pearson Education.

Thank You