Introduction to Logic, Logic Gates and Boolean Algebra

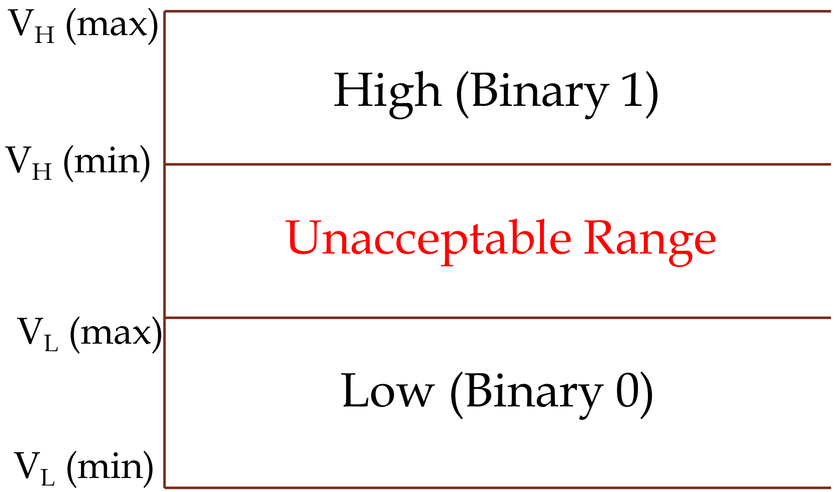
Topics to be covered

Logic Gates

- Logic Gates are the basic building blocks of any digital system. A logic gate can have one or more than one input but only one output. The relationship between the input/s and the output is based on a **certain logic**. The gates are named based on the logic.
- The names of the logic gates are:

- Basic Gates:
  - NOT Gate or Inverter
  - AND Gate
  - OR Gate
- Universal Gates:
  - NAND Gate
  - NOR Gate
- Exclusive Gates:
  - Exclusive-OR Gate
  - Exclusive-NOR

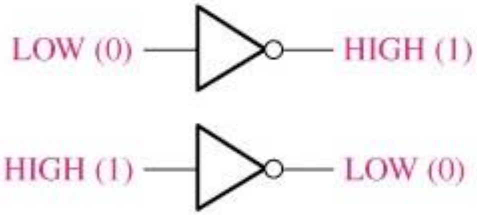
- **Bit:** in binary system we know that there are two digits 0 (low voltage) and 1(high voltage). The voltages used to represent a '1' or '0' are called logic levels



- Pulse is when clock frequency is applied to a circuit.
- Rising edge and falling edge.
- Periodic and non-periodic waveforms.
- Timing diagram

Inverter(NOT gate)

- Output of an inverter is opposite/complement of its input.



A	X
0	1
1	0

Truth table

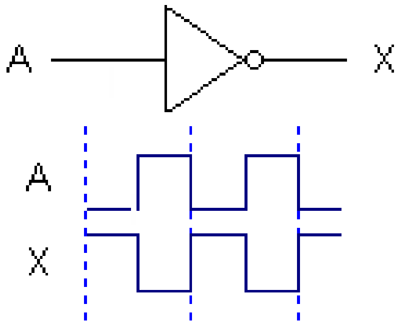
0 = LOW
1 = HIGH

$X = \overline{A}$

Boolean expression

When the input is LOW, the output is HIGH

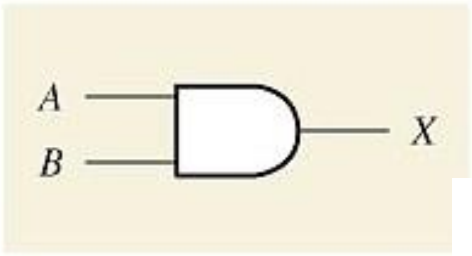
When the input is HIGH, the output is LOW



Pulsed waveforms

AND Gate

The output of an AND gate is HIGH only when all inputs are HIGH and LOW when any input is LOW.



Distinctive shape symbol

A	B	X
0	0	0
0	1	0
1	0	0
1	1	1

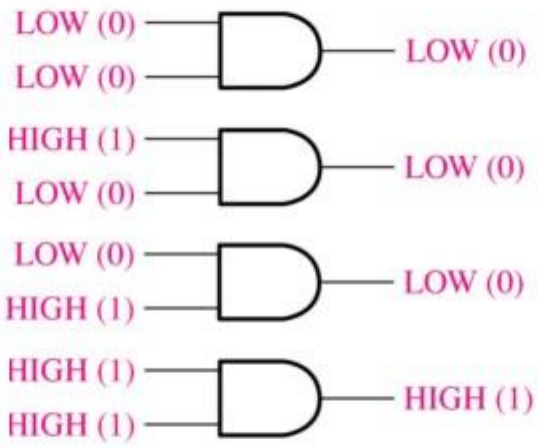
Truth table

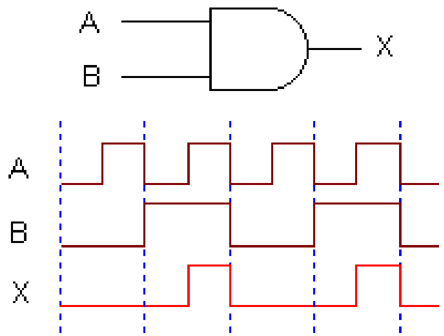
0 = LOW
1 = HIGH

$N = 2^n$

$X = AB$

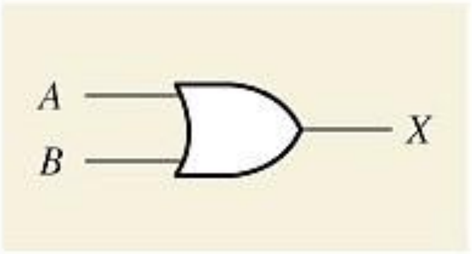
Boolean expression





Pulsed waveforms

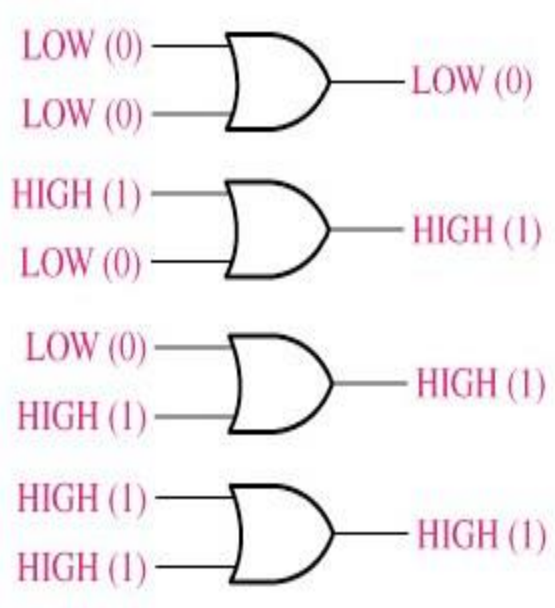
The OR Gate



Distinctive shape symbol

The output of an OR gate is HIGH whenever one or more inputs are HIGH and Low when all inputs are LOW

$N = 2^n$



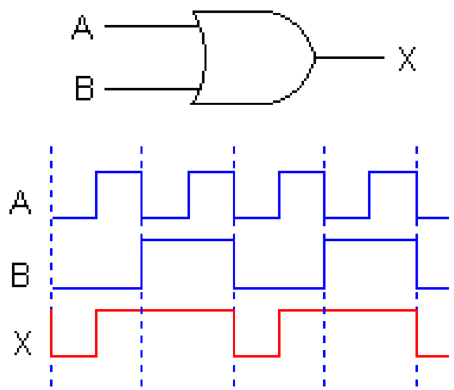
A	B	X
0	0	0
0	1	1
1	0	1
1	1	1

Truth table

$X = A + B$

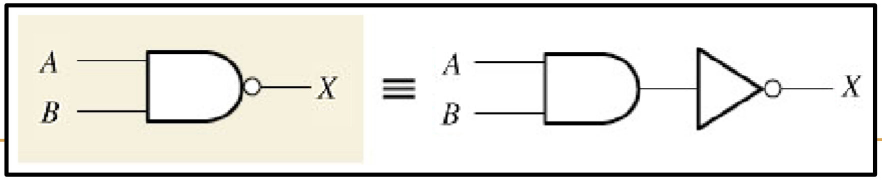
Boolean expression

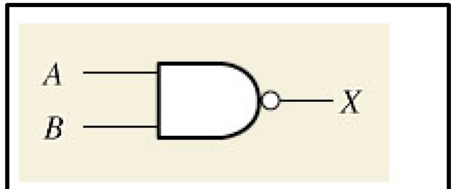
0 = LOW
1 = HIGH



Pulsed waveforms

The NAND Gate $= \overline{AND}$





Distinctive shape symbol

The output of a NAND gate is HIGH whenever one or more inputs are LOW.

A	B	X
0	0	1
0	1	1
1	0	1
1	1	0

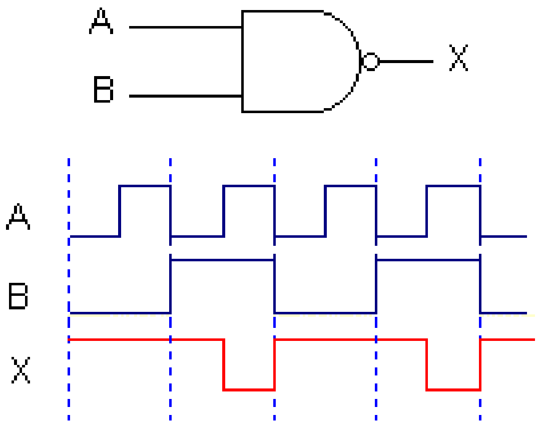
Truth table

0 = LOW
1 = HIGH

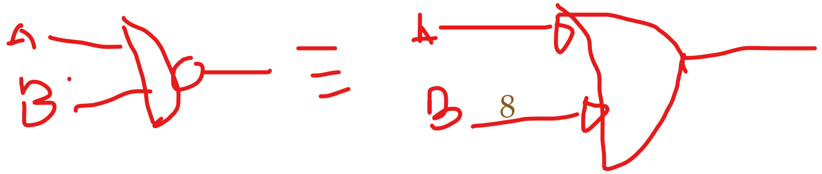
$$X = \overline{AB}$$

Boolean expression

$\boxed{\text{NAND} = \text{negative-OR}}$

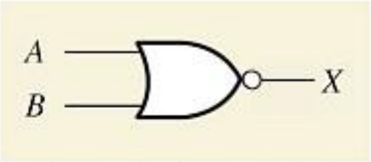


Pulsed waveforms

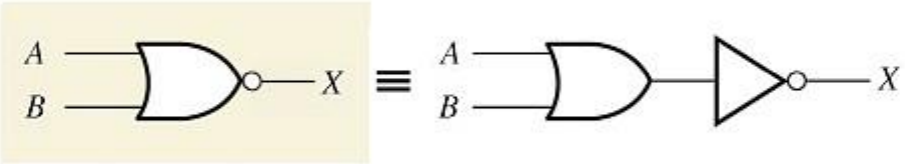


The NOR Gate

$\equiv \overline{OR}$



Distinctive shape symbol



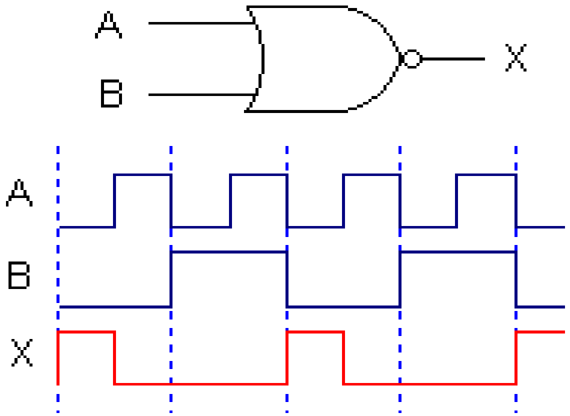
The output of a NOR gate is LOW whenever one or more inputs are HIGH.

A	B	X
0	0	1
0	1	0
1	0	0
1	1	0

Truth table

$$X = \overline{A + B}$$

Boolean expression

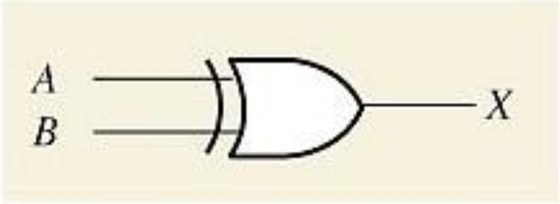


Pulsed waveforms

0 = LOW
1 = HIGH

NOR = Negative-AND

Exclusive-OR



Distinctive shape symbol

The output of an XOR gate is HIGH whenever the two inputs are different.

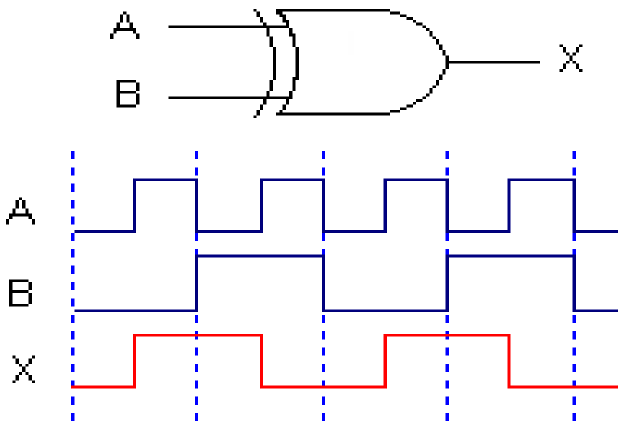
A	B	X
0	0	0
0	1	1
1	0	1
1	1	0

Truth table

0 = LOW
1 = HIGH

$$X = A \oplus B$$

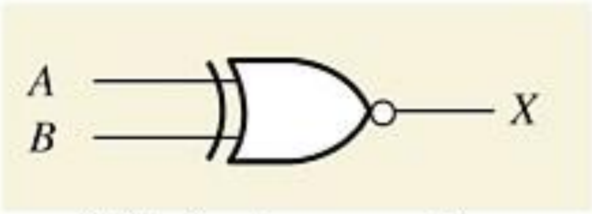
Boolean expression



Pulsed waveforms

The output of an XOR gate is HIGH when there are ODD number of 1's on the inputs to the gate

Exclusive-NOR Gate



Distinctive shape symbol

The output of an EX-NOR gate is HIGH whenever the two inputs are identical.

same

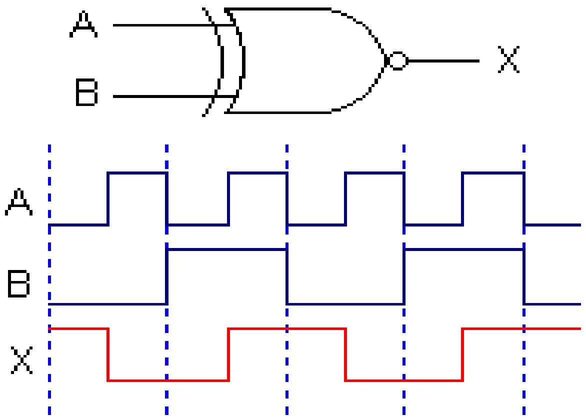
A	B	X
0	0	1
0	1	0
1	0	0
1	1	1

Truth table

0 = LOW
1 = HIGH

$$X = \overline{A \oplus B}$$

Boolean expression



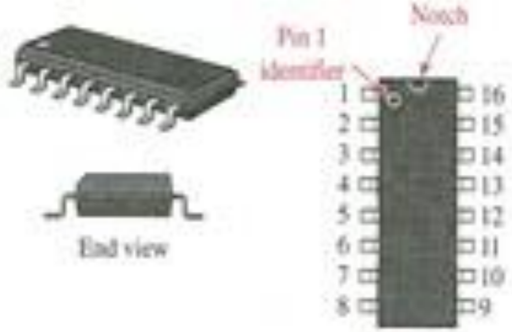
Pulsed waveforms

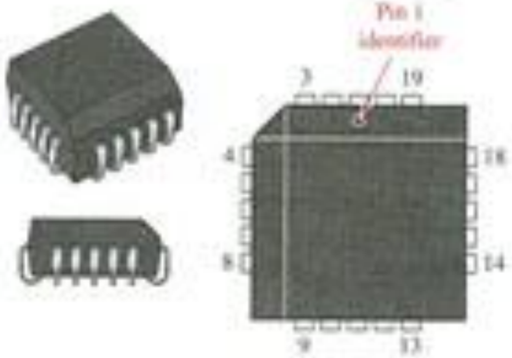
The output of an EX-NOR gate is HIGH when there are EVEN number of 1's on the inputs to the gate except when all its inputs are "LOW".

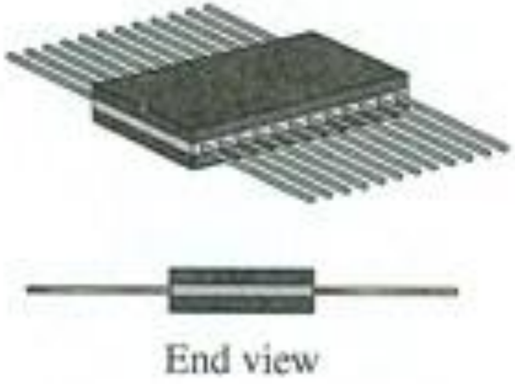
Fixed-Function Integrated Circuits

IC package styles

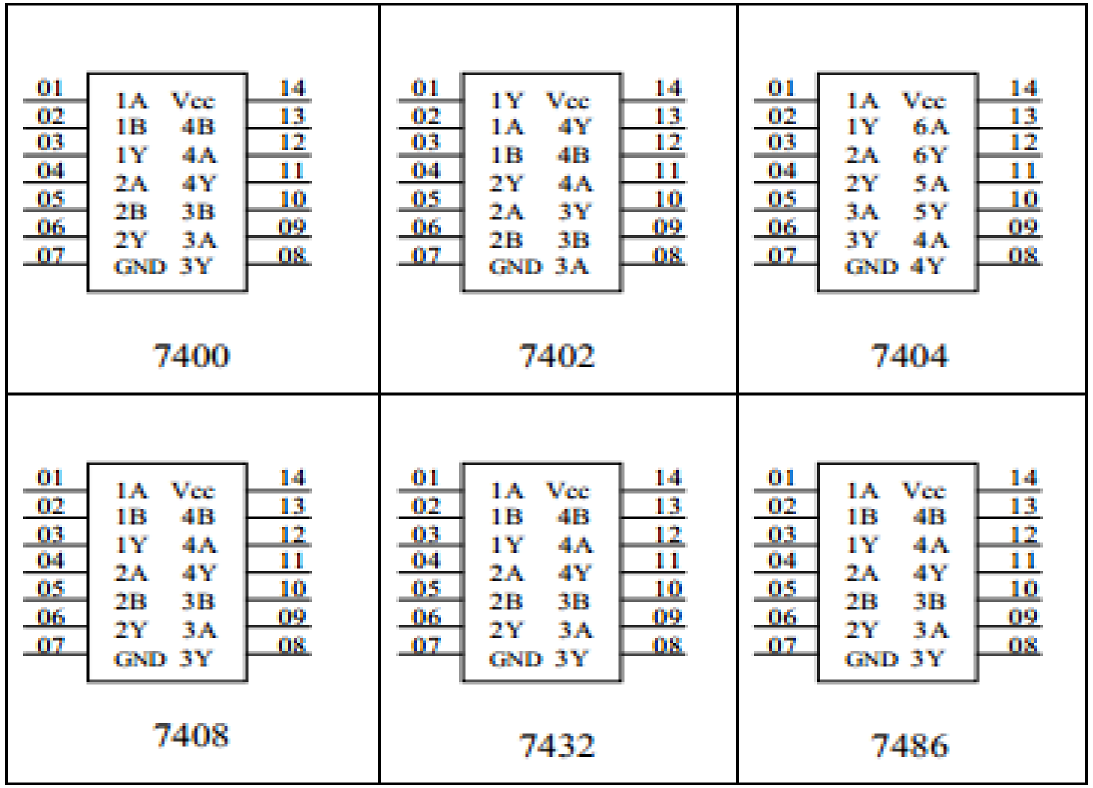
- Dual in-line package (DIP)
- Small-outline IC (SOIC)
- Flat pack (FP)
- Plastic-leaded chip carrier (PLCC)
- Leadless-ceramic chip carrier (LCCC)







IC configurations:



Integrated Circuits (ICs):

7400 :- Quad 2 I/p NAND.

7402 :- Quad 2 I/p NOR.

7404 :- Hex Inverter.

7408 :- Quad 2 I/p AND.

7432 :- Quad 2 I/p OR.

7486 :- Quad 2 I/p X-OR.

RULES OF BOOLEAN ALGEBRA

Basic rules of Boolean algebra.

1. $A + 0 = A$	**7.** $A \cdot A = A$
2. $A + 1 = 1$	**8.** $A \cdot \overline{A} = 0$
3. $A \cdot 0 = 0$	**9.** $\overline{\overline{A}} = A$
4. $A \cdot 1 = A$	**10.** $A + AB = A$
5. $A + A = A$	**11.** $A + \overline{A}B = A + B$
6. $A + \overline{A} = 1$	**12.** $(A + B)(A + C) = A + BC$

Rule 1. $A + 0 = A$

Rule 2. $A + 1 = 1$

Rule 3. $A \cdot 0 = 0$

Rule 4. $A \cdot 1 = A$

Rule 5. $A + A = A$

A	B	X
0	0	0
0	1	1
1	0	1
1	1	1

OR Truth Table

A	B	X
0	0	0
0	1	0
1	0	0
1	1	1

AND Truth Table

1. $A + 0 = A$
2. $A + 1 = 1$
3. $A \cdot 0 = 0$
4. $A \cdot 1 = A$
5. $A + A = A$
6. $A + \overline{A} = 1$
7. $A \cdot A = A$
8. $A \cdot \overline{A} = 0$
9. $\overline{\overline{A}} = A$
10. $A + AB = A$
11. $A + \overline{A}B = A + B$
12. $(A + B)(A + C) = A + BC$

Rule 6. $A + \bar{A} = 1$

Rule 7. $A \cdot A = A$

Rule 8. $A \cdot \bar{A} = 0$

1. $A + 0 = A$
2. $A + 1 = 1$
3. $A \cdot 0 = 0$
4. $A \cdot 1 = A$
5. $A + A = A$
6. $A + \bar{A} = 1$
7. $A \cdot A = A$
8. $A \cdot \bar{A} = 0$
9. $\bar{\bar{A}} = A$
10. $A + AB = A$
11. $A + \bar{A}B = A + B$
12. $(A + B)(A + C) = A + BC$

Rule 9. $\bar{\bar{A}} = A$

Rule 10. $A + AB = A$

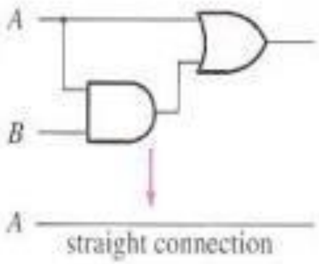
$A + AB = A(1 + B)$ Factoring (distributive law)

$= A \cdot 1$ Rule 2: $(1 + B) = 1$

$= A$ Rule 4: $A \cdot 1 = A$

A	B	AB	A + AB
0	0	0	0
0	1	0	0
1	0	0	1
1	1	1	1

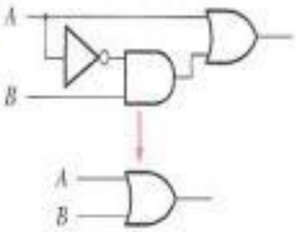
equal



Rule 11. $A + \bar{A}B = A + B$

$A + \bar{A}B$	$= (A + AB) + \bar{A}B$	Rule 10: $A = A + AB$
	$= (AA + AB) + \bar{A}B$	Rule 7: $A = AA$
	$= AA + AB + A\bar{A} + \bar{A}B$	Rule 8: adding $A\bar{A} = 0$
	$= (A + \bar{A})(A + B)$	Factoring
	$= 1 \cdot (A + B)$	Rule 6: $A + \bar{A} = 1$
	$= A + B$	Rule 4: drop the 1

A	B	$\bar{A}B$	$A + \bar{A}B$	$A + B$
0	0	0	0	0
0	1	1	1	1
1	0	0	1	1
1	1	0	1	1

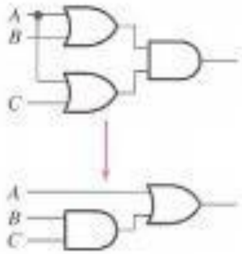


Rule 12. $(A + B)(A + C) = A + BC$

$(A + B)(A + C)$	$= AA + AC + AB + BC$	Distributive law
	$= A + AC + AB + BC$	Rule 7: $AA = A$
	$= A(1 + C) + AB + BC$	Factoring (distributive law)
	$= A \cdot 1 + AB + BC$	Rule 2: $1 + C = 1$
	$= A(1 + B) + BC$	Factoring (distributive law)
	$= A \cdot 1 + BC$	Rule 2: $1 + B = 1$
	$= A + BC$	Rule 4: $A \cdot 1 = A$

1. $A + 0 = A$
2. $A + 1 = 1$
3. $A \cdot 0 = 0$
4. $A \cdot 1 = A$
5. $A + A = A$
6. $A + \bar{A} = 1$
7. $A \cdot A = A$
8. $A \cdot \bar{A} = 0$
9. $\bar{\bar{A}} = A$
10. $A + AB = A$
11. $A + \bar{A}B = A + B$
12. $(A + B)(A + C) = A + BC$

A	B	C	$A + B$	$A + C$	$(A + B)(A + C)$	BC	$A + BC$
0	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0
0	1	0	1	0	0	0	0
0	1	1	1	1	1	1	1
1	0	0	1	1	1	0	1
1	0	1	1	1	1	0	1
1	1	0	1	1	1	0	1
1	1	1	1	1	1	1	1



Simplification using boolean algebra

Simplification means fewer gates
for the same function

EXAMPLE

$AB + A(B + C) + B(B + C)$ ▪ $B + AC$

$AB + A(B + C) + B(B + C)$

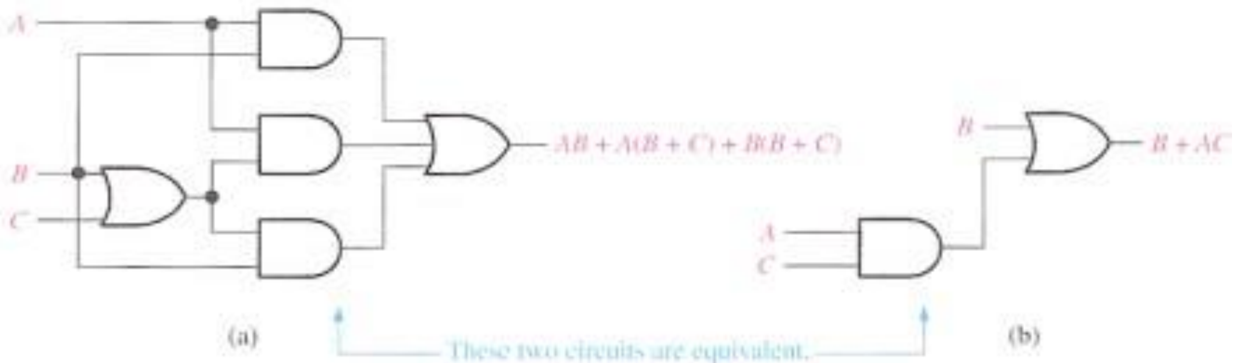
$AB + AB + AC + BB + BC$

$AB + AB + AC + B + BC$

$AB + AC + B + BC$

$AB + AC + B$

$B + AC$



EXAMPLE

$[A\overline{B}(C + BD) + \overline{A}\,\overline{B}]C$ $\qquad \overline{B}C$

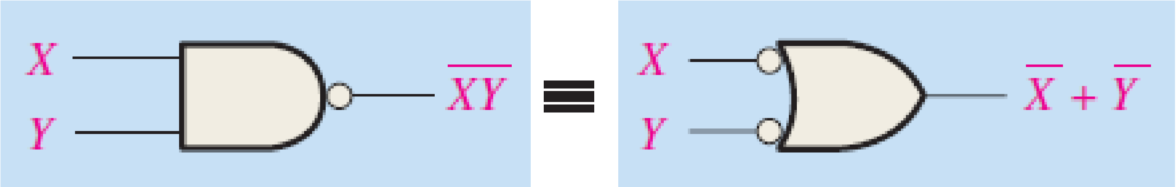
$\overline{A}BC + A\overline{B}\,\overline{C} + \overline{A}\,\overline{B}\,\overline{C} + A\overline{B}C + ABC$ $\qquad BC + A\overline{B} + \overline{B}\,\overline{C}$

▪

DEMORGAN'S THEOREMS

- The first theorem is stated as follows:
  The complement of a product of variables is equal to the sum of the complements of complements of the variable.
  The formula of this theorem for two variables is written as

$$\overline{XY} = \overline{X} + \overline{Y}$$



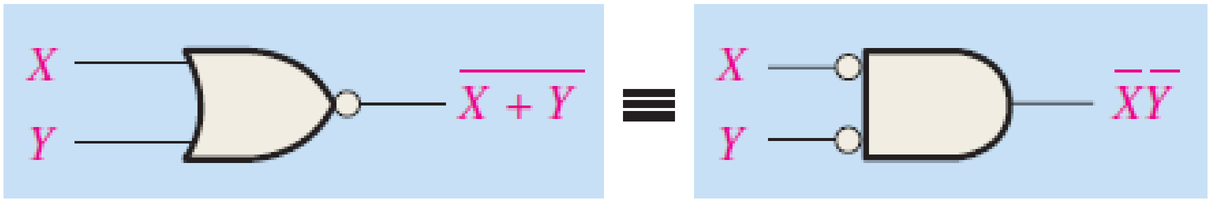
NAND   Negative-OR

Inputs		Output	
X	Y	$\overline{XY}$	$\overline{X} + \overline{Y}$
0	0	1	1
0	1	1	1
1	0	1	1
1	1	0	0

DEMORGAN'S THEOREMS

- The second theorem is stated as follows:
  The complement of a sum of variables is equal to the product of the complements of the variables.
  The formula of this theorem for two variables is written as

$$\overline{X + Y} = \overline{X}\,\overline{Y}$$



NOR Negative-AND

Inputs		Output	
X	Y	$\overline{X + Y}$	$\overline{X}\,\overline{Y}$
0	0	1	1
0	1	0	0
1	0	0	0
1	1	0	0

APPLICATION OF DEMORGAN'S THEOREM

Apply DeMorgan's theorems to the expressions $\overline{XYZ}$ and $\overline{X + Y + Z}$.

Solution

$$\overline{XYZ} = \overline{X} + \overline{Y} + \overline{Z}$$

$$\overline{X + Y + Z} = \overline{X}\,\overline{Y}\,\overline{Z}$$

Apply DeMorgan's theorems to the expressions $\overline{WXYZ}$ and $\overline{W + X + Y + Z}$.

Solution

$$\overline{WXYZ} = \overline{W} + \overline{X} + \overline{Y} + \overline{Z}$$

$$\overline{W + X + Y + Z} = \overline{W}\,\overline{X}\,\overline{Y}\,\overline{Z}$$

Apply DeMorgan's theorems to each expression:

(a) $\overline{\overline{(A + B)} + \overline{C}}$

(b) $\overline{(\overline{A} + B) + CD}$

(c) $\overline{(A + B)\overline{C}\,\overline{D} + E + \overline{F}}$

Solution

(a) $\overline{\overline{(A + B)} + \overline{C}} = \overline{\overline{(A + B)}}\,\overline{\overline{C}} = (A + B)C$

(b) $\overline{(\overline{A} + B) + CD} = \overline{(\overline{A} + B)}\,\overline{CD} = (\overline{\overline{A}}\,\overline{B})(\overline{C} + \overline{D}) = A\overline{B}(\overline{C} + \overline{D})$

(c) $\overline{(A + B)\overline{C}\,\overline{D} + E + \overline{F}} = \overline{((A + B)\overline{C}\,\overline{D})}\,\overline{(E + \overline{F})} = (\overline{A}\,\overline{B} + C + D)\overline{E}F$

APPLICATION OF DEMORGAN'S THEOREM

Apply DeMorgan's theorem to the expression $\overline{\overline{X} + \overline{Y} + \overline{Z}}$.

Apply DeMorgan's theorem to the expression $\overline{\overline{W}\,\overline{X}\,\overline{Y}\,\overline{Z}}$.

Apply DeMorgan's theorems to each of the following expressions:

(a) $\overline{(A + B + C)D}$

(b) $\overline{ABC + DEF}$

(c) $\overline{A\overline{B} + \overline{C}D + EF}$

The Boolean expression for an exclusive-OR gate is $A\overline{B} + \overline{A}B$. With this as a starting point, use DeMorgan's theorems and any other rules or laws that are applicable to develop an expression for the exclusive-NOR gate.

Starting with the expression for a 4-input NAND gate, use DeMorgan's theorems to develop an expression for a 4-input negative-OR gate.

Apply DeMorgan's theorems to the following expressions:

(a) $\overline{ABC} + (\overline{\overline{D} + E})$ **(b)** $\overline{(A + B)C}$ **(c)** $\overline{A + B + C} + \overline{\overline{D}E}$

Textbooks:

- [1] Thomas L. Floyd, “Digital Fundamentals” 11th edition, Prentice Hall.
- [2] M. Morris Mano, “Digital Logic & Computer Design” Prentice Hall.