# Karnaugh Map

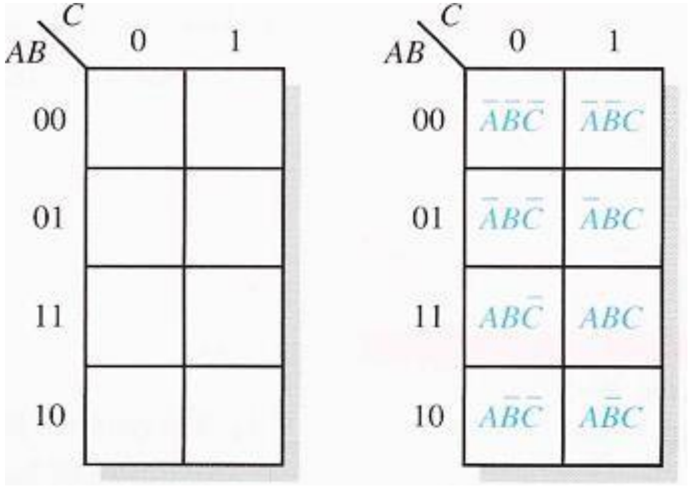
# The Karnaugh Map

A Karnaugh map provides a systematic method for simplifying Boolean expressions and, if properly used, will produce the simplest SOP or POS expression possible, known as the minimum expression. As you have seen, the effectiveness of algebraic simplification depends on your familiarity with all the laws, rules, and theorems of Boolean algebra and on your ability to apply them. The Karnaugh map, on the other hand, provides a "cookbook" method for simplification.

**A Kamaugh map is similar to a truth table**

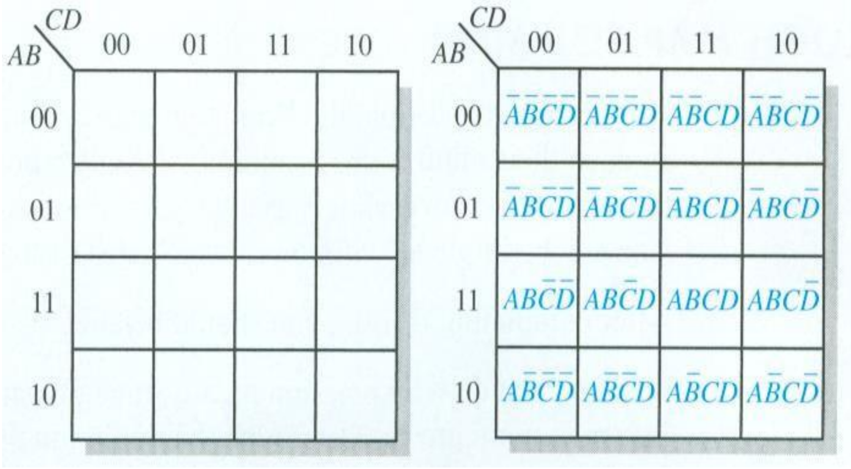
## The 3-Variable Karnaugh Map

The 3-variable Karnaugh map is an array of eight cells



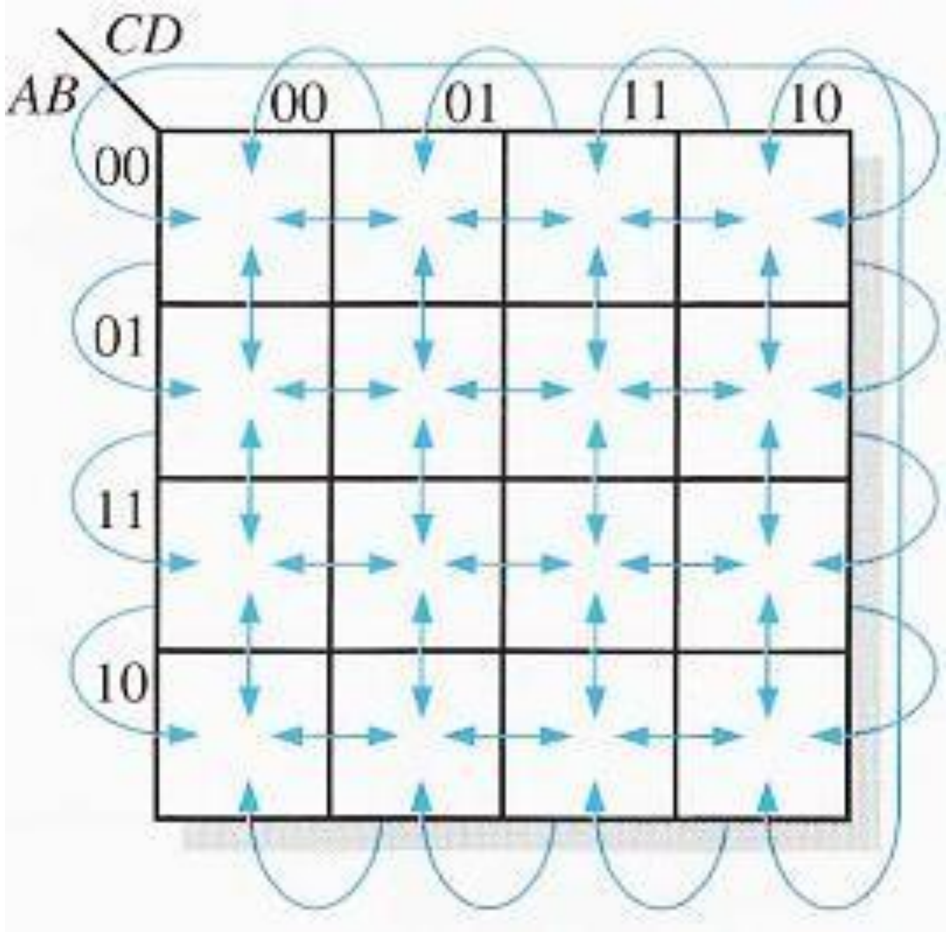

## The 4-Variable Karnaugh Map

The 4-variable Karnaugh map is an array of sixteen cells



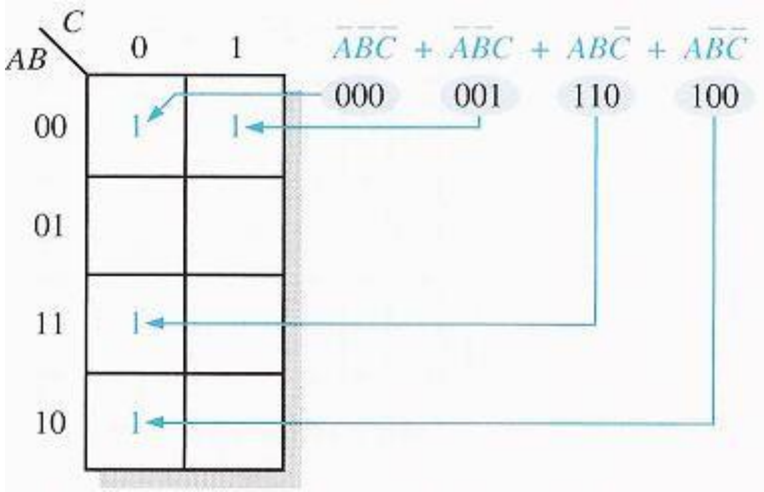
# Cell Adjacency

Cells that differ by only one variable are adjacent.



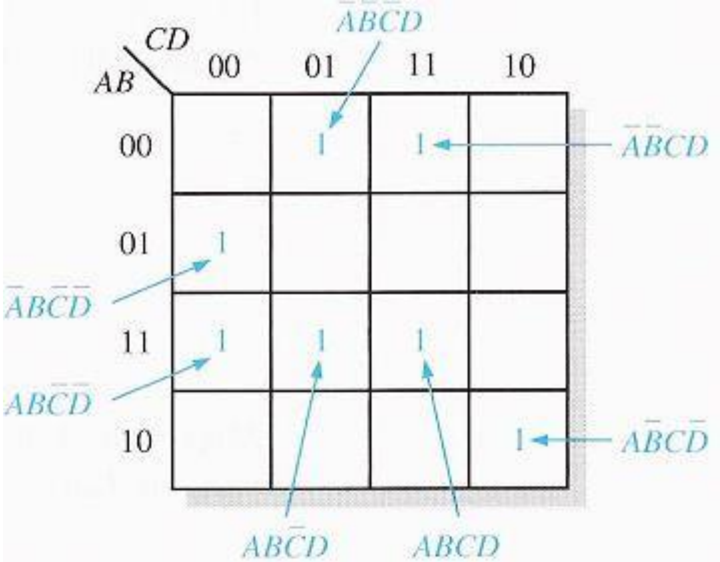
# KARNAUGH MAP SOP MINIMIZATION

## Mapping a Standard SOP Expression




$$\bar{A}\bar{B}CD + \bar{A}B\bar{C}\bar{D} + AB\bar{C}D + ABCD + AB\bar{C}\bar{D} + \bar{A}\bar{B}\bar{C}D + A\bar{B}C\bar{D}$$

0011 0100 1101 1111 1100 0001 1010



## Mapping a Nonstandard SOP Expression

A Boolean expression must first be in standard form before you use a Karnaugh map.

*Numerical Expansion of a Nonstandard Product Term*

**EXAMPLE**

Map the following SOP expression on a Karnaugh map: $\overline{A} + A\overline{B} + AB\overline{C}$.

| $\overline{A}$ | $+\ A\overline{B}$ | $+\ AB\overline{C}$ |
|---|---|---|
| 000 | 100 | 110 |
| 001 | 101 | |
| 010 | | |
| 011 | | |

| AB \ C | 0 | 1 |
|---|---|---|
| 00 | 1 | 1 |
| 01 | 1 | 1 |
| 11 | 1 | |
| 10 | 1 | 1 |

$\overline{B}\,\overline{C} + A\overline{B} + AB\overline{C} + A\overline{B}C\overline{D} + \overline{A}\,\overline{B}\,\overline{C}D + A\overline{B}CD$

| $\overline{B}\,\overline{C}$ | $A\overline{B}$ | $+\ AB\overline{C}$ | $+\ A\overline{B}C\overline{D}$ | $+\ \overline{A}\,\overline{B}\,\overline{C}D$ | $+\ A\overline{B}CD$ |
|---|---|---|---|---|---|
| 0000 | 1000 | 1100 | 1010 | 0001 | 1011 |
| 0001 | 1001 | 1101 | | | |
| 1000 | 1010 | | | | |
| 1001 | 1011 | | | | |

| AB \ CD | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 1 | 1 | | |
| 01 | | | | |
| 11 | 1 | 1 | | |
| 10 | 1 | 1 | 1 | 1 |

**Karnaugh Map Simplification of SOP Expressions**

a minimum SOP expression is obtained by grouping the 1s

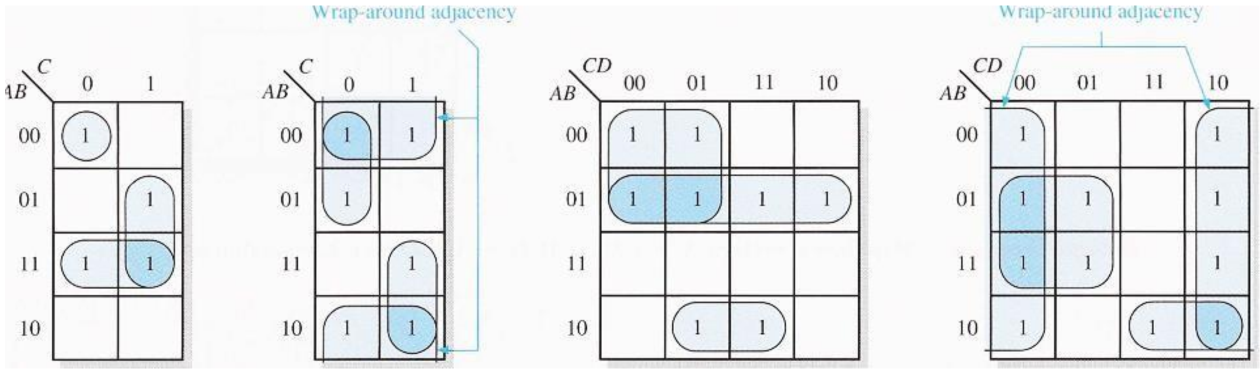
**Grouping the 1s:** You can group 1s on the Karnaugh map according to the following rules by enclosing those adjacent cells containing Is. The goal is to maximize the size of the groups and to minimize the number of groups.

1. A group must contain either 1, 2, 4, 8, or 16 cells, which are all powers of two. In the case of a 3-variable map, 2 3 = 8 cells is the maximum group.

2.Each cell in a group must be adjacent to one or more cells in that same group. but all cells in the group do not have to be adjacent to each other.

3. Always include the largest possible number of I s in a group in accordance with rule 1.

4.Each 1 on the map must be included in at least one group. The Is already in a group can be included in another group as long as the overlapping groups include Non-common 1s.



**Determining the Minimum SOP Expression from the Map:** When all the Is representing the standard product terms in an expression are properly mapped and grouped, the process of determining the resulting minimum SOP expression begins. The following rules are applied to find the minimum product terms and the minimum SOP expression:

1.Group the cells that have 1 s. Each group of cells containing I s creates one product term composed of all variables that occur in only one form (either un-complemented or complemented) within the group. Variables that occur both un-complemented and complemented within the group are eliminated. These are called contradictory variables.

2. Determine the minimum product term for each group.

**a. For a 3-variable map:**

(1) A 1-ceIl group yields a 3-variable product term

(2) A 2-cell group yields a 2-variable product term

(3) A 4-cell group yields a I-variable term

(4) An 8-cell group yields a value of I for the expression

**b. For a 4-variable map:**

(1) A 1-cell group yields a 4-variable product term

(2) A 2-cell group yields a 3-variable product term

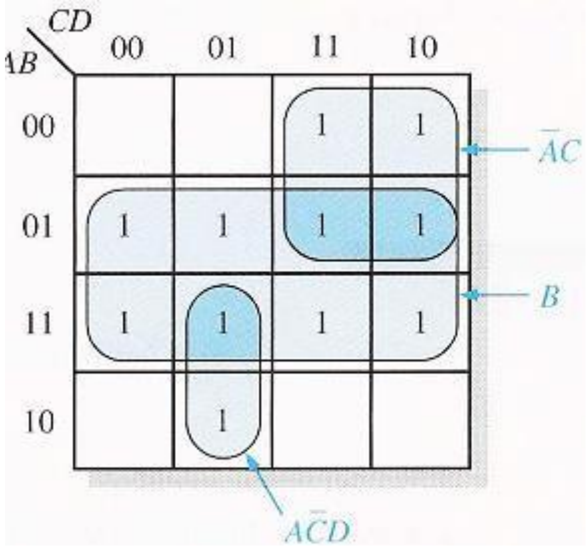
(3) A 4-cell group yields a 2-variable product term

(4) An 8-cell group yields a I-variable term

(5) A 16-cell group yields a value of I for the expression

3.When all the minimum product terms are derived from the Karnaugh map, they are summed to form the minimum SOP expression.

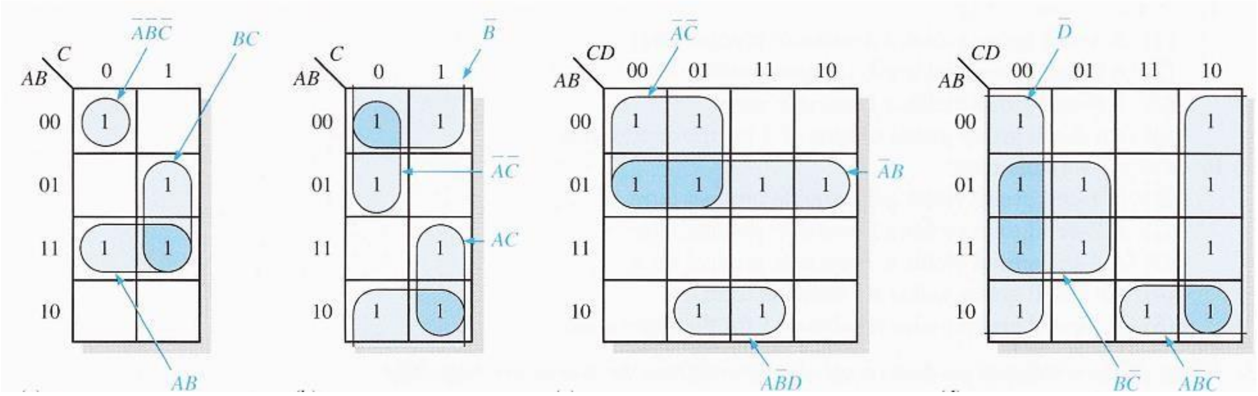
EXAMPLE




the sum of these product terms:

$$B + \bar{A}C + A\bar{C}D$$

EXAMPLE

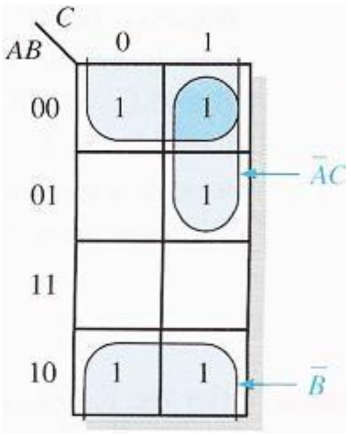


## EXAMPLE

Use a Karnaugh map to minimize the following standard SOP expression

$$A\overline{B}C + \overline{A}BC + \overline{A}\,\overline{B}C + \overline{A}\,\overline{B}\,\overline{C} + A\overline{B}\,\overline{C}$$

The binary values of the expression are



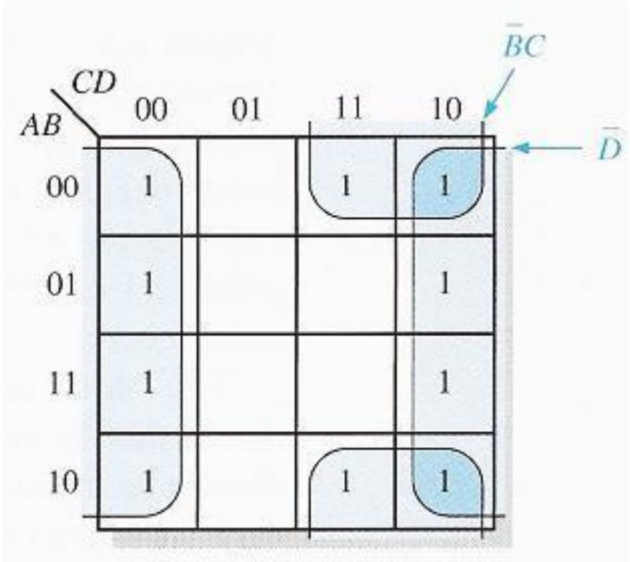

$$101 + 011 + 011 + 000 + 100$$

The resulting minimum SOP expression is

$$\overline{B} + \overline{A}C$$

## EXAMPLE

$$\overline{B}\,\overline{C}\,\overline{D} + \overline{A}B\overline{C}\,\overline{D} + AB\overline{C}\,\overline{D} + \overline{A}\,\overline{B}CD + A\overline{B}CD + \overline{A}\,\overline{B}C\overline{D} + \overline{A}BC\overline{D} + ABC\overline{D} + A\overline{B}C\overline{D}$$



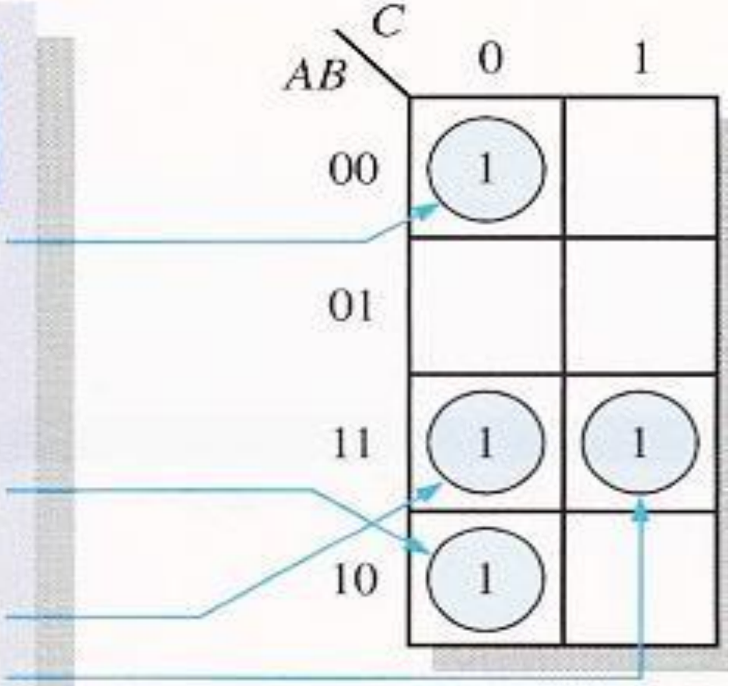

The resulting minimum SOP expression is

$$\overline{D} + \overline{B}C$$

# Mapping Directly from a Truth Table

$X = \overline{A}\,\overline{B}\,\overline{C} + A\overline{B}\,\overline{C} + AB\overline{C} + ABC$

| Inputs A | B | C | Output X |
|---|---|---|---|
| 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |

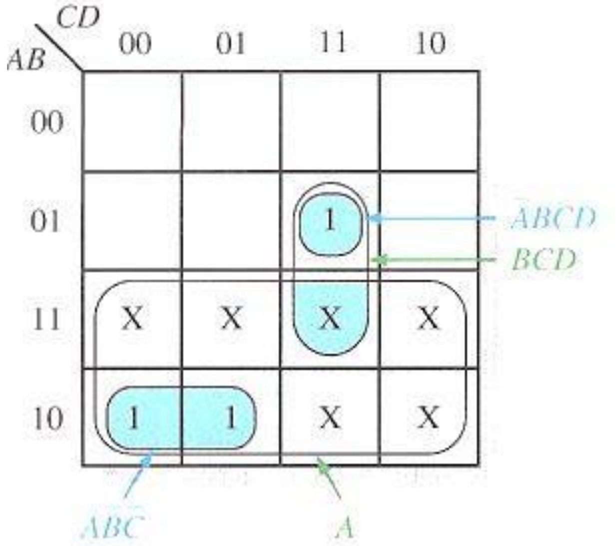


# "Don't Care" Conditions

Sometimes a situation arises in which some input variable combinations are not allowed. For example, in BCD code there are six invalid combinations: 1010, 1011, 1100, 1101, 1110, and 1111. Since these un-allowed states will never occur in an application involving the BCD code, they can be treated as "don't care" terms with respect to their effect on the output. That is, for these "don't care" terms either a 1 or a 0 may be assigned to the output: it really does not matter since they will never occur.

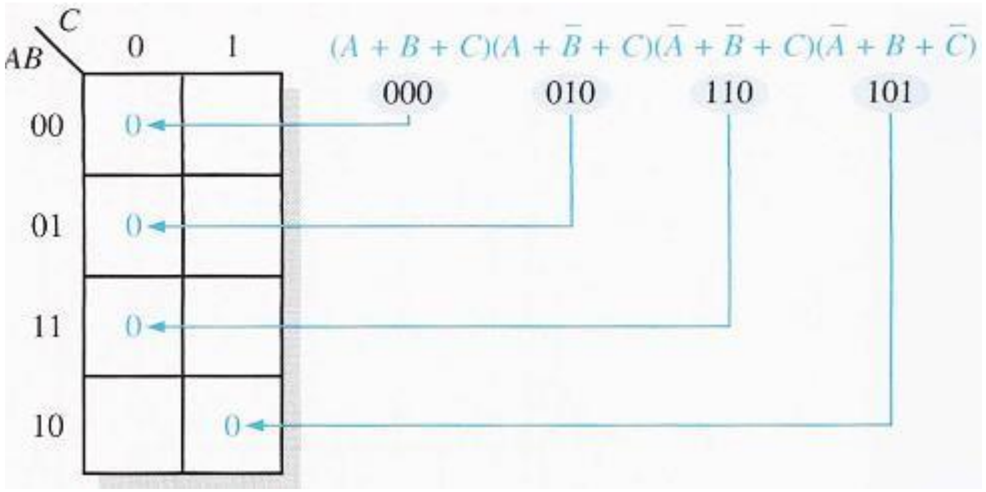
The "don't care" terms can be used to advantage on the Karnaugh map. Figure below shows that for each "don't care" term, an X is placed in the cell. When grouping the 1s, the Xs can be treated as 1s to make a larger grouping or as 0s if they cannot be used to advantage. The larger a group, the simpler the resulting term will be.

| Inputs ABCD | Output Y |
|---|---|
| 0 0 0 0 | 0 |
| 0 0 0 1 | 0 |
| 0 0 1 0 | 0 |
| 0 0 1 1 | 0 |
| 0 1 0 0 | 0 |
| 0 1 0 1 | 0 |
| 0 1 1 0 | 0 |
| 0 1 1 1 | 1 |
| 1 0 0 0 | 1 |
| 1 0 0 1 | 1 |
| 1 0 1 0 | X |
| 1 0 1 1 | X |
| 1 1 0 0 | X |
| 1 1 0 1 | X |
| 1 1 1 0 | X |
| 1 1 1 1 | X |




(b) Without “don’t cares” $Y = A\bar{B}\bar{C} + \bar{A}BCD$
With “don’t cares” $Y = A + BCD$

# KARNAUGH MAP POS MINIMIZATION




## Karnaugh Map Simplification of POS Expressions

The process for minimizing a POS expression is basically the same as for an SOP

**EXAMPLE**

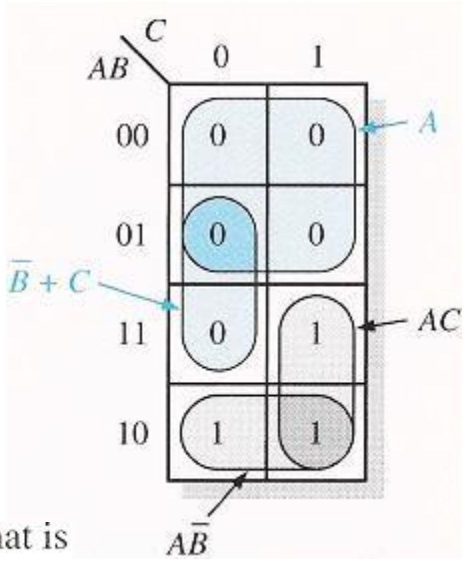
Use a Karnaugh map to minimize the following standard POS expression:

$$(A + B + C)(A + B + \overline{C})(A + \overline{B} + C)(A + \overline{B} + \overline{C})(\overline{A} + \overline{B} + C)$$

Also, derive the equivalent SOP expression.

*Solution* The combinations of binary values of the expression are

$$(0 + 0 + 0)\ (0 + 0 + 1)\ (0 + 1 + 0)\ (0 + 1 + 1)\ (1 + 1 + 0)$$




resulting minimum POS expression is

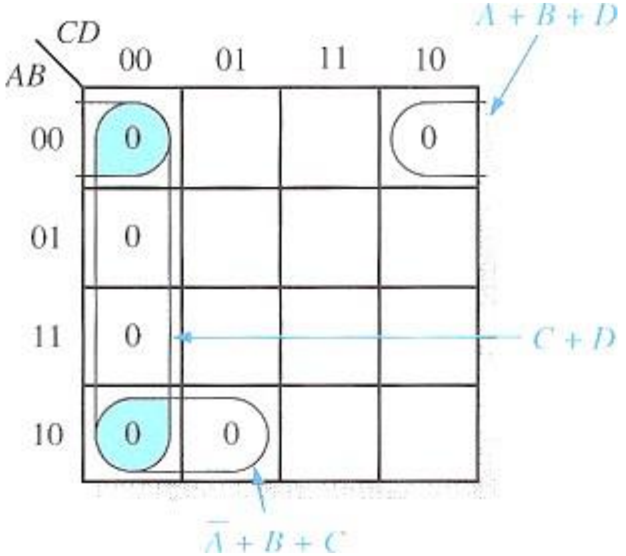
$$A(\overline{B} + C)$$

Grouping the 1s as shown by the gray areas yields an SOP expression that is equivalent to grouping the 0s.

$$AC + A\overline{B} = A(\overline{B} + C)$$

## EXAMPLE

$$(B + C + D)(A + B + \overline{C} + D)(\overline{A} + B + C + \overline{D})(A + \overline{B} + C + D)(\overline{A} + \overline{B} + C + D)$$



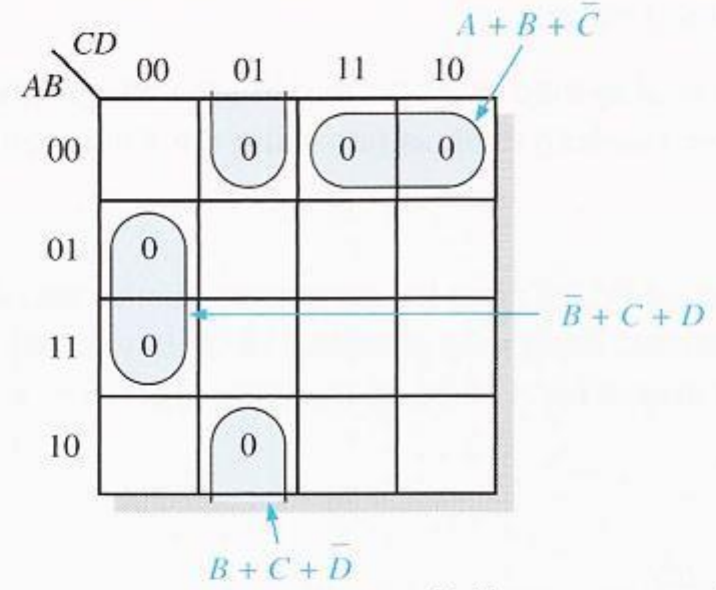

$$(C + D)\,(A + B + D)\,(\overline{A} + B + C)$$

## Converting Between POS and SOP Using the Karnaugh Map

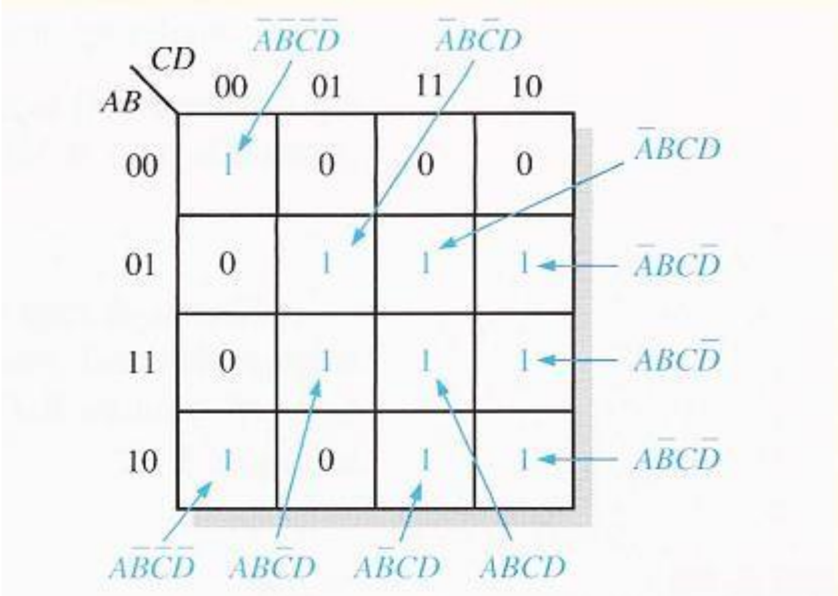
### EXAMPLE

Using a Karnaugh map, convert the following standard POS expression into a minimum POS expression, a standard SOP expression, and a minimum SOP expression.

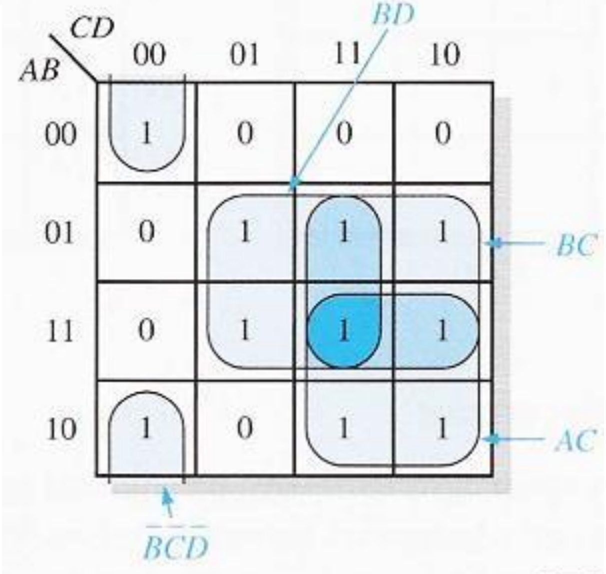
$$(\overline{A} + \overline{B} + C + D)(A + \overline{B} + C + D)(A + B + C + \overline{D})$$
$$(A + B + \overline{C} + \overline{D})(\overline{A} + B + C + \overline{D})(A + B + \overline{C} + D)$$




(a) Minimum POS: $(A + B + C)(\overline{B} + C + D)(B + C + \overline{D})$




(b) Standard SOP:
$\overline{A}\overline{B}\overline{C}\overline{D} + \overline{A}B\overline{C}D + \overline{A}BCD + \overline{A}BC\overline{D} + ABC\overline{D} + A\overline{B}C\overline{D} +$
$A\overline{B}\overline{C}\overline{D} + AB\overline{C}D + A\overline{B}CD + ABCD$




(c) Minimum SOP: $AC + BC + BD + \overline{B}\overline{C}\overline{D}$

Determine the simplifies POS for the following expression

F(A,B,C,D)= ε(308,1,3,5)

# Textbooks:


- [1] Thomas L. Floyd, “Digital Fundamentals” 11th edition, Prentice Hall.
- [2] M. Morris Mano, “Digital Logic & Computer Design” Prentice Hall.