Lecture -2
Sum-of-Products, Product-of-Sum

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Standardization

- All Boolean expression, regardless of their form, can be converted into either of the two standard forms.
- The two standard forms are: Sum-of-Products (SOP) and Product-of-Sum (POS).
- Standardization makes evaluation, simplification and implementation of Boolean expression much more systematic and easier.

Domain of a Boolean Expression

- The domain of a general Boolean expression is the set of variable contained in the Boolean expression in either complemented or uncomplemented form.
- For example, the domain of the expression $A\overline{B} + \overline{A}C$ is the set of variable A,B and C.
- And for example, the domain of the expression $A\overline{B}C + \overline{C}E + \overline{A}\overline{D}$ is the set of variables A,B,C,D and E.

Sum-of-Product (SOP)

- Sum of Products: A Product is defined as a term consisting of products of the literals. When two or more products are summed in a Boolean expression, it is called the Sum-of-Products (SOP).

$$\mathbf{A + \bar{A}B + B\bar{C}}$$

- Standard SOP: It is an expression where all the variables are present in each product terms. The products in a SSOP are called min terms ($\mathbf{m_i}$).

$$\mathbf{AB\bar{C} + \bar{A}BC + A\bar{B}C}$$

- Conversion of SOP to Standard SOP
  - Step1: Multiply each of the non-standard terms with a term made up of the sum of the missing variable and its complement. This do not change the function as we are just multiplying by 1.
  - Step2: Repeat step 1 until all the non-standard terms become standard terms.
- **Sum-of-Products is used to describe when the function is 1.**

Product-of-Sum (POS)

- Product of Sum: A Sum is defined as a term consisting of sum of the literals. When two or more sum terms are multiplied in a Boolean Expression, it is called the Product-of-Sum (POS).

$$(\mathbf{B}+\overline{\mathbf{C}})(\mathbf{A}+\overline{\mathbf{B}})$$

- Standard POS: It is an expression where all the variables are present in each sum terms. Each sum terms are called max terms ($\mathbf{M_i}$) .

$$(\overline{\mathbf{A}}+\mathbf{B}+\mathbf{C})(\mathbf{A}+\overline{\mathbf{B}}+\overline{\mathbf{C}})$$

- Conversion of POS to Standard POS
  - Step1: Add to each non-standard product terms a term made up of the product of the missing variable and its complement. This does not change the expression as we are just adding a 0.
  - Step2: Apply rule 12: $\mathbf{A}+\mathbf{BC}=(\mathbf{A}+\mathbf{B})(\mathbf{A}+\mathbf{C})$
  - Step3: Repeat step 1 until all the sum terms contain all the variable in the domain.
- **Product-of-Sum is used to describe when the function is 0.**

SOP & POS

Convert the following Boolean expression into standard SOP form:

$$A\overline{B}C + \overline{A}\,\overline{B} + AB\overline{C}D$$

Convert the expression $W\overline{X}Y + \overline{X}Y\overline{Z} + WX\overline{Y}$ to standard SOP form.

Convert the following Boolean expression into standard POS form:

$$(A + \overline{B} + C)(\overline{B} + C + \overline{D})(A + \overline{B} + \overline{C} + D)$$

Convert the expression $(A + \overline{B})(B + C)$ to standard POS form.

Min Terms and Max Terms

- Each variable in a Boolean expression is a literal.
- Boolean variable can appear in normal (A) or complemented ($\bar{A}$) form.
- Each product of all variables in the domain is called Min-Term.
- Each sum of all variables in the domain is called Max-Term.

A	B	C	Min-Terms	
0	0	0	$\bar{A}\bar{B}\bar{C}$	m_0
0	0	1	$\bar{A}\bar{B}C$	m_1
0	1	0	$\bar{A}B\bar{C}$	m_2
0	1	1	$\bar{A}BC$	m_3
1	0	0	$A\bar{B}\bar{C}$	m_4
1	0	1	$A\bar{B}C$	m_5
1	1	0	$AB\bar{C}$	m_6
1	1	1	ABC	m_7

A	B	C	Max-Terms	
0	0	0	$A + B + C$	M_0
0	0	1	$A + B + \bar{C}$	M_1
0	1	0	$A + \bar{B} + C$	M_2
0	1	1	$A + \bar{B} + \bar{C}$	M_3
1	0	0	$\bar{A} + B + C$	M_4
1	0	1	$\bar{A} + B + \bar{C}$	M_5
1	1	0	$\bar{A} + \bar{B} + C$	M_6
1	1	1	$\bar{A} + \bar{B} + \bar{C}$	M_7

For Min-Terms:
When 0→ Complemented Form
When 1→ Normal Form

For Max-Terms:
When 1→ Complemented Form
When 0→ Normal Form

Boolean Expression(SOP) to Truth-Table

- Truth-table can be formed for any Boolean expression.
- Converting a Boolean Expression to SSOP can make this task a lot easier.
- Find the truth-table for the following Boolean expression:

$$\mathbf{F(A,B,C) = AB + \bar{B}\bar{C}}$$

$$\mathbf{F(A,B,C) = AB(C+\bar{C}) + (A+\bar{A})\bar{B}\bar{C}}$$

$$\mathbf{F(A,B,C) = \underline{ABC + AB\bar{C} + A\bar{B}\bar{C} + \bar{A}\bar{B}\bar{C}}}$$

A	B	C	Min-Terms
0	0	0	$\bar{A}\bar{B}\bar{C}$
0	0	1	$\bar{A}\bar{B}C$
0	1	0	$\bar{A}B\bar{C}$
0	1	1	$\bar{A}BC$
1	0	0	$A\bar{B}\bar{C}$
1	0	1	$A\bar{B}C$
1	1	0	$AB\bar{C}$
1	1	1	ABC

A	B	C	F
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

Truth-Table to Function Implementation

- Find and implement the function from the following truth-table.

A	B	C	F
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

→

A	B	C	F
0	0	0	$\bar{A}\bar{B}\bar{C}$
0	0	1	
0	1	0	$\bar{A}B\bar{C}$
0	1	1	
1	0	0	
1	0	1	
1	1	0	$AB\bar{C}$
1	1	1	ABC

$$\mathbf{F(A,B,C) = \bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + AB\bar{C} + ABC}$$
$$\mathbf{F(A,B,C) = \bar{A}\bar{C} + AB}$$

- For the function $F = \sum(2,3,5,7)$:
  a) Construct the truth table.
  b) Implement the function.

Boolean Expression(POS) to Truth-Table

- Form a truth-table for the following Boolean expression.

$$\mathbf{F(A,B,C) = (A+B)\cdot(\overline{B}+\overline{C})}$$

- In the first step, we will convert the POS to SPOS:

$$\mathbf{F(A,B,C) = (A+B+C\overline{C})\cdot(\overline{B}+\overline{C}+A\overline{A})}$$

$$\mathbf{F(A,B,C) = (A+B+C)\cdot(A+B+\overline{C})\cdot(\overline{B}+\overline{C}+A)\cdot(\overline{B}+\overline{C}+\overline{A})}$$

- Once we have converted the expression to SPOS, now we can directly form the truth-table.

A	B	C	Max-Terms
0	0	0	$A+B+C$
0	0	1	$A+B+\overline{C}$
0	1	0	$A+\overline{B}+C$
0	1	1	$A+\overline{B}+\overline{C}$
1	0	0	$\overline{A}+B+C$
1	0	1	$\overline{A}+B+\overline{C}$
1	1	0	$\overline{A}+\overline{B}+C$
1	1	1	$\overline{A}+\overline{B}+\overline{C}$

A	B	C	Max-Terms
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

Function Implementation using POS

- For the function $F = \prod(1,2,5,6)$:
  a) Construct the truth table.
  b) Implement the function.

Solution:

Step 1

A	B	C	Max-Terms
0	0	0	
0	0	1	0
0	1	0	0
0	1	1	
1	0	0	
1	0	1	0
1	1	0	0
1	1	1	

Step 2

A	B	C	Max-Terms
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

Step 3 $\mathbf{F(A,B,C) = \bar{A}\bar{B}\bar{C} + \bar{A}BC + A\bar{B}\bar{C} + ABC}$

- Now that we have the Boolean expression, we can simplify it and implement it.

Connecting the Dots between SOP and POS

- From the truth table determine the standard SOP expression and the equivalent POS expression.

A	B	C	F
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

Standard SOP expression:

- Write down the sum of min terms of the combinations for which the function is 1.

$$\mathbf{F = \overline{A}BC + A\overline{B}\overline{C} + A\overline{B}C + AB\overline{C} + ABC}$$

Equivalent POS expression:

- Write down the product of max terms of the combinations for which the function is 0.

$$\mathbf{F = (A + B + C) \cdot (A + B + \overline{C}) \cdot (A + \overline{B} + C)}$$

HOW DOES THIS EVEN WORK!!!!!!!!

Connecting the Dots between SOP and POS

A	B	C	F
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

A	B	C	F	$\overline{F}$
0	0	0	0	1
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	1	0
1	0	1	1	0
1	1	0	1	0
1	1	1	1	0

$$\mathbf{F = \overline{A}BC + A\overline{B}\overline{C} + A\overline{B}C + AB\overline{C} + ABC}$$

$$\mathbf{\overline{F} = \overline{A}\overline{B}\overline{C} + \overline{A}\overline{B}C + \overline{A}B\overline{C}}$$

- Now we have the SOP expression of the complement of the function:

$$\mathbf{\overline{F} = \overline{A}\overline{B}\overline{C} + \overline{A}\overline{B}C + \overline{A}B\overline{C}}$$

- This means we have converted the 0s of the function 1s and vice versa.
- What if we turn the 0s back to 1, that is we convert $\overline{\mathrm{F}}$ to F

Connecting the Dots between SOP and POS

$$\bar{F} = \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}B\bar{C}$$

- Now to convert the 0s back to 1s we complement $\bar{F}$ that is $\bar{\bar{F}}$.

$$\bar{\bar{F}} = \overline{\bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}B\bar{C}}$$

$$F = \overline{\bar{A}\bar{B}\bar{C}} \cdot \overline{\bar{A}\bar{B}C} \cdot \overline{\bar{A}B\bar{C}}$$

$$F = (A + B + C) \cdot (A + B + \bar{C}) \cdot (A + \bar{B} + C)$$

- So we have reached the same expression of POS as we did earlier.
- SOP is the positive logic definition of the function.
- POS is the negative logic definition of the function.

What is positive logic and what is negative logic????

References

1. Thomas L. Floyd, “Digital Fundamentals” 11^{th} edition, Prentice Hall – Pearson Education.

Thank You