Lecture -4
Karnaugh Map

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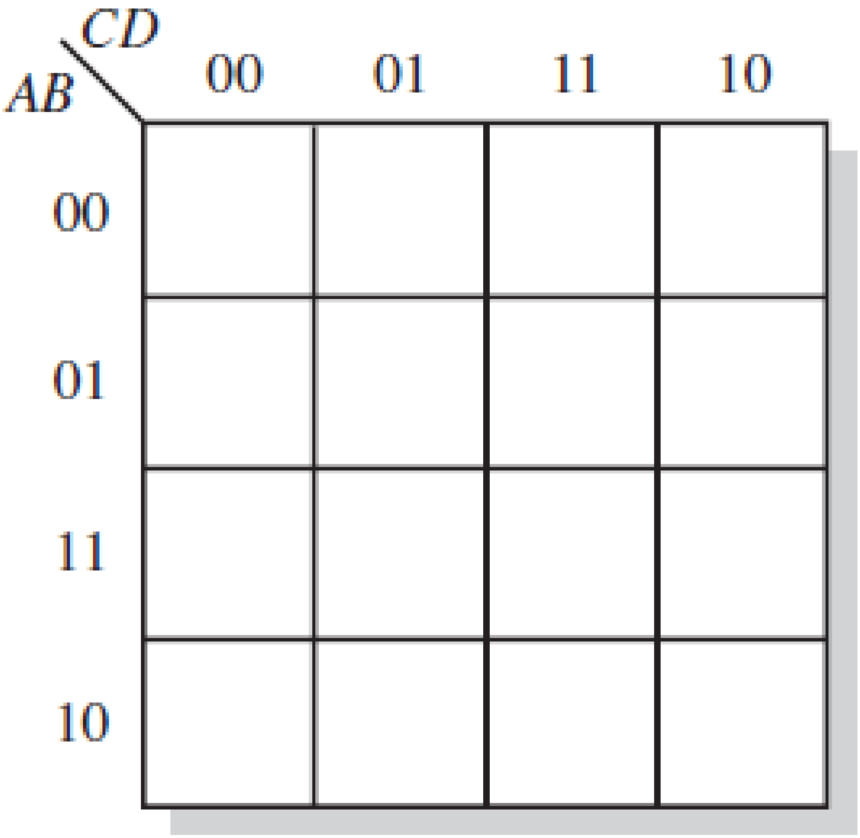
The Karnaugh Map (K-Map)

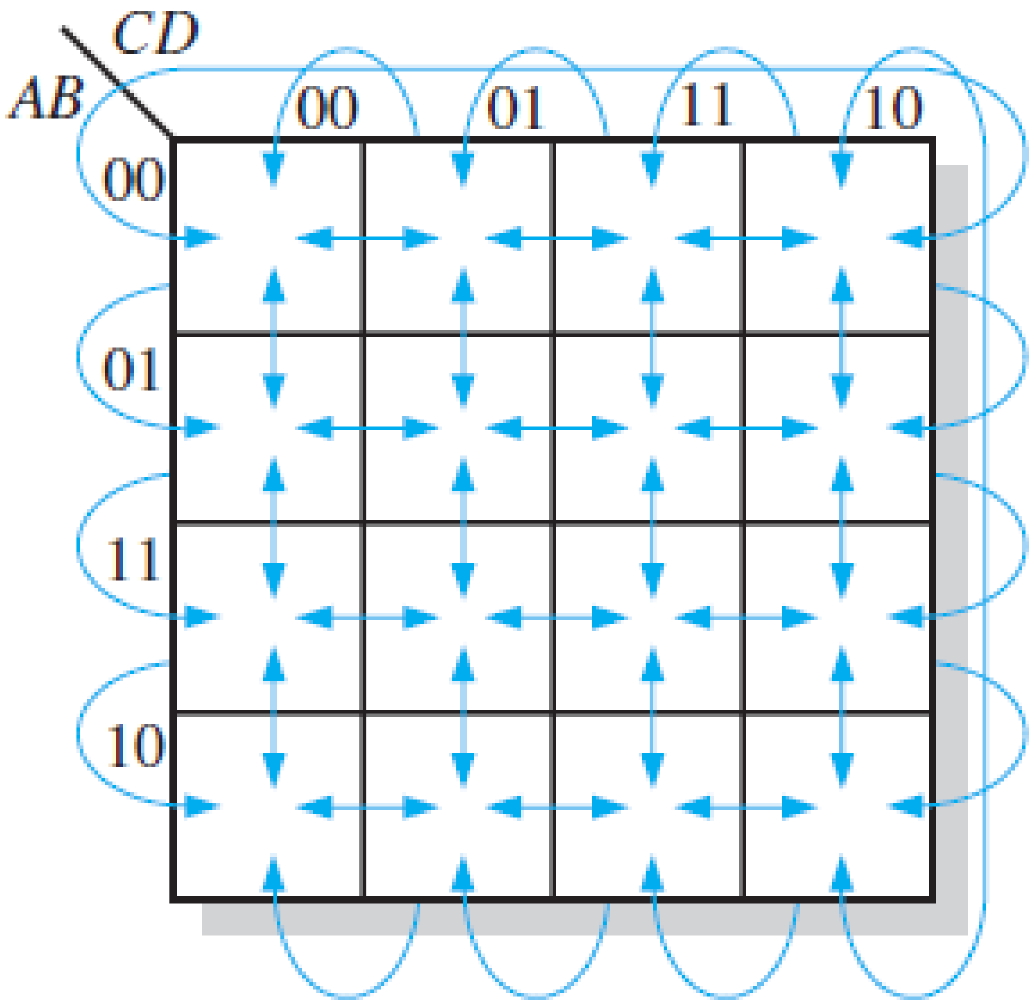
- A Karnaugh map is similar to a truth table because it presents all of the possible values of input variables and the resulting output for each value.
- The main purpose of K-Map is to simplify a Boolean expression.
- A Karnaugh map provides a systematic method for simplifying Boolean expressions and, if properly used, will produce the simplest SOP or POS expression possible, known as the minimum expression.
- The effectiveness of algebraic simplification depends on your familiarity with all the laws, rules, and theorems of Boolean algebra and on your ability to apply them.
- .The Karnaugh map, on the other hand, provides a "cookbook" method for simplification.



A 4-variable K-Map

Cell Adjacency

- The cells in a Karnaugh map are arranged so that there is only a single-variable change between adjacent cells.
- Adjacency is defined by a single-variable change.
- Physically, each cell is adjacent to the cells that are immediately next to it on any of the four sides.
- A cell is not adjacent to the cells that diagonally touch any of its corners.
- The cells in the top row are adjacent to the cells in the bottom row.
- The cells in the left column is adjacent to the cells in the right column.
- This is called "wrap-around" adjacency.



Truth Table to K-Map

- From the following truth table form a K-MAP

A	B	C	F
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	0

AB \ C	0	1
00	0	0
01	1	1
11	0	0
10	1	1

Truth Table to K-Map

- From the following truth table form a K-MAP

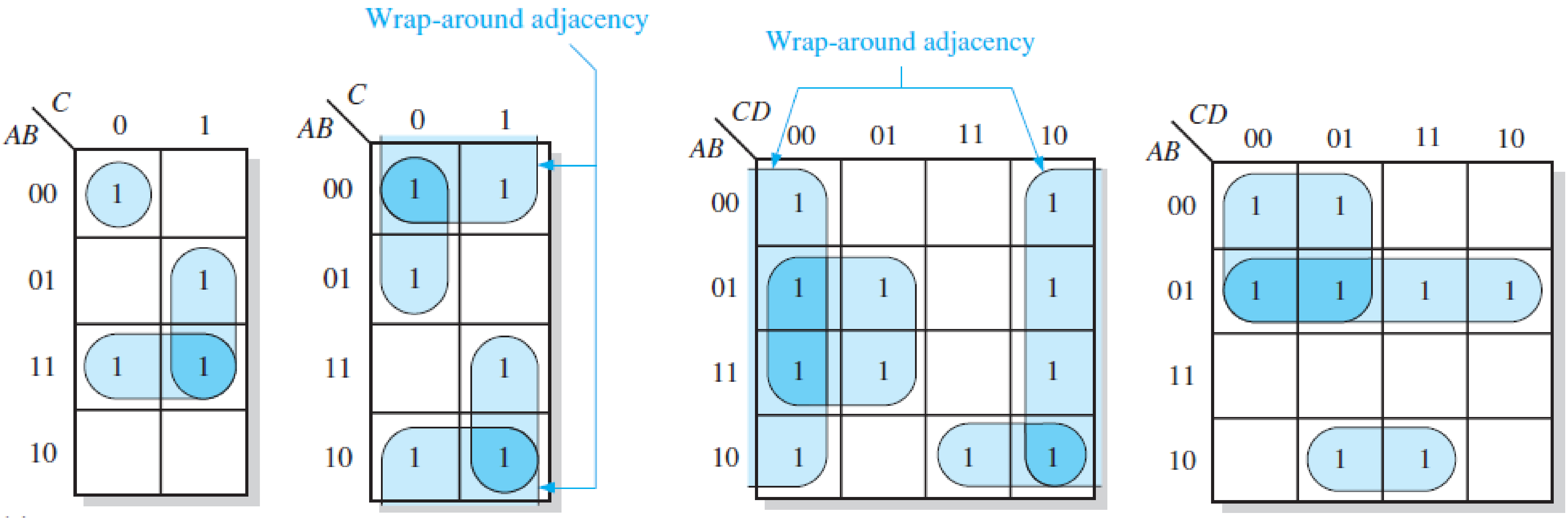
A	B	C	D	F
0	0	0	0	0
0	0	0	1	0
0	0	1	0	1
0	0	1	1	1
0	1	0	0	1
0	1	0	1	1
0	1	1	0	0
0	1	1	1	0
1	0	0	0	1
1	0	0	1	1
1	0	1	0	0
1	0	1	1	0
1	1	0	0	0
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

AB \ CD	00	01	11	10
00	0	0	1	1
01	1	1	0	0
11	0	1	1	1
10	1	1	0	0

Forming the Groups

- To find the minimal SOP/ simplified SOP expression we must first group the 1s that are adjacent.
- To group the 1s we should follow these rules:
  1. A group can contain number of cells that is only a power of 2 (2^n), i.e. 1,2,4,8 and 16.
  2. Each cell in a group must be adjacent to some other cell in the group, but not all cells need to be adjacent.
  3. Priority is to find the group containing maximum number of adjacent cells and to find the minimum no. of groups possible keeping in accordance to rule 1and 2.
  4. Each 1 in the map must be included in at least one group keeping accordance to the rules above.
  5. The 1s included in a group can be included in another group if the overlapping group includes ungrouped 1s keeping accordance to rules 1 to 4.

Forming the Groups



Forming the Groups

- Find the optimal groups for the following K-MAPs

AB \ CD	00	01	11	10
00	1	0	0	1
01	1	1	0	1
11	1	1	0	1
10	1	1	1	1

AB \ CD	00	01	11	10
00	1	0	0	1
01	1	1	0	1
11	1	1	1	1
10	1	1	1	1

AB \ CD	00	01	11	10
00	1	1	1	1
01	0	1	1	0
11	0	1	1	0
10	1	1	1	1

AB \ CD	00	01	11	10
00	1	0	0	1
01	0	1	1	0
11	0	1	1	0
10	1	0	0	1

AB \ CD	00	01	11	10
00	1	1	0	0
01	1	1	1	1
11	0	0	0	0
10	1	1	0	0

AB \ CD	00	01	11	10
00	1	0	1	0
01	0	1	0	1
11	1	0	1	0
10	0	1	0	1

1 - group

Finding the minimal SOP Form

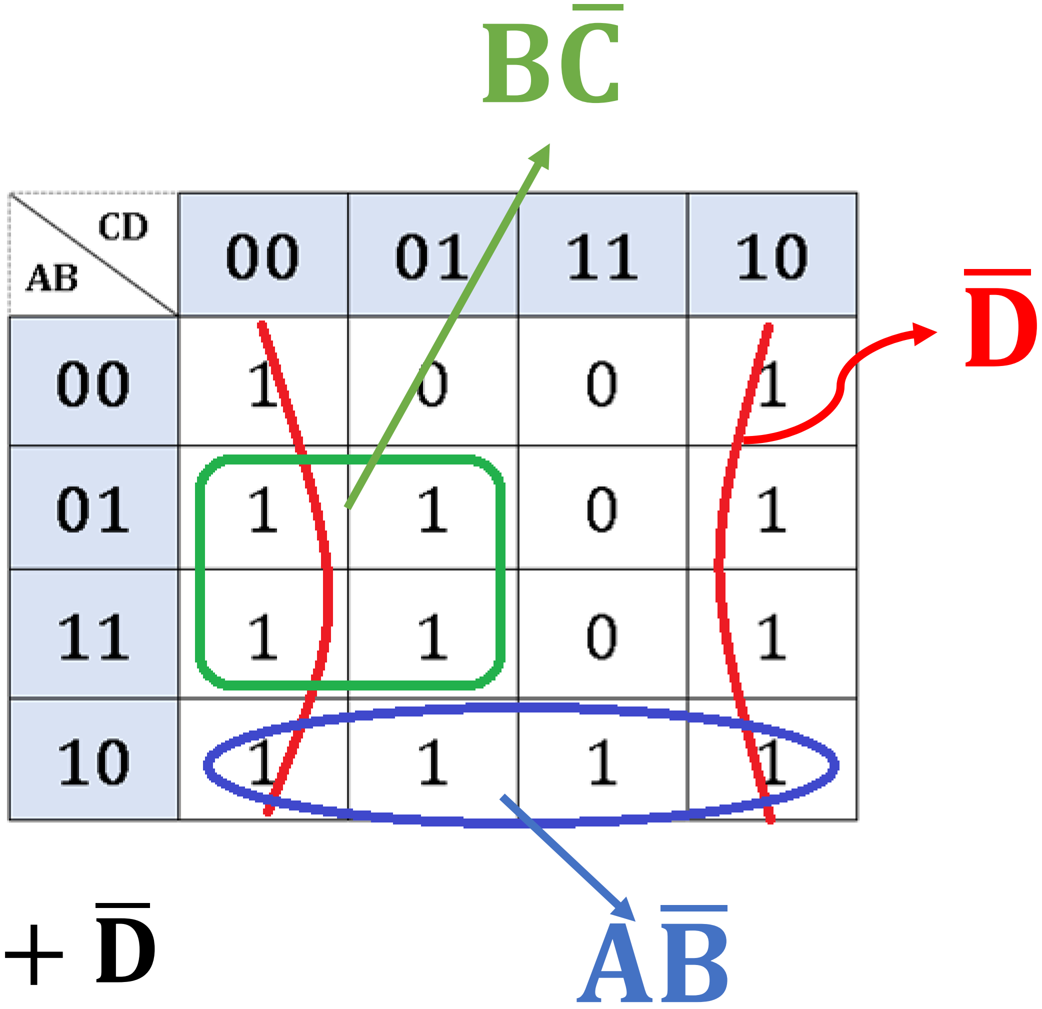
- After the 1s in the K-Maps are grouped, the process of minimizing the expression begins.
- For each group containing the 1s, there are variables which occur only in 1 form (complemented or uncomplemented form) form a product.
- The variables that occur in both complemented and uncomplemented forms are called contradictory variables.
- These variables are eliminated.
- For a 3 variable K-Map:
    - A 1-cell group yields a 3-variable product form.
    - A 2-cell group yields a 2-variable product form.
    - A 4-cell group yields a 1-variable product form.
    - An 8-cell group yields a value of 1 for the expression.
- For a 4 variable K-Map:
    - A 1-cell group yields a 4-variable product form.
    - A 2-cell group yields a 3-variable product form.
    - A 4-cell group yields a 2-variable product form.
    - An 8-cell group yields a 1-variable product form.
    - A 16-cell group yields a value of 1 for the expression.

Finding the minimal SOP Form

Example 1:

- First, we do the grouping.
- Then we find the products for each of the groups

AB \ CD	00	01	11	10
00	1	0	0	1
01	1	1	0	1
11	1	1	0	1
10	1	1	1	1



$$F = A\bar{B} + B\bar{C} + \bar{D}$$

Finding the minimal SOP Form

- Find the functions for the following K-MAPs

AB \ CD	00	01	11	10
00	1	0	0	1
01	1	1	0	1
11	1	1	0	1
10	1	1	1	1

AB \ CD	00	01	11	10
00	1	0	0	1
01	1	1	0	1
11	1	1	1	1
10	1	1	1	1

AB \ CD	00	01	11	10
00	1	1	1	1
01	0	1	1	0
11	0	1	1	0
10	1	1	1	1

AB \ CD	00	01	11	10
00	1	0	0	1
01	0	1	1	0
11	0	1	1	0
10	1	0	0	1

AB \ CD	00	01	11	10
00	1	1	0	0
01	1	1	1	1
11	0	0	0	0
10	1	1	0	0

AB \ CD	00	01	11	10
00	1	0	1	0
01	0	1	0	1
11	1	0	1	0
10	0	1	0	1

K-Map with Don’t Care

- Sometimes situation arises where we do not require all the input combinations, or they are simply not allowed.
- These are some combination that will never occur.
- These combinations can be treated as don’t care.
- A don’t care combination can be treated as 1 or 0, as per our simplification requirements.

A	B	C	D	F
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	X
1	0	1	1	X
1	1	0	0	X
1	1	0	1	X
1	1	1	0	X
1	1	1	1	X

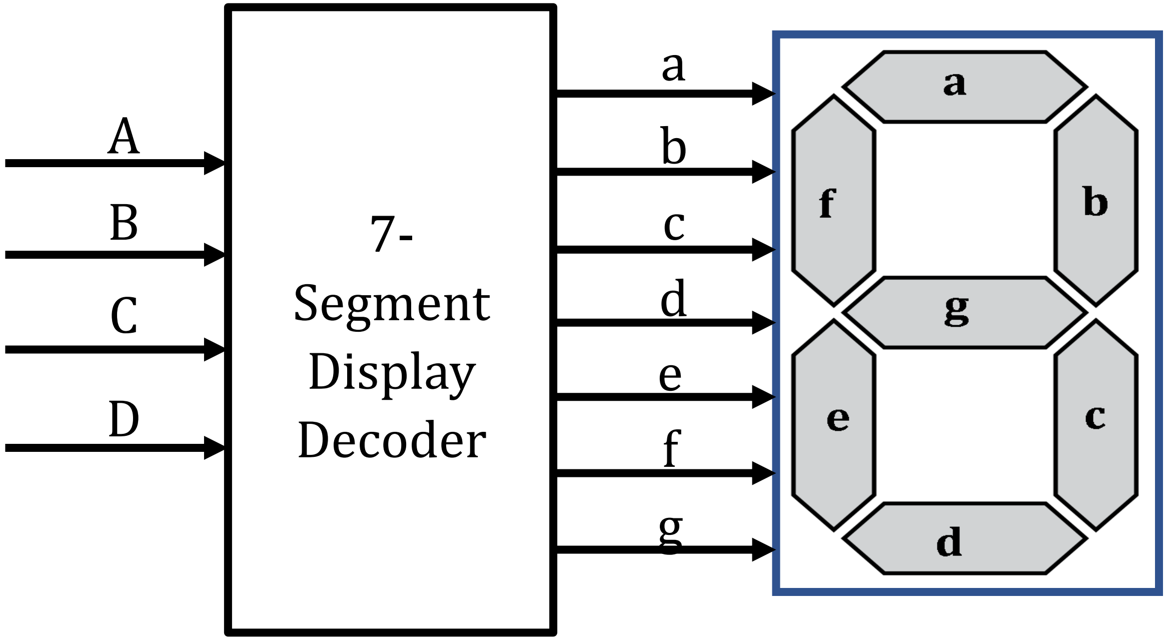
Required

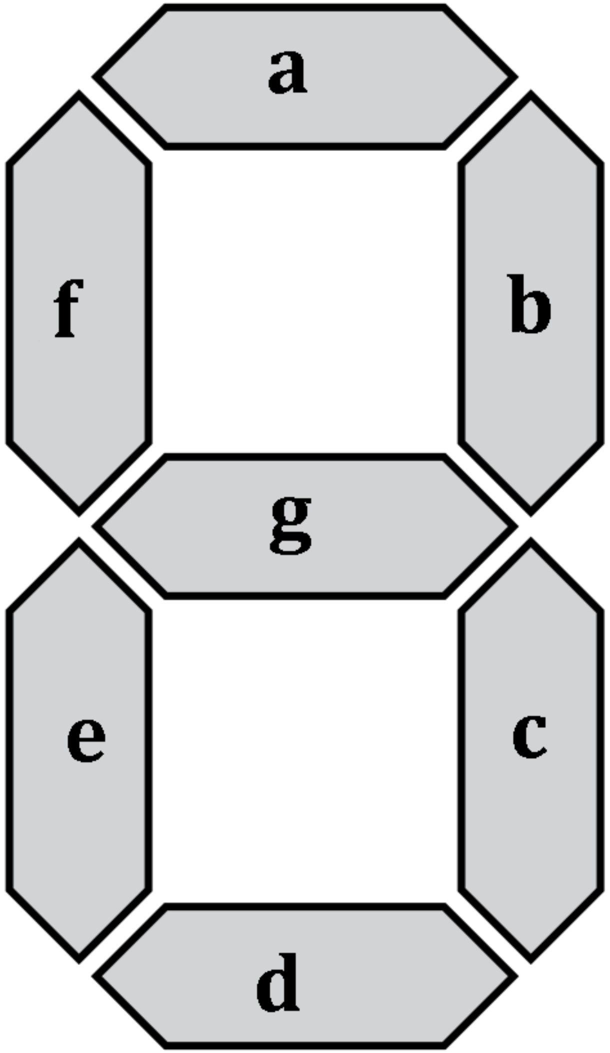
Not Required

AB \ CD	00	01	11	10
00	0	0	0	0
01	0	0	1	0
11	X	X	X	X
10	1	1	X	X

Application of K-Map with Don't Care

- An application where we need to use don't cares is the design of decoder for 7 segment displays.
- There are two types of 7-segment displays, namely Common Cathode and Common Anode.
- A decoder is needed to display the digits on a 7-segment display.
- We will design the decoder based on a common cathode 7-segment display.

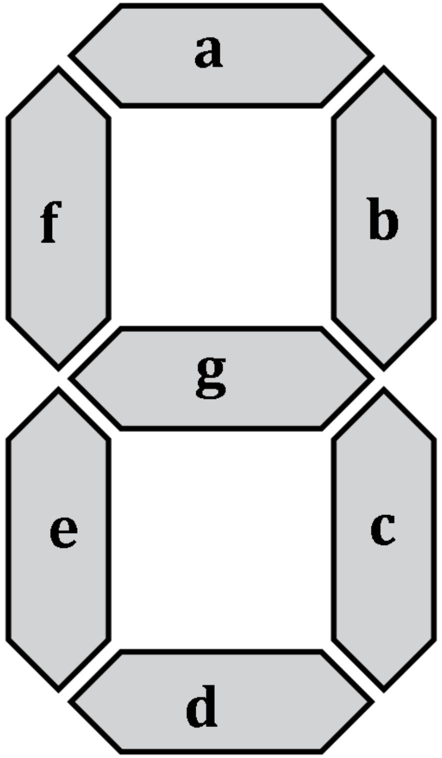




Application of K-Map with Don’t Care

Truth-table for 7-segment display decoder logical circuit.

Display Digits				
0	1	2	3	4
5	6	7	8	9



A	B	C	D	a	b	c	d	e	f	g
0	0	0	0	1	1	1	1	1	1	0
0	0	0	1	0	1	1	0	0	0	0
0	0	1	0	1	1	0	1	1	0	1
0	0	1	1	1	1	1	1	0	0	1
0	1	0	0	0	1	1	0	0	1	1
0	1	0	1	1	0	1	1	0	1	1
0	1	1	0	1	0	1	1	1	1	1
0	1	1	1	1	1	1	0	0	0	0
1	0	0	0	1	1	1	1	1	1	1
1	0	0	1	1	1	1	1	0	1	1
1	0	1	0	X	X	X	X	X	X	X
1	0	1	1	X	X	X	X	X	X	X
1	1	0	0	X	X	X	X	X	X	X
1	1	0	1	X	X	X	X	X	X	X
1	1	1	0	X	X	X	X	X	X	X
1	1	1	1	X	X	X	X	X	X	X

Problem Set for K-Map

1. For the function $F(A, B, C) = \sum(1,2,3,6)$
   a) Construct the truth table.
   b) Find the standard POS.
   c) Find the simplified SOP using KMAP.
   d) Find the simplified POS using KMAP.
   e) Implement the simplified SOP using basic logic gates.
   f) Implement the simplified SOP using universal NAND gates only.
2. For the function $F(A, B, C) = \prod(2,3,5,7)$
   a) Construct the truth table.
   b) Find the standard SOP.
   c) Find the simplified SOP using KMAP.
   d) Find the simplified POS using KMAP.
   e) Implement the simplified POS using basic logic gates.
   f) Implement the simplified SOP using universal NOR gates only.
3. For the function $F(A, B, C, D) = \sum(1,3,8,10)$ and $d(A, B, C, D) = (11,12,13,14,15)$ where, d(A,B,C,D) represents the don't care condition.
   a) Construct the truth table.
   b) Find the simplified SOP using KMAP.
   c) Find the simplified POS using KMAP.
   d) Implement the simplified POS using basic logic gates.
   e) Implement the simplified SOP using universal NAND gates only.

Problem Set for K-Map

4. For the following function $F(A,B,C,D) = \prod(2,4,7,9,11)$ and $d(A,B,C,D) = (1,3,12,13,14,15)$ where d(A,B,C,D) represents the don't care conditions.
   a) Construct the truth table.
   b) Find the simplified SOP using KMAP.
   c) Find the simplified POS using KMAP.
   d) Implement the simplified SOP using basic logic gates.
   e) Implement the simplified POS using universal NOR gates only.

References

1. Thomas L. Floyd, “Digital Fundamentals” 11th edition, Prentice Hall – Pearson Education.

Thank You