SET: A

AMERICAN INTERNATIONAL UNIVERSITY-BANGLADESH
Faculty of Science and Information Technology
Department of Mathematics
MAT3101 Numerical Methods for Science and Engineering (Section: All)

Midterm Examination Summer: 2022-23

Total Marks: 40 Time: 2 hours

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Instructions: 1. Results using programmable calculator without showing necessary steps will not be accepted.
2. Sharing or borrowing calculators and any other materials with others are not allowed.
3. Marks are indicated in the right margin.

Answer any **FIVE** of the following questions:

1. Given the linear system: $4x + 5y - 2z = -6, 2x - 5y + 2z = 24,$ $6x + 2y + 4z = 30.$

a. Use Gaussian elimination with partial pivoting to solve the system giving results to 3 decimal places. **6**

b. Write MATLAB codes to solve the above system by **backslash** operator. **2**

2. Given the linear system: $x + 2y + 10z = 10, x - 10y - z = 24,$ $11x + 5y + 8z = 31.$

a. Reduce the above system to diagonally dominant form. **2**

b. Write the corresponding Gauss- Seidel iteration formula. **2**

c. Compute **TWO** iterations to estimate the nearer solutions to 3 decimal places with $x_0 = 3.5, y_0 = -2.5, z_0 = 1.5.$ **4**

3. Given the equation $\sin x - x + 1 = 0.$

a. Find the number of real roots by using graphical method. **2**

b. Show that $[1.5, 2]$ holds at least one real root. **1**

c. Use the Bisection method **THREE** times in the interval $[1.5, 2]$ to get a new smaller interval, giving your answer to 3 decimal places. **3**

d. Use MATLAB command (**fzero**) to write a program to estimate the roots of the given equation $\sin x - x + 1 = 0$ in $[1.5, 2]$. **2**

4. Given the equation $x^3 - 5x + 1 = 0.$

a. Apply secant method **TWICE** to find the root to 3 decimal places of the given equation using the initial values $a = 2$ and $b = 2.2$. **3**

b. Write down an iteration formula based on Newton-Raphson method. **1**

c. Apply Newton-Raphson method **TWICE** to find the root to 3 decimal places of the given equation using the initial value $x_0 = -2$. **2**

d. Write down the MATLAB commands to find all the roots of the polynomial. **2**

5. Given the equation $xe^x - x - 3 = 0$.

a. The following iterative formulae are suggested to estimate the root of the above equation. **5**

$$i.\ x_{n+1} = \frac{1}{8}[9x_n - x_n e^{x_n} + 3], \qquad ii.\ x_{n+1} = \frac{1}{10}[11x_n - x_n e^{x_n} + 3].$$

State with reason which iterative formula will converge faster to the root near $x = 1.2$.

b. Use the suitable iterative formula from the above two (i) and (ii) and perform **THREE** iterations to find the root to 3 decimal places. **3**

6. Given the system of nonlinear equations $-2x^3 + 3y^2 + 42 = 0$,
$5x^2 + 3y^3 - 69 = 0$.

a. The following iterative formulas are suggested to estimate the root of the above equations. **4**

$$y_{n+1} = \left(\frac{-5{x_n}^2 + 69}{3}\right)^{1/3}, \qquad x_{n+1} = \left(\frac{3{y_n}^2 + 42}{2}\right)^{1/3}.$$

Verify whether the above iterative formula will converge to the root near $(x_0, y_0) = (1,1)$.

b. If converges, perform **TWO** iterations to find the root to 3 decimal places using the iterative formula. Otherwise write 'Not Convergent'. **4**

7. The table below gives the values of *x* and *f*(*x*):

x	-1	0	1
y(*x*)	3.2	4.5	3.4

a. Convert the nonlinear model $y = \frac{a}{x^2+b}$ into a linear form. **2**

b. Determine the coefficients a and b in the function $y = \frac{a}{x^2+b}$ that best fits the data. **5**

c. Hence find $y(-0.5)$. **1**

8. The table below gives the values of *x* and *f*(*x*):

x	0	1	2	3
f(*x*)	-4	-1	8	29

a. Construct a divided-difference table for the above data. **3**

b. Find the polynomial of least degree which passes through the points of the table and find $f(2.5)$. **2**

c. Use the inverse Lagrange interpolation formula to estimate the root of the equation *f*(*x*) = 0. **3**