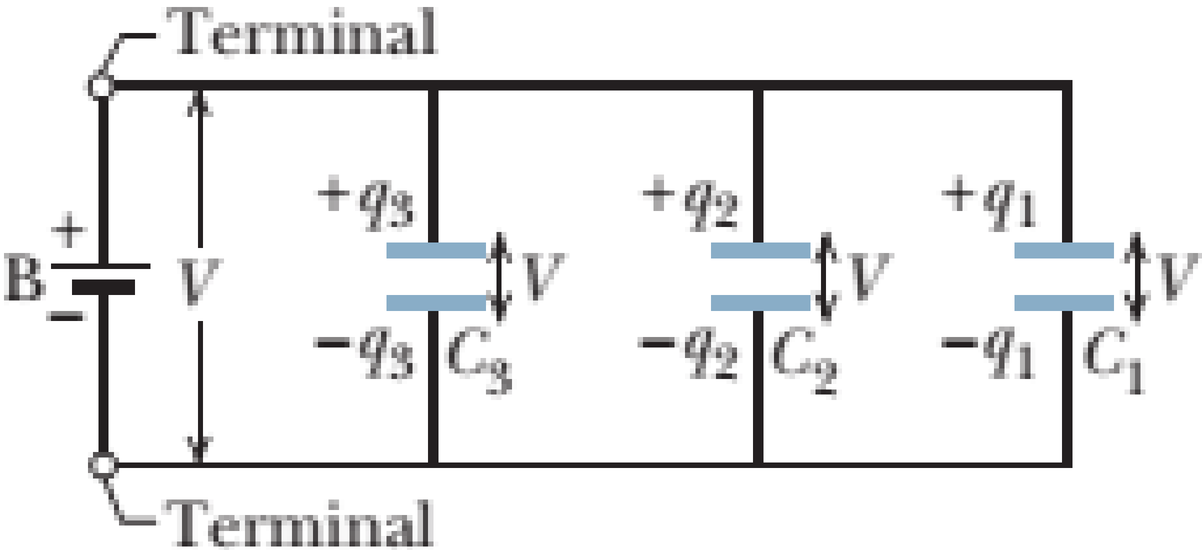
LESSON 7

BOOK CHAPTER 25

CAPACITANCE

Capacitors in parallel combination:



V is same across all capacitors

Charge on each capacitor:

$$q_1 = C_1 V$$

$$q_2 = C_2 V$$

$$q_3 = C_3 V$$

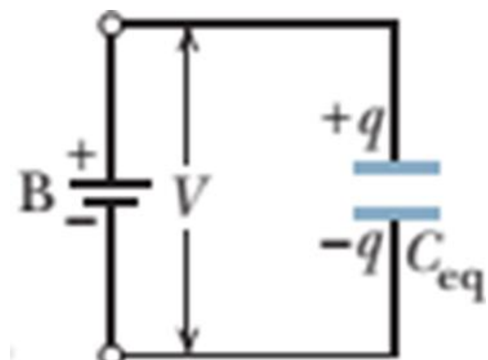
The total charge on the parallel combination is then

$$q = q_1 + q_2 + q_3 = (C_1 + C_2 + C_3)V$$

The equivalent capacitance, with the same total charge *q* and applied potential difference V as the combination, is then

$$C_{eq} = \frac{q}{V} = \frac{(C_1 + C_2 + C_3)V}{V} = C_1 + C_2 + C_3$$

$$\boxed{C_{eq} = C_1 + C_2 + C_3}$$



Capacitors in series combination:

q is same for all capacitors and voltages are different across capacitors.

Potential difference of each actual capacitor:

$$V_1 = \frac{q}{C_1} \qquad V_2 = \frac{q}{C_2} \qquad \text{And} \qquad V_3 = \frac{q}{C_3}$$

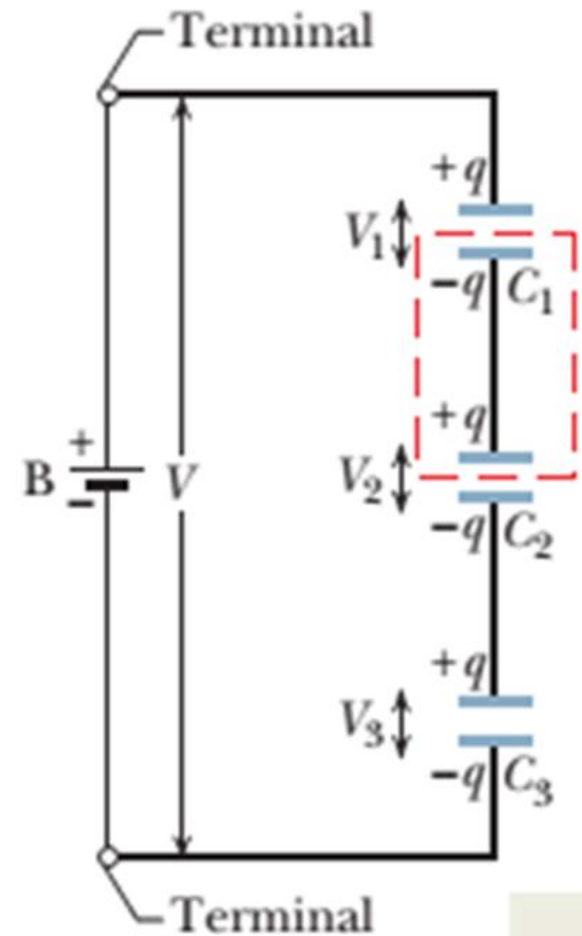
The total potential difference V due to the battery is the sum of these three potential differences. Thus,

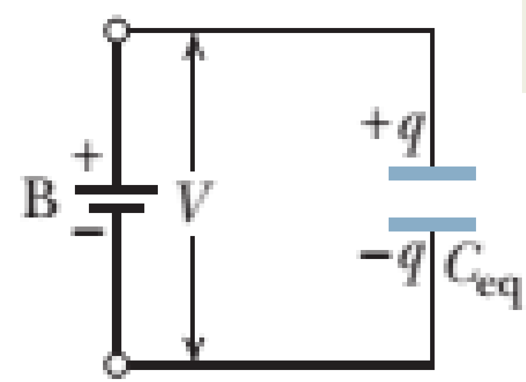
$$V = V_1 + V_2 + V_3 = q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)$$

The equivalent capacitance is then

$$C_{eq} = \frac{q}{V} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

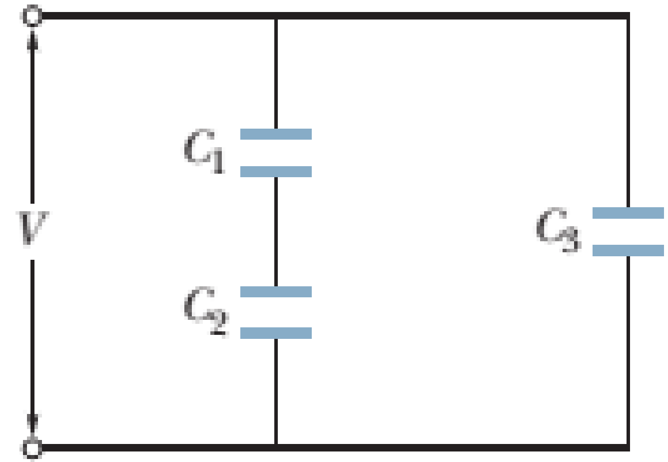
$$\boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$





Problem 10 (Book chapter 25):

In the adjacent Figure, find the equivalent capacitance of the combination. Assume that C_1 is $10.00\ \mu F$, C_2 is $5.00\ \mu F$, and C_3 is $4.00\ \mu F$.



Given

$$C_1 = 10\ \mu F \quad C_2 = 5\ \mu F$$

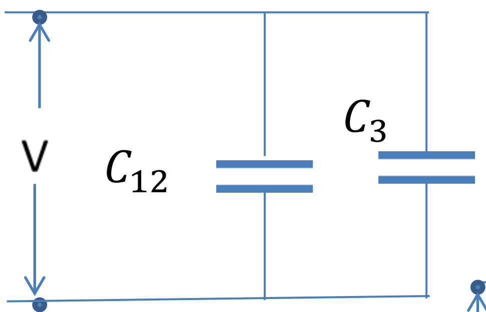
$$C_3 = 4\ \mu F$$

Answer:

Step1: $C_1\ and\ C_2\ are\ in\ series$

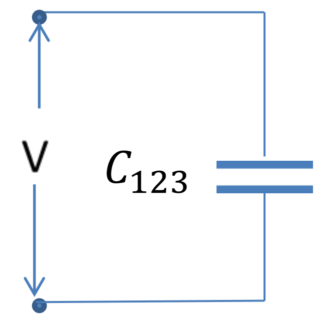
$$\frac{1}{C_{12}} = \frac{1}{C_1} + \frac{1}{C_2} \quad \text{or} \quad C_{12} = \frac{C_1 C_2}{C_2 + C_1}$$

$$C_{12} = \frac{(10)(5)}{5+10} = \frac{50}{15} = 3.33\ \mu F$$



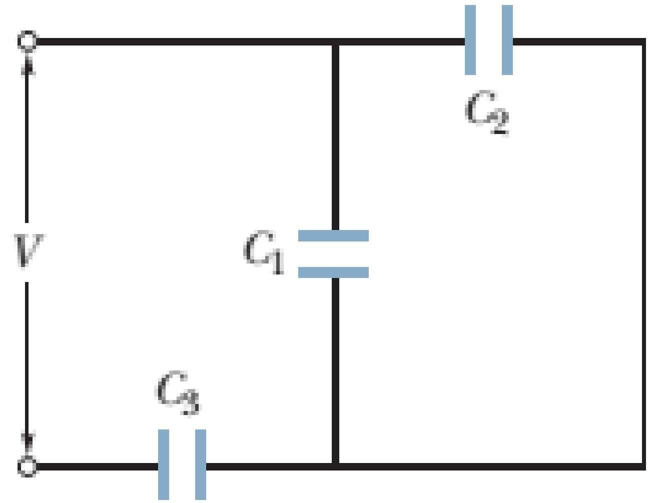
Step2: $C_{12}\ and\ C_3\ are\ in\ parellel$

$$C_{123} = C_{12} + C_3 = 3.33 + 4 = 7.33\ \mu F$$



Problem 11 (Book chapter 25):

In the adjacent Figure, find the equivalent capacitance of the combination. Assume that C_1 is $10.00\ \mu F$, C_2 is $5.00\ \mu F$, and C_3 is $4.00\ \mu F$.



Answer:

Step1: C_1 *and* C_2 *are in parellel*

$$C_{12} = C_1 + C_2 = 10 + 5 = 15\ \mu F$$

Step2: C_{12} *and* C_3 *are in series*

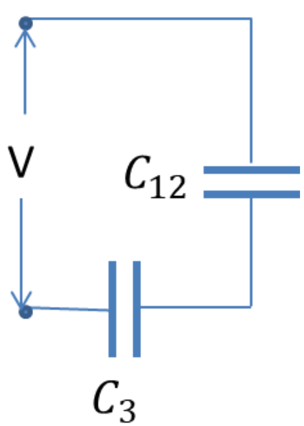
Given

$C_1 = 10\ \mu F$

$C_2 = 5\ \mu F$

$C_3 = 4\ \mu F$

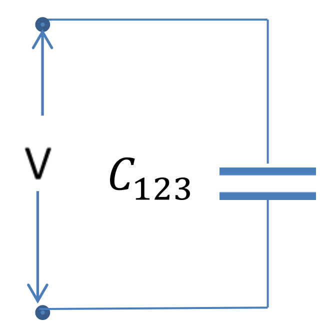
$C_{123} = ?$



$$\frac{1}{C_{123}} = \frac{1}{C_{12}} + \frac{1}{C_3} = \frac{C_3 + C_{12}}{(C_{12})(C_3)}$$

$$C_{123} = \frac{(C_{12})(C_3)}{C_3 + C_{12}}$$

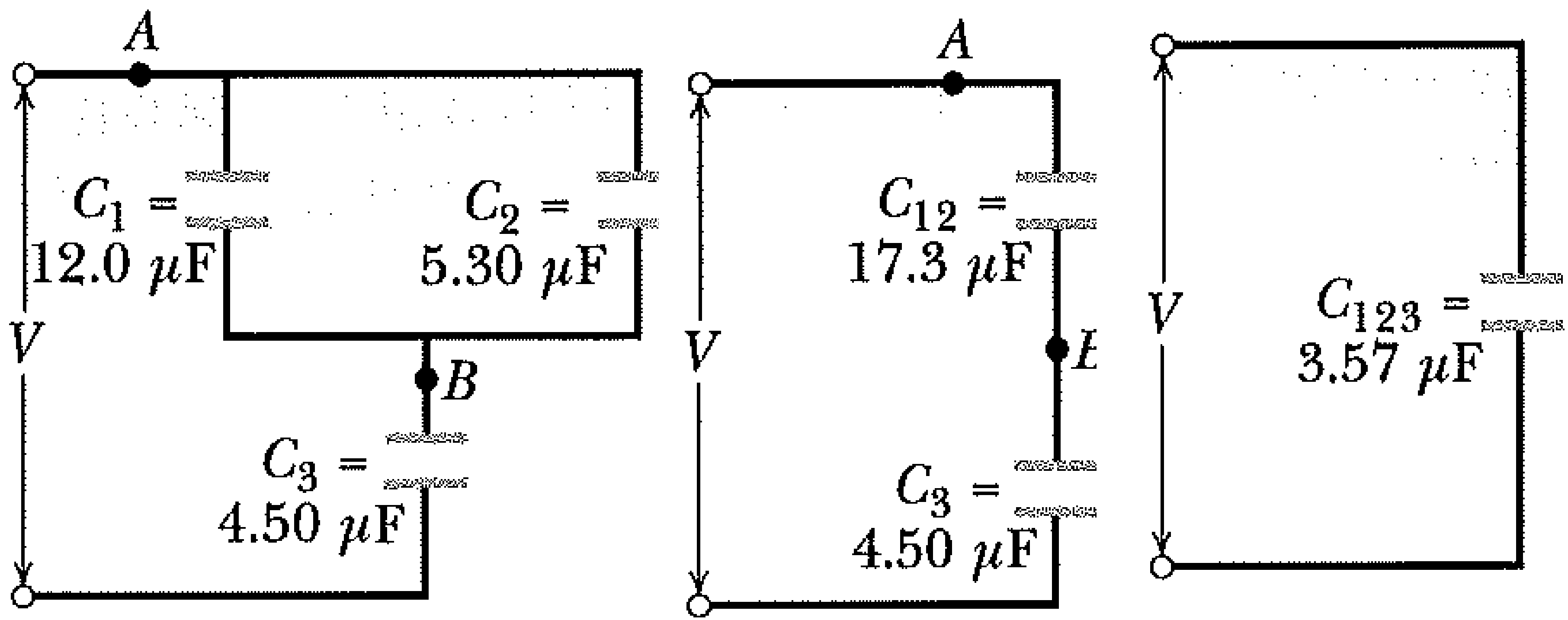
$$C_{123} = \frac{(15)(4)}{4 + 15} = \frac{60}{19} = 3.158\ \mu F$$



Sample Problem 25.02(a) (page 726): Find the equivalent capacitance for the combination of capacitances shown in Figure, across which potential difference V is applied.

Assume $C_1 = 12.0\ \mu F$, $C_2 = 5.30\ \mu F$, $and\ C_3 = 4.5\ \mu F$.

Home Task



Step1: $C_1\ and\ C_2\ are\ in\ parallel$

$$C_{12} = C_1 + C_2 = 12 + 5.3 = 17.3\ \mu F$$

Step2: $C_{12}\ and\ C_3\ are\ in$ series

$$\frac{1}{C_{123}} = \frac{1}{C_{12}} + \frac{1}{C_3} = \frac{C_3 + C_{12}}{(C_{12})(C_3)}$$

$$C_{123} = \frac{(C_{12})(C_3)}{C_3 + C_{12}}$$

$$C_{123} = \frac{(17.3)(4.5)}{4.5 + 17.3} = \frac{77.85}{21.8} = 3.57\ \mu F$$

Energy Stored in an Electric Field:

Suppose that, at a given instant, a charge q' has been transferred from one plate of a capacitor to the other. The potential difference V' between the plates at that instant will be $\frac{q'}{C}$. If an extra increment of charge dq' is then transferred, the increment of work required will be,

$$dW = V'dq' = \frac{q'}{C}dq'$$

The work required to bring the total capacitor charge up to a final value q is

$$W = \int_{q'=0}^{q'=q} dW = \frac{1}{C}\int_0^q q'dq' = \frac{q^2}{2C}$$

This work is stored as potential energy U in the capacitor, so

$$U = W = \frac{q^2}{2C} = \frac{(CV)^2}{2C} = \frac{1}{2}CV^2$$

Finally, $\boxed{U = \frac{q^2}{2C}}$ OR $\boxed{U = \frac{1}{2}CV^2}$

Energy Density:

In a parallel-plate capacitor, the electric field has the same value at all points between the plates. Thus, **the energy density u that is, the potential energy per unit volume** between the plates should also be uniform. We can find *u* by dividing the total potential energy by the volume *Ad* of the space between the plates. Thus

$$u = \frac{U}{Ad} = \left(\tfrac{1}{2}CV^2\right)\frac{1}{Ad}$$

For parallel plate capacitor, the capacitance can be expressed as

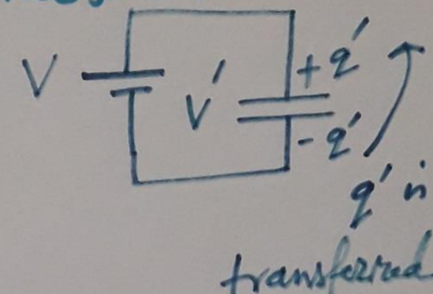
$$C = \frac{\varepsilon_0 A}{d}$$

Therefore,

$$u = \left(\tfrac{1}{2}CV^2\right)\frac{1}{Ad} = \left(\frac{V^2}{2Ad}\right)\left(\frac{\varepsilon_0 A}{d}\right) = \frac{\varepsilon_0 V^2}{2d^2}$$

Finally, $\boxed{u = \frac{1}{2}\varepsilon_0\left(\frac{V}{d}\right)^2}$ OR $\boxed{u = \frac{1}{2}\varepsilon_0 E^2}$ [since $V = Ed$]

Energy stored in an electric field:



$$q' = CV'$$

$$V' = \frac{q'}{C}$$

dW is needed for extra dq' is transferred bet^n the plates

$$V' = \frac{dW}{dq'}$$

$$dW = V'\,dq' = \frac{q'}{C}\,dq'$$

$$W = \int dW = \int_{q'=0}^{q'=q} \frac{q'}{C}\,dq' = \frac{1}{C}\int_0^q q'\,dq'$$

$$= \frac{1}{C}\left[\frac{q'^{\,1+1}}{1+1}\right]_0^q = \frac{1}{C}\left[\frac{q'^2}{2}\right]_0^q$$

$$= \frac{1}{2C}\left[q'^2\right]_0^q = \frac{1}{2C}\left(q^2 - 0^2\right)$$

$$W = \frac{1}{2C}q^2$$

$$U = W = \frac{1}{2C}q^2$$ [Work, W is stored as a potential energy, U]

$$U = \frac{q^2}{2C}$$

$$= \frac{(CV)^2}{2C}$$

$$= \frac{C^2V^2}{2C}$$

$$= \frac{CV^2}{2}$$

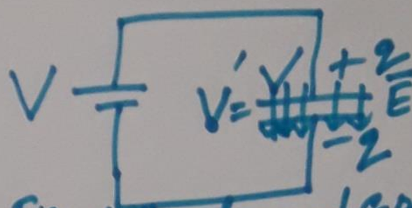


Fig fully charged capacitor

$q = CV$

$$U = \frac{1}{2}CV^2$$

$$\boxed{\therefore U = \frac{q^2}{2C} = \frac{1}{2}CV^2}$$

Energy density u: Energy density is the potential energy per unit volume betn the plates.

$$u = \frac{U}{V} = \frac{U}{Ad} = \left(\frac{1}{2}CV^2\right)\frac{1}{Ad}$$

$$= \left(\frac{V^2}{2Ad}\right)C = \frac{V^2}{2Ad}\left(\frac{\varepsilon_0 A}{d}\right) \qquad \left[C = \frac{\varepsilon_0 A}{d}\right]$$

$$= \frac{\varepsilon_0 V^2}{2d^2} = \frac{1}{2}\varepsilon_0\left(\frac{V}{d}\right)^2 \qquad \left[E = \frac{V}{d}\right]$$

$$\boxed{u = \frac{1}{2}\varepsilon_0 E^2}$$

Problem 29 (Book chapter 25):

What capacitance is required to store an energy of 10 kW.h at a potential difference of 1000 V?

Answer:

Given:

$$U = 10\ kW.\ h = 10000 \times 60 \times 60\ Watt - s$$

$$U = 36 \times 10^6\ \frac{J}{s}\,\mathrm{s} = 36 \times 10^6\ \mathrm{J}$$

[P $= \frac{W}{t}$

Unit: Watt $= \frac{J}{s}$]

Potential difference, $V = 1000\ volt$

$$\boxed{C = ?}$$

$$U = \frac{1}{2} C V^2$$

$$C = \frac{2U}{V^2} = \frac{2 \times 36 \times 10^6}{(1000)^2} = 72\ F$$

Problem 31 (Book chapter 25):

A 2.0 μF capacitor and a 4.0 μF capacitor are connected in parallel across a 300 V potential difference. Calculate the total energy stored in the capacitors.

Given

$$C_1 = 2\ \mu F \quad and\ C_2 = 4\ \mu F$$

$$V = 300\ volt$$

$$\boxed{U = ?}$$

Answer:

$C_1\ and\ C_2\ \ are\ in\ parallel$

$$C_{12} = C_1 + C_2 = 2 + 4 = 6\ \mu F = 6 \times 10^{-6}\ F$$

We know

$$U = \frac{1}{2} C_{12} V^2 = \frac{6 \times 10^{-6} \times (300)^2}{2}$$

$$U = 0.27\ J$$

Problem 33 (Book chapter 25):

A charged isolated metal sphere of diameter 10 cm has a potential of 8000 V relative to $V = 0$ at infinity. Calculate the energy density in the electric field near the surface of the sphere.

Answer: next slide

Problem 33 (Book chapter 25):

Given

For isolated sphere:

$Diameter, D = 10\ cm$

$Radius, R = 5\ cm = 0.05\ m$

$V = 8000\ volt$ $\quad \boxed{u = ?}$

Answer:

$$u = \frac{1}{2}\varepsilon_0\left(\frac{V}{R}\right)^2 = \frac{8.854 \times 10^{-12} \times (8000)^2}{2 \times (0.05)^2}$$

$$u = \frac{566.66 \times 10^{-6}}{5 \times 10^{-3}} = 113.33 \times 10^{-3}\ \frac{J}{m^3}$$

Sample problem 25.04 (page-730):

Given

For isolated sphere:

$Radius, R = 6.85\ cm = 0.0685\ m$

$q = 1.25\ nC = 1.25 \times 10^{-9}\ C$

(a) $\boxed{U = ?}$

(b) $\boxed{u = ?}$

Answer: We know $U = \dfrac{q^2}{2C}$

For isolated sphere, $C = 4\pi\epsilon_0 R$

$$U = \frac{q^2}{2 \times 4\pi\varepsilon_0 R} = \frac{9 \times 10^9 \times (1.25 \times 10^{-9})^2}{2 \times 0.0685}$$

$$\boxed{U = 102.64 \times 10^{-9}\ J}$$

$$u = \frac{1}{2}\varepsilon_0 E^2 = \frac{8.854 \times 10^{-12}}{2}\left(\frac{q}{4\pi\varepsilon_0 R^2}\right)^2 = \frac{8.854 \times 10^{-12}}{2}\left(\frac{9 \times 10^9 \times 1.25 \times 10^{-9}}{(0.0685)^2}\right)^2$$

$$\boxed{u = \frac{1120.58 \times 10^{-12}}{4.403 \times 10^{-5}} = 25.450 \times 10^{-6}\ \frac{J}{m^3}}$$

Thank You