LESSON 2

<u>BOOK CHAPTER 22</u>

<u>ELECTRIC FIELDS</u>

Electric Dipoles:

An electric dipole is a pair of point charges with equal magnitude and opposite sign (a positive charge $+q$ and a negative charge $-q$) separated by a small distance d.

Fig.1

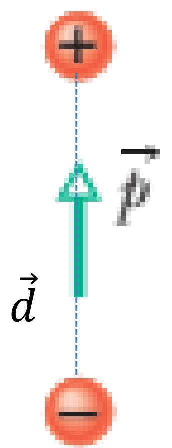
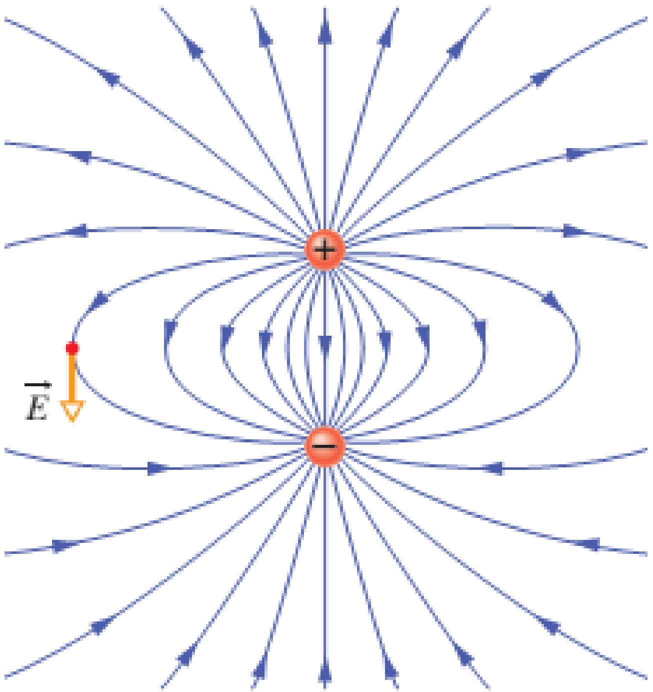


Fig.2



The product of the charge q and the separation d is the magnitude of a quantity called the **electric dipole moment,** denoted by p.

$$That\ is, \qquad p = qd$$

In vector form, $\boldsymbol{p} = q\mathrm{d}\,\hat{k}$

The direction of $\vec{p}$ is from negative charge to positive charge as shown in figure 1.

The pattern of electric field lines around an electric dipole, with an electric field vector $\vec{E}$ shown (Figure 2) at one point (tangent to the field line through that point).

The Electric Field Due to an Electric Dipole:

The net electric field at point P is

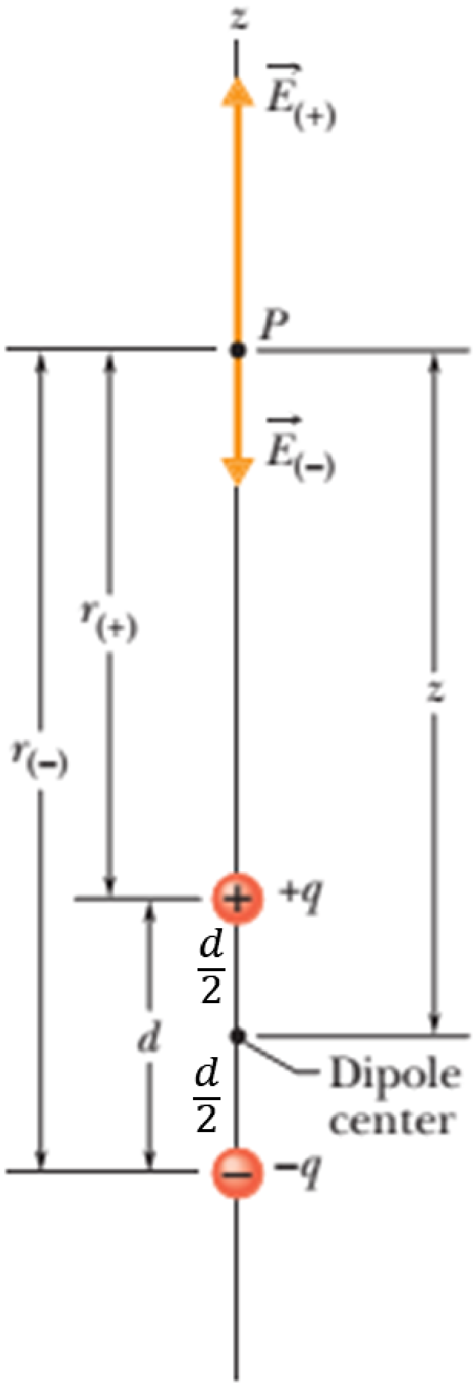
$$\boldsymbol{E} = \boldsymbol{E}_{(+)} + \boldsymbol{E}_{(-)}$$

$$E = +\frac{1}{4\pi\varepsilon_0}\frac{q}{r_{(+)}^2} + \left(-\frac{1}{4\pi\varepsilon_0}\frac{q}{r_{(-)}^2}\right)$$

$$E = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{\left(z-\frac{d}{2}\right)^2} - \frac{1}{\left(z+\frac{d}{2}\right)^2}\right]$$

$$E = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{\left\{z\left(1-\frac{d}{2z}\right)\right\}^2} - \frac{1}{\left\{z\left(1+\frac{d}{2z}\right)\right\}^2}\right]$$

$$E = \frac{q}{4\pi\varepsilon_0 z^2}\left[\frac{1}{\left(1-\frac{d}{2z}\right)^2} - \frac{1}{\left(1+\frac{d}{2z}\right)^2}\right]$$



$$E = \frac{q}{4\pi\varepsilon_0 z^2}\left[\left(\{1+(-\tfrac{d}{2z})\}^{-2} - \left(1+\tfrac{d}{2z}\right)^{-2}\right]\right.$$ For $z \gg d, we\ have\ \frac{d}{2z} \ll 1$

We use the form of binomial theorem,

$$(1+x)^n = 1 + \frac{nx}{1!} + \frac{n(n-1)x^2}{2!} + \ldots\ (x^2 < 1)$$

$$\boldsymbol{(1+x)^n \approx 1 + nx \quad for\ x \ll 1}$$

$$E = \frac{q}{4\pi\varepsilon_0 z^2}\left[\{\left(1+(-2)(-\tfrac{d}{2z}\right)\} - \{\left(1+(-2)(\tfrac{d}{2z}\right)\}\right]$$

$$E = \frac{q}{4\pi\varepsilon_0 z^2}\left[\left(1+\tfrac{d}{z}\right) - \left(1-\tfrac{d}{z}\right)\right]$$

$$E = \frac{q}{4\pi\varepsilon_0 z^2}\left[1+\tfrac{d}{z} - 1 + \tfrac{d}{z}\right]$$

$$E = \frac{q}{4\pi\varepsilon_0 z^2}\left[2\frac{d}{z}\right]$$

$$[p = qd]$$

$$\boxed{E = \frac{qd}{2\pi\varepsilon_0 z^3} = \frac{p}{2\pi\varepsilon_0 z^3}}$$

$$\boldsymbol{E} = \frac{p}{2\pi\varepsilon_0 z^3}\hat{k}$$

Linear charge density:

When charge is distributed along a line (such as a long, thin, charged plastic rod), we use (the Greek letter lambda λ) to represent the charge per unit length known as **linear charge density.**

That is

$$\lambda = \frac{Amount\ of\ charge\ distributed\ on\ the\ rod}{Length\ of\ the\ rod}$$ [For uniform linear charge density]

The SI unit of λ is Coulomb/meter; simply, we use C/m.

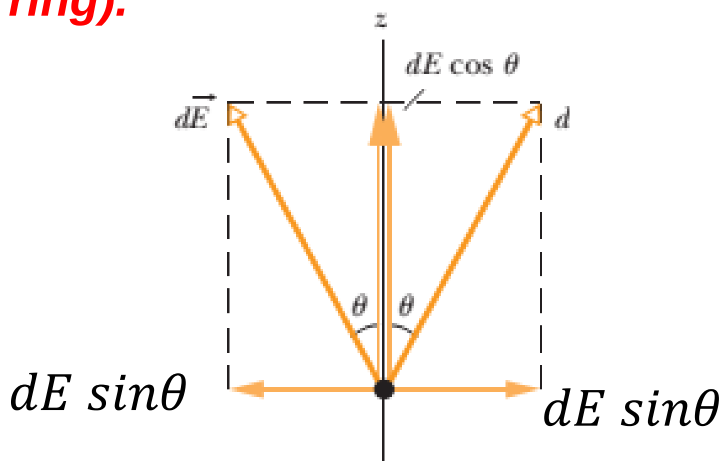
Note:

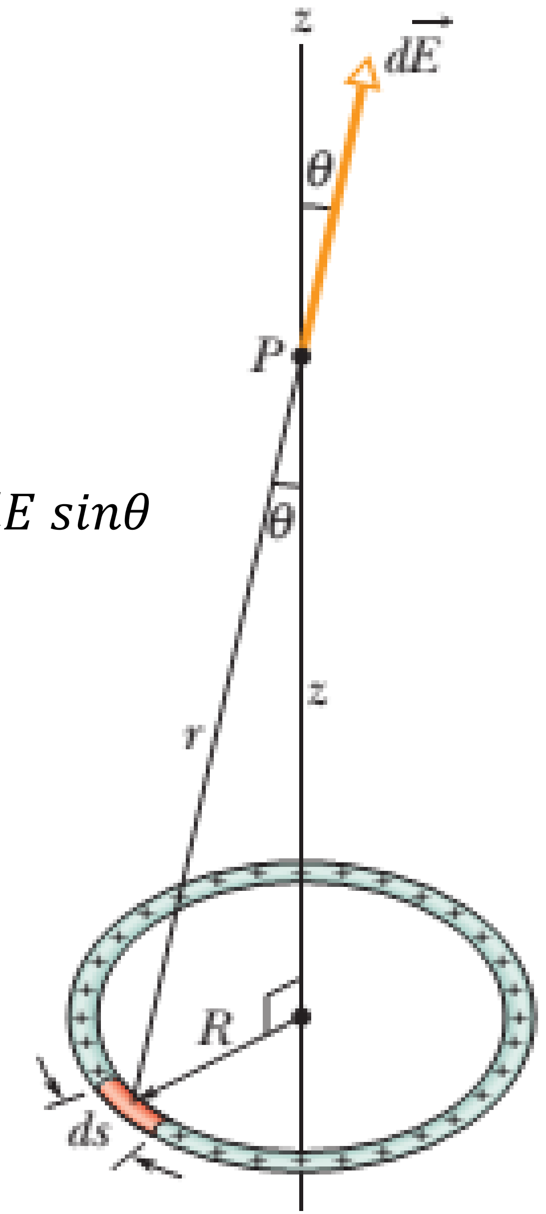
Some Measures of Electric Charge

Name	Symbol	SI Unit
Charge	q	C
Linear charge density	λ	C/m
Surface charge density	σ	C/m^2
Volume charge density	ρ	C/m^3

Electric field due to a line of charge (ring): $E = \frac{qz}{4\pi\varepsilon_0 (z^2+R^2)^{3/2}}$

For charge that is distributed uniformly over a ring, determine the net electric field at a given point on the axis of the ring distance z from the center of the ring).





Let *ds* be the (arc) length of any differential element of the ring. Since λ is the charge per unit (arc) length, the element has a charge of magnitude

$$Linear\ charge\ density, \lambda = dq/ds$$

$$dq = \lambda ds$$

This differential charge (dq) sets up a differential electric field $\mathrm{d}\vec{E}$ at point *P*, which is a distance *r* from the element. Treating the element as a point charge.

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda ds}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda ds}{\left(\sqrt{z^2+R^2}\right)^2} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda ds}{(z^2+R^2)}$$

Since the components perpendicular to the *z* axis(dE sinθ) cancel and the parallel components(dEcosθ) add, the net electric field along z-axis is

$$E_z = E = \int dE\ cos\theta = \int \frac{1}{4\pi\varepsilon_0}\frac{\lambda ds}{(z^2+R^2)}\left[\frac{z}{\sqrt{z^2+R^2}}\right] \qquad \left[cos\theta = \frac{z}{r} = \frac{z}{\sqrt{z^2+R^2}}\right]$$

$$E = \frac{\lambda z}{4\pi\varepsilon_0(z^2+R^2)^{3/2}}\int_{s=0}^{s=2\pi R} ds = \frac{\lambda z}{4\pi\varepsilon_0(z^2+R^2)^{3/2}}[s]_0^{2\pi R}$$

$$\mathrm{E} = \frac{\lambda z}{4\pi\varepsilon_0 (z^2+R^2)^{3/2}}(2\pi R\text{- }0)$$

$[\lambda = q/2\pi R]$

Total change of the ring,
q = $\lambda(2\pi R)$

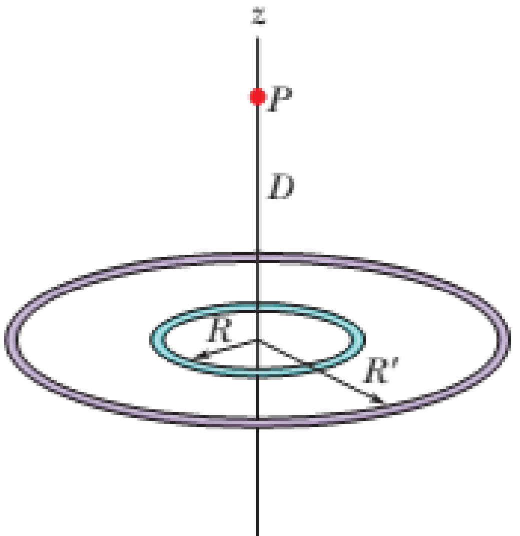
$$E = \frac{\{\lambda(2\pi R)\}\, z}{4\pi\varepsilon_0 (z^2 + R^2)^{3/2}}$$

$$\boxed{E = \frac{qz}{4\pi\varepsilon_0 (z^2 + R^2)^{3/2}}}$$

$$\boxed{\vec{E} = \frac{qz}{4\pi\varepsilon_0 (z^2+R^2)^{3/2}}\,\hat{k}}$$

Problem 30 (Book chapter 22)

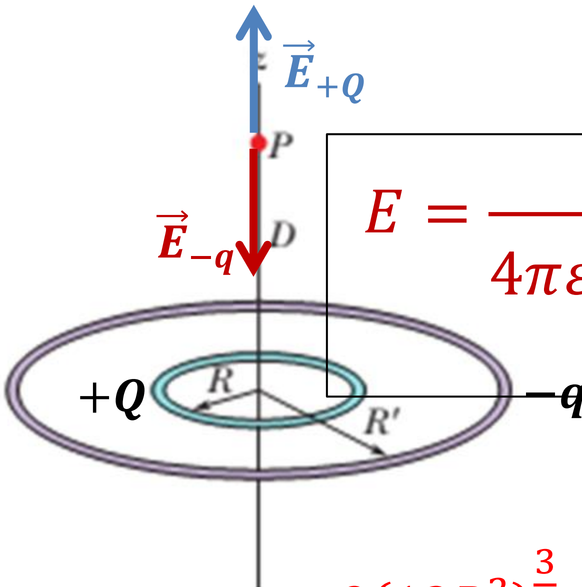
Figure shows two concentric rings, of radii R and $R' = 3R$, that lie on the same plane. Point P lies on the central z axis, at distance $D = 2R$ from the center of the rings. The smaller ring has uniformly distributed charge $+Q$. In terms of Q, what is the uniformly distributed charge on the larger ring if the net electric field at P is zero?



$$\boldsymbol{E}_{+Q} + \boldsymbol{E}_{-q} = 0$$

$$+E_{+Q} + (-E_{-q}) = 0$$

$$E_{+Q} = E_{-q}$$



$$\boxed{E = \frac{qz}{4\pi\varepsilon_0 (z^2 + R^2)^{3/2}}}$$

$$\frac{QD}{4\pi\varepsilon_0 (D^2 + R^2)^{3/2}} = \frac{qD}{4\pi\varepsilon_0 (D^2 + (3R)^2)^{3/2}}$$

$$\frac{Q(2R)}{(4R^2 + R^2)^{3/2}} = \frac{q(2R)}{(4R^2 + 9R^2)^{3/2}}$$

$$\frac{Q}{(5R^2)^{3/2}} = \frac{q}{(13R^2)^{3/2}}$$

$$q = \frac{Q(13R^2)^{\frac{3}{2}}}{(5R^2)^{\frac{3}{2}}} = \left(\frac{13R^2}{5R^2}\right)^{\frac{3}{2}} Q$$

$$q = \left(\frac{13}{5}\right)^{\frac{3}{2}} Q = 4.19Q$$

$$\boxed{q = -4.19Q}$$

THANK YOU