LESSON 9

BOOK CHAPTERS 25 and 26

CAPACITANCE

And

CURRENT and RESISTANCE

Energy Density:

In a parallel-plate capacitor, the electric field has the same value at all points between the plates. Thus, **the energy density u that is, the potential energy per unit volume** between the plates should also be uniform. We can find *u* by dividing the total potential energy by the volume *Ad* of the space between the plates. Thus

$$u = \frac{U}{Ad} = (\tfrac{1}{2}CV^2)\frac{1}{Ad}$$

For parallel plate capacitor, the capacitance can be expressed as

$$C = \frac{\varepsilon_0 A}{d}$$

Therefore,

$$u = (\tfrac{1}{2}CV^2)\frac{1}{Ad} = \left(\frac{V^2}{2Ad}\right)\left(\frac{\varepsilon_0 A}{d}\right) = \frac{\varepsilon_0 V^2}{2d^2}$$

Finally, $\boxed{u = \frac{1}{2}\varepsilon_0\left(\frac{V}{d}\right)^2}$ OR $\boxed{u = \frac{1}{2}\varepsilon_0 E^2}$ [since $V = Ed$]

Problem 29 (Book chapter 25):

What capacitance is required to store an energy of 10 kW.h at a potential difference of 1000 V?

Problem 31 (Book chapter 25):

A 2.0 μF capacitor and a 4.0 μF capacitor are connected in parallel across a 300 V potential difference. Calculate the total energy stored in the capacitors.

Problem 32 (Book chapter 25):

A parallel-plate air-filled capacitor having area 40 cm^2 and plate spacing 1.0 mm is charged to a potential difference of 600 V. Find (a) the capacitance, (b) the magnitude of the charge on each plate, (c) the stored energy, (d) the electric field between the plates, and (e) the energy density between the plates.

Problem 33 (Book chapter 25):

A charged isolated metal sphere of diameter 10 cm has a potential of 8000 V relative to $V = 0$ at infinity. Calculate the energy density in the electric field near the surface of the sphere.

Sample problem 25.04 (page-730):

An isolated conducting sphere whose radius R is 6.85 cm has a charge $q = 1.25$ nC. (a) How much potential energy is stored in the electric field of this charged conductor? (b) What is the energy density at the surface of the sphere?

Problem 29 (Book chapter 25):

What capacitance is required to store an energy of 10 kW.h at a potential difference of 1000 V?

Answer:

Given:

$$U = 10\,kW.\,h = 10000 \times 60 \times 60\,Watt - s$$

$$U = 36 \times 10^6\ \frac{J}{s}\,\mathrm{s} = 36 \times 10^6\ \mathrm{J}$$

[P $= \frac{W}{t}$

Unit: Watt $= \frac{J}{s}$]

Potential difference, $V = 1000\,volt$

$$\boxed{C = ?}$$

$$U = \frac{1}{2}CV^2$$

$$C = \frac{2U}{V^2} = \frac{2 \times 36 \times 10^6}{(1000)^2} = 72\,F$$

Problem 31 (Book chapter 25):

A 2.0 μF capacitor and a 4.0 μF capacitor are connected in parallel across a 300 V potential difference. Calculate the total energy stored in the capacitors.

Given

$$C_1 = 2\ \mu F \quad and\ C_2 = 4\ \mu F$$

$$V = 300\ volt$$

$$\boxed{U = ?}$$

Answer:

$C_1\ and\ C_2\ \ are\ in\ parallel$

$$C_{12} = C_1 + C_2 = 2 + 4 = 6\ \mu F = 6 \times 10^{-6}\ F$$

We know

$$U = \frac{1}{2} C_{12} V^2 = \frac{6 \times 10^{-6} \times (300)^2}{2}$$

$$U = 0.27\ J$$

Problem 32 (Book chapter 25):

Answer:

Given

$$A = 40\ cm^2 = 40 \times (10^{-2})^2\ m^2 = 4 \times 10^{-3}\ m^2$$

$$d = 1\ mm = 1 \times 10^{-3} m$$

$$V = 600\ volt$$

(a) $\boxed{C = ?}$

$$C = \frac{\varepsilon_0 A}{d} = \frac{8.854 \times 10^{-12} \times 4 \times 10^{-3}}{1 \times 10^{-3}}$$

$$\boxed{C = 35.416 \times 10^{-12}\ F}$$

(b) $\boxed{q = ?}$

$$\boxed{q = CV = 35.416 \times 10^{-12} \times 600 = 21.25 \times 10^{-9}\ C}$$

(c) $\boxed{U = ?}$

$$\boxed{U = \frac{1}{2} C V^2 = \frac{35.416 \times 10^{-12} \times (600)^2}{2} = 12.75 \times 10^{-6}\ J}$$

(d) $\boxed{E = ?}$

$$\boxed{E = \frac{V}{d} = \frac{600}{1 \times 10^{-3}} = 600 \times 10^3\ \frac{V}{m}}$$

(e) $\boxed{u = ?}$

$$\boxed{u = \frac{1}{2} \varepsilon_0 \left(\frac{V}{d}\right)^2 = \frac{8.854 \times 10^{-12} \times (600)^2}{2 \times (10^{-3})^2} = 1.59\ \frac{J}{m^3}}$$

Problem 33 (Book chapter 25):

Given

For isolated sphere:

$$Diameter, D = 10\ cm$$

$$Radius, R = 5\ cm = 0.05\ m$$

$$V = 8000\ volt \qquad \boxed{u = ?}$$

Answer:

$$u = \frac{1}{2}\varepsilon_0\left(\frac{V}{R}\right)^2 = \frac{8.854 \times 10^{-12} \times (8000)^2}{2 \times (0.05)^2}$$

$$u = \frac{566.66 \times 10^{-6}}{5 \times 10^{-3}} = 113.33 \times 10^{-3}\ \frac{J}{m^3}$$

Sample problem 25.04 (page-730):

Given

For isolated sphere:

$$Radius, R = 6.85\ cm = 0.0685\ m$$

$$q = 1.25\ nC = 1.25 \times 10^{-9}\ C$$

(a) $\boxed{U = ?}$

(b) $\boxed{u = ?}$

Answer: We know $U = \frac{q^2}{2C}$

For isolated sphere, $C = 4\pi\epsilon_0 R$

$$U = \frac{q^2}{2 \times 4\pi\varepsilon_0 R} = \frac{9 \times 10^9 \times (1.25 \times 10^{-9})^2}{2 \times 0.0685}$$

$$\boxed{U = 102.64 \times 10^{-9}\ J}$$

$$u = \frac{1}{2}\varepsilon_0 E^2 = \frac{8.854 \times 10^{-12}}{2}\left(\frac{q}{4\pi\varepsilon_0 R^2}\right)^2 = \frac{8.854 \times 10^{-12}}{2}\left(\frac{9 \times 10^9 \times 1.25 \times 10^{-9}}{(0.0685)^2}\right)^2$$

$$\boxed{u = \frac{1120.58 \times 10^{-12}}{4.403 \times 10^{-5}} = 25.450 \times 10^{-6}\ \frac{J}{m^3}}$$

Thank You