CHAPTER 23

Gauss' Law

Gauss' law relates the electric field at points on a (closed) Gaussian surface to the net charge enclosed by that surface

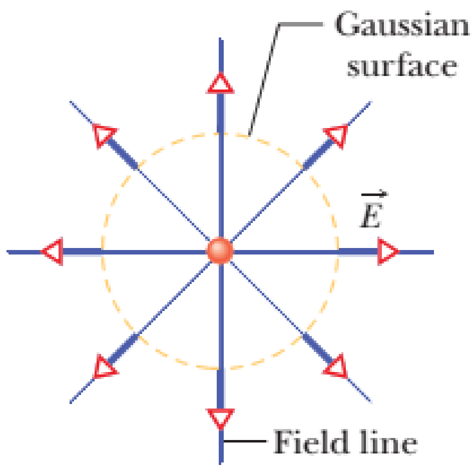


Figure 23-1 Electric field vectors and field lines pierce an imaginary, spherical Gaussian surface that encloses a particle with charge $+Q$.

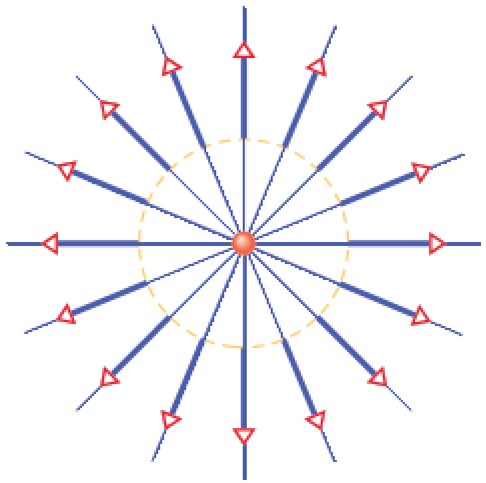


Figure 23-2 Now the enclosed particle has charge $+2Q$.

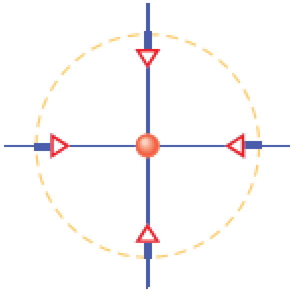


Figure 23-3 Can you tell what the enclosed charge is now?

Electric Flux:

$$\Delta\Phi = \vec{E} \cdot \Delta\vec{A}$$

$$\Delta\Phi = (E \cos\theta)\, \Delta A$$

$$\Phi = \sum \vec{E} \cdot \Delta\vec{A}$$

$$\Phi = \int \vec{E} \cdot d\vec{A} \qquad \text{(total flux)}$$

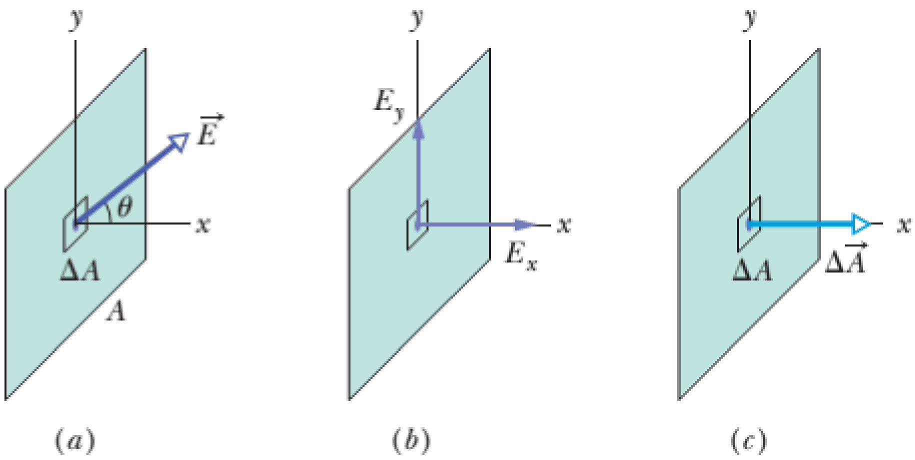


Figure 23-4 (*a*) An electric field vector pierces a small square patch on a flat surface. (*b*) Only the *x* component actually pierces the patch; the *y* component skims across it. (*c*) The area vector of the patch is perpendicular to the patch, with a magnitude equal to the patch's area.

An inward piercing field is negative flux. An outward piercing field is positive flux. A skimming field is zero flux.

$$\Phi = \oint \vec{E} \cdot d\vec{A} \quad \text{(net flux)}$$

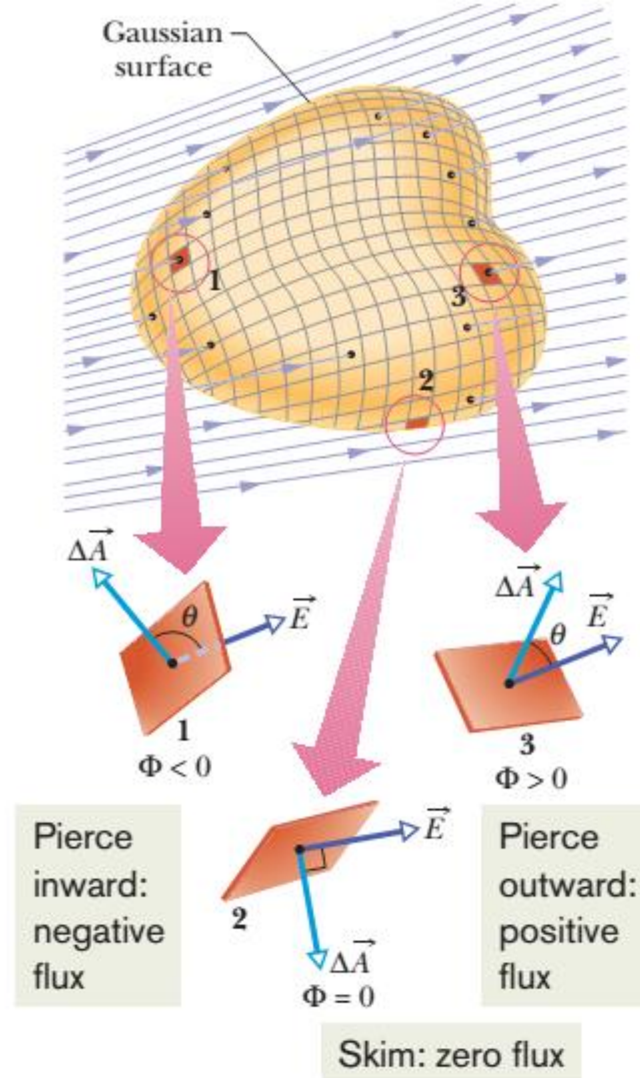


Figure 23-5 A Gaussian surface of arbitrary shape immersed in an electric field. The surface is divided into small squares of area ΔA. The electric field vectors $\vec{E}$ and the area vectors $\Delta\vec{A}$ for three representative squares, marked 1, 2, and 3, are shown.

Sample Problem 23-1

Figure 23-4 shows a Gaussian surface in the form of a cylinder of radius R immersed in a uniform electric field $\vec{E}$, with the cylinder axis parallel to the field. What is the flux Φ of the electric field through this closed surface?

$$\Phi = \oint \vec{E} \cdot d\vec{A}$$

$$= \int_a \vec{E} \cdot d\vec{A} + \int_b \vec{E} \cdot d\vec{A} + \int_c \vec{E} \cdot d\vec{A}. \qquad (23\text{-}5)$$

where $\int dA$ gives the cap's area A $(= \pi R^2)$.

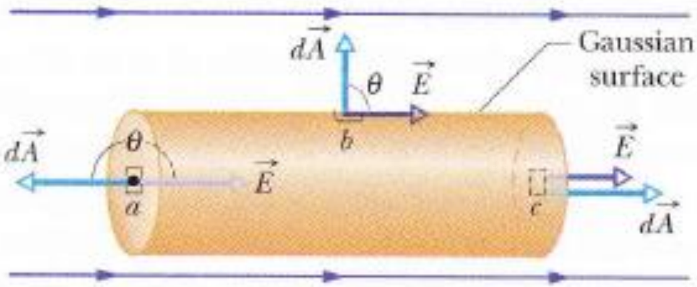


FIG. 23-4 A cylindrical Gaussian surface, closed by end caps, is immersed in a uniform electric field. The cylinder axis is parallel to the field direction.

Left cap: $\displaystyle\int_a \vec{E} \cdot d\vec{A} = \int E(\cos 180°)\, dA = -E \int dA = -EA,$

Right cap: $\displaystyle\int \vec{E} \cdot d\vec{A} = \int E(\cos 0)\, dA = EA.$

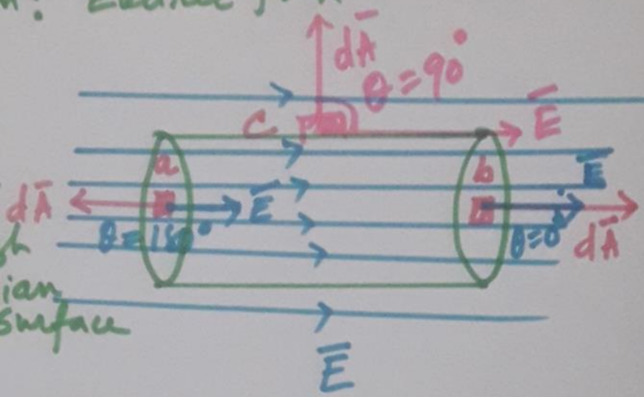
Finally, for the cylindrical surface, where the angle θ is 90° at all points,

$$\int_b \vec{E} \cdot d\vec{A} = \int E(\cos 90°)\, dA = 0.$$

This result is perhaps not surprising because the field lines that represent the electric field all pass entirely through the Gaussian surface, entering through the left end cap, leaving through the right end cap, and giving a net flux of zero.

$$\Phi = -EA + 0 + EA = 0. \qquad \text{(Answer)}$$

Sample problem: Electric flux



Total electric flux through the cylindrical Gaussian surface

$$\phi = \oint \bar{E}\cdot d\bar{A}$$

$$= \oint_a \bar{E}\cdot d\bar{A} + \oint_b \bar{E}\cdot d\bar{A} + \oint_c \bar{E}\cdot d\bar{A}$$

$$= \oint_a E\,dA\cos 180^\circ + \oint_b E\,dA\cos 0^\circ + \oint_c E\,dA\cos 90^\circ$$

$$= -\oint_a E\,dA + \oint_b E\,dA + \oint_c E\,dA\,(0)$$

$$= -E\oint_a dA + E\oint_b dA + 0$$

$$= -EA + EA$$

[A = area of left cap
A = area of right cap]

$$= 0$$

$$\therefore \phi = 0$$

23-4 Gauss' Law

Gauss' law relates the net flux Φ of an electric field through a closed surface (a Gaussian surface) to the *net* charge q_{enc} that is *enclosed* by that surface. It tells us that

$$\varepsilon_0 \Phi = q_{enc} \quad \text{(Gauss' law).} \qquad (23\text{-}6)$$

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q_{enc} \quad \text{(Gauss' law).} \qquad (23\text{-}7)$$

Net charged is located in a Vacuum or in air

If q_{enc} is positive, the net flux is *outward*; if q_{enc} is negative, the net flux is *inward*.

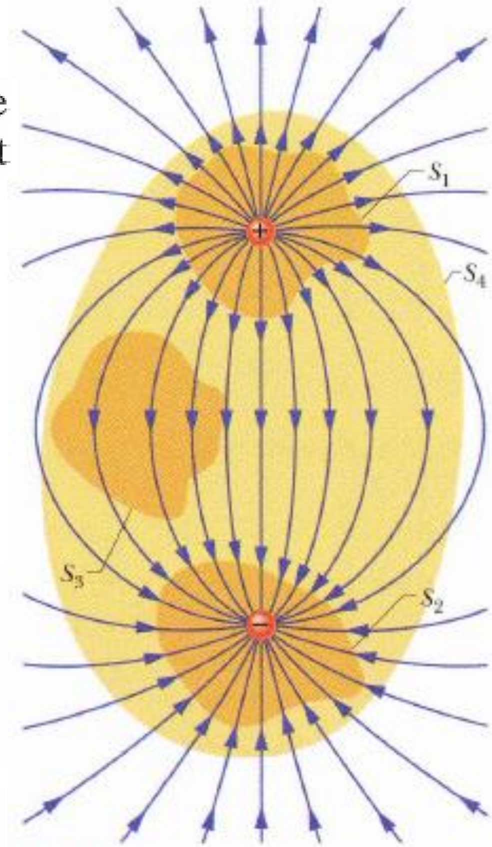


FIG. 23-6 Two point charges, equal in magnitude but opposite in sign, and the field lines that represent their net electric field. Four Gaussian surfaces are shown in cross section. Surface S_1 encloses the positive charge. Surface S_2 encloses the negative charge. Surface S_3 encloses no charge. Surface S_4 encloses both charges and thus no net charge.

Surface S_1. The electric field is outward for all points on this surface. Thus, the flux of the electric field through this surface is positive, and so is the net charge within the surface, as Gauss' law requires. (That is, in Eq. 23-6, if Φ is positive, q_{enc} must be also.)

Surface S_2. The electric field is inward for all points on this surface. Thus, the flux of the electric field is negative and so is the enclosed charge, as Gauss' law requires.

Surface S_3. This surface encloses no charge, and thus $q_{enc} = 0$. Gauss' law (Eq. 23-6) requires that the net flux of the electric field through this surface be zero. That is reasonable because all the field lines pass entirely through the surface, entering it at the top and leaving at the bottom.

Surface S_4. This surface encloses no *net* charge, because the enclosed positive and negative charges have equal magnitudes. Gauss' law requires that the net flux of the electric field through this surface be zero. That is reasonable because there are as many field lines leaving surface S_4 as entering it.

Large Q charge to S4: flux through any of them. The value of Q would not enter Gauss' law in any way, because Q lies outside all four of the Gaussian surfaces that we are considering

23-5 | Gauss' Law and Coulomb's Law

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = \varepsilon_0 \oint E\, dA = q_{\text{enc}}.$$

$$\varepsilon_0 E \oint dA = q. \tag{23-9}$$

$$\varepsilon_0 E(4\pi r^2) = q$$

$$E = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2}. \tag{23-10}$$

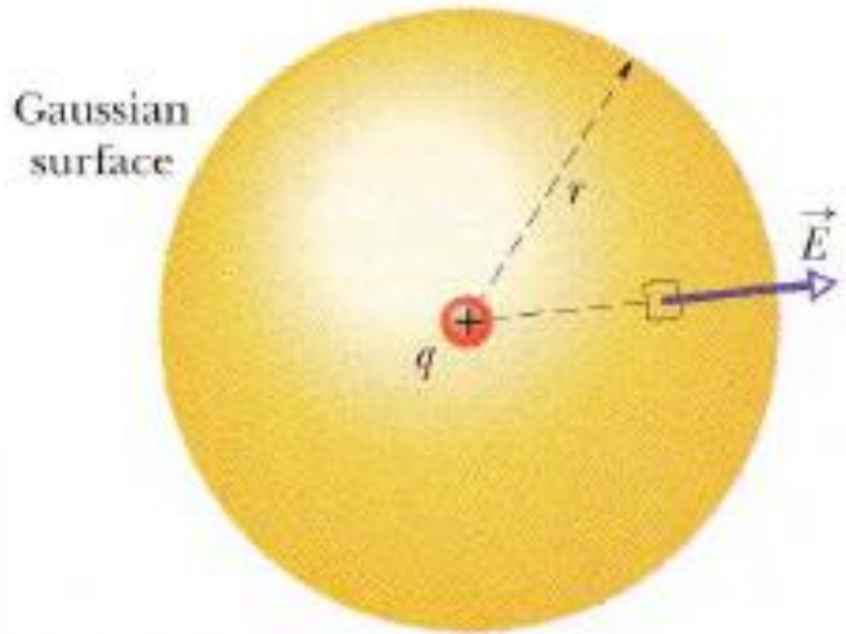
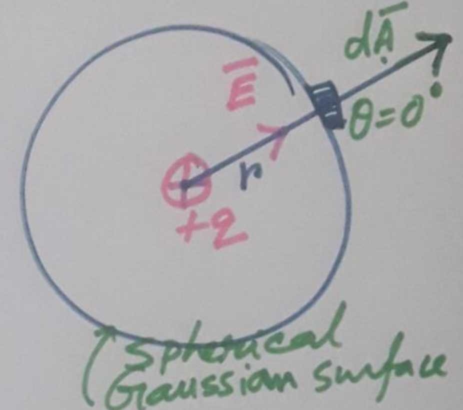


FIG. 23-8 A spherical Gaussian surface centered on a point charge q.

Coulomb's law from Gauss' law:

Gauss' law:

$$\epsilon_0 \oint \bar{E} \cdot d\bar{A} = q_{enc}$$



$$\epsilon_0 \oint E\, dA \cos 0° = q$$

$$\epsilon_0 \oint E\, dA = q$$

$$\epsilon_0 E \oint dA = q$$

[E = constant]

$$\epsilon_0 E (4\pi r^2) = q$$

[Surface area of a sphere = $4\pi r^2$]

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$$

[Coulomb's law]

23-7 | Applying Gauss' Law: Cylindrical Symmetry

Since $2\pi r$ is the cylinder's circumference and h is its height, the area A of the cylindrical surface is $2\pi rh$. The flux of $\vec{E}$ through this cylindrical surface is then

$$\Phi = EA\cos\theta = E(2\pi rh)\cos 0 = E(2\pi rh).$$

There is no flux through the end caps because $\vec{E}$, being radially directed, is parallel to the end caps at every point.

The charge enclosed by the surface is λh, which means Gauss' law,

$$\varepsilon_0\Phi = q_{enc},$$

reduces to

$$\varepsilon_0 E(2\pi rh) = \lambda h,$$

Type equation here.

yielding

$$E = \frac{\lambda}{2\pi\varepsilon_0 r} \quad \text{(line of charge).} \tag{23-12}$$

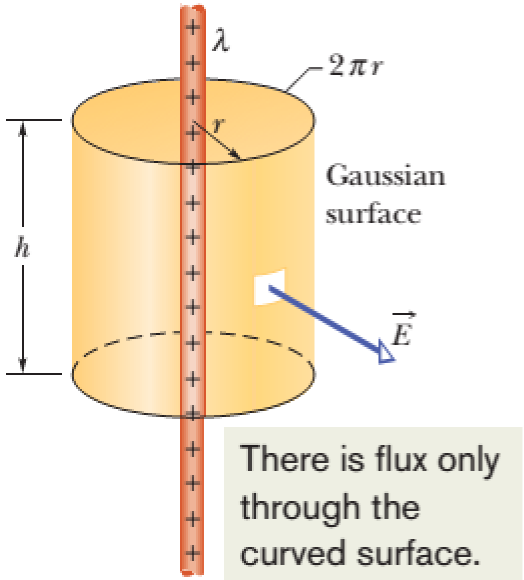


Figure 23-14 A Gaussian surface in the form of a closed cylinder surrounds a section of a very long, uniformly charged, cylindrical plastic rod.

Cylindrical symmetry: Apply Gauss' law

Gauss' law:

$$\epsilon_0 \oint \bar{E} \cdot d\bar{A} = q_{enc}$$

$$\epsilon_0 \oint_a \bar{E} \cdot d\bar{A} + \epsilon_0 \oint_b \bar{E} \cdot d\bar{A} + \epsilon_0 \oint_c \bar{E} \cdot d\bar{A} = q_{enc}$$

$$\epsilon_0 \oint_a E\,dA\cos 90^\circ + \epsilon_0 \oint_b E\,dA\cos 90^\circ + \epsilon_0 \oint_c E\,dA\cos 0^\circ = q_{enc}$$

$$0 + 0 + \epsilon_0 \oint_c E\,dA = \lambda h$$

$$\epsilon_0 E \oint_c dA = \lambda h$$

$$\epsilon_0 E (2\pi r \cancel{h}) = \lambda \cancel{h}$$

$$E = \frac{1}{2\pi\epsilon_0} \frac{\lambda}{r}$$

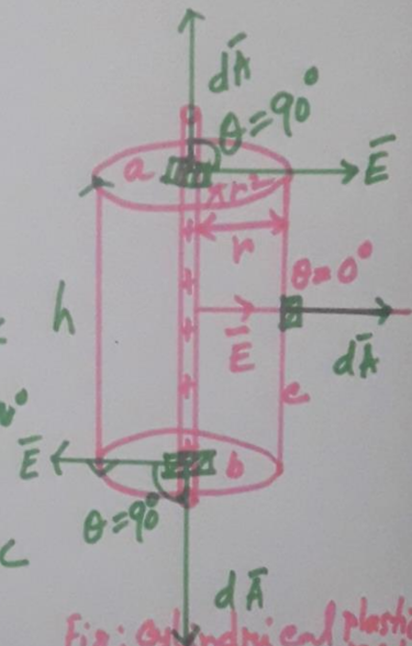


Fig: cylindrical plastic rod

a = top cap

b = bottom cap

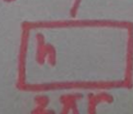
c = curved surface of a cylinder

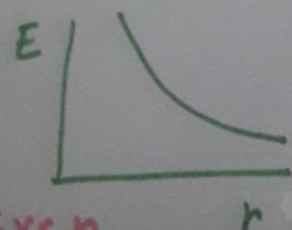
linear charge density,

$$\lambda = \frac{q_{enc}}{h}$$

$$q_{enc} = \lambda h$$

Area of curved surface $= 2\pi r h$

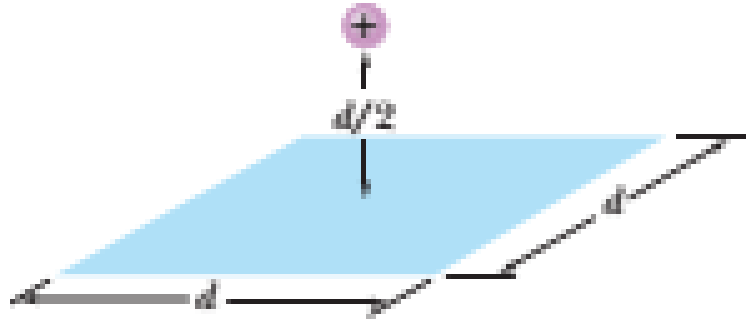




E vs r

Problem 5 (book chapter 23)

A proton is a distance *d*/*2* directly above the center of a square of side *d*. What is the magnitude of the electric flux through the square?



Answer:

Consider a cube consisting with six square faces; each of edge d. Thus, the proton is enclosed by the cubical Gaussian surface.

Electric flux through the cubical Gaussian surface is

$$\Phi_C = \frac{q}{\varepsilon_0} = \frac{e^+}{\varepsilon_0} = \frac{+1.6 \times 10^{-19}}{8.854 \times 10^{-12}} = 0.18 \times 10^{-7} = 1.80 \times 10^{-8}\ \frac{N-m^2}{C}$$

Electric flux through a face (a square) of a cube is

$$\Phi_s = \frac{\Phi_C}{6} = \frac{180 \times 10^{-9}}{6} = 3.01 \times 10^{-9}\ \frac{N-m^2}{C}$$

Problem 25 (Book chapter 23

An infinite line of charge produces a field of magnitude $4.5 \times 10^4\ N/C$ at distance 2.0 m. Find the linear charge density.

Given,

$$E = 4.5 \times 10^4\ N/C$$

$$r = 2.0\ m$$

$$\lambda = ?$$

$$E = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{r}$$

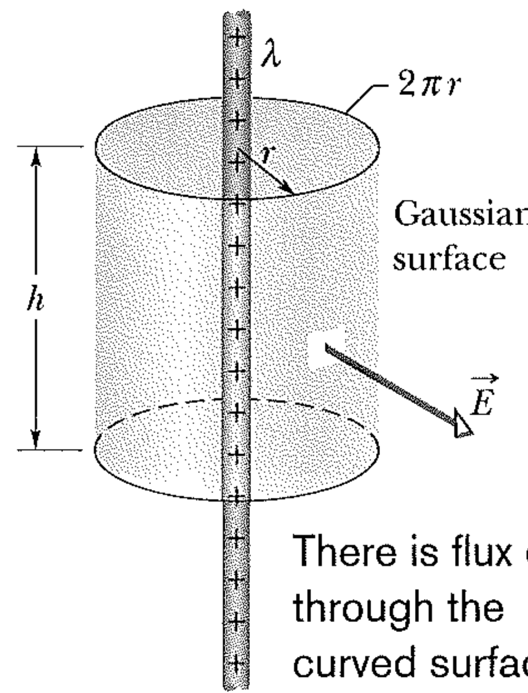
$$E = \frac{1}{4\pi\varepsilon_0}\frac{2\lambda}{r}$$

$$E = k\frac{2\lambda}{r}$$

$$\lambda = \frac{Er}{2k}$$

$$\lambda = \frac{(4.5 \times 10^4)(2)}{(2)(9 \times 10^9)} = 0.25 \times 10^{-5}$$

$$\lambda = 2.5 \times 10^{-6}\ \mathrm{C/m}$$



There is flux only through the curved surface.