LESSON 5

BOOK CHAPTER 24

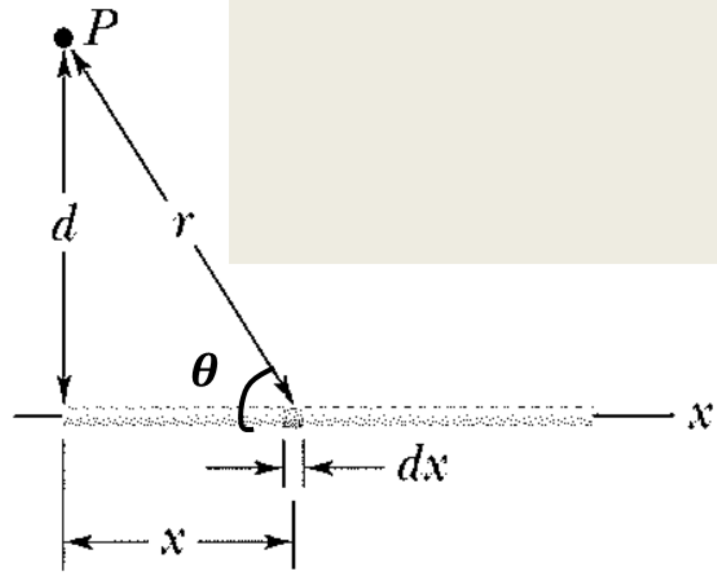
ELECTRIC POTENTIAL

Potential due to a Line of Charge:

A thin non-conducting rod of length *L* has a positive charge of uniform linear density *λ*. Let us determine the electric potential *V* due to the rod at point *P*, a perpendicular distance *d* from the left end of the rod.

Potential due to a Line of Charge

We consider a differential element *dx* of the rod as shown in Fig. This element of the rod has a differential charge *dq*.



$$\text{Linear charge density, } \lambda = \frac{dq}{dx}$$

$$dq = \lambda\, dx$$

$$r = \sqrt{d^2 + x^2}$$

Treating the element as a point charge, the potential dV at point P is

$$dV = \frac{1}{4\pi\epsilon_0}\frac{dq}{r} = \frac{1}{4\pi\epsilon_0}\frac{\lambda\, dx}{(d^2 + x^2)^{1/2}}$$

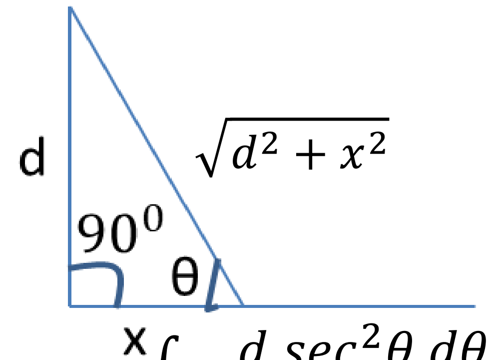
The total potential *V* produced by the rod at point *P* by integrating the above equation along the length of the rod, from *x* = 0 to *x* = *L*

$$V = \int dV = \int_{x=0}^{x=L} \frac{1}{4\pi\epsilon_0}\frac{\lambda\, dx}{(d^2 + x^2)^{1/2}} = \frac{\lambda}{4\pi\epsilon_0}\int_0^L \frac{dx}{(d^2 + x^2)^{1/2}}$$

Let $\tan\theta = \frac{x}{d}$

$$x = d\tan\theta$$

$$dx = d\,sec^2\theta\,d\theta$$



$$\int \frac{dx}{(d^2 + x^2)^{1/2}} = \int \frac{d\,sec^2\theta\,d\theta}{(d^2 + d^2tan^2\theta)^{1/2}} = \int \frac{d\,sec^2\theta\,d\theta}{\{d^2(1 + tan^2\theta)\}^{1/2}} = \int \frac{d\,sec^2\theta\,d\theta}{\{d^2(sec^2\theta)\}^{1/2}}$$

$$= \int \frac{d\,sec^2\theta\,d\theta}{d\sec\theta} = \int sec\theta\,d\theta = \ln|sec\,\theta + \tan\theta| = \ln\left|\frac{1}{cos\,\theta} + \tan\theta\right|$$

$$= \ln\left|\frac{1}{\frac{d}{\sqrt{d^2+x^2}}} + \frac{x}{d}\right| = \ln\left|\frac{x + \sqrt{d^2 + x^2}}{d}\right|$$

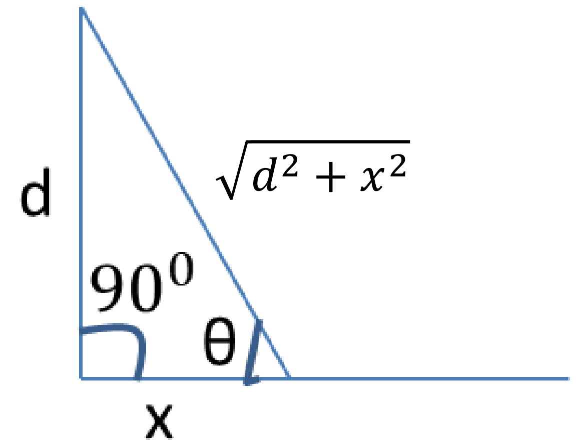
$$V = \frac{\lambda}{4\pi\epsilon_0}\int_{x=0}^{x=L} \frac{dx}{(d^2 + x^2)^{1/2}} = \frac{\lambda}{4\pi\varepsilon_0}[\ln\left|\frac{x + \sqrt{d^2 + x^2}}{d}\right|]_0^L$$

$$= \frac{\lambda}{4\pi\varepsilon_0}[\ln\left|\frac{L + \sqrt{d^2 + L^2}}{d}\right| - ln\left|\frac{0 + \sqrt{d^2 + 0^2}}{d}\right|]$$

$$= \frac{\lambda}{4\pi\varepsilon_0}[\ln\left|\frac{L + \sqrt{d^2 + L^2}}{d}\right| - ln1] = \frac{\lambda}{4\pi\varepsilon_0}[\ln\left|\frac{L + \sqrt{d^2 + L^2}}{d}\right| - 0]$$

$$\boxed{V = \frac{\lambda}{4\pi\varepsilon_0}\ln\left|\frac{L + \sqrt{d^2 + L^2}}{d}\right|}$$

Let $\tan\theta = \frac{d}{x}$ $\quad and \;\; x = d\cot\theta$



$$dx = -d\; cosec^2\theta\; d\theta$$

$$\int \frac{dx}{(d^2 + x^2)^{1/2}} = \int \frac{-d\; cosec^2\theta\;\; d\theta}{(d^2 + d^2 cot^2\theta)^{1/2}} = -\int cosec\theta\; d\theta = \ln|cosec\;\theta + \cot\theta|$$

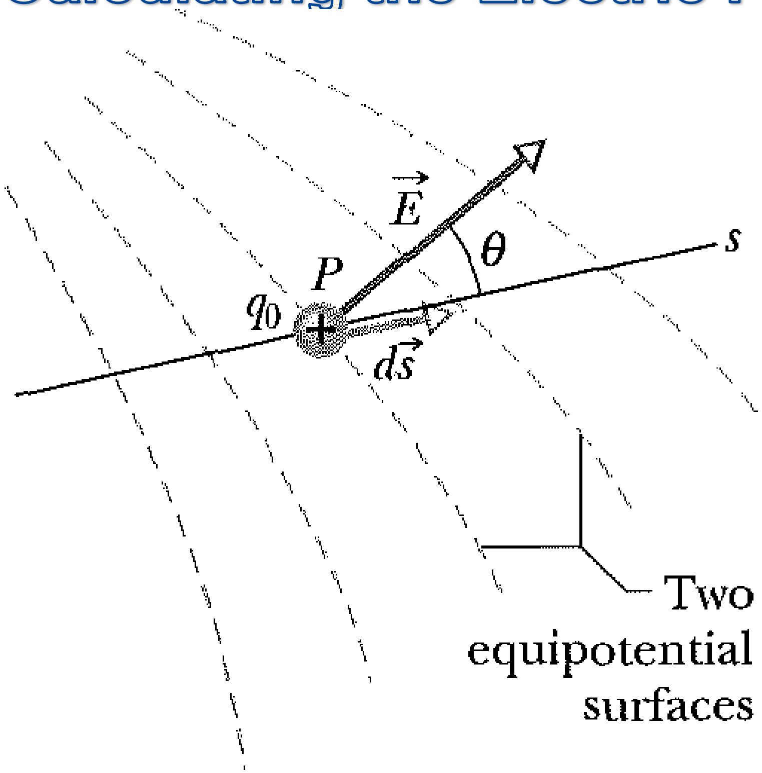
$$= \ln\left|\frac{\sqrt{d^2 + x^2}}{d} + \frac{x}{d}\right|$$

Now

$$V = \frac{\lambda}{4\pi\epsilon_0}\int_{x=0}^{x=L} \frac{dx}{(d^2 + x^2)^{1/2}} = \frac{\lambda}{4\pi\varepsilon_0}[\ln\left|\frac{\sqrt{d^2 + L^2}}{d} + \frac{L}{d}\right| - \ln\frac{d}{d}]$$

$$\boxed{V = \frac{\lambda}{4\pi\varepsilon_0}\ln\left|\frac{\sqrt{d^2 + L^2} + L}{d}\right|}$$

Calculating the Electric Field from the Electric Potential:



Suppose that a positive test charge q_0 moves through a displacement $d\vec{s}$ from one equipotential surface to the adjacent surface.

The differential work done in terms of electric potential difference dV is

$dW = -q_0 dV$ [external agent does not help]

The differential work done by the electric Field $\vec{E}$ is

$$dW = \vec{F}.d\vec{s} = q_0\vec{E}.d\vec{s} = q_0 E(cos\theta)\, ds$$

Hence, we get $$-q_0 dV = q_0 E(cos\theta)\, ds$$

$$E(cos\theta) = -\frac{dV}{ds}$$

Since $E\cos\theta$ *is the component of* $\vec{E}$ in the direction of $d\vec{s}$, then $E_s = -\dfrac{\partial V}{\partial s}$

If we take the s axis to be, in turn, the X, *y*, and z axes, we find that the X, *y*, and z components of $\vec{E}$ at any point are

$$\boxed{E_x = -\frac{\partial V}{\partial x}\ ; \qquad E_y = -\frac{\partial V}{\partial y}\ ; \qquad E_z = -\frac{\partial V}{\partial z}}$$

Field from the potential: $\bar{E} = -\bar{\nabla} V$

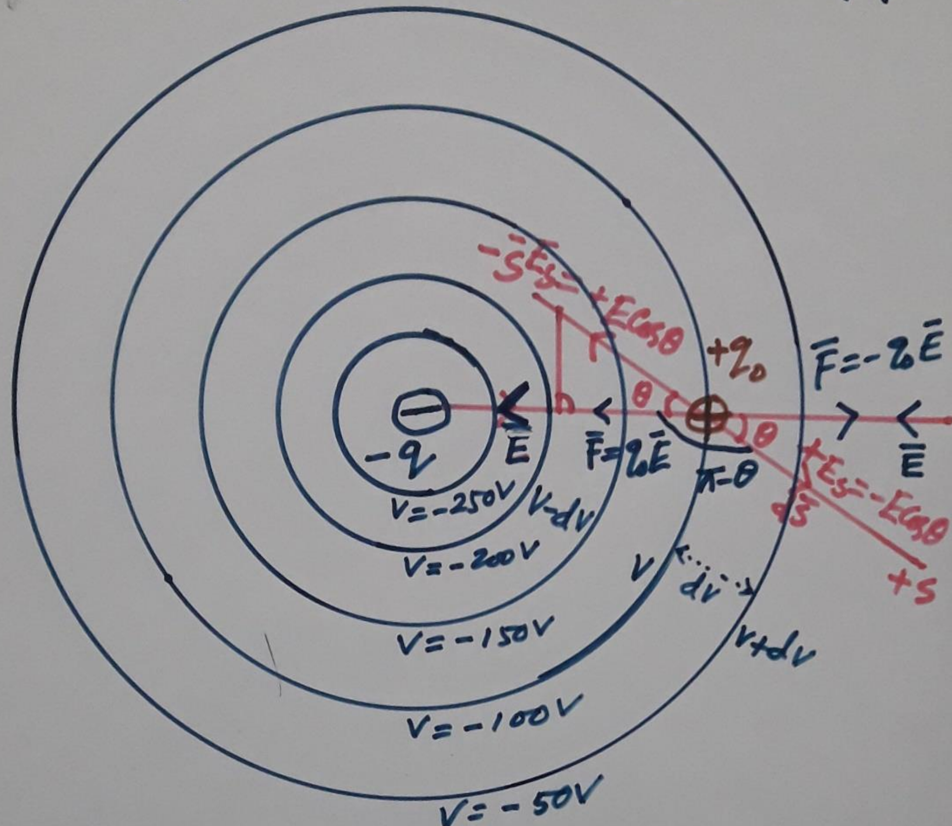


Fig: $-q$ produces the $\bar{E}$ and V.

$$dV = \frac{dW}{q_0} = \frac{\bar{F}\cdot d\bar{s}}{q_0} = \frac{(-q_0\bar{E})\cdot d\bar{s}}{q_0} = -\bar{E}\cdot d\bar{s}$$

$$= -E\,ds\cos(\pi-\theta) = -E\,ds(-\cos\theta) = E\,ds\cos\theta$$

$$= (E\cos\theta)\,ds$$

$$E\cos\theta = \frac{dV}{ds} \text{ or, } -E\cos\theta = -\frac{dV}{ds} \quad \begin{bmatrix} -E_s = +E\cos\theta \\ +E_s = -E\cos\theta \end{bmatrix}$$

$$\boxed{E_s = -\frac{dV}{ds}}$$

$$\boxed{E_s = -\frac{dV}{ds}}$$

$$E_x = -\frac{\partial V}{\partial x}$$

$$E_y = -\frac{\partial V}{\partial y}$$

$$E_z = -\frac{\partial V}{\partial z}$$

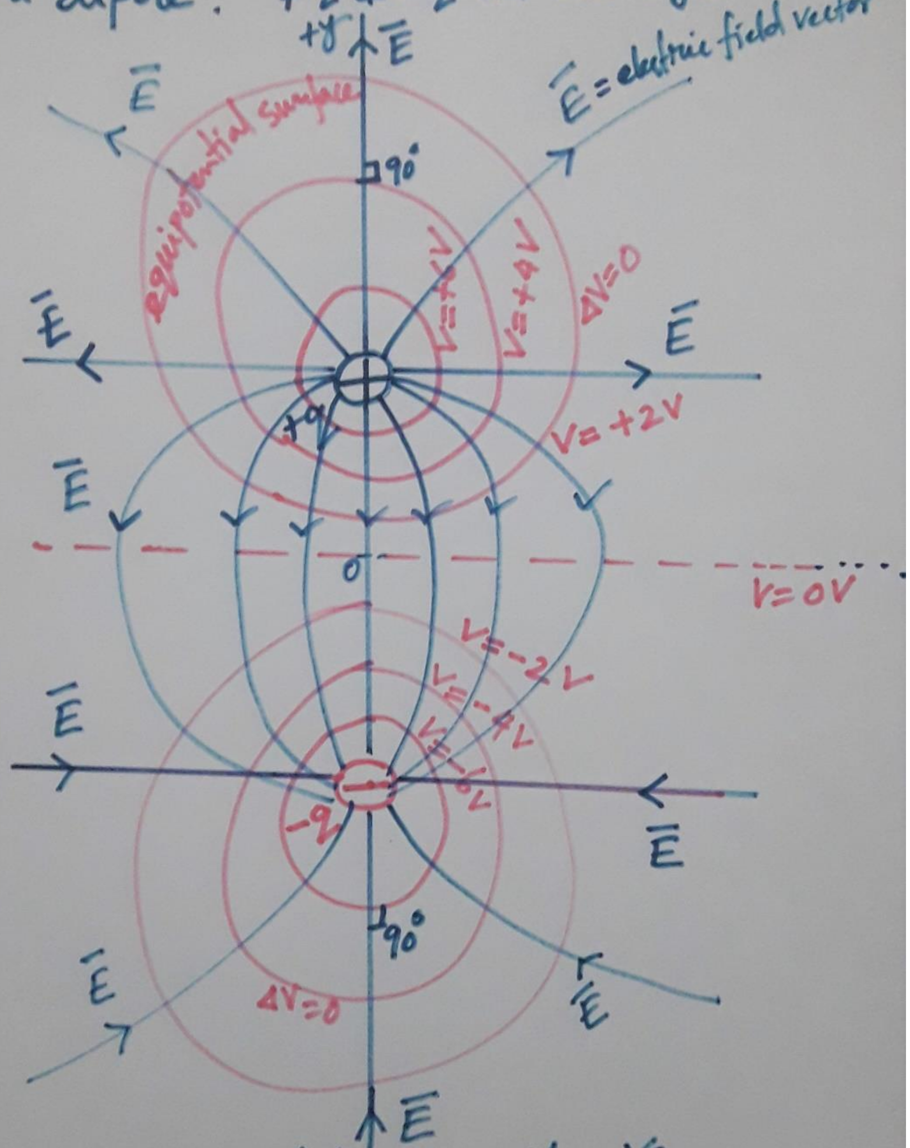
$$\bar{E} = -\left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)V$$

$$\boxed{\bar{E} = -\bar{\nabla} V}$$ [$\bar{\nabla} V$ = potential gradient]

E is the negative of the gradient of V.

Potential V decreases in the $\bar{E}$ direction.

e) For a dipole : +q & −q and d is very small

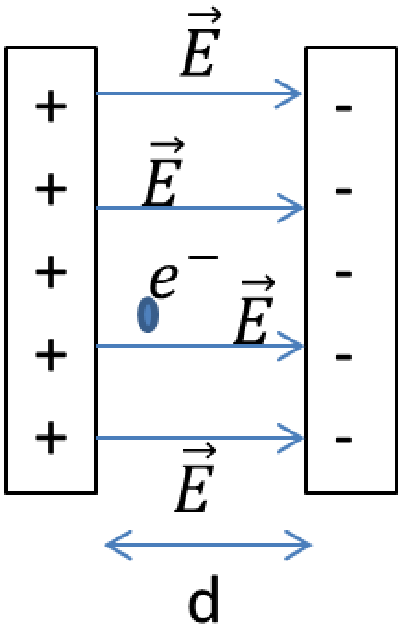


For +q, V=+ve and for −q, V=−ve

Fig: dipole shows $\bar{E}$ & V

Problem: 4 (Book chapter 24)

Two large, parallel, conducting plates are 12 cm apart and have charges of equal magnitude and opposite sign on their facing surfaces. An electric force of $3.9 \times 10^{-15}\ N$ acts on an electron placed anywhere between the two plates. (Neglect fringing.) (a) Find the electric field at the position of the electron. (b) What is the potential difference between the plates?



Given

d $=|\Delta x|=$ 12 cm = 0.12 m

$|F| = 3.9 \times 10^{-15} N$

$|e^-| = |q| = 1.6 \times 10^{-19}$C

(a) $|F| = qE$

$$E = \frac{3.9 \times 10^{-15}}{1.6 \times 10^{-19}} = 2.437 \times 10^4 \frac{N}{C}$$

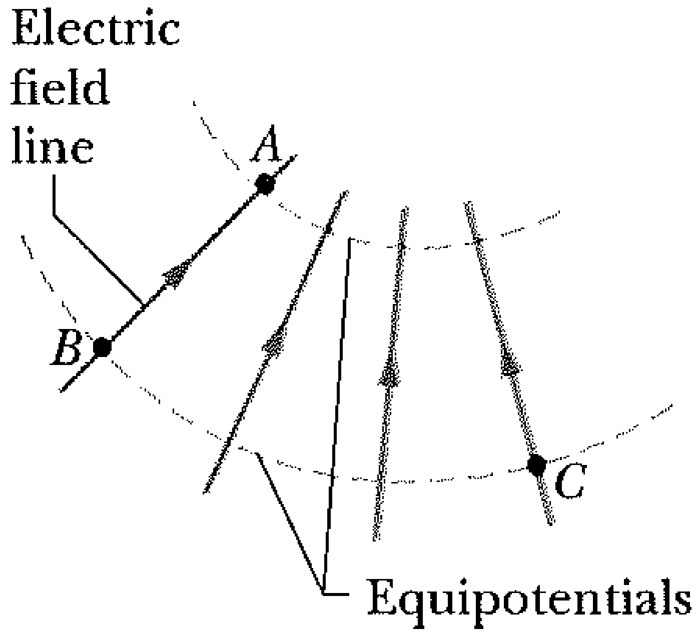
(b) The potential difference between the plates

$$E = \frac{|\Delta V|}{|\Delta x|}$$

$$|\Delta V| = E|\Delta x| = (2.437 \times 10^4)(0.12) = 0.293 \times 10^4\ Volt$$

Problem: 6 (Book chapter 24)

When an electron moves from A to B along an electric field line in the adjacent figure, the electric field does $3.94 \times 10^{-19}\, J$ of work on it. What are the electric potential differences (a) $V_B - V_A$, (b) $V_C - V_A$, and (c) $V_C - V_B$?



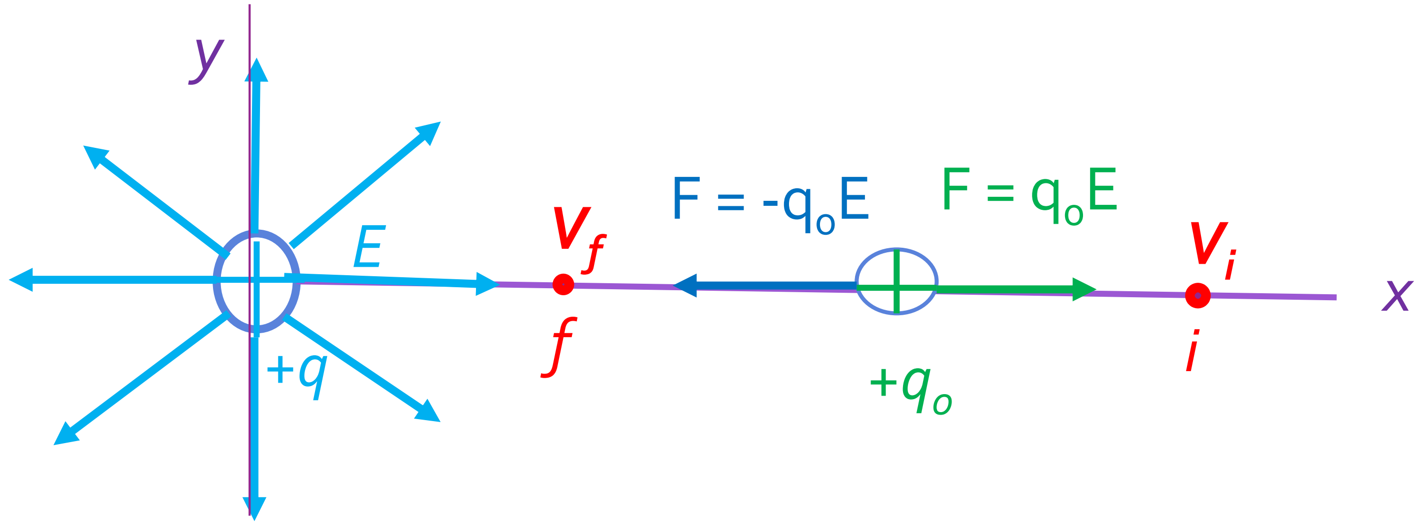
Answer:

(a) $$V_B - V_A = \frac{-W}{q} = \frac{-3.94 \times 10^{-19}}{-1.6 \times 10^{-19}} = 2.45\, Volt$$

(b) $$V_c - V_A = V_B - V_A = 2.45\, Volt$$

[Because points B and C are on the same equipotential surface, V_C=V_B]

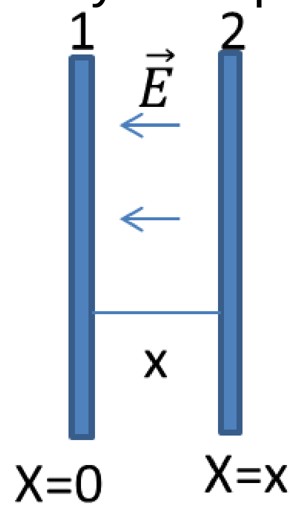
(c) $$V_c - V_B = 0\, Volt$$



Positive point charge: + q

Problem 36 (Book Chapter 24)

The electric potential *V* in the space between two flat parallel plates 1 and 2 is given (in volts) by $V = 1500x^2$, where *x* (in meters) is the perpendicular distance from plate 1. At $x = 1.3\ cm$, (a) what is the magnitude of the electric field and (b) is the field directed toward or away from plate 1?



Given

$$V = 1500\ x^2$$

At x = 1.3 cm= 0.013 m ,

$$E = ?$$

The direction of electric field,

$$\vec{E} = ?$$

(a) We have

$$E_x = -\frac{\partial V}{\partial x} = -\frac{\partial}{\partial x}(1500x^2) = -3000\ x$$

$$E_x = -3000 \times 0.013 = -39\ \frac{V}{m}$$

$$\vec{E} = E_x\hat{\imath} = 39(-\ \hat{\imath}\ )$$

Magnitude of $\vec{E}$ is

$$E = 39\ \frac{V}{m}$$

(b) The direction of electric field is toward plate 1, because $\vec{E} = 39(-\ \hat{\imath})\frac{V}{m}$

Problem 37 (Book Chapter 24)

What is the magnitude of the electric field at the point $(3.00\hat{\imath} - 2.00\hat{\jmath} + 4.00\hat{k})\ m$ if the electric potential in the region is given by $V = 2.00xyz^2$, where *V* is in volts and coordinates *x*, *y*, and *z* are in meters?

Answer:

We know

$$E_x = -\frac{\partial V}{\partial x} = -\frac{\partial}{\partial x}(2xyz^2) = -2yz^2 = -(2)(-2)(4^2) = 64\frac{V}{m}$$

$$E_y = -\frac{\partial V}{\partial y} = -\frac{\partial}{\partial y}(2xyz^2) = -2xz^2 = -(2)(3)(4^2) = -96\frac{V}{m}$$

$$E_z = -\frac{\partial V}{\partial z} = -\frac{\partial}{\partial z}(2xyz^2) = -4xyz = -(4)(3)(-2)(4) = 96\frac{V}{m}$$

$$\vec{E} = E_x\hat{\imath} + E_y\hat{\jmath} + E_z\hat{k} = 64\hat{\imath} - 96\hat{\jmath} + 96\hat{k}$$

Therefore,

$$|\vec{E}| = \sqrt{(64)^2+(-96)^2+(96)^2} = 150.09\frac{V}{m}$$

Given

$$V = 2xyz^2$$

And

$$(x, y, z) = (3, -2, 4)$$

$$|\vec{E}| = ?$$

Thank You