Experiment No. 1:

To determine the acceleration due to gravity applying linear least square regression method by using a simple pendulum.

1.1 Objectives:

The main objective of this lab is to determine the acceleration due to gravity in the lab with a simple pendulum. Also to learn how the linear least square regression method can be used to find the regression line for a set of data.

1.2 Prelab:

Student should read the lab manual and have clear idea about the objective, time frame and outcomes of the lab.

1.3 Outcomes:

After completing this experiment student should be able to answer the following questions:

- What is acceleration due to gravity?
- How a simple pendulum can be constructed and what are its criteria?
- How linear least square regression method (LLSRM) can be used to find the regression line? Why we need to learn it?
- How LLSRM method can be used to find slope and intersection for any number of data?
- How acceleration due to gravity can be calculated from the slope of the regression line?

1.4 Timing and Length of Investigation (Total 3 Hours):

- **Lab Preparation (15 minutes):**
    - Students will sit for the lab class with preparations and class attendance will be taken.
- **Lecture on Theory (30 minutes):**
    - Teacher will clarify the objective and theory of the experiment.
- **Lecture on Procedure (15 minutes):**
    - Students will learn about the procedure of the experiment through a video lecture.
- **Experimental Work (90 to 100 minutes):**
    - A sample data table will be provided to students and teacher will clarify every part of it.
    - Students will do all the calculations, draw graphs in excel and complete the result part.
- **Post Lab Discussion (15 to 20 minutes):**
    - Teacher will summarize the total lab work and have a discussion with the students related with the questions given in the outcomes part.
- **Report Submission:**
    - After completing the lab reports students will upload their lab reports as groups in the assignment section of MS Teams.

1.5 Theory:

The time period of small-angle oscillation of a simple pendulum (a metal bob attached by a light string and suspended vertically from a fixed support) can be shown to be

$$T = 2\pi\sqrt{\frac{L}{g}}$$

where L is the effective length (length from the point of suspension to the center of the bob) and time period (time of one complete oscillation) of a simple pendulum, respectively in a place where the acceleration due to gravity is g.

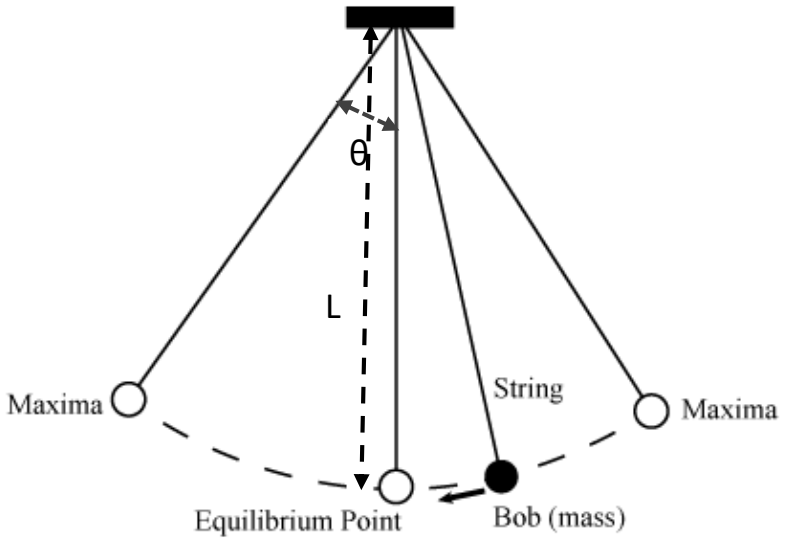

Figure 1.1: A swinging simple pendulum with an effective length L and amplitude θ.

The time period equation of a simple pendulum can be rearranged as

$$T^2 = \frac{4\pi^2}{g}L$$

Comparing this equation with the state line equation that goes through the origin (y = mx) the value of acceleration due to gravity can be determined by

$$g = \frac{4\pi^2}{m}$$

where m is the slope of the T^2 vs L graph.

For two types (independent and dependent) of variables x and y = f(x) the linear least square regression method can be used for N number of data points to find the best fitted line (regression line) as the fig. 1.2 shows.

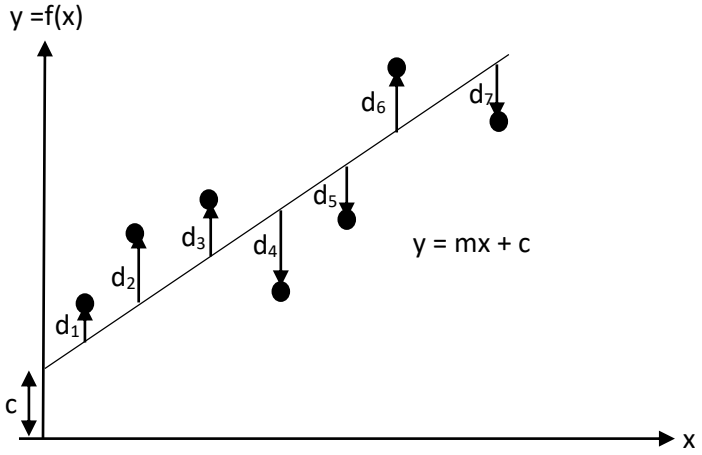

Figure 1.2: Way to get the best fitted line by finding the minimum value of $D = d_1^2 + d_2^2 + d_3^2 + d_4^2 + d_5^2 + d_6^2 + d_7^2$ according to the least square regression method. The equation for the best fitted line is y = mx + c, where m is the slope and c is the interception in the y axis. Here the number of data points is taken as N=7.

The formula for determining the slope of the regression line

$$m = \frac{\sum_i x_i y_i - \frac{(\sum_i x_i)(\sum_i y_i)}{N}}{\sum_i x_i^2 - \frac{(\sum_i x_i)^2}{N}} \qquad \text{(slope equation)}$$

and intercept $c = \bar{y} - m\bar{x}$, where $\bar{x}$ and $\bar{y}$ are mean value of x and y.

In the slope equation:

$$\sum_i x_i = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7,$$
$$\sum_i y_i = y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7,$$
$$\sum_i x_i y_i = x_1 y_1 + x_2 y_2 + x_3 y_3 + x_4 y_4 + x_5 y_5 + x_6 y_6 + x_7 y_7,$$
$$(\sum_i x_i)^2 = (x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7)^2,$$
$$\sum_i x_i^2 = x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2 + x_6^2 + x_7^2$$

1.6 Apparatus:

Metal bob, a piece of string, stand, clamp, meter scale and stop watch.

1.7 Procedure:

- Attach a light piece of string with the hook of the metal bob. Find the length L of the pendulum with a meter scale from the point of suspension to the mid-point of the bob.
- Give a small angle (**less than 10 degrees**) swing to the pendulum. Find the time period, T. To do it, measure the total time for 20 oscillations and divide it by 20. Repeat the procedure for different lengths and record the data in table 1.1.
- Using the Linear Least Square Regression Method (LLSRM) find the regression line and from the value of slope find g from the relation: slope = $4\pi^2/g$.
- Plot the same graph in Excel and also find the value of g from the equation of the graph.

1.8 Experimental Data:

Table 1.1: Time periods T for different lengths L of the simple pendulum.

No. of Obs.	Effective Length L (cm)	Time for 20 Oscillations t (s)	Time period T = t/20 (s)	T^2 (s^2)	L^2 (cm^2)	$L.T^2$ ($cm.s^2$)
1	150					
2	140					
3	130					
4	120					
5	110					
6	100					
7	90					
$\sum$						

1.9 Analysis:

Table 1.2: Finding the slope, m and intercept, c by using the linear least square regression method.

N	$\sum_i x_i$	$\sum_i y_i$	$\sum_i x_i y_i$	$\left(\sum_i x_i\right)^2$	$\sum_i x_i^2$	m	c
7							
Equation:							

A. The value of g using the LLSRM:

$$m = \frac{\sum_i x_i y_i - \frac{(\sum_i x_i)(\sum_i y_i)}{N}}{\sum_i x_i^2 - \frac{(\sum_i x_i)^2}{N}} =$$

$$\bar{x} = \frac{\sum_i x_i}{N} = \frac{840}{7} =$$

$$\bar{y} = \frac{\sum_i y_i}{N} =$$

Intercept, c = $\bar{y} - m\ \bar{x} =$

Acceleration due to gravity by LLSRM, $g_L = \frac{4\pi^2}{m} =$

B. The value of g from the graph of Excel:

Slope of the regression line, m =

Acceleration due to gravity by Excel, $g_E = \frac{4\pi^2}{m} =$

C. Percentage of difference in g between Excel and LLSRM:

$$\frac{g_E \sim g_L}{g_E} \times 100 =$$

1.10 Result:

Method	Value of g (m/s^2)	Comment
LLSRM		
Excel		

1.11 Resources:

For further understanding, students may go through the following resources:

- **Fundamentals of Physics**: Acceleration due to gravity (Chapter 13, page 360), Simple pendulum (Chapter 15, page 425-426)
- **Video Link**:
  - Simple pendulum: 1. https://www.youtube.com/watch?v=02w9lSii_Hs
    2. https://www.youtube.com/watch?v=bJKEN43695k
  - LLSRM: 1. https://www.youtube.com/watch?v=0T0z8d0_aY4
    2. https://www.youtube.com/watch?v=1C3olrs1CUw