LESSON 4

BOOK CHAPTER 24

ELECTRIC POTENTIAL

Electric potential, V:

If charge is +q:

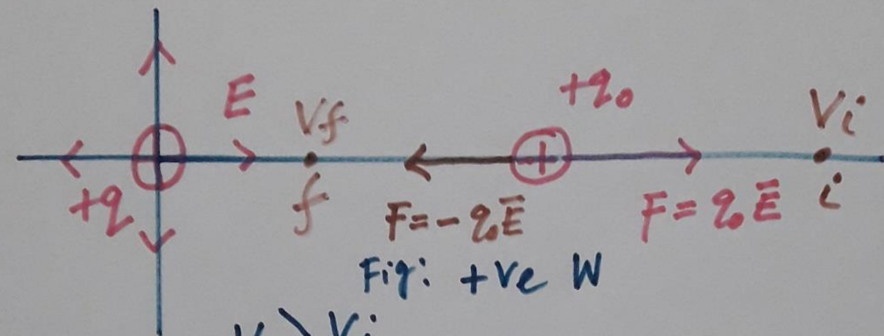


Fig: +ve W

$V_f > V_i$

$$V_f - V_i = \frac{W_{if}}{q_0}$$

at infinity: $V_f - 0 = \frac{W}{q_0}$

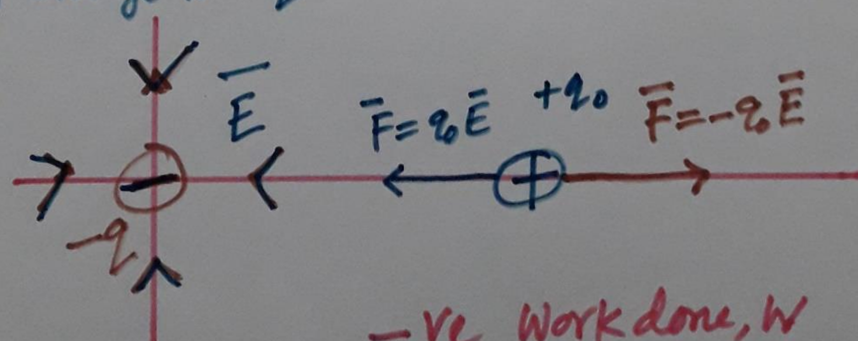
$V_i = 0$

if $V_f = V$ $\qquad \boxed{V = \frac{W}{q_0}}$ $\qquad \boxed{E = \frac{F}{q_0}}$ vector

similar

scalar

If charge is −q:



−ve Work done, W

$$V_f - V_i = \frac{W_{if}}{q_0}$$

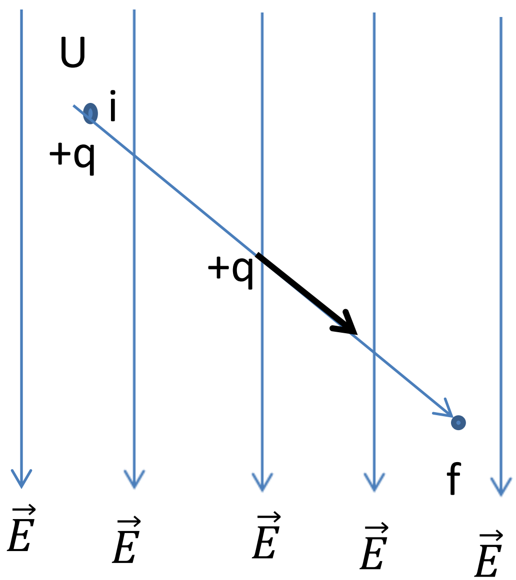
if $V_f > V_i$, $V_f - V_i = \frac{+W_{if}}{q_0}$ [W = +ve]

if $V_i > V_f$, $V_f - V_i = \frac{-W_{if}}{q_0}$ [W = -ve]

if $V_f = V_i$, $0 = \frac{W_{if}}{q_0}$ [W = 0]

$$W_{if} = 0$$

Electric Potential



The potential energy (U) per unit charge(q) at a point in an electric field ($\vec{E}$) is called the **electric potential V**.

$$V = \frac{U}{q} \quad \text{.............} \quad (i)$$

The electric potential difference between any two points i and f in an electric field is equal to the difference in potential energy per unit charge between the two points:

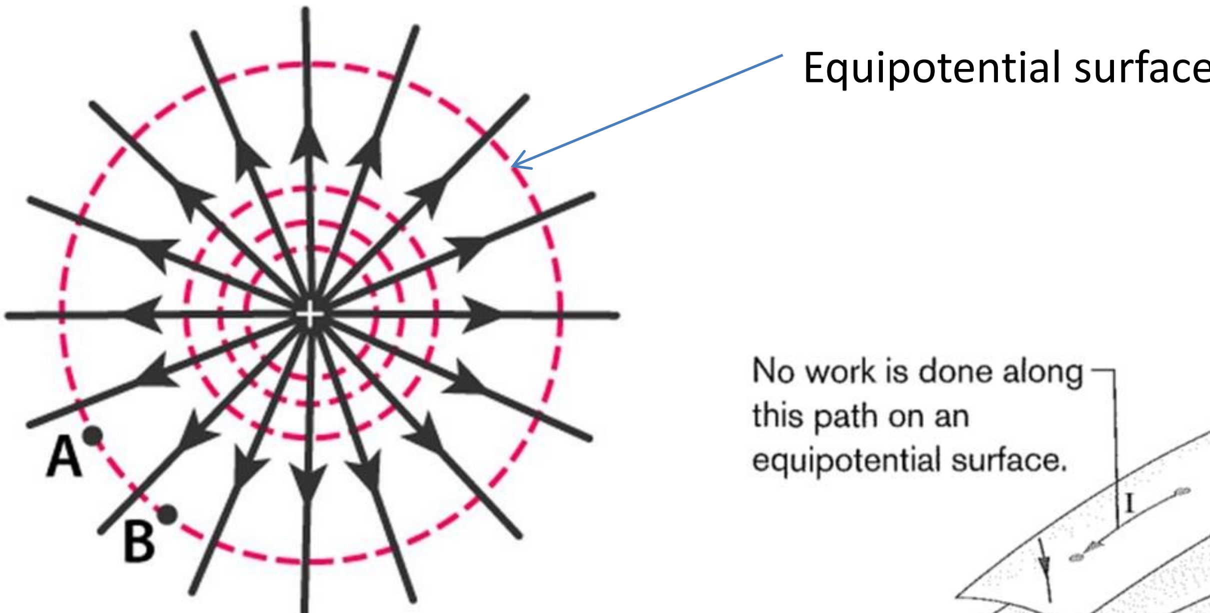
$$\Delta V = V_f - V_i = \frac{U_f}{q} - \frac{U_i}{q} = \frac{\Delta U}{q} \quad \text{.........} \ (ii)$$

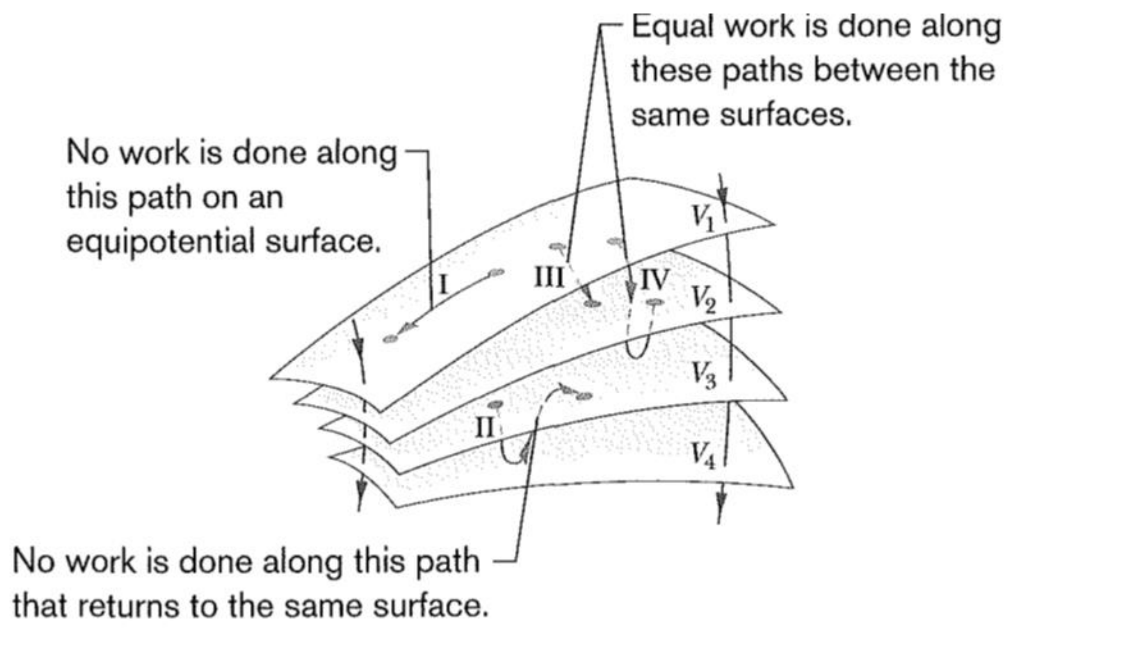
we can define the potential difference between points i and f as

$$\Delta V = V_f - V_i = \frac{-W}{q} \quad \text{...........} \ (iii)$$

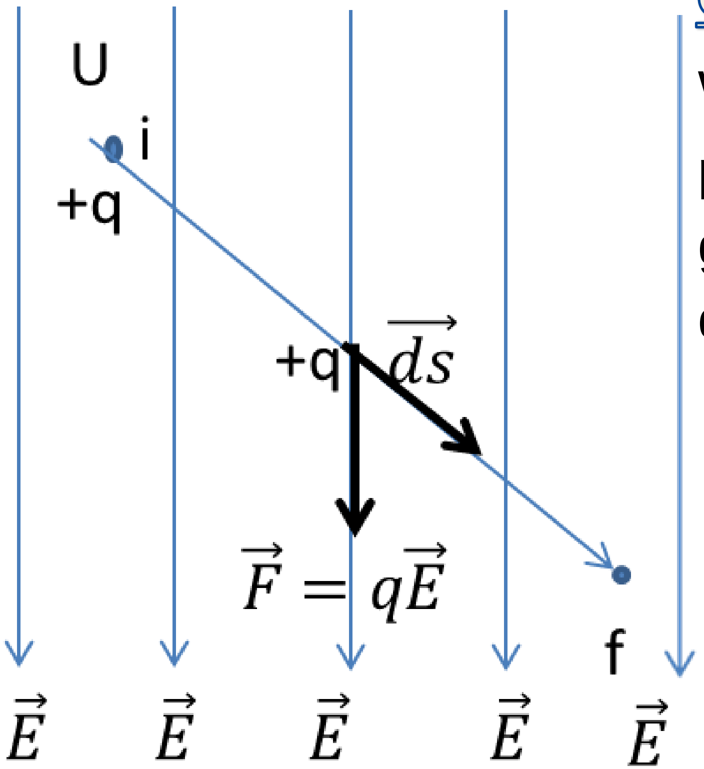
Equipotential Surfaces:

Adjacent points that have the same electric potential form an **equipoteutial** surface, which can be either an imaginary surface or a real, physical surface. No net work *W* is done on a charged particle by an electric field when the particle moves between two points A and *B* on the same equipotential surface.





Calculating the Potential from the Field



We know that the differential work *dW* done on a particle by a force $\vec{F}$ during a displacement $d\vec{s}$ is given by the dot product of the force and the displacement:

$$dW = \vec{F}.d\vec{s} = q\vec{E}.d\vec{s}$$

The total work *W* done on the particle by the field as the particle moves from point *i* to point *f*,

$$W = \int_i^f dW = q\int_i^f \vec{E}.d\vec{s}$$

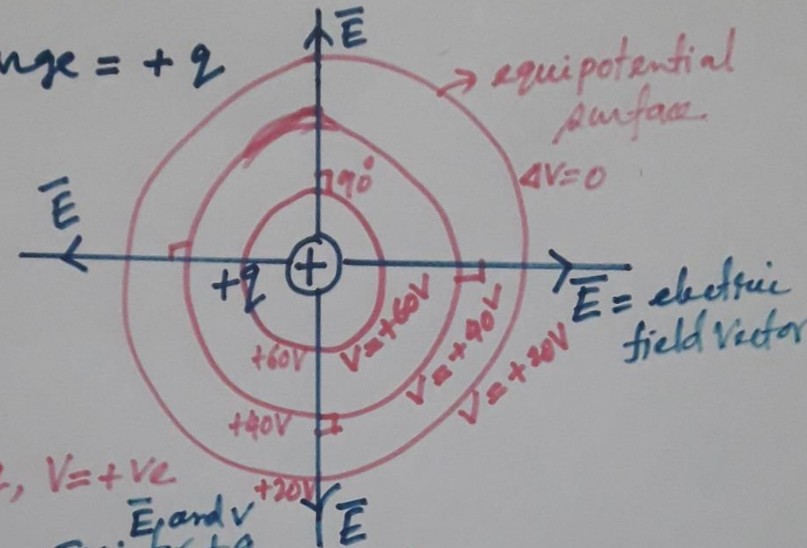
The work done by the electrostatic force in terms of potential difference:

$$\Delta V = V_f - V_i = \frac{-W}{q}$$

$$V_f - V_i = \frac{-q\int_i^f \vec{E}.d\vec{s}}{q} = -\int_i^f \vec{E}.d\vec{s} \qquad \left[\text{Using } W = \int_i^f \vec{E}.d\vec{s}\,\right]$$

Finally, $\boldsymbol{V_f - V_i = -\int_i^f \vec{E}.d\vec{s}}$ .... **(iv)**

(a) Change = +q



For +q, V=+ve

Fig: $\bar{E}$ and V for +q

(b) Change: −q

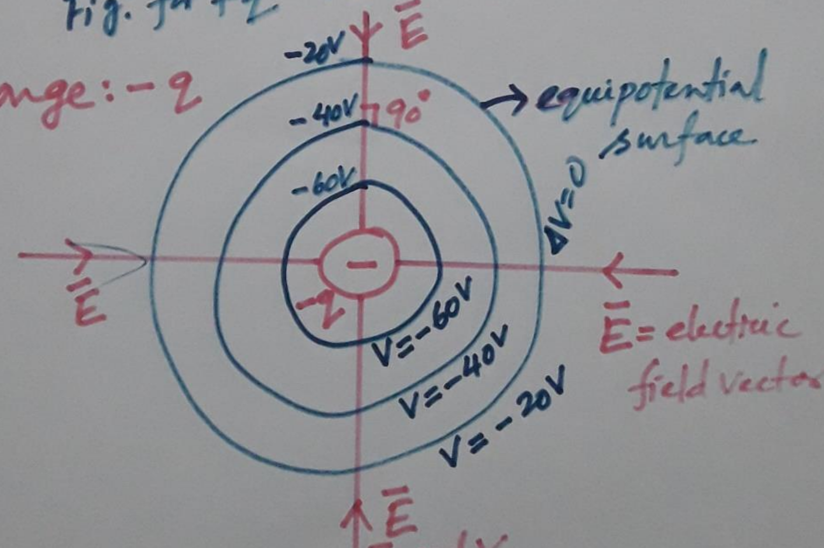
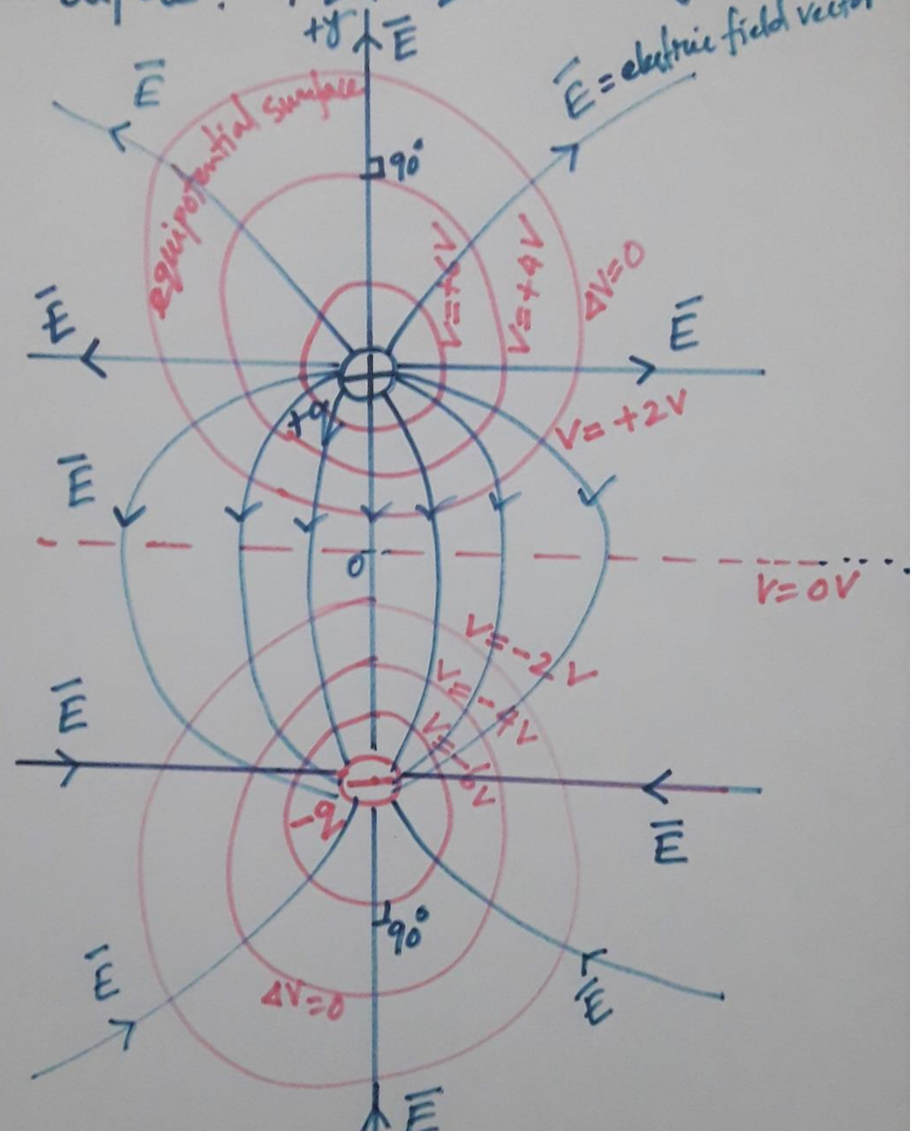


Fig: $\bar{E}$ and V for −q

For −q , V=−ve

(e) For a dipole: +q & −q and d is very small



For +q, V = +ve and for −q, V = −ve

Fig: dipole shows $\bar{E}$ & V

Electric potential due to a non-uniform electric field: $V_f - V_i = -\int_i^f \bar{E} \cdot d\bar{s}$

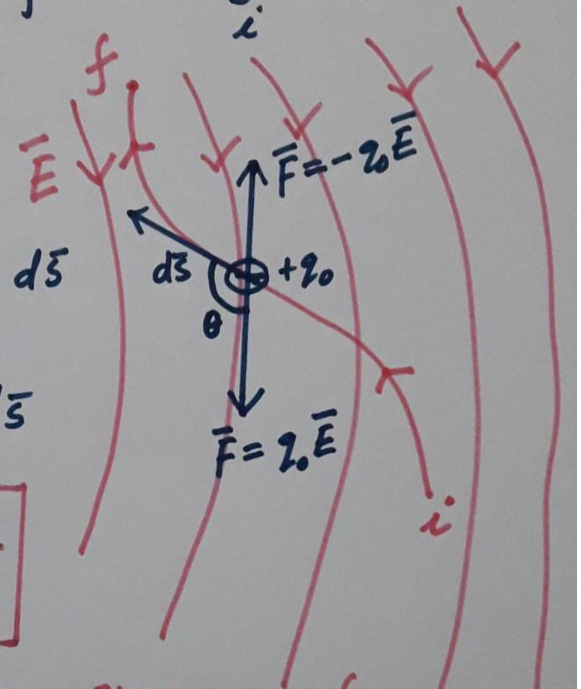
$$V_f - V_i = \frac{W_{if}}{q_0}$$

$$= \frac{1}{q_0}\int_i^f \bar{F} \cdot d\bar{s}$$

$$= \frac{1}{q_0}\int_i^f (-q_0\bar{E}) \cdot d\bar{s}$$

$$= \frac{-\cancel{q_0}}{\cancel{q_0}}\int_i^f \bar{E} \cdot d\bar{s}$$

$$\boxed{V_f - V_i = -\int_i^f \bar{E} \cdot d\bar{s}}$$



Potential difference between any two points (i and f) due to a non-uniform electric field

Fig: non-uniform electric field

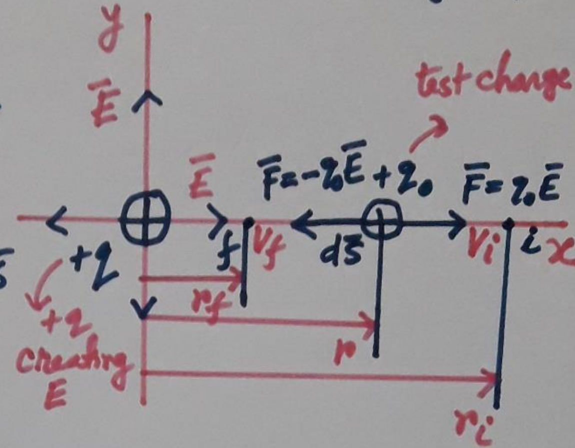
Electric potential V due to a point charge +q ①

$$V_f - V_i = \frac{W_{if}}{q_0}$$

$$= \frac{1}{q_0}\int_i^f \bar{F}\cdot d\bar{s}$$

$$= \frac{1}{q_0}\int_i^f (-q_0\bar{E})\cdot d\bar{s}$$

$$= \frac{-\cancel{q_0}}{\cancel{q_0}}\int_i^f \bar{E}\cdot d\bar{s}$$



$$V_f - V_i = -\int_i^f \bar{E}\cdot d\bar{s}$$

$$= -\int_i^f E\,ds\cos 180^\circ = +\int_i^f E\,ds$$

$$= \int_{r_i}^{r_f} E(-dr) \qquad [ds = -dr]$$

$$= -\int_{r_i}^{r_f} E\,dr = -\int_{r_i}^{r_f} \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\,dr \qquad \left[E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\right]$$

$$= \frac{-q}{4\pi\epsilon_0}\int_{r_i}^{r_f} r^{-2}\,dr = \frac{-q}{4\pi\epsilon_0}\left[\frac{r^{-2+1}}{-2+1}\right]_{r_i}^{r_f}$$

$$= \frac{-q}{4\pi\epsilon_0}\left[\frac{r^{-1}}{-1}\right]_{r_i}^{r_f} = \frac{+q}{4\pi\epsilon_0}\left[\frac{1}{r}\right]_{r_i}^{r_f}$$

$$V_f - V_i = \frac{q}{4\pi\epsilon_0}\left(\frac{1}{r_f} - \frac{1}{r_i}\right)$$

At infinity: $r_i \to \infty$, $V_i = 0$

$$V_f - V_i = \frac{q}{4\pi\epsilon_0}\left(\frac{1}{r_f} - \frac{1}{r_i}\right)$$

$$V_f - 0 = \frac{q}{4\pi\epsilon_0}\left(\frac{1}{r_f} - \frac{1}{\infty}\right)$$

$$V_f = \frac{q}{4\pi\epsilon_0}\frac{1}{r_f}$$

if $V_f = V$ and $r_f = r$

$$\boxed{V = \frac{1}{4\pi\epsilon_0}\frac{+q}{r}}$$ [if charge is +q]

if charge is $-q$:

$$\boxed{V = \frac{1}{4\pi\epsilon_0}\frac{-q}{r}}$$

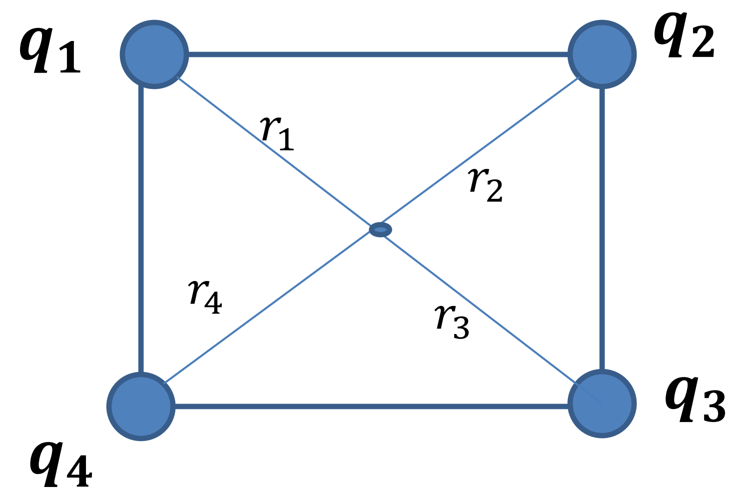
$$\boxed{\therefore\ V = \frac{1}{4\pi\epsilon_0}\frac{\pm q}{r}}$$

As V is a scalar quantity, it is written with the sign of the charge.

Potential Due to a Group of Point Charges:

General Formula:

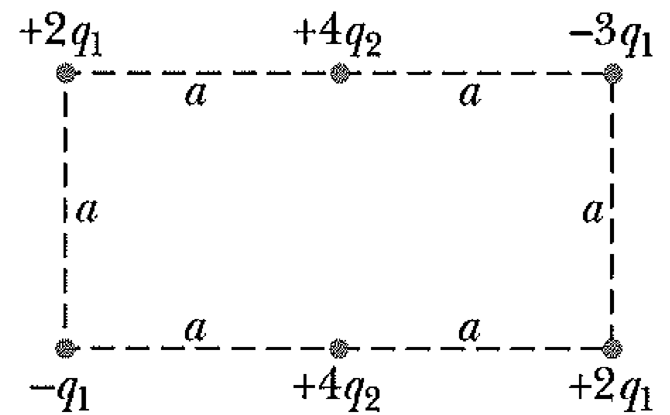
$$V = \sum_{i=1}^{n} V_i = \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i}$$



For an example: $V = \sum_{i=1}^{4} V_i = \frac{1}{4\pi\epsilon_0}\left[\frac{q_1}{r_1}+\frac{q_2}{r_2}+\frac{q_3}{r_3}+\frac{q_4}{r_4}\right]$

Problem: 16 (Book chapter 24)

Figure shows a rectangular array of charged particles fixed in place, with distance $a = 39\ cm$ and the charges shown as integer multiples of $q_1 = 3.4\ pC$ and $q_2 = 6.0\ pC$. With $V = 0$ at infinity, what is the net electric potential at the rectangle's center?



The net potential at the rectangle center is

Given $a = 39\ cm = 0.39\ m$

V = V1 + V2 + V3+V4+V5+V6 [algebraic sum]

$q_1 = 3.40\ pC = 3.4 \times 10^{-12}\ C$

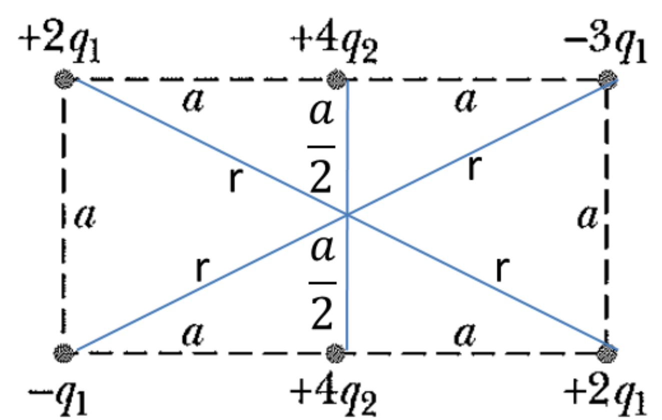
$q_2 = 6.0\ pC = 6.0 \times 10^{-12}\ C$

$$V = \frac{1}{4\pi\epsilon_0}\left[\frac{+2q_1}{r} + \frac{-3q_1}{r} + \frac{+2q_1}{r} + \frac{-q_1}{r}\right] + \frac{1}{4\pi\epsilon_0}\left[\frac{+4q_2}{\frac{a}{2}} + \frac{+4q_2}{\frac{a}{2}}\right]$$

$$V = 0 + \frac{1}{4\pi\epsilon_0}\left[\frac{4q_2}{0.195} + \frac{4q_2}{0.195}\right]$$

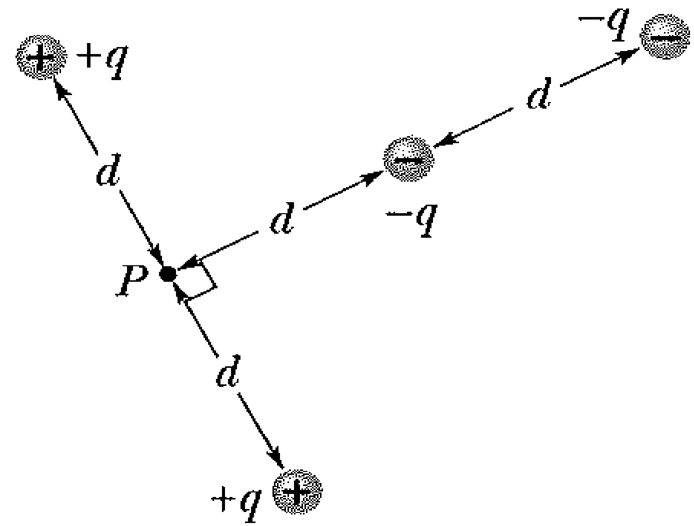
$$V = \frac{1}{4\pi\epsilon_0}\left[\frac{8q_2}{0.195}\right] = \frac{9 \times 10^9 \times 8 \times 6 \times 10^{-12}}{0.195}$$

V =2.215 Volt



Problem: 17 (Book chapter 24)

In the adjacent Figure, what is the net electric potential at point *P* due to the four particles if $V = 0$ at infinity, $q = 5\ fC$, and $d = 4\ cm$?



Answer:

Given

$q = 5\ fC = 5 \times 10^{-15}\ C$ and $d = 4\ cm = 0.04\ m$

The net electric potential at the point P is

V = V1 + V2 + V3+V4 [algebraic sum]

$$V = \frac{1}{4\pi\epsilon_0}\left[\frac{+q}{d} + \frac{+q}{d} + \frac{-q}{d} + \frac{-q}{2d}\right] = \frac{1}{4\pi\epsilon_0}\frac{q}{d}[1 + 1 - 1 - \frac{1}{2}]$$

$$V = \frac{9 \times 10^{9} \times 5 \times 10^{-15}}{0.04}(\frac{1}{2})$$

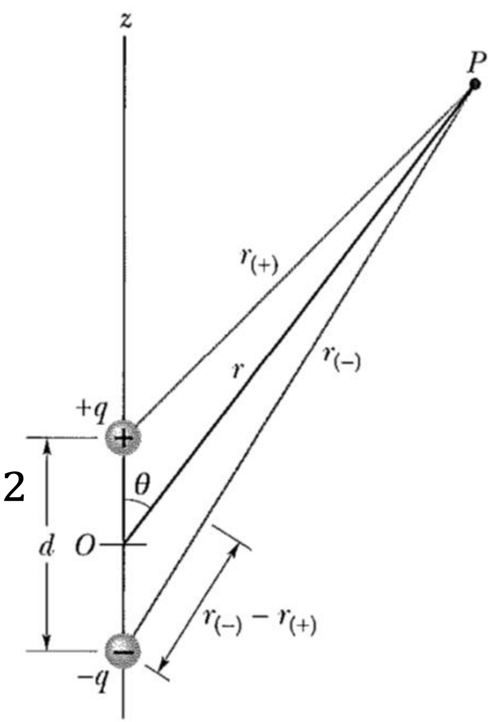
$$V = 562.5 \times 10^{-6}\ Volt$$

Potential Due to an Electric Dipole:

The net potential at *P* is given by

$$V = V_{(+)} + V_{(-)} = \frac{1}{4\pi\varepsilon_0}\left[\frac{+q}{r_{(+)}} + \frac{-q}{r_{(-)}}\right] = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r_{(+)}} - \frac{1}{r_{(-)}}\right]$$

$$V = \frac{q}{4\pi\varepsilon_0}\left(\frac{r_{(-)} - r_{(+)}}{r_{(-)}r_{(+)}}\right)$$

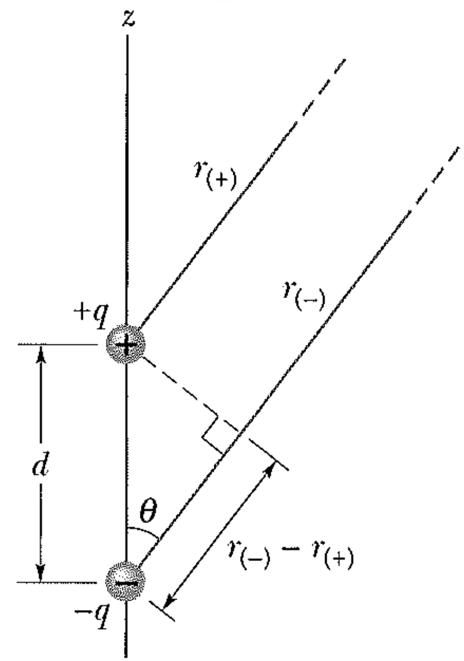


(1) If r ≫ *d*: $r_{(-)} - r_{(+)} = d\cos\theta \quad and \;\; r_{(-)}r_{(+)} \approx r^2$

$$V = \frac{q}{4\pi\varepsilon_0}\left(\frac{d\cos\theta}{r^2}\right)$$

Type equation here.

where, θ is measured from the direction of the electric dipole moment.



(2) V in terms of electric dipole moment: p = qd

$$\boldsymbol{V = \frac{1}{4\pi\varepsilon_0}\left(\frac{p\cos\theta}{r^2}\right)}$$

The vector $\vec{p}$ is directed along the dipole axis, from the negative to the positive charge.

Problem 21 (Book Chapter 24)

The ammonia molecule NH_3 has a permanent electric dipole moment equal to 1.47 D, where $1D = 1 debye\ unit = 3.34 \times 10^{-30} C - m$. Calculate the electric potential due to an ammonia molecule at a point 52.0 nm away along the axis of the dipole. (Set $V = 0$ at infinity.)

Given

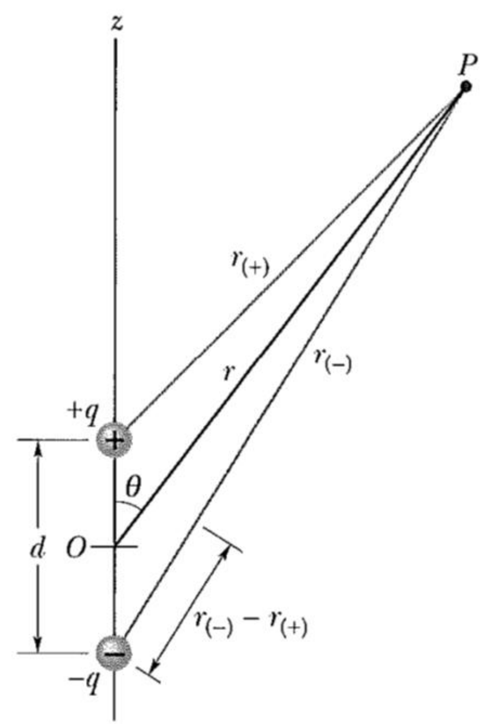
$$p = 1.47\ D = 1.47 \times 3.34 \times 10^{-30}\ C - m$$

$$r = 52\ nm = 52 \times 10^{-9}\ m$$

$$\theta = 0^0$$

$$V = ?$$

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{p\cos\theta}{r^2}\right)$$



$$V = \frac{9\times10^9\times1.47\times3.34\times10^{-30}\times\cos 0^0}{(52\times10^{-9})^2} = 16.34 \times 10^{-6}\ \text{Volt}$$

THANK YOU