Lecture 7

BOOK CHAPTER 7

(Kinetic energy and Work)
3rd quiz

Kinetic Energy:

***Kinetic energy** K is energy associated with the state of motion of an object. The faster the object moves, the greater is its kinetic energy. When the object is stationary, its kinetic energy is zero.*

For an object of mass *m* whose speed *v* is well below the speed of light,

$$K = \frac{1}{2}mv^2$$

*The SI unit of kinetic energy (and all types of energy) is the **joule** (J), named for James Prescott Joule, an English scientist of the 1800s and defined as*

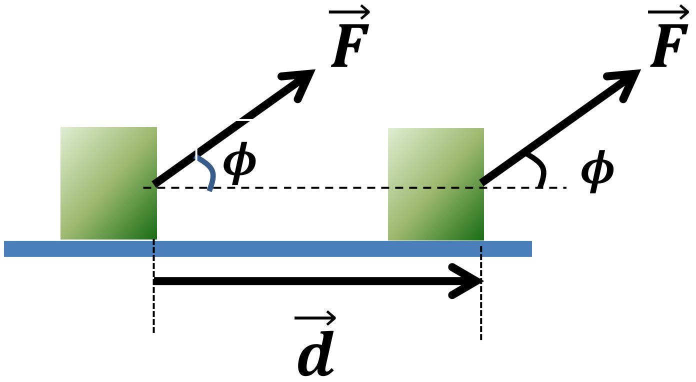
$$1\text{joule} = 1\ \text{J} = 1\text{kg}.\,m^2/s^2$$

Work:

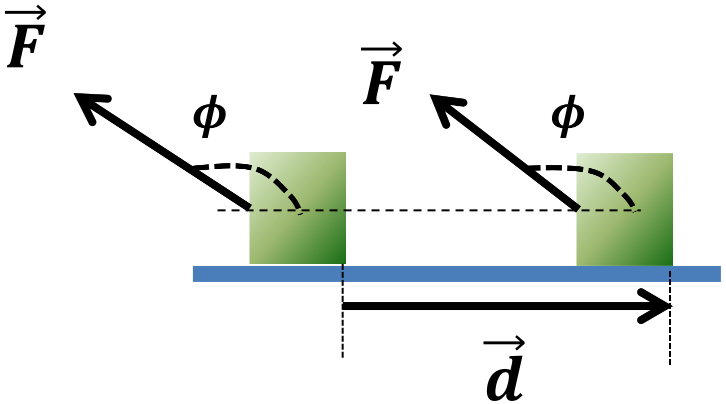
Work W is energy transferred to or from an object via a force acting on the object. Energy transferred to the object is positive work, and from the object, negative work.

The work done on a particle by a constant force $\vec{F}$ during displacement $\vec{d}$ is

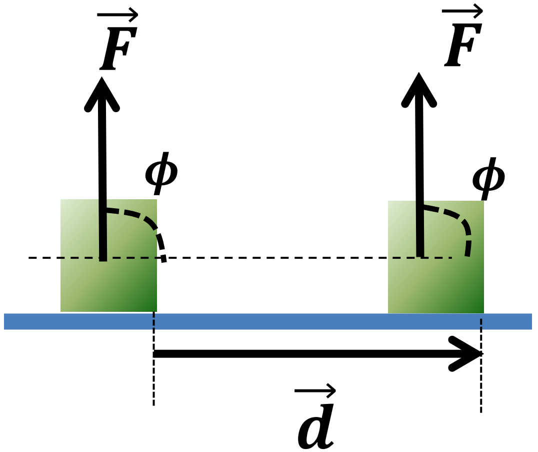
$$\boldsymbol{W = \vec{F}.\vec{d} = Fd\cos\phi}$$



Where ϕ is constant angle between the directions of $\vec{\boldsymbol{F}}$ and $\vec{\boldsymbol{d}}$**.** This is positive work, because $\boldsymbol{\phi < 90^0}$



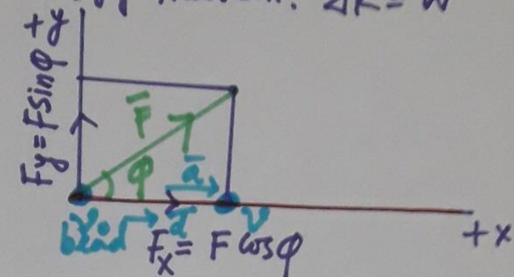
Work is Negative, because $\boldsymbol{90^0 < \phi}$



The force does *no* work on the object, because $\boldsymbol{\phi = 90^0}$

7-5 Work-Kinetic energy Theorem: $\Delta K = W$

Newton's 2nd law for component along x-axis,



$$F_x = m a_x$$

Equation of motion along x-axis,

$$v^2 = v_0^2 + 2a_x d$$

$$2a_x d = v^2 - v_0^2$$

$$a_x = \frac{v^2 - v_0^2}{2d}$$

$$\therefore F_x = m\left(\frac{v - v_0^2}{2d}\right)$$

$$F_x d = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$$

$$(F\cos\varphi)d = K_f - K_i$$

$$\bar{F}\cdot\bar{d} = \Delta K$$

$$W = \Delta K$$, Work-kinetic energy theorem

For a particle, a change ΔK in the kinetic energy equals the net work W done on the particle.

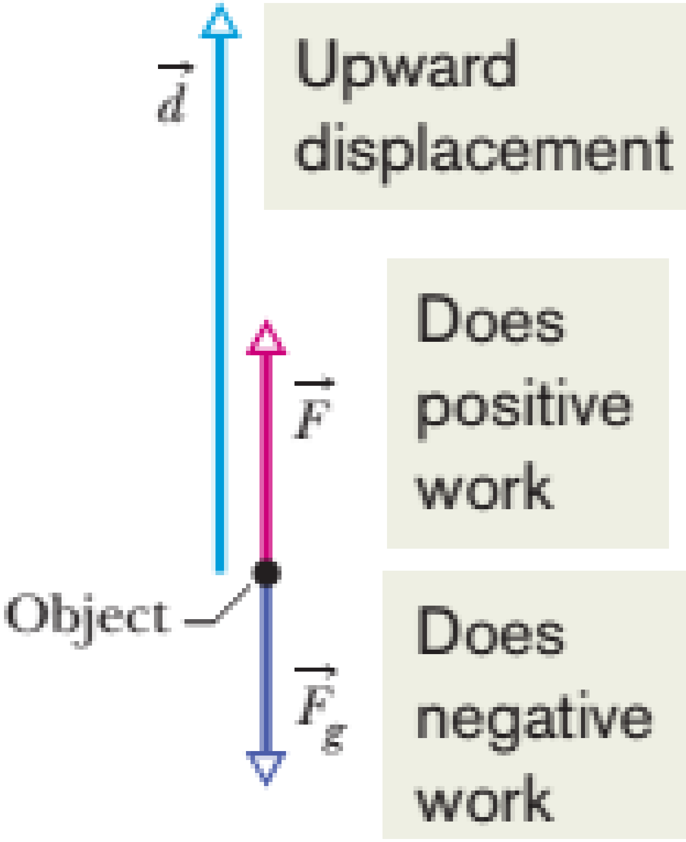
Work Done by the Gravitational Force:

We know that the work done on a particle by a constant force $\vec{F}$ during displacement $\vec{d}$ is

$$\boldsymbol{W = \vec{F}.\vec{d} = Fd\cos\phi}$$

Accordingly, the work W_g done by the gravitational force $\vec{F}_g$ on an object (particle/body) of mass *m* as the object moves through a displacement $\vec{d}$ is given by

$$W_g = F_g d\cos\phi$$



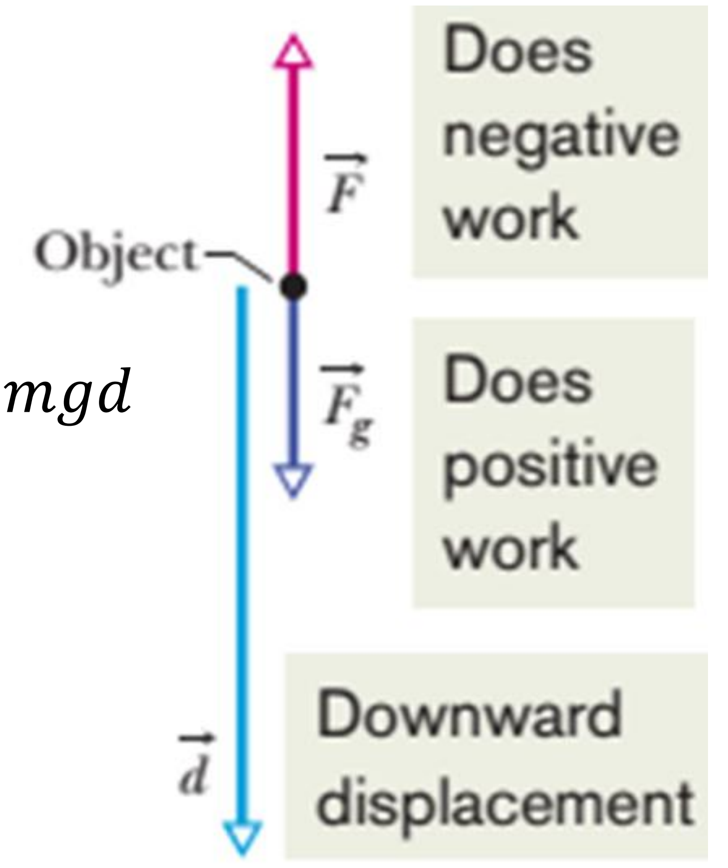
Where ϕ is the angle between $\vec{F}_g$ and $\vec{d}$.

❑ **For rising an object:** Force $\vec{F}_g$ is directed opposite the displacement $\vec{d}$. Hence, W_g becomes

$$W_g = \boldsymbol{F_g.d} = F_g d\boldsymbol{\cos\phi} = F_g\, d\cos 180^0 = mgd(-1) = -mgd$$

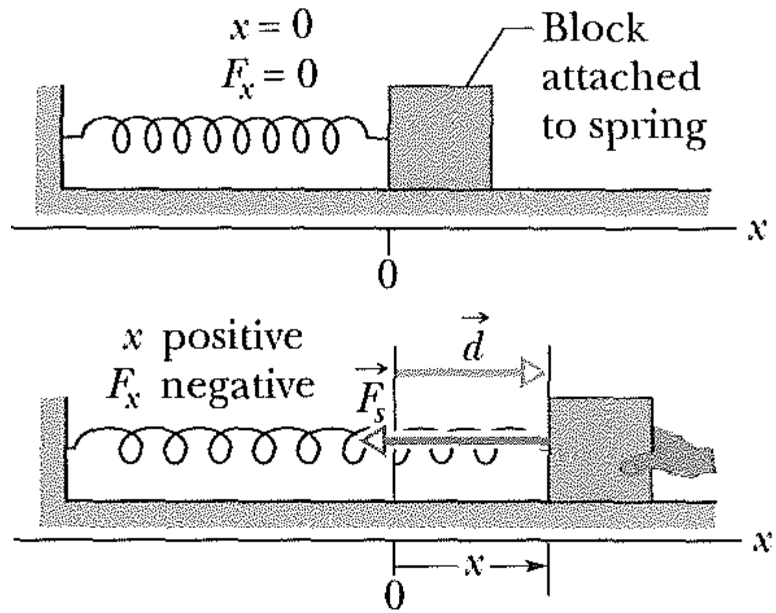
❑ **For lowering an object:** Force $\vec{F}_g$ is directed along the displacement $\vec{d}$. Hence, W_g becomes

$$W_g = \boldsymbol{F_g}.\boldsymbol{d} = F_g d\cos 0^0 = mgd(+1) = +mgd$$



The Spring Force:

The force $\vec{F}_s$ from a spring is proportional to the displacement $\vec{d}$ of the free end from its position when the spring is in its relaxed state (neither compressed nor extended).

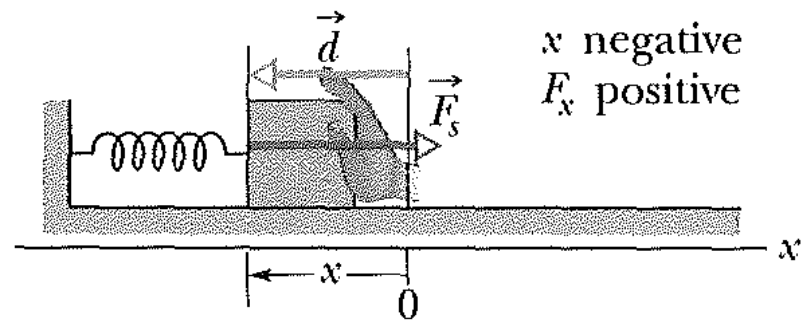


The *spring force* $\vec{F}_s$ is given by

$$\vec{F}_s = -k\vec{d}$$

which is known as **Hooke's law** after Robert Hooke, an English scientist of the late 1600s. The minus sign indicates that the direction of the spring force is always opposite the direction of the displacement of the spring's free end. The constant *k* is called the **spring constant** (or **force constant**) and is a measure of the stiffness of the spring.

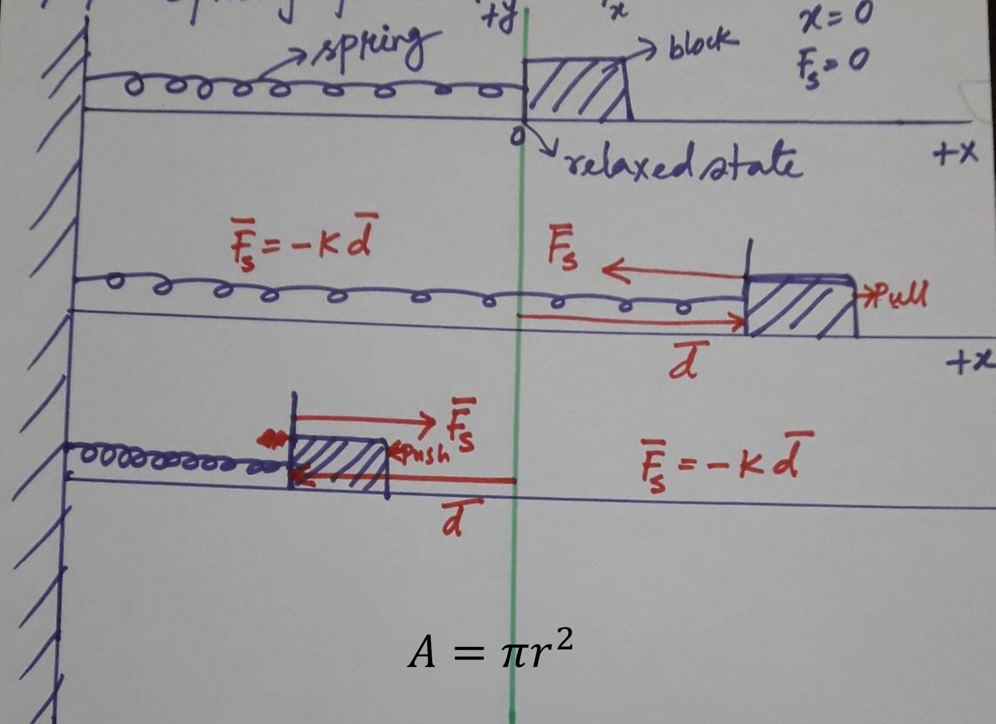
- The larger *k* is, the stiffer the spring; that is, the larger *k* is, the stronger the spring's pull or push for a given displacement.
- The SI unit for *k* is the newton per meter



If an *x* axis lies along the spring, with the origin at the location of the spring's free end when the spring is in its relaxed state, we can write

$$F_x = -kx$$

7-7 Spring force : $\bar{F}_x = -k\bar{x}$



$$A = \pi r^2$$

Hooke's law : Robert Hooke, English scientist, late 1600s

$$\bar{F}_s = -k\bar{d}$$

$\bar{F}_s$ → spring force (restoring force)

k → spring constant (force constant)

$\bar{d}$ → displacement

$$\bar{F}_x = -k\bar{x}$$

Spring force is a Variable force, $F(x)$

Hooke's law is a linear relationship between F_x and x.

$$k = \frac{F_x}{x}$$

The Work Done by a Spring Force:

The net work W_s done by the spring (from x_i to x_f) is

$$W_s = \int_{x_i}^{x_f} -F_x \, dx = \int_{x_i}^{x_f} -kx \, dx$$ [Where $|F_x| = kx$]

$$W_s = -k \int_{x_i}^{x_f} x \, dx$$

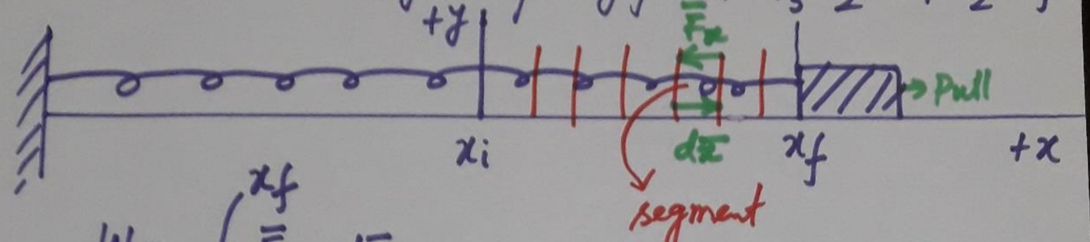
$$W_s = -k \left| \frac{x^2}{2} \right|_{x=x_i}^{x=x_f} = -\frac{1}{2} k\left(x_f^2 - x_i^2\right)$$

$$W_s = -\frac{1}{2} k x_f^2 + \frac{1}{2} k x_i^2$$

If $x_i = 0$ and if we assume that $x_f = x$, the above equation becomes

$$\boxed{\boldsymbol{W_s = -\frac{1}{2} k x^2}}$$

7-7 Work done by a spring force: $W_s = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$



$$W_s = \int_{x_i}^{x_f} \vec{F}_x \cdot d\vec{x}$$

$$= \int_{x_i}^{x_f} F_x \, dx \cos 180^\circ$$

$$= -\int_{x_i}^{x_f} F_x \, dx$$

Hooke's law:

$$\vec{F}_x = -k\vec{x}$$

$$|\vec{F}_x| = |-kx|$$

$$F_x = kx$$

$$= -\int_{x_i}^{x_f} kx \, dx$$

$$= -k\int_{x_i}^{x_f} x^1 dx = -k\left[\frac{x^{1+1}}{1+1}\right]_{x_i}^{x_f} = -k\left[\frac{x^2}{2}\right]_{x_i}^{x_f}$$

$$= -k\left(\frac{x_f^2}{2} - \frac{x_i^2}{2}\right) = -\frac{1}{2}kx_f^2 + \frac{1}{2}kx_i^2$$

$$W_s = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$$

Problem 1 (Book chapter 7)

A proton (mass $m = 1.67 \times 10^{-27} kg$) is being accelerated along a straight line at $3.6 \times 10^{15} m/s^2$ in a machine. If the proton has an initial speed of $2.4 \times 10^7\ m/s$ and travels 3.5 cm, what then is (a) its speed and (b) the increase in its kinetic energy?

Answer: Here, initial speed, $v_i = 2.4 \times 10^7\ m/s$ and the distance traveled by the proton, $s = 3.5\ cm = 0.035\ m$ and We assume final speed is v_f.

(a) We use the formula, $v_f^2 = v_i^2 + 2as = (2.4 \times 10^7)^2 + 2(3.6 \times 10^{15})(0.035)$

$$v_f^2 = 5.76 \times 10^{14} + 2.52 \times 10^{14} = 8.28 \times 10^{14}\ \frac{m^2}{s^2}$$

$$\boxed{\boldsymbol{v_f = 2.88 \times 10^7 m/s}}$$

(b) The change (increase) in kinetic energy,

$$\Delta K = K_f - K_i = \frac{1}{2} m\left(v_f^2 - v_i^2\right)$$

$$\Delta K = \frac{1.67 \times 10^{-27}}{2}\left((2.88 \times 10^7)^2 - (2.4 \times 10^7)^2\right)$$

$$\Delta K = \frac{1.67 \times 10^{-27}}{2}(8.29 \times 10^{14} - 5.76 \times 10^{14})$$

$$\Delta K = \frac{1.67 \times 10^{-27} \times 2.53 \times 10^{14}}{2} = \frac{4.23 \times 10^{-13}}{2}$$

$$\boxed{\boldsymbol{\Delta K = 2.115 \times 10^{-13}\ J}}$$

Problem 9 (Book chapter 7)

The only force acting on a 2.0 kg canister that is moving in an x-*y* plane has a magnitude of 5.0 N. The canister initially has a velocity of 4.0 m/s in the positive *x* direction and some time later has a velocity of 6.0 m/s in the positive *y* direction. How much work is done on the canister by the 5.0 N force during this time?

Answer: We use the formula for work-kinetic energy theorem, which is

$$W = \Delta K = K_f - K_i = \frac{1}{2} m\left(v_f^2 - v_i^2\right)$$

In the above formula, speed is required; whatever the directions.

$$W = \frac{2}{2}(6^2 - 4^2) = 36 - 16 = 20 \text{ J}$$

$$\boxed{\boldsymbol{W = 20} \textbf{ J}}$$

Given

$m = 2\ kg$

$v_i = 4\ m/s$

$v_f = 6\ m/s$

$W = ?$

Problem 11 (Book chapter 7)

A 12.0 N force with a fixed orientation does work on a particle as the particle moves through the three-dimensional displacement $\vec{d} = \left(2\hat{\imath} - 4\hat{\jmath} + 3\hat{k}\right) m$. What is the angle between the force and the displacement if the change in the particle's kinetic energy is (a) +30.0 J and (b) - 30.0 J?

Answer: Here, we use the work-kinetic energy relation, which is

$$\boldsymbol{\Delta K = W = \vec{F}.\vec{d} = Fd\cos\phi}$$

Where ϕ is the angle between force $\vec{F}$ and displacement $\vec{d}$.

(a) $\Delta K = Fd\cos\phi$

$$\cos\phi = \frac{\Delta K}{Fd}$$

$$\phi = \cos^{-1}\left(\frac{\Delta K}{Fd}\right) = \cos^{-1}\left[\frac{30}{(12)(5.385)}\right] = \cos^{-1}(0.464)$$

$$\boxed{\boldsymbol{\phi = 62.35^0}}$$

Given

$\Delta K = +30\ J$ for (a)

$\Delta K = -30\ J$ for (b)

$|F| = 12\ N$

$\vec{d} = \left(2\hat{\imath} - 4\hat{\jmath} + 3\hat{k}\right)$ m

$d = \sqrt{(2)^2 + (-4)^2 + (3)^2}$

$d = 5.385\ m$

(b) For $\Delta K = -30\ J$

$$\phi = \cos^{-1}\frac{\Delta K}{Fd} = \cos^{-1}[\frac{-30}{(12)(5.385)}] = \cos^{-1}(-0.464)$$

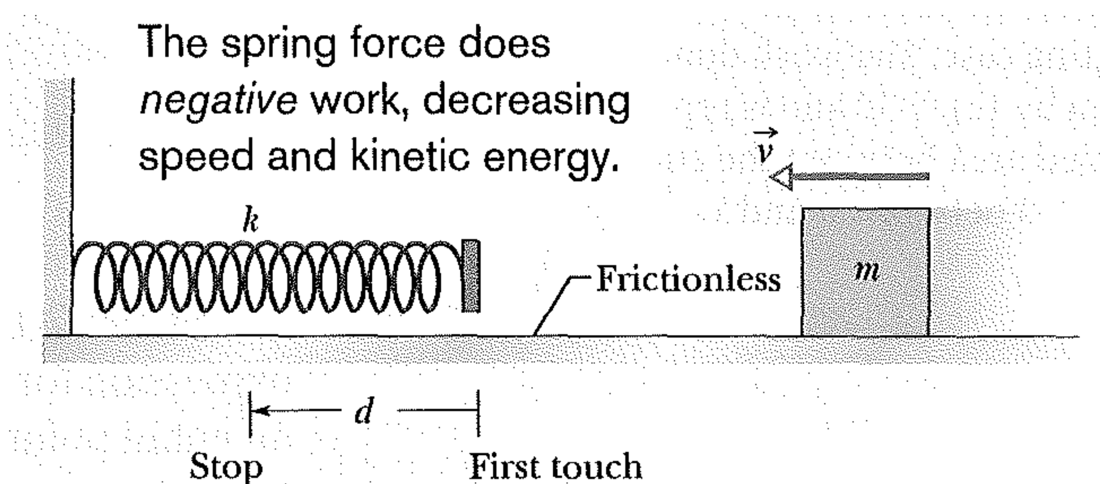
$$\boxed{\boldsymbol{\phi = 117.65^0}}$$

Sample Problem 7.06 (page 161) Home work

In the Figure below, a cumin canister of mass $m = 0.40\ kg$ slides across a horizontal frictionless counter with speed *v* = 0.50 *m/s.* It then runs into and compresses a spring of spring constant *k* = 750 *N/m.* When the canister is momentarily stopped by the spring, by what distance *d* is the spring compressed?

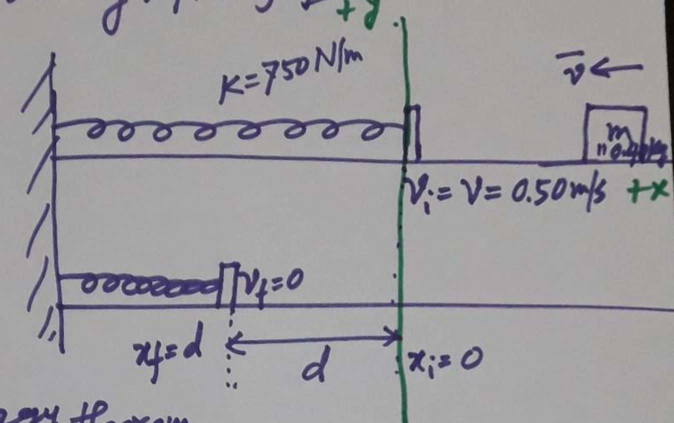
Hints: use the formula for work done by Spring.

$$W = K_f - K_i = -\frac{1}{2}kd^2$$



Sample problem

Fig. 7-10 : Work done by spring ~~Force~~ to change KE



Work-Kinetic energy theorem,

$$\Delta K = W_s$$

or, $K_f - K_i = \frac{1}{2} k x_i^2 - \frac{1}{2} k x_f^2$

or, $\frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 = \frac{1}{2} k (0)^2 - \frac{1}{2} k d^2$

or, $\frac{1}{2} m (0)^2 - \frac{1}{2} m v^2 = 0 - \frac{1}{2} k d^2$

or, $0 - \frac{1}{2} m v^2 = - \frac{1}{2} k d^2$

or, $m v^2 = k d^2$

or, $d^2 = \frac{m v^2}{k}$

or, $d = \sqrt{\frac{m v^2}{k}} = v \sqrt{\frac{m}{k}} = 0.50 \sqrt{\frac{0.40}{750}}$

$\therefore d = 1.2 \times 10^{-2}$ m $= 1.2$ cm Ans.

Here,
$x_i = 0$
$x_f = d$
$v_f = 0$
$v_i = v = 0.50$ m/s
$k = 750$ N/m

BOOK CHAPTER 8

Potential Energy and Conservation of Energy

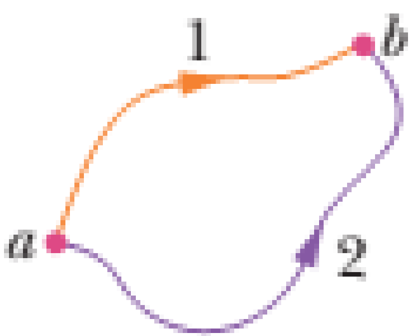
Conservative and non-conservative forces:

Conservative force:

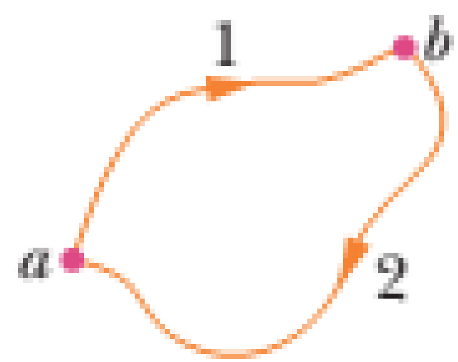
A force is a **conservative force** if the net work it does on a particle moving around any closed path, from an initial point and then back to that point, is zero. $W = \vec{F}.\vec{d}$

Equivalently, **a force is conservative** if the net work it does on a particle moving between two points does not depend on the path taken by the particle. Examples: The gravitational force and the spring force are conservative forces.

The term conservative force comes from the fact that when a conservative force exists, it conserves mechanical energy.

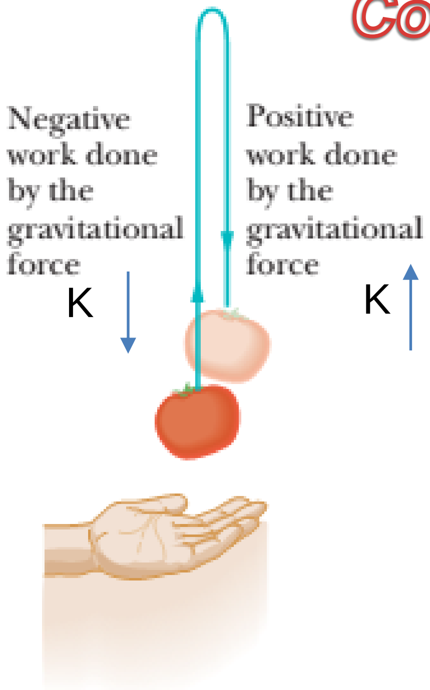


The force is conservative. Any choice of path between the points gives the same amount of work.

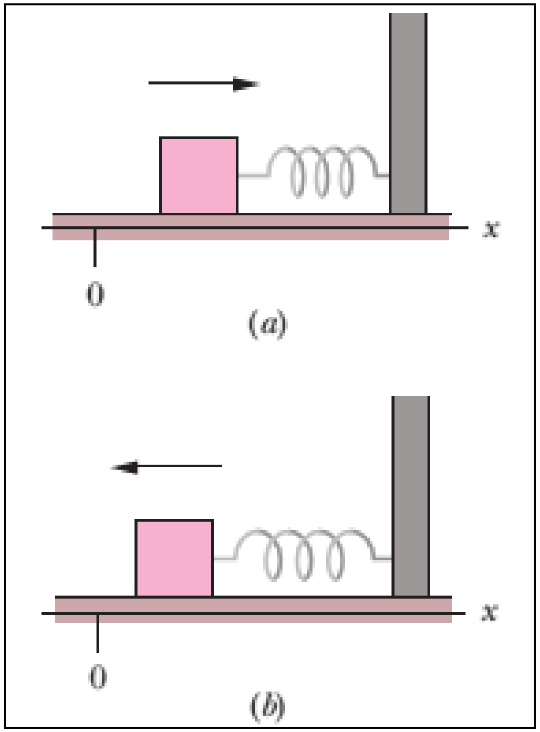


And a round trip gives a total work of zero.

Conservative force:



A tomato is thrown upward. As it rises, the gravitational force does negative work on it, decreasing its kinetic energy. As the tomato descends, the gravitational force does positive work on it, increasing its kinetic energy.



A block, attached to a spring and initially at rest at $x = 0$, is set in motion toward the right. (*a*) As the block moves rightward (as indicated by the arrow), the spring force does negative work on the block.
(*b*) Then, as the block moves back toward $x = 0$, the spring force does positive work on it.

For either rise or fall, the change ΔU in gravitational potential energy is defined as being equal to the negative of the work done on the tomato by the gravitational force. Using the general symbol *W* for work, we write this as

$$\Delta U = -W$$

Nonconservative force:

A force that is not conservative is called a **nonconservative force.** The kinetic frictional force and drag force are nonconservative.

For an example, let us send a block sliding across a floor that is not frictionless. During the sliding, a kinetic frictional force from the floor slows the block by transferring energy from its kinetic energy to a type of energy called *thermal energy* (which has to do with the random motions of atoms and molecules).We know from experiment that this energy transfer cannot be reversed (thermal energy cannot be transferred back to kinetic energy of the block by the kinetic frictional force).

The principle of Conservation of Mechanical Energy:

The mechanical energy E_{mec} of a system is the sum of its kinetic energy K and potential energy U of the objects within it. That is,

$$\boldsymbol{E_{mec} = K + U}$$

This conservation principle can also be written as

$$\boldsymbol{\Delta E_{mec} = \Delta K + \Delta U = 0}$$

Here $\Delta K = +W$

and $\Delta U = -W$