LESSON 6

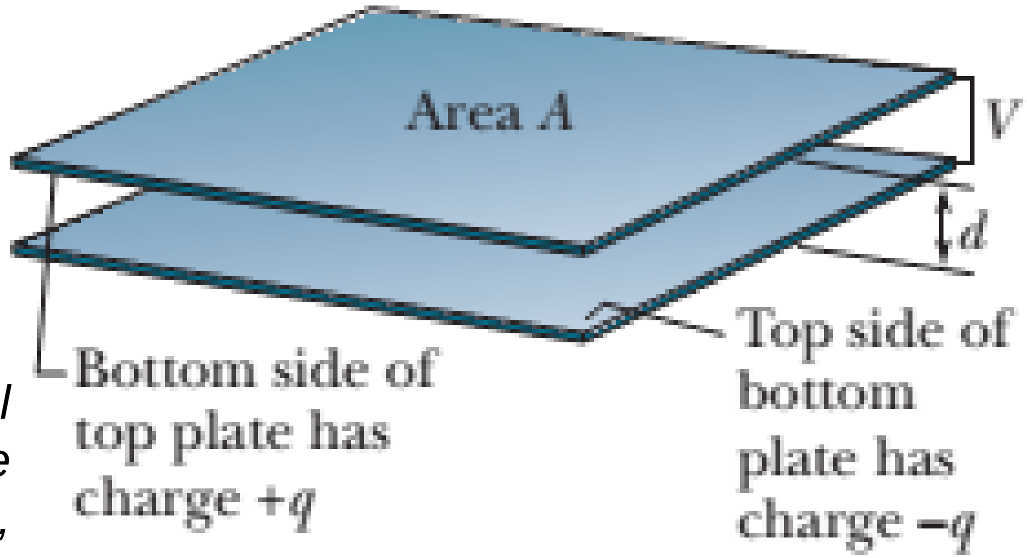
BOOK CHAPTER 25

CAPACITANCE

Paul Silvermann/Fundamental Photographs

An assortment of capacitors.

Capacitance



A capacitor consists of two isolated conductors (the plates) with charges $+q$ *and* $-q$.

The charge q and the potential difference V for a capacitor are proportional to each other; that is,

$$q \propto V$$

Therefore,

$$q = CV$$

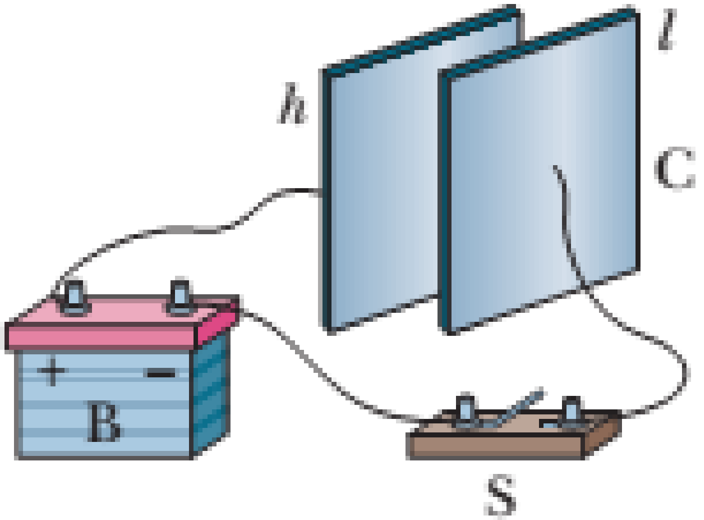
The proportionality constant C is called the capacitance of the capacitor.

The value of C depends only on the geometry of the plates and *not* on their charge or potential difference. The capacitance is a measure of how much charge must be put on the plates to produce a certain potential difference between them: The *greater the capacitance, the more charge is required.*

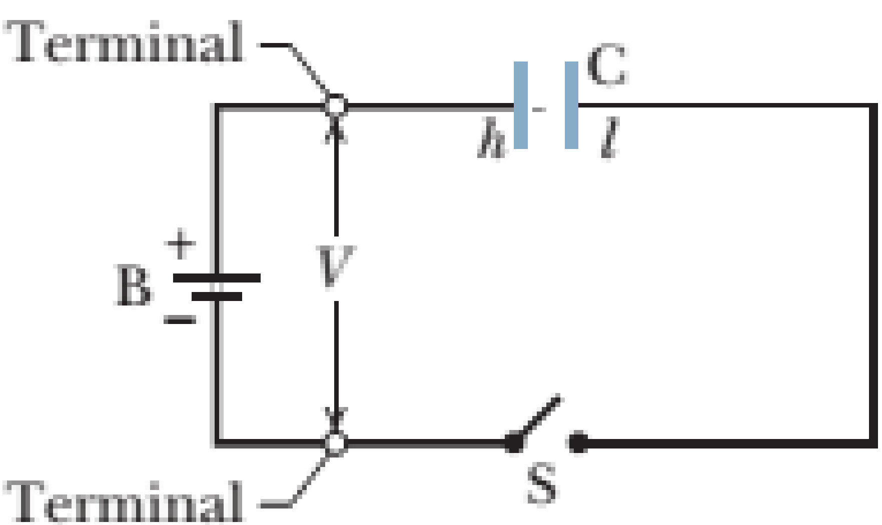
$$C = \frac{q}{V}$$

The SI unit of capacitance is the coulomb per volt. Common name is Farad (F):

Charging a Capacitor:



(a)

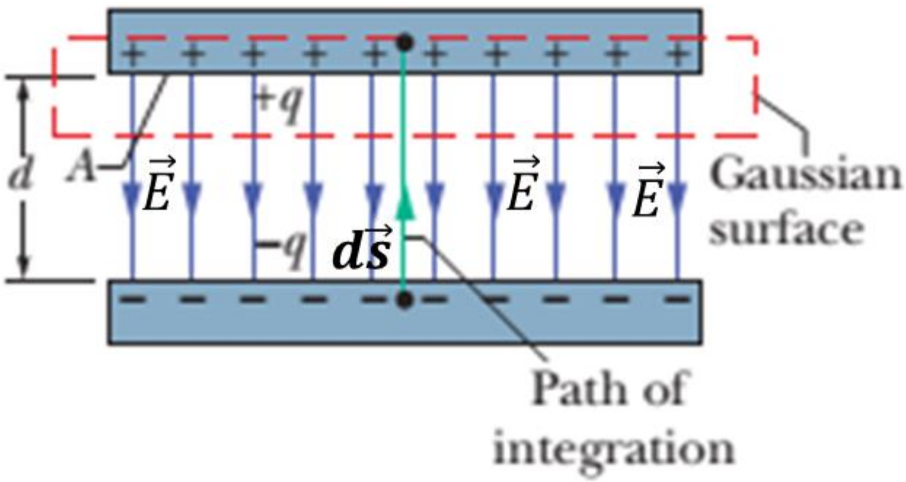


(b)

(*a*) Battery B, switch S, and plates *h* and *l* of capacitor C, connected in a circuit.

(*b*) A schematic diagram with the *circuit elements* represented by their symbols.

Calculating the Capacitance: A parallel-Plate Capacitor:



Applying Gauss' Law:

$$\varepsilon_0 \oint \vec{E}.d\vec{A} = q_{enc}$$

$$\varepsilon_0 \oint E(dA)cos0^0 = q$$

$$\varepsilon_0 \oint EdA = q$$

$$\varepsilon_0 E \oint dA = q$$

$$\varepsilon_0 EA = q$$

A is the area of that part of the Gaussian surface through which there is a flux.

The potential difference between the plates of a capacitor is related to the field $\vec{E}$ by

$$V_f - V_i = -\int_i^f \vec{E}.d\vec{s} = -\int_-^+ Edscos180$$

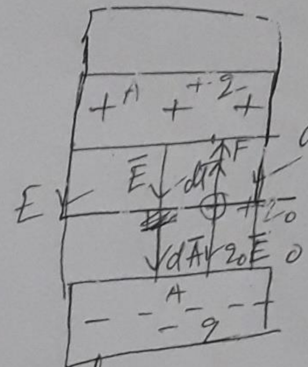
$$V = \int_-^+ E\, ds \qquad [V = V_f - V_i]$$

$$V = E \int_{s=0}^{s=d} ds = A = E[s]_0^d = E\,(d\, - 0) = Ed$$

$$C = \frac{q}{V} = \frac{\varepsilon_0 EA}{Ed} = \frac{\varepsilon_0 A}{d}$$

$$\boxed{C = \frac{\varepsilon_0 A}{d}}$$

1. Derive an expression for the capacitance of a parallel-plate capacitor. [12]



$$\epsilon_0 \oint \bar{E} \cdot d\bar{A} = q_{enc}$$

$$\epsilon_0 \oint E\, dA \cos 0^\circ = q$$

$$\epsilon_0 \oint E\, dA = q$$

$$\epsilon_0 E \oint dA = q$$

$$\epsilon_0 E A = q$$

$$E = \frac{q}{\epsilon_0 A}$$

$$V - V_A = \frac{W_{-+}}{q_0} = \frac{1}{q_0} \int_0^d \bar{F} \cdot d\bar{l} = \frac{1}{q_0} \int_0^d (-q_0 \bar{E}) \cdot d\bar{l} = -\int_0^d \bar{E} \cdot d\bar{l}$$

$$\cancel{V = -E \int_0^d dl = -E\, [l]_0^d = -E(d-0) = -Ed}$$

$$V = -\int_0^d E\, dl \cos 180^\circ = +\int_0^d E\, dl = E \int_0^d dl = E\, [l]_0^d$$

$$= E(d-0)$$

$$V = Ed = \frac{q}{\epsilon_0 A} d$$

$$V = \frac{qd}{\epsilon_0 A}$$

$$q = CV$$

$$q = C \frac{qd}{\epsilon_0 A}$$

$$\boxed{C = \frac{\epsilon_0 A}{d}}$$

Capacitance for a spherical capacitor: $C=4\pi\epsilon_0\left(\frac{ab}{b-a}\right)$ ①

(1) Gauss' law:

$$\epsilon_0 \oint \bar{E}\cdot d\bar{A} = q_{enc}$$

$$\epsilon_0 \oint E\,dA\cos 0^\circ = q$$

$$\epsilon_0 E \oint dA = q$$

$$\epsilon_0 E\left(4\pi r^2\right) = q$$

$$\boxed{E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}}$$

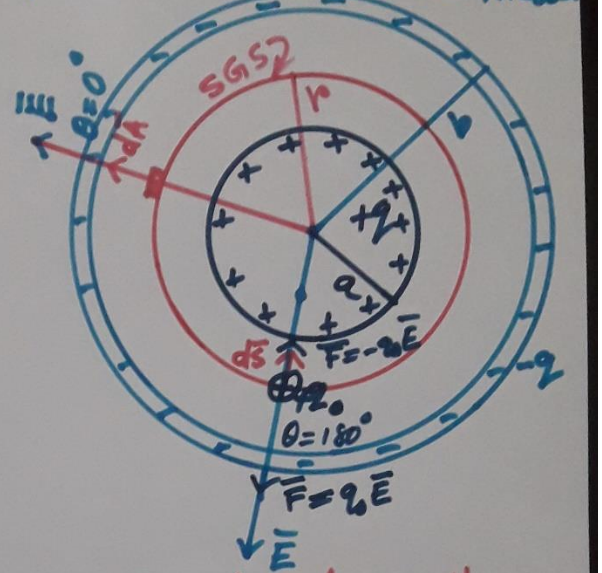


Fig. spherical capacitor

(2) $V = V_+ - V_- = -\int_-^+ \bar{E}\cdot d\bar{s}$

$$= -\int_-^+ E\,ds\cos 180^\circ = -\int_-^+ E\,ds(-1) = +\int_b^a E(-dr) \quad [ds=-dr]$$

$$= -\int_b^a E\,dr = -\int_b^a \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\,dr = \frac{-q}{4\pi\epsilon_0}\int_b^a r^{-2}\,dr$$

$$= \frac{-q}{4\pi\epsilon_0}\left[\frac{r^{-2+1}}{-2+1}\right]_b^a = \frac{-q}{4\pi\epsilon_0}\left[\frac{r^{-1}}{-1}\right]_b^a = \frac{+q}{4\pi\epsilon_0}\left[\frac{1}{r}\right]_b^a$$

$$= \frac{q}{4\pi\epsilon_0}\left(\frac{1}{a}-\frac{1}{b}\right) = \frac{q}{4\pi\epsilon_0}\left(\frac{b-a}{ab}\right)$$

$$\boxed{V = \frac{q}{4\pi\epsilon_0}\left(\frac{b-a}{ab}\right)}$$

(b) $q = CV$

$$C = \frac{q}{V} = \frac{q}{\frac{q}{4\pi\varepsilon_0}\left(\frac{b-a}{ab}\right)}$$

$$C = \cancel{q} \times \frac{4\pi\varepsilon_0}{\cancel{q}}\left(\frac{ab}{b-a}\right)$$

$$\therefore C = 4\pi\varepsilon_0\left(\frac{ab}{b-a}\right) \qquad [b > a]$$

An isolated sphere:

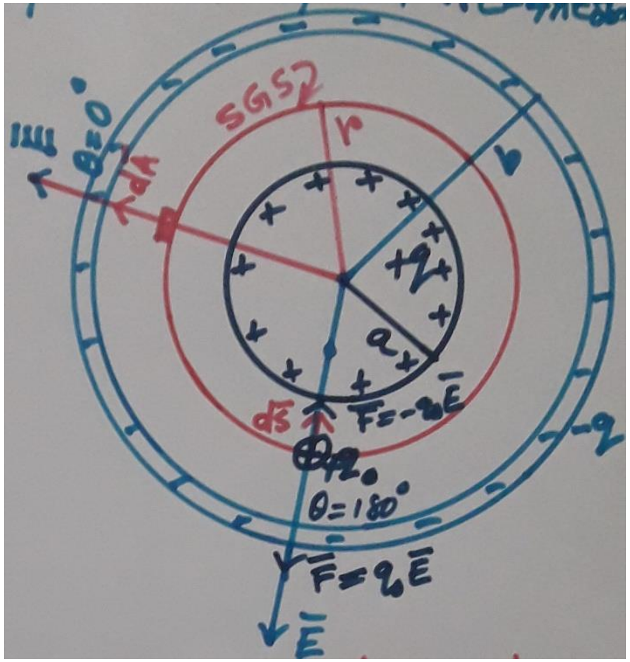
We can assign a capacitance to a *single* isolated spherical conductor of radius $a = R$ by assuming that the "missing plate" is a conducting sphere of infinite radius $b = \infty$.

The capacitance of the spherical capacitor,

$$C = \frac{4\pi\varepsilon_0\, ab}{b-a}$$ [Dividing both numerator and denominator by b]

$$C = \frac{4\pi\varepsilon_0\, a}{1-\frac{a}{b}}$$

If $b \to \infty$ (infinity), $C = \dfrac{4\pi\varepsilon_0\, a}{1-0} = 4\pi\varepsilon_0 a$

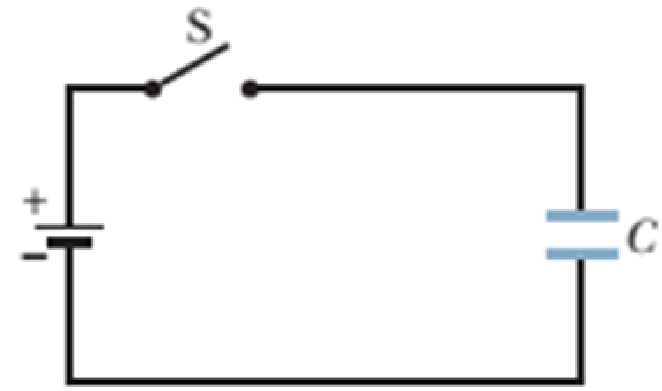


By substituting $a = R$,

$$\boxed{C = 4\pi\varepsilon_0 \mathrm{R}}$$ (capacitance for isolated sphere)

Problem 2 (Book chapter 25):

The capacitor in the adjacent Fig. has a capacitance of $25\,\mu F$ and is initially uncharged. The battery provides a potential difference of 120 V. After switch S is closed, how much charge will pass through it?



Given: $C = 25\,\mu F = 25 \times 10^{-6} F$

$V = 120\,V$

$q = ?$

$q = CV$

$q = 25 \times 10^{-6} \times 120 = 3 \times 10^{-3}$ C = 0.003 C

Problem 3 (Book chapter 25):

A parallel-plate capacitor has circular plates of 8.20 cm radius and 1.30 mm separation. (a) Calculate the capacitance. (b) Find the charge for a potential difference of 120 V.

Given: $r = 8.20\ cm = 0.082\ m$

$$d = 1.30\ mm = 1.3 \times 10^{-3}\ m$$

$$V = 120\ V$$

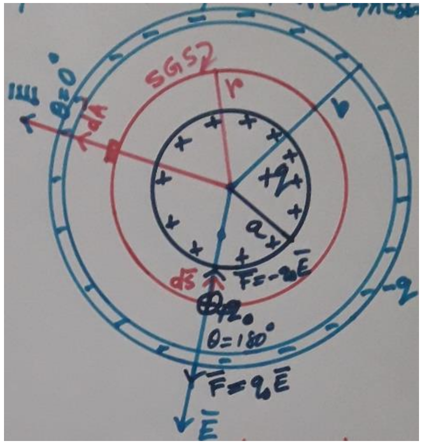
(a) $C = \dfrac{\varepsilon_0 A}{d} = \dfrac{\varepsilon_0(\pi r^2)}{d}$

$$C = \frac{8.854 \times 10^{-12} \times 3.1416 \times (0.082)^2}{1.3 \times 10^{-3}} = 143.87 \times 10^{-12}\ F$$

(b) $q = CV = 143.87 \times 10^{-12} \times 120 = 17.26 \times 10^{-9}\ C$

Problem 4 (Book chapter 25):

The plates of a spherical capacitor have radii 38.0 mm and 40.0 mm. (a) Calculate the capacitance. (b) What must be the plate area of a parallel-plate capacitor with the same plate separation and capacitance?



Given: $a = 38\, mm = 38 \times 10^{-3}\, m$

$$b = 40\, mm = 40 \times 10^{-3}\, m$$

$$b - a = (40 - 38) mm = 2\, mm = 2 \times 10^{-3}\, m$$

(a) Capacitance for a Spherical capacitor: $C = \dfrac{4\pi\varepsilon_0\, ab}{b - a}$

$$C = \frac{ab}{(1/4\pi\varepsilon_0)(b-a)} = \frac{1}{9 \times 10^9} \frac{38 \times 40 \times 10^{-6}}{2 \times 10^{-3}}$$

$$C = 84.44 \times 10^{-12}\, F$$

(b) Here: $d = b - a = 2\, mm = 2 \times 10^{-3}\, m$

$A = ?$

Parallel plate capacitor, $C = \dfrac{\varepsilon_0 A}{d}$

$$A = \frac{Cd}{\varepsilon_0} = \frac{84.44 \times 10^{-12} \times 2 \times 10^{-3}}{8.854 \times 10^{-12}}$$

$$A = 19.074 \times 10^{-3}\, m^2$$

Problem 6 (Book chapter 25):

You have two flat metal plates, each of area 1.00 m^2, with which to construct a parallel-plate capacitor. (a) If the capacitance of the device is to be 1.00 F, what must be the separation between the plates? (b) Could this capacitor actually be constructed?

Answer:

Given

$$A = 1.00\, m^2$$

$$C = 1.00\, F$$

(a) $d = ?$

(b) Could this capacitor actually be constructed?

(a) We know

$$C = \frac{\varepsilon_0 A}{d}$$

$$d = \frac{\varepsilon_0 A}{C} = \frac{8.854 \times 10^{-12} \times 1}{1} = 8.854 \times 10^{-12}\, m$$

d = 0.000000000008854 m

(b) No: It is not possible to construct a capacitor by the separation distance, $d = 8.854 \times 10^{-12}\, m$, because d value is less than the minimum size of an atom ($10^{-10}\, m$)

Thank You