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CG

Bezier Curve

* Start and end control point or anchor point वतन,

Properties of B.C. :-

(i) The curve always touches the first and last control point

(2) The degree of polynomial is always 1 less then the control points. (n-1) number of control points.

(3) The curve is contain within the convex hull of the control points.

(4) The curve exhibits global control poi

Equation of Bezier Curve:-

$$P(u) = \sum_{i=0}^{n} P_i \, B_{i,n}(u)$$

$$B_{i,n}(u) = \frac{n!}{i!\,(n-i)!} \cdot u^i \cdot (1-u)^{n-i}$$

n = degree of polynomial.

if $n = 3$

$$P(u) = \sum_{i=0}^{3} P_i \, B_{i,3}(u)$$

$$= P_0 B_{0,3}(u) + P_1 B_{1,3}(u) + P_2 B_{2,3}(u) + P_3 B_{3,3}(u)$$

$$= P_0 (1-u)^3 + P_1 3u(1-u)^2 + P_2 3u^2(1-u) + P_3 u^3$$

$$= x_0 (1-u)^3 + x_1 3u(1-u)^2 + x_2 3u^2(1-u) + x_3 u^3$$

$$= y_0 (1-u)^3 + y_1 3u(1-u)^2 + y_2 3u^2(1-u) + y_3 u^3$$

$$B_{0,3} = \frac{3!}{0!\,(3-0)!} \cdot u^0 \cdot (1-u)^{3-0}$$

$$= (1-u)^3$$

$$B_{1,3} = \frac{3!}{1!\,(3-1)!} \cdot u^1 \cdot (1-u)^{3-1}$$

$$= 3u(1-u)^2$$

$$B_{2,3} = \frac{3!}{2!\,(3-2)!} \cdot u^2 \cdot (1-u)^{3-2} = 3u^2(1-u)$$

$$B_{3,3} = \frac{3!}{3!(3-3)!} \cdot u^3 \cdot (1-u)^{3-3}$$

$$= u^3$$

A(1, 1), B(2, 3), C(4, 3), D(6, 4)

$x_0\ y_0$, $x_1\ y_1$, $x_2\ y_2$, $x_3\ y_3$

$\boxed{0 \le u \le 1 \quad 0.2 \text{ interval}}$

u	x(u)	y(u)
0.	1	1
0.2	1.71	0.98
0.4	2.62	2.63
0.6	3.66	3.09
0.8	4.81	4.50
1	6	4

if $n = 2$ then

$$P(u) = \sum_{i=0}^{2} P_i\, B_{i,2}(u)$$

$$= P_0 B_{0,2}(u) + P_1 B_{1,2}(u) + P_2 B_{2,2}(u)$$

$$= P_0 (1-u)^2 + P_1 2u(1-u)^1 + P_2 u^2$$

$$= x_0 (1-u)^2 + x_1 2u(1-u)^1 + x_2 u^2$$

$$= y_0 (1-u)^2 + y_1 2u(1-u)^1 + y_2 u^2$$