Lecture 6

Chapter 19: The Kinetic Theory of Gases

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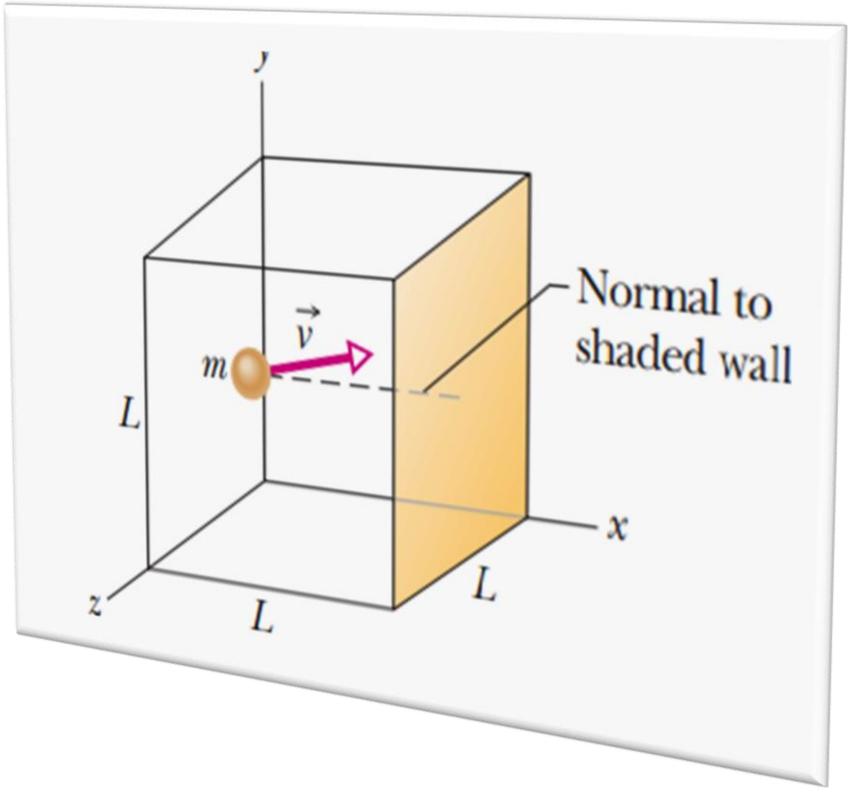
19-3 Pressure, temperature and rms speed :

Let n moles of an ideal gas be confined in a cubical box of volume $V = L^3$ at temperature T.



The molecules of gas in the box are moving in all directions and with various speeds and consider only elastic collisions with the walls.

One molecule: 1D (x-axis)

$$p = \frac{F_x}{A}$$

$$p = \frac{\frac{\Delta p_x}{\Delta t}}{A}$$

$$\Delta\overrightarrow{p_x} = \overrightarrow{p_{xf}} - \overrightarrow{p_{xi}} = \mathrm{m}(-v_x) - \mathrm{m}(+v_x) = -mv_x - \mathrm{m}v_x = -2mv_x$$

$$\Delta p_x = 2mv_x$$

$$v_x = \frac{\Delta x}{\Delta t}$$

$$\Delta t = \frac{2\mathrm{L}}{v_x}$$

$$A = L^2$$

$$p = \frac{\frac{\Delta p_x}{\Delta t}}{A} = \frac{\Delta p_x}{\Delta t}\left(\frac{1}{A}\right) = \frac{2mv_x}{\frac{2L}{v_x}}\left(\frac{1}{L^2}\right) = 2mv_x\left(\frac{v_x}{2L}\right)\left(\frac{1}{L^2}\right) = \frac{mv_x^2}{L^3} = \frac{mv_x^2}{V}$$

N molecules: 1D (x-axis)   N = nN_A and M = mN_A

$$p = \frac{m\,v_x^2}{V} = \frac{m}{V}(v_{x1}^2 + v_{x2}^2 + v_{x3}^2 + \cdots \qquad + v_{xN}^2)$$

$$p = \frac{m\,N}{V}\left(\frac{v_{x1}^2+v_{x2}^2+v_{x3}^2 + \cdots \qquad +v_{xN}^2}{N}\right)$$

$$p = \frac{m\,\mathrm{n}N_A}{V}\left(\frac{v_{x1}^2+v_{x2}^2+v_{x3}^2 + \cdots \qquad +v_{xN}^2}{N}\right)$$

$$p = \frac{\mathrm{n}(\mathrm{m}N_A)}{V}\left(v_x^2\right)_{avg}$$

$$p = \frac{nM}{V}\left(v_x^2\right)_{avg}$$

N molecules (not an ideal gas): 3D (x, y, z - axes)

$$v^2 = v_x^2 + v_y^2 + v_z^2$$

$$(v^2)_{avg} = (v_x^2)_{avg} + (v_y^2)_{avg} + (v_z^2)_{avg}$$

$$[v_x = v_y = v_z]$$

$$(v^2)_{avg} = (v_x^2)_{avg} + (v_x^2)_{avg} + (v_x^2)_{avg}$$

$$(v^2)_{avg} = 3(v_x^2)_{avg}$$

$$(v_x^2)_{avg} = \frac{(v^2)_{avg}}{3}$$

$$p = \frac{nM}{V}\frac{(v^2)_{avg}}{3}$$

$$p = \frac{nM}{3V}(v^2)_{avg}$$

$$v_{rms} = \sqrt{(v^2)_{avg}}$$

$$v_{rms}^2 = (v^2)_{avg}$$

$$p = \frac{nM}{3V}v_{rms}^2$$

$$v_{rms}^2 = \frac{3pV}{nM}$$

Ideal gas, $pV = nRT$

$$v_{rms} = \sqrt{\frac{3nRT}{nM}}$$

$$v_{rms} = \sqrt{\frac{3RT}{M}}$$

v_{rms} = microscopic property
T = macroscopic property

This is the relation between the rms speed of a microscopic property and the temperature of a macroscopic property.

$$(v_x^2)_{avg} + (v_x^2)_{avg} + (v_x^2)_{avg} = \left(\frac{v_{x1}^2 + v_{x2}^2 + v_{x3}^2 + \cdots \qquad + v_{xN}^2}{N}\right)$$

$$\left(\frac{v_{y1}^2 + v_{y2}^2 + v_{y3}^2 + \cdots \qquad + v_{xN}^2}{N}\right) + \left(\frac{v_{z1}^2 + v_{z2}^2 + v_{z3}^2 + \cdots \qquad + v_{xN}^2}{N}\right)$$

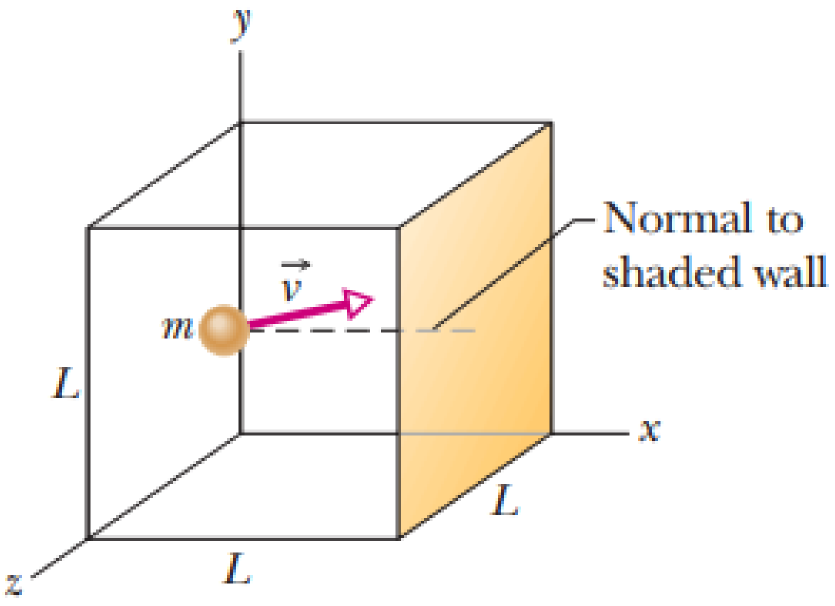
$$v^2 = v_x^2 + v_y^2 + v_z^2$$

$$v_{rms} = \sqrt{(v^2)_{avg}}$$

$$v_{rms}^2 = (v^2)_{avg}$$

19-4 Translational kinetic energy :

We consider a single molecule of an ideal gas as it moves around in the box but we now assume that its speed changes when it collides with other molecules. Its translational kinetic energy at any instant $\frac{1}{2}mv^2$. Its *average* translational kinetic energy over the time,



$$K_{avg} = \left(\tfrac{1}{2}mv^2\right)_{avg} = \tfrac{1}{2}m(v^2)_{avg} = \tfrac{1}{2}mv_{rms}^2,$$

$$v_{rms} = \sqrt{(v^2)_{avg}}$$

$$v_{rms}^2 = (v^2)_{avg}$$

$$K_{avg} = \left(\tfrac{1}{2}m\right)\frac{3RT}{M}.$$

$$v_{rms} = \sqrt{\frac{3RT}{M}}$$

$$K_{avg} = \left(\tfrac{1}{2}m\right)\frac{3RT}{M}.$$

$$K_{avg} = \frac{3RT}{2N_A}.$$

$$\boxed{K_{avg} = \tfrac{3}{2}kT.}$$

$M = mN_A$

$$\frac{m}{M} = \frac{1}{N_A}$$

Boltzmann constant, $k = R/N_A$

The average kinetic energy of an atom is a function of temperature.

Problem 18:

The temperature and pressure in the Sun's atmosphere are 2.00×10^6 K and 0.0300 Pa. Calculate the rms speed of free electrons (mass 9.11×10^{-31} kg) there, assuming they are an ideal gas.

Solution:

$$v_{rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3 \times 8.31 \times 2.00 \times 10^6}{5.49 \times 10^{-7}}} = 9.53 \times 10^6 \text{ m/s}$$

$$v_{rms} = 95,00000 \text{ m/s}$$

$M = mN_A =$ **9.11×10^{-31}** $(6.023 \times 10^{23}) = 5.49 \times 10^{-7}$ kg

Problem 25

Determine the average value of the translational kinetic energy of the molecules of an ideal gas at temperatures (a) 0.00 ^{0}C and (b) 100 ^{0}C. What is the translational kinetic energy per mole of an ideal gas at (c) 0.00 ^{0}C and (d) 100 ^{0}C?

Solution:

(a) K_{avg} per molecule $= \left(\frac{3}{2}\right) kT = \frac{3}{2}\left(\frac{R}{N_A}\right) T = \frac{3}{2} \times \frac{8.314}{6.022 \times 10^{23}} (273.0) = 5.654 \times 10^{-21}$J

(b) K_{avg} per molecule $= \left(\frac{3}{2}\right) kT = \frac{3}{2}\left(\frac{R}{N_A}\right) T = \frac{3}{2} \times \frac{8.314}{6.022 \times 10^{23}} (373.0) = 7.724 \times 10^{-21}$J

(c) K_{avg} per mole $= K_{avg} N_A = 5.654 \times 10^{-21}$ x 6.022×10^{23} = 3405 J

(d) K_{avg} per mole $= K_{avg} N_A = 7.724$ J $\times 10^{-21}$ x 6.022×10^{23} = 4651 J

Sample Problem 19.03 Average and rms values

Here are five numbers: 5, 11, 32, 67, and 89.

(a) What is the average value n_{avg} of these numbers?

(b) What is the rms value n_{rms} of these numbers?

(a) The average value, n_{avg} =

(b) The rms value n_{rms} =