Lecture 17

16-2: Wave speed on a stretched string, $v = \sqrt{\dfrac{\tau}{\mu}}$

➢ Speed of a wave is set by the properties of the medium (stretched string).

➢ If a wave is to travel through a medium, it must cause the particles of that stretched string (medium) to oscillate as it passes.

➢ It requires both mass (for kinetic energy, $K = \dfrac{1}{2}mv^2$) and elasticity (for potential energy, $U = \dfrac{1}{2}kx^2$) properties.

➢ Thus the mass and elasticity properties of the medium determine how fast the wave can travel in the medium.

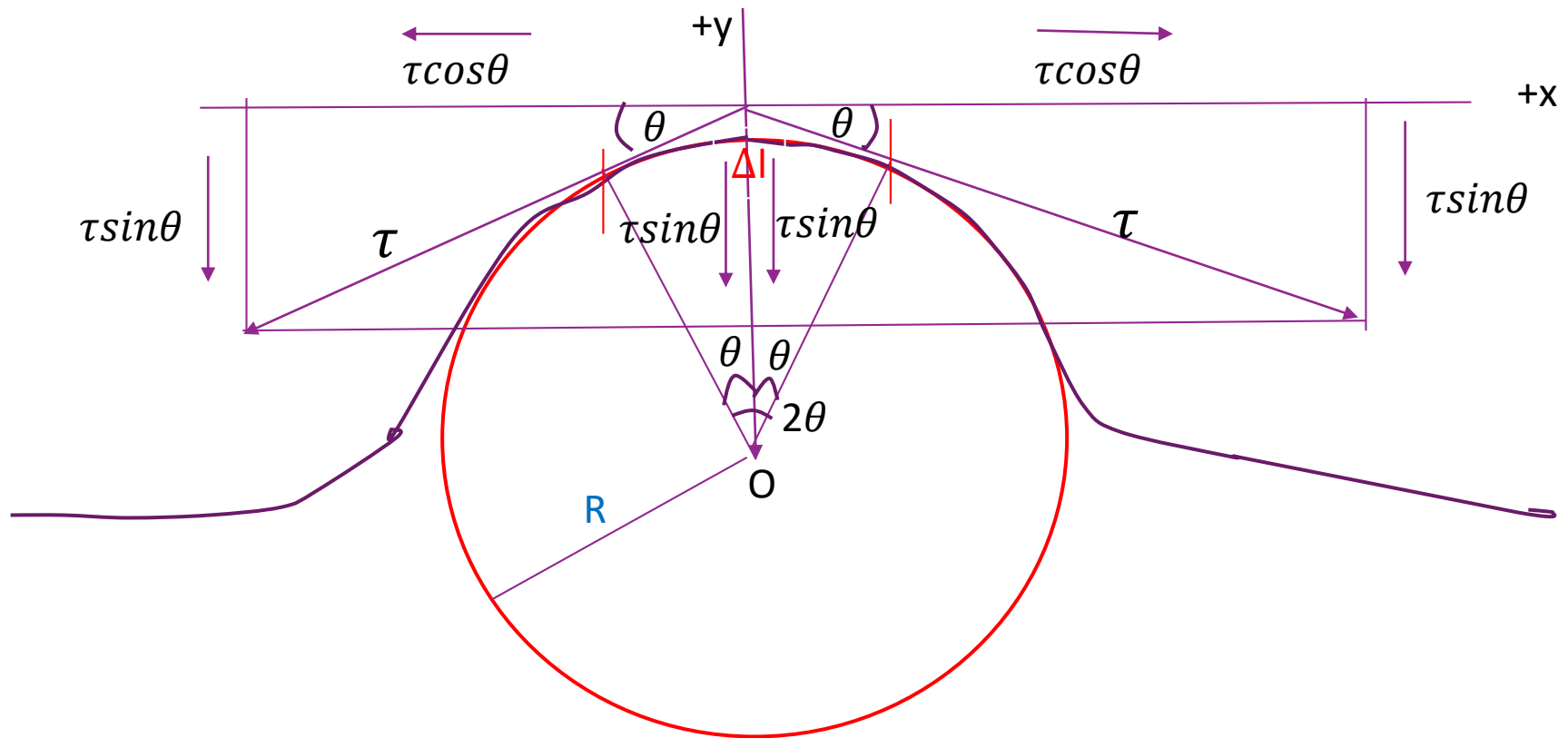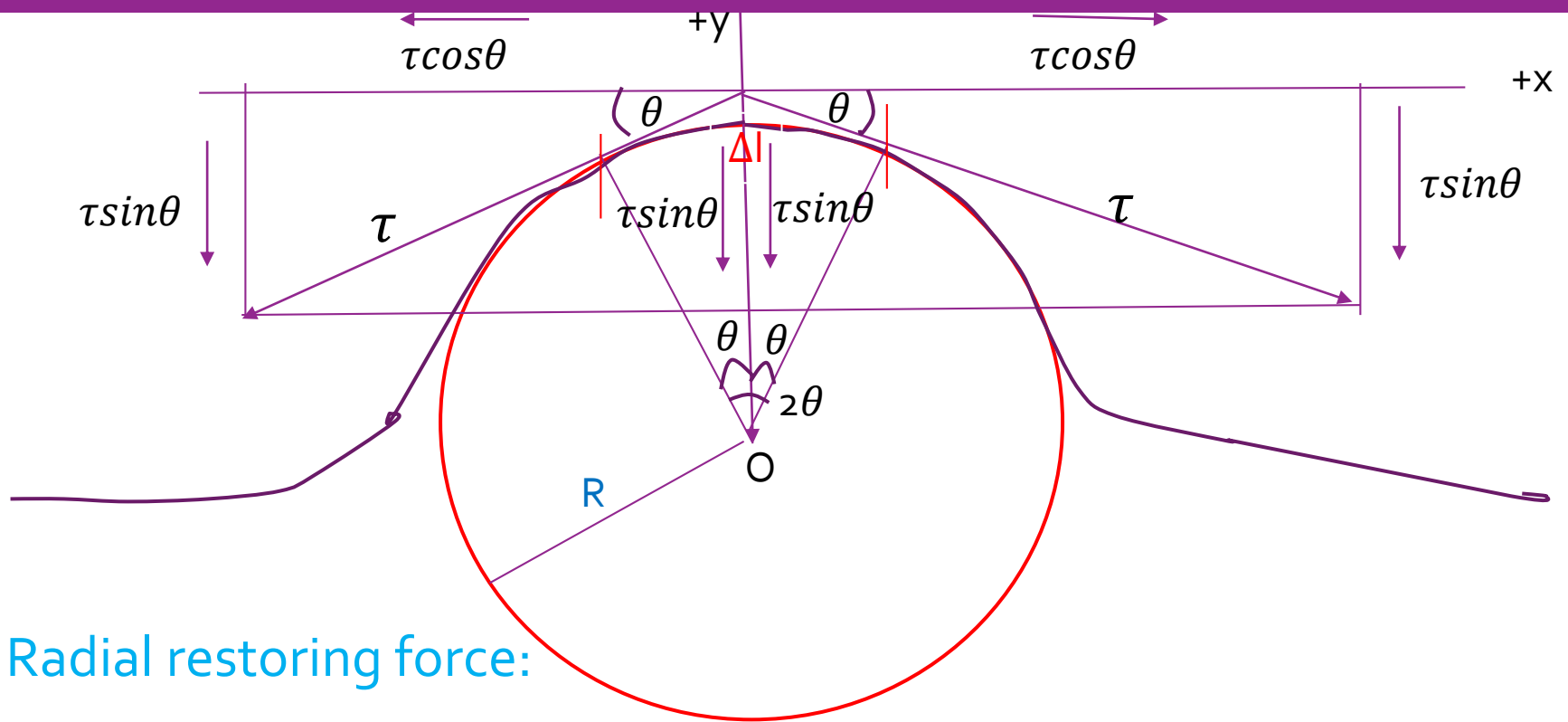# Derivation from Newton's second law of motion, $v = \sqrt{\dfrac{\tau}{\mu}}$

Fig.: A symmetrial pulse is stationary viewed from a reference frame.
String appears to move from right ot left with speed v.
String element of length Δl located at the top of the pulse

$\tau\cos\theta$ +y $\tau\cos\theta$ +x

θ θ

$\tau\sin\theta$ Δl $\tau\sin\theta$

τ $\tau\sin\theta$ $\tau\sin\theta$ τ $\tau\sin\theta$

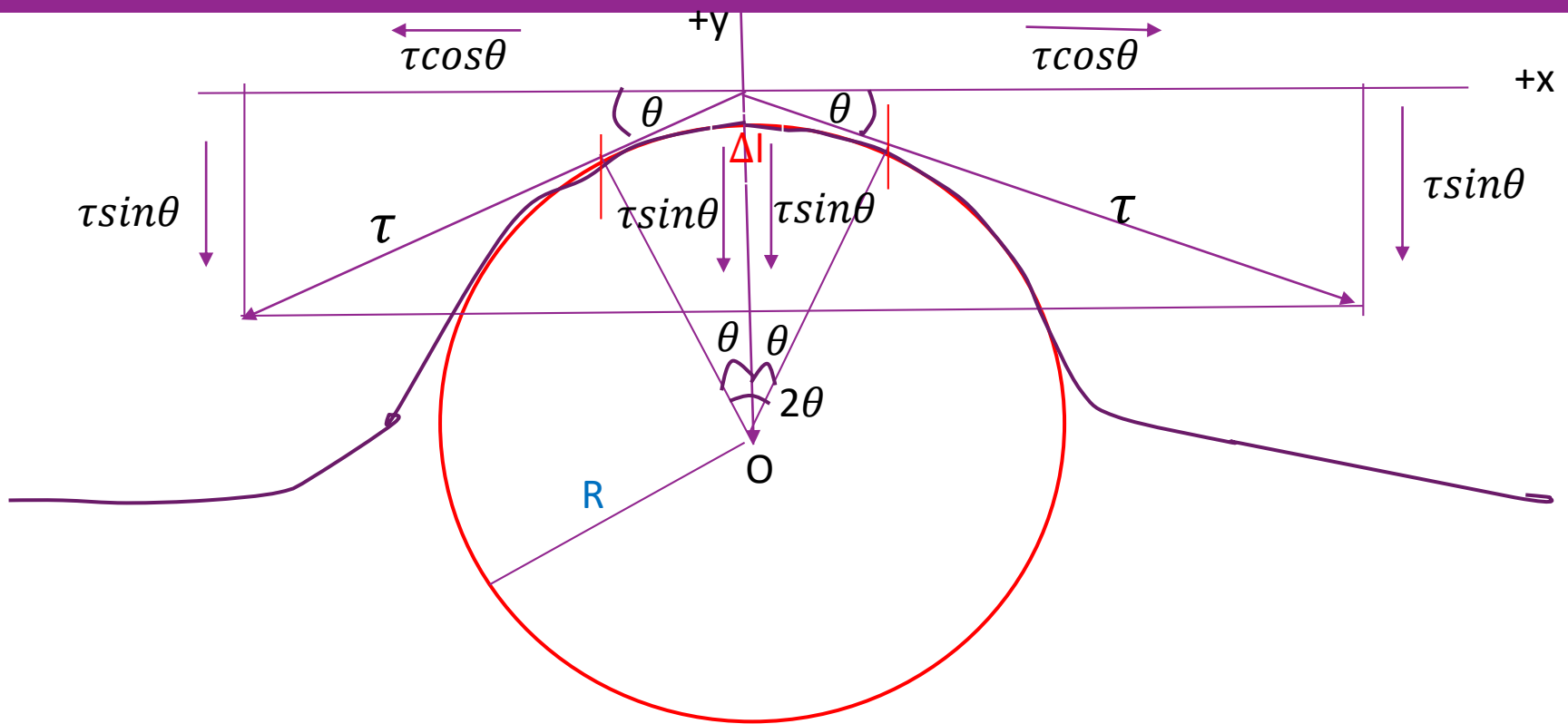θ θ

2θ

O

R

1. Radial restoring force:

$$F = \tau\sin\theta + \tau\sin\theta = 2\tau\sin\theta = 2\tau\theta = \tau(2\theta) = \tau\left(\frac{\Delta l}{R}\right)$$

[If θ is very small, $\sin\theta \cong \tan\theta \cong \theta \ and \ 2\theta = \frac{\Delta l}{R}$]

2. Mass of the element: *Linear density of the string* $= \dfrac{mass}{length}$
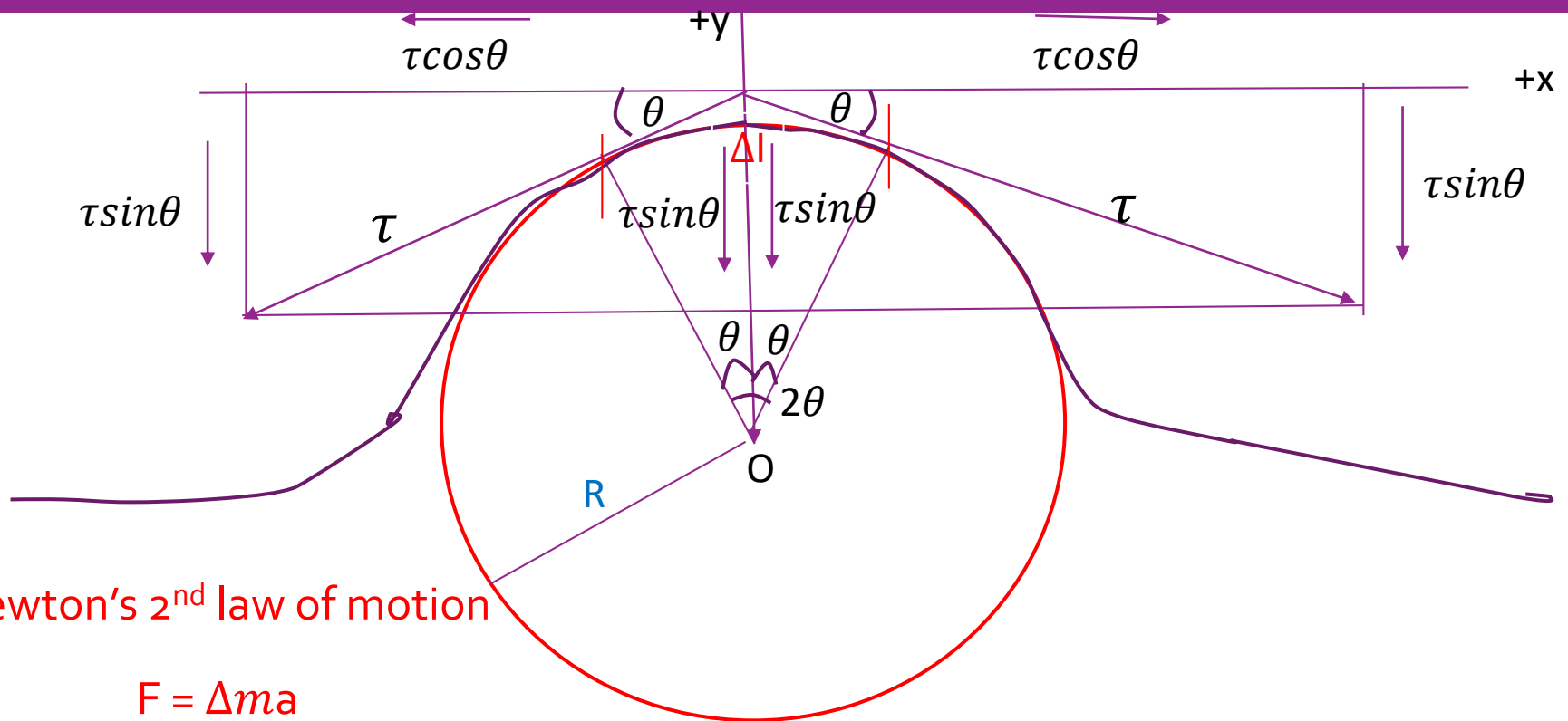
$$\mu = \frac{\Delta m}{\Delta l}$$

$$\Delta m = \mu\ \Delta l$$

$\tau \cos\theta$ +y $\tau \cos\theta$ +x

θ θ

Δl

$\tau \sin\theta$ $\tau \sin\theta$ $\tau \sin\theta$ $\tau \sin\theta$

τ τ

θ θ

2θ

O

R

3. Centripetal acceleration:

String element moves in an arc of a circle. It has a centripetal acceleration toward the center of the circle.

Centripetal acceleration is given by $a = \dfrac{v^2}{R}$

$\tau\cos\theta$ +y $\tau\cos\theta$ +x

θ θ

Δl

$\tau\sin\theta$ $\tau\sin\theta$ $\tau\sin\theta$ $\tau\sin\theta$

τ τ

θ θ

2θ

O

R

Newton's 2nd law of motion

F = Δma

$$\tau\left(\frac{\Delta l}{R}\right) = \mu\Delta l\left(\frac{v^2}{R}\right)$$

Type equation here.

$$\tau = \mu v^2$$

$$v^2 = \frac{\tau}{\mu}$$

$$v = \sqrt{\frac{\tau}{\mu}}$$

The speed of a wave along a stretched ideal string depends only on the tension and linear density of the string and not on the frequency of the wave.

14. The equation of a transverse wave on a string is
$y = (2.0\ m)\ sin\ [(20\ m^{-1})x - (600\ s^{-1})t]$. The tension in the string is 15 N.
(a) What is the wave speed? (b) Find the linear density of this string in grams per meter.

$y = (2.0\ m)\ sin\ [(20\ m^{-1})x - (600\ s^{-1})t]$

$y = y_m\ sin(kx - \omega t)$

Given, $y_m = 2.0\ m$

$k = 20\ rad/m$

$\omega = 600\ rad/s$

$\tau = 15\ N$

(a) $v = \dfrac{\omega}{k} = \dfrac{600}{20} = 30\ m/s$

(b) $v = \sqrt{\dfrac{\tau}{\mu}}$

$v^2 = \dfrac{\tau}{\mu}$

$\mu = \dfrac{\tau}{v^2} = \dfrac{15}{(30)^2} = 1.67 \times 10^{-2}\ kg/m = 16.7\ gm/m$