Lecture 19: Standing Waves

If two sinusoidal waves of the same amplitude and wavelength travel in opposite directions along a stretched string, their interference with each other produces a standing wave.

$$\left.\begin{aligned} y_1(x, t) &= y_m \sin(kx - \omega t) \\ y_2(x, t) &= y_m \sin(kx + \omega t) \end{aligned}\right\} \text{traveling waves}$$

Superposition principle, $y'(x, t) = y_1(x, t) + y_2(x, t)$

$$y'(x, t) = y_m \sin(kx - \omega t) + y_m \sin(kx + \omega t)$$

$$= y_m \{\sin(kx - \omega t) + \sin(kx + \omega t)\}$$

$$= y_m \{2 \sin\left(\frac{kx - \omega t + kx + \omega t}{2}\right) \cos\left(\frac{kx - \omega t - kx - \omega t}{2}\right)\}$$

$$= 2y_m \sin\left(\frac{2kx}{2}\right) \cos\left(\frac{-2\omega t}{2}\right)$$

$$y'(x, t) = [2y_m \sin kx]\cos\omega t \qquad \text{[standing wave]}$$

Resultant displacement = $y'(x, t)$

Amplitude at position x = $[2y_m \sin kx]$

Oscillating term = $\cos\omega t$

Nodes: **The string never moves. The amplitude of the resultant wave will be zero.**

Amplitude = $2y_m$ (sin kx) = $2y_m$ (0) = 0

If $\sin kx = 0$

$\sin kx = \sin(0, \pi, 2\pi, 3\pi, \ldots\ldots\ldots\ldots)$

$\sin kx = \sin n\pi$ for $n = 0, 1, 2, 3, \ldots\ldots$

$kx = n\pi$

$\frac{2\pi}{\lambda} x = n\pi$

$x = n\frac{\lambda}{2}$ for n = 0, 1,2,3..........

Antinodes: **The halfway between nodes are called antinodes, where the amplitude of the resultant wave will be maximum.**

Amplitude = $2y_m$ (sin kx) = $2y_m$ (1) = $2y_m$

If sin kx = 1

$\sin kx = \sin\left(1\frac{\pi}{2}, 3\frac{\pi}{2}, 5\frac{\pi}{2}...........\right)$

$\sin kx = \sin\left(n+\frac{1}{2}\right)\pi \qquad \text{for } n = 0, 1, 2, 3, \ldots\ldots$

$kx = \left(n+\frac{1}{2}\right)\pi$

$\frac{2\pi}{\lambda}x = \left(n+\frac{1}{2}\right)\pi$

$x = \left(n+\frac{1}{2}\right)\frac{\lambda}{2} \qquad \text{for } n = 0, 1, 2, 3, \ldots\ldots\ldots\ldots$

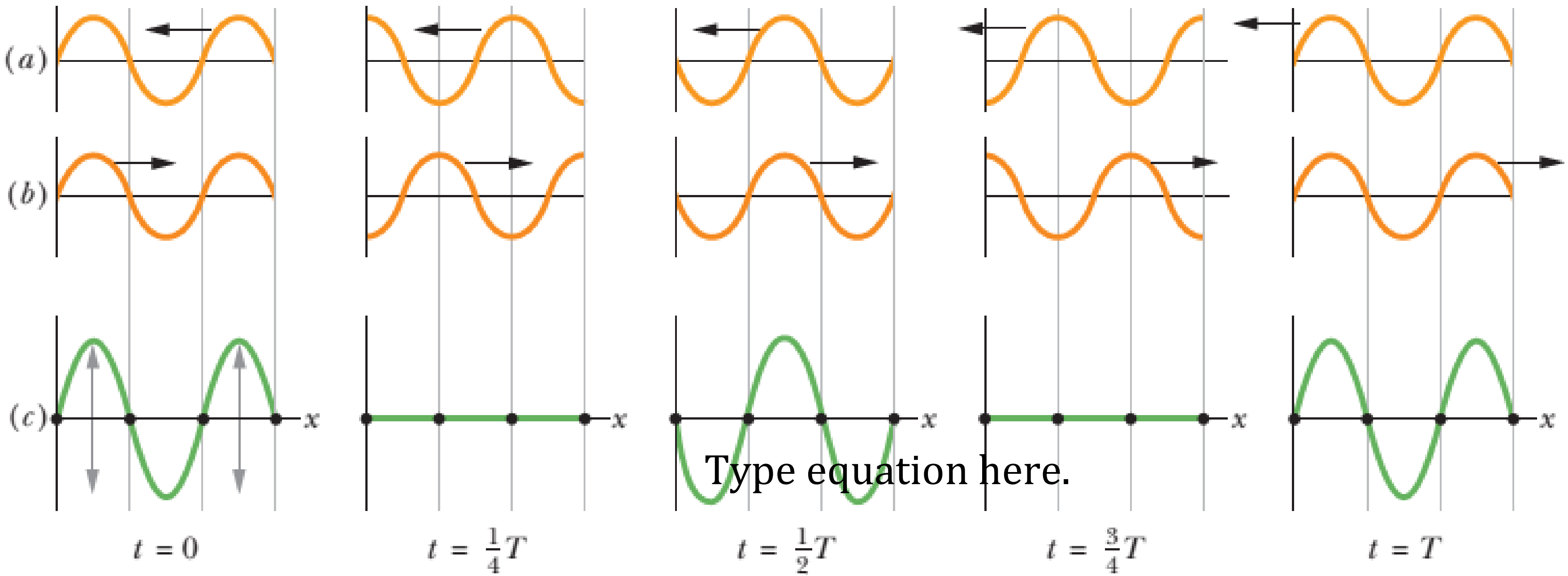

Fig. In phase: at t= 0, $\frac{1}{2}T, T, the\ fully\ constructive\ interference\ occurs\ because\ of\ the\ alignment\ at\ peaks\ with\ peaks\ and\ valleys\ with\ valleys$.

Fig. out of phase: at t= $\frac{1}{4}T, \frac{3}{4}T the\ fully\ destructive\ interference\ occurs\ because\ of\ the\ alignment\ of\ peaks\ \ and\ valleys$.

Nodes: some points never oscillate. Type equation here.

Antinodes: some points oscillate the most.

53. A string oscillate according to the equation $y' = (0.50\text{ cm}) \sin[(\pi/3\text{ cm}^{-1})x] \cos[(40\pi\text{ s}^{-1})t]$. What are (a) the amplitude and (b) the speed of the two waves (identical except for direction of travel) whose superposition gives this oscillation? (c) What is the distance between nodes? (d) What is the speed of a particle of the string at the position x = 1.5 cm when t = 9/8 s?

$$y' = [(0.50\text{ cm}) \sin\{(\pi/3\text{ cm}^{-1})x\}] \cos[(40\pi\text{ s}^{-1})t]$$

$$y' = [2y_m \sin kx] \cos\omega t$$

Given, $k = \frac{\pi}{3}$ rad/cm$= \frac{\pi}{0.03}$ rad/m; $\omega = 40\pi$ rad/s

(a) $2y_m = 0.50$ cm

$2y_m = 0.0050$ m

$y_m = 0.0050/2$ m

$y_m = 0.0025$ m Ans.

(b) $v = \frac{\omega}{k} = \frac{40\pi}{\frac{\pi}{0.03}} = 0.03(40) = 1.20$ m/s Ans.

(c) $\Delta x = \frac{\lambda}{2} = \frac{\frac{2\pi}{k}}{2} = \frac{\pi}{k} = \frac{\pi}{\frac{\pi}{0.03}} = 0.03$ m Ans.

(d) $y'(x, t) = [2y_m \sin kx] \cos\omega t$

The speed of a particle of a string is given by

$$u = \frac{\partial y'}{\partial t} = \frac{\partial}{\partial t}(2y_m \sin kx \cos\omega t)$$

$$= 2y_m \sin kx \frac{\partial}{\partial t}(\cos\omega t)$$

$$= 2y_m \sin kx \,(-\sin\omega t)\frac{\partial}{\partial t}(\omega t)$$

$$= 2y_m \sin kx \,(-\sin\omega t)(\omega)$$

$$= -\omega\, 2y_m \sin kx \,(\sin\omega t)$$

$$= -40\pi\, 2(0.0025) \sin\{(\tfrac{\pi}{0.03})(0.015)\} \sin\{40\pi(9/8)\}$$

$$= -40\pi\, 2(0.0025) \sin(\tfrac{\pi}{2}) \sin(45\pi)$$

$$= \{-40\pi\, 2(0.0025) \sin(\tfrac{\pi}{2})\}(0)$$

$$= 0$$

56. A standing wave pattern on a string is described by y(x,t)= 0.040 (sin 5πx)(cos 40πt), where x and y are in meters and t is in seconds. For x = 0, what is the location of the node with the (i) smallest, (ii) second smallest, and (iii) third smallest value of x?

Solution:

Given, y(x,t)= 0.040 (sin 5πx)(cos 40πt)

Again, y (x,t) $= [2y_m \sin kx] \cos\omega t$

Now, $2y_m = 0.40$ m K= 5π rad/m ω = 40 π rad/s

For node, Sin 5πx = 0

$5\pi x = n\pi$

$x = n/5$

(i) Smallest value, x= 0

(ii) Second smallest value, x= 1/5

(iii) Third smallest value, x= 2/5