

# Lesson 9

## Chapter 20: Entropy and the second law of thermodynamics

### 20-2 Entropy in the Real World: Engines

**Heat Engine:** A heat engine, or more simply, **an engine**, is a device that **extracts energy from its environment** in the form of **heat** and **does useful work**. At the **heart of every engine** is a **working substance**.

**Carnot Engine:** Although **an ideal gas does not exist**, any **real gas** approaches ideal behavior if its **density is low enough**. Similarly, we can **study real engines** by analyzing the behavior of an **ideal engine**.

“An **ideal engine** where all processes are **reversible** and **no wasteful energy transfers occur** due to, say, friction and turbulence”.

## Schematic diagram of a Carnot Engine:

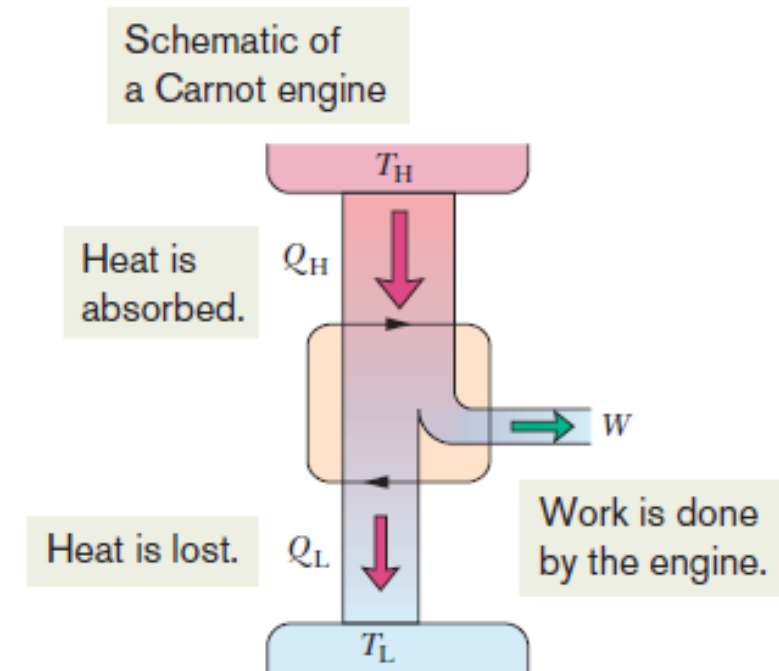
We shall focus on a particular ideal engine called a Carnot engine after the French scientist and engineer N. L. Sadi Carnot (pronounced “cah-no”), who first proposed the engine’s concept in 1824. This ideal engine turns out to be the best (in principle) at using energy as heat to do useful work. Surprisingly, Carnot was able to analyze the performance of this engine before the first law of thermodynamics and the concept of entropy had been discovered.

### pV plot of the Carnot cycle:

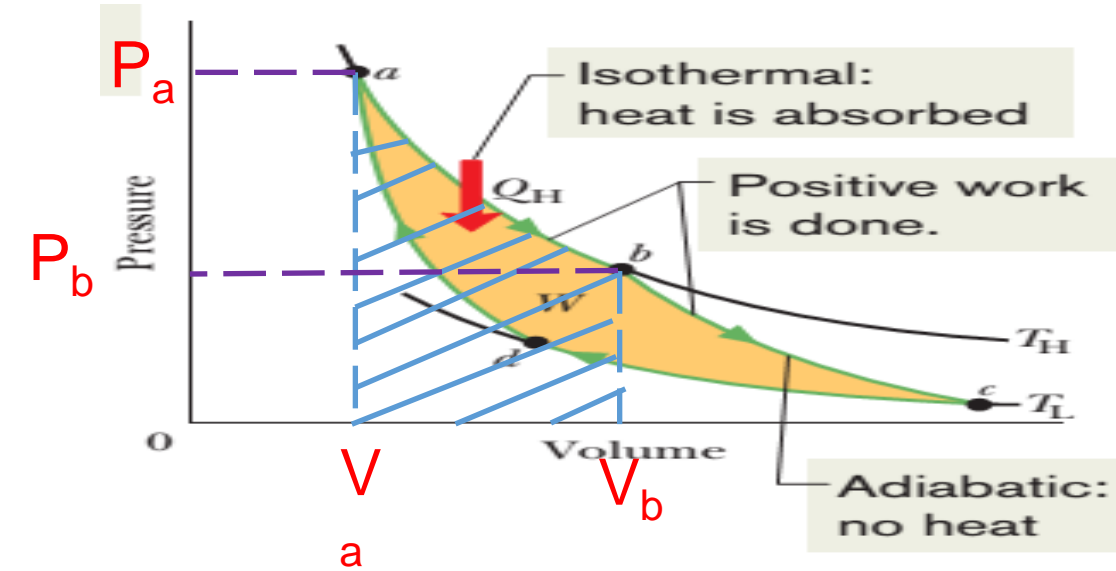
If we place the cylinder in contact with the high temperature reservoir at temperature  $T_H$ , heat  $Q_H$  is transferred to the working substance from this reservoir as the gas undergoes an isothermal expansion from volume  $V_a$  to volume  $V_b$ .

Similarly, with the working substance in contact with the low-temperature reservoir at temperature  $T_L$ , heat  $Q_L$  is transferred from the working substance to the low-temperature reservoir as the gas undergoes an isothermal compression from volume  $V_c$  to volume  $V_d$ .

Thus the heat transfers to or from the working substance only during the isothermal processes  $ab$  and  $cd$ .



**Isothermal expansion** at constant  $T_H$  from a to b states:



Heat  $Q_H$  is absorbed by the working substance.

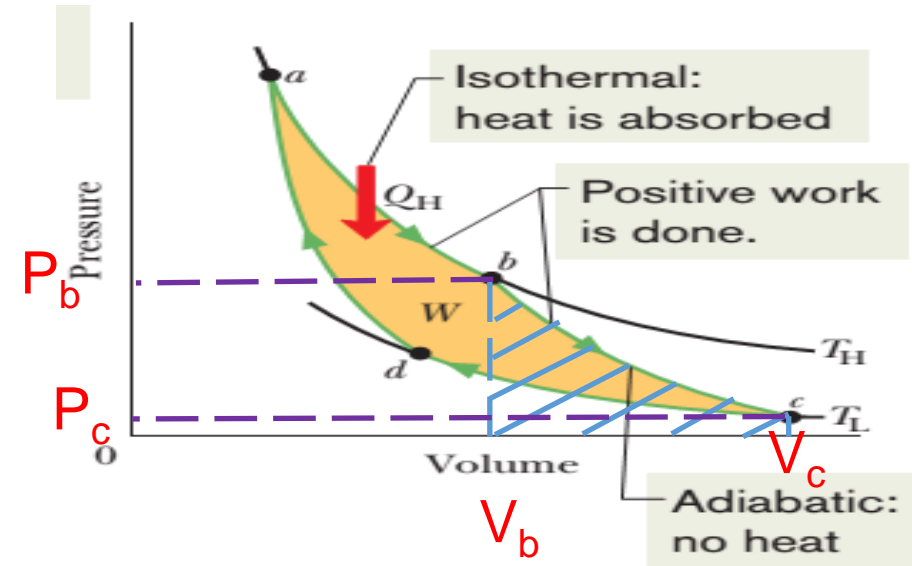
Pressure decreases from  $P_a$  to  $P_b$

Volume increases from  $V_a$  to  $V_b$

$$W = nRT_H \ln \frac{V_b}{V_a} \quad [\text{positive}]$$

During the processes ab and bc, the working substance is expanding and thus doing positive work as it raises the weighted piston. This work is represented by the area under curve abc.

**Adiabatic expansion** from b to c states:



Heat  $Q$  is zero.

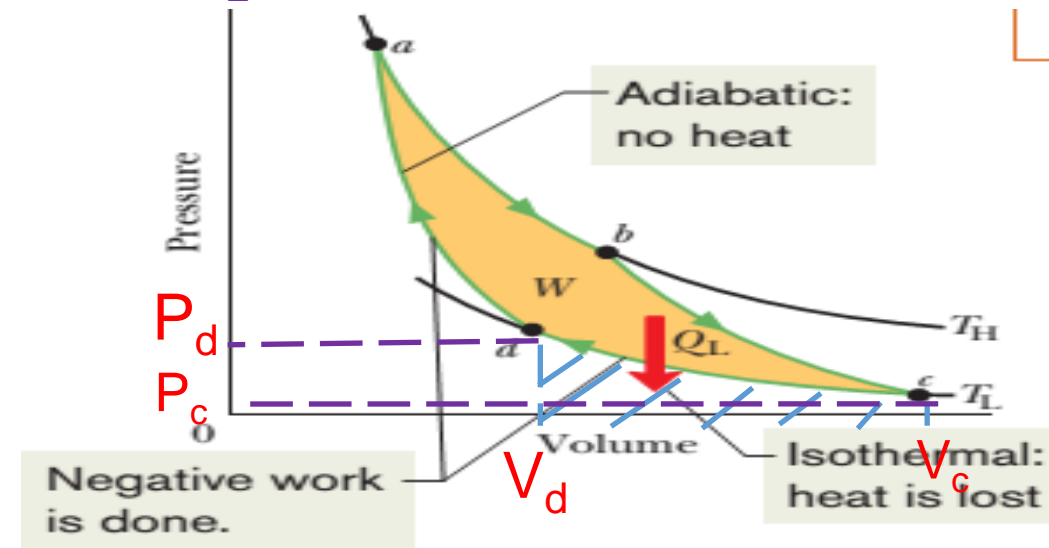
Pressure decreases from  $P_b$  to  $P_c$

Volume increases from  $V_b$  to  $V_c$

$$W = \frac{P_b V_b - P_c V_c}{\gamma - 1}$$

Isothermal compression at constant  $T_L$  from c to d states:

Heat  $Q_L$  is released.



Pressure increases from  $P_c$  to  $P_d$

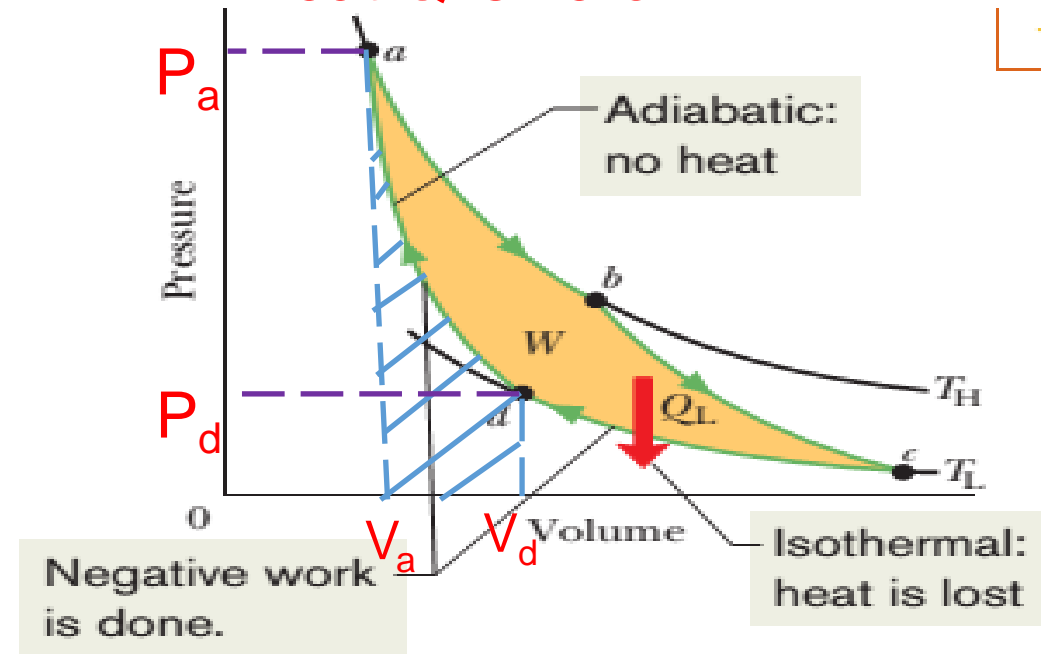
Volume decreases from  $V_c$  to  $V_d$

$$W = nRT_L \ln \frac{V_d}{V_c} \quad [\text{negative}]$$

During the processes cd and da, the working substance is being **compressed**, which means that it is doing **negative work** on its environment. This work is represented by the **area under curve cda**.

Adiabatic compression from d to a states:

Heat  $Q$  is zero.



Pressure increases from  $P_d$  to  $P_a$

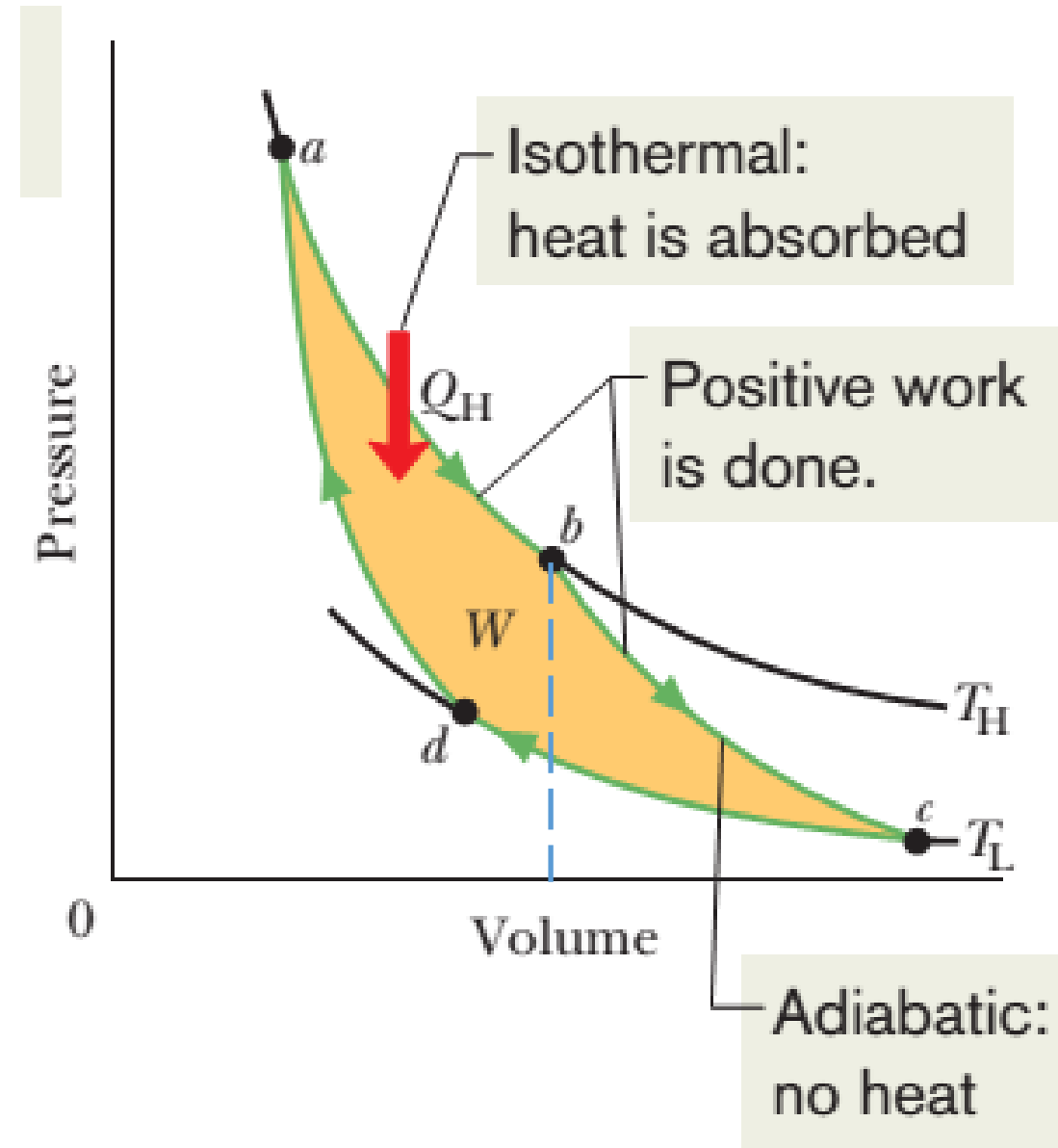
Volume decreases from  $V_d$  to  $V_a$

$$W = \frac{P_d V_d - P_a V_a}{\gamma - 1} \quad [\text{negative}]$$

## Carnot cycle:

$W = \text{abcd area inside the curve}$  [positive]

The **net work per cycle** is the **difference between these two areas** and is a **positive quantity equal to the area enclosed by cycle abcd** (yellow area).



## Carnot cycle on a temperature–entropy diagram

In the temperature–entropy ( $T - S$ ) diagram the lettered points  $a$ ,  $b$ ,  $c$ , and  $d$  correspond to the lettered points in the  $p - V$  diagram for the Carnot cycle.

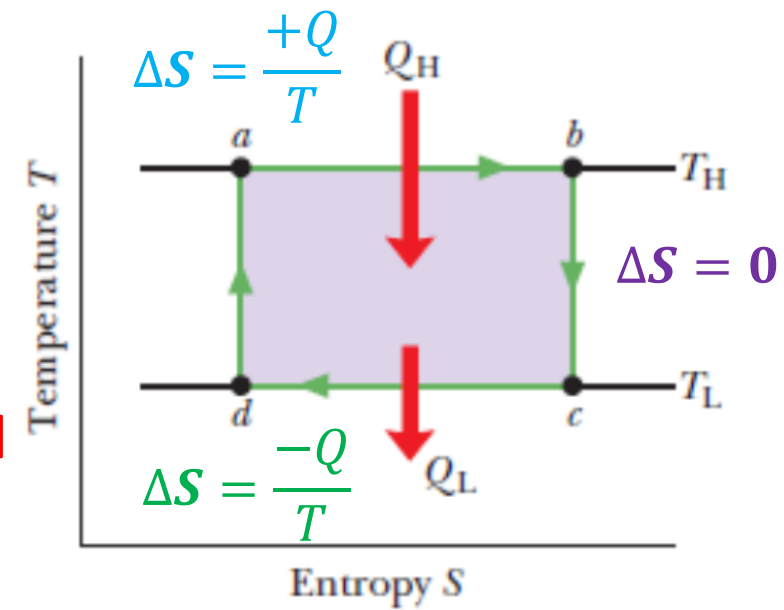
The two horizontal lines in Fig. correspond to the two isothermal processes of the cycle. Process  $ab$  is the isothermal expansion and process  $bc$  is the isothermal compression of the cycle.

As the working substance (reversibly) absorbs energy  $Q_H$  as heat at constant temperature  $T_H$  during the expansion, its entropy increases.

$$\Delta S = \frac{+Q}{T}$$

Similarly, during the isothermal compression  $cd$ , the working substance (reversibly) loses energy  $Q_L$  as heat at constant temperature  $T_L$ , and its entropy decreases.  $\Delta S = \frac{-Q}{T}$

The two vertical lines correspond to the two adiabatic processes of the Carnot cycle. Because no energy is transferred as heat during the two processes ( $Q = 0$ ), the entropy of the working substance is constant during them.  $\Delta S = 0$



**The Work:** In a Carnot engine, the working substance completes reversible cycles.

For a complete cycle of the working substance, the net internal energy change,  $\Delta E_{int} = 0$

In each cycle of a Carnot engine, the heat  $Q_H$  is transferred to the working substance from the high temperature reservoir  $T_H$  and the heat  $Q_L$  is transferred from the working substance to the low temperature reservoir  $T_L$ .

So, the net heat transfer per cycle,

$$Q = |Q_H| - |Q_L|$$

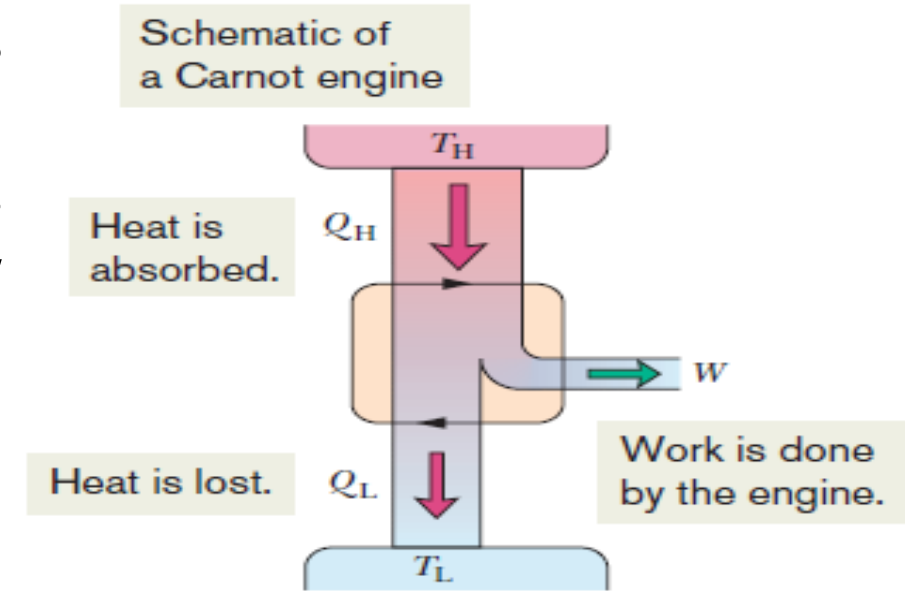
The first law of thermodynamics for the Carnot cycle,

$$\Delta E_{int} = Q - W$$

$$0 = Q - W$$

$$W = Q$$

$W = |Q_H| - |Q_L|$  This is the net work done by a Carnot engine during a cycle.



## Entropy Changes:

There are **two isothermal processes** in each cycle of a Carnot engine.

During the **isothermal expansion**, the **working substance absorbs heat**  $|Q_H|$  at temperature  $T_H$ .

The **increase in entropy**,  $\Delta S_H = \frac{+|Q_H|}{T_H}$

Again during the **isothermal compression**, the working substance **releases heat**  $|Q_H|$  at constant temperature  $T_L$ .

The **decrease in entropy**,  $\Delta S_L = \frac{-|Q_L|}{T_L}$

Thus the **net entropy change per cycle**,

$$\Delta S = \Delta S_H + \Delta S_L$$

$$\Delta S = \frac{+|Q_H|}{T_H} + \frac{-|Q_L|}{T_L}$$

For a complete cycle,  $\Delta S = 0$

$$0 = \frac{+|Q_H|}{T_H} + \frac{-|Q_L|}{T_L}$$

$$\frac{|Q_H|}{T_H} = \frac{|Q_L|}{T_L} \quad \frac{|Q_H|}{|Q_L|} = \frac{T_H}{T_L}$$



## Efficiency of a Carnot Engine:

Thermal efficiency of any engine is defined as,

$$\varepsilon = \frac{\text{energy we get}}{\text{energy we provide}} = \frac{|W|}{|Q_H|}$$

$$\varepsilon = \frac{|Q_H| - |Q_L|}{|Q_H|}$$

$$\varepsilon = 1 - \frac{|Q_L|}{|Q_H|} \quad [\text{any engine}]$$

$$\varepsilon = 1 - \frac{T_L}{T_H} \quad [\text{Carnot engine}] \quad [T_L < T_H]$$

Because  $T_L < T_H$ , the efficiency of Carnot engine is less than unity or less than 100%. Thus only a **part** of the extracted heat is available to do **work** and the **rest** is delivered to the low temperature **reservoir**.

**23. A Carnot engine whose low-temperature reservoir is at 17 °C has an efficiency of 40%. By how much should the temperature of the high-temperature reservoir be increased to increase the efficiency to 50%?**

**Solution:**

Given,

$$T_L = 17^\circ\text{C} = 290\text{ K}$$

$$\text{Initial efficiency, } \varepsilon_c = 40\%$$

$$\text{Final efficiency, } \varepsilon'_c = 50\%$$

$$\Delta T_H = ?$$

For the initial state,

$$\varepsilon_c = 1 - \frac{T_L}{T_H}$$

$$\Rightarrow 40\% = 1 - \frac{T_L}{T_H}$$

$$\Rightarrow \frac{T_L}{T_H} = 1 - 0.40$$

$$\therefore T_H = 483.33\text{ K}$$

For the final state,

$$\varepsilon'_c = 1 - \frac{T_L}{T'_H}$$

$$\Rightarrow 50\% = 1 - \frac{T_L}{T'_H}$$

$$\Rightarrow \frac{T_L}{T'_H} = 1 - 0.50$$

$$\therefore T'_H = 580\text{ K}$$

**So the increased temperature of the high temperature reservoir,**

$$\Delta T_H = T'_H - T_H$$

$$= (580 - 483.33)\text{ K}$$

$$= 96.67\text{ K}$$

**24. A Carnot engine absorbs 52 kJ as heat and exhausts 36 kJ as heat in each cycle. Calculate (a) the engine's efficiency and (b) the work done per cycle in kilojoules.**

**Solution:**

**Given,**

$$|Q_H| = 52 \text{ KJ} = 52 \times 10^3 \text{ J}$$

$$|Q_L| = 36 \text{ KJ} = 36 \times 10^3 \text{ J}$$

$$(a) \varepsilon_c = ?$$

**We know,**

$$\varepsilon_c = \left( 1 - \frac{|Q_L|}{|Q_H|} \right) \times 100\%$$

$$= \left( 1 - \frac{36 \times 10^3}{52 \times 10^3} \right) \times 100\%$$

$$= 30.77 \%$$

$$(b) W = ?$$

**We know**

$$W = |Q_H| - |Q_L|$$

$$= 52 \text{ KJ} - 36 \text{ KJ}$$

$$= 16 \text{ KJ}$$

## Chapter 20: Entropy and the second law of thermodynamics

### 20-2 No Perfect Engines:

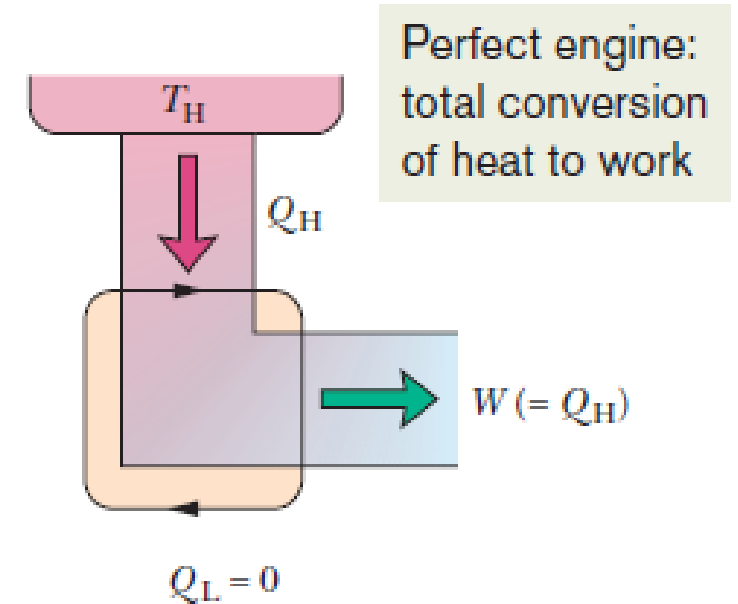
**Inventors** continually try to improve engine efficiency by reducing the energy  $Q_L$  that is “thrown away” during each cycle. The inventor’s dream is to produce the perfect engine, diagrammed in Fig, in which  $Q_L$  is reduced to zero and  $Q_H$  is converted completely into work.

Also, a perfect engine is only a dream: impossible requirements

$$\varepsilon = 1 - \frac{T_L}{T_H} = 1 - \frac{0}{\infty} = 1 = 100\%$$

Instead, experience gives the following alternative version of the second law of thermodynamics, which says in short, there are no perfect engines:

“No series of processes is possible whose sole result is the transfer of energy as heat from a thermal reservoir and the complete conversion of this energy to work”.



**25. A Carnot engine has an efficiency of 22.0%. It operates between constant-temperature reservoirs differing in temperature by 75.0 C°. What is the temperature of the (a) lower-temperature and (b) higher-temperature reservoir?**

**Given,**

**Efficiency,  $\varepsilon_c = 22.0\% = 0.22$**

**Difference in temperature,  $T_H - T_L = 75^0C = 75\text{ K}$**

$$(T_H + 273) - (T_L + 273) = 75K$$

**(a)  $T_L = ?$**

**We know,**

$$\begin{aligned}\varepsilon_c &= 1 - \frac{T_L}{T_H} \\ \Rightarrow 22\% &= 1 - \frac{T_L}{75 + T_L} \quad [\text{as } T_H = 75 + T_L] \\ \Rightarrow 0.22 &= \frac{75 + T_L - T_L}{75 + T_L}\end{aligned}$$

$$\Rightarrow 0.22 = \frac{75}{75 + T_L}$$

$$\Rightarrow 0.22(75 + T_L) = 75$$

$$\therefore T_L = 266\text{ K}$$

**(b)  $T_H = ?$**

**We have,**

$$T_H - T_L = 75\text{ K}$$

$$\therefore T_H = 341\text{ K}$$

**27. A Carnot engine operates between 235 °C and 115 °C, absorbing  $6.30 \times 10^4$  J per cycle at the higher temperature. (a) What is the efficiency of the engine? (b) How much work per cycle is this engine capable of performing?**

**Given,**

$$T_H = 235^\circ\text{C} = 508\text{ K}$$

$$T_L = 115^\circ\text{C} = 388\text{ K}$$

$$Q_H = 6.3 \times 10^4\text{ J}$$

$$(a) \ \varepsilon_c = ?$$

We know,

$$\begin{aligned}\varepsilon_c &= \left(1 - \frac{T_L}{T_H}\right) \times 100\% \\ &= \left(1 - \frac{388}{508}\right) \times 100\% \\ &= 23.62\%\end{aligned}$$

$$(b) \quad \text{The work done per cycle,}$$

$$W = \varepsilon_c |Q_H| \quad \left[\varepsilon_c = \frac{W}{|Q_H|}\right]$$

$$= 23.62\% \times (6.3 \times 10^4)$$

$$= 1.48 \times 10^4\text{ J}$$