Introduction to Trees

Course Code: 00090 Course Title: Discrete Mathematics

Dept. of Computer Science
Faculty of Science and Technology

Lecturer No:	**20**	**Week No:**	**11**	**Semester:**	
Lecturer:	*Md. Mahmudur Rahman (mahmudur@aiub.edu)*				

Lecture Outline

1. Introduction of Tree

Quiz-3 today

Objectives and Outcomes

- Objectives: To understand the definition of tree and different tree terminologies, to understand the theorems related to tree.
- Outcomes: The students are expected to be able to explain tree terminologies and the theorems related to tree.
- ter this class the student will be able to define different kind Tree. The student will be capable of demonstrating theorem related to Tree. Finally, by using the theorem the student will be able to sort out mathematical problems linked with Tree.

What is a Tree?

- **Definition**: A ***tree*** is a connected undirected graph with no simple circuits.

Example 1

- **Which of the graphs shown in Figure 2 are trees?**

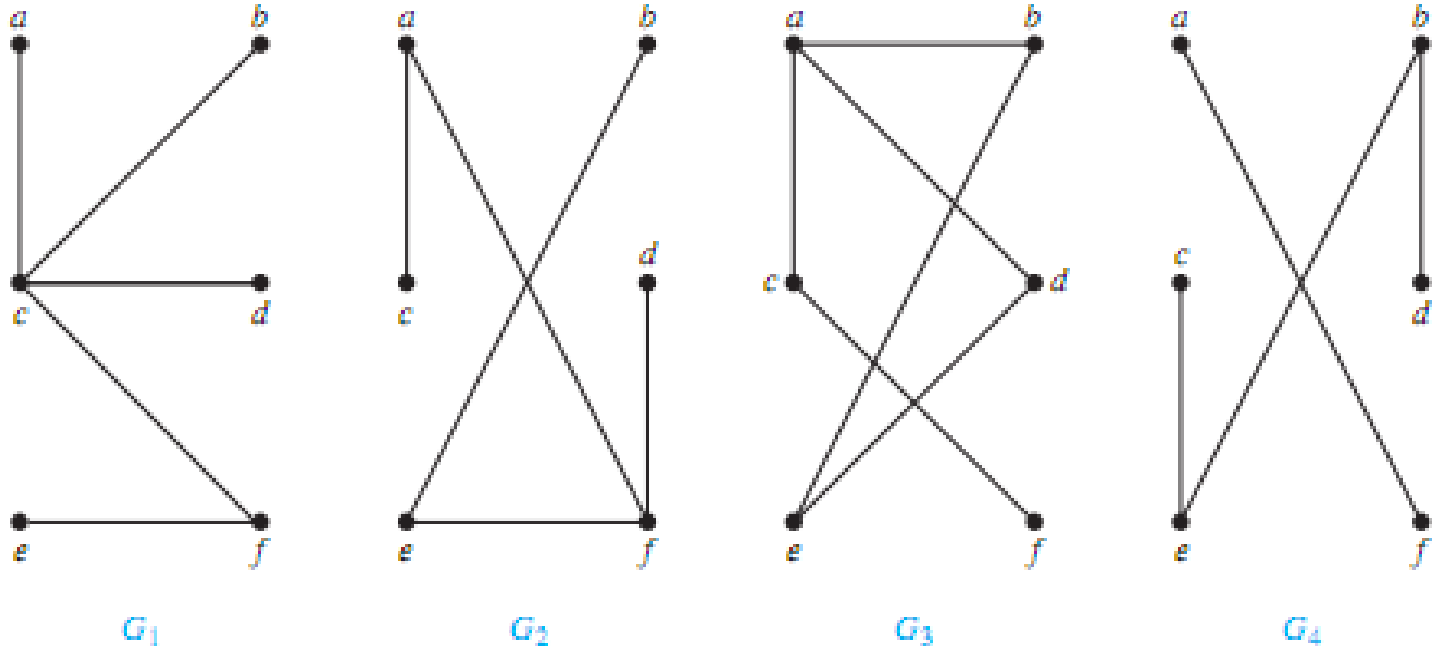


FIGURE 2 Examples of Trees and Graphs That Are Not Trees.

Example 1

- **Solution**:
- **G1 and G2 are trees**, because both are connected graphs with no simple circuits.

- **G3 is not a tree** because e, b, a, d, e is a simple circuit in this graph.

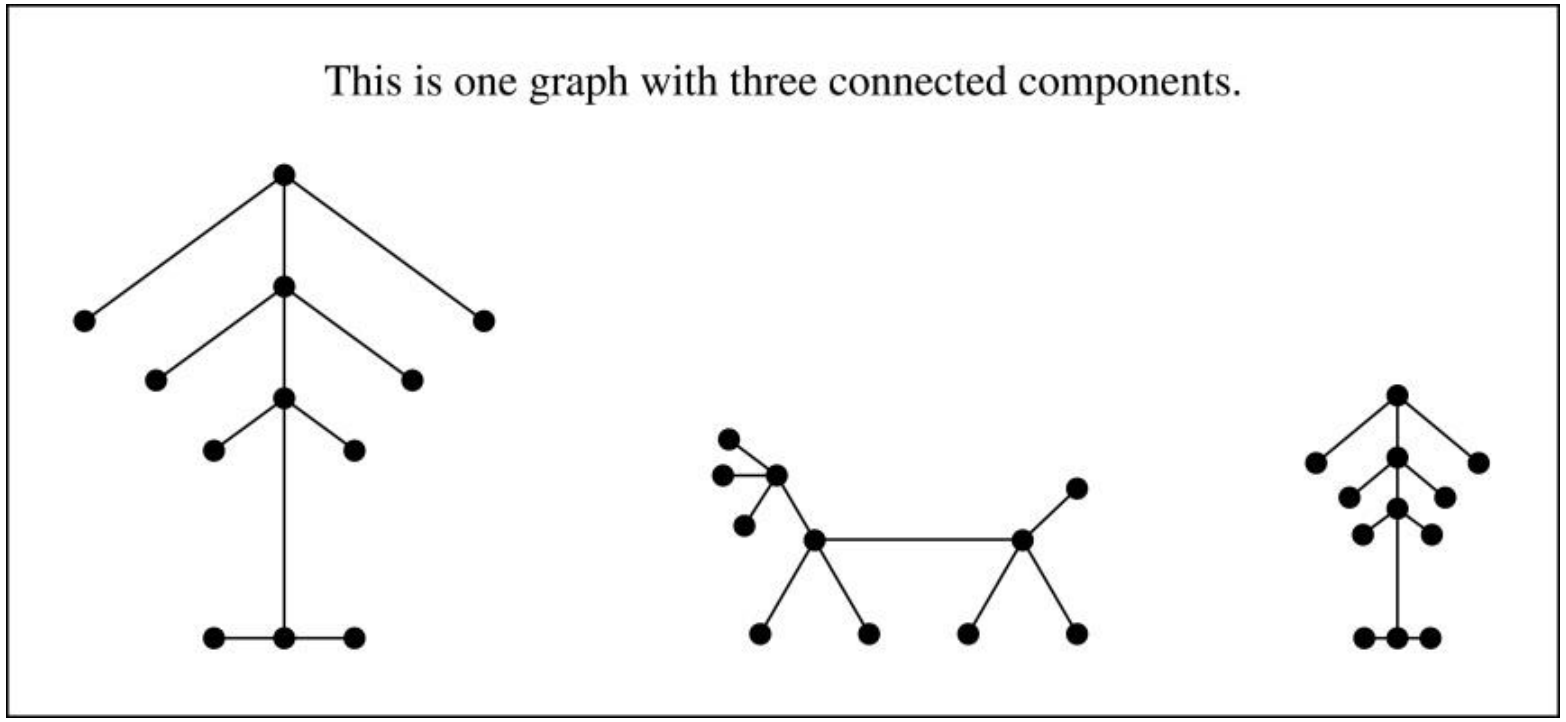
- Finally, **G4 is not a tree** because it is not connected.

Theorem 1

- An undirected graph is a **tree** if and only if there is a unique simple path *between* any two of its vertices.

Forest

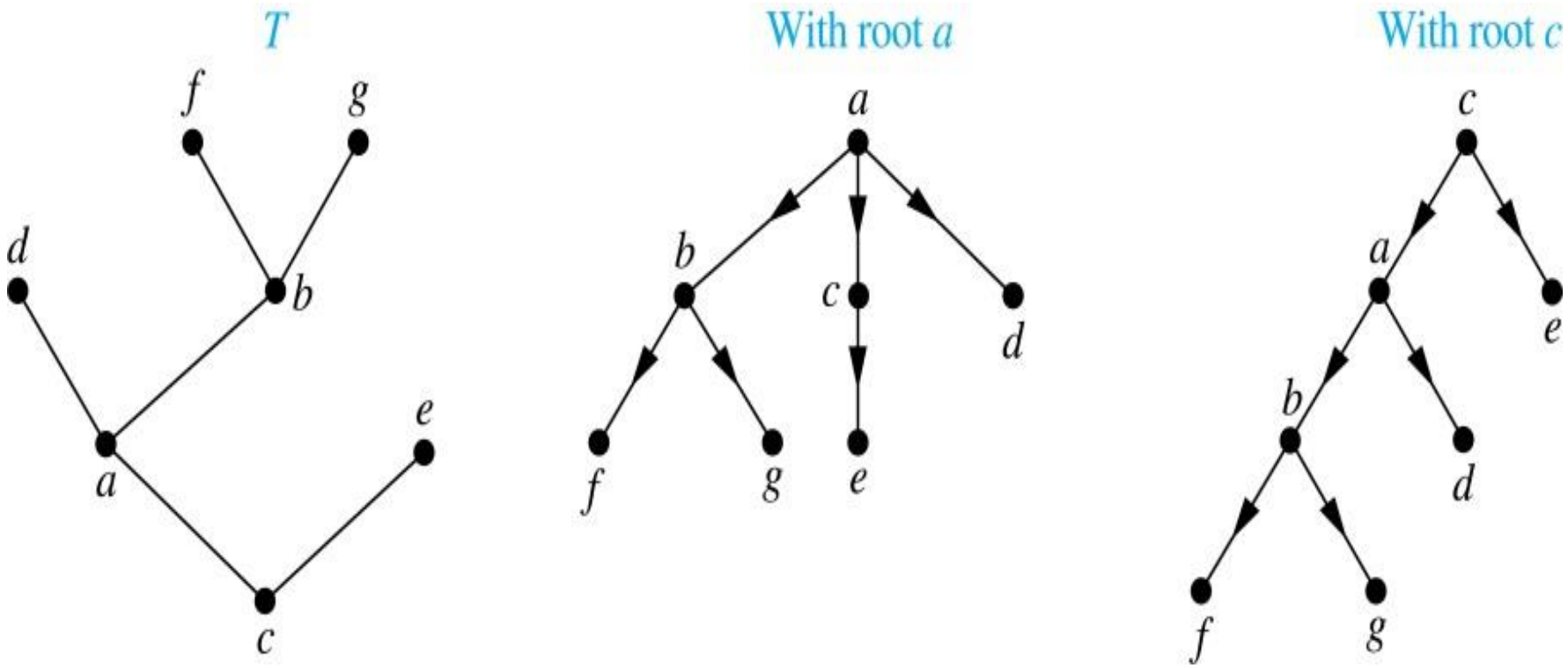
- A *forest* is a graph that has no simple circuit, but is not connected. Each of the connected components in a forest is a tree.



Rooted Tree

- **Rooted tree**: A rooted tree is a tree in which **one vertex** has been designated as the **root** and every edge is directed away from the root.
  - We can change an unrooted tree into a rooted tree by choosing any vertex as the root
  - Different choices of the root produce different rooted trees

A Tree and Rooted Trees Formed by Designating Two Roots



The rooted trees formed by designating *a* to be root and *c* to be the root

Ordered Rooted Tree

- **Definition**: An ***ordered rooted tree*** is a rooted tree where the children of each internal vertex are ordered.
  - We draw ordered rooted trees so that the children of each internal vertex are shown in order from left to right
- **Definition**: A ***binary tree*** is an ordered rooted where each internal vertex has at most two children. If an internal vertex of a binary tree has two children, the first is called the ***left child*** and the second the ***right child***. The tree rooted at the left child of a vertex is called the ***left subtree*** of this vertex, and the tree rooted at the right child of a vertex is called the ***right subtree*** of this vertex.

Example 4 @page 627

Consider the binary tree *T*.

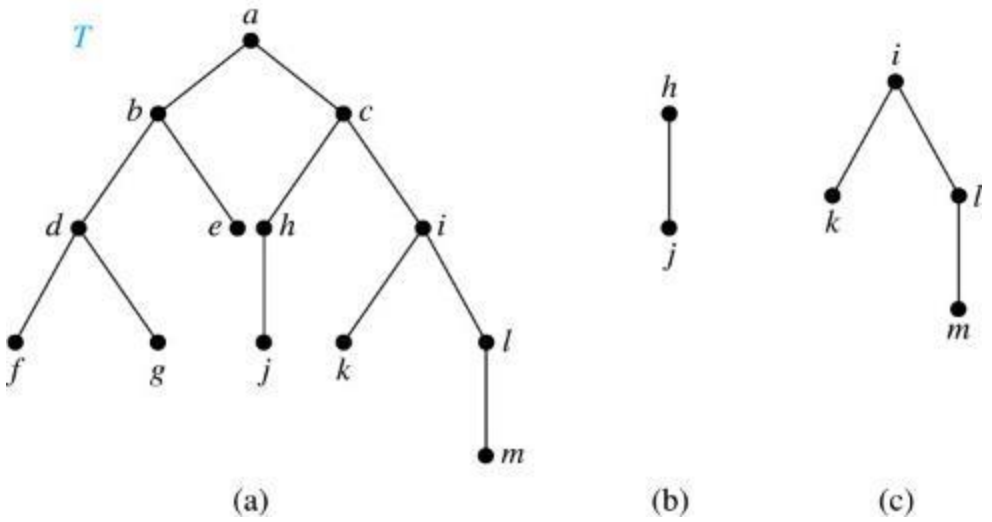
(*i*) What are the left and right children of *d*?

(*ii*) What are the left and right subtrees of *c*?

Solution:

(*i*) The left child of *d* is *f* and the right child is *g*.

(*ii*) The left and right subtrees of *c* are displayed in (b) and (c).



Rooted Tree Terminologies

- **Parent**: If *v* is a vertex of a rooted tree other than the root, the *parent* of *v* is the unique vertex *u* such that there is a directed edge from *u* to *v.*
  - Note that such a vertex is unique
- **Child**: When *u* is a parent of *v*, *v* is called a ***child*** of *u*.
- **Siblings**: Vertices with the same parent are called ***siblings***.
- **Leaf**: A vertex of a rooted tree is called a ***leaf*** if it has no children.
- **Internal vertex:** A vertex that has children is called ***internal vertex***.

- **Note**: *The root is an internal vertex unless it is the only vertex in the graph, in which case it is a leaf.*

Rooted Tree Terminologies

- **Ancestor(s):** The **ancestors** of a vertex other than the root are the vertices in the path from the root to this vertex, excluding the vertex itself and including the root.
- **Descendant(s) :** The **descendants** of a vertex ***v*** are those vertices that have ***v*** as an ancestor.
- **Subtree**: If *a* is a vertex in a tree, the **subtree with *a* as its root** is the subgraph of the tree consisting of *a* and its descendants and all edges incident to these descendants.

Illustration of Tree Terminology

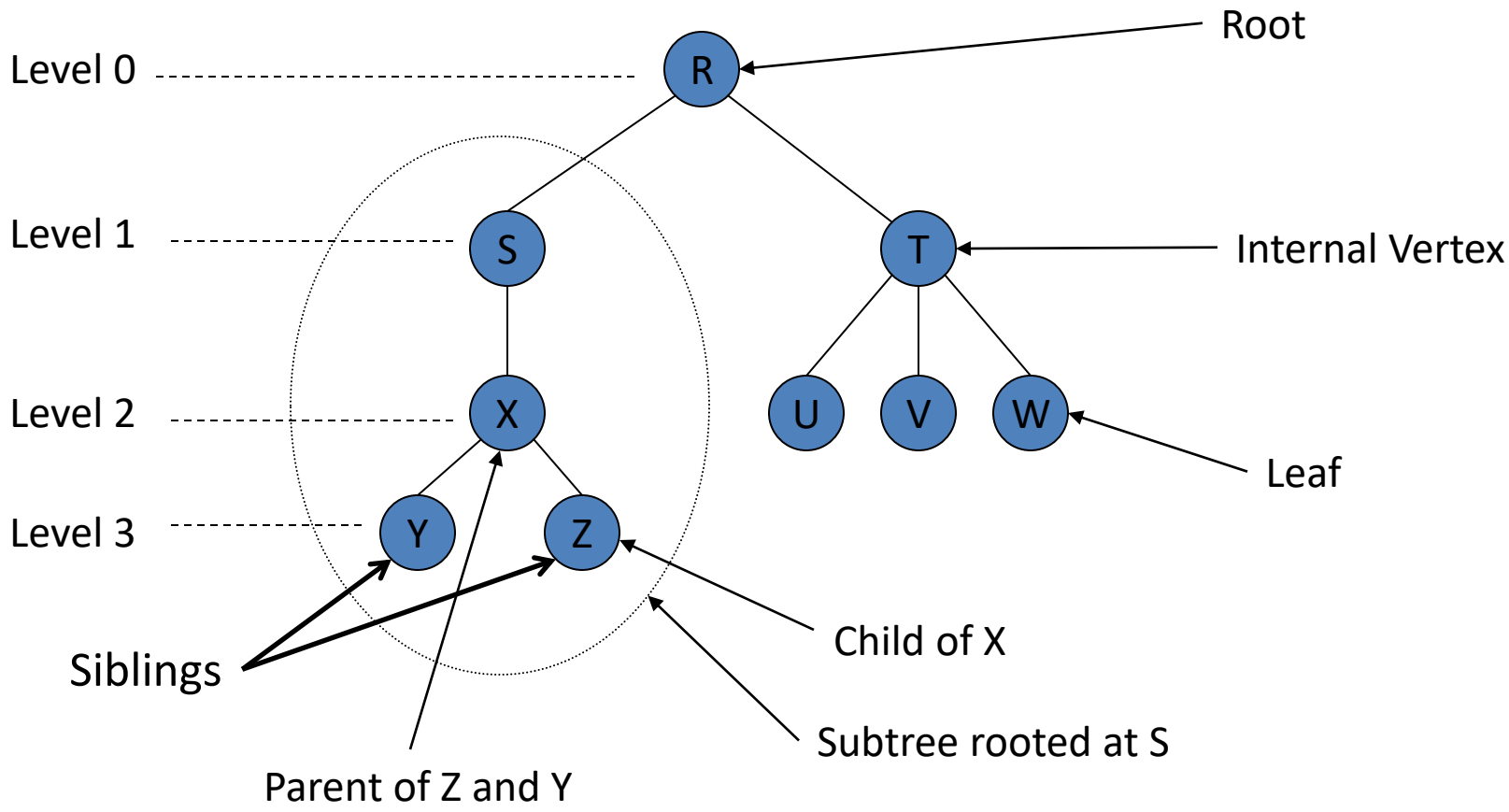
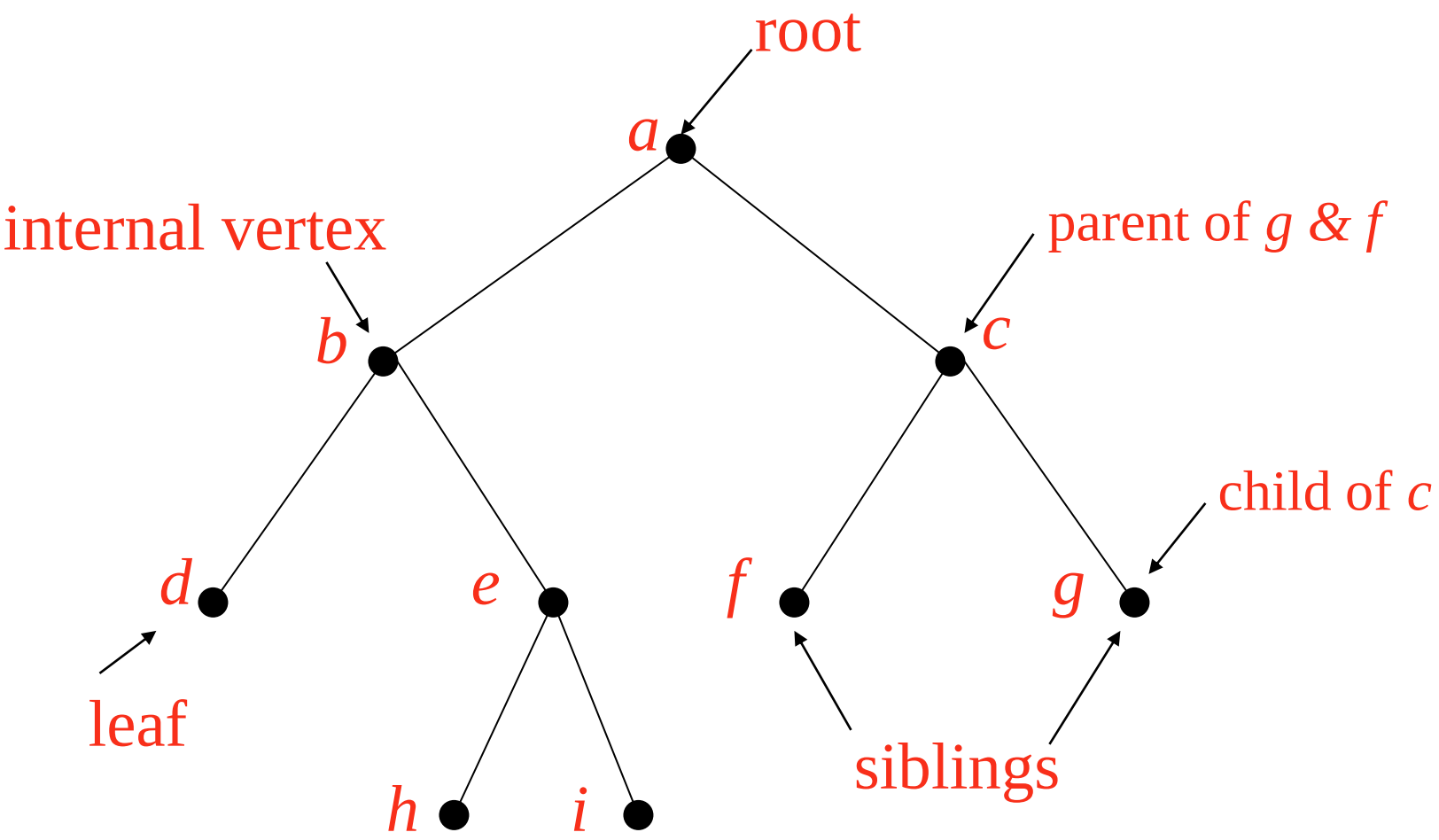


Illustration of Tree Terminology



Books

- Rosen, K. H., & Krithivasan, K. (2012). Discrete mathematics and its applications: with combinatorics and graph theory. Tata McGraw-Hill Education. (7th Edition)

References

1. Discrete Mathematics, *Richard Johnsonbaugh*, Pearson education, Inc.
2. Discrete Mathematical Structures, *Bernard Kolman*, *Robert C. Busby*, *Sharon Ross,* Prentice-Hall, Inc.
3. *SCHAUM'S outlines Discrete Mathematics(2nd edition)*, by *Seymour Lipschutz*, *Marc Lipson*

- University of Wisconsin-Madison

http://pages.cs.wisc.edu/~deppeler/cs367-common/readings/Trees/intro.html

- Bradfield School of Computer Science

https://bradfieldcs.com/algos/trees/introduction/