Lecture 13

Chapter 15: Oscillations

15-1 Simple Harmonic Motion :

Periodic Motion :

If the motion of a body is such that it crosses from the same direction a particular point in its path of motion at regular interval, then the motion is called periodic motion or harmonic motion

Simple Harmonic Motion :

If acceleration of a body executing periodic motion acts along a fixed point in its path of motion in a such a way that its magnitude from that point is proportional to its displacement, then the motion of the body is called Simple Harmonic Motion.

Such motion is a sinusoidal function of time t . That is, it can be written as a sine or a cosine of time t. Here we arbitrarily choose the cosine function and write the displacement (or position) of the particle in Fig.15-1 as

$$x(t) = x_m \cos(\omega t + \varphi) \quad \text{; (displacement) .... (1)}$$

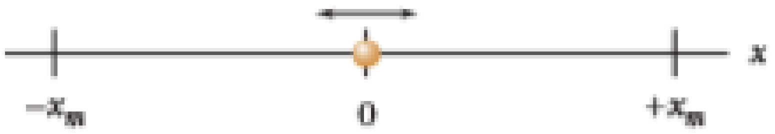

Figure 15-1 A particle repeatedly oscillates left and right along an x axis, between extreme points x_m and $-x_m$.

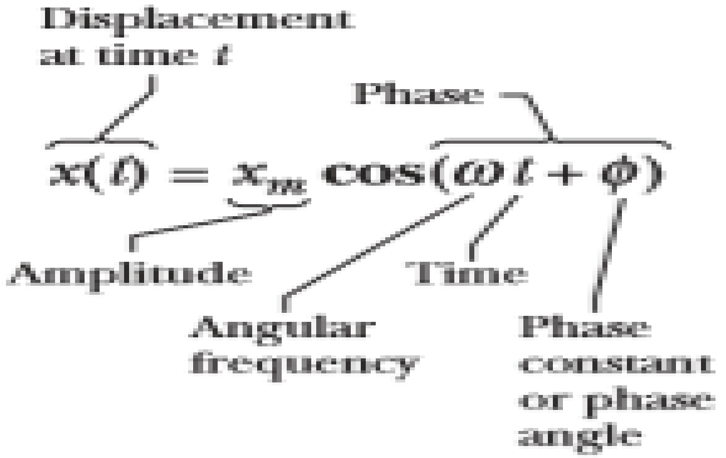

Figure 15-3 A handy guide to the quantities in Eq. 15-3 for simple harmonic motion.

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Frequency :

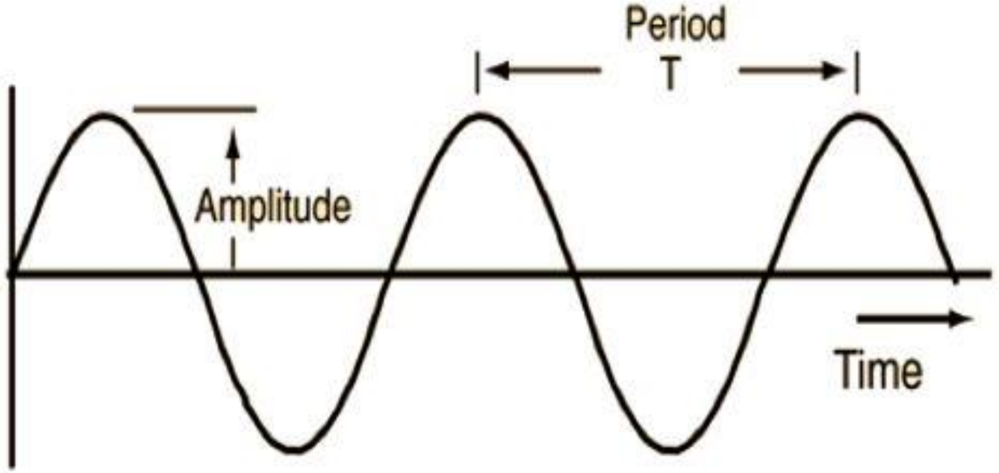
The frequency f of the oscillation is the number of times per second that it completes a full oscillation (a cycle) and has the unit of hertz (abbreviated Hz),

where 1 hertz = 1 Hz = 1 oscillation per second = 1 s^{-1} ………. (2)

Time Period :

The time for one full cycle is the period T of the oscillation , which is

$$\boldsymbol{T = \frac{1}{f}} \qquad \text{……….} \; (3)$$



Question : Proof that , *ω = 2πf*

Solution :

- To relate it to the frequency f and the period T, let's first note that the position x(t) of the particle must (by definition) return to its initial value at the end of a period.
- That is, if x(t) is the position at some chosen time t, then the particle must return to that same position at time t + T.

➢ Let’s use Eq.1 to express this condition, but let’s also just set φ = 0 to get it out of the way.

Returning to the same position can then be written as

$$x\,(t) = x_m \cos\,(\omega t + \varphi)$$

At t=t

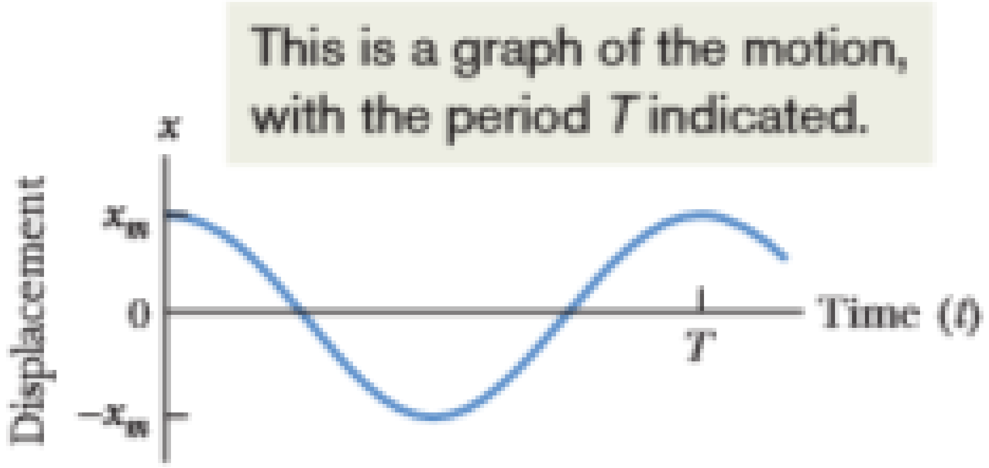
$$x1 = x_m \cos \omega t$$

At t = t+T

$$x2 = x_m \cos \omega\,(t + T)$$

$$x1 = x2$$

Type equation he



$$x_m \cos \omega t = x_m \cos \omega\,(t + T)$$

The cosine function first repeats itself when its argument (the phase , remember) has increased by 2 π rad. So, Eq. 4 tells us that

$$\omega\,(t + T) = \omega\,t + 2\,\pi$$

$$\text{or,} \qquad \omega T = 2\,\pi \text{ rad}$$

Thus , from Eq.2 the angular frequency is , $\omega = \frac{2\,\pi}{T} = 2\pi f$ ……. (5)

The **SI unit** of angular frequency is the ***radian per second*****.**

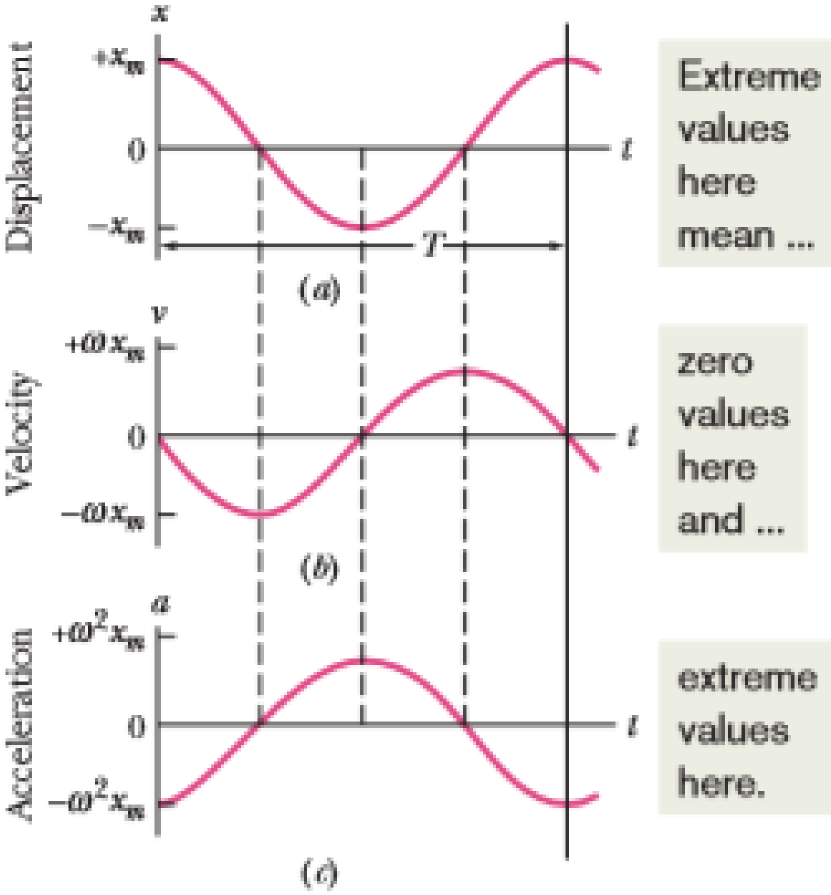
The Velocity of SHM : To find the velocity v(t) as a function of time , let's take a time derivative of the position function x(t) in Eq. 1 :

$$x(t) = x_m \cos(\omega t + \varphi)$$

$$v(t) = \frac{d}{dt} x(t) = \frac{d}{dt}\left[x_m \cos(\omega t + \varphi) \right]$$

$$v(t) = -\omega x_m \sin(\omega t + \varphi) \quad \text{(velocity)} \quad \ldots\ldots.. \quad (6)$$

- The velocity depends on time because the sine function varies with time, between the values of +1 and -1.
- The quantities in front of the sine function determine the extent of the variation in the velocity, between + ωx_m and - ωx_m . We say that ωx_m is the velocity amplitude v_m of the velocity variation. $v_m = \omega x_m$.
- When the particle is moving rightward through x = 0, its velocity is positive and the magnitude is at this greatest value.
- When it is moving leftward through x = 0,its velocity is negative and the magnitude is again at this greatest value.
- This variation with time (a negative sine function) is displayed in the graph of Fig. b for a phase constant of φ = 0, which corresponds to the cosine function for the displacement versus time shown in Fig. a.

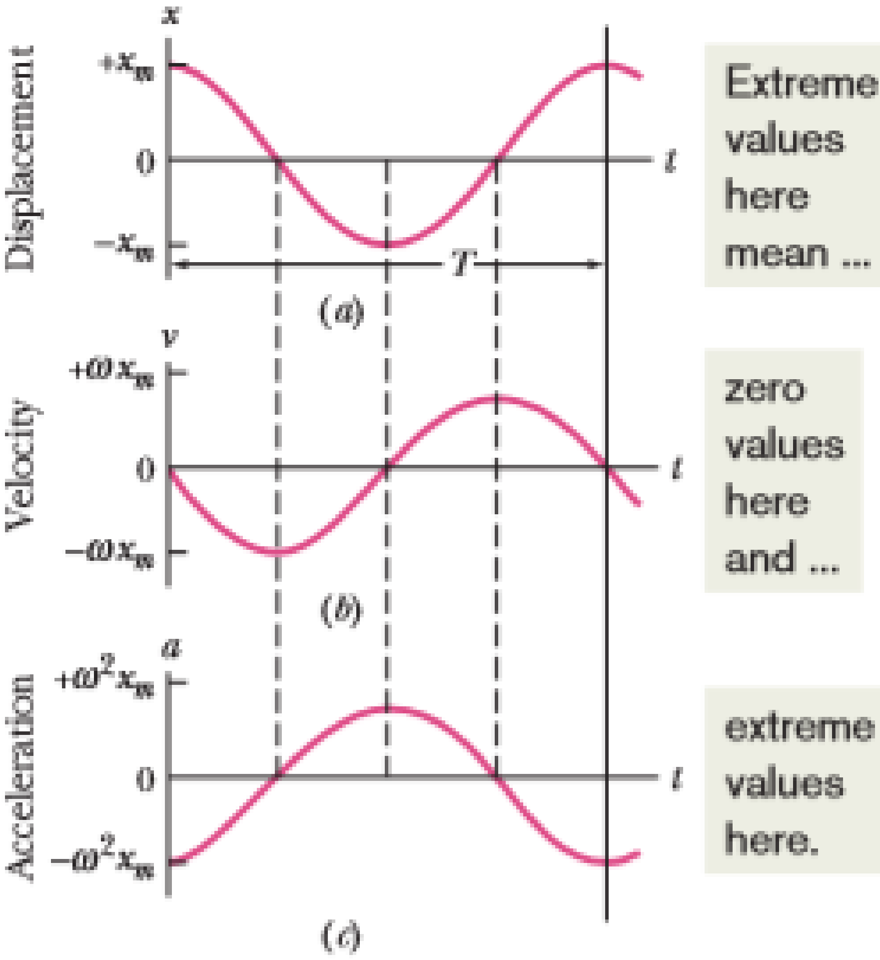


The Acceleration of SHM :

It can be found by differentiating the velocity function of Eq. 6 with respect to time to get the acceleration function of the particle in simple harmonic motion:

$$a(t) = \frac{d}{dt} v(t) = \frac{d}{dt} [- \omega x_m \sin(\omega t + \varphi)]$$

$$a(t) = -\omega^2 x_m \cos(\omega t + \varphi) \quad \text{(acceleration)} \;\ldots..\; (7)$$



- The acceleration varies because the cosine function varies with time, between +1 and -1.The variation in the magnitude of the acceleration is set by the acceleration amplitude a_m, which is the product $\omega^2 x_m$ that multiplies the cosine function. $a_m = \omega^2 x_m$

- Figure c displays **Eq. 7** for a phase constant $\varphi = 0$, consistent with Figs. a and b. Note that the acceleration magnitude is zero when the cosine is zero , which is when the particle is at $x = 0$.

- And the acceleration magnitude is maximum when the cosine magnitude is maximum, which is when the particle is at an extreme point, where it has been slowed to a stop so that its motion can be reversed.

$$x(t) = x_m \cos(\omega t + \varphi) \quad ; \quad \textit{(displacement)} \; \ldots\ldots. \; (1)$$

$$a(t) = -\omega^2 x_m \cos(\omega t + \varphi) \quad \textit{(acceleration)} \; \ldots\ldots.. \; (7)$$

Comparing Eqs. 1 and 7 we see an extremely neat relationship:

$$a(t) = -\omega^2 x(t) \quad \ldots\ldots.. \; (8)$$

In SHM, the acceleration a is proportional to the displacement x but opposite in sign, and the two quantities are related by the square of the angular frequency ω.

Linear simple harmonic oscillator [undamped oscillator] :The force law for simple harmonic motion

Let us assume that there is no friction.

Using Eq 8 we can apply Newton's second law to describe the force responsible for SHM:

$$F = ma = m(-\omega^2 x) = -(m\omega^2)x \quad \ldots\ldots\ldots\ldots (9)$$

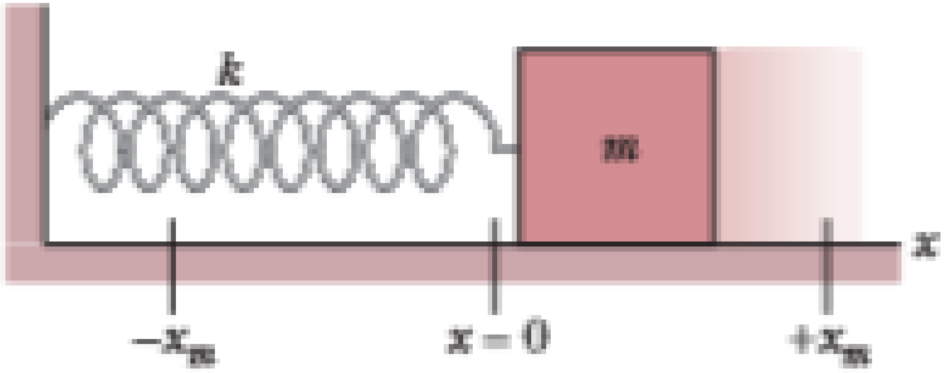
The minus sign means that the direction of the force on the particle is opposite the direction of the displacement of the particle.

That is , in SHM the force is a restoring force in the sense that it fights against the displacement , attempting to restore the particle to the center point at **x = 0** .

Now for a block on a spring as in Fig. we know from Hooke's law,

$$F = -kx \quad \ldots\ldots\ldots\ldots (10)$$

for the force acting on the block.



$$F = -kx$$

$$-(m\omega^2)x = -kx$$

$$k = m\omega^2$$

Comparing Eqs.9 and 10,we can now relate the spring constant k (a measure of the stiffness of the spring) to the mass of the block and the resulting angular frequency of the SHM:

$$k = m\,\omega^2 \qquad \text{.........} \; (11)$$

Then the angular frequency ,

$$\omega = \sqrt{\frac{k}{m}} \qquad \text{........} \; (12)$$

the period of the motion can be found by combining Eqs.5 $[\omega = \dfrac{2\,\pi}{T}]$ and Eq. 12 to write

$$\omega = \frac{2\,\pi}{T}$$

$$\sqrt{\frac{k}{m}} = \frac{2\,\pi}{T}$$

$$\mathbf{T = 2\,\pi}\,\sqrt{\frac{m}{k}} \qquad \text{......} \quad \text{.......} \; (13)$$

3 : What is the maximum acceleration of a platform that oscillates at amplitude 2.20 cm and frequency 6.60 Hz?

$x_m = 2.20 \text{ cm} = 0.0220 \text{ m}$

$f = 6.60 \text{ Hz}$

$$x(t) = x_m \cos(\omega t + \varphi)$$

$$v(t) = -\omega x_m \sin(\omega t + \varphi)$$

$$a(t) = -\omega^2 x_m \cos(\omega t + \varphi)$$

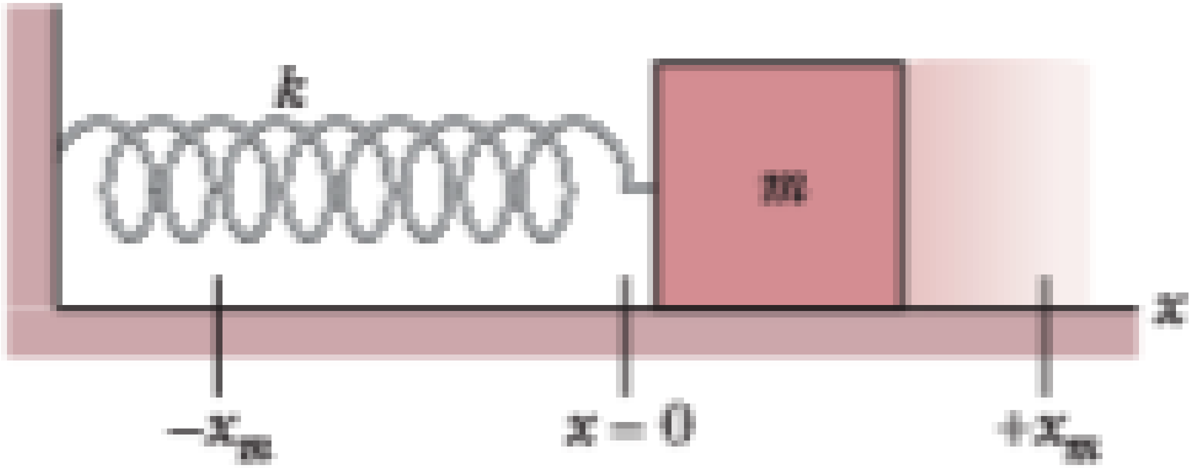
$a_m = \omega^2 x_m = (2\pi f)^2 x_m = 4\pi^2 (6.60)^2 (0.0220) = 37.8 \text{ m/s}^2 = 37.8 \text{ m/s-s}$

13 : An oscillator consists of a block of mass 0.500 kg connected to a spring. When set into oscillation with amplitude 35.0 cm, the oscillator repeats its motion every 0.500 s. Find the (a) period, (b) frequency, (c) angular frequency, (d) spring constant, (e) maximum speed, and (f) magnitude of the maximum force on the block from the spring.

Given: $m = 0.500$ kg

$x_m = 35.0$ cm $= 0.35$ m

$T = 0.500$ s



(a) $T = 0.500$ s

(b) $f = \frac{1}{T} = \frac{1}{0.500} = 2.00$ Hz [2 oscillations/s]

(c) $\omega = 2\pi f = 2\pi(2.00) = 12.6$ rad/s

(d) $\omega = \sqrt{\frac{k}{m}}.$

$k = m\,\omega^2 = (0.500)(12.6)^2 = 79.0$ N/m

(e) $v(t) = -\,\omega\, x_m \sin(\omega t + \varphi)$

$v_m = \omega x_m = (12.6)(0.350) = 4.40$ m/s

(f) $\vec{\mathbf{F}} = -\,k\,\vec{\mathbf{x}}$

$F_s = k\,x_m = (79.0)(0.350) = 27.6$ N

Newton's third law, $F_s = F_m = 27.6$ N

Additional problem:
Sample problem 15.01; page 420