Find the angle to any fringe:

Lesson-21

bright fringe: $d \sin\theta = m\lambda$ for $m = 0, 1, 2, 3 \ldots$

(1) m = 0: central maximum

$d \sin\theta = (0)\lambda$ $\quad \sin\theta = 0$ $\quad \theta = \sin^{-1} 0$ $\quad \theta = 0$

(2) m = 1: first bright fringe/ first maxima

$d \sin\theta = 1\lambda$ $\quad \sin\theta = \frac{\lambda}{d}$ $\quad \theta = \sin^{-1}(\frac{\lambda}{d})$

(3) m = 2: second bright fringe/ second maxima

$d \sin\theta = 2\lambda$ $\quad \sin\theta = \frac{2\lambda}{d}$ $\quad \theta = \sin^{-1}(\frac{2\lambda}{d})$

dark fringe: $d \sin\theta = (m + \frac{1}{2})\lambda$ for $m = 0, 1, 2, 3 \ldots$

(1) m = 0: first dark fringe/ first minima

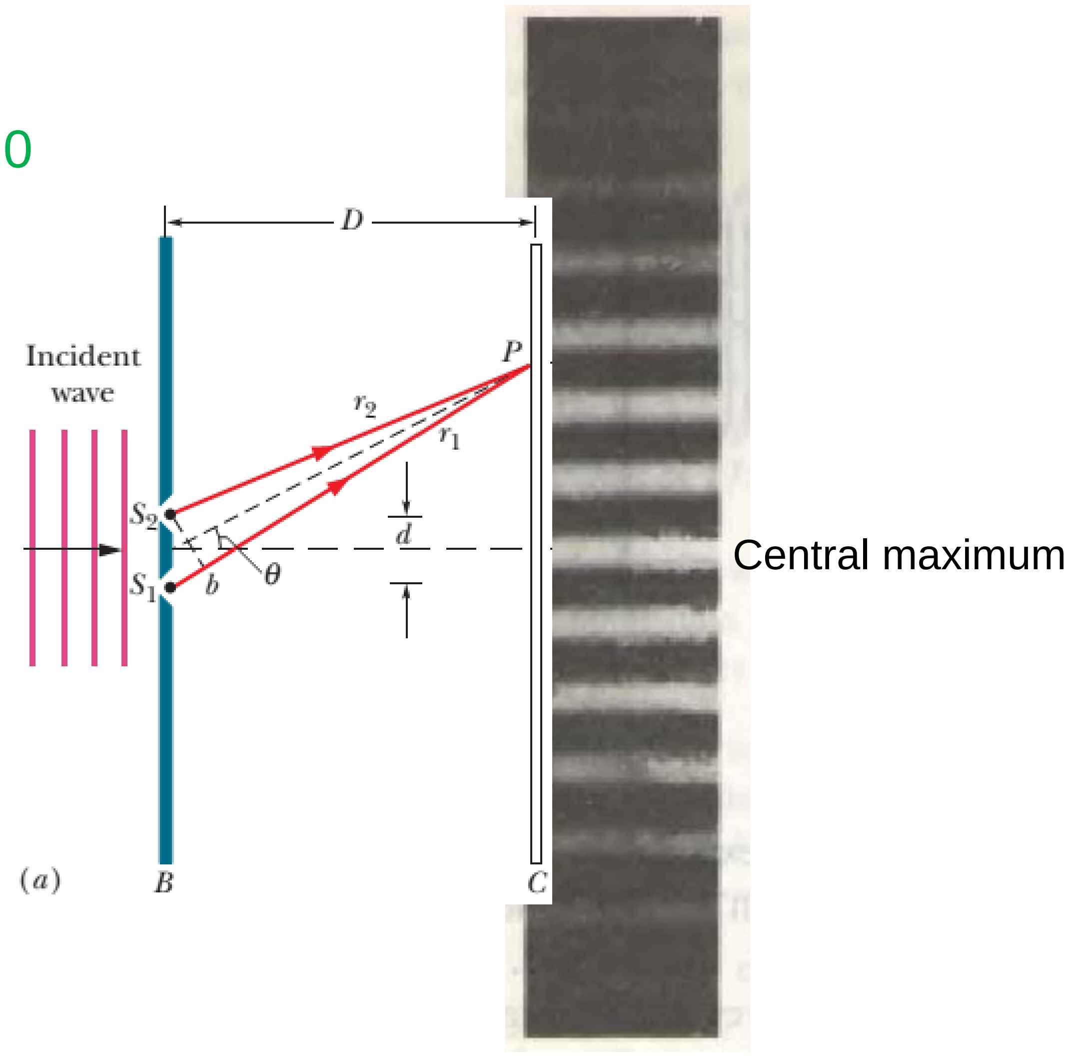
$d \sin\theta = (0 + \frac{1}{2})\lambda$ $\quad \sin\theta = \frac{\lambda}{2d}$ $\quad \theta = \sin^{-1}(\frac{\lambda}{2d})$

(2) m = 1: second dark fringe/ second minima

$d \sin\theta = (1 + \frac{1}{2})\lambda$ $\quad d \sin\theta = (\frac{3\lambda}{2})$ $\quad \theta = \sin^{-1}(\frac{3\lambda}{2d})$

(3) m = 2: third dark fringe/ third minima

$d \sin\theta = (2 + \frac{1}{2})\lambda$ $\quad d \sin\theta = (\frac{5\lambda}{2})$ $\quad \theta = \sin^{-1}(\frac{5\lambda}{2d})$



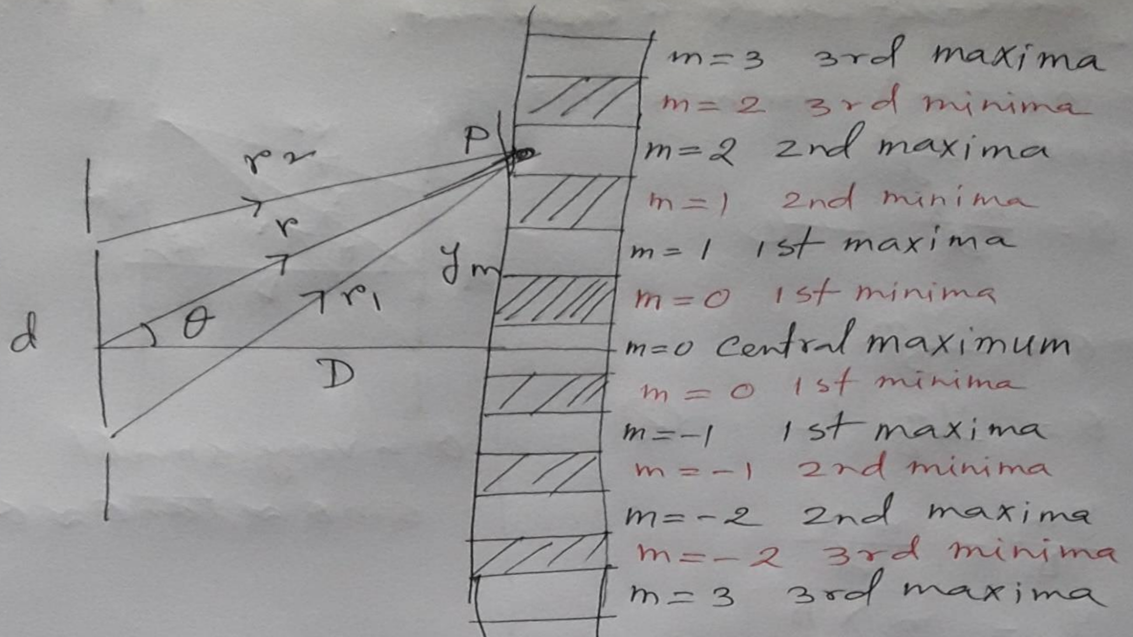


Fig: Young's double slit experiment

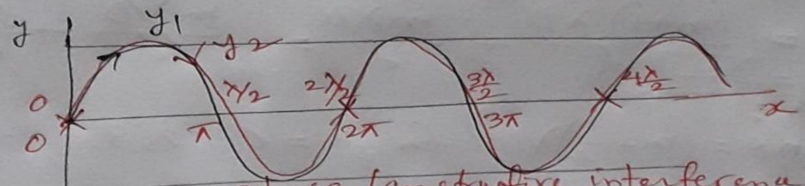


Fig: in phase (constructive interference)

Maxima:

$\Delta L = d\sin\theta = 0, 2\frac{\lambda}{2}, 4\frac{\lambda}{2}, 6\frac{\lambda}{2}$

$d\sin\theta = 0, \lambda, 2\lambda, 3\lambda, \ldots$

$d\sin\theta = m\lambda$ for $m = 0, 1, 2, 3, \ldots$

Minima:

$\Delta L = d\sin\theta = 1\frac{\lambda}{2}, 3\frac{\lambda}{2}, 5\frac{\lambda}{2}, \ldots$

$d\sin\theta = (m+\frac{1}{2})\lambda$ for $m = 0, 1, 2, 3, \ldots$

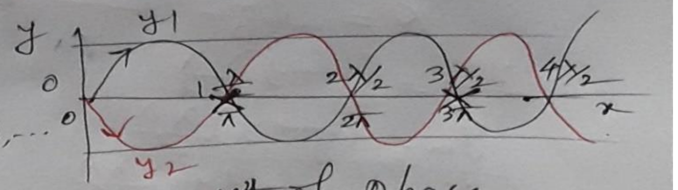


Fig: out of phase (destructive interference)

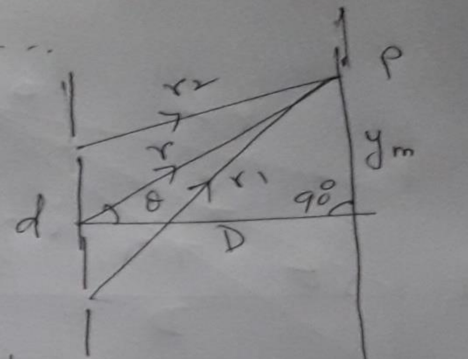
T4: Derive the positions of mth maxima and minima for Young's double-slit interference experiment.

Maxima:

$$d\sin\theta = m\lambda \quad \text{for } m = 0, 1, 2, 3, \ldots$$

$$\sin\theta = \frac{m\lambda}{d}$$

$$\theta = \frac{m\lambda}{d} \quad \text{—— (1)}$$



$$\tan\theta = \frac{y_m}{D}$$

$$\theta = \frac{y_m}{D} \quad \text{—— (2)}$$

θ is very small

$\sin\theta \approx \tan\theta \approx \theta$

$$\therefore \frac{y_m}{D} = \frac{m\lambda}{d}$$

$$\boxed{y_m = \frac{m\lambda D}{d}}$$

$$\therefore y_{m+1} = \frac{(m+1)\lambda D}{d}$$

$$\Delta y = \frac{(m+1)\lambda D}{d} - \frac{m\lambda D}{d} = \cancel{\frac{m\lambda D}{d}} + \frac{\lambda D}{d} - \cancel{\frac{m\lambda D}{d}} = \frac{\lambda D}{d}$$

$$\boxed{\Delta y = \frac{\lambda D}{d}}$$

Minima:

$$d\sin\theta = \left(m+\tfrac{1}{2}\right)\lambda \quad \text{for } m = 0, 1, 2, 3, \ldots$$

$$\sin\theta = \left(m+\tfrac{1}{2}\right)\frac{\lambda}{d}$$

$$\theta = \left(m+\tfrac{1}{2}\right)\frac{\lambda}{d} \quad \text{—— (1)}$$

$$\tan\theta = \frac{y_m}{D}$$

$$\theta = \frac{y_m}{D} \quad \text{—— (2)}$$

$$\frac{y_m}{D} = \left(m+\tfrac{1}{2}\right)\frac{\lambda}{d}$$

$$\boxed{y_m = \left(m+\tfrac{1}{2}\right)\frac{\lambda D}{d}}$$

$$y_{m+1} = \left(m+\tfrac{1}{2}+1\right)\frac{\lambda D}{d}$$

$$\Delta y = \left(m+\tfrac{1}{2}+1\right)\frac{\lambda D}{d} - \left(m+\tfrac{1}{2}\right)\frac{\lambda D}{d}$$

$$= \cancel{\left(m+\tfrac{1}{2}\right)\frac{\lambda D}{d}} + \frac{\lambda D}{d} - \cancel{\left(m+\tfrac{1}{2}\right)\frac{\lambda D}{d}}$$

$$\boxed{\Delta y = \frac{\lambda D}{d}}$$

20. Monochromatic green light, of wavelength 550 nm, illuminates two parallel narrow slits 7.70 mm apart. Calculate the angular deviation (θ in Fig. 35-10) of the third-order bright fringe (a) in radians and (b) in degrees.

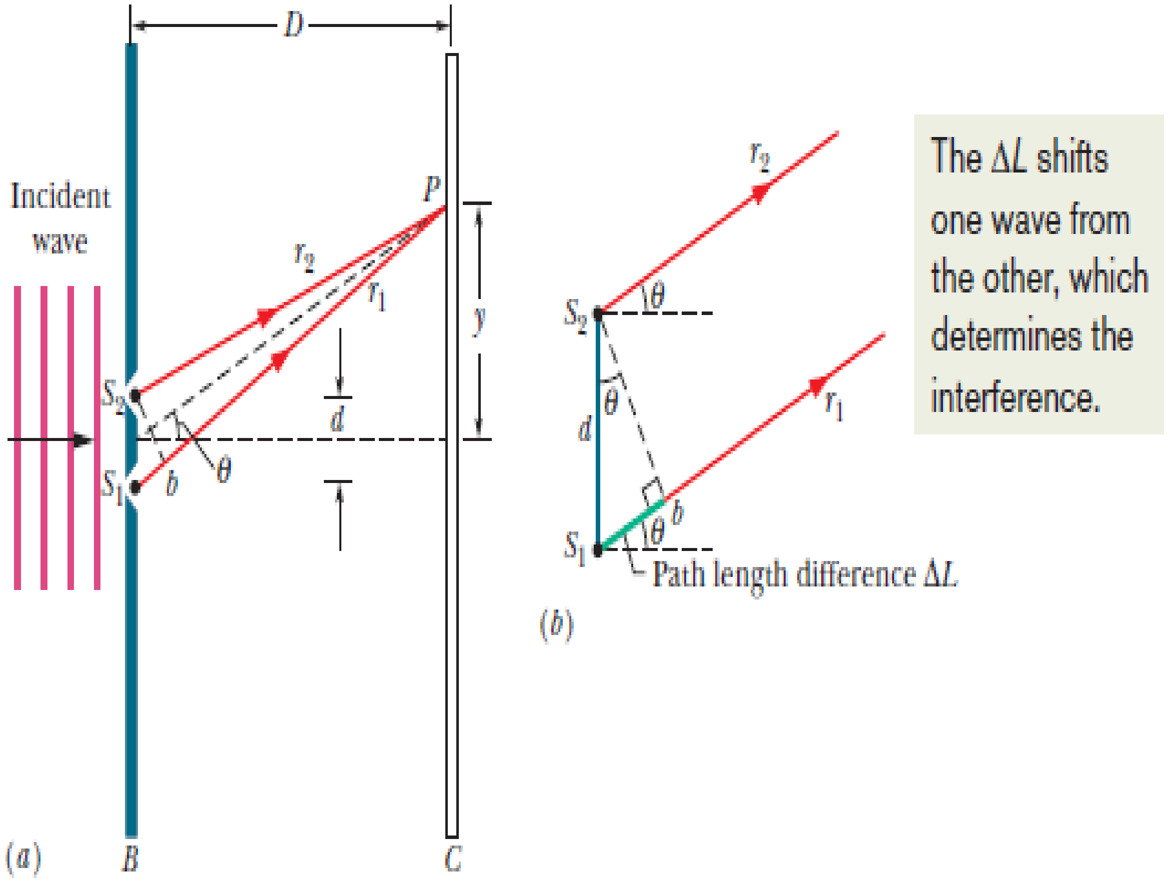


Figure 35-10 (*a*) Waves from slits S_1 and S_2 (which extend into and out of the page) combine at P, an arbitrary point on screen C at distance y from the central axis. The angle θ serves as a convenient locator for P. (*b*) For $D \gg d$, we can approximate rays r_1 and r_2 as being parallel, at angle θ to the central axis.

20. (a) we know,

$$d \sin\theta = n\lambda$$

$$\sin\theta = \frac{n\lambda}{d}$$

$$\theta = \sin^{-1}\left(\frac{3 \times 550 \times 10^{-9}}{7.70 \times 10^{-3}}\right)$$

$$\theta = 0.216 \text{ rad}$$

Ans:

(b) $\theta = \dfrac{0.216}{\pi/180} = 12.4^{\circ}$

Ans.

93. If the distance between the first and tenth minima of a double-slit pattern is 18.0 mm and the slits are separated by 0.150 mm with the screen 50.0 cm from the slits, what is the wavelength of the light used?

93. we know, double slit experiment, the minimum occurs,

$$d \sin\theta = \left(m + \tfrac{1}{2}\right)\lambda$$

Here, angle θ is very small

$$d\theta = \left(m+\tfrac{1}{2}\right)\lambda \quad \text{so,} \quad \theta = \left(m+\tfrac{1}{2}\right)\frac{\lambda}{d} \qquad (1)$$

Again, the distance from the minimum to the central fringe is,

$$y = D\tan\theta = D\theta.$$

(Using equation (1))

$$y = \left(m+\tfrac{1}{2}\right)\frac{D\lambda}{d}$$

$$y = \frac{mD\lambda}{d} + \frac{1}{2}\frac{D\lambda}{d}$$

take the derivative, we get,

$$\Delta y = \frac{\Delta m \, D\lambda}{d}$$

$$\therefore \lambda = \frac{\Delta y \, d}{\Delta m \, D}$$

$$= \frac{18\times10^{-3} \times 0.150\times10^{-3}}{9\times 0.50}$$

$$= 6.00\times10^{-7}\,m$$

$$= 600\ nm$$

Ans:

Here,
First minimum, $m = 0$
Tenth minimum, $m = 9$
$\Delta m = 9$
$d = 0.150 \times 10^{-3}\,m$
$D = 0.50\,m$
$\Delta y = 18 \times 10^{-3}\,m$

__Sample Problem 35.02.__ **In a double-slit interference pattern, what is the distance on screen C between adjacent maxima near the center of the interference pattern? The wavelength λ of the light is 546 nm, the slit separation d is 0.12 mm, and the slit–screen separation D is 55 cm.**

Given,

The wavelength of the light is $\lambda = 546\ nm = 546 \times 10^{-9} m$

The slit separation $d = 0.12\ mm = 0.12 \times 10^{-3} m$

The slit–screen separation $D = 55\ cm = 0.55 m$

The position of the mth maxima, $y_m = \frac{m\lambda D}{d}$

And the position of the next maxima, $y_{m+1} = \frac{(m+1)\lambda D}{d}$

So, the distance between the adjacent maxima, $\Delta y = Y_{m+1} - Y_m$

$$\Delta y = \frac{(m+1)\lambda D}{d} - \frac{m\lambda D}{d}$$

$$\Delta y = \frac{m\lambda D + \lambda D - m\lambda D}{d}$$

$$\Delta y = \frac{\lambda D}{d} = \frac{(546 \times 10^{-9}) \times 0.55m}{0.12 \times 10^{-3}}$$

$$\therefore \Delta y = 2.50 \times 10^{-3} m$$