CSC 2221: Algorithms

Lecture 3

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1. Bubble Sort
2. Insertion sort
3. Selection sort
4. Linear Search

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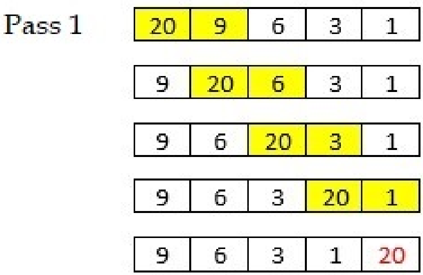
Bubble Sort

- Running time $O(n^2)$
- $a[i] > a[i+1]$ $swap(a[i], a[i+1])$
- in the loop, index i=0 to ?



Pass 1

20 9 6 3 1

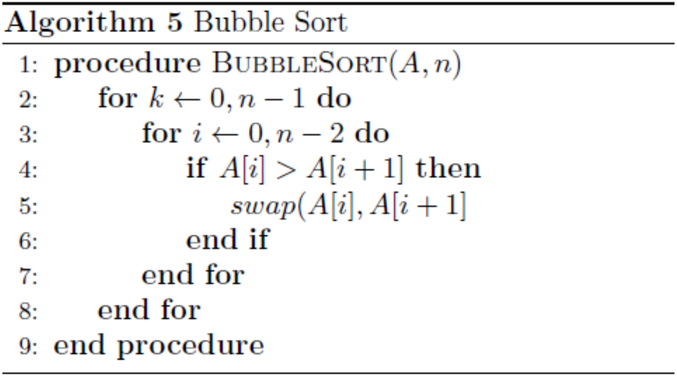
9 20 6 3 1

9 6 20 3 1

9 6 3 20 1

9 6 3 1 20

Bubble Sort Algorithm



Algorithm 5 Bubble Sort

```
procedure BubbleSort($A, n$)
    for $k \leftarrow 0, n-1$ do
        for $i \leftarrow 0, n-2$ do
            if $A[i] > A[i+1]$ then
                $swap(A[i], A[i+1]$
            end if
        end for
    end for
end procedure
```

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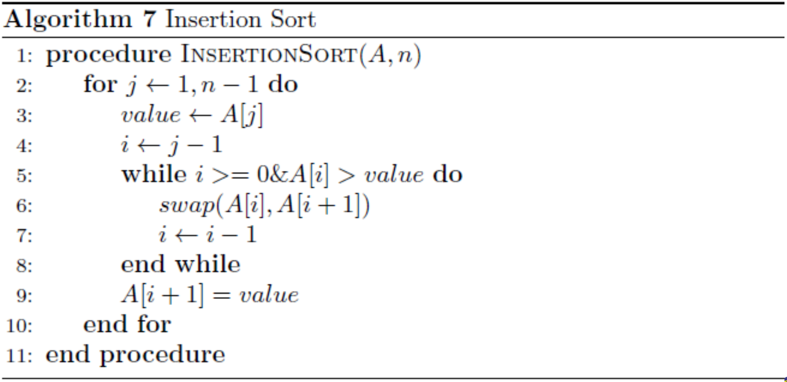
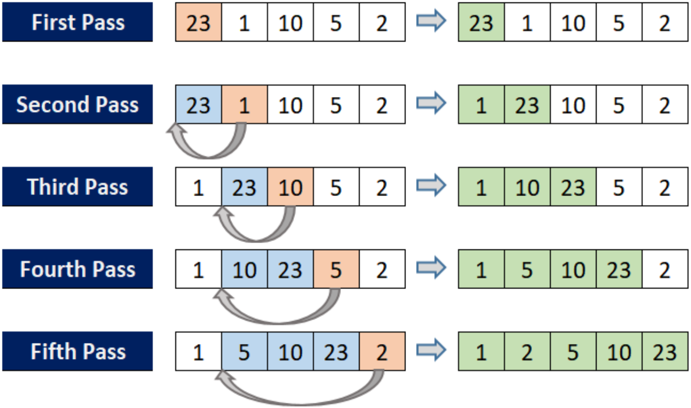
Insertion sort



Algorithm 7 Insertion Sort

```
procedure InsertionSort($A, n$)
    for $j \leftarrow 1, n - 1$ do
        $value \leftarrow A[j]$
        $i \leftarrow j - 1$
        while $i >= 0 \& A[i] > value$ do
            $swap(A[i], A[i + 1])$
            $i \leftarrow i - 1$
        end while
        $A[i + 1] = value$
    end for
end procedure
```

Insertion sort simulation



First Pass	23	1	10	5	2	⇨	23	1	10	5	2
Second Pass	23	1	10	5	2	⇨	1	23	10	5	2
Third Pass	1	23	10	5	2	⇨	1	10	23	5	2
Fourth Pass	1	10	23	5	2	⇨	1	5	10	23	2
Fifth Pass	1	5	10	23	2	⇨	1	2	5	10	23

Insertion sort complexity

```
INSERTION-SORT(A)                                   cost     times
for $j = 2$ to $A.length$                           $c_1$    $n$
    $key = A[j]$                                    $c_2$    $n-1$
    // Insert $A[j]$ into the sorted
            sequence $A[1..j-1]$.                   $0$      $n-1$
    $i = j - 1$                                     $c_4$    $n-1$
    while $i > 0$ and $A[i] > key$                  $c_5$    $\sum_{j=2}^{n} t_j$
        $A[i+1] = A[i]$                             $c_6$    $\sum_{j=2}^{n} (t_j - 1)$
        $i = i - 1$                                 $c_7$    $\sum_{j=2}^{n} (t_j - 1)$
    $A[i+1] = key$                                  $c_8$    $n-1$
```

Insertion sort complexity

$$T(n)=c_1 n+c_2(n-1)+c_4(n-1)+c_5\sum_{j=2}^{n} t_j+c_6\sum_{j=2}^{n}(t_j-1)+c_7\sum_{j=2}^{n}(t_j-1)+c_8(n-1)$$

- Best Case : List is sorted

$$T(n)=c_1 n+c_2(n-1)+c_4(n-1)+c_5\sum_{j=2}^{n} 1+c_8(n-1)$$

$$=c_1 n+c_2(n-1)+c_4(n-1)+c_5(n-1)+c_8(n-1)$$

$$=(c_1+c_2+c_4+c_5+c_8)n-(c_2+c_4+c_5+c_8)$$

$$=an+b$$

Insertion sort complexity

$$T(n)=c_1 n+c_2(n-1)+c_4(n-1)+c_5\sum_{j=2}^{n} t_j+c_6\sum_{j=2}^{n}(t_j-1)+c_7\sum_{j=2}^{n}(t_j-1)+c_8(n-1)$$

$$\sum_{j=2}^{n} j=(\sum_{j=1}^{n} j)-1=\frac{n(n+1)}{2}-1$$

Worst Case : List is reversed order

$$T(n)=c_1 n+c_2(n-1)+c_4(n-1)+c_5\left(\frac{n(n+1)}{2}-1\right)+c_6\left(\frac{(n-1)n}{2}\right)+c_7\left(\frac{(n-1)n}{2}\right)+c_8(n-1)$$

$$=\left(\frac{c_5}{2}+\frac{c_6}{2}+\frac{c_7}{2}\right)n^2+\left(c_1+c_2+c_4+\frac{c_5}{2}-\frac{c_6}{2}-\frac{c_7}{2}+c_8\right)n-(c_2+c_4+c_5+c_8)$$

$$=an^2+bn+c$$

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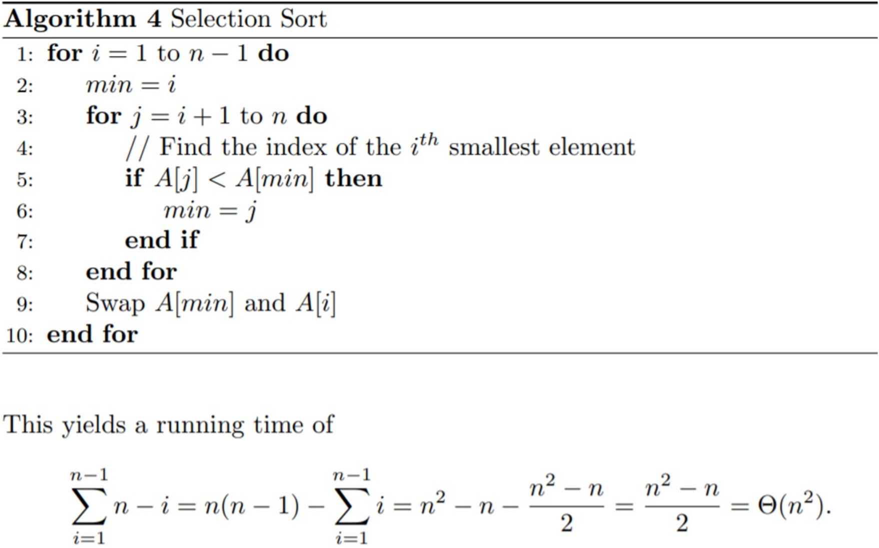
Selection sort



Algorithm 4 Selection Sort

```
for $i = 1$ to $n - 1$ do
    $min = i$
    for $j = i + 1$ to $n$ do
        // Find the index of the $i^{th}$ smallest element
        if $A[j] < A[min]$ then
            $min = j$
        end if
    end for
    Swap $A[min]$ and $A[i]$
end for
```

This yields a running time of

$$\sum_{i=1}^{n-1} n - i = n(n-1) - \sum_{i=1}^{n-1} i = n^2 - n - \frac{n^2 - n}{2} = \frac{n^2 - n}{2} = \Theta(n^2).$$

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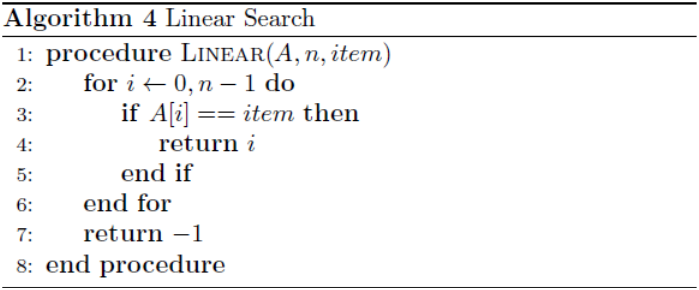
Linear Search

- Time complexity: $O(n)$



Algorithm 4 Linear Search

```
procedure LINEAR($A, n, item$)
    for $i \leftarrow 0, n-1$ do
        if $A[i] == item$ then
            return $i$
        end if
    end for
    return $-1$
end procedure
```

References

Introduction to Algorithms, Third Edition, Thomas H. Cormen, Charle E. Leiserson, Ronald L. Rivest, Clifford Stein (clrs).