

Time Analysis

1. Swap (a, b) {

$$\begin{array}{l} \text{-temp} = a \quad | \\ a = b \quad | \\ b = \text{-temp} \quad | \\ \} \end{array}$$

$$\therefore f(n) = 3$$

Space

$$\begin{array}{r} a = 1 \\ b = 1 \\ \text{-temp} = 1 \\ \hline S(n) = 3 \end{array}$$

$\therefore O(1)$

$\therefore O(1) = \text{constant}$

2. void sum($A[], n$) {

$$s = 0 \quad |$$

$$\text{for } (i=0; i < n; i++) \{ \quad |$$

$$s = s + A[i] \quad |$$

}

$$\begin{array}{r} A[] = n \\ n = 1 \\ i = 1 \\ s = 1 \end{array}$$

$$\begin{array}{l} f(n) = n + n + 1 + 1 \\ = 2n + 2 \end{array}$$

$$\therefore S(n) = n + 3$$

$\therefore O(n)$

$\therefore O(n) =$

3. Add (A, B, n) {

$$n+1 \quad | \quad \text{for } (i=0; i < n; i++) \{$$

$$n(n+1) \quad | \quad \text{for } (j=0; j < n; j++) \{$$

$$n \times n \quad | \quad C[i, j] = A[i, j] + B[i, j]$$

$$\begin{array}{l} f(n) = n + 1 + n^2 + n + n^2 \\ = 2n^2 + 2n + 1 \end{array}$$

$\therefore O(n^2)$

$$A = n^2$$

$$B = n^2$$

$$n = 1$$

$$i = 1$$

$$j = 1$$

$$C = n^2$$

$$S(n) = 3n^2 + 3$$

$\therefore O(n^2)$

4. $\text{for}(i=0; i < n; i++) \{$

statement

$i = 1$
 $s(n) = 2$
 $\therefore O(1)$

$n+1$
 n

$\}$

$\text{for}(i=0; i < n; i++) \{$

statement

$f(n) = n+1+n+n+1+n$
 $= 4n+2$
 $\therefore O(n)$

$n+1$
 n

$\}$

5. $\text{for}(i=n; i > 0; i--) \{$

statement

$i = 1$
 $s(n) = 2$
 $\therefore O(1)$

$n+1$
 n

$\}$

$f(n) = n+1+n$
 $= 2n+1$
 $\therefore O(n)$

6. $\text{for}(i=1; i < n; i=i+2) \{$

statement

$\frac{n}{2}$

$\}$

$O(n)$

7. $\text{for}(i=0; i < n; i++) \{$

statement

$j = 1$
 $n - 1$
 $s(n) = 3$
 $\therefore O(1)$

$n+1$
 $n(n+1)$
 n^2

$\}$

$f(n) = n+1+n^2+n+n^2$
 $= 2n^2+2n+1$
 $\therefore O(n^2)$

8. $\text{for } (i=0; i < n; i++) \{$
 $\quad \text{for } (j=0; j < i; j++) \{$
 $\quad \quad \text{statements;}$
 $\quad \}$
 $\}$

i	j	no. of time
0	0	0
1	0, 1	1 } 1
2	0, 1, 2	2 } 2
n		

$$\therefore 1+2+\dots+n = \frac{n(n+1)}{2}$$

$$\therefore f(n) = \frac{n^2 + n}{2}$$

$$\therefore O(n^2)$$

9. $p=0;$
 $\text{for } (i=1; p \leq n; i++) \{$
 $\quad p = p+i;$
 $\}$

i	p
1	0+1
2	0+1+2
3	0+1+2+3
\vdots	
k	$0+1+2+3+\dots+k$ $= \frac{k(k+1)}{2}$

let, $p \geq n$

$$\Rightarrow \frac{k(k+1)}{2} \geq n$$

$$\Rightarrow \frac{k^2 + k}{2} \geq n$$

$$\Rightarrow k^2 \geq n$$

$$\therefore k \geq \sqrt{n}$$

$$\therefore O(\sqrt{n})$$

10. $\text{for } (i=1; i < n; i = i \times 2)$
{ statement }

Assume, $i > n$

$$\Rightarrow 2^k > n$$

$$\therefore k > \log_2 n$$

$$\therefore O(\log_2 n)$$

$$\begin{array}{rcl} \frac{i}{1} & = & 2^0 \\ \frac{i}{1 \times 2} & = & 2^1 \\ \frac{i}{1 \times 2 \times 2} & = & 2^2 \\ \frac{i}{1 \times 2 \times 2 \times 2} & = & 2^3 \\ \vdots & & \\ \frac{i}{2^k} & & \end{array}$$

11. $\text{for } (i=n; i >= 1; i = i/2) \{$
start
{}

Assume,

$$i < 1$$

$$\Rightarrow \frac{n}{2^k} < 1$$

$$\Rightarrow n < 2^k$$

$$\therefore k > \log_2 n$$

$$\therefore O(\log_2 n)$$

$$\begin{array}{rcl} \frac{i}{n} & & \\ \frac{i}{\frac{n}{2}} & = & \frac{n}{2^1} \\ \frac{i}{\frac{n}{2 \times 2}} & = & \frac{n}{2^2} \\ \vdots & & \\ \frac{i}{\frac{n}{2^k}} & = & \frac{n}{2^k} \end{array}$$

12. $\text{for}(i=0; i*i < n; i++) \{$

start ————— n
}

$\text{for}(j=0; j < n; j++) \{$

start ————— n
}

$$\begin{array}{c} \xrightarrow{\quad} \\ f(n) = n + n \\ = 2n \end{array}$$

$\therefore O(n)$

13. $p = 0$

$\text{for}(i=1; i < n; i = i * 2) \{$

$p++;$ ————— $\log_2 n$

}

$\text{for}(j=1; j < p; j = j * 2) \{$

start ————— $\log_2 p$

}

$$p = \log_2 n$$

~~O~~ $(\log \log n)$

14. $\text{for}(i=0; i < n; i++) \{$ $\dots n+1$
 $\quad \quad \quad \text{for}(j=1; j < n; j=j*2) \{$ $\dots (n+1)\log_2 n$
 $\quad \quad \quad \quad \quad \text{start};$ $\dots n \times \log_2 n$
 $\quad \quad \quad \}$

$$\begin{aligned} f(n) &= n+1 + \log_2 n + n \log_2 n + n \log_2 n \\ &= 2n \log_2 n + \log_2 n + n+1 \end{aligned}$$

$\therefore O(n \log n)$

15. $i = 1$ $\begin{array}{c} i \quad k \\ \hline 1 & 1 \end{array}$
 $k = 1$
 $\text{while } (k < n) \{$
 $\quad \quad \quad \text{start}$
 $\quad \quad \quad k = k + i$
 $\quad \quad \quad i++$
 $\quad \quad \}$

2	$1+2$
3	$1+2+3$
\vdots	\vdots
m	$1+2+3+\dots+m$

$$= \frac{m(m+1)}{2}$$

Assume,

$$k > n$$

$$\Rightarrow \frac{m(m+1)}{2} > n$$

$$\Rightarrow \frac{m^2+m}{2} > n$$

$$\Rightarrow m^2 > n$$

$$\therefore m > \sqrt{n}$$

$$\therefore O(\sqrt{n})$$

16. $\text{while } (m \neq n) \{$

$\text{if } (m > n)$ (wt) $\frac{\overline{\begin{array}{c} m - n \\ 6 \\ 3 \\ 3 \end{array}}}{\overline{\begin{array}{c} 3 \\ 3 \end{array}}} \} 2 \therefore O(1)$
 $m = m - n$

else (wt) $\frac{\overline{\begin{array}{c} 16 \\ 14 \\ 12 \\ 2 \end{array}}}{\overline{\begin{array}{c} 2 \\ 2 \\ 2 \\ 2 \end{array}}} \} n \therefore O(n)$
 $n = n - m$

$\}$ (wt) $\frac{\overline{\begin{array}{c} 2 \\ 2 \end{array}}}{\overline{\begin{array}{c} 2 \end{array}}} \} \text{worst case}$

Asymptotic notation

$$1 < \log n < \sqrt{n} < n < n \log n < n^2 < n^3 < \dots < 2^n < 3^n < \dots < n^n$$

- Big - oh (O) upper bound \rightarrow (worst case)
- Big - Omega (Ω) Lower bound \rightarrow (Best case)
- Theta (Θ) Average bound \rightarrow (Avg case)

1. $f(n) = 2n^2 + 3n + 1$

$$2n^2 + 3n + 1 \leq 2n^2 + 3n^2 + 1/n^2$$

$$\Rightarrow 2n^2 + 3n + 1 \leq 9n^2 \quad n \geq 1$$

$$\therefore f(n) = O(n^2)$$

$$2n^2 + 3n + 1 \geq 1 \times n^2$$

$$\therefore f(n) = \Omega(n^2)$$

$$n^2 \leq 2n^2 + 3n + 1 \leq 9n^2$$

$$\therefore f(n) = \Theta(n^2)$$

2. $f(n) = n^2 \log n + n$

$$n^2 \log n \leq n^2 \log n + n \leq 10 n^2 \log n$$

$\therefore O(n^2 \log n)$

$\Omega(n^2 \log n)$

$\Theta(n^2 \log n)$

3. $f(n) = n!$

$$= n \times (n-1) \times (n-2) \cdots \times 3 \times 2 \times 1$$

$$= 1 \times 2 \times 3 \times \cdots \times n$$

$$1 \times 1 \cdots 1 \leq 1 \times 2 \times 3 \times \cdots \times n \leq n \times n \times n \times \cdots \times n$$

$$1 \leq n! \leq n^n$$

1 & n^n are not same, so we can't take Θ .

$\Omega(1)$

$O(n^n)$

4. $f(n) = \log n!$

$$\log(1 \times 1 \cdots 1) \leq \log(1 \times 2 \times \cdots \times n) \leq \log(n \times n \times \cdots \times n)$$

$$1 \leq \log n! \leq \log n^n \text{ or } n \log n$$

$O(n \log n)$

$\Omega(1)$

Sorting Analysis & Simulation

Bubble Sort

```
1. BubbleSort(A[], n)
2.   for k<-0, n-1 do      —— n
3.     for i<-0, n-2 do —— (n-1)n
4.       if A[i] > A[i+1] then
5.         swap(A[i], A[i+1]) —— n(n-1)(n-1)
6.       end if
7.     end for
8.   end for
9. end procedure
```

$f(n) = n + n^2 - n + n^2 - 2n + 1$
 $= 2n^2 - 2n + 1$

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Recursive Equation: $T(n) = T(n-1) + n \quad \therefore T(n) = \begin{cases} 1 & ; n=0 \\ T(n-1) + n & ; n>0 \end{cases}$

Substitution $\rightarrow T(n) = T(n-1) + n$

$$= [T(n-2) + n-1] + n$$

$$\begin{aligned} &= T(n-2) + 2n-1 \\ &= [T(n-3) + (n-2)] + 2n-1 \\ &= T(n-3) + 3n-3 \\ &\vdots \\ &= T(n-k) + kn - k \end{aligned}$$

Assume,

$$n-k=0$$

$$\therefore n=k$$

$$= T(n-2) + (n-1) + n$$

$$= T(n-3) + (n-2) + (n-1) + n$$

$$\vdots$$

$$= T(n-k) + (n-k+1) + (n-k+2) + \dots + (n-1) + n$$

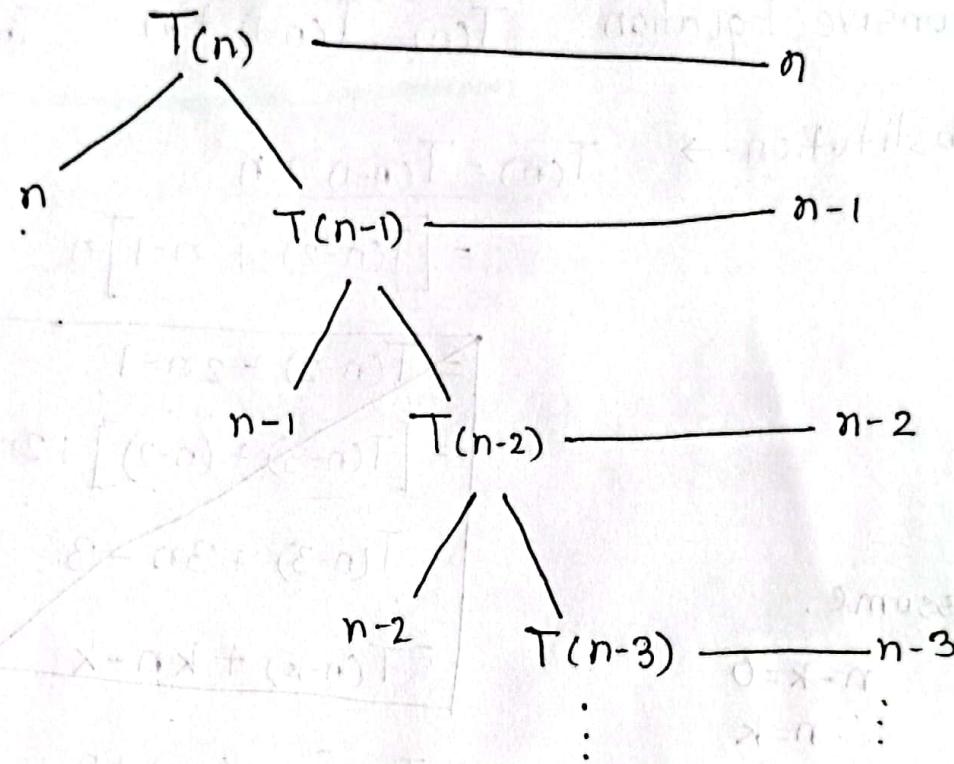
$$\stackrel{z}{=} T(n-n) + (n-n+1) + (n-n+2) + \dots + (n-n) + n$$

$$= T(0) + 1 + 2 + \dots + (n-1) + n$$

$$= 1 + \frac{n(n+1)}{2}$$

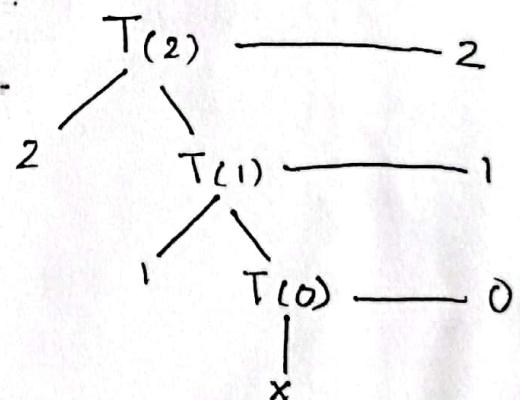
$$\therefore \mathcal{O}(n^2)$$

Tree →



$$\therefore 0+1+2+\dots+(n-3)+(n-2)+(n-1)+n \\ = \frac{n(n+1)}{2} = T(n)$$

∴ $O(n^2)$



Insertion sort

```

1. InsertionSort(A[], n)
2.   for j ← 1, n-1 do
3.     value ← A[j]
4.     i ← j-1
5.     while i >= 0 & A[i] > value do
6.       swap(A[i], A[i+1])
7.       i ← i-1
8.     end while
9.     A[i+1] = value
10.    end for
11. end procedure

```

1	23	1	10	52

2	1	23	10	52

3	1	10	23	52

4	1	5	10	23	2

5	1	2	5	10	23
sorted					

(a) [TIA] base case

n = 1, t₁ = 1

t₁ = 1, t₂ = 2

t₂ = 2, t₃ = 3

t₃ = 3, t₄ = 4

t₄ = 4, t₅ = 5

t₅ = 5, t₆ = 6

t₆ = 6, t₇ = 7

t₇ = 7, t₈ = 8

t₈ = 8, t₉ = 9

t₉ = 9, t₁₀ = 10

t₁₀ = 10, t₁₁ = 11

t₁₁ = 11, t₁₂ = 12

t₁₂ = 12, t₁₃ = 13

t₁₃ = 13, t₁₄ = 14

t₁₄ = 14, t₁₅ = 15

t₁₅ = 15, t₁₆ = 16

t₁₆ = 16, t₁₇ = 17

t₁₇ = 17, t₁₈ = 18

t₁₈ = 18, t₁₉ = 19

t₁₉ = 19, t₂₀ = 20

t₂₀ = 20, t₂₁ = 21

t₂₁ = 21, t₂₂ = 22

t₂₂ = 22, t₂₃ = 23

t₂₃ = 23, t₂₄ = 24

t₂₄ = 24, t₂₅ = 25

t₂₅ = 25, t₂₆ = 26

t₂₆ = 26, t₂₇ = 27

t₂₇ = 27, t₂₈ = 28

t₂₈ = 28, t₂₉ = 29

t₂₉ = 29, t₃₀ = 30

t₃₀ = 30, t₃₁ = 31

t₃₁ = 31, t₃₂ = 32

t₃₂ = 32, t₃₃ = 33

t₃₃ = 33, t₃₄ = 34

t₃₄ = 34, t₃₅ = 35

t₃₅ = 35, t₃₆ = 36

t₃₆ = 36, t₃₇ = 37

t₃₇ = 37, t₃₈ = 38

t₃₈ = 38, t₃₉ = 39

t₃₉ = 39, t₄₀ = 40

t₄₀ = 40, t₄₁ = 41

t₄₁ = 41, t₄₂ = 42

t₄₂ = 42, t₄₃ = 43

t₄₃ = 43, t₄₄ = 44

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t₅₁ = 51, t₅₂ = 52

t₅₂ = 52, t₅₃ = 53

t₅₃ = 53, t₅₄ = 54

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t₅₉ = 59, t₆₀ = 60

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t₁₀₃ = 103, t₁₀₄ = 104

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t₁₃₃ = 133, t₁₃₄ = 134

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t₁₅₁ = 151, t₁₅₂ = 152

t₁₅₂ = 152, t₁₅₃ = 153

t₁₅₃ = 153, t₁₅₄ = 154

t₁₅₄ = 154, t₁₅₅ = 155

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t₁₅₆ = 156, t₁₅₇ = 157

t₁₅₇ = 15

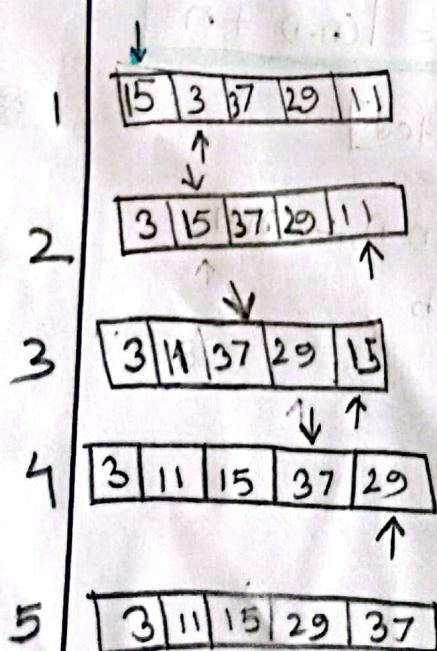
Selection sort

```
1 SelectionSort(A[], n)           (n-1) (n-1) (n-1) (n-1) (n-1) (n-1)
2   for i = 1 to n-1 do          n
3     min = i                   (n-1)
4       for j = i+1 to n do      (n-1) n
5         if A[j] < A[min] then  (n-1) (n-1)
6           min = j             (n-1) (n-1)
7         end if
8       end for
9       Swap A[min] and A[i]    (n-1) (n-1) (n-1) (n-1) (n-1) (n-1)
10      end for
11    end procedure
```

1 2 3 4 5 6 7 8 9 10 11

$f(n) = 3n - 2 + 3n^2 - 5n + 2$
 $= 3n^2 - 2n$
 $\therefore O(n^2)$

Recursive Equation: $T(n) = T(n-1) + n$



Linear Search

```
1 LinearSearch (A, n, item) -  
2   for i<=0 , n-1 do  
3     if A[i] == item then  
4       return i  
5     end if  
6   end for  
7   return -1  
8 end procedure
```

$$f(n) = n + n-1 + n-1 + 1
= 3n + 1$$

$O(n)$

item = 5

1.

6	4	5	3
---	---	---	---

↑
x

2.

6	4	5	13
---	---	---	----

↑
x

3.

6	4	5	3
---	---	---	---

↑
✓

Quick sort

```
1. void quickSort (Arr[], low, high) {  
2.     if (low < high) then  
3.         (C1) pi = partition (Arr, low, high), ————— n  
4.         quickSort (Arr, low, pi-1) ————— n/2  
5.         quickSort (Arr, pi+1, high), ————— n/2  
6.     end if  
7. end procedure  
  
8. partition (Arr[], low, high)  
9.     pivot = Arr[low]  
10.    i = low  
11.    for (j=low+1 to j<=high) do  
12.        if Arr[j] < pivot then  
13.            i++  
14.            swap (Arr[i], Arr[j])  
15.        end if  
16.    end for  
17.    swap (Arr[i], Arr[low])  
18. end procedure
```

(partition mid)

Best case:

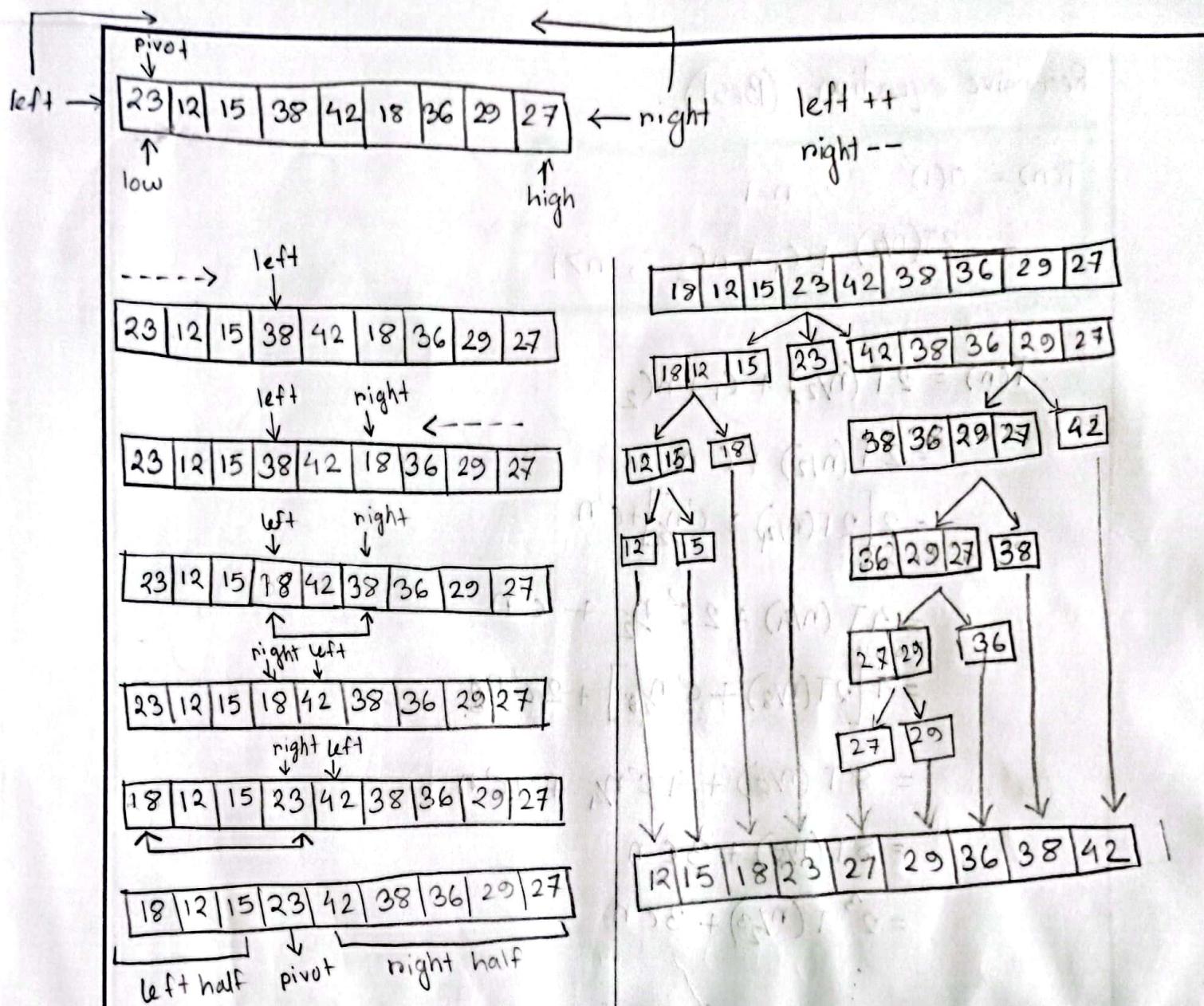
$O(n \log n)$

$\frac{n(n+1)}{2}$

Worst case

$O(n^2)$

(already sorted)



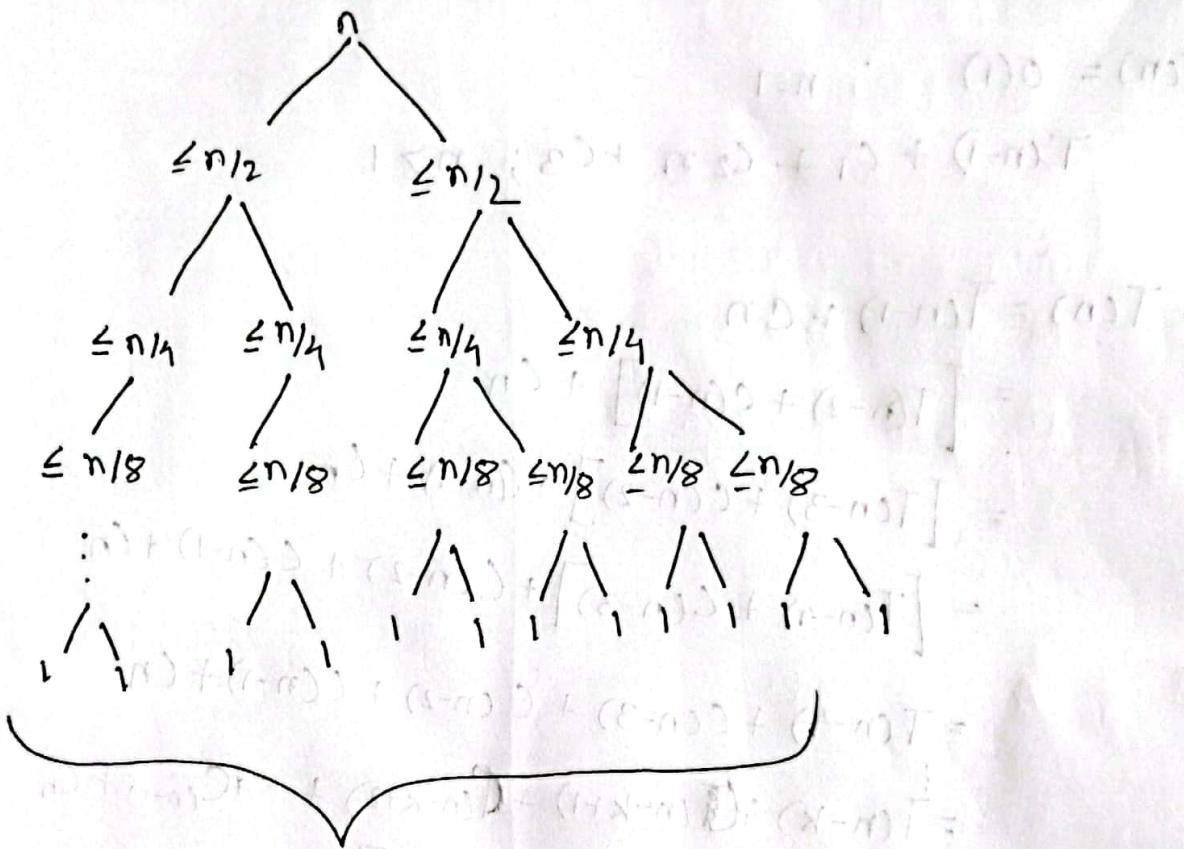
Recursive equation: (Best)

$$T(n) = O(1) \quad ; \quad n=1$$

$$2T(n/2) + c_1 + nc_2 \quad ; \quad n > 1$$

$$\begin{aligned} \therefore T(n) &= 2T(n/2) + c_1 + nc_2 \\ &= 2T(n/2) + c'n \\ &= 2 \left[2T(n/4) + c'n/2 \right] + c'n \\ &= 4T(n/4) + 2c'n/2 + c'n \\ &= 4 \left[2T(n/8) + c'n/4 \right] + 2c'n \\ &= 8T(n/8) + 4c'n/4 + 2c'n \\ &= 8T(n/8) + 3c'n \\ &= 2^3 T(n/2^3) + 3c'n \\ &\vdots \\ &= 2^k T(n/2^k) + k c'n \end{aligned}$$

$$\begin{array}{l|l} \text{Assume, } \frac{n}{2^k} = 1 & = 2^{\log_2 n} T(1) + c'n \log_2 n \\ \Rightarrow n = 2^k & = nC + c'n \log_2 n \\ \therefore k = \log_2 n & \therefore O(n \log n) \end{array}$$



cn

$$\leq 2cn/2 = cn$$

$$\leq 4cn/4 = cn$$

$$\leq 8 \cdot n/8 = cn$$

•

$$\angle nC = Cn$$

$$\therefore O(\log_2 n)$$

(Worst)

$$T(n) = O(1) ; n=1$$

$$T(n-1) + C_1 + C_2 n + C_3 ; n > 1$$

$$\therefore T(n) = T(n-1) + Cn$$

$$= [T(n-2) + C(n-1)] + Cn$$

$$= [T(n-3) + C(n-2)] + C(n-1) + Cn$$

$$= [T(n-4) + C(n-3)] + C(n-2) + C(n-1) + Cn$$

$$= T(n-4) + C(n-3) + C(n-2) + C(n-1) + Cn$$

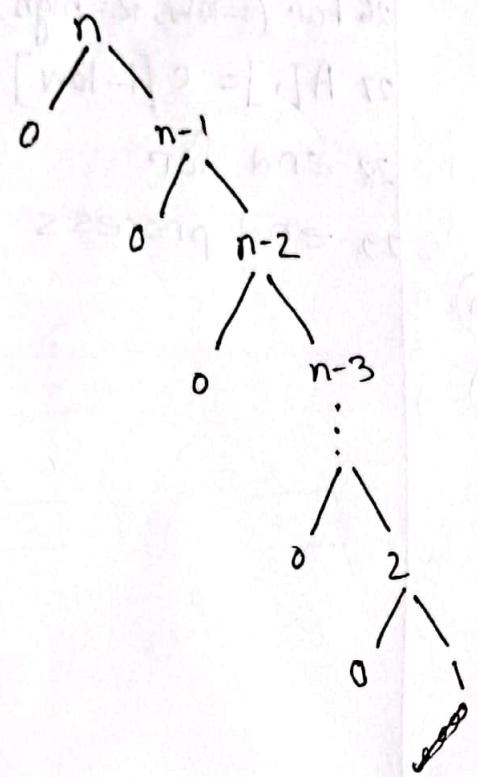
$$= T(n-k) + C(n-k+1) + C(n-k+2) + \dots + C_{(n-1)} + Cn$$

Assume,
 $k=n-1$

$$= T(1) + C[1+2+\dots+(n-1)+n]$$

$$= T(1) + C \frac{n(n+1)}{2}$$

$$\therefore O(n^2)$$



$$\therefore cn$$

$$c(n-1)$$

$$c(n-2)$$

$$c(n-3)$$

:

$$2c$$

$$c$$

$$0$$

$$\therefore cn + c(n-1) + c(n-2) + c(n-3) + \dots + 2c + c = \cancel{c(n-1+n-2+n-3+\dots+4)} - c(4n-4)$$

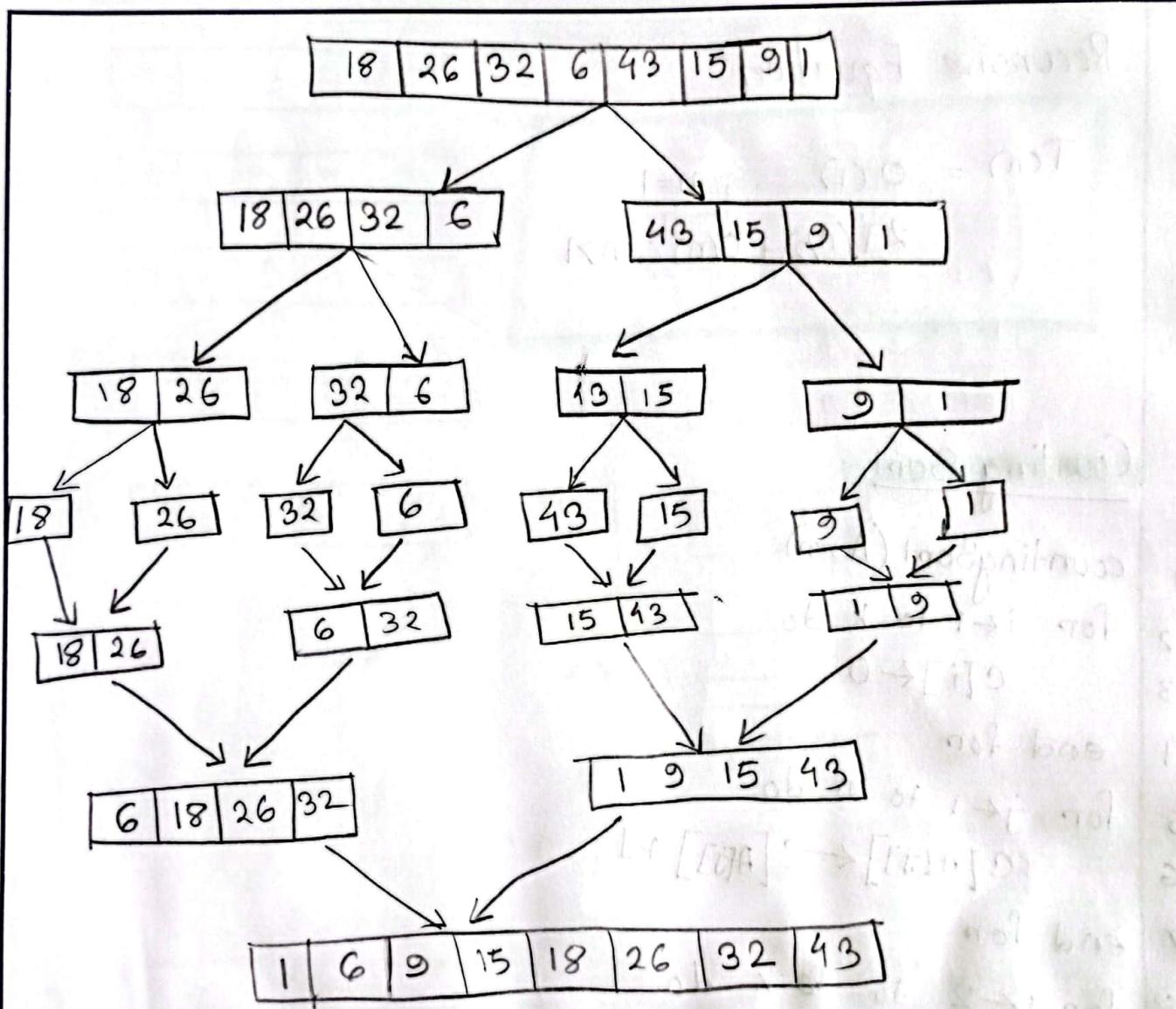
$$= c \frac{n(n-1)}{2}$$

$$\therefore O(n^2)$$

MergeSort

```
1 MergeSort(A, low, high)
2     mid = (low + high) / 2 — 1
3     if low < high then — 1
4         MergeSort(A, low, mid) -  $\frac{n}{2}$ 
5         MergeSort(A, mid+1, high) -  $\frac{n}{2}$ 
6         Merge(A, low, high, mid) - n
7     endif
8 end process

9 Merge(A, low, high, mid)
10    i = low
11    j = mid + 1
12    k = 0
13    C = [high - low + 1]
14    while i <= mid and j <= high do
15        if A[i] < A[j] then
16            C[k++] = A[i++]
17        else
18            C[k++] = A[j++]
19        end while
20    while (i <= mid) do
21        C[k++] = A[i++]
22    end while
23    while j <= high do
24        C[k++] = A[j++]
25    end while
```



Recursive Equation:

$$T(n) = \begin{cases} O(1) & ; n=1 \\ 2T(n/2) + O(n) & ; n>1 \end{cases}$$

Counting Sort

```
1 countingSort(A, n)
2   for i ← 1 to k do
3     C[i] ← 0
4   end for
5   for j ← 1 to n do
6     C[A[j]] ← C[A[j]] + 1
7   end for
8   for i ← 2 to k do
9     C[i] ← C[i] + C[i-1]
10  end for
11  for j ← n to 1 do
12    B[C[A[j]]] ← A[j]
13    C[A[j]] ← C[A[j]] - 1
14  end for
15 end process
```

$O(n+k)$ [worst case]

	1	2	3	4	5
1.	4	1	3	4	3
2.	4	1	3	4	3
3.	4	1	3	4	3
4.	4	1	3	4	3
5.	4	1	3	4	3

	1	2	3	4
C.	0	0	0	0
	0	0	0	1
	1	0	0	1
	1	0	1	1
	1	0	1	2
	1	0	2	2

frequency

	1	2	3	4	5
A:	4	1	3	4	3
B:	0	1	2	3	4
A:	4	1	3	4	3
B:			3	4	
A:	4	1	3	4	3
B:			3	4	
A:	4	1	3	4	3
B:	1	3	3	4	
A:	4	1	3	4	3
B:	1	3	3	4	

	1	0	2	2
C':				

	1	1	2	2
6.				
7.				

	1	1	3	5
8.				

	1	2	3	4
C':	1	1	2	5

	4	1	3	4
A:				

	1	3	3	4
B:				

sorted

	1	1	2	4
C':				

	1	1	1	4
C':				

	0	1	1	4
C':				

Binary Search

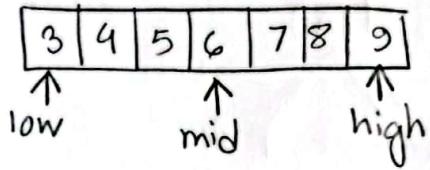
(Iterative)

```
1 binarySearch(A, x, low, high)
2     repeat till low = high
3         mid = (low+high)/2
4         if (x = A[mid]) then
5             return mid
6         end if
7         else if (x > A[mid]) then
8             low = mid + 1
9         end else if
10        else
11            high = mid - 1
12    end process
```

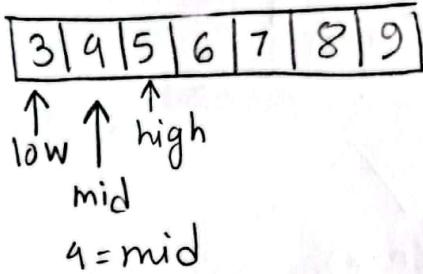
(Recursive)

```
1 binarySearch(A, x, low, high)
2     if (low > high) then
3         return false
4     end if
5     else
6         mid = (low + high)/2
7         if x = A[mid] then
8             return mid
9         end if
10        else if x > A[mid] then
11            return binarySearch(A, x, mid + 1, high)
12        end else if
13        else
14            return binarySearch(A, x, low, mid - 1)
```

$x = 4$



$4 < \text{mid}$



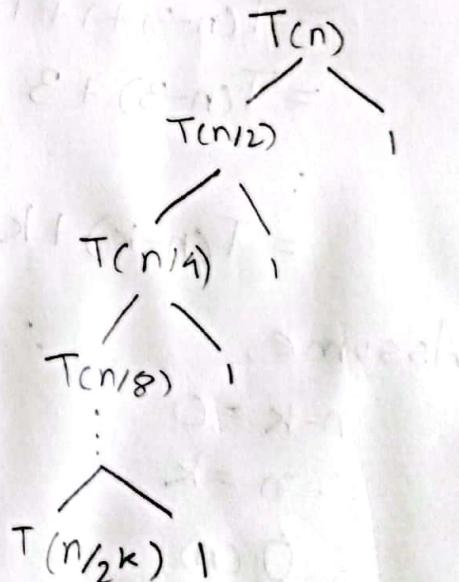
$4 = \text{mid}$

Recursive Equation:

$$T(n) = T(n/2) + 1 \quad ; \quad n > 1$$

$$1 \quad ; \quad n = 1$$

$$\begin{aligned} \therefore T(n) &= T(n/2) + 1 \\ &= [T(n/4) + 1] + 1 \\ &= [T(n/8) + 1] + 1 + 1 \\ &= T(n/8) + 3 \\ &= T(n/2^3) + 3 \\ &= T(n/2^k) + k \end{aligned}$$



Assume,

$$\frac{n}{2^k} = 1$$

$$\Rightarrow n = 2^k$$

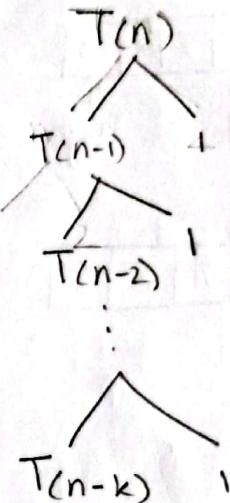
$$\therefore k = \log_2 n$$

$\therefore O(\log n)$

Recurrence

1 void Test (int n) {
 if (n > 0) {
 cout << n;
 }
 Test (n - 1);
}

$T(n) = T(n-1) + 1$
 $O(n)$



$$\begin{aligned}T(n) &= T(n-1) + 1 \\&= T(n-2) + 1 + 1 \\&= T(n-3) + 1 + 1 + 1 \\&= T(n-3) + 3 \\&\vdots \\&= T(n-k) + k\end{aligned}$$

Assume, $\therefore T(n) = T(0) + n$

$$n - k = 0 \quad = 1 + n$$

$$\therefore n = k$$

$$\therefore O(n)$$

2. `Void Test(int n) {`

$$\text{if } (n > 0) \{ \quad \dots \}$$

$$\quad \text{for}(\text{int } i=0; i < n; i++) \{ \quad \dots \}$$

$$\quad \quad \text{cout} \ll n; \quad \dots \}$$

$$\quad \}$$

$$\text{Test}(n-1); \quad \dots \}$$

$$\}$$

$$T(n) = T(n-1) + 2n + 2 \quad ; \quad n=0$$

$$= T(n-1) + n \quad \therefore T(n) = \begin{cases} 1 & ; \quad n=0 \\ T(n-1) + n & ; \quad n > 0 \end{cases}$$

3. `void Test(int n) {`

$$\text{if } (n > 0) \{$$

$$\quad \text{for}(\text{int } i=1; i < n; i=i*2) \{ \quad \dots \}$$

$$\quad \quad \text{cout} \ll \dots \}$$

$$\quad \}$$

$$\text{Test}(n-1); \quad \dots \}$$

$$\}$$

$$T(n) = T(n-1) + \log n \quad ; \quad n > 0$$

$$= 1 \quad ; \quad n=0$$

$$\begin{aligned}
 T(n) &= T(n-1) + \log n \\
 &= T(n-2) + \log(n-1) + \log n \\
 &= T(n-3) + \log(n-2) + \log(n-1) + \log n \\
 &\vdots \\
 &= T(n-k) + \log(n-k+1) + \log(n-k) + \cdots + \log n
 \end{aligned}$$

Assume, $n-k=0$
 $\therefore n=k$

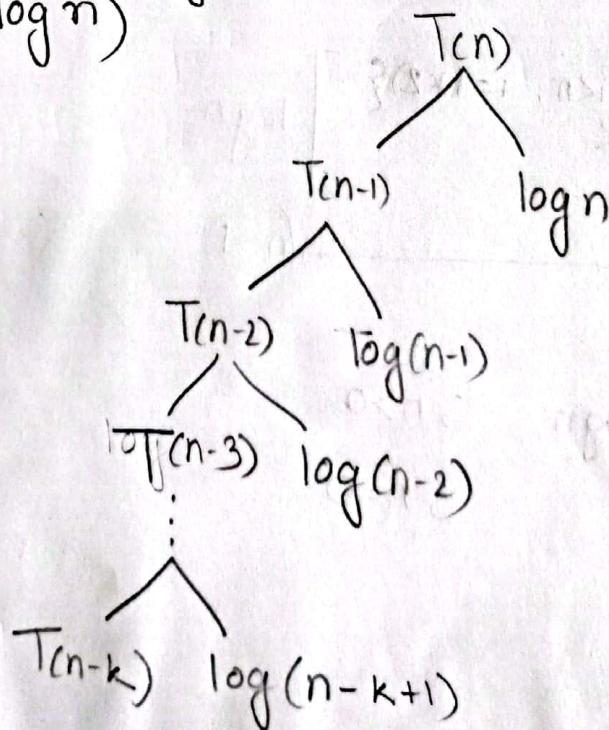
$$\therefore T(n) = T(0) + \log 1 + \log 2 + \cdots + \log n$$

$$= 1 + \log(1 \times 2 \times \cdots \times n)$$

$$= 1 + \log n!$$

$$= 1 + n \log n$$

$$\therefore O(n \log n)$$



$$4. T(n) = \begin{cases} 1 & ; n=0 \\ 2T(n-1) + 1 & ; n>0 \end{cases}$$

$$\begin{aligned} T(n) &= 2T(n-1) + 1 \\ &= 2[2T(n-2) + 1] + 1 \\ &= 2^2 T(n-2) + 2 + 1 \\ &= 2^2 [2T(n-3) + 1] + 2 + 1 \\ &= 2^3 T(n-3) + 2^2 + 2^1 + 2^0 \\ &\vdots \\ &= 2^k T(n-k) + 2^{k-1} + 2^{k-2} + \dots + 2^1 + 2^0 \end{aligned}$$

Assume, $n-k=0$
 $\therefore n=k$

$$\begin{aligned} \therefore T(n) &= 2^n T(0) + 2^{n-1} + 2^{n-2} + \dots + 2^1 + 2^0 \\ &= 2^n + 2^{n-1} + 2^{n-2} + \dots + 2^1 + 2^0 \\ &= 2^{n+1} - 1 \\ \therefore O(2^n) \end{aligned}$$

$$\frac{2^0 + 2^1 + 2^2 + \dots + 2^k}{2^k} = 2^{k+1} - 1$$

5.

$$T(n) = \begin{cases} 1 & ; n=1 \\ T(n/2) + 1 & ; n>1 \end{cases}$$

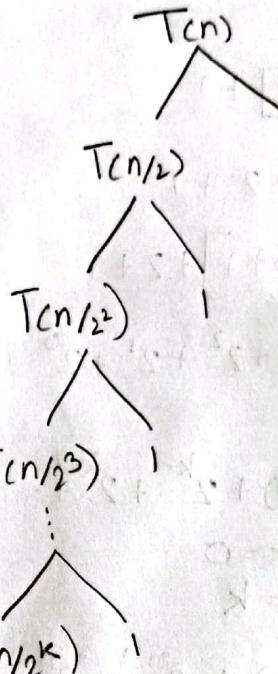
$$\begin{aligned} T(n) &= T(n/2) + 1 \\ &= T(n/4) + 1 + 1 \\ &= T(n/8) + 1 + 1 + 1 \\ &= T(n/2^3) + 3 \\ &\vdots \\ &= T(n/2^k) + k \end{aligned}$$

Assume, $2^k = \log n$

$$\Rightarrow k = \log_2 n$$

$$\begin{aligned} \therefore T(n) &= T(1) + \log_2 n \\ &= 1 + \log_2 n \end{aligned}$$

$$\therefore O(\log n)$$



6.

$$T(n) = \begin{cases} 1 & n=1 \\ T(n/2) + n & n>1 \end{cases}$$

$$\begin{aligned} T(n) &= T(n/2) + n \\ &= T(n/4) + n/2 + n \\ &= T(n/8) + n/4 + n/2 + n = T(n/2^3) + n/2^2 + n/2 + n \\ &\vdots \\ &= T(n/2^k) + n/2^{k-1} + n/2^{k-2} + \dots + n/2 + n \end{aligned}$$

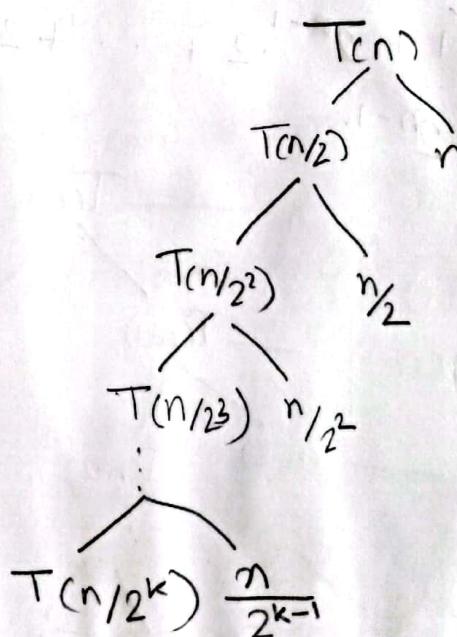
Assume, $\frac{n}{2^k} = 1$

$$\Rightarrow n = 2^k$$

$$\therefore k = \log_2 n$$

$$\begin{aligned} \therefore T(n) &= T(1) + n \left(\frac{1}{2^{n-1}} + \frac{1}{2^{n-2}} + \dots + \frac{1}{2^1} + \frac{1}{2^0} \right) \\ &= 1 + n \left(\cancel{\frac{1}{2^{n-1}}} + \cancel{\frac{1}{2^{n-2}}} + \dots + \cancel{\frac{1}{2^1}} + \cancel{\frac{1}{2^0}} \right) \\ &= 1 + n \\ &= 1 + 2n \end{aligned}$$

$$\therefore O(n)$$



7.

$$T(n) = 1 \quad ; n=0$$

$$2T(n-1) + n \quad ; n>1$$

$$T(n) = 2T(n-1) + n$$

$$= 2^2 T(n-2) + 2(n-1) + n$$

$$= 2^3 T(n-3) + 2^2(n-2) + 2(n-1) + n$$

:

$$= 2^k T(n-k) + 2^{k-1}(n-k+1) + \dots + 2(n-1) + n$$

Assume, $n-k=1$

$$\therefore \cancel{n-k} + k = n-1$$

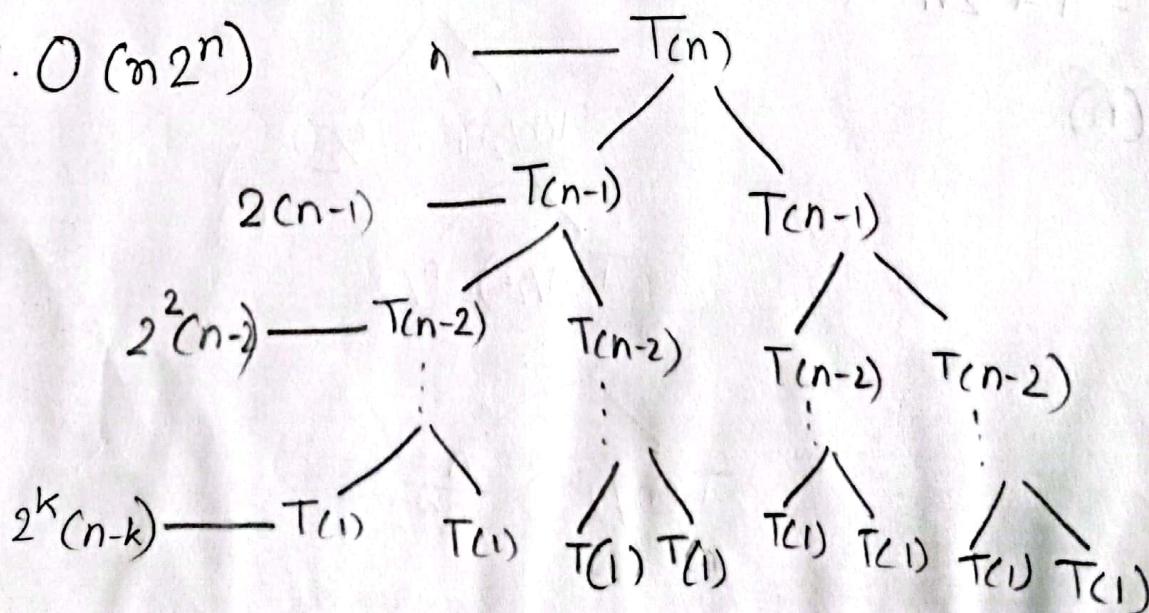
$$\therefore T(n) = 2^{n-1} T(1) + 2^{n-2}(2) + \dots + 2(n-1) + n$$

$$= 2^{n-1} \cdot 1 + 2^{n-2} \cdot 2 + \dots + 2(n-1) + 2^0(n)$$

$$= 2^{n-1} \left(1 + 2^{n-2} \cdot 2 + \dots + 2^{n-n}(n+1) + 2^{n-n+1}n \right)$$

$$= n \cdot 2^{n-1}$$

$\therefore O(n2^n)$



Masters Theorem

(Decreasing order)

$$T(n) = aT(n-b) + f(n)$$

$a > 0, b > 0, f(n) = O(n^k)$ where $k \geq 0$

Case: 1

$$a=1 ; O(n \cdot f(n))$$

$$\text{Eq: (i)} \quad T(n) = T(n-1) + 1$$

$$a=1, b=1, f(n)=1$$

$$\therefore O(n \cdot 1) = O(n)$$

$$\text{(ii)} \quad T(n) = T(n-1) + n$$

$$a=1, b=1, f(n)=n$$

$$\therefore O(n \cdot n) = O(n^2)$$

$$\text{(iii)} \quad T(n) = T(n-1) + \log n$$

$$a=1, b=1, f(n)=\log n$$

$$\therefore O(n \cdot \log n) = O(n \log n)$$

Case: 2

$$a > 1 \therefore O(f(n) \cdot a^n b)$$

$$\text{eg: (i)} \quad T(n) = 2T(n-1) + 1$$

$$a=2, b=1, f(n)=1$$

$$\therefore O(1 \cdot 2^n) = O(2^n)$$

$$(ii) T(n) = 3T(n-1) + 1$$

$$a=3, b=1, f(n)=1$$

$$\therefore O(1 \cdot 3^n) = O(3^n)$$

$$(iii) T(n) = 2T(n-1) + n$$

$$a=2, b=1, f(n)=n$$

$$\therefore O(n \cdot 2^n) = O(n2^n)$$

Case: 3

$$a < 1; O(f(n))$$

$$\text{eg: } T(n) = \frac{1}{2}T(n-1) + n$$

$$\therefore a = \frac{1}{2}, b=1, f(n)=n$$

$$\therefore O(n)$$

(Dividing function)

$$T(n) = aT(n/b) + f(n)$$

$$a \geq 1, b > 1, f(n) = \Theta(n^k \log^p n)$$

Case: 1

$$\log_b a > k, \Theta(n \log_b a)$$

$$\text{eg: (i)} T(n) = 2T(n/2) + 1$$

$$a=2, b=2, f(n) = \Theta(1)$$

$$= \Theta(n^0 \log^0 n)$$

$$\therefore K=0, p=0$$

$$\therefore \log_2 2 = 1 > k$$

$$\therefore \Theta(n \log_2^2) = \Theta(n^1) = \Theta(n)$$

$$\text{(ii)} T(n) \leq T(n/2) + n$$

$$a=1, b=2, f(n) = \Theta(n)$$

$$= \Theta(n^1 \log^0 n)$$

$$\therefore K=1, p=0$$

$$\therefore \log_2 4 = 2 > k$$

$$\therefore \Theta(n \log_2^4) = \Theta(n^2)$$

$$(iii) T(n) = 8T(n/2) + n^2$$

$$\therefore a=8, b=2, f(n)=\Theta(n^2 \cdot \log^0 n)$$

$$\therefore k=2, p=0$$

$$\therefore \log_2 8 = 3 > k$$

$$\therefore \Theta(n^{\log_2 8}) = \Theta(n^3)$$

$$(iv) T(n) = 9T(n/3) + 1$$

$$a=9, b=3, f(n)=\Theta(1) \\ = \Theta(n^0 \cdot \log^0 n)$$

$$\therefore k=0, p=0$$

$$\therefore \log_b a = \log_3 9 = 2 > 0$$

$$\therefore \Theta(n^{\log_3 9}) = \Theta(n^2)$$

Case: 2

$$\log_b a = k, p > -1, \Theta(n^k \log^{p+1} n)$$

$$p = -1, \Theta(n^k \log \log n)$$

$$p < -1, \Theta(n^k)$$

$$\text{eg: (i) } T(n) = 9T(n/3) + n^2$$

$$a=9, \quad b=3, \quad f(n)=\Theta(n^2)$$

$$= \Theta(n^2 \log^0 n)$$

$$\therefore P=0, \quad k=2$$

$$\therefore \log_3 9 = 2 = k \quad \& \quad P > -1$$

$$\therefore \Theta(n^2 \log^0 n) = \Theta(n^2 \log n)$$

$$(ii) \quad T(n) = 2T(n/2) + n$$

$$a=2, \quad b=2, \quad f(n)=\Theta(n)$$

$$= \Theta(n^1 \log^0 n)$$

$$\therefore k=1, \quad p=0$$

$$\therefore \log_2 2 = 1 = k \quad \& \quad p > -1$$

$$\therefore \Theta(n^1 \log^0 n) = \Theta(n \log n)$$

$$(iii) \quad T(n) = 4T(n/2) + n^2$$

$$\therefore a=4, \quad b=2, \quad f(n)=\Theta(n^2)$$

$$= \Theta(n^2 \log^0 n)$$

$$\therefore k=\underline{2}, \quad p=0$$

$$\therefore \log_2 4 = 2 = k \quad \& \quad p > -1$$

$$\therefore \Theta(n^2 \log n)$$

$$(iv) T(n) = 4T(n/2) + n^2 \log n$$

$$\therefore a=4, b=2, f(n)=\Theta(n^2 \log n)$$
$$= \Theta(n^2 \log^1 n)$$

$$\therefore k=2, p=1$$

$$\therefore \log_2 4 = 2 = k \quad \& \quad p > -1$$

$$\therefore \Theta(n^2 \log^2 n)$$

$$(v) T(n) = 2T(n/2) + \frac{n}{\log n}$$

$$\therefore a=2, b=2, f(n)=\Theta\left(\frac{n}{\log n}\right)$$

$$= \Theta(n^1 \log^{-1} n)$$

$$\therefore p=-1, k=1$$

$$\therefore \log_2 2 = 1 = k \quad \& \quad p=-1$$

$$\therefore \Theta(n \log \log n)$$

$$(vi) T(n) = 2T(n/2) + n \log^{-2} n$$

$$\therefore a=2 \quad b=2 \quad f(n)=\Theta(n^1 \log^{-2} n)$$

$$\therefore p=-2 \quad k=1$$

$$\therefore \log_a b = \log_2 2 = 1 = k \quad \& \quad p < -1$$

$$\therefore \Theta(n)$$

Case: 3

$$\log_b^a < k, \quad p \geq 0, \quad \Theta(n^k \log^p n)$$

$$p < 0, \quad \Theta(n^k)$$

e.g: (i) $T(n) = T(n/2) + n^2$

$$a=1, \quad b=2, \quad f(n)=\Theta(n^2 \log^0 n)$$

$$\therefore k=2, \quad p=0$$

$$\therefore \log_b^a = \log_2^1 < k \quad \& \quad p=0$$

$$\therefore \Theta(n^2 \log^0 n) = \Theta(n^2)$$

(ii) $T(n) = 2T(n/2) + n^2$

$$a=2, \quad b=2, \quad f(n)=\Theta(n^2 \log^0 n)$$

$$\therefore k=2, \quad p=0$$

$$\therefore \log_2^2 = 1 < k \quad \& \quad p=0$$

$$\therefore \Theta(n^2 \log^0 n) = \Theta(n^2)$$

(iii) $T(n) = T(n/2) + \frac{n^3}{\log n}$

$$a=1, \quad b=2, \quad f(n)=\Theta(n^3 \log^{-1} n)$$

$$\therefore k=3, \quad p=-1$$

$$\therefore \log_2^1 < k \quad \& \quad p < 0$$

$$\therefore \Theta(n^3)$$

General

Greedy method

Greedy(a, n) {

for $i \leftarrow 1$ to n do

$x = \text{select}(a)$

if Feasible(x) then

solution = solution + x

end if

end for

}

$a \boxed{a_0 | a_1 | a_2 | a_3 | a_4}$

$n=5$

Knapsack Problem

$n=7$

Obj 0 1 2 3 4 5 6 7

$m=15$

profit p 10 5 15 7 6 18 3 $0 \leq x \leq 1$

kg

weight w 2 3 5 7 1 4 1

$\frac{p}{w}$ 5 1.66 3 1 6 4.5 3

$x (x_1 \ x_2 \ x_3 \ x_4 \ x_5 \ x_6 \ x_7)$

Satisfication:

$$\sum x_i w_i = 1x_2 + \frac{2}{3}x_3 + 1x_5 + 0x_7 + 1x_1 + 1x_4 + 1x_1$$

$$= 15 \text{ kg}$$

$$15 - 1 = 14$$

$$14 - 2 = 12$$

$$\sum x_i p_i = 1x_10 + \frac{2}{3}x_5 + 1x_15 + 0x_7 + 1x_6 + 1x_18 + 1x_3$$

$$= 54.6$$

$$12 - 4 = 8$$

$$8 - 5 = 3$$

$$3 - 1 = 2$$

$$2 - 2 = 0 \text{ kg}$$

Constraint: $\sum x_i w_i \leq m$

Object: $\sum x_i p_i \max$

$$W = 16 \text{ kg}$$

item	weight	value	v/w	chosen	
i ₁	6	6	1	1	$16 - 1 = 15$
i ₂	10	2	0.2	0	$15 - 3 = 12$
i ₃	3	1	0.33	$\frac{1}{3}$	$12 - 5 = 7$
i ₄	5	8	1.6	1	$7 - 6 = 1$
i ₅	1	3	3	1	$1 - 1 = 0$
i ₆	3	5	1.66	0.16	

~~Decision~~

$$\sum i_i w_i = 1 \times 6 + 0 \times 10 + \frac{1}{3} \times 3 + 1 \times 5 + 1 \times 1 + 1 \times 3$$

$$= 6 + 0 + 1 + 5 + 1 + 3$$

$$= 16$$

$$\sum i_i v_i = 1 \times 6 + 0 \times 2 + \frac{1}{3} \times 1 + 1 \times 8 + 1 \times 3 + 1 \times 5$$

$$= 6 + 0 + \frac{1}{3} + 8 + 3 + 5$$

$$= 22.33$$