Error Control Codes

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Dept. of Computer Science
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Lecture Outline

1. Cyclic redundancy check
2. Linear block code

Cyclic Redundancy Check

Introduction

- What if the transmitted bits get altered on the way?
  - Is there any technique to detect the error?

Yes, using Cyclic Redundancy Check (CRC)

- CRC
  - In CRC, some redundant bits are sent in addition to the message bits.
  - The purpose of the redundant bits is to facilitate detecting error.
  - *The redundant bits are called frame check sequence (FCS)*

How is FCS generated?

Cyclic Redundancy Check….

Introduction….

- Strength of the CRC depends on the number of redundant bits (that is, FCS length)
- Longer FCS length results in better accuracy in detecting error

- Required two sequence
  - *Message sequence, M*
    - The desired data to be sent
    - Can be of any length
  - *Pattern sequence, P*
    - Known to both sender and receiver
    - If we want to use *K* bits FCS, we need a pattern bit sequence, *P*, of length *K+1* bits.

Cyclic Redundancy Check….

Generation of FCS

1. Decide how many FCS bits, *K*, you are going to use.
2. Append K zeros at the end of the message bits to generate *M+K* bits long sequence *S*.
3. Select a *K+1* bits long pattern sequence, *P*.
4. Divide the sequence *S* by the pattern sequence *P* to find the *K* bits of the remainder, R.
5. Remove the appended zeros from S and append the calculated remainder *R* Thus, the *N* bits message bits and *K* bits remainder constitutes the transmitting sequence, *T*.

Cyclic Redundancy Check….

Error detection at the receiver

1. At the destination, the received sequence, T', is divided by the same patter sequence, P.
2. If at this step there is no remainder, the data unit is assumed to be correct and is therefore accepted.
3. A remainder indicates that the data unit has been damaged on the way and therefore must be rejected.

Cyclic Redundancy Check….

Example 1

- Generate FCS if the message polynomial and generator polynomial are $X^3 + X^2 + 1$ And $X^3 + X + 1$, respectively.

Let M(x) be the **message polynomial**

Let P(x) be the **generator polynomial/Pattern sequence**

Let M(x) = $X^3 + X^2 + 1 \longrightarrow$ **1 1 0 1**

Let P(x) = $X^3 + X + 1 \longrightarrow$ **1 0 1 1**

1. Consider the case where M=*1101 and P=1011*.
2. *Since P consists of 4 bits, append K=3 bits zeros (000) at the end of M, S=1101000*
3. *Divide S by P to get 3 bits remainder.*

Cyclic Redundancy Check….

Example 1

At sender

```
                  1111
             ___________
P → 1011  |  1101000   ← S
             1011
             ________
              1100
              1011
              ________
               1110
               1011
               ________
                1010
                1011
                ________
                 001   ← R
```

Transmit sequence or codeword

T = 1 1 0 1 0 0 1

Cyclic Redundancy Check….

Example 1

At Receiver

```
1011 | 1 1 0 1 0 0 1
       1 0 1 1
       -------------
         1 1 0 0
         1 0 1 1
       -------------
           1 1 1 0
           1 0 1 1
       -------------
             1 0 1 1
             1 0 1 1
       -------------
             0 0 0 0
```

Since the remainder is zero, there is no error in the received sequence

Cyclic Redundancy Check….

Example 1

What if any bit gets altered in the channel?

Suppose that the second bit (red) has altered from 1 to 0.

```
1011 | 1 0 0 1 0 0 1
       1 0 1 1
       ---------------
           1 0 0 0
           1 0 1 1
       ---------------
               1 1 1
```

The nonzero remainder indicates an erroneous reception.

The frame will not be acknowledged.

The sender will resend the frame.

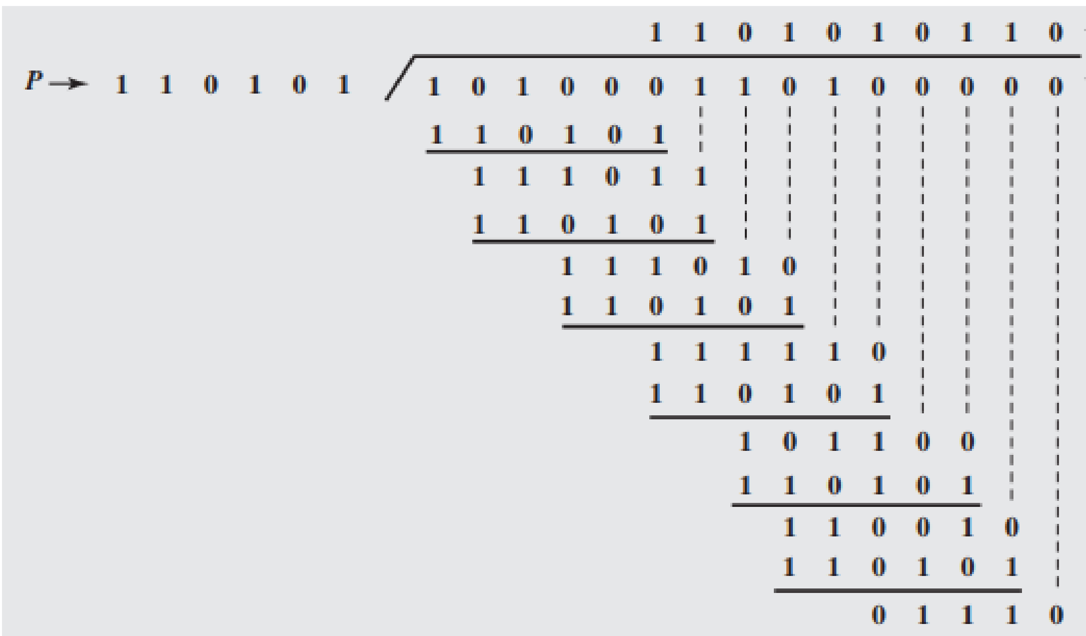
Cyclic Redundancy Check….

Example 2

- Message M = 1010001101
- Pattern P = 110101
- Length of P=6
- Append K=6-1=5 zeros at the end of M
- S=101000110100000
- Now divide S by P to find 5 bits remainder [1].

Cyclic Redundancy Check….

Example 2



- Transmitted sequence, *T*=101000110101110
- At the receiving end, *T* is divided by *P* to see if the remainder is zero. The zero remainder indicates error free reception.

Cyclic Redundancy Check….

Example 1

```
                                        1  1  0  1  0  1  0  1  1  0
                        _____________________________________________
P → 1  1  0  1  0  1  /  1  0  1  0  0  0  1  1  0  1  0  1  1  1  0
                         1  1  0  1  0  1
                         ----------------
                            1  1  1  0  1  1
                            1  1  0  1  0  1
                            ----------------
                                  1  1  1  0  1  0
                                  1  1  0  1  0  1
                                  ----------------
                                        1  1  1  1  1  0
                                        1  1  0  1  0  1
                                        ----------------
                                              1  0  1  1  1  1
                                              1  1  0  1  0  1
                                              ----------------
                                                    1  1  0  1  0  1
                                                    1  1  0  1  0  1
                                                    ----------------
                                                                   0
```

Because there is no remainder, it is assumed that there have been no errors.

Homework

1. Detect whether the received sequence 101110101 is error free if the pattern sequence is 1010.

Linear Block Code

Generator Matrix

Linear Block Code: A code in which addition of any two codewords gives another codeword [2].

Message, M: k bits long

Redundant bits, Q: q bits long

Codeword length, N: $k+q$ bits long

Generator matrix, $G = [P_{k\times q} I_k]$

For $k = 3$ and q= 3,

$$G = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$P_{3\times 3} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

$$I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Then, it is a (n, k) =(6, 3) block code

Linear Block Code….

Codeword calculation

The codeword for the message [0 1 1] is

$$C = M \times G$$

$$C = \begin{bmatrix} 0 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$C = [\underbrace{1 \quad 1 \quad 0}_{Q} \quad \underbrace{0 \quad 1 \quad 1}_{M}]$$

Modulo-2 summation

$$0 \times 1 \oplus 1 \times 0 \oplus 1 \times 1 = 1$$

$$0 \times 1 \oplus 1 \times 1 \oplus 1 \times 0 = 1$$

$$0 \times 0 \oplus 1 \times 1 \oplus 1 \times 1 = 0$$

$$0 \times 1 \oplus 1 \times 0 \oplus 1 \times 0 = 0$$

$$0 \times 0 \oplus 1 \times 1 \oplus 1 \times 0 = 1$$

$$0 \times 0 \oplus 1 \times 0 \oplus 1 \times 1 = 1$$

Linear Block Code….

Error-detection

Receiving end

Parity check matrix, $\mathrm{H} = \left[I_q \;\; P^T_{k\times q} \right]$

$$\mathrm{H} = [I_3 \;\; P^T_{3\times 3}]$$

$$\mathrm{H} = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

$$P_{3\times 3} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

$$P^T_{3\times 3} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$P^T_{3\times 3}$ is the transpose of $P_{3\times 3}$

Linear Block Code….

Error-detection….

Suppose that there is no error in the received sequence.

Hence the received sequence, r, is the same as the transmit sequence, C.

$$r = C$$

$$r = [1 \quad 1 \quad 0 \quad 0 \quad 1 \quad 1]$$

Syndrome, $s = rH^T$

$$s = [1 \quad 1 \quad 0 \quad 0 \quad 1 \quad 1]\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

$$s = [0 \quad 0 \quad 0]$$

The all-zero syndrome indicates a correct reception !

$$\mathrm{H} = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

$$H^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

Linear Block Code....

Error-detection....

Suppose that there is an error in the received sequence.
The second bit (from left side) has altered from 1 to 0

$$r = [1 \quad 0 \quad 0 \quad 0 \quad 1 \quad 1]$$

Syndrome, $s = rH^T$

$$s = [1 \quad 0 \quad 0 \quad 0 \quad 1 \quad 1]\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

$$s = [0 \quad 1 \quad 0]$$

The non-zero syndrome indicates an erroneous reception !

Linear Block Code….

Error-correction

How to correct the error?

$$H^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

1. Syndrome , $s = [0 \quad 1 \quad 0]$
2. Locate the syndrome in H^T
3. It is in second row
4. So, the second element in the received sequence, $r = [1 \quad 0 \quad 0 \quad 0 \quad 1 \quad 1]$ is erroneous.
4. Alter the second bit from 0 to 1.
5. So, the correct received sequence is [1 1 0 0 1 1].

Note: The given generator matrix enables correction of at most 1 bits.

It is possible to correct more bits , but it requires quite a lot work! No Free Lunch!

Homework

- Consider a (7, 4) code whose generator matrix is given by

Message bit [0 1 0]

$$G = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

(a) Find all the codewords of the code

(b) Find the parity-check matrix

(c) Find the syndrome for the received vector [1 1 0 1 0 1 0]. Is it a valid codeword?

References

[1] W. Stallings, *Data and Computer Communication*, 10th ed., Pearson Education, Inc., 2014, USA, pp. 194 - 196.

[2] B. Sklar, *Digital Communications*, 2nd ed., Prentice Hall. 2017, USA, pp. 328 - 345.

Recommended Books

1. **Data Communications and Networking**, *B. A. Forouzan*, McGraw-Hill, Inc., Fourth Edition, 2007, USA.
2. **Computer Networking: A Top-Down Approach**, *J. F., Kurose, K. W. Ross*, Pearson Education, Inc., Sixth Edition, USA.
3. **Official Cert Guide CCNA 200-301 , vol. 1**, *W. Odom*, Cisco Press, First Edition, 2019, USA.
4. **CCNA Routing and Switching**, *T. Lammle*, John Wily & Sons, Second Edition, 2016, USA.
5. **TCP/IP Protocol Suite**, *B. A. Forouzan*, McGraw-Hill, Inc., Fourth Edition, 2009, USA.
6. **Data and Computer Communication**, *W. Stallings*, Pearson Education, Inc., Tenth Education, 2013, USA.