# USING WAVELET ANALYSIS TO TRACK PERIODICITIES

A short guide

# Introduction & Contents

This document presents an overview of CWT analysis applied to solar data (namely helioseismic GONG and MDI/HMI data and the solar activity proxy F10.7cm index), and a user guide on using wavelet_analysis to extract significant periodicities.

**Section 1** presents the code wavelet_analysis.py and explains how to use the code, how it works, and describes the outputs.

**Section 2** applied this code to helioseismic data that is m-averaged (i.e. it has the azimuthal contribution averaged out). In this section I present a short study on the significant periodicities of helioseismic v1y GONG and MDI/HMI data across Solar cycles 23 and 24. We also separate these datasets but average frequency. We also compare these results to analysis of the F10.7 cm index.

**Section 3** covers the application of this code to helioseismic data once more, this time to explore the latitudinal dependance of significant periodicities. To do this, we use the v1f GONG data, which has azimuthal (m) dependance. We again apply the CWT across different latitude bands and investigate how the periodicities change between solar cycle 23 and 24.

Common vocab used in this document:

- Harmonic degree (also called l or ell)
- Radial degree (also called n)
- Azimuthal degree (also called m)
- CWT= Continuous Wavelet Transform
- GWT= Global Wavelet Transform
- GONG= Global Oscillation Network Group
- MDI= Micheallson Doppler Imager
- HMI= Helioseismic and Magnetic Imager
- QBO= Quasi-Biennial Oscillation
- modes/ multiplet: A specific oscillation mode, characterised by a given m, n, l.

Good places for further reading:

Global Helioseismology: [https://gong.nso.edu/](https://gong.nso.edu/), T. Mehta Thesis 2023
CWT: [https://psl.noaa.gov/people/gilbert.p.compo/Torrence_compo1998.pdf](https://psl.noaa.gov/people/gilbert.p.compo/Torrence_compo1998.pdf)
QBO: Broomhall 2012, 2017, Mehta 2022,

Any questions? Feel free to contact me at [tishtrya.mehta@gmail.com](mailto:tishtrya.mehta@gmail.com) 😁

# Section 1. Performing Wavelet analysis on user inputted data, using wavelet_analysis.py

Wavelet_analysis.py allows a user to read in data in csv or txt format and extract the significant periodicity from the normalised data's continuous wavelet transform (CWT) and Global Wavelet Transform (GWT) with error bars corresponding to 98% confidence levels.

The code works as follows:

1. The data is read in using `wavelet_data_reader` , and reads the column indicated by variable **`index`**, reading in the **filename** of the data, where the data has been saved in **DATA_PATH**.
2. The data is preprocessed by
   a. removing the nan values from the data
   b. if necessary removing the final data point to ensure the data is even
   c. Normalising the data by finding the best fitting savgol (Savitzky-Golay) envelope for the data, and dividing the data by it. The savgol envelope can be manipulated by changing the variables **savgol_wl** and **savgol_pol**. This should be done manually, by eye, until a good fit is found. A good fit should result in the normalised data having a roughly constant amplitude (Shown in the second panel of the produced figure).
   Savgol_wl determines the window length of the Savgol envelope. In general, a larger value smooths the envelope over broader length scales. This value must be less than/equal to the length of the data
   Savgol_pol is the polynomial order of the savgol envelope. A larger value will try to fit to the smaller scale features, whereas a lower value will create a more smoothed profile. Play around with these values and you'll get the hang of it.

   This process uses cadence **dt**, and start time **starttime.**
   Note: We normalise the data because we may want to only examine the overall periodicities in the data and ignore the effects of amplitude modulation. Wavelet analysis is sensitive to the amplitude of the data by design and assessing non-normalised data may mean you miss observing periodicities at low amplitudes/ your signal may be dominated by large scale oscillations. However if you wish to examine amplitude modulation you can force your savgol envelope to be constant ([1,1,1,..]) which will remove the normalisation effect.
3. The normalised data is assessed by `wavelet_analysis_finder` , producing a CWT, GWT and associated coi, etc,
4. We find the period corresponding to the maximum power in the CWT. We also obtain the corresponding errors on this period. We do this by assessing the 98% confidence contour, and finding the minimum and maximum periods that correspond to the 98% contour at the time of the maximum power period. We only search over the range specified by the user inputted variables **seg1_index_min**, **seg1_index_max.**
5. The above is then plotted using `spectrum_plotter`
6. The above is then saved as a figure, with **title** as a user defined variable

Alongside downloading the input data, the user will have to change the following variables:

```
#    User chosen variables
```

```
#    ----------
#    filename : string
#        Name of the file containing the data (csv or txt format)
#    index : int
#        The index of where the data is saved in the input file
#    savgol_wl : int
#        The window length for the savgol filter
#    savgol_pol : int
#        The polynomial order for the savgol filter
#    starttime : datetime.datetime
#        The start time of the data (YYYY, M, D)
#    dt : int
#        The time step of the data in days
#    title : string
#        The title of the output plot and csv file
#    seg1_index_min : int                                          ****
#        The minimum index of the data to be examined
#    seg1_index_max : int
#        The maximum index of the data to be examined
#    DATA_PATH : string
#        The path to where the input data is saved
#    SAVE_PATH : string
#        Where the output plot and csv file are saved
```

**** To examine the full duration of the data seg1_index_min should =0 and seg1_index_max should = -1.

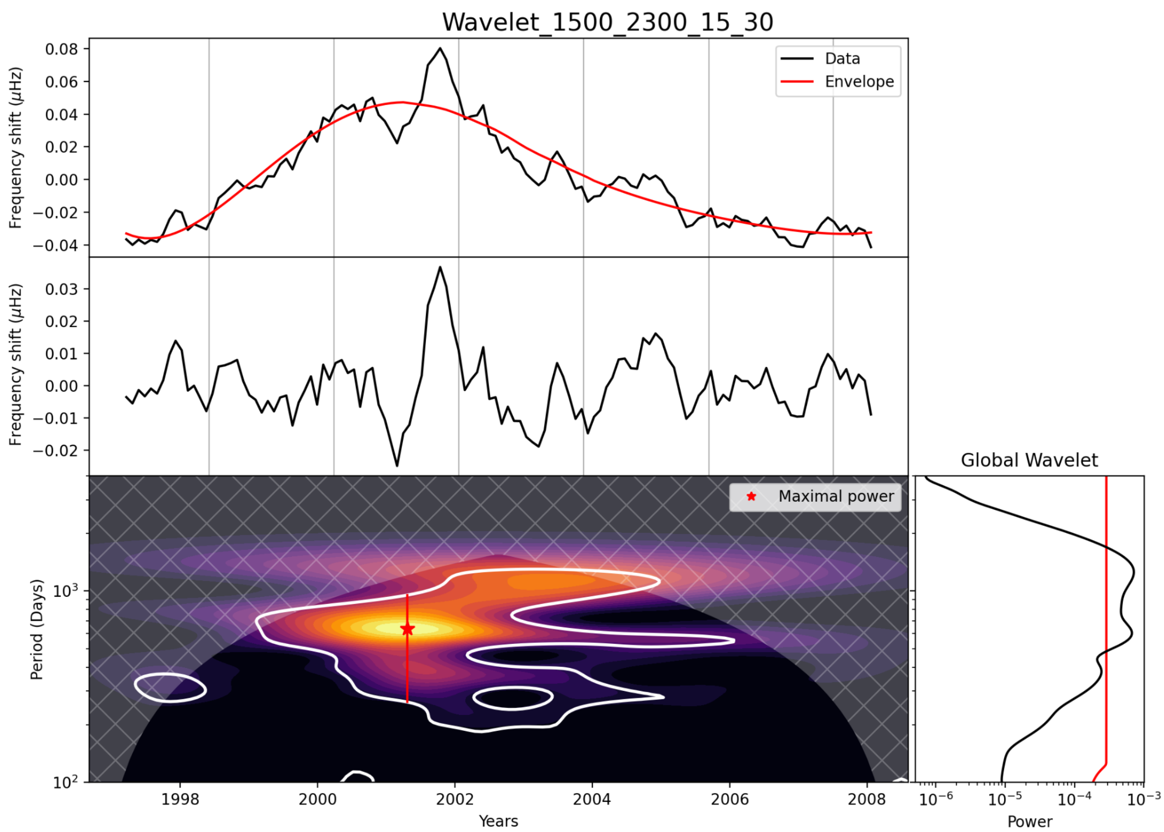
```
E.g.:
filename="v1f_1000-2000.csv"
savgol_wl=31
savgol_pol=5
starttime=datetime.datetime(1996,9,3)
dt=36
title='All_modes_1000-2000uHz'
seg1_index_min,seg1_index_max = 0,-1
index=0
DATA_PATH =  os.path.join("/", "home", "space", "phrsnz", "Desktop", "Academic",
"Programs","PhD", "QBO_evolution", "CWT_analysis_fmodes", "Output")
sAVE_PATH =  os.path.join("/", "home", "space", "phrsnz", "Desktop", "Academic",
"Programs","PhD", "QBO_evolution", "CWT_analysis_fmodes", "Output")
#---------------------------
```

After implementing these variables to your preference, the code can be run. It relies on

- V1f_data_sorting.py
- Wavelet_spectrum_maker.py
- Wavelet_errorbars.py
- Wavelet_spectrum_plotter.py
- Wavelet_data_saver.py
- Constants.py

which work in the order above to read in the data and preprocess the data, perform wavelet analysis on the preprocessed data, find the corresponding errors, create a plot/ figure of the analysis, and save the spectrum and the results into a csv file.

Your output will look like this:
Figure:




CSV:

```
#    2. A csv file containing [columns are in order]
        a. The time coordinate of when the input data reaches its maximal value
        b. The time coordinate of when the input data reaches its minimal value
        c. The period corresponding to the maximal power observed in the continuous
wavelet spectrum
        d. The period corresponding to the lower bound of the 98% confidence level
for the maximal power period described in c.
        e. The period corresponding to the upper bound of the 98% confidence level
for the maximal power period described in c.
        f. The time coordinate of when the input data reaches its maximal value at
the 98% confidence level
        g. The period at the time described in a.
        h. The period corresponding to the lower bound of the 98% confidence level
for the maximal power period described in g.
```

```
        i. The period corresponding to the upper bound of the 98% confidence level
for the maximal power period described in g.
#    """
```

e.g.

"T_[MaxAmp]","T_[MinAmp]","max_pow_period","min_pow","max_pow","t_max_pow_period","period_solar_max","min_pow_solar_max","max_pow_solar_max"
2001-09-13 00:00:00, 2007-07-08 00:00:00, 637.74, 269.88, 877.36,
2001-04-22 00:00:00,623.17,74.38,4441.48

# Section 2. Assessing the temporal evolution of the QBO using CWT

## Aims:

The aim of this study is to track the evolution in periodicity of the solar quasi-biennial oscillation. This could help us understand the generation mechanism of the QBO and better characterise the solar dynamo. We use helioseismic data over solar cycles 23 and 24 and assess it using the Continuous Wavelet Transform to fit the evolution in periodicity. We may split up the helioseismic data according to low and high and all frequencies, and use m-averaged frequencies. We may also compare the helioseismic findings with data from the solar activity proxy, the F10.7cm index.

## Data: Defining the solar cycle start and end times

All source data has been uploaded to Warwick Files. We use the start and end dates of the solar cycles determined by SILSO (Sunspot Index and Long-term Solar Observations) which are derived from the maxima and minima of the 13-month smoothed monthly sunspot number time series.

The SILSO solar cycle min and max dates: https://www.sidc.be/silso/cyclesminmax
The 13 month smoothed monthly sunspot number time series:
https://www.sidc.be/silso/infosnmstot

Here I take the first day of the month noted on the SILSO cycle max and min dates to mark the start of a given cycle, and take the last day of the cycle as the day prior to the first day of the next cycle, giving the following dates.

Cycle 23: 1/8/1996 – 30/11/2008

Cycle 24: 1/12/2008 – 30/11/2019

I use these dates to dictate my choice of the time interval for helioseismic datasets (see below). Because helioseismic data is binned into 36/72 day bins, we cannot exactly match the above cycle dates, but instead choose the closest possible bins to the above.

## Datasources

Helioseismic:

GONG (Global Oscillation Network Group):
Website: https://gong2.nso.edu/archive/patch.pl?menutype=t
We use the v1y datafiles
Cadence: 36 days

Cycle 23 using GONG v1y data files:
03/09/1996 – 23/11/2008
Number of datafiles: 125
Common modes: 459
Ranges for n, l (inclusive, respectively); 1-19, 19-135

Cycle 24 using GONG v1y data files:
29/12/2008 – 02/11/2019
Number of datafiles: 111
Common modes: 734
Ranges for n, l (inclusive, respectively); 1–20, 19–145

Combined (literally the combination of the above two cycles):
03/09/1996 – 02/11/2019
Number of datafiles: 236
Common modes: 396
Ranges for n, l (inclusive, respectively); 1-19, 21–132

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MDI/HMI (Michelson Doppler Imager, Helioseismic and Magnetic Imager)
Website: http://jsoc.stanford.edu/ajax/lookdata.html
Differences between the MDI and HMI mode frequencies are within 1-sigma (where sigma is the uncertainty in frequency determination) for the five overlapping 72-day datasets.
Cadence: 72 days
MDI/HMI files are named as: MDI_freq.**** where **** is a 4 digit number corresponding to the start time of the interval beginning **** days since: 1/1/1993. Therefore MDI_freq.3232 refers to the dataset corresponding to 3232 days after 1/1/1993 which is 7/12/2001. As MDI/HMI datasets have a cadence of 72 days, MDI_freq.3232 covers the time interval of 72 days starting 7/12/2001, so 7/12/2001– 16/2/2002.

Therefore to calculate the corresponding MDI/HMI datasets for Cycles 23, 24 I find the closest datasets to the above time ranges.

For Cycle 23 (1/8/1996 – 30/11/2008):
MDI_freq.1288 – MDI_freq.5752
Which corresponds to intervals, of 72 days, starting:
12/7/1996 – 1/10/2008
Number of datafiles: 61
Common modes: 1289
Ranges for n, l (inclusive, respectively); 1–21, 5–150

Cycle 24: 1/12/2008 – 30/11/2019
MDI_freq.5824 – HMI_freq.9856
Which corresponds to intervals, of 72 days, starting:
12/12/2008 – 27/12/2019
Number of datafiles: 57
Common modes: 1333
Ranges for n, l (inclusive, respectively); 1-22, 3–150

These two cycles are adjacent, independent and non-overlapping, so the two above cycles can be linked together to produce a continuous time series of constant cadence:

Combined:
12/7/1996 –27/12/2019
MDI_freq.1288–HMI_freq.9856
Number of datafiles: 118
Common modes: 1224
Ranges for n, l (inclusive, respectively); 1–21, 6–150

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Solar Activity Proxy

F10.7
Website: nrc.canada.ca/en
Cadence: Non-rebinned 1 day

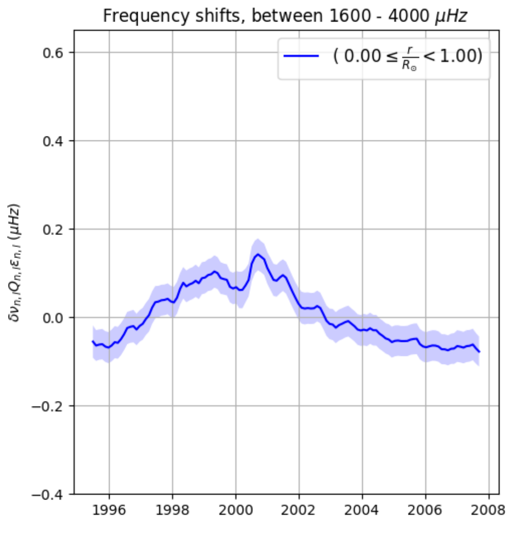
**Sorting and distribution of data:**

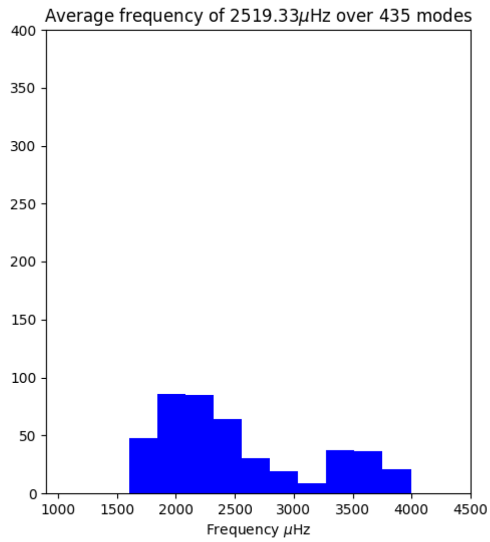
Now that the data has been obtained, we may sort the modes by average frequency and cycle. This uses the same technique described in Section 3, by using code similar to `v1f_plots_sorting.py`. We obtain the average frequency of each mode and produce three bins for each helioseismic data source;

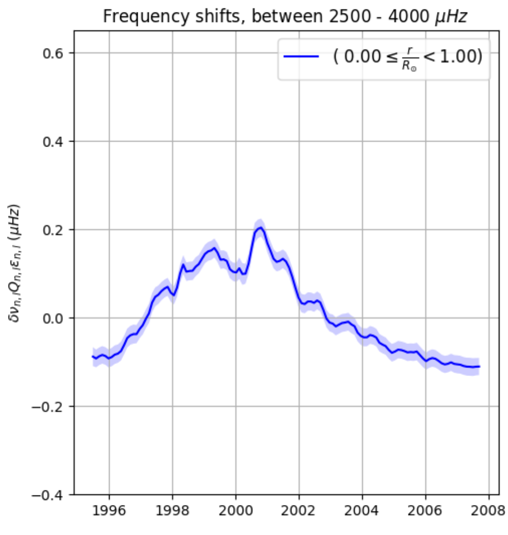
- Low frequency for all modes with avg. frequencies between 1600-2500 uHz,
- High frequency for avg. frequencies between 2500-4000 uHz,
- and all frequencies for avg freq. between 1600-4000uHz.

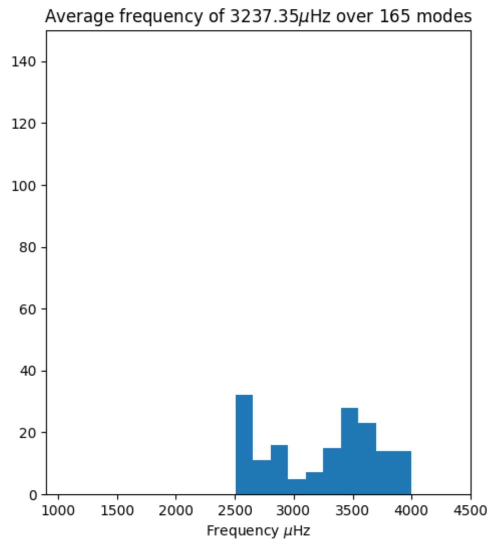
The F10.7 is not filtered in this way as it is a 'singular' data source, rather than the helioseismic data that are the result of averaging over many modes.

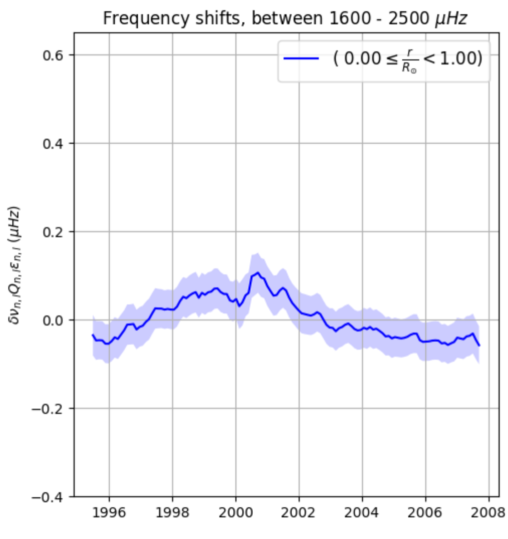
Below are plots showing (Left) the average mode produced from averaging over all the modes in a given cycle and frequency bin and (Right) the associated histogram of the bin showing its distribution in frequency space. Note that we only accept modes that have 100% fill (no nans), and are therefore generally limited to a small number of modes.  GONG results are shown in Blue, MDI/HMI in Red.

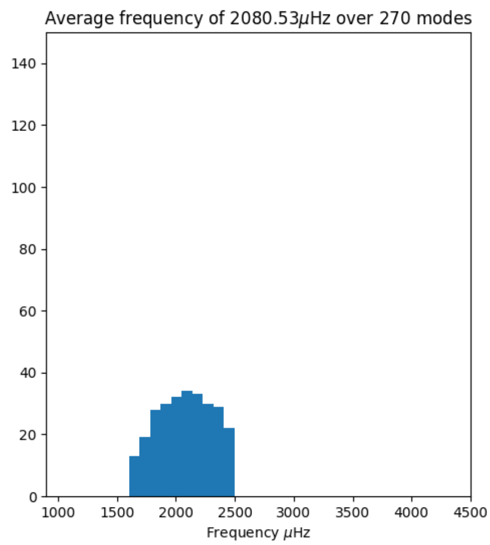


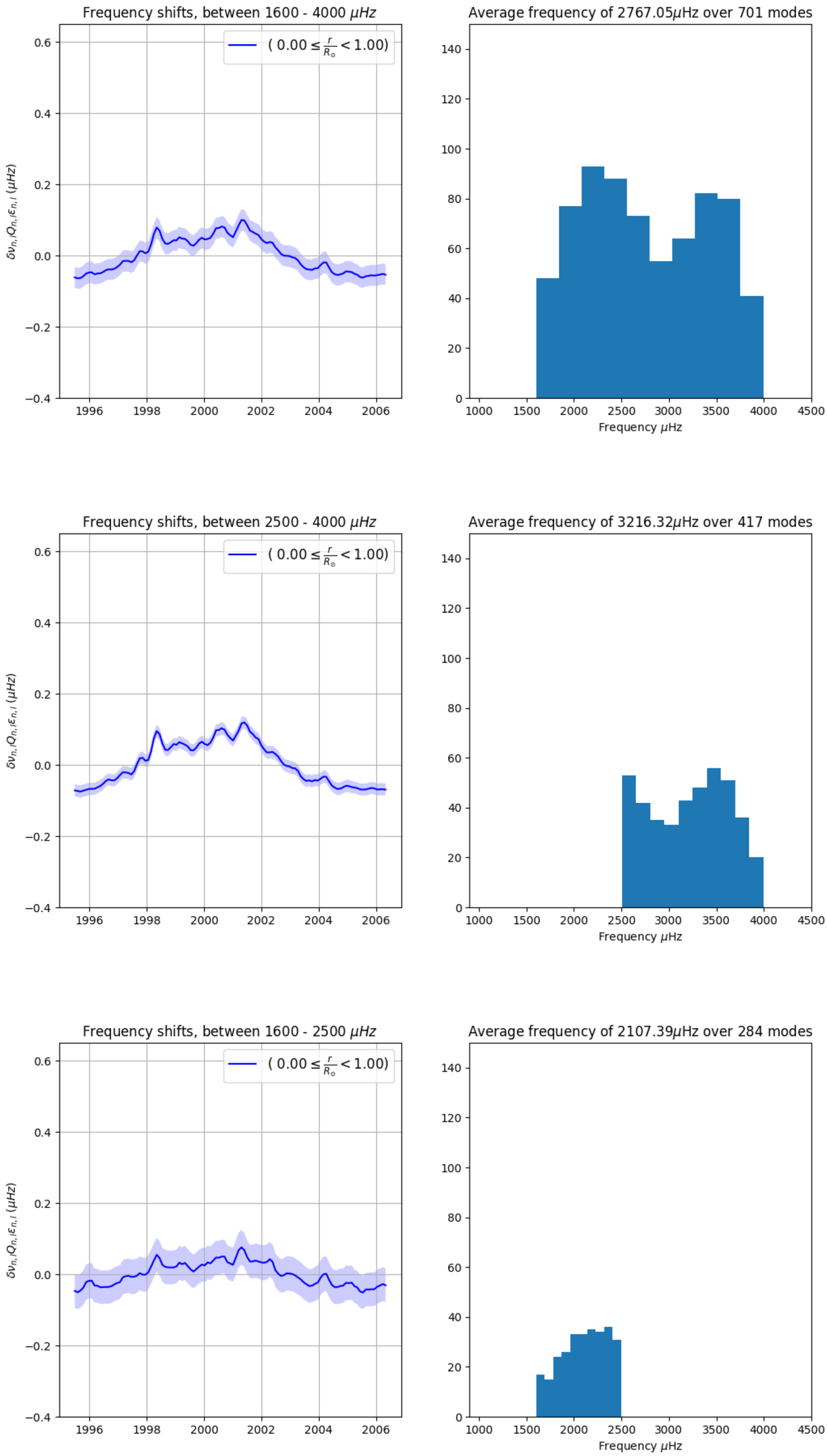



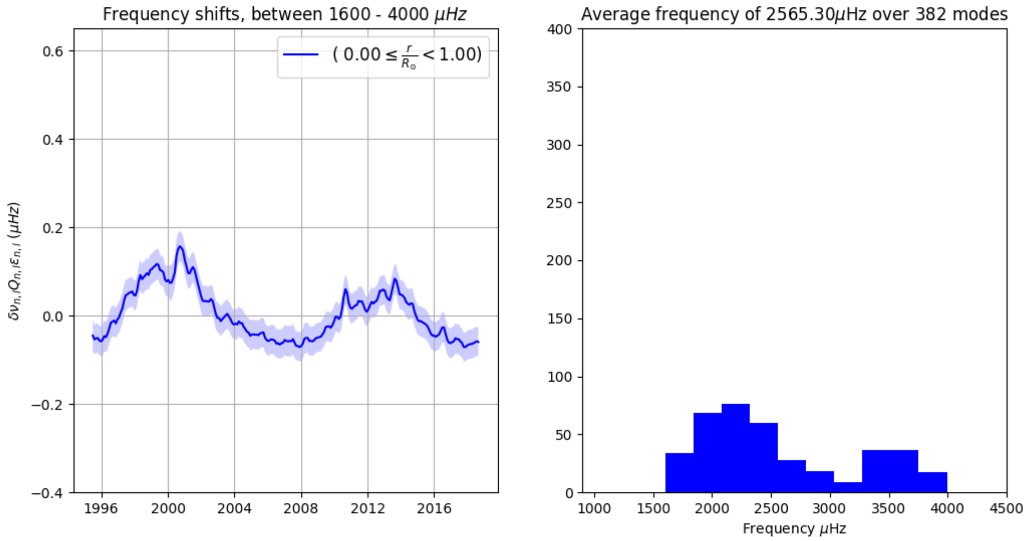



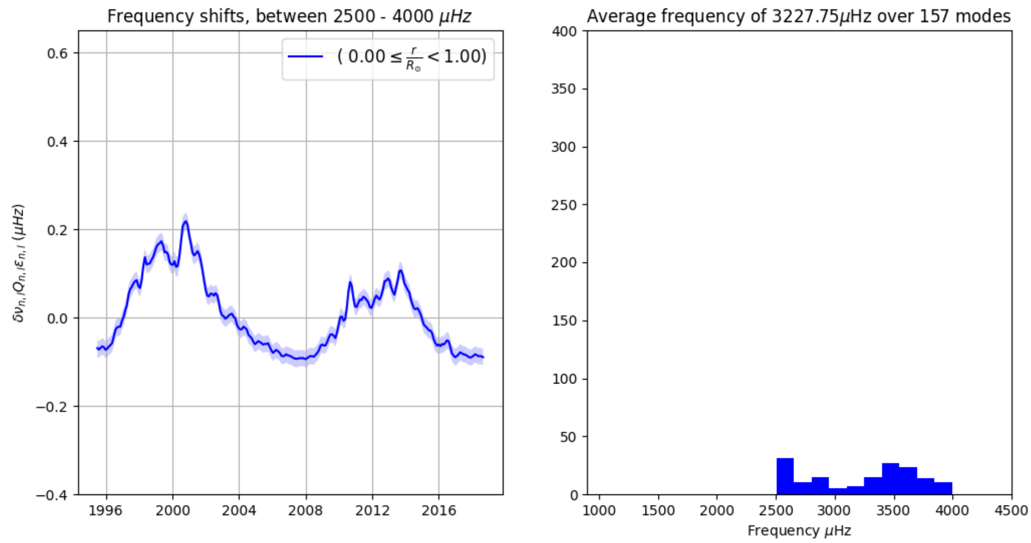



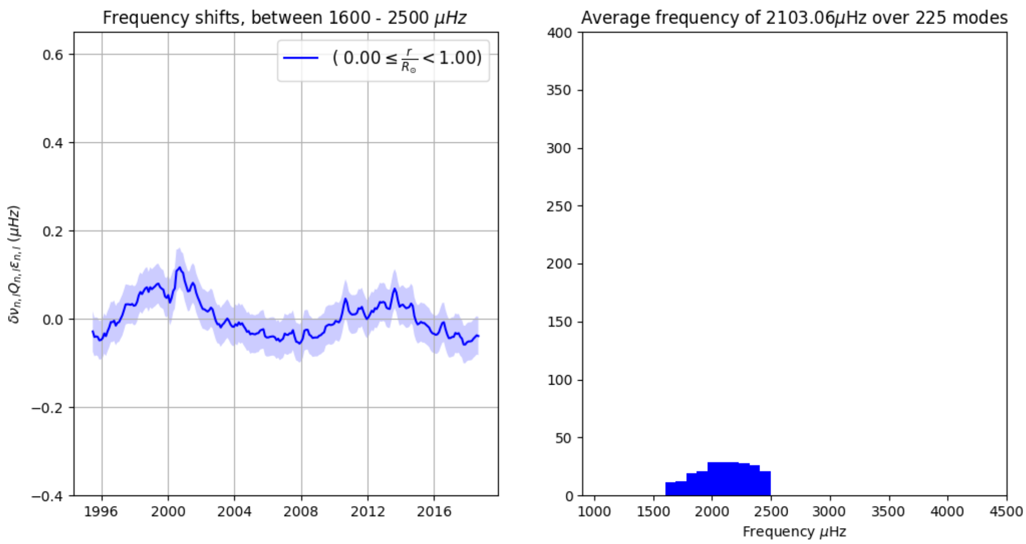

**GONG; CYCLE 23**



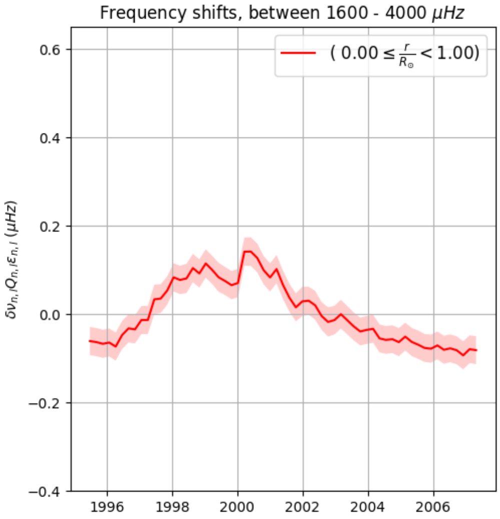

**GONG; CYCLE 24**

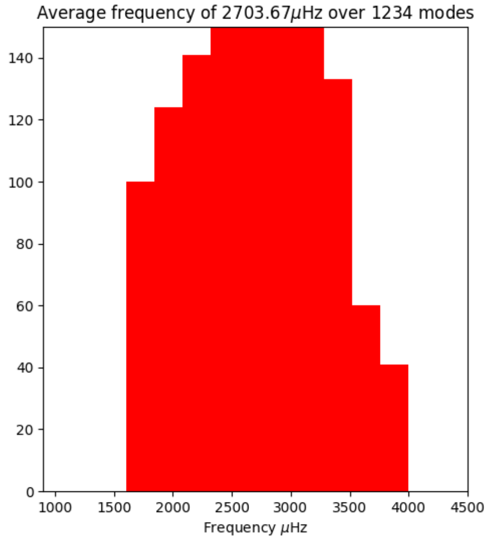


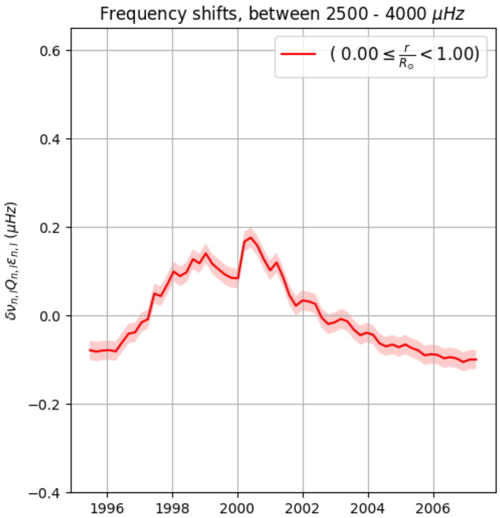



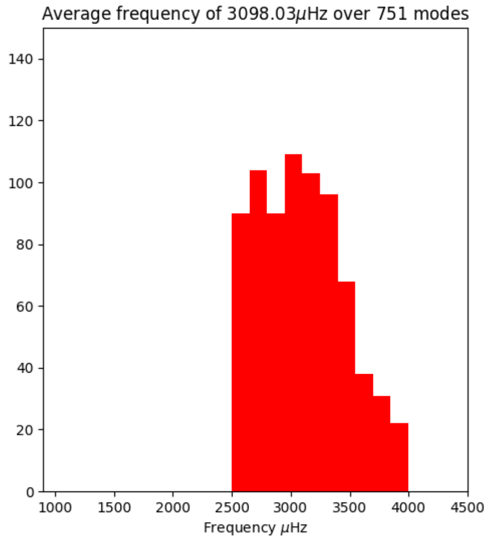



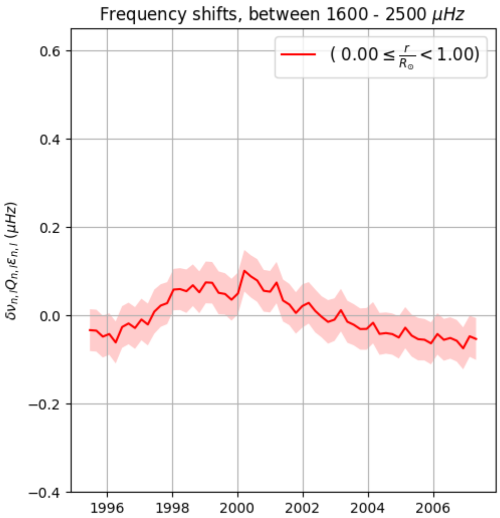

**GONG CYCLES 23+24**

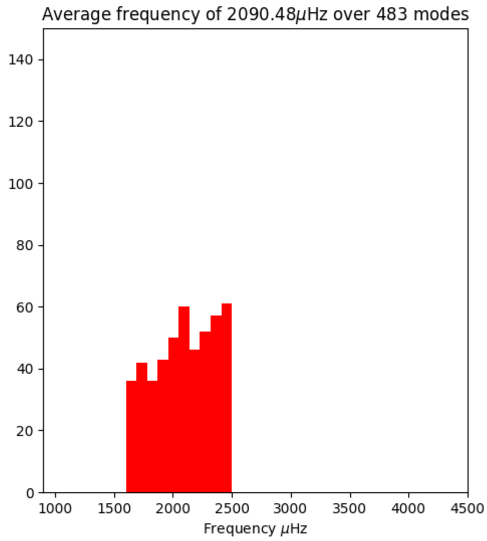


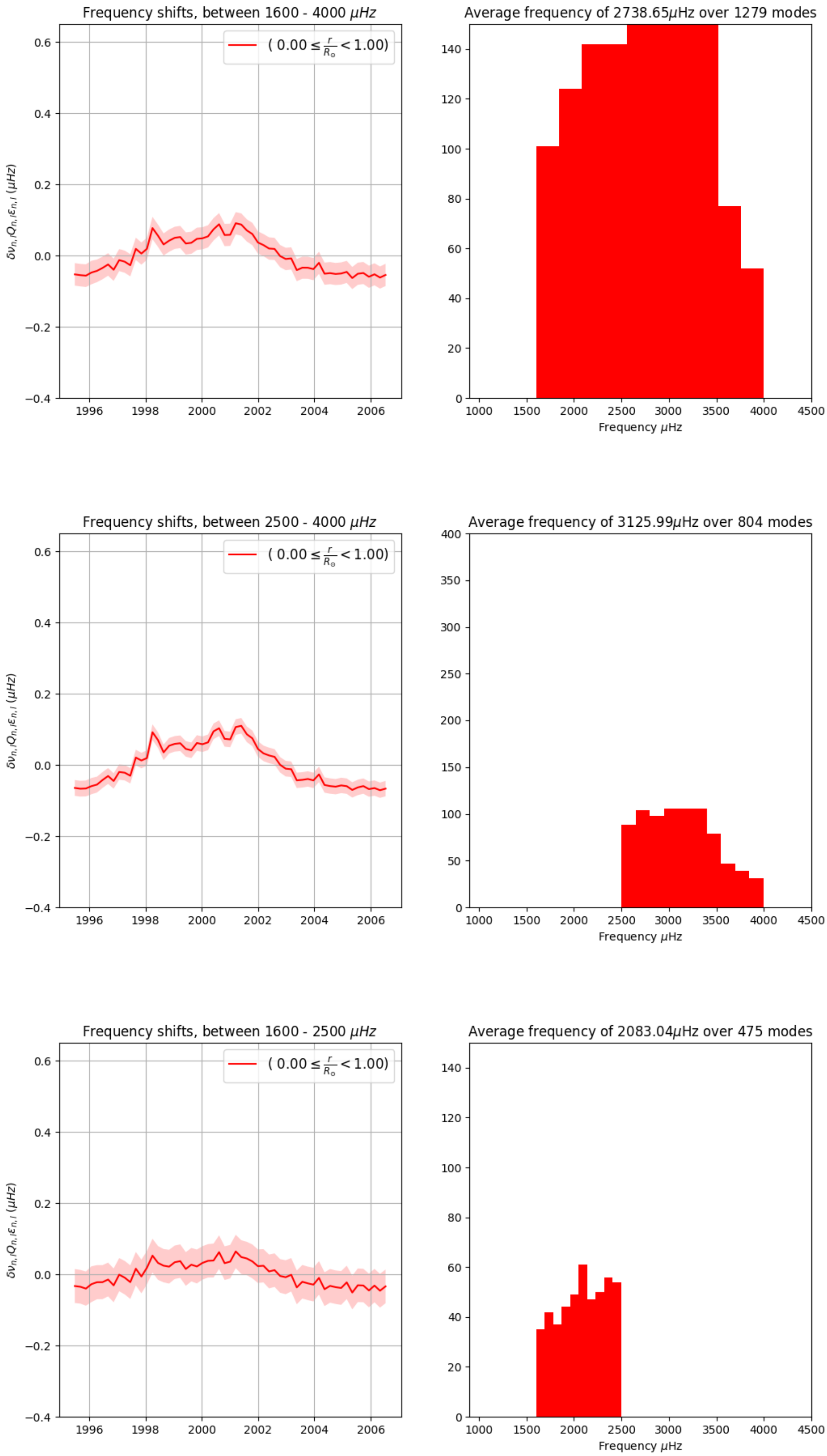



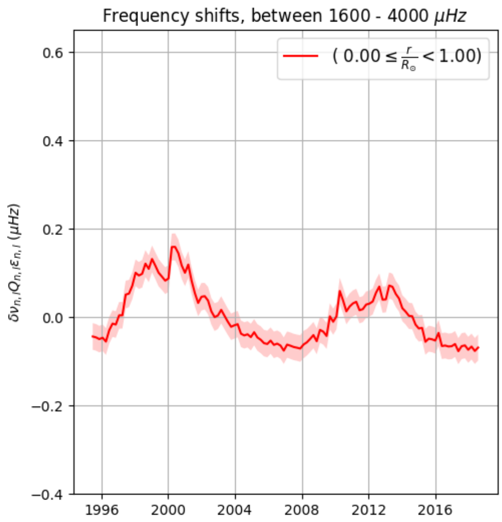



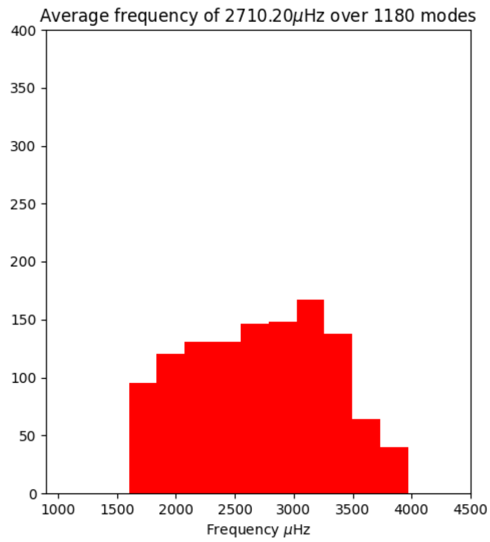



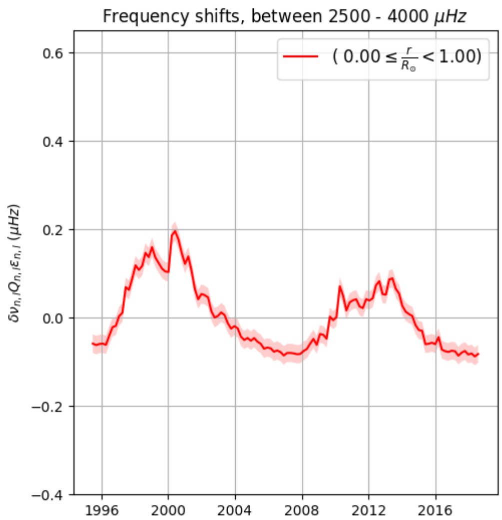

**MDIHMI- CYCLE 23**

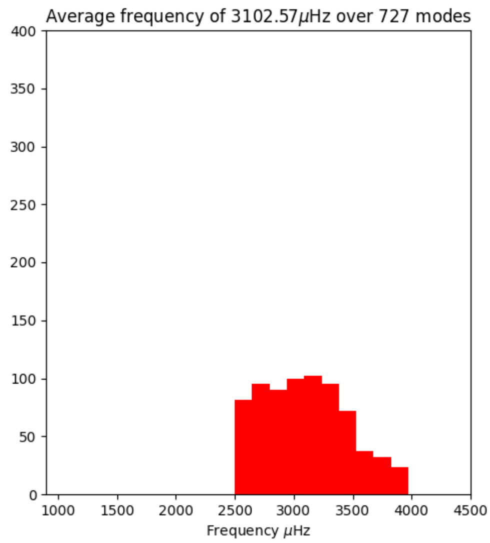


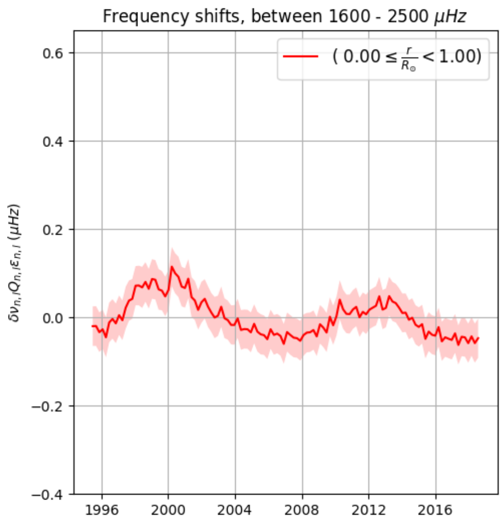

**MDIHMI CYCLE 24**

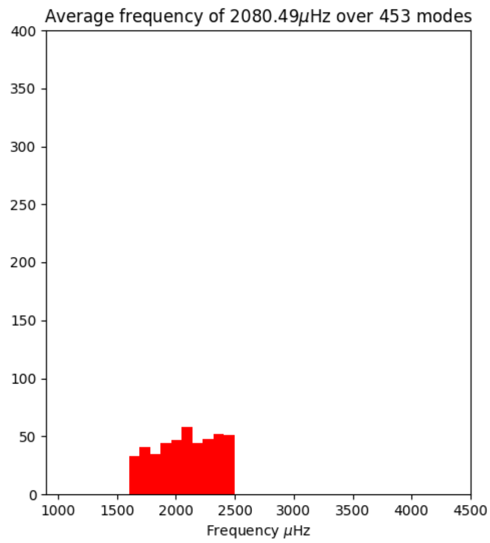


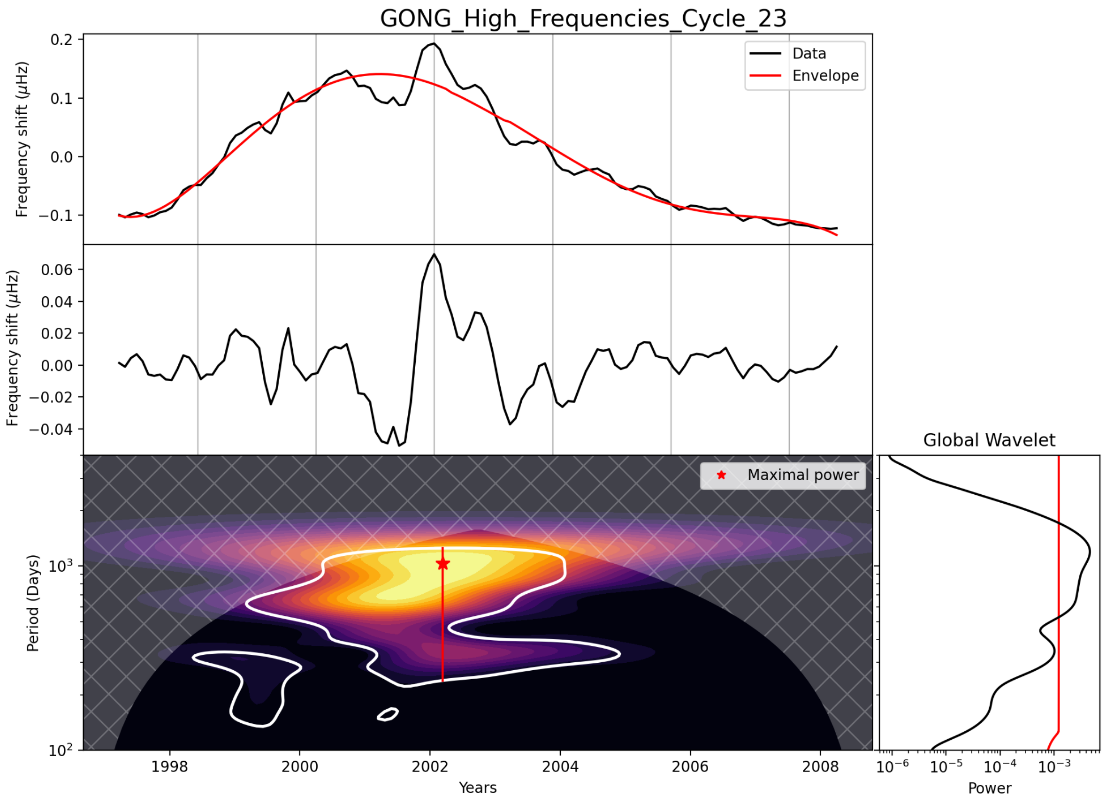

**MDIHMI CYCLES 23 + 24**

**Preliminary findings:**

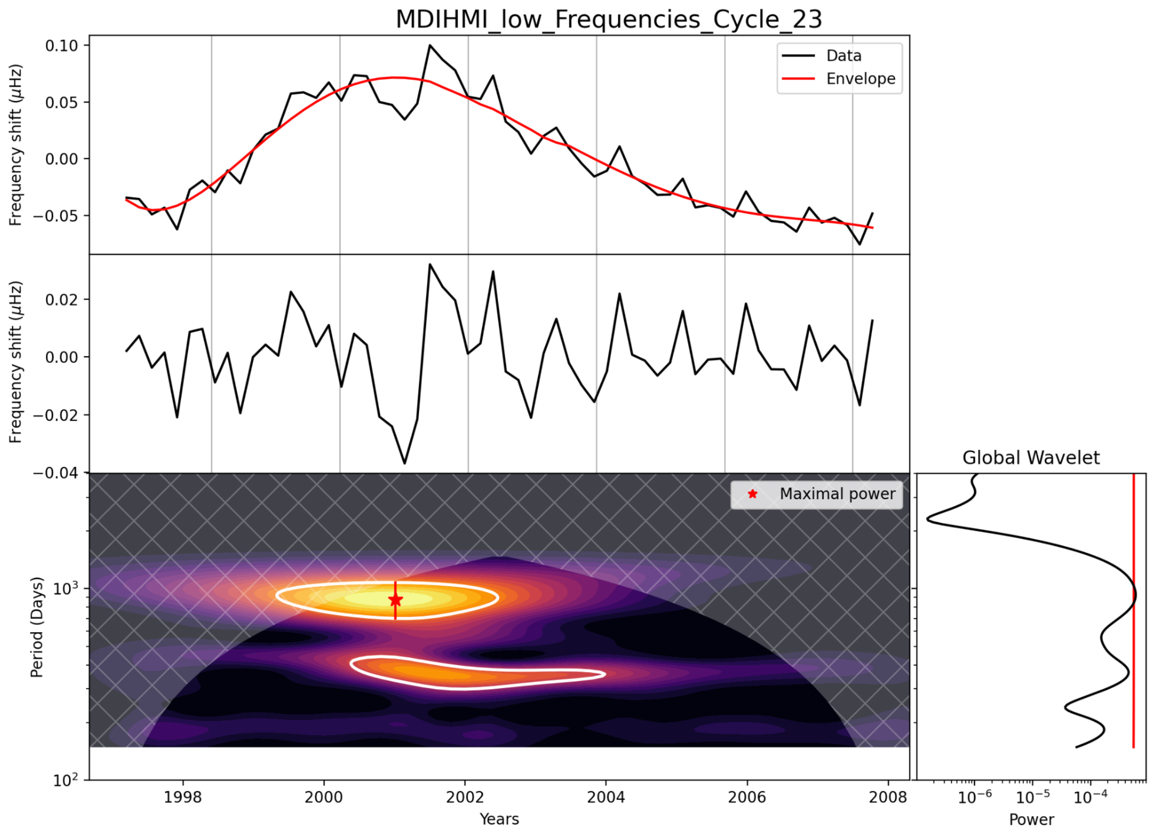
Now the data has been processed, we are able to assess the data using the method described in Section 1. If you would like to look at the raw data, you can download it from the attached **Section_2_data** zip file. Note that the data is labelled GONG_all_23.csv for GONG data, across all frequencies, in Cycle 23. The format follows for all datasets.

The analysis was carried out using `GONG_MDIHMI_wavelet_analysis.py` (full descriptions of similar codes are in Sections 1 and 3)

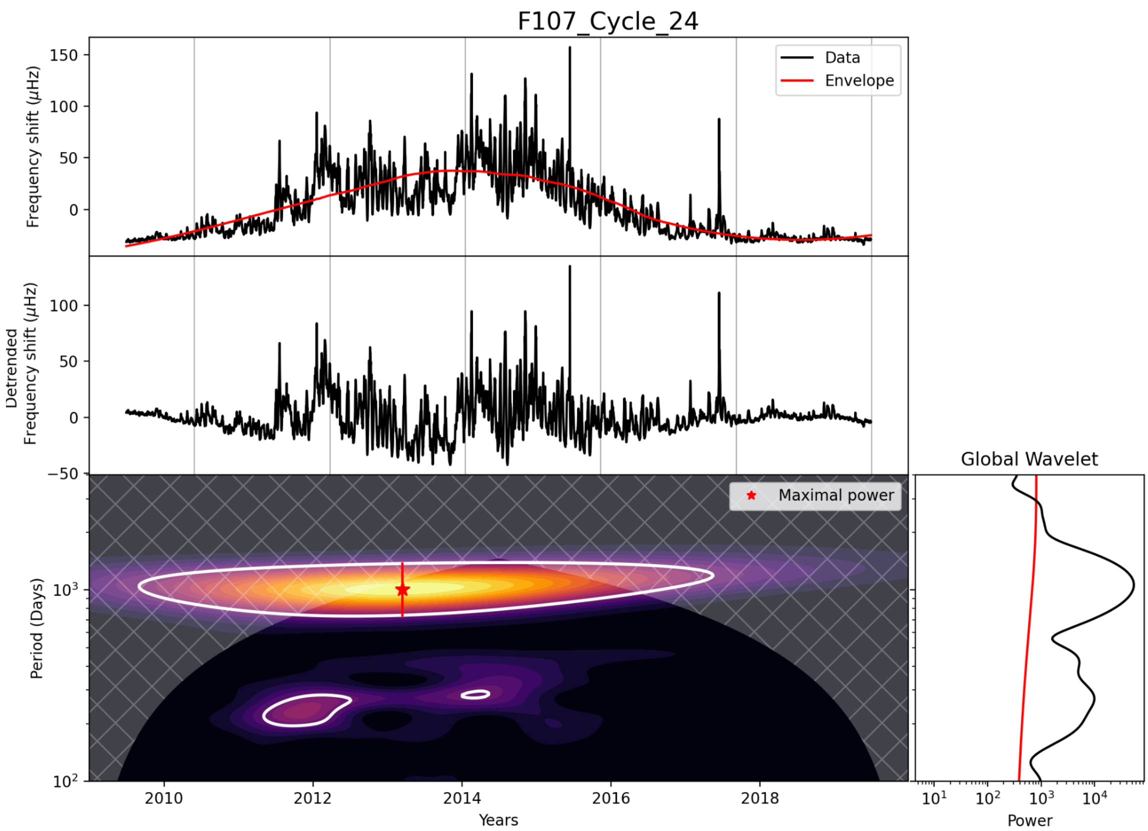
Here are some example results for GONG (High Frequencies, Cycle 23). Note that we can see significant periodicities still within the 98% confidence contour from periods as low as 200 days- this is likely due to the annual oscillation.



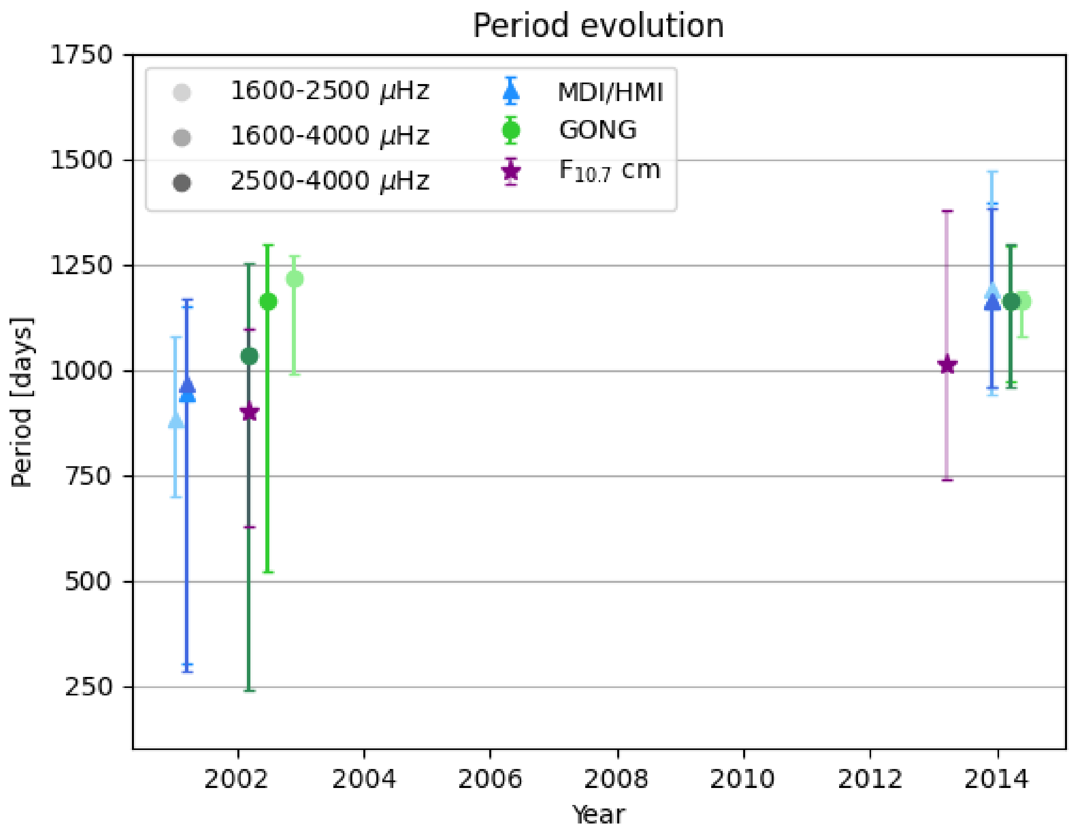

MDI/HMI (Low frequencies, Cycle 23). Again we see a significant secondary structure at around 200-400 days, most likely attributed to the annual oscillation.




And the F10.7 cm data for Cycle 24. We see a small signature of the annual oscillation again in two contours around 2012 and 2014.



We can visualise all the results from Cycles 23 and 24 in the three frequency bands for the helioseismic data, and the solar activity proxy, in the below figure.
Here we have shown the different frequency bands by shade; with the lightest shade showing the low frequency, mid shade showing the entire frequency range, and the darkest shade showing the high frequency band. We separate the instruments by color, showing GONG data in shades of green circles and MDI/HMI data in shades of blue triangles. Finally the F10.7 data is shown as purple stars.




The above plot shows that the average period of the QBO increases from Cycle 23 to Cycle 24. This is most significantly seen in the F10.7 cm and MDIHMI results.

Below is a table of results; where Time_max is the time that the max power period occurs. Period is the max power period, and Low err/ High err are the bottom/top of the 98% confidence error bar respectively. I.e. For the GONG, All, 23 result, we could write the period as 1163 days + 133 days, - 643 days.

| Instrum | Freq | Cycle | Time_max | Period | Low err | High Err |
|---|---|---|---|---|---|---|
| GONG | all | 23 | 28/06/2002 | 1163 | 520 | 1296 |
| GONG | low | 23 | 19/11/2002 | 1218 | 989 | 1273 |
| GONG | high | 23 | 12/03/2002 | 1036 | 239 | 1253 |
| GONG | all | 24 | 21/03/2014 | 1163 | 972 | 1293 |
| GONG | low | 24 | 01/06/2014 | 1163 | 1079 | 1186 |
| GONG | high | 24 | 21/03/2014 | 1163 | 958 | 1298 |
| MDIHMI | all | 23 | 17/03/2001 | 945 | 301 | 1148 |
| MDIHMI | low | 23 | 04/01/2001 | 881 | 699 | 1077 |
| MDIHMI | high | 23 | 17/03/2001 | 967 | 285 | 1170 |
| MDIHMI | all | 24 | 03/12/2013 | 1163 | 958 | 1396 |
| MDIHMI | low | 24 | 03/12/2013 | 1190 | 942 | 1469 |
| MDIHMI | high | 24 | 03/12/2013 | 1163 | 957 | 1380 |
| F107 | all | 23 | 08/03/2002 | 900 | 626 | 1097 |
| F107 | all | 24 | 15/03/2013 | 1010 | 738 | 1376 |

This means that, averaging over frequencies:

From Cycle 23 to 24 the avg. periodicity from GONG data went from:
$1139^{+135}_{-556}$ days to $1163^{+96}_{-160}$ days. An increase of 24 days. (Not stat sig as dt=36)

From Cycle 23 to 24 the avg. periodicity from MDI/HMI data went from:
$931^{+201}_{-502}$ days to $1172^{+243}_{-219}$ days. **An increase of 241 days**.

From Cycle 23 to 24 the avg. periodicity from F10.7cm data went from:
$900^{+197}_{-273}$ days to $1010^{+366}_{-272}$ days. **An increase of 110 days**.

**Conclusion**

We see strong evidence of an increase in periodicity in the QBO over solar cycles 23 to 24 using the CWT, which supports the findings of Mehta 2022.

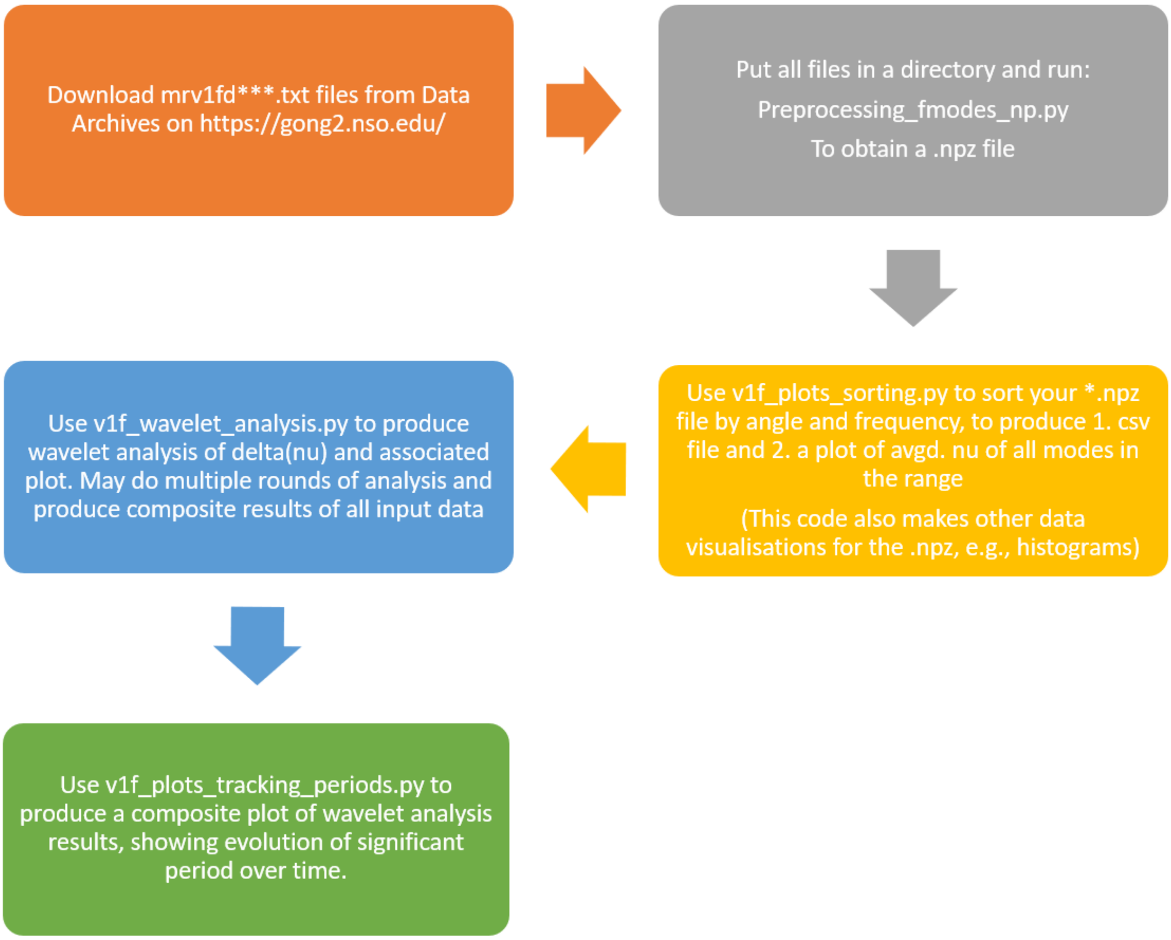
# Section 3: Examining the latitudinal dependance of the QBO using CWT

# Aim:

The motivation for this study was to investigate the latitudinal dependence of the QBO and examine whether this dependance had any cycle variance. To perform this study, we turn to the v1f GONG data files and sort the modes by frequency and angle. The process of obtaining the mode angle is discussed in Step 2 of the methodology. By running these modes through the same process discussed in Sections 1 and 2, we are able to track the periodicity of the significant oscillations in the data (the QBO) and see how it changes with latitude.

# Data and Methodology:

The below section outlines how a reader may replicate the results discussed here. **If you have already downloaded the *.npz file, skip to step 3.** All relevant data/ codes are uploaded to either Warwick Files or GitHub under TishtryaMehta (excluding the raw v1f data which should be manually downloaded as it's around 18GB for both cycles).



1. **Step one: Obtaining the data**

We investigated the v1f data from Cycle 23, 24 (+ the start of cycle 25). Data had cadence of 36 d

Cycle 23:
From mrv1f960903d***.txt –mrv1f081123d***.txt
Equivalent to 03/09/1996 - 23/11/2008 (inclusive)
With d*** indicating the harmonic degree from 001–150 (although we later restrict this from l=20 –150).
Data is structured as mrv1fYYMMDDd***.txt
For data ranges:
l=20 -150,
m =-l to l
n = 1–35 (all values are inclusive)

Cycle 24:
From mrv1f081229d***.txt –mrv230202d***.txt
As above further details.

Combined cycles:
From mrv1f960903d***.txt —mrv230202d***.txt

Data can be downloaded manually as the "Mode Frequency Tables" from
https://gong2.nso.edu/archive/patch.pl?menutype=globalHelio
Common issues:

- Can't download data across the millennium (2000/01/01). Must manually do in batches of 20th and 21st century
- Can't seem to download for specific harmonic degrees so must download all values of ell.

Once data is downloaded, separate data into directories corresponding to Cycle 23/ Cycle 24/ Cycle 2324 [combined]. You should have about 17k files for Cycle 23, 26k for Cycle 24

2. **Step Two: Obtain all frequency/angle data for all multiplets**

As we have so much data it's easier to obtain one massive file once then read/ manipulate that file, rather than rerunning it each time. So we use the python file `preprocessing_fmodes.py` to do this.

The code takes in all possible multiplet combinations from ell=20 - 150, m=-150- 150, and n=1-35 and for each combination looks for the corresponding frequency in all the downloaded mrv1f*.txt files in the given directory. We exclude data with 'bad fits' (columns 19 and 21 in the mrv1f*.txt files). We then take these frequencies in chronological order to construct a time series of frequencies (inclusive of nan values where bad data appeared). We also obtain the average frequency of this time series frequency, calling it avg nu. We also obtain the mode angle for each multiplet using the relation: theta= arccos(mod(m)/sqrt(l*(l+1))), from Simoniello 2016 (`doi:10.3847/0004-637X/828/1/41`). For some of these m, l combinations we get a nan result for the angle, and that multiplet is non physical and therefore excluded.

This code has been parallelised for time and usually takes 1-2hrs. Could be faster but to be honest only needs to be run once per directory, so isn't much of an issue. This outputs a large .npz file containing the frequency time series, the error on frequency time series, avg frequency, mode angle, for each possible multiplet combination.

Common issues:

- The header changes length at `mrv1f080702d***` from 119 rows to 120; this should be manually reflected in the 'reading in stage' of the txt files by changing the index where the skip rows value goes from 119 to 120 (it's currently at -5 for Cycle 23 data as `mrv1f080702d***` is the 5th to last file in the Cycle 3 directory for a given harmonic degree). Change this at will or keep at 120 and risk omitting valuable data.
- The values you run this code over (l,m,n) are hard coded in '`constants.py`'. If you just want to run this over e.g. l=5-10, all m, all n, go to `constants.py` and change L_min and L_max.

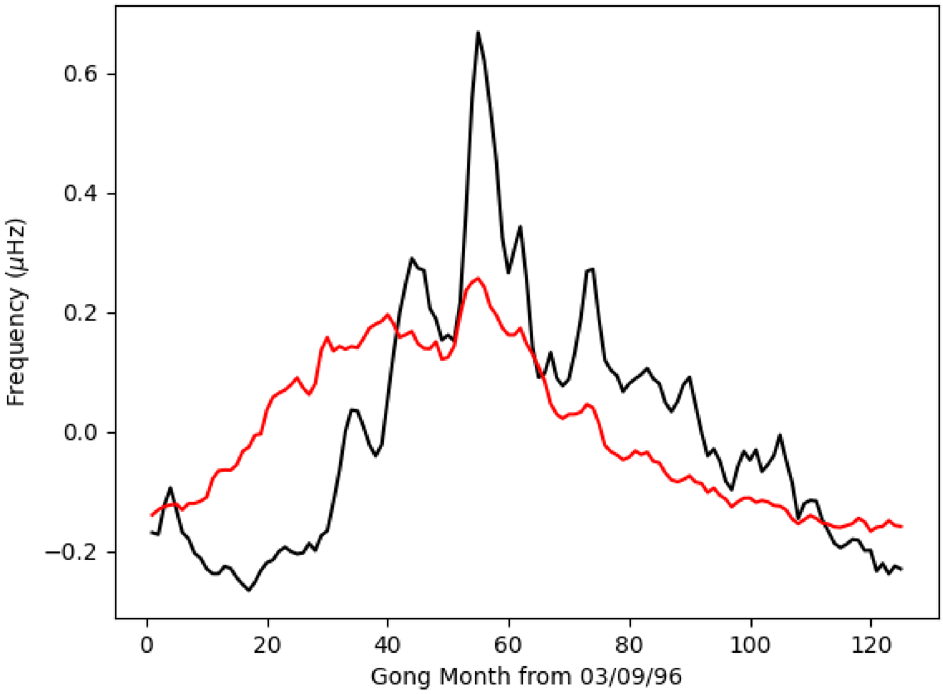
### 3. Step Three: Sorting the data

Once we have the .npz file the fun begins! Now we can sort the data into the frequency/ angle ranges that we care about and perform our analysis on them. To do this, I use the `v1f_plots_sorting.py` code.

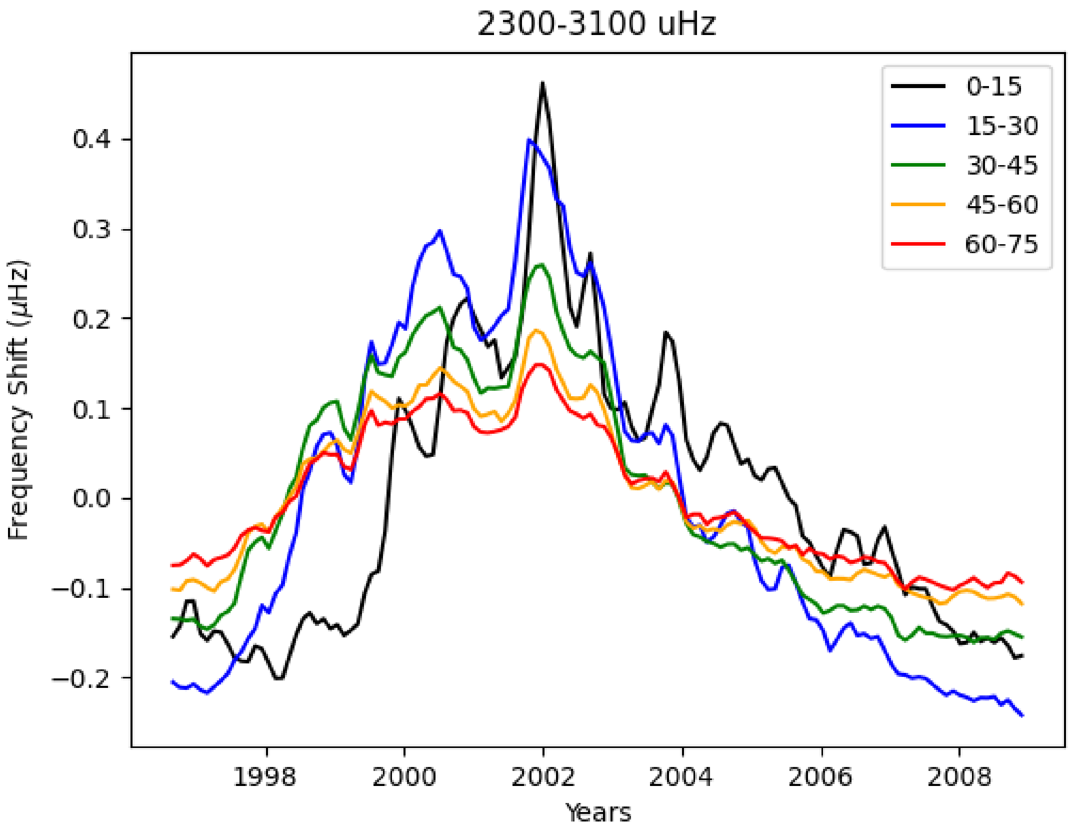
Using the `frequency_sorted_chooser` function, called from `v1f_data_sorting.py`, specify lower (lower frequency), higher (higher frequency), lower theta (lower angle) and upper theta (higher angle), and the code will run through all the multiplets that fit that criteria. Crucially the code will only select modes that have 100% fill, aka no nans within their frequency time array. This decision was the result of multiple preliminary studies which were unable to account for the biassing of the results that arose when nans were included. The code will then average over all of these modes to produce a single frequency time series which it will save as a png and as a csv in your directory of choice.

Here are some example results:

This figure shows an example of two frequency shifts with avg frequencies between 2300-3100uHz from Cycle 23 across two different latitude bands. The black shows low latitude (0-15 degrees) and the red shows high latitude (60-75 degrees).

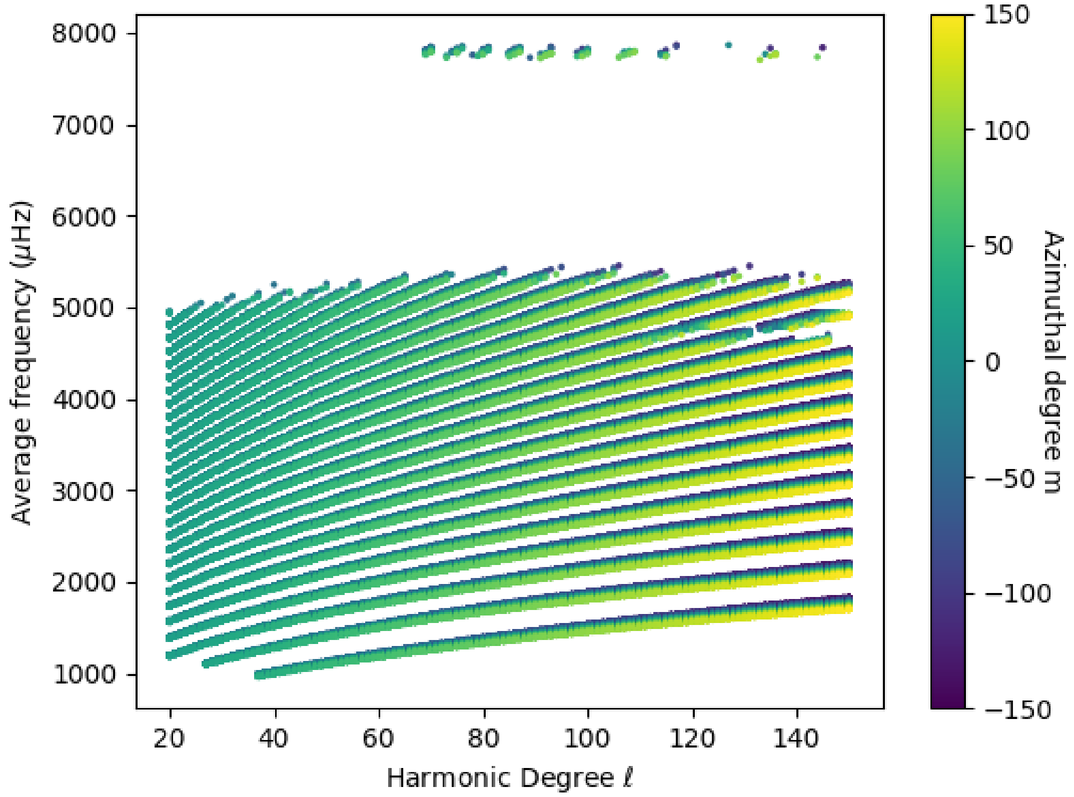


Frequency (μHz)
0.6
0.4
0.2
0.0
−0.2
0
20
40
60
80
100
120
Gong Month from 03/09/96

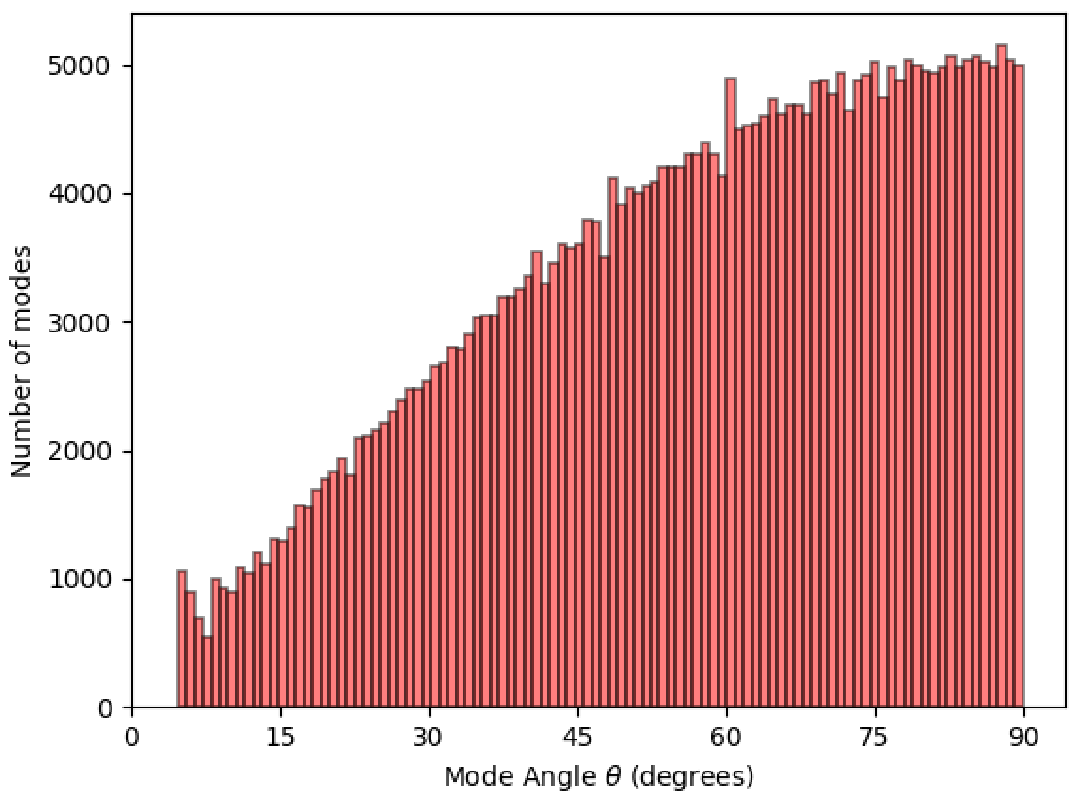



The figure shows the same plot over many more latitude bands.

The next figure shows a typical ‘ell-nu’ diagram for all modes from Cycle 23




Finally the final figure shows a histogram of mode angles from all multiplets from Cycle 23.



In this study, I sorted the data into three frequency bands; low ( 1500-2300 uHz), medium (2300-3100 uHz) and high (3100-3900uHz). I didn’t perform over all frequencies as my machine struggles with the processing of so many modes. I also sorted according to latitude using overlapping bins of width 15 degrees: [0-15, 7-22, 15-30, 22-37]... and so on to 67-82 degrees. As there were in general more modes at the high frequencies and high latitudes, this resulted in some averaged frequency time series that were avgd over mode mode at high freq/latitude, compared with low freq/latitude.

Here I detail the bins and how many modes each bin avgd. over:

****************************************************************************************************

**Data: Cycle 23:**

```
Low freq: 1500-2300
********************************************************************
Between 1500 -- 2300 uHz and  7 -- 22 deg, we averaged over  288
modes, with avg angle: 15.117347661067466
********************************************************************
Between 1500 -- 2300 uHz and  22 -- 37 deg, we averaged over  495
modes, with avg angle: 30.235338829558643
********************************************************************
Between 1500 -- 2300 uHz and  37 -- 52 deg, we averaged over  731
modes, with avg angle: 44.78155614862163
********************************************************************
Between 1500 -- 2300 uHz and  52 -- 67 deg, we averaged over  870
modes, with avg angle: 59.6105329263203
********************************************************************
Between 1500 -- 2300 uHz and  67 -- 82 deg, we averaged over  1061
modes, with avg angle: 74.70071992430529
********************************************************************
Between 1500 -- 2300 uHz and  0 -- 15 deg, we averaged over  138
modes, with avg angle: 10.322911150759651
********************************************************************
Between 1500 -- 2300 uHz and  15 -- 30 deg, we averaged over  387
modes, with avg angle: 23.115031533982037
********************************************************************
Between 1500 -- 2300 uHz and  30 -- 45 deg, we averaged over  638
modes, with avg angle: 37.993037967856985
********************************************************************
Between 1500 -- 2300 uHz and  45 -- 60 deg, we averaged over  818
modes, with avg angle: 52.68713942923868
********************************************************************
```

Between 1500 -- 2300 uHz and  60 -- 75 deg, we averaged over  952 modes, with avg angle: 67.79076084832103

Mid freq: 2300-3100
*******************************************************************
Between 2300 -- 3100 uHz and  7 -- 22 deg, we averaged over  317 modes, with avg angle: 15.076873756978848
*******************************************************************
Between 2300 -- 3100 uHz and  22 -- 37 deg, we averaged over  553 modes, with avg angle: 30.03225599565631
*******************************************************************
Between 2300 -- 3100 uHz and  37 -- 52 deg, we averaged over  744 modes, with avg angle: 44.96667877648566
*******************************************************************
Between 2300 -- 3100 uHz and  52 -- 67 deg, we averaged over  1097 modes, with avg angle: 60.1592848269565
*******************************************************************
Between 2300 -- 3100 uHz and  67 -- 82 deg, we averaged over  1607 modes, with avg angle: 74.93100459751474
*******************************************************************
Between 2300 -- 3100 uHz and  0 -- 15 deg, we averaged over  145 modes, with avg angle: 10.738394470910668
*******************************************************************
Between 2300 -- 3100 uHz and  15 -- 30 deg, we averaged over  438 modes, with avg angle: 23.178510267415195
*******************************************************************
Between 2300 -- 3100 uHz and  30 -- 45 deg, we averaged over  649 modes, with avg angle: 37.846645181925425
*******************************************************************
Between 2300 -- 3100 uHz and  45 -- 60 deg, we averaged over  882 modes, with avg angle: 52.823672897498554
*******************************************************************
Between 2300 -- 3100 uHz and  60 -- 75 deg, we averaged over  1396 modes, with avg angle: 68.02976868096661

High: 3100-3900

*******************************************************************

Between 3100 -- 3900 uHz and  7 -- 22 deg, we averaged over  709 modes, with avg angle: 15.155519811982657
********************************************************************
Between 3100 -- 3900 uHz and  22 -- 37 deg, we averaged over  1335 modes, with avg angle: 30.066845251760387
********************************************************************
Between 3100 -- 3900 uHz and  37 -- 52 deg, we averaged over  1890 modes, with avg angle: 44.811841169622504
********************************************************************
Between 3100 -- 3900 uHz and  52 -- 67 deg, we averaged over  2232 modes, with avg angle: 59.63412817613952
********************************************************************
Between 3100 -- 3900 uHz and  67 -- 82 deg, we averaged over  2600 modes, with avg angle: 74.6232063647283
********************************************************************
Between 3100 -- 3900 uHz and  0 -- 15 deg, we averaged over  348 modes, with avg angle: 10.310888448242757
********************************************************************
Between 3100 -- 3900 uHz and  15 -- 30 deg, we averaged over  1028 modes, with avg angle: 23.320371936718974
********************************************************************
Between 3100 -- 3900 uHz and  30 -- 45 deg, we averaged over  1644 modes, with avg angle: 37.911537274269854
********************************************************************
Between 3100 -- 3900 uHz and  45 -- 60 deg, we averaged over  2109 modes, with avg angle: 52.66464180210526
********************************************************************
Between 3100 -- 3900 uHz and  60 -- 75 deg, we averaged over  2428 modes, with avg angle: 67.772170699921

**Cycle 24**

Low:
********************************************************************
Between 1500 -- 2300 uHz and  7 -- 22 deg, we averaged over  351 modes, with avg angle: 15.564617549075791
********************************************************************
Between 1500 -- 2300 uHz and  22 -- 37 deg, we averaged over  659 modes, with avg angle: 29.90561274503452

```
********************************************************************
Between 1500 -- 2300 uHz and  37 -- 52 deg, we averaged over  882
modes, with avg angle: 44.60119669240029
********************************************************************
Between 1500 -- 2300 uHz and  52 -- 67 deg, we averaged over  1049
modes, with avg angle: 59.883098505577216
********************************************************************
Between 1500 -- 2300 uHz and  67 -- 82 deg, we averaged over  1228
modes, with avg angle: 74.62464397376648
********************************************************************
Between 1500 -- 2300 uHz and  0 -- 15 deg, we averaged over  167
modes, with avg angle: 10.171016120354855
********************************************************************
Between 1500 -- 2300 uHz and  15 -- 30 deg, we averaged over  534
modes, with avg angle: 23.189535021994867
********************************************************************
Between 1500 -- 2300 uHz and  30 -- 45 deg, we averaged over  792
modes, with avg angle: 37.90581393684451
********************************************************************
Between 1500 -- 2300 uHz and  45 -- 60 deg, we averaged over  941
modes, with avg angle: 52.57918937553004
********************************************************************
Between 1500 -- 2300 uHz and  60 -- 75 deg, we averaged over  1177
modes, with avg angle: 67.69415074878681


Mid:
********************************************************************
Between 2300 -- 3100 uHz and  7 -- 22 deg, we averaged over  358
modes, with avg angle: 14.85945258112775
********************************************************************
Between 2300 -- 3100 uHz and  22 -- 37 deg, we averaged over  616
modes, with avg angle: 30.03631748678053
********************************************************************
Between 2300 -- 3100 uHz and  37 -- 52 deg, we averaged over  848
modes, with avg angle: 44.97500009360447
********************************************************************
Between 2300 -- 3100 uHz and  52 -- 67 deg, we averaged over  1232
modes, with avg angle: 60.00566319423251
********************************************************************
```

```
Between 2300 -- 3100 uHz and  67 -- 82 deg, we averaged over  1758
modes, with avg angle: 74.91134588617616
********************************************************************
Between 2300 -- 3100 uHz and  0 -- 15 deg, we averaged over  166
modes, with avg angle: 10.549208307361656
********************************************************************
Between 2300 -- 3100 uHz and  15 -- 30 deg, we averaged over  489
modes, with avg angle: 23.226637697830146
********************************************************************
Between 2300 -- 3100 uHz and  30 -- 45 deg, we averaged over  729
modes, with avg angle: 37.82094496397235
********************************************************************
Between 2300 -- 3100 uHz and  45 -- 60 deg, we averaged over  1031
modes, with avg angle: 52.93928700235007
********************************************************************
Between 2300 -- 3100 uHz and  60 -- 75 deg, we averaged over  1503
modes, with avg angle: 67.92593419588322


High:
********************************************************************
Between 3100 -- 3900 uHz and  7 -- 22 deg, we averaged over  813
modes, with avg angle: 15.158122087141987
********************************************************************
Between 3100 -- 3900 uHz and  22 -- 37 deg, we averaged over  1519
modes, with avg angle: 30.041469479399083
********************************************************************
Between 3100 -- 3900 uHz and  37 -- 52 deg, we averaged over  2117
modes, with avg angle: 44.78452497199429
********************************************************************
Between 3100 -- 3900 uHz and  52 -- 67 deg, we averaged over  2580
modes, with avg angle: 59.70556194062578
********************************************************************
Between 3100 -- 3900 uHz and  67 -- 82 deg, we averaged over  3050
modes, with avg angle: 74.74988376540912
********************************************************************
Between 3100 -- 3900 uHz and  0 -- 15 deg, we averaged over  408
modes, with avg angle: 10.363296680320026
********************************************************************
Between 3100 -- 3900 uHz and  15 -- 30 deg, we averaged over  1182
modes, with avg angle: 23.38686189249401
```

*********************************************************************
Between 3100 -- 3900 uHz and  30 -- 45 deg, we averaged over  1854
modes, with avg angle: 37.92528156481975
*********************************************************************
Between 3100 -- 3900 uHz and  45 -- 60 deg, we averaged over  2366
modes, with avg angle: 52.73156219310228
*********************************************************************
Between 3100 -- 3900 uHz and  60 -- 75 deg, we averaged over  2820
modes, with avg angle: 67.71149572815895

**Both cycles:**

Low:

*********************************************************************
Between 1500 -- 2300 uHz and  7 -- 22 deg, we averaged over  104
modes, with avg angle: 15.078487824280101
*********************************************************************
Between 1500 -- 2300 uHz and  22 -- 37 deg, we averaged over  146
modes, with avg angle: 30.19751785426234
*********************************************************************
Between 1500 -- 2300 uHz and  37 -- 52 deg, we averaged over  221
modes, with avg angle: 44.83265538070395
*********************************************************************
Between 1500 -- 2300 uHz and  52 -- 67 deg, we averaged over  297
modes, with avg angle: 59.925188684852394
*********************************************************************
Between 1500 -- 2300 uHz and  67 -- 82 deg, we averaged over  420
modes, with avg angle: 74.8160396454441
*********************************************************************
Between 1500 -- 2300 uHz and  0 -- 15 deg, we averaged over  49
modes, with avg angle: 10.627055609446849
*********************************************************************
Between 1500 -- 2300 uHz and  15 -- 30 deg, we averaged over  121
modes, with avg angle: 22.64248160422636
*********************************************************************
Between 1500 -- 2300 uHz and  30 -- 45 deg, we averaged over  192
modes, with avg angle: 37.920230971984466
*********************************************************************

```
Between 1500 -- 2300 uHz and  45 -- 60 deg, we averaged over  258
modes, with avg angle: 52.898215586272194
*********************************************************************
Between 1500 -- 2300 uHz and  60 -- 75 deg, we averaged over  361
modes, with avg angle: 68.09793780179376


Mid:
*********************************************************************
Between 2300 -- 3100 uHz and  7 -- 22 deg, we averaged over  144
modes, with avg angle: 15.254223867303267
*********************************************************************
Between 2300 -- 3100 uHz and  22 -- 37 deg, we averaged over  236
modes, with avg angle: 29.97834893154028
*********************************************************************
Between 2300 -- 3100 uHz and  37 -- 52 deg, we averaged over  315
modes, with avg angle: 44.93352626155754
*********************************************************************
Between 2300 -- 3100 uHz and  52 -- 67 deg, we averaged over  541
modes, with avg angle: 60.31998898824521
*********************************************************************
Between 2300 -- 3100 uHz and  67 -- 82 deg, we averaged over  946
modes, with avg angle: 75.21940433975945
*********************************************************************
Between 2300 -- 3100 uHz and  0 -- 15 deg, we averaged over  62
modes, with avg angle: 10.857471093033563
*********************************************************************
Between 2300 -- 3100 uHz and  15 -- 30 deg, we averaged over  197
modes, with avg angle: 23.029271817920108
*********************************************************************
Between 2300 -- 3100 uHz and  30 -- 45 deg, we averaged over  280
modes, with avg angle: 37.91678504817219
*********************************************************************
Between 2300 -- 3100 uHz and  45 -- 60 deg, we averaged over  394
modes, with avg angle: 53.162728914732874
*********************************************************************
Between 2300 -- 3100 uHz and  60 -- 75 deg, we averaged over  740
modes, with avg angle: 68.20597357951698


High:
```

```
*******************************************************************
Between 3100 -- 3900 uHz and  7 -- 22 deg, we averaged over  558
modes, with avg angle: 15.090546469397312
*******************************************************************
Between 3100 -- 3900 uHz and  22 -- 37 deg, we averaged over  1057
modes, with avg angle: 30.101326403181417
*******************************************************************
Between 3100 -- 3900 uHz and  37 -- 52 deg, we averaged over  1485
modes, with avg angle: 44.708794661536544
*******************************************************************
Between 3100 -- 3900 uHz and  52 -- 67 deg, we averaged over  1700
modes, with avg angle: 59.61958325229774
*******************************************************************
Between 3100 -- 3900 uHz and  67 -- 82 deg, we averaged over  2015
modes, with avg angle: 74.67772290067457
*******************************************************************
Between 3100 -- 3900 uHz and  0 -- 15 deg, we averaged over  275
modes, with avg angle: 10.275446840423253
*******************************************************************
Between 3100 -- 3900 uHz and  15 -- 30 deg, we averaged over  810
modes, with avg angle: 23.37266693448823
*******************************************************************
Between 3100 -- 3900 uHz and  30 -- 45 deg, we averaged over  1316
modes, with avg angle: 37.95686487768894
*******************************************************************
Between 3100 -- 3900 uHz and  45 -- 60 deg, we averaged over  1614
modes, with avg angle: 52.67163700341243
*******************************************************************
Between 3100 -- 3900 uHz and  60 -- 75 deg, we averaged over  1846
modes, with avg angle: 67.78035631866557
```

4. **Step Four: Assessing the averaged frequency shifts with wavelet analysis**

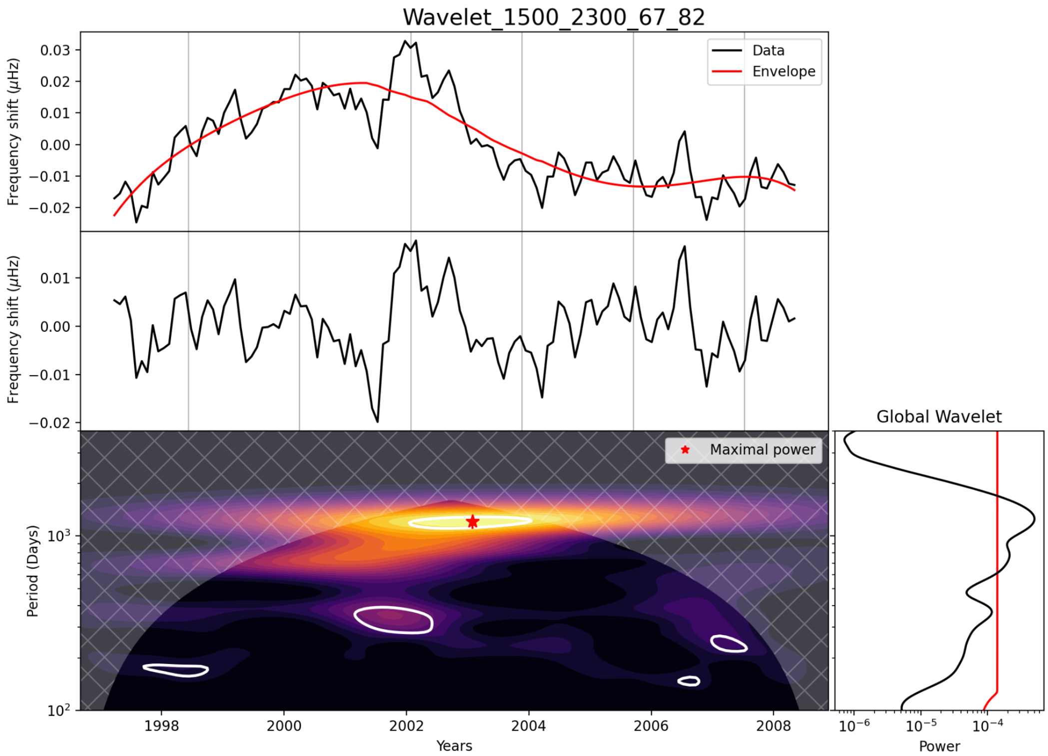
Now you can assess each averaged mode using wavelet analysis. Using this code you can actually run in any csv data to assess, so the possibilities are endless. See the user manual earlier in this document, titled '**PERFORMING WAVELET ANALYSIS ON USER INPUTTED DATA, USING wavelet_analysis.py**' to see how you can use this code for your own data.

Anyway, `v1f_wavelet_analysis.py` does the following:

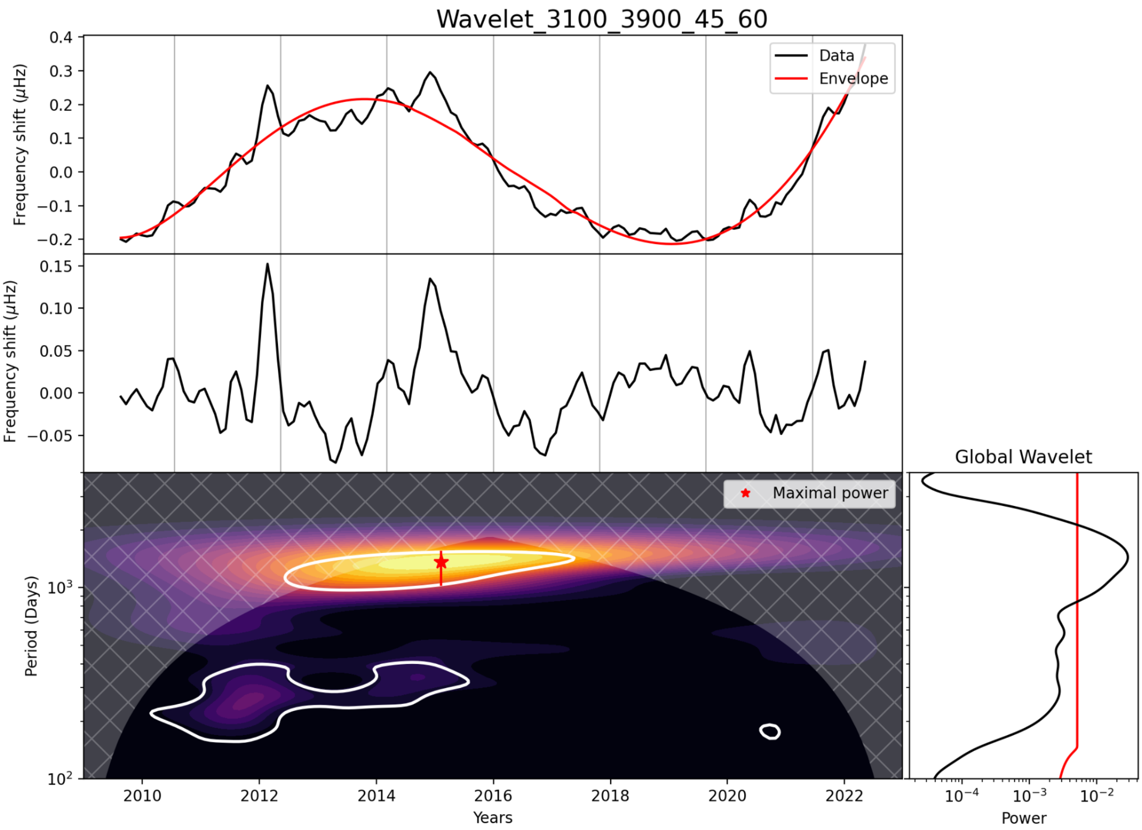
a. Takes the frequency shift of the data by subtracting the average frequency of the data. Typical frequency shifts are between +/- 0.2 uHz
b. Normalises the amplitude of the frequency shift by dividing through by a savgol window (https://docs.scipy.org/doc/scipy/reference/generated/scipy.signal.savgol_filter.html) Input savgol parameters are prescribed as default, but if you're using differently shaped data you must change the window length and poly order values so the envelope is a 'good fit' such that the normalised signal is approximately flat. If you don't want to use the savgol envelope, just set it as [1,1,1..].
c. Performs a CWT on the normalised amplitude frequency shift
d. Performs a GWT on the normalised amplitude frequency shift
e. Obtains the period corresponding to the period with max power in the CWT*, and the upper and lower errors on this period. The errors are found by tracing up and down in period space (respectfully) from the max power period to the period corresponding to the 98% confidence contour for the time_{max power period}.

*We can select the segment of the data we want to examine for the max power period. FOr example, this is by default set to [0,-1], to examine the entire duration. When examining the combined cycles, we may examine e.g. [0,130] (where the total input data has length e.g. 260) to examine the first half of the data only.

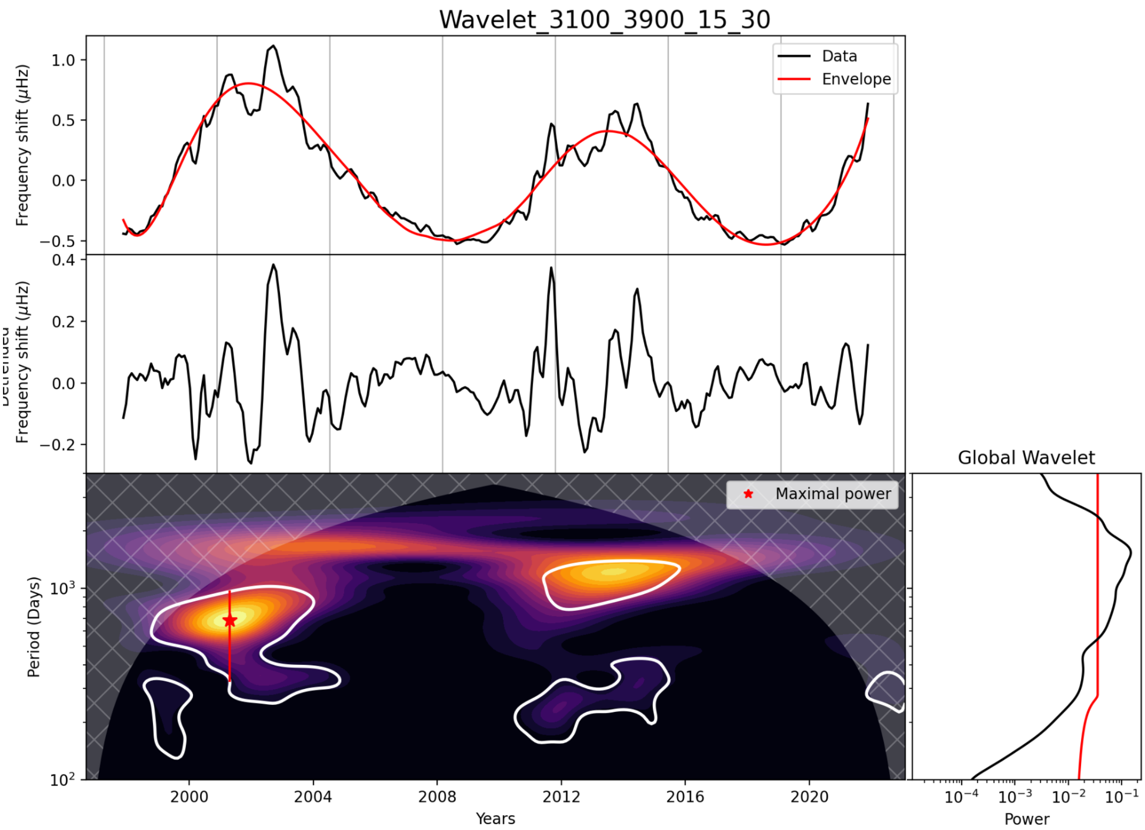
This is from Cycle 23, avgd modes between 1500-2300 uHz, latitude between 67-82 degrees.




This is from Cycle 24, avgd modes between 3100-3900uHz, 45-60 degs




Combined Cycles 23&24, 3100-3900uHz, 15-30 degs.



For each of these plots, we also obtain a .csv file with numerical results, including the time of max power, and the time of solar max (which is the time corresponding to maximal amplitude in the raw frequency shift).

Common problems

- On occasion the max power appears outside the 98% contours. I don't know why. In these cases manually override

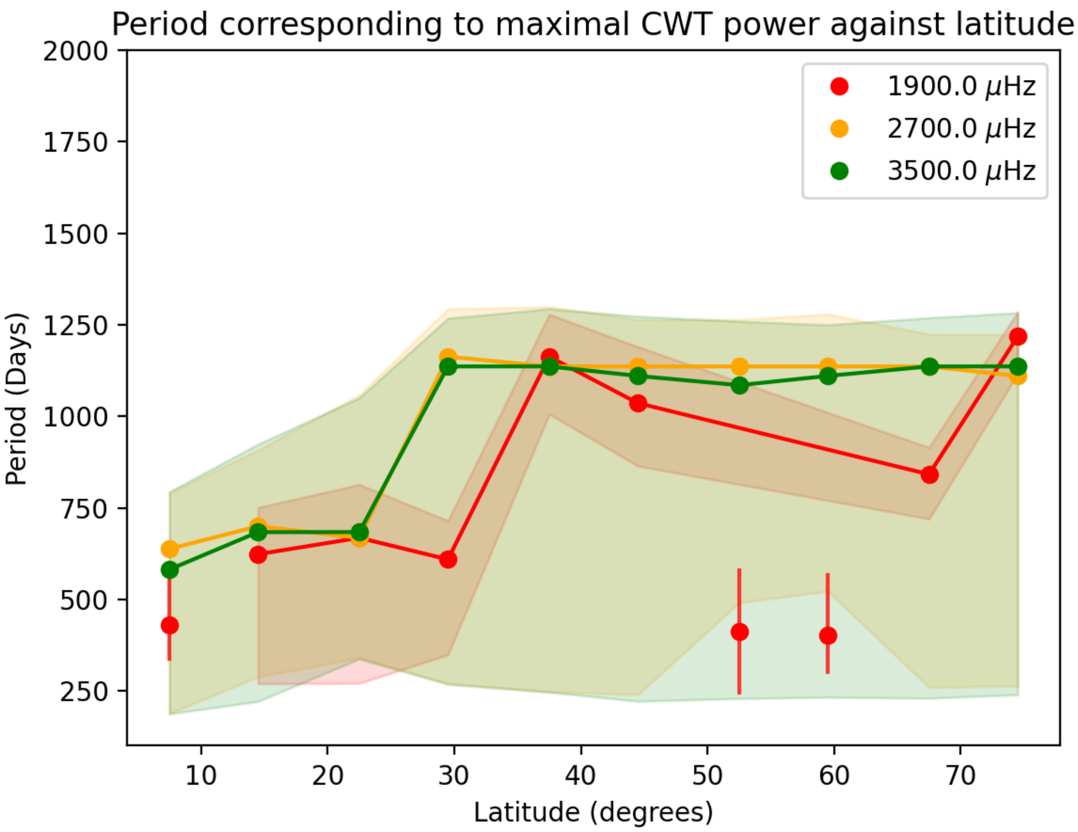
### 5. Step Five: Track the period evolution

Using `v1f_plots_tracking_periods.py`, we can make a plot of the periods corresponding to maximum power in the CWT, plotted against some changing variable; i.e. frequency of latitude bands.

Here is a plot showing the evolution of maximal period in Cycle 23 plotted against latitude bands for each of the three frequency bands:

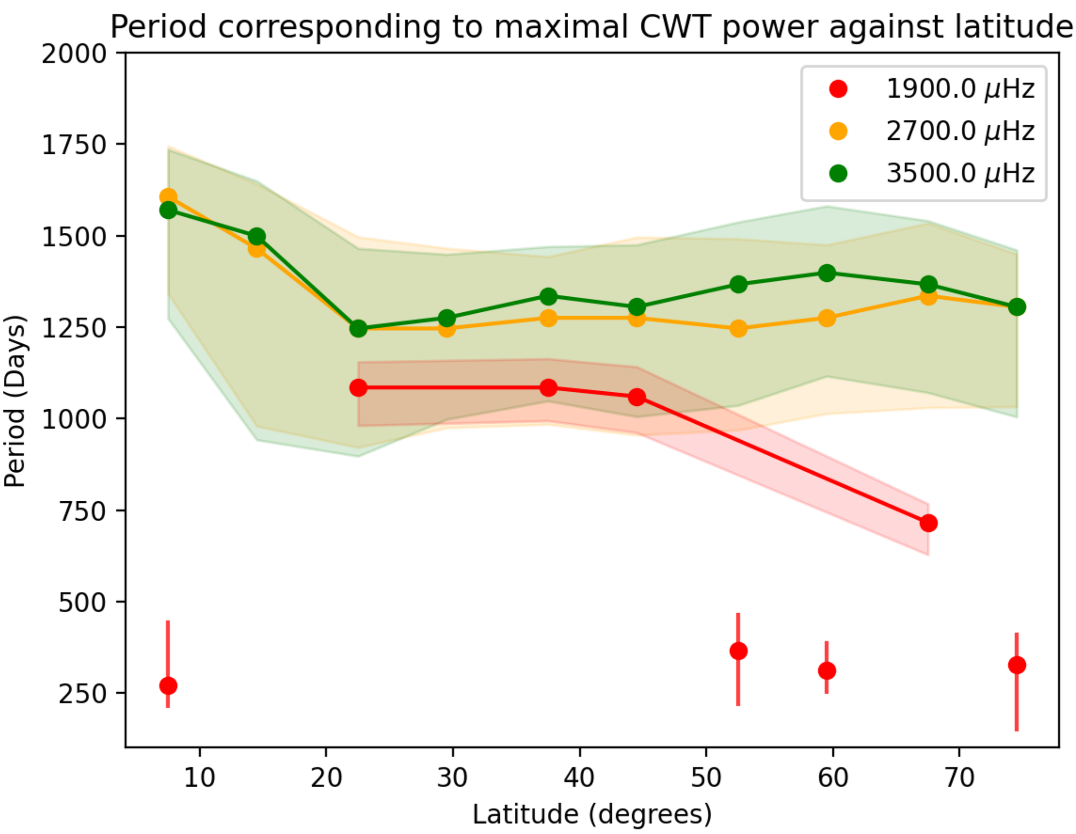
## Conclusions

Here's a plot of the significant periodicities in each mode for Cycle 23, filtered by average latitude and frequency band.



Error bars are the same as given in the CWT plots in step 4. We see that across all frequency bands we see a jump in the max power period from ~20- 30 degrees. At low latitudes (<30), the max power period is around 400-600 (which could be due to mixing of the annual oscillation and the QBO), whereas at high latitudes the maximum period is around 1000-1250 days. The scatter points are the periods that were <548 days (where 1.5 * 365 days = 1.5 years), which we considered to be part of the annual oscillation

Below is a similar plot, for Cycle 24.




We see that for Cycle 24 we obtain consistently longer periods from the CWT across all latitudes, with a slight decrease in periodicity for higher latitudes. The increase in error bars for low frequency bands is a methodology failing and not of ‘real’ origin. This finding is in agreement from the findings of Mehta 2022..

<u>Common issues:</u>

- When reading in the csv file produced by Step Four, the csv file header has square brackets surrounding the header, and excess spaces. These must be manually removed
- The header also uses single quotation marks (‘). These should be replaced by speech marks (“).

Table of results from latitude study:
Here Max Power Period is the period obtained at the time where the maximum power occurs and the lower and upper errors are the periods which correspond to the 98% confidence contour. So the result in the first row corresponds to a period of 430 days [+136 days], [-93 days]. The solar max and solar min are the dates that corresponded with the min and max amplitude in the raw input data respectively.

Cycle 23:

| Average Frequency [microHz] | Avg Latitude [degrees] | Max Power Period [d] | Lower Error [d] | Upper Error [d] | Solarmax | Solarmin |
|---|---|---|---|---|---|---|
| 1900 | 7.5 | 430.6 | 337.4 | 566.0 | 04/02/2002 | 25/02/1998 |
| 1900 | 14.5 | 623.2 | 269.3 | 750.8 | 30/12/2001 | 24/10/2007 |
| 1900 | 22.5 | 667.9 | 270.5 | 813.8 | 24/11/2001 | 13/08/2007 |
| 1900 | 29.5 | 608.9 | 348.5 | 714.6 | 19/10/2001 | 02/03/1997 |
| 1900 | 37.5 | 1162.9 | 1005.8 | 1278.4 | 19/10/2001 | 29/11/2007 |
| 1900 | 44.5 | 1036.0 | 864.2 | 1190.2 | 30/12/2001 | 20/12/1996 |
| 1900 | 52.5 | 411.1 | 244.8 | 579.7 | 30/12/2001 | 02/05/2006 |
| 1900 | 59.5 | 401.8 | 302.5 | 566.3 | 19/10/2001 | 24/10/2007 |
| 1900 | 67.5 | 841.5 | 719.7 | 915.0 | 24/11/2001 | 29/11/2007 |
| 1900 | 74.5 | 1217.9 | 1115.8 | 1284.8 | 24/11/2001 | 25/01/1997 |
| 2700 | 7.5 | 637.7 | 188.3 | 790.9 | 30/12/2001 | 25/02/1998 |
| 2700 | 14.5 | 699.5 | 288.0 | 907.7 | 30/12/2001 | 18/10/2008 |
| 2700 | 22.5 | 667.9 | 340.2 | 1058.4 | 19/10/2001 | 18/10/2008 |
| 2700 | 29.5 | 1162.9 | 270.7 | 1293.8 | 24/11/2001 | 16/03/2008 |
| 2700 | 37.5 | 1136.3 | 247.4 | 1299.0 | 30/12/2001 | 16/03/2008 |
| 2700 | 44.5 | 1136.3 | 239.2 | 1263.9 | 24/11/2001 | 18/09/2007 |
| 2700 | 52.5 | 1136.3 | 491.0 | 1264.2 | 24/11/2001 | 18/09/2007 |
| 2700 | 59.5 | 1136.3 | 522.1 | 1279.5 | 24/11/2001 | 27/05/2008 |
| 2700 | 67.5 | 1136.3 | 259.4 | 1224.2 | 30/12/2001 | 29/11/2007 |
| 2700 | 74.5 | 1110.4 | 263.4 | 1223.3 | 30/12/2001 | 29/11/2007 |
| 3500 | 7.5 | 581.4 | 186.8 | 794.0 | 30/12/2001 | 02/04/1998 |
| 3500 | 14.5 | 683.5 | 221.0 | 924.0 | 30/12/2001 | 07/08/2008 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 3500 | 22.5 | 683.5 | 337.1 | 1050.7 | 30/12/2001 | 02/07/2008 |
| 3500 | 29.5 | 1136.3 | 267.6 | 1268.0 | 19/10/2001 | 02/07/2008 |
| 3500 | 37.5 | 1136.3 | 246.1 | 1293.8 | 30/12/2001 | 12/09/2008 |
| 3500 | 44.5 | 1110.4 | 221.5 | 1273.3 | 30/12/2001 | 12/09/2008 |
| 3500 | 52.5 | 1085.0 | 228.8 | 1259.7 | 30/12/2001 | 12/09/2008 |
| 3500 | 59.5 | 1110.4 | 232.3 | 1250.2 | 30/12/2001 | 12/09/2008 |
| 3500 | 67.5 | 1136.3 | 229.7 | 1269.5 | 30/12/2001 | 27/05/2008 |
| 3500 | 74.5 | 1136.3 | 239.3 | 1283.1 | 24/11/2001 | 27/05/2008 |

Cycle 24:

| Average Frequency [microHz] | Avg Latitude [degrees] | Max Power Period [d] | Lower Error [d] | Upper Error [d] | Solarmax | Solarmin |
|---|---|---|---|---|---|---|
| 1900 | 7.5 | 271.3 | 213.8 | 443.6 | 17/09/2014 | 18/11/2009 |
| 1900 | 22.5 | 1085.0 | 981.1 | 1155.2 | 17/09/2014 | 29/01/2010 |
| 1900 | 37.5 | 1085.0 | 994.2 | 1163.8 | 12/08/2014 | 11/05/2018 |
| 1900 | 44.5 | 1060.2 | 961.8 | 1141.9 | 17/09/2014 | 18/11/2009 |
| 1900 | 52.5 | 366.3 | 218.0 | 464.6 | 17/09/2014 | 24/12/2009 |
| 1900 | 59.5 | 311.6 | 251.4 | 386.4 | 17/09/2014 | 29/12/2008 |
| 1900 | 67.5 | 715.8 | 627.9 | 767.0 | 03/10/2011 | 22/06/2010 |
| 1900 | 74.5 | 326.3 | 150.2 | 409.2 | 03/10/2011 | 29/01/2010 |
| 2700 | 7.5 | 1607.0 | 1342.0 | 1745.2 | 28/11/2014 | 31/05/2021 |
| 2700 | 14.5 | 1465.1 | 979.7 | 1641.7 | 17/09/2014 | 02/08/2009 |
| 2700 | 22.5 | 1246.3 | 921.4 | 1496.8 | 17/09/2014 | 29/12/2008 |
| 2700 | 29.5 | 1246.3 | 974.9 | 1465.8 | 22/11/2022 | 29/12/2008 |
| 2700 | 37.5 | 1275.5 | 984.4 | 1442.8 | 22/11/2022 | 18/02/2020 |
| 2700 | 44.5 | 1275.5 | 955.0 | 1496.2 | 22/11/2022 | 18/01/2019 |
| 2700 | 52.5 | 1246.3 | 968.6 | 1491.7 | 22/11/2022 | 18/01/2019 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 2700 | 59.5 | 1275.5 | 1013.7 | 1475.1 | 17/09/2014 | 29/12/2008 |
| 2700 | 67.5 | 1335.8 | 1029.9 | 1534.4 | 17/09/2014 | 03/02/2009 |
| 2700 | 74.5 | 1305.3 | 1032.5 | 1449.7 | 22/11/2022 | 11/03/2009 |
| 3500 | 7.5 | 1570.3 | 1273.7 | 1735.9 | 28/11/2014 | 31/05/2021 |
| 3500 | 14.5 | 1499.4 | 942.1 | 1649.9 | 17/09/2014 | 18/02/2020 |
| 3500 | 22.5 | 1246.3 | 897.3 | 1465.5 | 23/10/2014 | 13/01/2020 |
| 3500 | 29.5 | 1275.5 | 998.2 | 1449.2 | 22/11/2022 | 13/12/2018 |
| 3500 | 37.5 | 1335.8 | 1048.3 | 1470.4 | 22/11/2022 | 03/02/2009 |
| 3500 | 44.5 | 1305.3 | 1005.5 | 1475.2 | 22/11/2022 | 03/02/2009 |
| 3500 | 52.5 | 1367.0 | 1036.0 | 1537.4 | 22/11/2022 | 03/02/2009 |
| 3500 | 59.5 | 1399.0 | 1116.1 | 1581.4 | 22/11/2022 | 29/12/2008 |
| 3500 | 67.5 | 1367.0 | 1071.2 | 1540.9 | 22/11/2022 | 29/12/2008 |
| 3500 | 74.5 | 1305.3 | 1004.7 | 1461.0 | 22/11/2022 | 28/02/2018 |

**<u>Overall discussion:</u>**

Both Sections 2 and 3 suggest an increase in QBO periodicity between Cycles 23 and 24. This is not only seen in the helioseismic data, but also in solar activity proxy F10.7 cm index. We also see a latitudinal influence on the perceived presence of the QBO in Cycle 23, where the QBO is identified in modes with latitudes > 30 degrees, and only the annual oscillation is seen for latitudes <30 degrees. However for Cycle 24 this dependence is not seen. QBO periodicities are higher in Cycle 24 compared to Cycle 23 across all latitude bands.

**<u>Python codes used in this study:</u>**

All of the below can be found on GitHub under:
https://github.com/TishtryaMehta/QBO_evolution

Note to warwick files

`Preprocessing_fmodes.py`
`constants.py`
`Data_reader.py`
`Data_analyser.py`

`V1f_plots_sorting.py`
`V1f_data_sorting.py`
`v1f_plots_tracking_periods.py`
`v1f_wavelet_analysis.py`
`Wavelet_spectrum_maker.py`
`Wavelet_errorbars.py`
`Wavelet_spectrum_plotter.py`
`Wavelet_data_saver.py`
`WaveletFunctions`
`Wavelet_analysis.py`

Other codes are available which perform the collection and sorting of modes in Sections 2 and 3. They are less well documented, but work! If you have any questions feel free to drop me an email (tishtrya.mehta@gmail.com) with a specific query 🙂