The random Variable X represents the amount of memory in a Purchased flash drive ; takes values: 1 2 4 8 16

Stat 408: Homework 1

1.) a.) $E(x) = \sum_{x=1,2,4,8,16} x P_X(x)$

$= (1 \times P(x=1)) + (2 \times P(x=2)) + (4 \times P(x=4)) + (8 \times P(x=8)) + (16 \times P(x=16))$

$= (1 \times 0.05) + (2 \times 0.10) + (4 \times 0.35) + (8 \times 0.40) + (16 \times 0.10)$

$= 0.05 + 0.2 + 1.4 + 3.2 + 1.6$

$= \boxed{6.45}$ ← Expected Value of random Valuable X

b.) $E(x^2) = \sum x^2 P_X(x)$

$= \left[(1^2 \times P(x=1)) + (2^2 \times P(x=2)) + (4^2 \times P(x=4)) + (8^2 \times P(x=8)) + (16^2 \times P(x=16)) \right]$

$= (1^2 \times 0.05) + (2^2 \times 0.10) + (4^2 \times 0.35) + (8^2 \times 0.40) + (16^2 \times 0.10)$

$= 0.05 + 0.4 + 5.6 + 25.6 + 25.6$

$= \boxed{57.25}$ ← Value of $E(x^2)$

$V(x) = E(x^2) - [E(x)]^2$

$= 57.25 - (6.45)^2$

$= \boxed{15.6475}$ ← $v(x)$

c.) $\sigma = \sqrt{V(x)}$

$= \sqrt{15.6475}$

$= \boxed{3.9557}$ ← standard deviation of X

The standard deviation of the random Variable X is obtained by taking square root of the Variance.

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i-\bar{x})^2$$

Stat 408 : Homework 1 (continued)

2.) a.)

$$\mu = \frac{\begin{pmatrix}0.83 + 0.88 + 0.88 + 1.04 + 1.09 + 1.12 + 1.29 + 1.31 + \\ 1.48 + 1.49 + 1.59 + 1.62 + 1.65 + 1.71 + 1.76 + 1.83\end{pmatrix}}{16}$$

$$= \frac{21.57}{16} = \boxed{1.348}$$ ← Point estimate of the mean value

↳ The point estimate of the mean value of coating thickness is 1.348

b.)

$(.83-1.348)^2 = .268$	$(1.48-1.348)^2 = .017$
$(.88-1.348)^2 = .219$	$(1.49-1.348)^2 = .020$
$(.88-1.348)^2 = .219$	$(1.59-1.348)^2 = .059$
$(1.04-1.348)^2 = .095$	$(1.62-1.348)^2 = .074$
$(1.09-1.348)^2 = .067$	$(1.65-1.348)^2 = .091$
$(1.12-1.348)^2 = .052$	$(1.71-1.348)^2 = .131$
$(1.29-1.348)^2 = .003$	$(1.76-1.348)^2 = .170$
$(1.31-1.348)^2 = .001$	$(1.83-1.348)^2 = .232$

↳ summed up = 1.718

$$\sum_{i=1}^{n}(x_i-\bar{x})^2 \approx 1.718$$

↳ dividing the sum by (n-1) to get the Sample Variance (s^2)

$$s^2 = \frac{1.718}{15} = \boxed{.115}$$ ← Point estimate of the Variance

The point estimate of the variance of coating thickness is .115 (I used the sample variance estimator)

3) a.) $\bar{x} (\pm) \frac{\alpha}{2} \times \frac{\sigma}{\sqrt{n}}$ $\quad$ $Z_{\alpha/2} = Z_{0.05/2}$

$= 58.3 (\pm) 1.96 \times \frac{3}{\sqrt{25}} \rightarrow = Z_{0.025}$

$= (57.124, 59.476)$ $\quad$ $= \pm 1.96$

b.) $\bar{x} (\pm) \frac{\alpha}{2} \times \frac{\sigma}{\sqrt{n}}$ $\quad$ $Z_{\alpha/2} = Z_{0.05/2}$

$= 58.3 (\pm) 1.96 \times \frac{3}{\sqrt{100}} \rightarrow = Z_{0.025}$

$= (57.712, 58.888)$ $\quad$ $= \pm 1.96$

c.) $\bar{x} (\pm) \frac{\alpha}{2} \times \frac{\sigma}{\sqrt{n}}$

$= 58.3 (\pm) 2.576 \times \frac{3}{\sqrt{25}}$

$= (56.7544, 59.8456)$

4.) a.) • specifications state that mean strength of Weld should exceed 100 lb/in², therefore alternate Hypothesis should be

↳ $H_a: \mu \geq 100$

It is more preferable to use this than $H_a: \mu \leq 100$ because $\mu \leq 100$ will support the null hypothesis.

• The alternative & null hypothesis are:

$H_0: \mu \leq 100$ vs $H_a: \mu > 100$

The alternative hypothesis was chosen to be $H_a: \mu > 0$ rather than $H_a: \mu < 100$, since the team wanted sufficient evidence to support that the mean exceeds 100 lb/in²

b.) A type I Error would consist of rejecting the null hypothesis H_0 when it is true. Type II is not rejecting H_0 when false.