

Question 3:

$$a.) L(\beta_0, \beta_1) = \prod [P(Y_i=1)^{y_i} \times P(Y_i=0)^{1-y_i}]$$

$\prod$  represents the product over all observations ( $i = 1 \text{ to } 4$ )

$\therefore P(Y_i=1)$  is the probability of  $Y_i$  being 1.

$$\hookrightarrow P(Y_i=1) = \frac{1}{1+e^{-(\beta_0+\beta_1 x_i)}}$$

Likelihood function

$$\hookrightarrow L(\beta_0, \beta_1) = \prod [P(Y_i=1)^{y_i} \times P(Y_i=0)^{1-y_i}]$$

$$\hookrightarrow \log \beta_1 \{ L(\beta_0) = \sum [y_i \times \log(P(Y_i=1)) + (1-y_i) \times \log(1-P(Y_i=1))] \}$$

$\sum$  represents the sum over all observations

a. Likelihood function:

$$L(\beta_0, \beta_1) = \left( \frac{1}{1+e^{-(\beta_0+\beta_1 x_4)}} \right) \times \left( \frac{1}{1+e^{-(\beta_0+\beta_1 x_5)}} \right) \times \\ \left( \frac{1}{1+e^{-(\beta_0+\beta_1 x_2)}} \right) \times \left( \frac{1}{1+e^{-(\beta_0+\beta_1 x_1)}} \right)$$

b. Likelihood functions:

$$\log \beta_1 \{ L(\beta_0) = \log \left( \frac{1}{1+e^{(\beta_0+\beta_1 x_4)}} \right) \times \left( \frac{1}{1+e^{(\beta_0+\beta_1 x_5)}} \right)$$

$$\times \left( 1 - \frac{1}{1+e^{(\beta_0+\beta_1 x_2)}} \right) \times \left( 1 - \frac{1}{1+e^{(\beta_0+\beta_1 x_1)}} \right)$$