AdMU Progvar

Team Notebook

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Contents

1. Code Templates

```
#include <bits/stdc++.h> ---------------------------------//001
typedef long long ll; -----------------------------------//002
typedef unsigned long long ull; -------------------------//003
typedef std::pair<int, int> ii; -------------------------//004
typedef std::pair<int, ii> iii; -------------------------//005
typedef std::vector<int> vi; ----------------------------//006
typedef std::vector<vi> vvi; ----------------------------//007
typedef std::vector<ii> vii; ----------------------------//008
typedef std::vector<iii> viii; --------------------------//009
const int INF = ~(1<<31); -------------------------------//00a
const ll LINF = (1LL << 60); ----------------------------//00b
const int MAXN = 1e5+1; ---------------------------------//00c
const double EPS = 1e-9; --------------------------------//00d
const double pi = acos(-1); -----------------------------//00e
```

2. Data Structures

2.1. Union Find.

```
struct union_find { -------------------------------------//335
- vi p; union_find(int n) : p(n, -1) { } ----------------//336
- int find(int x) { return p[x] < 0 ? x : p[x] = find(p[x]); }
- bool unite(int x, int y) { ----------------------------//338
--- int xp = find(x), yp = find(y); ---------------------//339
--- if (xp == yp)       return false; -------------------//33a
--- if (p[xp] > p[yp])  std::swap(xp,yp); ---------------//33b
```

```
--- p[xp] += p[yp], p[yp] = xp; ---------------------------//33c
--- return true; ------------------------------------------//33d
- } -------------------------------------------------------//33e
- int size(int x) { return -p[find(x)]; } -----------------//33f
}; --------------------------------------------------------//340
```

2.2. Fenwick Tree.

2.2.1. *Fenwick Tree w/ Point Queries.*

```
struct fenwick { ------------------------------------------//0fc
- vi ar; --------------------------------------------------//0fd
- fenwick(vi &_ar) : ar(_ar.size(), 0) { ------------------//0fe
--- for (int i = 0; i < ar.size(); ++i) { -----------------//0ff
----- ar[i] += _ar[i]; ------------------------------------//100
----- int j = i | (i+1); ----------------------------------//101
----- if (j < ar.size()) ----------------------------------//102
------- ar[j] += ar[i]; -----------------------------------//103
--- } -----------------------------------------------------//104
- } -------------------------------------------------------//105
- int sum(int i) { ----------------------------------------//106
--- int res = 0; ------------------------------------------//107
--- for (; i >= 0; i = (i & (i+1)) - 1) -------------------//108
----- res += ar[i]; ---------------------------------------//109
--- return res; -------------------------------------------//10a
- } -------------------------------------------------------//10b
- int sum(int i, int j) {  return sum(j) - sum(i-1);  } --//10c
- void add(int i, int val) { ------------------------------//10d
--- for (; i < ar.size(); i |= i+1) -----------------------//10e
----- ar[i] += val; ---------------------------------------//10f
- } -------------------------------------------------------//110
- int get(int i) { ----------------------------------------//111
--- int res = ar[i]; --------------------------------------//112
--- if (i) { ----------------------------------------------//113
----- int lca = (i & (i+1)) - 1; --------------------------//114
----- for (--i; i != lca; i = (i&(i+1))-1) ----------------//115
------- res -= ar[i]; -------------------------------------//116
--- } -----------------------------------------------------//117
--- return res; -------------------------------------------//118
- } -------------------------------------------------------//119
- void set(int i, int val) {  add(i, -get(i) + val);  } --//11a
- // range update, point query // -------------------------//11b
- void add(int i, int j, int val) { -----------------------//11c
--- add(i, val); ------------------------------------------//11d
--- add(j+1, -val); ---------------------------------------//11e
- } -------------------------------------------------------//11f
- int get1(int i) { return sum(i); } ----------------------//120
- ////////////////////////////// --------------------------//121
}; --------------------------------------------------------//122
```

2.2.2. *Fenwick Tree w/ Max Queries.*

```
struct fenwick { ------------------------------------------//123
- vi ar; --------------------------------------------------//124
- fenwick(vi &_ar) : ar(_ar.size(), 0) { ------------------//125
--- for (int i = 0; i < ar.size(); ++i) { -----------------//126
----- ar[i] = std::max(ar[i], _ar[i]); --------------------//127
----- int j = i | (i+1); ----------------------------------//128
----- if (j < ar.size()) ----------------------------------//129
------- ar[j] = std::max(ar[j], ar[i]); -------------------//12a
--- } -----------------------------------------------------//12b
- } -------------------------------------------------------//12c
- void set(int i, int v) { --------------------------------//12d
--- for (; i < ar.size(); i |= i+1) -----------------------//12e
----- ar[i] = std::max(ar[i], v); -------------------------//12f
- } -------------------------------------------------------//130
- // max[0..i] ---------------------------------------------//131
- int max(int i) { ----------------------------------------//132
--- int res = -INF; ---------------------------------------//133
--- for (; i >= 0; i = (i & (i+1)) - 1) -------------------//134
----- res = std::max(res, ar[i]); -------------------------//135
--- return res; -------------------------------------------//136
- } -------------------------------------------------------//137
}; --------------------------------------------------------//138
```

2.3. Segment Tree.

2.3.1. *Recursive, Point-update Segment Tree.*

```
struct segtree { ------------------------------------------//221
- int i, j, val; ------------------------------------------//222
- segtree *l, *r; -----------------------------------------//223
- segtree(vi &ar, int _i, int _j) : i(_i), j(_j) { -------//224
--- if (i == j) { -----------------------------------------//225
----- val = ar[i]; ----------------------------------------//226
----- l = r = NULL; ---------------------------------------//227
--- } else { ----------------------------------------------//228
----- int k = (i+j) >> 1; ---------------------------------//229
----- l = new segtree(ar, i, k); --------------------------//22a
----- r = new segtree(ar, k+1, j); ------------------------//22b
----- val = l->val + r->val; ------------------------------//22c
--- } -----------------------------------------------------//22d
- } -------------------------------------------------------//22e
- void update(int _i, int _val) { -------------------------//22f
--- if (_i <= i and j <= _i) { ----------------------------//230
----- val += _val; ----------------------------------------//231
--- } else if (_i < i or j < _i) { ------------------------//232
----- // do nothing -----------------------------------------//233
--- } else { ----------------------------------------------//234
----- l->update(_i, _val); --------------------------------//235
----- r->update(_i, _val); --------------------------------//236
----- val = l->val + r->val; ------------------------------//237
--- } -----------------------------------------------------//238
- } -------------------------------------------------------//239
- int query(int _i, int _j) { -----------------------------//23a
--- if (_i <= i and j <= _j) { ----------------------------//23b
----- return val; -----------------------------------------//23c
--- } else if (_j < i or j < _i) { ------------------------//23d
----- return 0; -------------------------------------------//23e
--- } else { ----------------------------------------------//23f
----- return l->query(_i, _j) + r->query(_i, _j); --------//240
--- } -----------------------------------------------------//241
- } -------------------------------------------------------//242
}; --------------------------------------------------------//243
```

2.3.2. *Iterative, Point-update Segment Tree.*

```
struct segtree { ------------------------------------------//1a9
- int n; --------------------------------------------------//1aa
- int *vals; ----------------------------------------------//1ab
- segtree(vi &ar, int n) { --------------------------------//1ac
--- this->n = n; ------------------------------------------//1ad
--- vals = new int[2*n]; ----------------------------------//1ae
--- for (int i = 0; i < n; ++i) --------------------------//1af
----- vals[i+n] = ar[i]; ----------------------------------//1b0
--- for (int i = n-1; i > 0; --i) -------------------------//1b1
----- vals[i] = vals[i<<1] + vals[i<<1|1]; ----------------//1b2
- } -------------------------------------------------------//1b3
- void update(int i, int v) { -----------------------------//1b4
--- for (vals[i += n] += v; i > 1; i >>= 1) ---------------//1b5
----- vals[i>>1] = vals[i] + vals[i^1]; -------------------//1b6
- } -------------------------------------------------------//1b7
- int query(int l, int r) { -------------------------------//1b8
--- int res = 0; ------------------------------------------//1b9
--- for (l += n, r += n+1; l < r; l >>= 1, r >>= 1) { ----//1ba
----- if (l&1)  res += vals[l++]; -------------------------//1bb
----- if (r&1)  res += vals[--r]; -------------------------//1bc
--- } -----------------------------------------------------//1bd
--- return res; -------------------------------------------//1be
- } -------------------------------------------------------//1bf
}; --------------------------------------------------------//1c0
```

2.3.3. *Pointer-based, Range-update Segment Tree.*

```
struct segtree { ------------------------------------------//1f1
- int i, j, val, temp_val = 0; ----------------------------//1f2
- segtree *l, *r; -----------------------------------------//1f3
- segtree(vi &ar, int _i, int _j) : i(_i), j(_j) { -------//1f4
--- if (i == j) { -----------------------------------------//1f5
----- val = ar[i]; ----------------------------------------//1f6
----- l = r = NULL; ---------------------------------------//1f7
--- } else { ----------------------------------------------//1f8
----- int k = (i + j) >> 1; -------------------------------//1f9
----- l = new segtree(ar, i, k); --------------------------//1fa
----- r = new segtree(ar, k+1, j); ------------------------//1fb
----- val = l->val + r->val; ------------------------------//1fc
--- } -----------------------------------------------------//1fd
- } -------------------------------------------------------//1fe
- void visit() { ------------------------------------------//1ff
--- if (temp_val) { ---------------------------------------//200
----- val += (j-i+1) * temp_val; --------------------------//201
----- if (l) { --------------------------------------------//202
------- l->temp_val += temp_val; --------------------------//203
------- r->temp_val += temp_val; --------------------------//204
----- } ---------------------------------------------------//205
----- temp_val = 0; ---------------------------------------//206
--- } -----------------------------------------------------//207
- } -------------------------------------------------------//208
- void increase(int _i, int _j, int _inc) { ---------------//209
--- visit(); ----------------------------------------------//20a
--- if (_i <= i && j <= _j) { -----------------------------//20b
----- temp_val += _inc; -----------------------------------//20c
----- visit(); --------------------------------------------//20d
--- } else if (_j < i or j < _i) { ------------------------//20e
```

```
----- // do nothing
--- } else {
----- l->increase(_i, _j, _inc);
----- r->increase(_i, _j, _inc);
----- val = l->val + r->val;
--- }
- }
- int query(int _i, int _j) {
--- visit();
--- if (_i <= i and j <= _j) {
----- return val;
--- } else if (_j < i || j < _i) {
----- return 0;
--- } else {
----- return l->query(_i, _j) + r->query(_i, _j);
--- }
- }
};
```

2.3.4. *Array-based, Range-update Segment Tree.*

```
struct segtree {
- int n, *vals, *deltas;
- segtree(vi &ar) {
--- n = ar.size();
--- vals = new int[4*n];
--- deltas = new int[4*n];
--- build(ar, 1, 0, n-1);
- }
- void build(vi &ar, int p, int i, int j) {
--- deltas[p] = 0;
--- if (i == j)
----- vals[p] = ar[i];
--- else {
----- int k = (i + j) / 2;
----- build(ar, p<<1, i, k);
----- build(ar, p<<1|1, k+1, j);
----- pull(p);
--- }
- }
- void pull(int p) {
--- vals[p] = vals[p<<1] + vals[p<<1|1];
- }
- void push(int p, int i, int j) {
--- if (deltas[p]) {
----- vals[p] += (j - i + 1) * deltas[p];
----- if (i != j) {
------- deltas[p<<1] += deltas[p];
------- deltas[p<<1|1] += deltas[p];
----- }
----- deltas[p] = 0;
--- }
- }
- void update(int _i, int _j, int v,
------------ int p, int i, int j) {
--- push(p, i, j);
--- if (_i <= i && j <= _j) {
----- deltas[p] += v;
----- push(p, i, j);
--- } else if (_j < i || j < _i) {
----- // do nothing
--- } else {
----- int k = (i + j) / 2;
----- update(_i, _j, v, p<<1, i, k);
----- update(_i, _j, v, p<<1|1, k+1, j);
----- pull(p);
--- }
- }
- int query(int _i, int _j,
---------- int p, int i, int j) {
--- push(p, i, j);
--- if (_i <= i and j <= _j) {
----- return vals[p];
--- } else if (_j < i || j < _i) {
----- return 0;
--- } else {
----- int k = (i + j) / 2;
----- return query(_i, _j, p<<1, i, k) +
----------- query(_i, _j, p<<1|1, k+1, j);
--- }
- }
};
```

2.3.5. *Persistent Segment Tree (Point-update).*

```
struct node {  int l, r, lid, rid, val;  };
struct segtree {
- node *nodes;
- int n, node_cnt = 0;
- segtree(int n, int capacity) {
--- this->n = n;
--- nodes = new node[capacity];
- }
- int build (vi &ar, int l, int r) {
--- if (l > r)  return -1;
--- int id = node_cnt++;
--- nodes[id].l = l;
--- nodes[id].r = r;
--- if (l == r) {
----- nodes[id].lid = -1;
----- nodes[id].rid = -1;
----- nodes[id].val = ar[l];
--- } else {
----- int m = (l + r) / 2;
----- nodes[id].lid = build(ar, l, m);
----- nodes[id].rid = build(ar, m+1, r);
----- nodes[id].val = nodes[nodes[id].lid].val +
--------------------- nodes[nodes[id].rid].val;
--- }
--- return id;
- }
- int update(int id, int idx, int delta) {
--- if (id == -1)
----- return -1;
--- if (idx < nodes[id].l or nodes[id].r < idx)
----- return id;
--- int nid = node_cnt++;
--- nodes[nid].l = nodes[id].l;
--- nodes[nid].r = nodes[id].r;
--- nodes[nid].lid = update(nodes[id].lid, idx, delta);
--- nodes[nid].rid = update(nodes[id].rid, idx, delta);
--- nodes[nid].val = nodes[id].val + delta;
--- return nid;
- }
- int query(int id, int l, int r) {
--- if (r < nodes[id].l or nodes[id].r < l)
----- return 0;
--- if (l <= nodes[id].l and nodes[id].r <= r)
----- return nodes[id].val;
--- return query(nodes[id].lid, l, r) +
--------- query(nodes[id].rid, l, r);
-- }
 };
```

2.3.6. *2D Segment Tree.*

```
struct segtree_2d {
- int n, m, **ar;
- segtree_2d(int n, int m) {
--- this->n = n;    this->m = m;
--- ar = new int[n];
--- for (int i = 0; i < n; ++i) {
----- ar[i] = new int[m];
----- for (int j = 0; j < m; ++j)
------- ar[i][j] = 0;
--- }
- }
- void update(int x, int y, int v) {
--- ar[x + n][y + m] = v;
--- for (int i = x + n; i > 0; i >>= 1) {
----- for (int j = y + m; j > 0; j >>= 1) {
------- ar[i>>1][j] = min(ar[i][j], ar[i^1][j]);
------- ar[i][j>>1] = min(ar[i][j], ar[i][j^1]);
- }}} // just call update one by one to build
- int query(int x1, int x2, int y1, int y2) {
--- int s = INF;
--- if(~x2) for(int a=x1+n, b=x2+n+1; a<b; a>>=1, b>>=1) {
----- if (a & 1) s = min(s, query(a++, -1, y1, y2));
----- if (b & 1) s = min(s, query(--b, -1, y1, y2));
--- } else for (int a=y1+m, b=y2+m+1; a<b; a>>=1, b>>=1) {
----- if (a & 1) s = min(s, ar[x1][a++]);
----- if (b & 1) s = min(s, ar[x1][--b]);
--- } return s;
- }
};
```

2.4. **Treap.**

2.4.1. *Explicit Treap.*

2.4.2. *Implicit Treap.*

```
struct cartree {
- typedef struct _Node {
```

```
--- int node_val, subtree_val, delta, prio, size; --------//2cf
--- _Node *l, *r; ----------------------------------------//2d0
--- _Node(int val) : node_val(val), subtree_val(val), ----//2d1
------- delta(0), prio((rand()<<16)^rand()), size(1), ----//2d2
------- l(NULL), r(NULL) {} ------------------------------//2d3
--- ~_Node() { delete l; delete r; } ---------------------//2d4
- } *Node; -----------------------------------------------//2d5
- int get_subtree_val(Node v) { --------------------------//2d6
--- return v ? v->subtree_val : 0;  } --------------------//2d7
- int get_size(Node v) { return v ? v->size : 0; } -------//2d8
- void apply_delta(Node v, int delta) { ------------------//2d9
--- if (!v) return; --------------------------------------//2da
--- v->delta += delta; -----------------------------------//2db
--- v->node_val += delta; --------------------------------//2dc
--- v->subtree_val += delta * get_size(v); ---------------//2dd
- } ------------------------------------------------------//2de
- void push_delta(Node v) { ------------------------------//2df
--- if (!v) return; --------------------------------------//2e0
--- apply_delta(v->l, v->delta); -------------------------//2e1
--- apply_delta(v->r, v->delta); -------------------------//2e2
--- v->delta = 0; ----------------------------------------//2e3
- } ------------------------------------------------------//2e4
- void update(Node v) { ----------------------------------//2e5
--- if (!v) return; --------------------------------------//2e6
--- v->subtree_val = get_subtree_val(v->l) + v->node_val -//2e7
----------------- + get_subtree_val(v->r); --------------//2e8
--- v->size = get_size(v->l) + 1 + get_size(v->r); -------//2e9
- } ------------------------------------------------------//2ea
- Node merge(Node l, Node r) { ---------------------------//2eb
--- push_delta(l);    push_delta(r); ---------------------//2ec
--- if (!l || !r)   return l ? l : r; --------------------//2ed
--- if (l->size <= r->size) { ----------------------------//2ee
----- l->r = merge(l->r, r); -----------------------------//2ef
----- update(l); -----------------------------------------//2f0
----- return l; ------------------------------------------//2f1
--- } else { ---------------------------------------------//2f2
----- r->l = merge(l, r->l); -----------------------------//2f3
----- update(r); -----------------------------------------//2f4
----- return r; ------------------------------------------//2f5
--- } ----------------------------------------------------//2f6
- } ------------------------------------------------------//2f7
- void split(Node v, int key, Node &l, Node &r) { --------//2f8
--- push_delta(v); ---------------------------------------//2f9
--- l = r = NULL; ----------------------------------------//2fa
--- if (!v)     return; ----------------------------------//2fb
--- if (key <= get_size(v->l)) { -------------------------//2fc
----- split(v->l, key, l, v->l); -------------------------//2fd
----- r = v; ---------------------------------------------//2fe
--- } else { ---------------------------------------------//2ff
----- split(v->r, key - get_size(v->l) - 1, v->r, r); ----//300
----- l = v; ---------------------------------------------//301
--- } ----------------------------------------------------//302
--- update(v); -------------------------------------------//303
- } ------------------------------------------------------//304
- Node root; ---------------------------------------------//305
public: --------------------------------------------------//306
- cartree() : root(NULL) {} ------------------------------//307
- ~cartree() { delete root; } ----------------------------//308
- int get(Node v, int key) { -----------------------------//309
--- push_delta(v); ---------------------------------------//30a
--- if (key < get_size(v->l)) ----------------------------//30b
----- return get(v->l, key); -----------------------------//30c
--- else if (key > get_size(v->l)) -----------------------//30d
----- return get(v->r, key - get_size(v->l) - 1); --------//30e
--- return v->node_val; ----------------------------------//30f
- } ------------------------------------------------------//310
- int get(int key) { return get(root, key); } ------------//311
- void insert(Node item, int key) { ----------------------//312
--- Node l, r; -------------------------------------------//313
--- split(root, key, l, r); ------------------------------//314
--- root = merge(merge(l, item), r); ---------------------//315
- } ------------------------------------------------------//316
- void insert(int key, int val) { ------------------------//317
--- insert(new _Node(val), key); -------------------------//318
- } ------------------------------------------------------//319
- void erase(int key) { ----------------------------------//31a
--- Node l, m, r; ----------------------------------------//31b
--- split(root, key + 1, m, r); --------------------------//31c
--- split(m, key, l, m); ---------------------------------//31d
--- delete m; --------------------------------------------//31e
--- root = merge(l, r); ----------------------------------//31f
- } ------------------------------------------------------//320
- int query(int a, int b) { ------------------------------//321
--- Node l1, r1; -----------------------------------------//322
--- split(root, b+1, l1, r1); ----------------------------//323
--- Node l2, r2; -----------------------------------------//324
--- split(l1, a, l2, r2); --------------------------------//325
--- int res = get_subtree_val(r2); -----------------------//326
--- l1 = merge(l2, r2); ----------------------------------//327
--- root = merge(l1, r1); --------------------------------//328
--- return res; -------------------------------------------//329
- } ------------------------------------------------------//32a
- void update(int a, int b, int delta) { -----------------//32b
--- Node l1, r1; -----------------------------------------//32c
--- split(root, b+1, l1, r1); ----------------------------//32d
--- Node l2, r2; -----------------------------------------//32e
--- split(l1, a, l2, r2); --------------------------------//32f
--- apply_delta(r2, delta); ------------------------------//330
--- l1 = merge(l2, r2); ----------------------------------//331
--- root = merge(l1, r1); --------------------------------//332
- } ------------------------------------------------------//333
- int size() { return get_size(root); }  }; --------------//334
```

2.4.3. *Persistent Treap.*

2.5. Splay Tree.

```
struct node *null; ---------------------------------------//278
struct node { --------------------------------------------//279
- node *left, *right, *parent; ---------------------------//27a
- bool reverse; int size, value; -------------------------//27b
- node*& get(int d) {return d == 0 ? left : right;} ------//27c
- node(int v=0): reverse(0), size(0), value(v) { ---------//27d
- left = right = parent = null ? null : this; ------------//27e
- }}; ----------------------------------------------------//27f
struct SplayTree { ---------------------------------------//280
- node *root; --------------------------------------------//281
- SplayTree(int arr[] = NULL, int n = 0) { ---------------//282
--- if (!null) null = new node(); ------------------------//283
--- root = build(arr, n); --------------------------------//284
- } // build a splay tree based on array values ----------//285
- node* build(int arr[], int n) { ------------------------//286
--- if (n == 0) return null; -----------------------------//287
--- int mid = n >> 1; ------------------------------------//288
--- node *p = new node(arr ? arr[mid] : 0); --------------//289
--- link(p, build(arr, mid), 0); -------------------------//28a
--- link(p, build(arr? arr+mid+1 : NULL, n-mid-1), 1); ---//28b
--- pull(p); return p; -----------------------------------//28c
- } // pull information from children (editable) ---------//28d
- void pull(node *p) { -----------------------------------//28e
--- p->size = p->left->size + p->right->size + 1; --------//28f
- } // push down lazy flags to children (editable) -------//290
- void push(node *p) { -----------------------------------//291
--- if (p != null && p->reverse) { -----------------------//292
----- swap(p->left, p->right); ---------------------------//293
----- p->left->reverse ^= 1; -----------------------------//294
----- p->right->reverse ^= 1; ----------------------------//295
----- p->reverse ^= 1; -----------------------------------//296
--- }} // assign son to be the new child of p ------------//297
- void link(node *p, node *son, int d) { -----------------//298
--- p->get(d) = son; -------------------------------------//299
--- son->parent = p; } -----------------------------------//29a
- int dir(node *p, node *son) { --------------------------//29b
--- return p->left == son ? 0 : 1;} ----------------------//29c
- void rotate(node *x, int d) { --------------------------//29d
--- node *y = x->get(d), *z = x->parent; -----------------//29e
--- link(x, y->get(d ^ 1), d); ---------------------------//29f
--- link(y, x, d ^ 1); -----------------------------------//2a0
--- link(z, y, dir(z, x)); -------------------------------//2a1
--- pull(x); pull(y);} -----------------------------------//2a2
- node* splay(node *p) { // splay node p to root ---------//2a3
--- while (p->parent != null) { --------------------------//2a4
----- node *m = p->parent, *g = m->parent; ---------------//2a5
----- push(g); push(m); push(p); -------------------------//2a6
----- int dm = dir(m, p), dg = dir(g, m); ----------------//2a7
----- if (g == null) rotate(m, dm); ----------------------//2a8
----- else if (dm == dg) rotate(g, dg), rotate(m, dm); ---//2a9
----- else rotate(m, dm), rotate(g, dg); -----------------//2aa
--- } return root = p; } ---------------------------------//2ab
- node* get(int k) { // get the node at index k ----------//2ac
--- node *p = root; --------------------------------------//2ad
--- while (push(p), p->left->size != k) { ----------------//2ae
------- if (k < p->left->size) p = p->left; --------------//2af
------- else k -= p->left->size + 1, p = p->right; -------//2b0
--- } ----------------------------------------------------//2b1
--- return p == null ? null : splay(p); ------------------//2b2
- } // keep the first k nodes, the rest in r -------------//2b3
- void split(node *&r, int k) { --------------------------//2b4
--- if (k == 0) {r = root; root = null; return;} ---------//2b5
--- r = get(k - 1)->right; -------------------------------//2b6
```

```
--- root->right = r->parent = null; ----------------------//2b7
--- pull(root); } ------------------------------------------//2b8
- void merge(node *r) { //merge current tree with r ------//2b9
--- if (root == null) {root = r; return;} ----------------//2ba
--- link(get(root->size - 1), r, 1); --------------------//2bb
--- pull(root); } ------------------------------------------//2bc
- void assign(int k, int val) { // assign arr[k]= val ----//2bd
--- get(k)->value = val; pull(root); } -------------------//2be
- void reverse(int L, int R) {// reverse arr[L...R] ------//2bf
--- node *m, *r; split(r, R + 1); split(m, L); -----------//2c0
--- m->reverse ^= 1; push(m); merge(m); merge(r); --------//2c1
- } // insert a new node before the node at index k ------//2c2
- node* insert(int k, int v) { ---------------------------//2c3
--- node *r; split(r, k); --------------------------------//2c4
--- node *p = new node(v); p->size = 1; ------------------//2c5
--- link(root, p, 1); merge(r); --------------------------//2c6
--- return p; } ------------------------------------------//2c7
- void erase(int k) { // erase node at index k -----------//2c8
--- node *r, *m; -----------------------------------------//2c9
--- split(r, k + 1); split(m, k); ------------------------//2ca
--- merge(r); delete m;} ---------------------------------//2cb
}; -------------------------------------------------------//2cc
```

2.6. Ordered Statistics Tree.

```
#include <ext/pb_ds/assoc_container.hpp> -----------------//146
#include <ext/pb_ds/tree_policy.hpp> ---------------------//147
using namespace __gnu_pbds; ------------------------------//148
template <typename T> ------------------------------------//149
using indexed_set = std::tree<T, null_type, less<T>, -----//14a
splay_tree_tag, tree_order_statistics_node_update>; ------//14b
// indexed_set<int> t; t.insert(...); --------------------//14c
// t.find_by_order(index); // 0-based --------------------//14d
// t.order_of_key(key); ----------------------------------//14e
```

2.7. Sparse Table.

2.7.1. *1D Sparse Table.*

```
int lg[MAXN+1], spt[20][MAXN]; ---------------------------//244
void build(vi &arr, int n) { -----------------------------//245
- for (int i = 2; i <= n; ++i) lg[i] = lg[i>>1] + 1; -----//246
- for (int i = 0; i < n; ++i) spt[0][i] = arr[i]; --------//247
- for (int j = 0; (2 << j) <= n; ++j) --------------------//248
--- for (int i = 0; i + (2 << j) <= n; ++i) --------------//249
----- spt[j+1][i] = std::min(spt[j][i], spt[j][i+(1<<j)]);//24a
} --------------------------------------------------------//24b
int query(int a, int b) { --------------------------------//24c
- int k = lg[b-a+1], ab = b - (1<<k) + 1; ----------------//24d
- return std::min(spt[k][a], spt[k][ab]); ----------------//24e
} --------------------------------------------------------//24f
```

2.7.2. *2D Sparse Table.*

```
const int N = 100, LGN = 20; -----------------------------//250
int lg[N], A[N][N], st[LGN][LGN][N][N]; ------------------//251
void build(int n, int m) { -------------------------------//252
- for(int k=2; k<=std::max(n,m); ++k) lg[k] = lg[k>>1]+1; //253
- for(int i = 0; i < n; ++i) -----------------------------//254
--- for(int j = 0; j < m; ++j) ---------------------------//255
----- st[0][0][i][j] = A[i][j]; --------------------------//256
- for(int bj = 0; (2 << bj) <= m; ++bj) ------------------//257
--- for(int j = 0; j + (2 << bj) <= m; ++j) --------------//258
----- for(int i = 0; i < n; ++i) -------------------------//259
------- st[0][bj+1][i][j] = ------------------------------//25a
--------- std::max(st[0][bj][i][j], ----------------------//25b
------------------ st[0][bj][i][j + (1 << bj)]); ---------//25c
- for(int bi = 0; (2 << bi) <= n; ++bi) ------------------//25d
--- for(int i = 0; i + (2 << bi) <= n; ++i) --------------//25e
----- for(int j = 0; j < m; ++j) -------------------------//25f
------- st[bi+1][0][i][j] = ------------------------------//260
--------- std::max(st[bi][0][i][j], ----------------------//261
------------------ st[bi][0][i + (1 << bi)][j]); ---------//262
- for(int bi = 0; (2 << bi) <= n; ++bi) ------------------//263
--- for(int i = 0; i + (2 << bi) <= n; ++i) --------------//264
----- for(int bj = 0; (2 << bj) <= m; ++bj) --------------//265
------- for(int j = 0; j + (2 << bj) <= m; ++j) { --------//266
--------- int ik = i + (1 << bi); ------------------------//267
--------- int jk = j + (1 << bj); ------------------------//268
--------- st[bi+1][bj+1][i][j] = -------------------------//269
----------- std::max(std::max(st[bi][bj][i][j], ----------//26a
---------------------------- st[bi][bj][ik][j]), ---------//26b
------------------ std::max(st[bi][bj][i][jk], -----------//26c
---------------------------- st[bi][bj][ik][jk])); ------//26d
------- } ------------------------------------------------//26e
} --------------------------------------------------------//26f
int query(int x1, int x2, int y1, int y2) { --------------//270
- int kx = lg[x2 - x1 + 1],   ky = lg[y2 - y1 + 1]; ------//271
- int x12 = x2 - (1<<kx) + 1, y12 = y2 - (1<<ky) + 1; ----//272
- return std::max(std::max(st[kx][ky][x1][y1], -----------//273
------------------------- st[kx][ky][x1][y12]), ---------//274
----------------- std::max(st[kx][ky][x12][y1], ----------//275
------------------------- st[kx][ky][x12][y12])); -------//276
} --------------------------------------------------------//277
```

2.8. Misof Tree.

A simple tree data structure for inserting, erasing, and querying the nth largest element.

```
#define BITS 15 ------------------------------------------//139
struct misof_tree { --------------------------------------//13a
- int cnt[BITS][1<<BITS]; --------------------------------//13b
- misof_tree() { memset(cnt, 0, sizeof(cnt)); } ---------//13c
- void insert(int x) { -----------------------------------//13d
--- for (int i = 0; i < BITS; cnt[i++][x]++, x >>= 1); } -//13e
- void erase(int x) { ------------------------------------//13f
--- for (int i = 0; i < BITS; cnt[i++][x]--, x >>= 1); } -//140
- int nth(int n) { ---------------------------------------//141
--- int res = 0; -----------------------------------------//142
--- for (int i = BITS-1; i >= 0; i--) --------------------//143
----- if (cnt[i][res <<= 1] <= n) n -= cnt[i][res], res |= 1;
--- return res; } }; -------------------------------------//145
```

3. Graphs

Using adjacency list:

```
struct graph { -------------------------------------------//584
- int n, *dist; ------------------------------------------//585
- vii *adj; ----------------------------------------------//586
- graph(int n) { -----------------------------------------//587
--- this->n = n; -----------------------------------------//588
--- adj = new vii[n]; ------------------------------------//589
--- dist = new int[n]; -----------------------------------//58a
- } ------------------------------------------------------//58b
- void add_edge(int u, int v, int w) { -------------------//58c
--- adj[u].push_back({v, w}); ----------------------------//58d
--- // adj[v].push_back({u, w}); -------------------------//58e
- } ------------------------------------------------------//58f
}; -------------------------------------------------------//590
```

Using adjacency matrix:

```
struct graph { -------------------------------------------//591
- int n, **mat; ------------------------------------------//592
- graph(int n) { -----------------------------------------//593
--- this->n = n; -----------------------------------------//594
--- mat = new int*[n]; -----------------------------------//595
--- for (int i = 0; i < n; ++i) { ------------------------//596
----- mat[i] = new int[n]; -------------------------------//597
----- for (int j = 0; j < n; ++j) ------------------------//598
------- mat[i][j] = INF; ---------------------------------//599
----- mat[i][i] = 0; -------------------------------------//59a
--- } ----------------------------------------------------//59b
- } ------------------------------------------------------//59c
- void add_edge(int u, int v, int w) { -------------------//59d
--- mat[u][v] = std::min(mat[u][v], w); ------------------//59e
--- // mat[v][u] = std::min(mat[v][u], w); ---------------//59f
- } ------------------------------------------------------//5a0
}; -------------------------------------------------------//5a1
```

Using edge list:

```
struct graph { -------------------------------------------//5a2
- int n; -------------------------------------------------//5a3
- std::vector<iii> edges; --------------------------------//5a4
- graph(int n) : n(n) {} ---------------------------------//5a5
- void add_edge(int u, int v, int w) { -------------------//5a6
--- edges.push_back({w, {u, v}}); ------------------------//5a7
- } ------------------------------------------------------//5a8
}; -------------------------------------------------------//5a9
```

3.1. Single-Source Shortest Paths.

3.1.1. *Dijkstra.*

```
#include "graph_template_adjlist.cpp" --------------------//7e3
// insert inside graph; needs n, dist[], and adj[] -------//7e4
void dijkstra(int s) { -----------------------------------//7e5
- for (int u = 0; u < n; ++u) ----------------------------//7e6
--- dist[u] = INF; ---------------------------------------//7e7
- dist[s] = 0; -------------------------------------------//7e8
- std::priority_queue<ii, vii, std::greater<ii> > pq; ----//7e9
- pq.push({0, s}); ---------------------------------------//7ea
- while (!pq.empty()) { ----------------------------------//7eb
--- int u = pq.top().second; -----------------------------//7ec
--- int d = pq.top().first; ------------------------------//7ed
--- pq.pop(); --------------------------------------------//7ee
--- if (dist[u] < d) -------------------------------------//7ef
----- continue; ------------------------------------------//7f0
--- dist[u] = d; -----------------------------------------//7f1
--- for (auto &e : adj[u]) { -----------------------------//7f2
```

```
----- int v = e.first; ------------------------------------//7f3
----- int w = e.second; -----------------------------------//7f4
----- if (dist[v] > dist[u] + w) { ------------------------//7f5
------- dist[v] = dist[u] + w; ----------------------------//7f6
------- pq.push({dist[v], v}); ----------------------------//7f7
----- } ---------------------------------------------------//7f8
--- } -----------------------------------------------------//7f9
- } -------------------------------------------------------//7fa
} ---------------------------------------------------------//7fb
```

3.1.2. *Bellman-Ford.*

```
#include "graph_template_adjlist.cpp" --------------------//7cf
// insert inside graph; needs n, dist[], and adj[] -------//7d0
void bellman_ford(int s) { -------------------------------//7d1
- for (int u = 0; u < n; ++u) ----------------------------//7d2
--- dist[u] = INF; ---------------------------------------//7d3
- dist[s] = 0; -------------------------------------------//7d4
- for (int i = 0; i < n-1; ++i) --------------------------//7d5
--- for (int u = 0; u < n; ++u) --------------------------//7d6
----- for (auto &e : adj[u]) -----------------------------//7d7
------- if (dist[u] + e.second < dist[e.first]) ----------//7d8
--------- dist[e.first] = dist[u] + e.second; ------------//7d9
} --------------------------------------------------------//7da
// you can call this after running bellman_ford() --------//7db
bool has_neg_cycle() { -----------------------------------//7dc
- for (int u = 0; u < n; ++u) ----------------------------//7dd
--- for (auto &e : adj[u]) -------------------------------//7de
----- if (dist[e.first] > dist[u] + e.second) ------------//7df
------- return true; -------------------------------------//7e0
- return false; ------------------------------------------//7e1
} --------------------------------------------------------//7e2
```

3.1.3. *SPFA.*

```
struct edge { --------------------------------------------//805
- int v; long long cost; ---------------------------------//806
- edge(int v, long long cost): v(v), cost(cost) {} -------//807
}; -------------------------------------------------------//808
long long dist[N]; int vis[N]; bool inq[N]; --------------//809
void spfa(vector<edge*> adj[], int n, int s) { -----------//80a
- fill(dist, dist + n, LLONG_MAX); -----------------------//80b
- fill(vis, vis + n, 0); ---------------------------------//80c
- fill(inq, inq + n, false); -----------------------------//80d
- queue<int> q; q.push(s); -------------------------------//80e
- for (dist[s] = 0; !q.empty(); q.pop()) { ---------------//80f
--- int u = q.front(); inq[u] = false; -------------------//810
--- if (++vis[u] >= n) dist[u] = LLONG_MIN; --------------//811
--- for (int i = 0; i < adj[u].size(); ++i) { ------------//812
----- edge& e = *adj[u][i]; ------------------------------//813
----- // uncomment below for min cost max flow -----------//814
----- // if (e.cap <= e.flow) continue; ------------------//815
----- int v = e.v; ---------------------------------------//816
----- long long w = vis[u] >= n ? 0LL : e.cost; ----------//817
----- if (dist[u] + w < dist[v]) { -----------------------//818
------- dist[v] = dist[u] + w; ---------------------------//819
------- if (!inq[v]) { -----------------------------------//81a
--------- inq[v] = true; ---------------------------------//81b
--------- q.push(v); -------------------------------------//81c
------- }}}}} --------------------------------------------//81d
```

3.2. All-Pairs Shortest Paths.

3.2.1. *Floyd-Washall.*

```
#include "graph_template_adjmat.cpp" ---------------------//7fc
// insert inside graph; needs n and mat[][] --------------//7fd
void floyd_warshall() { ----------------------------------//7fe
- for (int k = 0; k < n; ++k) ----------------------------//7ff
--- for (int i = 0; i < n; ++i) --------------------------//800
----- for (int j = 0; j < n; ++j) ------------------------//801
------- if (mat[i][k] + mat[k][j] < mat[i][j]) -----------//802
--------- mat[i][j] = mat[i][k] + mat[k][j]; -------------//803
} --------------------------------------------------------//804
```

3.3. Strongly Connected Components.

3.3.1. *Kosaraju.*

```
struct kosaraju_graph { ----------------------------------//790
- int n; -------------------------------------------------//791
- int *vis; ----------------------------------------------//792
- vi **adj; ----------------------------------------------//793
- std::vector<vi> sccs; ----------------------------------//794
- kosaraju_graph(int n) { --------------------------------//795
--- this->n = n; -----------------------------------------//796
--- vis = new int[n]; ------------------------------------//797
--- adj = new vi*[2]; ------------------------------------//798
--- for (int dir = 0; dir < 2; ++dir) --------------------//799
----- adj[dir] = new vi[n]; ------------------------------//79a
- } ------------------------------------------------------//79b
- void add_edge(int u, int v) { --------------------------//79c
--- adj[0][u].push_back(v); ------------------------------//79d
--- adj[1][v].push_back(u); ------------------------------//79e
- } ------------------------------------------------------//79f
- void dfs(int u, int p, int dir, vi &topo) { ------------//7a0
--- vis[u] = 1; ------------------------------------------//7a1
--- for (int v : adj[dir][u]) ----------------------------//7a2
----- if (!vis[v] && v != p) -----------------------------//7a3
------- dfs(v, u, dir, topo); ----------------------------//7a4
--- topo.push_back(u); -----------------------------------//7a5
- } ------------------------------------------------------//7a6
- void kosaraju() { --------------------------------------//7a7
--- vi topo; ---------------------------------------------//7a8
--- for (int u = 0; u < n; ++u)  vis[u] = 0; -------------//7a9
--- for (int u = 0; u < n; ++u) --------------------------//7aa
----- if (!vis[u]) ---------------------------------------//7ab
------- dfs(u, -1, 0, topo); -----------------------------//7ac
--- for (int u = 0; u < n; ++u)  vis[u] = 0; -------------//7ad
--- for (int i = n-1; i >= 0; --i) { ---------------------//7ae
----- if (!vis[topo[i]]) { -------------------------------//7af
------- sccs.push_back({}); ------------------------------//7b0
------- dfs(topo[i], -1, 1, sccs.back()); ----------------//7b1
----- } --------------------------------------------------//7b2
--- } ----------------------------------------------------//7b3
- } ------------------------------------------------------//7b4
}; -------------------------------------------------------//7b5
```

3.3.2. *Tarjan's Offline Algorithm.*

```
int n, id[N], low[N], st[N], in[N], TOP, ID; -------------//7b6
int scc[N], SCC_SIZE; // 0 <= scc[u] < SCC_SIZE ----------//7b7
vector<int> adj[N]; // 0-based adjlist -------------------//7b8
void dfs(int u) { ----------------------------------------//7b9
--- id[u] = low[u] = ID++; -------------------------------//7ba
--- st[TOP++] = u; in[u] = 1; ----------------------------//7bb
--- for (int v : adj[u]) { -------------------------------//7bc
------- if (id[v] == -1) { -------------------------------//7bd
----------- dfs(v); --------------------------------------//7be
----------- low[u] = min(low[u], low[v]); ----------------//7bf
------- } else if (in[v] == 1) ---------------------------//7c0
----------- low[u] = min(low[u], id[v]); -----------------//7c1
--- } ----------------------------------------------------//7c2
--- if (id[u] == low[u]) { -------------------------------//7c3
------- int sid = SCC_SIZE++; ----------------------------//7c4
------- do { ---------------------------------------------//7c5
----------- int v = st[--TOP]; ---------------------------//7c6
----------- in[v] = 0; scc[v] = sid; ---------------------//7c7
------- } while (st[TOP] != u); --------------------------//7c8
--- }} ---------------------------------------------------//7c9
void tarjan() { // call tarjan() to load SCC -------------//7ca
--- memset(id, -1, sizeof(int) * n); ---------------------//7cb
--- SCC_SIZE = ID = TOP = 0; -----------------------------//7cc
--- for (int i = 0; i < n; ++i) --------------------------//7cd
------- if (id[i] == -1) dfs(i); } -----------------------//7ce
```

3.4. Minimum Mean Weight Cycle.

Run this for each strongly connected component

```
double min_mean_cycle(vector<vector<pair<int,double>>> adj){
- int n = size(adj); double mn = INFINITY; ---------------//5f4
- vector<vector<double> > arr(n+1, vector<double>(n, mn));//5f5
- arr[0][0] = 0; -----------------------------------------//5f6
- rep(k,1,n+1) rep(j,0,n) iter(it,adj[j]) ----------------//5f7
--- arr[k][it->first] = min(arr[k][it->first], -----------//5f8
---------------------------- it->second + arr[k-1][j]); ---//5f9
- rep(k,0,n) { -------------------------------------------//5fa
--- double mx = -INFINITY; -------------------------------//5fb
--- rep(i,0,n) mx = max(mx, (arr[n][i]-arr[k][i])/(n-k)); //5fc
--- mn = min(mn, mx); } ----------------------------------//5fd
- return mn; } -------------------------------------------//5fe
```

3.5. Cut Points and Bridges.

```
vii bridges; ---------------------------------------------//516
vi adj[MAXN], disc, low, articulation_points; ------------//517
int TIME; ------------------------------------------------//518
void bridges_artics (int u, int p) { ---------------------//519
- disc[u] = low[u] = TIME++; -----------------------------//51a
- int children = 0; --------------------------------------//51b
- bool has_low_child = false; ----------------------------//51c
- for (int v : adj[u]) { ---------------------------------//51d
--- if (disc[v] == -1) { ---------------------------------//51e
---- bridges_artics(v, u); -------------------------------//51f
---- children++; -----------------------------------------//520
----- if (disc[u] < low[v]) ------------------------------//521
------- bridges.push_back({u, v}); -----------------------//522
----- if (disc[u] <= low[v]) -----------------------------//523
```

```
------- has_low_child = true; ----------------------------//524
----- low[u] = min(low[u], low[v]); ----------------------//525
--- } else if (v != p) -----------------------------------//526
----- low[u] = min(low[u], disc[v]); ---------------------//527
- } ------------------------------------------------------//528
- if ((p == -1 && children >= 2) || ----------------------//529
----- (p != -1 && has_low_child)) ------------------------//52a
--- articulation_points.push_back(u); --------------------//52b
} --------------------------------------------------------//52c
```

3.6. Biconnected Components.

3.6.1. *Bridge Tree.*

3.6.2. *Block-Cut Tree.*

3.7. Minimum Spanning Tree.

3.7.1. *Kruskal.*

```
#include "graph_template_edgelist.cpp" -------------------//76e
#include "union_find.cpp" --------------------------------//76f
// insert inside graph; needs n, and edges ---------------//770
void kruskal(viii &res) ----------------------------------//771
- viii().swap(res); // or use res.clear(); ---------------//772
- std::priority_queue<iii, viii, std::greater<iii> > pq; -//773
- for (auto &edge : edges) -------------------------------//774
--- pq.push(edge); ---------------------------------------//775
- union_find uf(n); --------------------------------------//776
- while (!pq.empty()) { ----------------------------------//777
--- auto node = pq.top();   pq.pop(); --------------------//778
--- int u = node.second.first; ---------------------------//779
--- int v = node.second.second; --------------------------//77a
--- if (uf.unite(u, v)) ----------------------------------//77b
----- res.push_back(node); -------------------------------//77c
- } ------------------------------------------------------//77d
} --------------------------------------------------------//77e
```

3.7.2. *Prim.*

```
#include "graph_template_adjlist.cpp" --------------------//77f
// insert inside graph; needs n, vis[], and adj[] --------//780
void prim(viii &res, int s=0) { --------------------------//781
- viii().swap(res); // or use res.clear(); ---------------//782
- std::priority_queue<ii, vii, std::greater<ii> > pq; ----//783
- pq.push{{0, s}}; ---------------------------------------//784
- while (!pq.empty()) { ----------------------------------//785
--- int u = pq.top().second;    pq.pop(); ----------------//786
--- vis[u] = true; ---------------------------------------//787
--- for (auto &[v, w] : adj[u]) { ------------------------//788
----- if (v == u)   continue; ----------------------------//789
----- if (vis[v])   continue; ----------------------------//78a
----- res.push_back({w, {u, v}}); ------------------------//78b
----- pq.push({w, v}); -----------------------------------//78c
--- } ----------------------------------------------------//78d
- } ------------------------------------------------------//78e
} --------------------------------------------------------//78f
```

3.8. Euler Path/Cycle.

3.8.1. *Euler Path/Cycle in a Directed Graph.*

```
#define MAXV 1000 ----------------------------------------//556
#define MAXE 5000 ----------------------------------------//557
vi adj[MAXV]; --------------------------------------------//558
int n, m, indeg[MAXV], outdeg[MAXV], res[MAXE + 1]; ------//559
ii start_end() { -----------------------------------------//55a
- int start = -1, end = -1, any = 0, c = 0; --------------//55b
- rep(i,0,n) { -------------------------------------------//55c
--- if (outdeg[i] > 0) any = i; --------------------------//55d
--- if (indeg[i] + 1 == outdeg[i]) start = i, c++; -------//55e
--- else if (indeg[i] == outdeg[i] + 1) end = i, c++; ----//55f
--- else if (indeg[i] != outdeg[i]) return ii(-1,-1); } --//560
- if ((start == -1) != (end == -1) || (c != 2 && c != 0)) //561
--- return ii(-1,-1); ------------------------------------//562
- if (start == -1) start = end = any; --------------------//563
- return ii(start, end); } -------------------------------//564
bool euler_path() { --------------------------------------//565
- ii se = start_end(); -----------------------------------//566
- int cur = se.first, at = m + 1; ------------------------//567
- if (cur == -1) return false; ---------------------------//568
- stack<int> s; ------------------------------------------//569
- while (true) { -----------------------------------------//56a
--- if (outdeg[cur] == 0) { ------------------------------//56b
----- res[--at] = cur; -----------------------------------//56c
----- if (s.empty()) break; ------------------------------//56d
----- cur = s.top(); s.pop(); ----------------------------//56e
--- } else s.push(cur), cur = adj[cur][--outdeg[cur]]; } -//56f
- return at == 0; } --------------------------------------//570
```

3.8.2. (. Euler Path/Cycle in an Undirected Graph)

```
multiset<int> adj[1010]; ---------------------------------//571
list<int> L; ---------------------------------------------//572
list<int>::iterator euler(int at, int to, ----------------//573
--- list<int>::iterator it) { ----------------------------//574
- if (at == to) return it; -------------------------------//575
- L.insert(it, at), --it; --------------------------------//576
- while (!adj[at].empty()) { -----------------------------//577
--- int nxt = *adj[at].begin(); --------------------------//578
--- adj[at].erase(adj[at].find(nxt)); --------------------//579
--- adj[nxt].erase(adj[nxt].find(at)); -------------------//57a
--- if (to == -1) { --------------------------------------//57b
----- it = euler(nxt, at, it); ---------------------------//57c
----- L.insert(it, at); ----------------------------------//57d
----- --it; ----------------------------------------------//57e
--- } else { ---------------------------------------------//57f
----- it = euler(nxt, to, it); ---------------------------//580
----- to = -1; } } ---------------------------------------//581
- return it; } -------------------------------------------//582
// euler(0,-1,L.begin()) ---------------------------------//583
```

3.9. Bipartite Matching.

3.9.1. *Alternating Paths Algorithm.*

```
vi* adj; -------------------------------------------------//630
bool* done; ----------------------------------------------//631
int* owner; ----------------------------------------------//632
int alternating_path(int left) { -------------------------//633
- if (done[left]) return 0; ------------------------------//634
- done[left] = true; -------------------------------------//635
- rep(i,0,size(adj[left])) { -----------------------------//636
--- int right = adj[left][i]; ----------------------------//637
--- if (owner[right] == -1 || ----------------------------//638
------- alternating_path(owner[right])) { ----------------//639
----- owner[right] = left; return 1; } } ----------------//63a
- return 0; } --------------------------------------------//63b
```

3.9.2. *Hopcroft-Karp Algorithm.*

```
#define MAXN 5000 ----------------------------------------//64b
int dist[MAXN+1], q[MAXN+1]; -----------------------------//64c
#define dist(v) dist[v == -1 ? MAXN : v] -----------------//64d
struct bipartite_graph { ---------------------------------//64e
- int N, M, *L, *R; vi *adj; -----------------------------//64f
- bipartite_graph(int _N, int _M) : N(_N), M(_M), --------//650
--- L(new int[N]), R(new int[M]), adj(new vi[N]) {} ------//651
- ~bipartite_graph() { delete[] adj; delete[] L; delete[] R; }
- bool bfs() { -------------------------------------------//653
--- int l = 0, r = 0; ------------------------------------//654
--- rep(v,0,N) if(L[v] == -1) dist(v) = 0, q[r++] = v; ---//655
----- else dist(v) = INF; --------------------------------//656
--- dist(-1) = INF; --------------------------------------//657
--- while(l < r) { ---------------------------------------//658
----- int v = q[l++]; ------------------------------------//659
----- if(dist(v) < dist(-1)) { ---------------------------//65a
------- iter(u, adj[v]) if(dist(R[*u]) == INF) -----------//65b
--------- dist(R[*u]) = dist(v) + 1, q[r++] = R[*u]; } } -//65c
--- return dist(-1) != INF; } ----------------------------//65d
- bool dfs(int v) { --------------------------------------//65e
--- if(v != -1) { ----------------------------------------//65f
----- iter(u, adj[v]) ------------------------------------//660
------- if(dist(R[*u]) == dist(v) + 1) -------------------//661
--------- if(dfs(R[*u])) { -------------------------------//662
----------- R[*u] = v, L[v] = *u; ------------------------//663
----------- return true; } -------------------------------//664
----- dist(v) = INF; -------------------------------------//665
----- return false; } ------------------------------------//666
--- return true; } ---------------------------------------//667
- void add_edge(int i, int j) { adj[i].push_back(j); } ---//668
- int maximum_matching() { -------------------------------//669
--- int matching = 0; ------------------------------------//66a
--- memset(L, -1, sizeof(int) * N); ----------------------//66b
--- memset(R, -1, sizeof(int) * M); ----------------------//66c
--- while(bfs()) rep(i,0,N) ------------------------------//66d
----- matching += L[i] == -1 && dfs(i); ------------------//66e
--- return matching; } }; --------------------------------//66f
```

3.9.3. *Minimum Vertex Cover in Bipartite Graphs.*

```
#include "hopcroft_karp.cpp" -----------------------------//63c
vector<bool> alt; ----------------------------------------//63d
void dfs(bipartite_graph &g, int at) { -------------------//63e
- alt[at] = true; ----------------------------------------//63f
- iter(it,g.adj[at]) { -----------------------------------//640
--- alt[*it + g.N] = true; -------------------------------//641
--- if (g.R[*it] != -1 && !alt[g.R[*it]]) ----------------//642
----- dfs(g, g.R[*it]); } } ------------------------------//643
```

```
vi mvc_bipartite(bipartite_graph &g) { -------------------//644
- vi res; g.maximum_matching(); --------------------------//645
- alt.assign(g.N + g.M,false); ---------------------------//646
- rep(i,0,g.N) if (g.L[i] == -1) dfs(g, i); --------------//647
- rep(i,0,g.N) if (!alt[i]) res.push_back(i); ------------//648
- rep(i,0,g.M) if (alt[g.N + i]) res.push_back(g.N + i); -//649
- return res; } ------------------------------------------//64a
```

3.10. Maximum Flow.

3.10.1. *Edmonds-Karp.*

```
struct flow_network { -----------------------------------//6f2
- int n, s, t, *par, **c, **f; --------------------------//6f3
- vi *adj; ----------------------------------------------//6f4
- flow_network(int n, int s, int t) : n(n), s(s), t(t) { -//6f5
--- adj = new std::vector<int>[n]; ----------------------//6f6
--- par = new int[n]; -----------------------------------//6f7
--- c = new int*[n]; ------------------------------------//6f8
--- f = new int*[n]; ------------------------------------//6f9
--- for (int i = 0; i < n; ++i) { -----------------------//6fa
----- c[i] = new int[n]; --------------------------------//6fb
----- f[i] = new int[n]; --------------------------------//6fc
----- for (int j = 0; j < n; ++j) -----------------------//6fd
------- c[i][j] = f[i][j] = 0; --------------------------//6fe
--- } ---------------------------------------------------//6ff
- } -----------------------------------------------------//700
- void add_edge(int u, int v, int w) { ------------------//701
--- adj[u].push_back(v); --------------------------------//702
--- adj[v].push_back(u); --------------------------------//703
--- c[u][v] += w; ---------------------------------------//704
- } -----------------------------------------------------//705
- int res(int i, int j) { return c[i][j] - f[i][j]; } ---//706
- bool bfs() { ------------------------------------------//707
--- std::queue<int> q; ----------------------------------//708
--- q.push(this->s); ------------------------------------//709
--- while (!q.empty()) { --------------------------------//70a
----- int u = q.front();  q.pop(); ----------------------//70b
----- for (int v : adj[u]) { ----------------------------//70c
------- if (res(u, v) > 0 and par[v] == -1) { -----------//70d
--------- par[v] = u; -----------------------------------//70e
--------- if (v == this->t) -----------------------------//70f
----------- return true; --------------------------------//710
--------- q.push(v); ------------------------------------//711
------- } -----------------------------------------------//712
----- } -------------------------------------------------//713
--- } ---------------------------------------------------//714
--- return false; ---------------------------------------//715
- } -----------------------------------------------------//716
- bool aug_path() { -------------------------------------//717
--- for (int u = 0; u < n; ++u) -------------------------//718
----- par[u] = -1; --------------------------------------//719
--- par[s] = s; -----------------------------------------//71a
--- return bfs(); ---------------------------------------//71b
- } -----------------------------------------------------//71c
- int calc_max_flow() { ---------------------------------//71d
--- int ans = 0; ----------------------------------------//71e
--- while (aug_path()) { --------------------------------//71f
----- int flow = INF; -----------------------------------//720
----- for (int u = t; u != s; u = par[u]) ---------------//721
------- flow = std::min(flow, res(par[u], u)); ----------//722
----- for (int u = t; u != s; u = par[u]) ---------------//723
------- f[par[u]][u] += flow, f[u][par[u]] -= flow; -----//724
----- ans += flow; --------------------------------------//725
--- } ---------------------------------------------------//726
--- return ans; -----------------------------------------//727
- } -----------------------------------------------------//728
}; ------------------------------------------------------//729
```

3.10.2. *Dinic.*

```
struct flow_network { -----------------------------------//6a4
- int n, s, t, *adj_ptr, *dist, *par, **c, **f; ---------//6a5
- vi *adj; ----------------------------------------------//6a6
- flow_network(int n, int s, int t) : n(n), s(s), t(t) { -//6a7
--- adj = new std::vector<int>[n]; ----------------------//6a8
--- adj_ptr = new int[n]; -------------------------------//6a9
--- dist = new int[n]; ----------------------------------//6aa
--- par = new int[n]; -----------------------------------//6ab
--- c = new int*[n]; ------------------------------------//6ac
--- f = new int*[n]; ------------------------------------//6ad
--- for (int i = 0; i < n; ++i) { -----------------------//6ae
----- c[i] = new int[n]; --------------------------------//6af
----- f[i] = new int[n]; --------------------------------//6b0
----- for (int j = 0; j < n; ++j) -----------------------//6b1
------- c[i][j] = f[i][j] = 0; --------------------------//6b2
--- } ---------------------------------------------------//6b3
- } -----------------------------------------------------//6b4
- void add_edge(int u, int v, int w) { ------------------//6b5
--- adj[u].push_back(v); --------------------------------//6b6
--- adj[v].push_back(u); --------------------------------//6b7
--- c[u][v] += w; ---------------------------------------//6b8
- } -----------------------------------------------------//6b9
- int res(int i, int j) { return c[i][j] - f[i][j]; } ---//6ba
- void reset(int *ar, int val) { ------------------------//6bb
--- for (int i = 0; i < n; ++i) -------------------------//6bc
----- ar[i] = val; --------------------------------------//6bd
- } -----------------------------------------------------//6be
- bool make_level_graph() { -----------------------------//6bf
--- reset(dist, -1); ------------------------------------//6c0
--- std::queue<int> q; ----------------------------------//6c1
--- q.push(s); ------------------------------------------//6c2
--- dist[s] = 0; ----------------------------------------//6c3
--- while (!q.empty()) { --------------------------------//6c4
----- int u = q.front();  q.pop(); ----------------------//6c5
----- for (int v : adj[u]) { ----------------------------//6c6
------- if (res(u, v) > 0 and dist[v] == -1) { ----------//6c7
--------- dist[v] = dist[u] + 1; ------------------------//6c8
--------- q.push(v); ------------------------------------//6c9
------- } -----------------------------------------------//6ca
----- } -------------------------------------------------//6cb
--- } ---------------------------------------------------//6cc
--- return dist[t] != -1; -------------------------------//6cd
- } -----------------------------------------------------//6ce
- bool next(int u, int v) { -----------------------------//6cf
--- return dist[v] == dist[u] + 1; ----------------------//6d0
- } -----------------------------------------------------//6d1
- bool dfs(int u) { -------------------------------------//6d2
--- if (u == t)   return true; --------------------------//6d3
--- for (int &i = adj_ptr[u]; i < adj[u].size(); ++i) { --//6d4
----- int v = adj[u][i]; --------------------------------//6d5
----- if (next(u, v) and res(u, v) > 0 and dfs(v)) { -----//6d6
------- par[v] = u; -------------------------------------//6d7
------- return true; ------------------------------------//6d8
----- } -------------------------------------------------//6d9
--- } ---------------------------------------------------//6da
--- dist[u] = -1; ---------------------------------------//6db
--- return false; ---------------------------------------//6dc
- } -----------------------------------------------------//6dd
- bool aug_path() { -------------------------------------//6de
--- reset(par, -1); -------------------------------------//6df
--- par[s] = s; -----------------------------------------//6e0
--- return dfs(s);  } -----------------------------------//6e1
- int calc_max_flow() { ---------------------------------//6e2
--- int ans = 0; ----------------------------------------//6e3
--- while (make_level_graph()) { ------------------------//6e4
----- reset(adj_ptr, 0); --------------------------------//6e5
----- while (aug_path()) { ------------------------------//6e6
------- int flow = INF; ---------------------------------//6e7
------- for (int u = t; u != s; u = par[u]) -------------//6e8
--------- flow = std::min(flow, res(par[u], u)); --------//6e9
------- for (int u = t; u != s; u = par[u]) -------------//6ea
--------- f[par[u]][u] += flow, f[u][par[u]] -= flow; ----//6eb
------- ans += flow; ------------------------------------//6ec
----- } -------------------------------------------------//6ed
--- } ---------------------------------------------------//6ee
--- return ans; -----------------------------------------//6ef
- } -----------------------------------------------------//6f0
}; ------------------------------------------------------//6f1
```

3.11. All-pairs Maximum Flow.

3.11.1. *Gomory-Hu.*

```
#define MAXV 2000 -------------------------------------//72a
int q[MAXV], d[MAXV]; --------------------------------//72b
struct flow_network { --------------------------------//72c
- struct edge { int v, nxt, cap; ---------------------//72d
--- edge(int _v, int _cap, int _nxt) -----------------//72e
----- : v(_v), nxt(_nxt), cap(_cap) { } }; -----------//72f
- int n, *head, *curh; vector<edge> e, e_store; -------//730
- flow_network(int _n) : n(_n) { ---------------------//731
--- curh = new int[n]; -------------------------------//732
--- memset(head = new int[n], -1, n*sizeof(int)); } ----//733
- void reset() { e = e_store; } ----------------------//734
- void add_edge(int u, int v, int uv, int vu=0) { ------//735
--- e.push_back(edge(v,uv,head[u])); head[u]=(int)size(e)-1;
--- e.push_back(edge(u,vu,head[v])); head[v]=(int)size(e)-1;}
- int augment(int v, int t, int f) { -----------------//738
--- if (v == t) return f; ----------------------------//739
--- for (int &i = curh[v], ret; i != -1; i = e[i].nxt) ---//73a
----- if (e[i].cap > 0 && d[e[i].v] + 1 == d[v]) --------//73b
------- if ((ret = augment(e[i].v, t, min(f, e[i].cap))) > 0)
```

```
--------- return (e[i].cap -= ret, e[i^1].cap += ret, ret);
--- return 0; } ------------------------------------------//73e
- int max_flow(int s, int t, bool res=true) { ------------//73f
--- e_store = e; -----------------------------------------//740
--- int l, r, f = 0, x; ----------------------------------//741
--- while (true) { ---------------------------------------//742
----- memset(d, -1, n*sizeof(int)); ----------------------//743
----- l = r = 0, d[q[r++] = t] = 0; ----------------------//744
----- while (l < r) --------------------------------------//745
------- for (int v = q[l++], i = head[v]; i != -1; i=e[i].nxt)
--------- if (e[i^1].cap > 0 && d[e[i].v] == -1) ---------//747
----------- d[q[r++] = e[i].v] = d[v]+1; -----------------//748
----- if (d[s] == -1) break; -----------------------------//749
----- memcpy(curh, head, n * sizeof(int)); ---------------//74a
----- while ((x = augment(s, t, INF)) != 0) f += x; } ----//74b
--- if (res) reset(); ------------------------------------//74c
--- return f; } }; ---------------------------------------//74d
bool same[MAXV]; -----------------------------------------//74e
pair<vii, vvi> construct_gh_tree(flow_network &g) { ------//74f
- int n = g.n, v; ----------------------------------------//750
- vii par(n, ii(0, 0)); vvi cap(n, vi(n, -1)); -----------//751
- rep(s,1,n) { -------------------------------------------//752
--- int l = 0, r = 0; ------------------------------------//753
--- par[s].second = g.max_flow(s, par[s].first, false); --//754
--- memset(d, 0, n * sizeof(int)); -----------------------//755
--- memset(same, 0, n * sizeof(bool)); -------------------//756
--- d[q[r++] = s] = 1; -----------------------------------//757
--- while (l < r) { --------------------------------------//758
----- same[v = q[l++]] = true; ---------------------------//759
----- for (int i = g.head[v]; i != -1; i = g.e[i].nxt) ---//75a
------- if (g.e[i].cap > 0 && d[g.e[i].v] == 0) ----------//75b
--------- d[q[r++] = g.e[i].v] = 1; } --------------------//75c
--- rep(i,s+1,n) -----------------------------------------//75d
----- if (par[i].first == par[s].first && same[i]) -------//75e
------- par[i].first = s; --------------------------------//75f
--- g.reset(); } -----------------------------------------//760
- rep(i,0,n) { -------------------------------------------//761
--- int mn = INF, cur = i; -------------------------------//762
--- while (true) { ---------------------------------------//763
----- cap[cur][i] = mn; ----------------------------------//764
----- if (cur == 0) break; -------------------------------//765
----- mn = min(mn, par[cur].second), cur = par[cur].first; } }
- return make_pair(par, cap); } --------------------------//767
int compute_max_flow(int s, int t, const pair<vii, vvi> &gh) {
- int cur = INF, at = s; ----------------------------------//769
- while (gh.second[at][t] == -1) -------------------------//76a
--- cur = min(cur, gh.first[at].second), -----------------//76b
--- at = gh.first[at].first; -----------------------------//76c
- return min(cur, gh.second[at][t]); } -------------------//76d
```

3.12. **Minimum Arborescence.** Given a weighted directed graph, finds a subset of edges of minimum total weight so that there is a unique path from the root r to each vertex. Returns a vector of size n, where the ith element is the edge for the ith vertex. The answer for the root is undefined!

```
#include "../data-structures/union_find.cpp" ------------//49d
struct arborescence { ------------------------------------//49e
- int n; union_find uf; ----------------------------------//49f
- vector<vector<pair<ii,int> > > adj; --------------------//4a0
- arborescence(int _n) : n(_n), uf(n), adj(n) { } --------//4a1
- void add_edge(int a, int b, int c) { -------------------//4a2
--- adj[b].push_back(make_pair(ii(a,b),c)); } ------------//4a3
- vii find_min(int r) { ----------------------------------//4a4
--- vi vis(n,-1), mn(n,INF); vii par(n); -----------------//4a5
--- rep(i,0,n) { -----------------------------------------//4a6
----- if (uf.find(i) != i) continue; ---------------------//4a7
----- int at = i; ----------------------------------------//4a8
----- while (at != r && vis[at] == -1) { -----------------//4a9
------- vis[at] = i; -------------------------------------//4aa
------- iter(it,adj[at]) if (it->second < mn[at] && ------//4ab
----------- uf.find(it->first.first) != at) --------------//4ac
--------- mn[at] = it->second, par[at] = it->first; ------//4ad
------- if (par[at] == ii(0,0)) return vii(); ------------//4ae
------- at = uf.find(par[at].first); } -------------------//4af
----- if (at == r || vis[at] != i) continue; -------------//4b0
----- union_find tmp = uf; vi seq; -----------------------//4b1
----- do { seq.push_back(at); at = uf.find(par[at].first);//4b2
----- } while (at != seq.front()); -----------------------//4b3
----- iter(it,seq) uf.unite(*it,seq[0]); -----------------//4b4
----- int c = uf.find(seq[0]); ---------------------------//4b5
----- vector<pair<ii,int> > nw; --------------------------//4b6
----- iter(it,seq) iter(jt,adj[*it]) ---------------------//4b7
------- nw.push_back(make_pair(jt->first, ----------------//4b8
------------ jt->second - mn[*it])); ---------------------//4b9
----- adj[c] = nw; ---------------------------------------//4ba
----- vii rest = find_min(r); ----------------------------//4bb
----- if (size(rest) == 0) return rest; ------------------//4bc
----- ii use = rest[c]; ----------------------------------//4bd
----- rest[at = tmp.find(use.second)] = use; -------------//4be
----- iter(it,seq) if (*it != at) ------------------------//4bf
------- rest[*it] = par[*it]; ----------------------------//4c0
----- return rest; } -------------------------------------//4c1
--- return par; } }; -------------------------------------//4c2
```

3.13. **Blossom algorithm.** Finds a maximum matching in an arbitrary graph in $O(|V|^4)$ time. Be vary of loop edges.

```
#define MAXV 300 -----------------------------------------//4c3
bool marked[MAXV], emarked[MAXV][MAXV]; ------------------//4c4
int S[MAXV]; ---------------------------------------------//4c5
vi find_augmenting_path(const vector<vi> &adj,const vi &m){
- int n = size(adj), s = 0; ------------------------------//4c7
- vi par(n,-1), height(n), root(n,-1), q, a, b; ----------//4c8
- memset(marked,0,sizeof(marked)); -----------------------//4c9
- memset(emarked,0,sizeof(emarked)); ---------------------//4ca
- rep(i,0,n) if (m[i] >= 0) emarked[i][m[i]] = true; -----//4cb
----------- else root[i] = i, S[s++] = i; ----------------//4cc
- while (s) { --------------------------------------------//4cd
--- int v = S[--s]; --------------------------------------//4ce
--- iter(wt,adj[v]) { ------------------------------------//4cf
----- int w = *wt; ---------------------------------------//4d0
----- if (emarked[v][w]) continue; -----------------------//4d1
----- if (root[w] == -1) { -------------------------------//4d2
------- int x = S[s++] = m[w]; ---------------------------//4d3
------- par[w]=v, root[w]=root[v], height[w]=height[v]+1; //4d4
------- par[x]=w, root[x]=root[w], height[x]=height[w]+1; //4d5
----- } else if (height[w] % 2 == 0) { -------------------//4d6
------- if (root[v] != root[w]) { ------------------------//4d7
--------- while (v != -1) q.push_back(v), v = par[v]; ----//4d8
--------- reverse(q.begin(), q.end()); -------------------//4d9
--------- while (w != -1) q.push_back(w), w = par[w]; ----//4da
--------- return q; --------------------------------------//4db
------- } else { -----------------------------------------//4dc
--------- int c = v; -------------------------------------//4dd
--------- while (c != -1) a.push_back(c), c = par[c]; ----//4de
--------- c = w; -----------------------------------------//4df
--------- while (c != -1) b.push_back(c), c = par[c]; ----//4e0
--------- while (!a.empty()&&!b.empty()&&a.back()==b.back())
----------- c = a.back(), a.pop_back(), b.pop_back(); ----//4e2
--------- memset(marked,0,sizeof(marked)); ---------------//4e3
--------- fill(par.begin(), par.end(), 0); ---------------//4e4
--------- iter(it,a) par[*it] = 1; iter(it,b) par[*it] = 1;
--------- par[c] = s = 1; --------------------------------//4e6
--------- rep(i,0,n) root[par[i] = par[i] ? 0 : s++] = i; //4e7
--------- vector<vi> adj2(s); ----------------------------//4e8
--------- rep(i,0,n) iter(it,adj[i]) { -------------------//4e9
----------- if (par[*it] == 0) continue; -----------------//4ea
----------- if (par[i] == 0) { ---------------------------//4eb
------------- if (!marked[par[*it]])  { ------------------//4ec
--------------- adj2[par[i]].push_back(par[*it]); --------//4ed
--------------- adj2[par[*it]].push_back(par[i]); --------//4ee
--------------- marked[par[*it]] = true; } ---------------//4ef
----------- } else adj2[par[i]].push_back(par[*it]); } ---//4f0
--------- vi m2(s, -1); ----------------------------------//4f1
--------- if (m[c] != -1) m2[m2[par[m[c]]] = 0] = par[m[c]];
--------- rep(i,0,n) if(par[i]!=0&&m[i]!=-1&&par[m[i]]!=0)//4f3
----------- m2[par[i]] = par[m[i]]; ----------------------//4f4
--------- vi p = find_augmenting_path(adj2, m2); ---------//4f5
--------- int t = 0; -------------------------------------//4f6
--------- while (t < size(p) && p[t]) t++; ---------------//4f7
--------- if (t == size(p)) { ----------------------------//4f8
----------- rep(i,0,size(p)) p[i] = root[p[i]]; ----------//4f9
----------- return p; } ----------------------------------//4fa
--------- if (!p[0] || (m[c] != -1 && p[t+1] != par[m[c]]))
----------- reverse(p.begin(), p.end()), t=(int)size(p)-t-1;
--------- rep(i,0,t) q.push_back(root[p[i]]); ------------//4fd
--------- iter(it,adj[root[p[t-1]]]) { -------------------//4fe
----------- if (par[*it] != (s = 0)) continue; -----------//4ff
----------- a.push_back(c), reverse(a.begin(), a.end()); -//500
----------- iter(jt,b) a.push_back(*jt); -----------------//501
----------- while (a[s] != *it) s++; ---------------------//502
----------- if((height[*it]&1)^(s<(int)size(a)-(int)size(b)))
------------- reverse(a.begin(),a.end()), s=(int)size(a)-s-1;
----------- while(a[s]!=c)q.push_back(a[s]),s=(s+1)%size(a);
----------- q.push_back(c); ------------------------------//506
----------- rep(i,t+1,size(p)) q.push_back(root[p[i]]); --//507
----------- return q; } } } ------------------------------//508
----- emarked[v][w] = emarked[w][v] = true; } ------------//509
```

```
--- marked[v] = true; } return q; }
vii max_matching(const vector<vi> &adj) {
- vi m(size(adj), -1), ap; vii res, es;
- rep(i,0,size(adj)) iter(it,adj[i]) es.emplace_back(i,*it);
- random_shuffle(es.begin(), es.end());
- iter(it,es) if (m[it->first] == -1 && m[it->second] == -1)
--- m[it->first] = it->second, m[it->second] = it->first;
- do { ap = find_augmenting_path(adj, m);
------ rep(i,0,size(ap)) m[m[ap[i^1]]] = ap[i] = ap[i^1];
- } while (!ap.empty());
- rep(i,0,size(m)) if (i < m[i]) res.emplace_back(i, m[i]);
- return res; }
```

3.14. **Maximum Density Subgraph.** Given (weighted) undirected graph G. Binary search density. If g is current density, construct flow network: (S,u,m), $(u,T,m+2g-d_u)$, $(u,v,1)$, where m is a large constant (larger than sum of edge weights). Run floating-point max-flow. If minimum cut has empty S-component, then maximum density is smaller than g, otherwise it's larger. Distance between valid densities is at least $1/(n(n-1))$. Edge case when density is 0. This also works for weighted graphs by replacing d_u by the weighted degree, and doing more iterations (if weights are not integers).

3.15. **Maximum-Weight Closure.** Given a vertex-weighted directed graph G. Turn the graph into a flow network, adding weight ∞ to each edge. Add vertices S, T. For each vertex v of weight w, add edge (S,v,w) if $w \geq 0$, or edge $(v,T,-w)$ if $w < 0$. Sum of positive weights minus minimum $S-T$ cut is the answer. Vertices reachable from S are in the closure. The maximum-weight closure is the same as the complement of the minimum-weight closure on the graph with edges reversed.

3.16. **Maximum Weighted Independent Set in a Bipartite Graph.** This is the same as the minimum weighted vertex cover. Solve this by constructing a flow network with edges $(S,u,w(u))$ for $u \in L$, $(v,T,w(v))$ for $v \in R$ and (u,v,∞) for $(u,v) \in E$. The minimum S,T-cut is the answer. Vertices adjacent to a cut edge are in the vertex cover.

3.17. **Synchronizing word problem.** A DFA has a synchronizing word (an input sequence that moves all states to the same state) iff. each pair of states has a synchronizing word. That can be checked using reverse DFS over pairs of states. Finding the shortest synchronizing word is NP-complete.

3.18. **Max flow with lower bounds on edges.** Change edge $(u,v,l \leq f \leq c)$ to $(u,v,f \leq c-l)$. Add edge (t,s,∞). Create super-nodes S, T. Let $M(u) = \sum_v l(v,u) - \sum_v l(u,v)$. If $M(u) < 0$, add edge $(u,T,-M(u))$, else add edge $(S,u,M(u))$. Max flow from S to T. If all edges from S are saturated, then we have a feasible flow. Continue running max flow from s to t in original graph.

3.19. **Tutte matrix for general matching.** Create an $n \times n$ matrix A. For each edge (i,j), $i < j$, let $A_{ij} = x_{ij}$ and $A_{ji} = -x_{ij}$. All other entries are 0. The determinant of A is zero iff. the graph has a perfect matching. A randomized algorithm uses the Schwartz–Zippel lemma to check if it is zero.

3.20. **Heavy Light Decomposition.**

```
#include "segment_tree.cpp"
struct heavy_light_tree {
- int n;
- std::vector<int> *adj;
- segtree *segment_tree;
- int *par, *heavy, *dep, *path_root, *pos;
- heavy_light_tree(int n) {
--- this->n = n;
--- this->adj = new std::vector<int>[n];
--- segment_tree = new segtree(0, n-1);
--- par = new int[n];
--- heavy = new int[n];
--- dep = new int[n];
--- path_root = new int[n];
--- pos = new int[n];
- }
- void add_edge(int u, int v) {
--- adj[u].push_back(v);
--- adj[v].push_back(u);
- }
- void build(int root) {
--- for (int u = 0; u < n; ++u)
----- heavy[u] = -1;
--- par[root] = root;
--- dep[root] = 0;
--- dfs(root);
--- for (int u = 0, p = 0; u < n; ++u) {
----- if (par[u] == -1 or heavy[par[u]] != u) {
------- for (int v = u; v != -1; v = heavy[v]) {
--------- path_root[v] = u;
--------- pos[v] = p++;
------- }
----- }
--- }
- }
- int dfs(int u) {
--- int sz = 1;
--- int max_subtree_sz = 0;
--- for (int v : adj[u]) {
----- if (v != par[u]) {
------- par[v] = u;
------- dep[v] = dep[u] + 1;
------- int subtree_sz = dfs(v);
------- if (max_subtree_sz < subtree_sz) {
--------- max_subtree_sz = subtree_sz;
--------- heavy[u] = v;
------- }
------- sz += subtree_sz;
----- }
--- }
--- return sz;
- }
- int query(int u, int v) {
--- int res = 0;
--- while (path_root[u] != path_root[v]) {
----- if (dep[path_root[u]] > dep[path_root[v]])
------- std::swap(u, v);
----- res += segment_tree->sum(pos[path_root[v]], pos[v]);
----- v = par[path_root[v]];
--- }
--- res += segment_tree->sum(pos[u], pos[v]);
--- return res;
- }
- void update(int u, int v, int c) {
--- for (; path_root[u] != path_root[v];
--------- v = par[path_root[v]]) {
----- if (dep[path_root[u]] > dep[path_root[v]])
------- std::swap(u, v);
----- segment_tree->increase(pos[path_root[v]], pos[v], c);
--- }
--- segment_tree->increase(pos[u], pos[v], c);
- }
};
```

3.21. **Centroid Decomposition.**

```
#define MAXV 100100
#define LGMAXV 20
int jmp[MAXV][LGMAXV],
- path[MAXV][LGMAXV],
- sz[MAXV], seph[MAXV],
- shortest[MAXV];
struct centroid_decomposition {
- int n; vvi adj;
- centroid_decomposition(int _n) : n(_n), adj(n) { }
- void add_edge(int a, int b) {
--- adj[a].push_back(b); adj[b].push_back(a); }
- int dfs(int u, int p) {
--- sz[u] = 1;
--- rep(i,0,size(adj[u]))
----- if (adj[u][i] != p) sz[u] += dfs(adj[u][i], u);
--- return sz[u]; }
- void makepaths(int sep, int u, int p, int len) {
--- jmp[u][seph[sep]] = sep, path[u][seph[sep]] = len;
--- int bad = -1;
--- rep(i,0,size(adj[u])) {
----- if (adj[u][i] == p) bad = i;
----- else makepaths(sep, adj[u][i], u, len + 1);
--- }
--- if (p == sep)
----- swap(adj[u][bad], adj[u].back()), adj[u].pop_back(); }
- void separate(int h=0, int u=0) {
--- dfs(u,-1); int sep = u;
--- down: iter(nxt,adj[sep])
----- if (sz[*nxt] < sz[sep] && sz[*nxt] > sz[u]/2) {
------- sep = *nxt; goto down; }
--- seph[sep] = h, makepaths(sep, sep, -1, 0);
--- rep(i,0,size(adj[sep])) separate(h+1, adj[sep][i]); }
- void paint(int u) {
--- rep(h,0,seph[u]+1)
----- shortest[jmp[u][h]] = min(shortest[jmp[u][h]],
                                path[u][h]); }
```

```
- int closest(int u) { ----------------------------------//551
--- int mn = INF/2; -------------------------------------//552
--- rep(h,0,seph[u]+1) ----------------------------------//553
----- mn = min(mn, path[u][h] + shortest[jmp[u][h]]); ---//554
--- return mn; } }; -------------------------------------//555
```

3.22. Least Common Ancestor.

3.22.1. *Binary Lifting.*

```
struct graph { ------------------------------------------//670
- int n; ------------------------------------------------//671
- int logn; ---------------------------------------------//672
- std::vector<int> *adj; --------------------------------//673
- int *dep; ---------------------------------------------//674
- int **par; --------------------------------------------//675
- graph(int n, int logn=20) { ---------------------------//676
--- this->n = n; ----------------------------------------//677
--- this->logn = logn; ----------------------------------//678
--- adj = new std::vector<int>[n]; ----------------------//679
--- dep = new int[n]; -----------------------------------//67a
--- par = new int*[n]; ----------------------------------//67b
--- for (int i = 0; i < n; ++i) -------------------------//67c
----- par[i] = new int[logn]; ---------------------------//67d
- } -----------------------------------------------------//67e
- void dfs(int u, int p, int d) { -----------------------//67f
--- dep[u] = d; -----------------------------------------//680
--- par[u][0] = p; --------------------------------------//681
--- for (int v : adj[u]) --------------------------------//682
----- if (v != p) ---------------------------------------//683
------- dfs(v, u, d+1); ---------------------------------//684
- } -----------------------------------------------------//685
- int ascend(int u, int k) { ----------------------------//686
--- for (int i = 0; i < logn; ++i) ----------------------//687
----- if (k & (1 << i)) ---------------------------------//688
------- u = par[u][i]; ----------------------------------//689
--- return u; -------------------------------------------//68a
- } -----------------------------------------------------//68b
- int lca(int u, int v) { -------------------------------//68c
--- if (dep[u] > dep[v])  u = ascend(u, dep[u] - dep[v]); //68d
--- if (dep[v] > dep[u])  v = ascend(v, dep[v] - dep[u]); //68e
--- if (u == v)           return u; ---------------------//68f
--- for (int k = logn-1; k >= 0; --k) { -----------------//690
----- if (par[u][k] != par[v][k]) { ---------------------//691
------- u = par[u][k]; ----------------------------------//692
------- v = par[v][k]; ----------------------------------//693
----- } -------------------------------------------------//694
--- } ---------------------------------------------------//695
--- return par[u][0]; -----------------------------------//696
- } -----------------------------------------------------//697
- bool is_anc(int u, int v) { ---------------------------//698
--- if (dep[u] < dep[v]) --------------------------------//699
----- std::swap(u, v); ----------------------------------//69a
--- return ascend(u, dep[u] - dep[v]) == v; -------------//69b
- } -----------------------------------------------------//69c
- void prep_lca(int root=0) { ---------------------------//69d
--- dfs(root, root, 0); ---------------------------------//69e
--- for (int k = 1; k < logn; ++k) ----------------------//69f
----- for (int u = 0; u < n; ++u) -----------------------//6a0
------- par[u][k] = par[par[u][k-1]][k-1]; --------------//6a1
- } -----------------------------------------------------//6a2
}; ------------------------------------------------------//6a3
```

3.23. Counting Spanning Trees.

Kirchoff's Theorem: The number of spanning trees of any graph is the determinant of any cofactor of the Laplacian matrix in $O(n^3)$.

(1) Let A be the adjacency matrix.
(2) Let D be the degree matrix (matrix with vertex degrees on the diagonal).
(3) Get $D - A$ and delete exactly one row and column. Any row and column will do. This will be the cofactor matrix.
(4) Get the determinant of this cofactor matrix using Gauss-Jordan.
(5) Spanning Trees = $|\text{cofactor}(D - A)|$

3.24. Erdős-Gallai Theorem.

A sequence of non-negative integers $d_1 \geq \cdots \geq d_n$ can be represented as the degree sequence of finite simple graph on n vertices if and only if $d_1 + \cdots + d_n$ is even and the following holds for $1 \leq k \leq n$:

$$\sum_{i=1}^{n} d_i \leq k(k-1) + \sum_{i=k+1}^{n} \min(d_i, k)$$

3.25. Tree Isomorphism.

```
// REQUIREMENT: list of primes pr[], see prime sieve -----//5ff
typedef long long LL; -----------------------------------//600
int pre[N], q[N], path[N]; bool vis[N]; -----------------//601
// perform BFS and return the last node visited ---------//602
int bfs(int u, vector<int> adj[]) { ---------------------//603
--- memset(vis, 0, sizeof(vis)); ------------------------//604
--- int head = 0, tail = 0; -----------------------------//605
--- q[tail++] = u; vis[u] = true; pre[u] = -1; ----------//606
--- while (head != tail) { ------------------------------//607
------- u = q[head]; if (++head == N) head = 0; ---------//608
------- for (int i = 0; i < adj[u].size(); ++i) { -------//609
----------- int v = adj[u][i]; --------------------------//60a
----------- if (!vis[v]) { ------------------------------//60b
--------------- vis[v] = true; pre[v] = u; --------------//60c
--------------- q[tail++] = v; if (tail == N) tail = 0; --//60d
----------- }}} -----------------------------------------//60e
--- return u; -------------------------------------------//60f
} // returns the list of tree centers -------------------//610
vector<int> tree_centers(int r, vector<int> adj[]) { -----//611
--- int size = 0; ---------------------------------------//612
--- for (int u=bfs(bfs(r, adj), adj); u!=-1; u=pre[u]) ---//613
------- path[size++] = u; -------------------------------//614
--- vector<int> med(1, path[size/2]); -------------------//615
--- if (size % 2 == 0) med.push_back(path[size/2-1]); ----//616
--- return med; -----------------------------------------//617
} // returns "unique hashcode" for tree with root u ------//618
LL rootcode(int u, vector<int> adj[], int p=-1, int d=15){//619
--- vector<LL> k; int nd = (d + 1) % primes; ------------//61a
--- for (int i = 0; i < adj[u].size(); ++i) -------------//61b
------- if (adj[u][i] != p) -----------------------------//61c
----------- k.push_back(rootcode(adj[u][i], adj, u, nd)); //61d
--- sort(k.begin(), k.end()); ---------------------------//61e
--- LL h = k.size() + 1; --------------------------------//61f
--- for (int i = 0; i < k.size(); ++i) ------------------//620
------- h = h * pr[d] + k[i]; ---------------------------//621
--- return h; -------------------------------------------//622
} // returns "unique hashcode" for the whole tree --------//623
LL treecode(int root, vector<int> adj[]) { --------------//624
--- vector<int> c = tree_centers(root, adj); ------------//625
--- if (c.size()==1) ------------------------------------//626
------- return (rootcode(c[0], adj) << 1) | 1; ----------//627
--- return (rootcode(c[0],adj)*rootcode(c[1],adj))<<1; ---//628
} // checks if two trees are isomorphic -----------------//629
bool isomorphic(int r1, vector<int> adj1[], int r2, ------//62a
--------------- vector<int> adj2[], bool rooted = false) {//62b
--- if (rooted) -----------------------------------------//62c
------- return rootcode(r1, adj1) == rootcode(r2, adj2); -//62d
--- return treecode(r1, adj1) == treecode(r2, adj2); -----//62e
} -------------------------------------------------------//62f
```

4. Strings

4.1. Knuth-Morris-Pratt.

Count and find all matches of string f in string s in $O(n)$ time.

```
int par[N]; // parent table -----------------------------//b34
void buildKMP(string& f) { ------------------------------//b35
--- par[0] = -1, par[1] = 0; ----------------------------//b36
--- int i = 2, j = 0; -----------------------------------//b37
--- while (i <= f.length()) { ---------------------------//b38
------- if (f[i-1] == f[j]) par[i++] = ++j; -------------//b39
------- else if (j > 0) j = par[j]; ---------------------//b3a
------- else par[i++] = 0; }} ---------------------------//b3b
vector<int> KMP(string& s, string& f) { -----------------//b3c
--- buildKMP(f); // call once if f is the same ----------//b3d
--- int i = 0, j = 0; vector<int> ans; ------------------//b3e
--- while (i + j < s.length()) { ------------------------//b3f
------- if (s[i + j] == f[j]) { -------------------------//b40
----------- if (++j == f.length()) { --------------------//b41
--------------- ans.push_back(i); -----------------------//b42
--------------- i += j - par[j]; ------------------------//b43
--------------- if (j > 0) j = par[j]; ------------------//b44
----------- } -------------------------------------------//b45
------- } else { ----------------------------------------//b46
----------- i += j - par[j]; ----------------------------//b47
----------- if (j > 0) j = par[j]; ----------------------//b48
------- } -----------------------------------------------//b49
--- } return ans; } -------------------------------------//b4a
```

4.2. Trie.

```
template <class T> --------------------------------------//bbd
struct trie { -------------------------------------------//bbe
- struct node { -----------------------------------------//bbf
--- map<T, node*> children; -----------------------------//bc0
--- int prefixes, words; --------------------------------//bc1
--- node() { prefixes = words = 0; } }; -----------------//bc2
- node* root; -------------------------------------------//bc3
- trie() : root(new node()) {  } ------------------------//bc4
- template <class I> ------------------------------------//bc5
- void insert(I begin, I end) { -------------------------//bc6
```

```
--- node* cur = root; -----------------------------------//bc7
--- while (true) { --------------------------------------//bc8
----- cur->prefixes++; ----------------------------------//bc9
----- if (begin == end) { cur->words++; break; } --------//bca
----- else { --------------------------------------------//bcb
------- T head = *begin; --------------------------------//bcc
------- typename map<T, node*>::const_iterator it; ------//bcd
------- it = cur->children.find(head); ------------------//bce
------- if (it == cur->children.end()) { ----------------//bcf
--------- pair<T, node*> nw(head, new node()); ----------//bd0
--------- it = cur->children.insert(nw).first; ----------//bd1
------- } begin++, cur = it->second; } } } --------------//bd2
- template<class I> -------------------------------------//bd3
- int countMatches(I begin, I end) { --------------------//bd4
--- node* cur = root; -----------------------------------//bd5
--- while (true) { --------------------------------------//bd6
----- if (begin == end) return cur->words; --------------//bd7
----- else { --------------------------------------------//bd8
------- T head = *begin; --------------------------------//bd9
------- typename map<T, node*>::const_iterator it; ------//bda
------- it = cur->children.find(head); ------------------//bdb
------- if (it == cur->children.end()) return 0; --------//bdc
------- begin++, cur = it->second; } } } ----------------//bdd
- template<class I> -------------------------------------//bde
- int countPrefixes(I begin, I end) { -------------------//bdf
--- node* cur = root; -----------------------------------//be0
--- while (true) { --------------------------------------//be1
----- if (begin == end) return cur->prefixes; -----------//be2
----- else { --------------------------------------------//be3
------- T head = *begin; --------------------------------//be4
------- typename map<T, node*>::const_iterator it; ------//be5
------- it = cur->children.find(head); ------------------//be6
------- if (it == cur->children.end()) return 0; --------//be7
------- begin++, cur = it->second; } } } }; -------------//be8
```

4.3. **Suffix Array.** Construct a sorted catalog of all substrings of s in $O(n \log n)$ time using counting sort.

```
// sa[i]: ith smallest substring at s[sa[i]:] -----------//ba5
// pos[i]: position of s[i:] in suffix array ------------//ba6
int sa[N], pos[N], va[N], c[N], gap, n; -----------------//ba7
bool cmp(int i, int j) // reverse stable sort -----------//ba8
--- {return pos[i]!=pos[j] ? pos[i] < pos[j] : j < i;} ---//ba9
bool equal(int i, int j) --------------------------------//baa
--- {return pos[i] == pos[j] && i + gap < n && ----------//bab
---------- pos[i + gap / 2] == pos[j + gap / 2];} -------//bac
void buildSA(string s) { --------------------------------//bad
--- s += '$'; n = s.length(); ---------------------------//bae
--- for (int i = 0; i < n; i++){sa[i]=i; pos[i]=s[i];} ---//baf
--- sort (sa, sa + n, cmp); -----------------------------//bb0
--- for (gap = 1; gap < n * 2; gap <<= 1) { -------------//bb1
------- va[sa[0]] = 0; ----------------------------------//bb2
------- for (int i = 1; i < n; i++) { -------------------//bb3
----------- int prev = sa[i - 1], next = sa[i]; ---------//bb4
----------- va[next] = equal(prev, next) ? va[prev] : i; -//bb5
------- } -----------------------------------------------//bb6
------- for (int i = 0; i < n; ++i) ---------------------//bb7
----------- { pos[i] = va[i]; va[i] = sa[i]; c[i] = i; } -//bb8
------- for (int i = 0; i < n; i++) { -------------------//bb9
----------- int id = va[i] - gap; -----------------------//bba
----------- if (id >= 0) sa[c[pos[id]]++] = id; ---------//bbb
------- }}} ---------------------------------------------//bbc
```

4.4. **Longest Common Prefix.** Find the length of the longest common prefix for every substring in $O(n)$.

```
int lcp[N]; // lcp[i] = LCP(s[sa[i]:], s[sa[i+1]:]) ------//b4b
void buildLCP(string s) {// build suffix array first -----//b4c
--- for (int i = 0, k = 0; i < n; i++) { ----------------//b4d
------- if (pos[i] != n - 1) { --------------------------//b4e
----------- for(int j = sa[pos[i]+1]; s[i+k]==s[j+k];k++);//b4f
----------- lcp[pos[i]] = k; if (k > 0) k--; ------------//b50
--- } else { lcp[pos[i]] = 0; }}} -----------------------//b51
```

4.5. **Aho-Corasick Trie.** Find all multiple pattern matches in $O(n)$ time. This is KMP for multiple strings.

```
class Node { --------------------------------------------//af3
--- HashMap<Character, Node> next = new HashMap<>(); -----//af4
--- Node fail = null; -----------------------------------//af5
--- long count = 0; -------------------------------------//af6
--- public void add(String s) { // adds string to trie ---//af7
------- Node node = this; -------------------------------//af8
------- for (char c : s.toCharArray()) { ----------------//af9
----------- if (!node.contains(c)) ----------------------//afa
--------------- node.next.put(c, new Node()); -----------//afb
----------- node = node.get(c); -------------------------//afc
------- } node.count++; } -------------------------------//afd
--- public void prepare() { -----------------------------//afe
------- // prepares fail links of Aho-Corasick Trie ------//aff
------- Node root = this; root.fail = null; -------------//b00
------- Queue<Node> q = new ArrayDeque<Node>(); ---------//b01
------- for (Node child : next.values()) // BFS ---------//b02
----------- { child.fail = root; q.offer(child); } ------//b03
------- while (!q.isEmpty()) { --------------------------//b04
----------- Node head = q.poll(); -----------------------//b05
----------- for (Character letter : head.next.keySet()) { //b06
----------- // traverse upwards to get nearest fail link -//b07
--------------- Node p = head; --------------------------//b08
--------------- Node nextNode = head.get(letter); -------//b09
--------------- do { p = p.fail; } ----------------------//b0a
--------------- while(p != root && !p.contains(letter)); -//b0b
--------------- if (p.contains(letter)) { // fail link found
------------------- p = p.get(letter); ------------------//b0d
------------------- nextNode.fail = p; ------------------//b0e
------------------- nextNode.count += p.count; ----------//b0f
--------------- } else { nextNode.fail = root; } --------//b10
--------------- q.offer(nextNode); ----------------------//b11
----------- }}} -----------------------------------------//b12
--- public BigInteger search(String s) { ----------------//b13
------- // counts the words added in trie present in s ---//b14
------- Node root = this, p = this; ---------------------//b15
------- BigInteger ans = BigInteger.ZERO; ---------------//b16
------- for (char c : s.toCharArray()) { ----------------//b17
----------- while (p != root && !p.contains(c)) p = p.fail;
----------- if (p.contains(c)) { ------------------------//b19
--------------- p = p.get(c); ---------------------------//b1a
--------------- ans = ans.add(BigInteger.valueOf(p.count));
----------- } -------------------------------------------//b1c
------- } return ans; } ---------------------------------//b1d
--- // helper methods ------------------------------------//b1e
--- private Node get(char c) { return next.get(c); } -----//b1f
--- private boolean contains(char c) { ------------------//b20
------- return next.containsKey(c); ---------------------//b21
}} // Usage: Node trie = new Node(); --------------------//b22
// for (String s : dictionary) trie.add(s); -------------//b23
// trie.prepare(); BigInteger m = trie.search(str); ------//b24
```

4.6. **Palindromic Tree.** Find lengths and frequencies of all palindromic substrings of a string in $O(n)$ time.

Theorem: there can only be up to n unique palindromic substrings for any string.

```
int par[N*2+1], child[N*2+1][128]; ----------------------//b52
int len[N*2+1], node[N*2+1], cs[N*2+1], size; -----------//b53
long long cnt[N + 2]; // count can be very large --------//b54
int newNode(int p = -1) { -------------------------------//b55
--- cnt[size] = 0; par[size] = p; -----------------------//b56
--- len[size] = (p == -1 ? 0 : len[p] + 2); -------------//b57
--- memset(child[size], -1, sizeof child[size]); --------//b58
--- return size++; --------------------------------------//b59
} -------------------------------------------------------//b5a
int get(int i, char c) { --------------------------------//b5b
--- if (child[i][c] == -1) child[i][c] = newNode(i); -----//b5c
--- return child[i][c]; ---------------------------------//b5d
} -------------------------------------------------------//b5e
void manachers(char s[]) { ------------------------------//b5f
--- int n = strlen(s), cn = n * 2 + 1; ------------------//b60
--- for (int i = 0; i < n; i++) -------------------------//b61
------- {cs[i * 2] = -1; cs[i * 2 + 1] = s[i];} ---------//b62
--- size = n * 2; ---------------------------------------//b63
--- int odd = newNode(), even = newNode(); --------------//b64
--- int cen = 0, rad = 0, L = 0, R = 0; -----------------//b65
--- size = 0; len[odd] = -1; ----------------------------//b66
--- for (int i = 0; i < cn; i++) ------------------------//b67
------- node[i] = (i % 2 == 0 ? even : get(odd, cs[i])); -//b68
--- for (int i = 1; i < cn; i++) { ----------------------//b69
------- if (i > rad) { L = i - 1; R = i + 1; } ----------//b6a
------- else { ------------------------------------------//b6b
----------- int M = cen * 2 - i; // retrieve from mirror -//b6c
----------- node[i] = node[M]; --------------------------//b6d
----------- if (len[node[M]] < rad - i) L = -1; ---------//b6e
----------- else { --------------------------------------//b6f
--------------- R = rad + 1; L = i * 2 - R; -------------//b70
--------------- while (len[node[i]] > rad - i) ----------//b71
------------------- node[i] = par[node[i]]; -------------//b72
----------- } -------------------------------------------//b73
------- } // expand palindrome --------------------------//b74
------- while (L >= 0 && R < cn && cs[L] == cs[R]) { -----//b75
----------- if (cs[L] != -1) node[i] = get(node[i],cs[L]);//b76
----------- L--, R++; -----------------------------------//b77
------- } -----------------------------------------------//b78
------- cnt[node[i]]++; ---------------------------------//b79
```

```
------- if (i + len[node[i]] > rad) ---------------------//b7a
------- { rad = i + len[node[i]]; cen = i; } ------------//b7b
--- } ---------------------------------------------------//b7c
--- for (int i = size - 1; i >= 0; --i) -----------------//b7d
--- cnt[par[i]] += cnt[i]; // update parent count -------//b7e
} -------------------------------------------------------//b7f
int countUniquePalindromes(char s[]) --------------------//b80
--- {manachers(s); return size;} ------------------------//b81
int countAllPalindromes(char s[]) { ---------------------//b82
--- manachers(s); int total = 0; ------------------------//b83
--- for (int i = 0; i < size; i++) total += cnt[i]; -----//b84
--- return total;} --------------------------------------//b85
// longest palindrome substring of s --------------------//b86
string longestPalindrome(char s[]) { --------------------//b87
--- manachers(s); ---------------------------------------//b88
--- int n = strlen(s), cn = n * 2 + 1, mx = 0; ----------//b89
--- for (int i = 1; i < cn; i++) ------------------------//b8a
------- if (len[node[mx]] < len[node[i]]) ---------------//b8b
----------- mx = i; -------------------------------------//b8c
--- int pos = (mx - len[node[mx]]) / 2; -----------------//b8d
--- return string(s + pos, s + pos + len[node[mx]]); } --//b8e
```

4.7. **Z Algorithm.** Find the longest common prefix of all substrings of s with itself in $O(n)$ time.

```
int z[N]; // z[i] = lcp(s, s[i:]) -----------------------//be9
void computeZ(string s) { -------------------------------//bea
--- int n = s.length(), L = 0, R = 0; z[0] = n; ---------//beb
--- for (int i = 1; i < n; i++) { -----------------------//bec
------- if (i > R) { ------------------------------------//bed
----------- L = R = i; ----------------------------------//bee
----------- while (R < n && s[R - L] == s[R]) R++; ------//bef
----------- z[i] = R - L; R--; --------------------------//bf0
------- } else { ----------------------------------------//bf1
----------- int k = i - L; ------------------------------//bf2
----------- if (z[k] < R - i + 1) z[i] = z[k]; ----------//bf3
----------- else { --------------------------------------//bf4
--------------- L = i; ----------------------------------//bf5
--------------- while (R < n && s[R - L] == s[R]) R++; --//bf6
--------------- z[i] = R - L; R--; ----------------------//bf7
----------- }}}} ----------------------------------------//bf8
```

4.8. **Booth's Minimum String Rotation.** Booth's Algo: Find the index of the lexicographically least string rotation in $O(n)$ time.

```
int f[N * 2]; -------------------------------------------//b25
int booth(string S) { -----------------------------------//b26
--- S.append(S); // concatenate itself ------------------//b27
--- int n = S.length(), i, j, k = 0; --------------------//b28
--- memset(f, -1, sizeof(int) * n); ---------------------//b29
--- for (j = 1; j < n; j++) { ---------------------------//b2a
------- i = f[j-k-1]; -----------------------------------//b2b
------- while (i != -1 && S[j] != S[k + i + 1]) { -------//b2c
----------- if (S[j] < S[k + i + 1]) k = j - i - 1; -----//b2d
----------- i = f[i]; -----------------------------------//b2e
------- } if (i == -1 && S[j] != S[k + i + 1]) { --------//b2f
----------- if (S[j] < S[k + i + 1]) k = j; -------------//b30
----------- f[j - k] = -1; ------------------------------//b31
------- } else f[j - k] = i + 1; ------------------------//b32
--- } return k; } ---------------------------------------//b33
```

4.9. **Hashing.**

4.9.1. *Polynomial Hashing.*

```
int MAXN = 1e5+1, MOD = 1e9+7; --------------------------//b8f
struct hasher { -----------------------------------------//b90
- int n; ------------------------------------------------//b91
- std::vector<ll> *p_pow; -------------------------------//b92
- std::vector<ll> *h_ans; -------------------------------//b93
- hash(vi &s, vi primes) { ------------------------------//b94
--- n = primes.size(); ----------------------------------//b95
--- p_pow = new std::vector<ll>[n]; ---------------------//b96
--- h_ans = new std::vector<ll>[n]; ---------------------//b97
--- for (int i = 0; i < n; ++i) { -----------------------//b98
----- p_pow[i] = std::vector<ll>(MAXN); -----------------//b99
----- p_pow[i][0] = 1; ----------------------------------//b9a
----- for (int j = 0; j+1 < MAXN; ++j) ------------------//b9b
------- p_pow[i][j+1] = (p_pow[i][j] * primes[i]) % MOD; -//b9c
----- h_ans[i] = std::vector<ll>(MAXN); -----------------//b9d
----- h_ans[i][0] = 0; ----------------------------------//b9e
----- for (int j = 0; j < s.size(); ++j) ----------------//b9f
------- h_ans[i][j+1] = (h_ans[i][j] + ------------------//ba0
------------------------ s[j] * p_pow[i][j]) % MOD; -----//ba1
--- } ---------------------------------------------------//ba2
- } -----------------------------------------------------//ba3
}; ------------------------------------------------------//ba4
```

5. Number Theory

5.1. **Eratosthenes Prime Sieve.**

```
bitset<N> is; // #include <bitset> ----------------------//8e2
int pr[N], primes = 0; ----------------------------------//8e3
void sieve() { ------------------------------------------//8e4
--- is[2] = true; pr[primes++] = 2; ---------------------//8e5
--- for (int i = 3; i < N; i += 2) is[i] = 1; -----------//8e6
--- for (int i = 3; i*i < N; i += 2) --------------------//8e7
------- if (is[i]) --------------------------------------//8e8
----------- for (int j = i*i; j < N; j += i) ------------//8e9
--------------- is[j]= 0; -------------------------------//8ea
--- for (int i = 3; i < N; i += 2) ----------------------//8eb
------- if (is[i]) --------------------------------------//8ec
----------- pr[primes++] = i;} --------------------------//8ed
```

5.2. **Divisor Sieve.**

```
int divisors[N]; // initially 0 -------------------------//86b
void divisorSieve() { -----------------------------------//86c
--- for (int i = 1; i < N; i++) -------------------------//86d
------- for (int j = i; j < N; j += i) ------------------//86e
----------- divisors[j]++;} -----------------------------//86f
```

5.3. **Number/Sum of Divisors.** If a number n is prime factorized where $n = {p_1}^{e_1} \times {p_2}^{e_2} \times \cdots \times {p_k}^{e_k}$, where σ_0 is the number of divisors while σ_1 is the sum of divisors:

$$\sum_{d|n} d^k = \sigma_k(n) = \prod \frac{{p_i}^{k(e_i)+1} - 1}{p_i - 1}$$

$$\text{Product: } \prod_{d|n} d = n^{\frac{\sigma_1(n)}{2}}$$

5.4. **Möbius Sieve.** The Möbius function μ is the Möbius inverse of e such that $e(n) = \sum_{d|n} \mu(d)$.

```
bitset<N> is; int mu[N]; --------------------------------//8d0
void mobiusSieve() { ------------------------------------//8d1
--- for (int i = 1; i < N; ++i) mu[i] = 1; --------------//8d2
--- for (int i = 2; i < N; ++i) if (!is[i]) { -----------//8d3
------- for (int j = i; j < N; j += i){ -----------------//8d4
----------- is[j] = 1; ----------------------------------//8d5
----------- mu[j] *= -1; --------------------------------//8d6
------- } -----------------------------------------------//8d7
------- for (long long j = 1LL*i*i; j < N; j += i*i) ----//8d8
----------- mu[j] = 0;} ---------------------------------//8d9
```

5.5. **Möbius Inversion.** Given arithmetic functions f and g:

$$g(n) = \sum_{d|n} f(d) \quad \Leftrightarrow \quad f(n) = \sum_{d|n} \mu(d)\, g\left(\frac{n}{d}\right)$$

5.6. **GCD Subset Counting.** Count number of subsets $S \subseteq A$ such that $\gcd(S) = g$ (modifiable).

```
int f[MX+1]; // MX is maximum number of array -----------//886
long long gcnt[MX+1]; // gcnt[G]: answer when gcd==G ----//887
long long C(int f) {return (1ll << f) - 1;} -------------//888
// f: frequency count -----------------------------------//889
// C(f): # of subsets of f elements (YOU CAN EDIT) ------//88a
void gcd_counter(int a[], int n) { ----------------------//88b
--- memset(f, 0, sizeof f); -----------------------------//88c
--- memset(gcnt, 0, sizeof gcnt); -----------------------//88d
--- int mx = 0; -----------------------------------------//88e
--- for (int i = 0; i < n; ++i) { -----------------------//88f
------- f[a[i]] += 1; -----------------------------------//890
------- mx = max(mx, a[i]); -----------------------------//891
--- } ---------------------------------------------------//892
--- for (int i = mx; i >= 1; --i) { ---------------------//893
------- int add = f[i]; ---------------------------------//894
------- long long sub = 0; ------------------------------//895
------- for (int j = 2*i; j <= mx; j += i) { ------------//896
----------- add += f[j]; --------------------------------//897
----------- sub += gcnt[j]; -----------------------------//898
------- } -----------------------------------------------//899
------- gcnt[i] = C(add) - sub; -------------------------//89a
--- }} // Usage: int subsets_with_gcd_1 = gcnt[1]; ------//89b
```

5.7. **Euler Totient.** Counts all integers from 1 to n that are relatively prime to n in $O(\sqrt{n})$ time.

```
LL totient(LL n) { --------------------------------------//906
--- if (n <= 1) return 1; -------------------------------//907
--- LL tot = n; -----------------------------------------//908
--- for (int i = 2; i * i <= n; i++) { ------------------//909
------- if (n % i == 0) tot -= tot / i; -----------------//90a
------- while (n % i == 0) n /= i; ----------------------//90b
--- } ---------------------------------------------------//90c
--- if (n > 1) tot -= tot / n; --------------------------//90d
--- return tot; } ---------------------------------------//90e
```

5.8. Euler Phi Sieve. Sieve version of Euler totient, runs in $O(N \log N)$ time. Note that $n = \sum_{d|n} \varphi(d)$.

```
bitset<N> is; int phi[N]; ---------------------------------//8da
void phiSieve() { -----------------------------------------//8db
--- for (int i = 1; i < N; ++i) phi[i] = i; --------------//8dc
--- for (int i = 2; i < N; ++i) if (!is[i]) { ------------//8dd
------- for (int j = i; j < N; j += i) { -----------------//8de
----------- phi[j] -= phi[j] / i; -------------------------//8df
----------- is[j] = true; ---------------------------------//8e0
------- }}} -----------------------------------------------//8e1
```

5.9. Extended Euclidean. Assigns x, y such that $ax + by = \gcd(a, b)$ and returns $\gcd(a, b)$.

```
typedef long long LL; -------------------------------------//870
typedef pair<LL, LL> PAIR; --------------------------------//871
LL mod(LL x, LL m) { // use this instead of x % m --------//872
--- if (m == 0) return 0; ---------------------------------//873
--- if (m < 0) m *= -1; -----------------------------------//874
--- return (x%m + m) % m; // always nonnegative ----------//875
} ---------------------------------------------------------//876
LL extended_euclid(LL a, LL b, LL &x, LL &y) { -----------//877
--- if (b==0) {x = 1; y = 0; return a;} ------------------//878
--- LL g = extended_euclid(b, a%b, x, y); ----------------//879
--- LL z = x - a/b*y; ------------------------------------//87a
--- x = y; y = z; return g; ------------------------------//87b
} ---------------------------------------------------------//87c
```

5.10. Modular Exponentiation. Find $b^e \pmod m$ in $O(loge)$ time.

```
template <class T> ----------------------------------------//8c9
T mod_pow(T b, T e, T m) { -------------------------------//8ca
- T res = T(1); -------------------------------------------//8cb
- while (e) { ---------------------------------------------//8cc
--- if (e & T(1)) res = smod(res * b, m); ----------------//8cd
--- b = smod(b * b, m), e >>= T(1); } --------------------//8ce
- return res; } -------------------------------------------//8cf
```

5.11. Modular Inverse. Find unique x such that $ax \equiv 1 \pmod m$. Returns 0 if no unique solution is found. Please use modulo solver for the non-unique case.

```
LL modinv(LL a, LL m) { -----------------------------------//8bf
--- LL x, y; LL g = extended_euclid(a, m, x, y); ---------//8c0
--- if (g == 1 || g == -1) return mod(x * g, m); ---------//8c1
--- return 0; // 0 if invalid ----------------------------//8c2
} ---------------------------------------------------------//8c3
```

5.12. Modulo Solver. Solve for values of x for $ax \equiv b \pmod m$. Returns $(-1, -1)$ if there is no solution. Returns a pair (x, M) where solution is $x \bmod M$.

```
PAIR modsolver(LL a, LL b, LL m) { -----------------------//8c4
--- LL x, y; LL g = extended_euclid(a, m, x, y); ---------//8c5
--- if (b % g != 0) return PAIR(-1, -1); -----------------//8c6
--- return PAIR(mod(x*b/g, m/g), abs(m/g)); --------------//8c7
} ---------------------------------------------------------//8c8
```

5.13. Linear Diophantine. Computes integers x and y such that $ax + by = c$, returns $(-1, -1)$ if no solution. Tries to return positive integer answers for x and y if possible.

```
PAIR null(-1, -1); // needs extended euclidean -----------//8a2
PAIR diophantine(LL a, LL b, LL c) { ---------------------//8a3
--- if (!a && !b) return c ? null : PAIR(0, 0); ----------//8a4
--- if (!a) return c % b ? null : PAIR(0, c / b); --------//8a5
--- if (!b) return c % a ? null : PAIR(c / a, 0); --------//8a6
--- LL x, y; LL g = extended_euclid(a, b, x, y); ---------//8a7
--- if (c % g) return null; ------------------------------//8a8
--- y = mod(y * (c/g), a/g); -----------------------------//8a9
--- if (y == 0) y += abs(a/g); // prefer positive sol. ---//8aa
--- return PAIR((c - b*y)/a, y); -------------------------//8ab
} ---------------------------------------------------------//8ac
```

5.14. Chinese Remainder Theorem. Solves linear congruence $x \equiv b_i \pmod{m_i}$. Returns $(-1, -1)$ if there is no solution. Returns a pair (x, M) where solution is $x \bmod M$.

```
PAIR chinese(LL b1, LL m1, LL b2, LL m2) { ---------------//85d
--- LL x, y; LL g = extended_euclid(m1, m2, x, y); -------//85e
--- if (b1 % g != b2 % g) return PAIR(-1, -1); -----------//85f
--- LL M = abs(m1 / g * m2); -----------------------------//860
--- return PAIR(mod(mod(x*b2*m1+y*b1*m2, M*g)/g,M),M); ---//861
} ---------------------------------------------------------//862
PAIR chinese_remainder(LL b[], LL m[], int n) { ----------//863
--- PAIR ans(0, 1); --------------------------------------//864
--- for (int i = 0; i < n; ++i) { ------------------------//865
------- ans = chinese(b[i],m[i],ans.first,ans.second); ---//866
------- if (ans.second == -1) break; ---------------------//867
------- } ------------------------------------------------//868
--- return ans; ------------------------------------------//869
} ---------------------------------------------------------//86a
```

5.14.1. *Super Chinese Remainder.* Solves linear congruence $a_i x \equiv b_i \pmod{m_i}$. Returns $(-1, -1)$ if there is no solution.

```
PAIR super_chinese(LL a[], LL b[], LL m[], int n) { ------//8fb
--- PAIR ans(0, 1); --------------------------------------//8fc
--- for (int i = 0; i < n; ++i) { ------------------------//8fd
------- PAIR two = modsolver(a[i], b[i], m[i]); ----------//8fe
------- if (two.second == -1) return two; ----------------//8ff
------- ans = chinese(ans.first, ans.second, -------------//900
------- two.first, two.second); --------------------------//901
------- if (ans.second == -1) break; ---------------------//902
--- } ----------------------------------------------------//903
--- return ans; ------------------------------------------//904
} ---------------------------------------------------------//905
```

5.15. Primitive Root.

```
#include "mod_pow.cpp" ------------------------------------//8ee
ll primitive_root(ll m) { --------------------------------//8ef
- vector<ll> div; ----------------------------------------//8f0
- for (ll i = 1; i*i <= m-1; i++) { ----------------------//8f1
--- if ((m-1) % i == 0) { --------------------------------//8f2
----- if (i < m) div.push_back(i); -----------------------//8f3
----- if (m/i < m) div.push_back(m/i); } } ---------------//8f4
- rep(x,2,m) { -------------------------------------------//8f5
--- bool ok = true; --------------------------------------//8f6
--- iter(it,div) if (mod_pow<ll>(x, *it, m) == 1) { ------//8f7
----- ok = false; break; } -------------------------------//8f8
--- if (ok) return x; } ----------------------------------//8f9
- return -1; } -------------------------------------------//8fa
```

5.16. Josephus. Last man standing out of n if every kth is killed. Zero-based, and does not kill 0 on first pass.

```
int J(int n, int k) { ------------------------------------//89c
- if (n == 1) return 0; ----------------------------------//89d
- if (k == 1) return n-1; --------------------------------//89e
- if (n < k) return (J(n-1,k)+k)%n; ----------------------//89f
- int np = n - n/k; --------------------------------------//8a0
- return k*((J(np,k)+np-n%k%np)%np) / (k-1); } ----------//8a1
```

5.17. Number of Integer Points under a Lines. Count the number of integer solutions to $Ax + By \leq C$, $0 \leq x \leq n$, $0 \leq y$. In other words, evaluate the sum $\sum_{x=0}^{n} \left\lfloor \frac{C - Ax}{B} + 1 \right\rfloor$. To count all solutions, let $n = \left\lfloor \frac{c}{a} \right\rfloor$. In any case, it must hold that $C - nA \geq 0$. Be very careful about overflows.

6. Algebra

6.1. Fast Fourier Transform. Compute the Discrete Fourier Transform (DFT) of a polynomial in $O(n \log n)$ time.

```
struct poly { --------------------------------------------//01d
--- double a, b; -----------------------------------------//01e
--- poly(double a=0, double b=0): a(a), b(b) {} ----------//01f
--- poly operator+(const poly& p) const { ----------------//020
------- return poly(a + p.a, b + p.b);} ------------------//021
--- poly operator-(const poly& p) const { ----------------//022
------- return poly(a - p.a, b - p.b);} ------------------//023
--- poly operator*(const poly& p) const { ----------------//024
------- return poly(a*p.a - b*p.b, a*p.b + b*p.a);} ------//025
}; -------------------------------------------------------//026
void fft(poly in[], poly p[], int n, int s) { ------------//027
--- if (n < 1) return; -----------------------------------//028
--- if (n == 1) {p[0] = in[0]; return;} ------------------//029
--- n >>= 1; fft(in, p, n, s << 1); ----------------------//02a
--- fft(in + s, p + n, n, s << 1); -----------------------//02b
--- poly w(1), wn(cos(M_PI/n), sin(M_PI/n)); -------------//02c
--- for (int i = 0; i < n; ++i) { ------------------------//02d
------- poly even = p[i], odd = p[i + n]; ----------------//02e
------- p[i] = even + w * odd; ---------------------------//02f
------- p[i + n] = even - w * odd; -----------------------//030
------- w = w * wn; --------------------------------------//031
--- } ----------------------------------------------------//032
} --------------------------------------------------------//033
void fft(poly p[], int n) { ------------------------------//034
--- poly *f = new poly[n]; fft(p, f, n, 1); --------------//035
--- copy(f, f + n, p); delete[] f; -----------------------//036
} --------------------------------------------------------//037
void inverse_fft(poly p[], int n) { ----------------------//038
--- for(int i=0; i<n; i++) {p[i].b *= -1;} fft(p, n); ----//039
--- for(int i=0; i<n; i++) {p[i].a/=n; p[i].b/= -1*n;} ---//03a
} --------------------------------------------------------//03b
```

6.2. FFT Polynomial Multiplication. Multiply integer polynomials a, b of size an, bn using FFT in $O(n \log n)$. Stores answer in an array c, rounded to the nearest integer (or double).

```
// note: c[] should have size of at least (an+bn) --------//00f
int mult(int a[],int an,int b[],int bn,int c[]) { --------//010
--- int n, degree = an + bn - 1; -------------------------//011
--- for (n = 1; n < degree; n <<= 1); // power of 2 ------//012
--- poly *A = new poly[n], *B = new poly[n]; -------------//013
--- copy(a, a + an, A); fill(A + an, A + n, 0); ----------//014
--- copy(b, b + bn, B); fill(B + bn, B + n, 0); ----------//015
--- fft(A, n); fft(B, n); --------------------------------//016
--- for (int i = 0; i < n; i++) A[i] = A[i] * B[i]; ------//017
--- inverse_fft(A, n); -----------------------------------//018
--- for (int i = 0; i < degree; i++) ---------------------//019
------- c[i] = int(A[i].a + 0.5); // same as round(A[i].a)//01a
--- delete[] A, B; return degree; ------------------------//01b
} --------------------------------------------------------//01c
```

6.3. Number Theoretic Transform. Other possible moduli: $2113929217(2^{25}), 2013265920268435457(2^{28}, with g = 5)$

```
#include "../mathematics/primitive_root.cpp" -------------//063
int mod = 998244353, g = primitive_root(mod), ------------//064
- ginv = mod_pow<ll>(g, mod-2, mod), ---------------------//065
- inv2 = mod_pow<ll>(2, mod-2, mod); ---------------------//066
#define MAXN (1<<22) -------------------------------------//067
struct Num { ---------------------------------------------//068
- int x; -------------------------------------------------//069
- Num(ll _x=0) { x = (_x%mod+mod)%mod; } -----------------//06a
- Num operator +(const Num &b) { return x + b.x; } -------//06b
- Num operator -(const Num &b) const { return x - b.x; } -//06c
- Num operator *(const Num &b) const { return (ll)x * b.x; }
- Num operator /(const Num &b) const { -------------------//06e
--- return (ll)x * b.inv().x; } --------------------------//06f
- Num inv() const { return mod_pow<ll>((ll)x, mod-2, mod); }
- Num pow(int p) const { return mod_pow<ll>((ll)x, p, mod); }
} T1[MAXN], T2[MAXN]; ------------------------------------//072
void ntt(Num x[], int n, bool inv = false) { -------------//073
- Num z = inv ? ginv : g; --------------------------------//074
- z = z.pow((mod - 1) / n); ------------------------------//075
- for (ll i = 0, j = 0; i < n; i++) { --------------------//076
--- if (i < j) swap(x[i], x[j]); -------------------------//077
--- ll k = n>>1; -----------------------------------------//078
--- while (1 <= k && k <= j) j -= k, k >>= 1; -----------//079
--- j += k; } --------------------------------------------//07a
- for (int mx = 1, p = n/2; mx < n; mx <<= 1, p >>= 1) { -//07b
--- Num wp = z.pow(p), w = 1; ----------------------------//07c
--- for (int k = 0; k < mx; k++, w = w*wp) { ------------//07d
----- for (int i = k; i < n; i += mx << 1) { ------------//07e
------- Num t = x[i + mx] * w; ---------------------------//07f
------- x[i + mx] = x[i] - t; ----------------------------//080
------- x[i] = x[i] + t; } } } ---------------------------//081
- if (inv) { ---------------------------------------------//082
--- Num ni = Num(n).inv(); -------------------------------//083
--- rep(i,0,n) { x[i] = x[i] * ni; } } } -----------------//084
void inv(Num x[], Num y[], int l) { ----------------------//085
- if (l == 1) { y[0] = x[0].inv(); return; } -------------//086
- inv(x, y, l>>1); ---------------------------------------//087
- // NOTE: maybe l<<2 instead of l<<1 --------------------//088
- rep(i,l>>1,l<<1) T1[i] = y[i] = 0; --------------------//089
- rep(i,0,l) T1[i] = x[i]; -------------------------------//08a
- ntt(T1, l<<1); ntt(y, l<<1); ---------------------------//08b
- rep(i,0,l<<1) y[i] = y[i]*2 - T1[i] * y[i] * y[i]; -----//08c
- ntt(y, l<<1, true); } ----------------------------------//08d
void sqrt(Num x[], Num y[], int l) { ---------------------//08e
- if (l == 1) { assert(x[0].x == 1); y[0] = 1; return; } -//08f
- sqrt(x, y, l>>1); --------------------------------------//090
- inv(y, T2, l>>1); --------------------------------------//091
- rep(i,l>>1,l<<1) T1[i] = T2[i] = 0; -------------------//092
- rep(i,0,l) T1[i] = x[i]; -------------------------------//093
- ntt(T2, l<<1); ntt(T1, l<<1); --------------------------//094
- rep(i,0,l<<1) T2[i] = T1[i] * T2[i]; ------------------//095
- ntt(T2, l<<1, true); -----------------------------------//096
- rep(i,0,l) y[i] = (y[i] + T2[i]) * inv2; } -------------//097
```

6.4. Polynomial Long Division. Divide two polynomials A and B to get Q and R, where $\frac{A}{B} = Q + \frac{R}{B}$.

```
typedef vector<double> Poly; -----------------------------//098
Poly Q, R; // quotient and remainder ---------------------//099
void trim(Poly& A) { // remove trailing zeroes -----------//09a
--- while (!A.empty() && abs(A.back()) < EPS) ------------//09b
--- A.pop_back(); ----------------------------------------//09c
} --------------------------------------------------------//09d
void divide(Poly A, Poly B) { ----------------------------//09e
--- if (B.size() == 0) throw exception(); ----------------//09f
--- if (A.size() < B.size()) {Q.clear(); R=A; return;} ---//0a0
--- Q.assign(A.size() - B.size() + 1, 0); ----------------//0a1
--- Poly part; -------------------------------------------//0a2
--- while (A.size() >= B.size()) { -----------------------//0a3
------- int As = A.size(), Bs = B.size(); ----------------//0a4
------- part.assign(As, 0); ------------------------------//0a5
------- for (int i = 0; i < Bs; i++) ---------------------//0a6
----------- part[As-Bs+i] = B[i]; ------------------------//0a7
------- double scale = Q[As-Bs] = A[As-1] / part[As-1]; --//0a8
------- for (int i = 0; i < As; i++) ---------------------//0a9
----------- A[i] -= part[i] * scale; ---------------------//0aa
------- trim(A); -----------------------------------------//0ab
--- } R = A; trim(Q); } ----------------------------------//0ac
```

6.5. Matrix Multiplication. Multiplies matrices $A_{p \times q}$ and $B_{q \times r}$ in $O(n^3)$ time, modulo `MOD`.

```
long[][] multiply(long A[][], long B[][]) { --------------//052
--- int p = A.length, q = A[0].length, r = B[0].length; --//053
--- // if(q != B.length) throw new Exception(":((("); ----//054
--- long AB[][] = new long[p][r]; ------------------------//055
--- for (int i = 0; i < p; i++) --------------------------//056
--- for (int j = 0; j < q; j++) --------------------------//057
--- for (int k = 0; k < r; k++) --------------------------//058
------- (AB[i][k] += A[i][j] * B[j][k]) %= MOD; ----------//059
--- return AB; } -----------------------------------------//05a
```

6.6. Matrix Power. Computes for B^e in $O(n^3 \log e)$ time. Refer to Matrix Multiplication.

```
long[][] power(long B[][], long e) { ---------------------//05b
--- int n = B.length; ------------------------------------//05c
--- long ans[][]= new long[n][n]; ------------------------//05d
--- for (int i = 0; i < n; i++) ans[i][i] = 1; -----------//05e
--- while (e > 0) { --------------------------------------//05f
------- if (e % 2 == 1) ans = multiply(ans, b); ----------//060
------- b = multiply(b, b); e /= 2; ----------------------//061
--- } return ans;} ---------------------------------------//062
```

6.7. Fibonacci Matrix. Fast computation for nth Fibonacci $\{F_1, F_2, \ldots, F_n\}$ in $O(\log n)$:

$$\begin{bmatrix} F_n \\ F_{n-1} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^n \times \begin{bmatrix} F_2 \\ F_1 \end{bmatrix}$$

6.8. Gauss-Jordan/Matrix Determinant. Row reduce matrix A in $O(n^3)$ time. Returns `true` if a solution exists.

```
boolean gaussJordan(double A[][]) { ----------------------//03c
--- int n = A.length, m = A[0].length; -------------------//03d
--- boolean singular = false; ----------------------------//03e
--- // double determinant = 1; ---------------------------//03f
--- for (int i=0, p=0; i<n && p<m; i++, p++) { -----------//040
------- for (int k = i + 1; k < n; k++) { ----------------//041
----------- if (Math.abs(A[k][p]) > EPS) { // swap -------//042
----------- // determinant *= -1; ------------------------//043
----------- double t[]=A[i]; A[i]=A[k]; A[k]=t; ----------//044
----------- break; ---------------------------------------//045
----------- } --------------------------------------------//046
------- } ------------------------------------------------//047
------- // determinant *= A[i][p]; -----------------------//048
------- if (Math.abs(A[i][p]) < EPS) ---------------------//049
----------- { singular = true; i--; continue; } ----------//04a
------- for (int j = m-1; j >= p; j--) A[i][j]/= A[i][p]; //04b
------- for (int k = 0; k < n; k++) { --------------------//04c
----------- if (i == k) continue; ------------------------//04d
----------- for (int j = m-1; j >= p; j--) ---------------//04e
--------------- A[k][j] -= A[k][p] * A[i][j]; ------------//04f
------- } ------------------------------------------------//050
--- } return !singular; } --------------------------------//051
```

7. Combinatorics

7.1. Lucas Theorem. Compute $\binom{n}{k} \bmod p$ in $O(p + \log_p n)$ time, where p is a prime.

```
LL f[P], lid; // P: biggest prime ------------------------//0f2
LL lucas(LL n, LL k, int p) { ----------------------------//0f3
--- if (k == 0) return 1; --------------------------------//0f4
--- if (n < p && k < p) { --------------------------------//0f5
------- if (lid != p) { ----------------------------------//0f6
----------- lid = p; f[0] = 1; ---------------------------//0f7
----------- for (int i = 0; i < p; ++i) f[i]=f[i-1]*i%p; -//0f8
------- } ------------------------------------------------//0f9
------- return f[n] * modpow(f[n-k]*f[k]%p, p-2, p) % p;} //0fa
--- return lucas(n/p,k/p,p) * lucas(n%p,k%p,p) % p; } ----//0fb
```

7.2. Granville's Theorem. Compute $\binom{n}{k} \bmod m$ (for any m) in $O(m^2 \log^2 n)$ time.

```
def fprime(n, p): ---------------------------------------//0bd
--- # counts the number of prime divisors of n! ----------//0be
--- pk, ans = p, 0 --------------------------------------//0bf
--- while pk <= n: --------------------------------------//0c0
------- ans += n // pk ----------------------------------//0c1
------- pk *= p -----------------------------------------//0c2
--- return ans ------------------------------------------//0c3
def granville(n, k, p, E): ------------------------------//0c4
--- # n choose k (mod p^E) ------------------------------//0c5
--- prime_pow = fprime(n,p)-fprime(k,p)-fprime(n-k,p) ----//0c6
--- if prime_pow >= E: return 0 -------------------------//0c7
--- e = E - prime_pow -----------------------------------//0c8
--- pe = p ** e -----------------------------------------//0c9
--- r, f = n - k, [1]*pe --------------------------------//0ca
--- for i in range(1, pe): ------------------------------//0cb
------- x = i -------------------------------------------//0cc
------- if x % p == 0: ----------------------------------//0cd
----------- x = 1 ---------------------------------------//0ce
------- f[i] = f[i-1] * x % pe --------------------------//0cf
--- numer, denom, negate, ptr = 1, 1, 0, 0 --------------//0d0
--- while n: --------------------------------------------//0d1
------- if f[-1] != 1 and ptr >= e: ---------------------//0d2
----------- negate ^= (n&1) ^ (k&1) ^ (r&1) -------------//0d3
------- numer = numer * f[n%pe] % pe --------------------//0d4
------- denom = denom * f[k%pe] % pe * f[r%pe] % pe ------//0d5
------- n, k, r = n//p, k//p, r//p ----------------------//0d6
------- ptr += 1 ----------------------------------------//0d7
--- ans = numer * modinv(denom, pe) % pe ----------------//0d8
--- if negate and (p != 2 or e < 3): --------------------//0d9
------- ans = (pe - ans) % pe ---------------------------//0da
--- return mod(ans * p**prime_pow, p**E) ----------------//0db
def choose(n, k, m): # generalized (n choose k) mod m ----//0dc
--- factors, x, p = [], m, 2 ----------------------------//0dd
--- while p*p <= x: -------------------------------------//0de
------- e = 0 -------------------------------------------//0df
------- while x % p == 0: -------------------------------//0e0
----------- e += 1 --------------------------------------//0e1
----------- x //= p -------------------------------------//0e2
------- if e: factors.append((p, e)) --------------------//0e3
------- p += 1 ------------------------------------------//0e4
--- if x > 1: factors.append((x, 1)) --------------------//0e5
--- crt_array = [granville(n,k,p,e) for p, e in factors] -//0e6
--- mod_array = [p**e for p, e in factors] --------------//0e7
--- return chinese_remainder(crt_array, mod_array)[0] ----//0e8
```

7.3. **Derangements.** Compute the number of permutations with n elements such that no element is at their original position:

$$D(n) = (n-1)\left(D(n-1) + D(n-2)\right) = nD(n-1) + (-1)^n$$

7.4. **Factoradics.** Convert a permutation of n items to factoradics and vice versa in $O(n \log n)$.

```
// use fenwick tree add, sum, and low code ---------------//0ad
typedef long long LL; -----------------------------------//0ae
void factoradic(int arr[], int n) { // 0 to n-1 ----------//0af
--- for (int i = 0; i <=n; i++) fen[i] = 0; --------------//0b0
--- for (int i = 1; i < n; i++) add(i, 1); ---------------//0b1
--- for (int i = 0; i < n; i++) { -----------------------//0b2
--- int s = sum(arr[i]); --------------------------------//0b3
--- add(arr[i], -1); arr[i] = s; ------------------------//0b4
--- }} --------------------------------------------------//0b5
void permute(int arr[], int n) { // factoradic to perm ---//0b6
--- for (int i = 0; i <=n; i++) fen[i] = 0; --------------//0b7
--- for (int i = 1; i < n; i++) add(i, 1); ---------------//0b8
--- for (int i = 0; i < n; i++) { -----------------------//0b9
--- arr[i] = low(arr[i] - 1); ---------------------------//0ba
--- add(arr[i], -1); ------------------------------------//0bb
--- }} --------------------------------------------------//0bc
```

7.5. **kth Permutation.** Get the next kth permutation of n items, if exists, using factoradics. All values should be from 0 to $n-1$. Use factoradics methods as discussed above.

```
bool kth_permutation(int arr[], int n, LL k) { -----------//0e9
--- factoradic(arr, n); // values from 0 to n-1 ---------//0ea
--- for (int i = n-1; i >= 0 && k > 0; --i){ ------------//0eb
------- LL temp = arr[i] + k; ---------------------------//0ec
------- arr[i] = temp % (n - i); ------------------------//0ed
------- k = temp / (n - i); -----------------------------//0ee
--- } ---------------------------------------------------//0ef
--- permute(arr, n); ------------------------------------//0f0
--- return k == 0; } ------------------------------------//0f1
```

7.6. **Catalan Numbers.**

$$C_n = \frac{1}{n+1}\binom{2n}{n} = \binom{2n}{n} - \binom{2n}{n+1}$$

(1) The number of non-crossing partitions of an n-element set
(2) The number of expressions with n pairs of parentheses
(3) The number of ways $n+1$ factors can be parenthesized
(4) The number of full binary trees with $n+1$ leaves
(5) The number of monotonic lattice paths of an $n \times n$ grid (5-SAT problem)
(6) The number of triangulations of a convex polygon with $n+2$ sides (non-rotational)
(7) The number of permutations $\{1, \ldots, n\}$ without a 3-term increasing subsequence
(8) The number of ways to form a mountain range with n ups and n downs

7.7. **Stirling Numbers.** s_1: Count the number of permutations of n elements with k disjoint cycles

s_2: Count the ways to partition a set of n elements into k nonempty subsets

$$s_1(n,k) = \begin{cases} 1 & n = k = 0 \\ s_1(n-1,k-1) - (n-1)s_1(n-1,k) & n,k > 0 \\ 0 & \text{elsewhere} \end{cases}$$

$$s_2(n,k) = \begin{cases} 1 & n = k = 0 \\ s_2(n-1,k-1) + ks_2(n-1,k) & n,k > 0 \\ 0 & \text{elsewhere} \end{cases}$$

7.8. **Partition Function.** Pregenerate the number of partitions of positive integer n with n positive addends.

$$p(n,k) = \begin{cases} 1 & n = k = 0 \\ 0 & n < k \\ p(n-1,k-1) + p(n-k,k) & n \geq k \end{cases}$$

8. Geometry

```
#include <complex> --------------------------------------//37e
#define x real() ----------------------------------------//37f
#define y imag() ----------------------------------------//380
typedef std::complex<double> point; // 2D point only -----//381
const double PI = acos(-1.0), EPS = 1e-7; ---------------//382
```

8.1. **Dots and Cross Products.**

```
double dot(point a, point b) ----------------------------//3cc
- {return a.x * b.x + a.y * b.y;} // + a.z * b.z; --------//3cd
double cross(point a, point b) --------------------------//3ce
- {return a.x * b.y - a.y * b.x;} -----------------------//3cf
double cross(point a, point b, point c) -----------------//3d0
- {return cross(a, b) + cross(b, c) + cross(c, a);} ------//3d1
double cross3D(point a, point b) { ----------------------//3d2
- return point(a.x*b.y - a.y*b.x, a.y*b.z - -------------//3d3
-------------- a.z*b.y, a.z*b.x - a.x*b.z);} ------------//3d4
```

8.2. **Angles and Rotations.**

```
double angle(point a, point b, point c) { ---------------//341
- // angle formed by abc in radians: PI < x <= PI --------//342
- return abs(remainder(arg(a-b) - arg(c-b), 2*PI));} -----//343
point rotate(point p, point a, double d) { --------------//344
- //rotate point a about pivot p CCW at d radians --------//345
- return p + (a - p) * point(cos(d), sin(d));} -----------//346
```

8.3. **Spherical Coordinates.**

$$\begin{aligned} x &= r\cos\theta\cos\phi & r &= \sqrt{x^2+y^2+z^2} \\ y &= r\cos\theta\sin\phi & \theta &= \cos^{-1} x/r \\ z &= r\sin\theta & \phi &= \operatorname{atan2}(y,x) \end{aligned}$$

8.4. **Point Projection.**

```
point proj(point p, point v) { --------------------------//459
- // project point p onto a vector v (2D & 3D) -----------//45a
- return dot(p, v) / norm(v) * v;} ----------------------//45b
point projLine(point p, point a, point b) { -------------//45c
- // project point p onto line ab (2D & 3D) --------------//45d
- return a + dot(p-a, b-a) / norm(b-a) * (b-a);} ---------//45e
point projSeg(point p, point a, point b) { --------------//45f
- // project point p onto segment ab (2D & 3D) -----------//460
- double s = dot(p-a, b-a) / norm(b-a); -----------------//461
- return a + min(1.0, max(0.0, s)) * (b-a);} ------------//462
point projPlane(point p, double a, double b, ------------//463
--------------- double c, double d) { -------------------//464
- // project p onto plane ax+by+cz+d=0 (3D) --------------//465
- // same as: o + p - project(p - o, n); ----------------//466
- double k = -d / (a*a + b*b + c*c); --------------------//467
- point o(a*k, b*k, c*k), n(a, b, c); -------------------//468
- point v(p.x-o.x, p.y-o.y, p.z-o.z); -------------------//469
- double s = dot(v, n) / dot(n, n); ---------------------//46a
```

```
- return point(o.x + p.x + s * n.x, o.y + ----------------//46b
-------------- p.y +s * n.y, o.z + p.z + s * n.z);} ------//46c
```

8.5. Great Circle Distance.

```
double greatCircleDist(double lat1, double long1, --------//3d5
--- double lat2, double long2, double R) { ---------------//3d6
- long1 *= PI / 180; lat1 *= PI / 180; // to radians -----//3d7
- long2 *= PI / 180; lat2 *= PI / 180; -------------------//3d8
- return R*acos(sin(lat1)*sin(lat2) + --------------------//3d9
-------- cos(lat1)*cos(lat2)*cos(abs(long1 - long2))); ---//3da
} --------------------------------------------------------//3db
// another version, using actual (x, y, z) ---------------//3dc
double greatCircleDist(point a, point b) { ---------------//3dd
- return atan2(abs(cross3D(a, b)), dot3D(a, b)); ---------//3de
} --------------------------------------------------------//3df
```

8.6. Point/Line/Plane Distances.

```
double distPtLine(point p, double a, double b, -----------//3b2
--- double c) { ------------------------------------------//3b3
- // dist from point p to line ax+by+c=0 -----------------//3b4
- return abs(a*p.x + b*p.y + c) / sqrt(a*a + b*b);} ------//3b5
double distPtLine(point p, point a, point b) { -----------//3b6
- // dist from point p to line ab ------------------------//3b7
- return abs((a.y - b.y) * (p.x - a.x) + -----------------//3b8
-------- (b.x - a.x) * (p.y - a.y)) / --------------------//3b9
-------- hypot(a.x - b.x, a.y - b.y);} -------------------//3ba
double distPtPlane(point p, double a, double b, ----------//3bb
----- double c, double d) { ------------------------------//3bc
- // distance to 3D plane ax + by + cz + d = 0 -----------//3bd
- return (a*p.x+b*p.y+c*p.z+d)/sqrt(a*a+b*b+c*c); --------//3be
} /*! // distance between 3D lines AB & CD (untested) ----//3bf
double distLine3D(point A,point B,point C,point D){ ------//3c0
- point u = B - A, v = D - C, w = A - C; -----------------//3c1
- double a = dot(u, u), b = dot(u, v); -------------------//3c2
- double c = dot(v, v), d = dot(u, w); -------------------//3c3
- double e = dot(v, w), det = a*c - b*b; -----------------//3c4
- double s = det < EPS ? 0.0 : (b*e - c*d) / det; --------//3c5
- double t = det < EPS -----------------------------------//3c6
--- ? (b > c ? d/b : e/c) // parallel --------------------//3c7
--- : (a*e - b*d) / det; ---------------------------------//3c8
- point top = A + u * s, bot = w - A - v * t; ------------//3c9
- return dist(top, bot); ---------------------------------//3ca
} // dist<EPS: intersection     */ -----------------------//3cb
```

8.7. Intersections.

8.7.1. *Line-Segment Intersection.* Get intersection points of 2D lines/segments $\overline{ab}$ and $\overline{cd}$.

```
point null(HUGE_VAL, HUGE_VAL); --------------------------//40b
point line_inter(point a, point b, point c, --------------//40c
---------------- point d, bool seg = false) { ------------//40d
- point ab(b.x - a.x, b.y - a.y); ------------------------//40e
- point cd(d.x - c.x, d.y - c.y); ------------------------//40f
- point ac(c.x - a.x, c.y - a.y); ------------------------//410
- double D = -cross(ab, cd); // determinant --------------//411
- double Ds = cross(cd, ac); -----------------------------//412
- double Dt = cross(ab, ac); -----------------------------//413
- if (abs(D) < EPS) { // parallel ------------------------//414
--- if (seg && abs(Ds) < EPS) { // collinear -------------//415
----- point p[] = {a, b, c, d}; --------------------------//416
----- sort(p, p + 4, [](point a, point b) { --------------//417
------- return a.x < b.x-EPS || --------------------------//418
-------------- (dist(a,b) < EPS && a.y < b.y-EPS); -------//419
----- }); ------------------------------------------------//41a
----- return dist(p[1], p[2]) < EPS ? p[1] : null; -------//41b
--- } ----------------------------------------------------//41c
--- return null; -----------------------------------------//41d
- } ------------------------------------------------------//41e
- double s = Ds / D, t = Dt / D; -------------------------//41f
- if (seg && (min(s,t)<-EPS||max(s,t)>1+EPS)) ------------//420
--- return null; -----------------------------------------//421
- return point(a.x + s * ab.x, a.y + s * ab.y); ----------//422
}/* double A = cross(d-a, b-a), B = cross(c-a, b-a); -----//423
return (B*d - A*c)/(B - A); */ ---------------------------//424
```

8.7.2. *Circle-Line Intersection.* Get intersection points of circle at center c, radius r, and line $\overline{ab}$.

```
std::vector<point> CL_inter(point c, double r, -----------//358
--- point a, point b) { ----------------------------------//359
- point p = projLine(c, a, b); ---------------------------//35a
- double d = abs(c - p); vector<point> ans; --------------//35b
- if (d > r + EPS); // none ------------------------------//35c
- else if (d > r - EPS) ans.push_back(p); // tangent -----//35d
- else if (d < EPS) { // diameter ------------------------//35e
--- point v = r * (b - a) / abs(b - a); ------------------//35f
--- ans.push_back(c + v); --------------------------------//360
--- ans.push_back(c - v); --------------------------------//361
- } else { -----------------------------------------------//362
--- double t = acos(d / r); ------------------------------//363
--- p = c + (p - c) * r / d; -----------------------------//364
--- ans.push_back(rotate(c, p, t)); ----------------------//365
--- ans.push_back(rotate(c, p, -t)); ---------------------//366
- } return ans; ------------------------------------------//367
} --------------------------------------------------------//368
```

8.7.3. *Circle-Circle Intersection.*

```
std::vector<point> CC_intersection(point c1, -------------//347
--- double r1, point c2, double r2) { --------------------//348
- double d = dist(c1, c2); -------------------------------//349
- vector<point> ans; -------------------------------------//34a
- if (d < EPS) { -----------------------------------------//34b
--- if (abs(r1-r2) < EPS); // inf intersections ----------//34c
- } else if (r1 < EPS) { ---------------------------------//34d
--- if (abs(d - r2) < EPS) ans.push_back(c1); ------------//34e
- } else { -----------------------------------------------//34f
--- double s = (r1*r1 + d*d - r2*r2) / (2*r1*d); ---------//350
--- double t = acos(max(-1.0, min(1.0, s))); -------------//351
--- point mid = c1 + (c2 - c1) * r1 / d; -----------------//352
--- ans.push_back(rotate(c1, mid, t)); -------------------//353
--- if (abs(sin(t)) >= EPS) ------------------------------//354
----- ans.push_back(rotate(c2, mid, -t)); ----------------//355
- } return ans; ------------------------------------------//356
} --------------------------------------------------------//357
```

8.8. Polygon Areas.

Find the area of any 2D polygon given as points in $O(n)$.

```
double area(point p[], int n) { --------------------------//43f
- double a = 0; ------------------------------------------//440
- for (int i = 0, j = n - 1; i < n; j = i++) -------------//441
--- a += cross(p[i], p[j]); ------------------------------//442
- return abs(a) / 2; } -----------------------------------//443
```

8.8.1. *Triangle Area.* Find the area of a triangle using only their lengths. Lengths must be valid.

```
double area(double a, double b, double c) { --------------//478
- double s = (a + b + c) / 2; ----------------------------//479
- return sqrt(s*(s-a)*(s-b)*(s-c)); } --------------------//47a
```

Cyclic Quadrilateral Area. Find the area of a cyclic quadrilateral using only their lengths. A quadrilateral is cyclic if its inner angles sum up to 360°.

```
double area(double a, double b, double c, double d) { ----//3af
- double s = (a + b + c + d) / 2; ------------------------//3b0
- return sqrt((s-a)*(s-b)*(s-c)*(s-d)); } ----------------//3b1
```

8.9. Polygon Centroid.

Get the centroid/center of mass of a polygon in $O(m)$.

```
point centroid(point p[], int n) { -----------------------//444
- point ans(0, 0); ---------------------------------------//445
- double z = 0; ------------------------------------------//446
- for (int i = 0, j = n - 1; i < n; j = i++) { -----------//447
--- double cp = cross(p[j], p[i]); -----------------------//448
--- ans += (p[j] + p[i]) * cp; ---------------------------//449
--- z += cp; ---------------------------------------------//44a
- } return ans / (3 * z); } ------------------------------//44b
```

8.10. Convex Hull.

Get the convex hull of a set of points using Graham-Andrew's scan. This sorts the points at $O(n \log n)$, then performs the Monotonic Chain Algorithm at $O(n)$.

```
// counterclockwise hull in p[], returns size of hull ----//383
bool xcmp(const point& a, const point& b) ----------------//384
- {return a.x < b.x || (a.x == b.x && a.y < b.y);} -------//385
int convex_hull(point p[], int n) { ----------------------//386
- sort(p, p + n, xcmp); if (n <= 1) return n; ------------//387
- int k = 0; point *h = new point[2 * n]; ----------------//388
- double zer = EPS; // -EPS to include collinears --------//389
- for (int i = 0; i < n; h[k++] = p[i++]) ----------------//38a
--- while (k >= 2 && cross(h[k-2],h[k-1],p[i]) < zer) ----//38b
----- --k; -----------------------------------------------//38c
- for(int i = n-2, t = k; i >= 0; h[k++] = p[i--]) -------//38d
--- while (k > t && cross(h[k-2],h[k-1],p[i]) < zer) -----//38e
----- --k; -----------------------------------------------//38f
- k -= 1 + (h[0].x==h[1].x&&h[0].y==h[1].y ? 1 : 0); -----//390
- copy(h, h + k, p); delete[] h; return k; } -------------//391
```

8.11. **Point in Polygon.** Check if a point is strictly inside (or on the border) of a polygon in $O(n)$.

```
bool inPolygon(point q, point p[], int n) { --------------//44c
- bool in = false; -----------------------------------------//44d
- for (int i = 0, j = n - 1; i < n; j = i++) --------------//44e
--- in ^= (((p[i].y > q.y) != (p[j].y > q.y)) && ---------//44f
----- q.x < (p[j].x - p[i].x) * (q.y - p[i].y) / ---------//450
----- (p[j].y - p[i].y) + p[i].x); -----------------------//451
- return in; } --------------------------------------------//452
bool onPolygon(point q, point p[], int n) { --------------//453
- for (int i = 0, j = n - 1; i < n; j = i++) --------------//454
- if (abs(dist(p[i], q) + dist(p[j], q) - -----------------//455
--------- dist(p[i], p[j])) < EPS) ------------------------//456
--- return true; ------------------------------------------//457
- return false; } -----------------------------------------//458
```

8.12. **Cut Polygon by a Line.** Cut polygon by line $\overline{ab}$ to its left in $O(n)$, such that $\angle abp$ is counter-clockwise.

```
vector<point> cut(point p[],int n,point a,point b) { -----//3a6
- vector<point> poly; -------------------------------------//3a7
- for (int i = 0, j = n - 1; i < n; j = i++) { -----------//3a8
--- double c1 = cross(a, b, p[j]); -----------------------//3a9
--- double c2 = cross(a, b, p[i]); -----------------------//3aa
--- if (c1 > -EPS) poly.push_back(p[j]); -----------------//3ab
--- if (c1 * c2 < -EPS) ----------------------------------//3ac
----- poly.push_back(line_inter(p[j], p[i], a, b)); ------//3ad
- } return poly; } ----------------------------------------//3ae
```

8.13. **Triangle Centers.**

```
point bary(point A, point B, point C, --------------------//47b
---------- double a, double b, double c) { ---------------//47c
- return (A*a + B*b + C*c) / (a + b + c);} ---------------//47d
point trilinear(point A, point B, point C, ---------------//47e
--------------- double a, double b, double c) { ----------//47f
- return bary(A,B,C,abs(B-C)*a, --------------------------//480
------------- abs(C-A)*b,abs(A-B)*c);} -------------------//481
point centroid(point A, point B, point C) { --------------//482
- return bary(A, B, C, 1, 1, 1);} ------------------------//483
point circumcenter(point A, point B, point C) { ----------//484
- double a=norm(B-C), b=norm(C-A), c=norm(A-B); ----------//485
- return bary(A,B,C,a*(b+c-a),b*(c+a-b),c*(a+b-c));} -----//486
point orthocenter(point A, point B, point C) { -----------//487
- return bary(A,B,C, tan(angle(B,A,C)), ------------------//488
------------- tan(angle(A,B,C)), tan(angle(A,C,B)));} ----//489
point incenter(point A, point B, point C) { --------------//48a
- return bary(A,B,C,abs(B-C),abs(A-C),abs(A-B));} --------//48b
// incircle radius given the side lengths a, b, c --------//48c
double inradius(double a, double b, double c) { ----------//48d
- double s = (a + b + c) / 2; ----------------------------//48e
- return sqrt(s * (s-a) * (s-b) * (s-c)) / s;} -----------//48f
point excenter(point A, point B, point C) { --------------//490
- double a = abs(B-C), b = abs(C-A), c = abs(A-B); -------//491
- return bary(A, B, C, -a, b, c); ------------------------//492
- // return bary(A, B, C, a, -b, c); ---------------------//493
- // return bary(A, B, C, a, b, -c); ---------------------//494
} ---------------------------------------------------------//495
point brocard(point A, point B, point C) { ---------------//496
- double a = abs(B-C), b = abs(C-A), c = abs(A-B); -------//497
- return bary(A,B,C,c/b*a,a/c*b,b/a*c); // CCW -----------//498
- // return bary(A,B,C,b/c*a,c/a*b,a/b*c); // CW ---------//499
} ---------------------------------------------------------//49a
point symmedian(point A, point B, point C) { -------------//49b
- return bary(A,B,C,norm(B-C),norm(C-A),norm(A-B));} -----//49c
```

8.14. **Convex Polygon Intersection.** Get the intersection of two convex polygons in $O(n^2)$.

```
std::vector<point> convex_polygon_inter(point a[], -------//392
--- int an, point b[], int bn) { -------------------------//393
- point ans[an + bn + an*bn]; ----------------------------//394
- int size = 0; -------------------------------------------//395
- for (int i = 0; i < an; ++i) ---------------------------//396
--- if (inPolygon(a[i],b,bn) || onPolygon(a[i],b,bn)) ----//397
----- ans[size++] = a[i]; --------------------------------//398
- for (int i = 0; i < bn; ++i) ---------------------------//399
--- if (inPolygon(b[i],a,an) || onPolygon(b[i],a,an)) ----//39a
----- ans[size++] = b[i]; --------------------------------//39b
- for (int i = 0, I = an - 1; i < an; I = i++) -----------//39c
--- for (int j = 0, J = bn - 1; j < bn; J = j++) { -------//39d
----- try { ----------------------------------------------//39e
------- point p=line_inter(a[i],a[I],b[j],b[J],true); ----//39f
------- ans[size++] = p; ---------------------------------//3a0
----- } catch (exception ex) {} --------------------------//3a1
--- } ----------------------------------------------------//3a2
- size = convex_hull(ans, size); -------------------------//3a3
- return vector<point>(ans, ans + size); -----------------//3a4
} --------------------------------------------------------//3a5
```

8.15. **Pick's Theorem for Lattice Points.** Count points with integer coordinates inside and on the boundary of a polygon in $O(n)$ using Pick's theorem: Area $= I + B/2 - 1$.

```
int interior(point p[], int n) ---------------------------//438
- {return area(p,n) - boundary(p,n) / 2 + 1;} -----------//439
int boundary(point p[], int n) { -------------------------//43a
- int ans = 0; --------------------------------------------//43b
- for (int i = 0, j = n - 1; i < n; j = i++) -------------//43c
--- ans += gcd(p[i].x - p[j].x, p[i].y - p[j].y); --------//43d
- return ans;} -------------------------------------------//43e
```

8.16. **Minimum Enclosing Circle.** Get the minimum bounding ball that encloses a set of points (2D or 3D) in Θn.

```
pair<point, double> bounding_ball(point p[], int n){ -----//425
- random_shuffle(p, p + n); ------------------------------//426
- point center(0, 0); double radius = 0; -----------------//427
- for (int i = 0; i < n; ++i) { --------------------------//428
--- if (dist(center, p[i]) > radius + EPS) { -------------//429
----- center = p[i]; radius = 0; -------------------------//42a
----- for (int j = 0; j < i; ++j) ------------------------//42b
------- if (dist(center, p[j]) > radius + EPS) { ---------//42c
--------- center.x = (p[i].x + p[j].x) / 2; --------------//42d
--------- center.y = (p[i].y + p[j].y) / 2; --------------//42e
--------- // center.z = (p[i].z + p[j].z) / 2; -----------//42f
--------- radius = dist(center, p[i]); // midpoint -------//430
--------- for (int k = 0; k < j; ++k) --------------------//431
----------- if (dist(center, p[k]) > radius + EPS) { -----//432
------------- center=circumcenter(p[i], p[j], p[k]); -----//433
------------- radius = dist(center, p[i]); ---------------//434
----------- }}}} -----------------------------------------//435
- return make_pair(center, radius); ----------------------//436
} --------------------------------------------------------//437
```

8.17. **Shamos Algorithm.** Solve for the polygon diameter in $O(n \log n)$.

```
double shamos(point p[], int n) { ------------------------//46d
- point *h = new point[n+1]; copy(p, p + n, h); ----------//46e
- int k = convex_hull(h, n); if (k <= 2) return 0; -------//46f
- h[k] = h[0]; double d = HUGE_VAL; ----------------------//470
- for (int i = 0, j = 1; i < k; ++i) { -------------------//471
--- while (distPtLine(h[j+1], h[i], h[i+1]) >= -----------//472
---------- distPtLine(h[j], h[i], h[i+1])) { -------------//473
----- j = (j + 1) % k; -----------------------------------//474
--- } ----------------------------------------------------//475
--- d = min(d, distPtLine(h[j], h[i], h[i+1])); ----------//476
- } return d; } ------------------------------------------//477
```

8.18. **kD Tree.** Get the k-nearest neighbors of a point within pruned radius in $O(k \log k \log n)$.

```
#define cpoint const point& ------------------------------//3e0
bool cmpx(cpoint a, cpoint b) {return a.x < b.x;} --------//3e1
bool cmpy(cpoint a, cpoint b) {return a.y < b.y;} --------//3e2
struct KDTree { ------------------------------------------//3e3
- KDTree(point p[],int n): p(p), n(n) {build(0,n);} ------//3e4
- priority_queue< pair<double, point*> > pq; -------------//3e5
- point *p; int n, k; double qx, qy, prune; --------------//3e6
- void build(int L, int R, bool dvx=false) { -------------//3e7
--- if (L >= R) return; ----------------------------------//3e8
--- int M = (L + R) / 2; ---------------------------------//3e9
--- nth_element(p + L, p + M, p + R, dvx?cmpx:cmpy); -----//3ea
--- build(L, M, !dvx); build(M + 1, R, !dvx); ------------//3eb
- } ------------------------------------------------------//3ec
- void dfs(int L, int R, bool dvx) { ---------------------//3ed
--- if (L >= R) return; ----------------------------------//3ee
--- int M = (L + R) / 2; ---------------------------------//3ef
--- double dx = qx - p[M].x, dy = qy - p[M].y; -----------//3f0
--- double delta = dvx ? dx : dy; ------------------------//3f1
--- double D = dx * dx + dy * dy; ------------------------//3f2
--- if(D<=prune && (pq.size()<k||D<pq.top().first)){ -----//3f3
----- pq.push(make_pair(D, &p[M])); ----------------------//3f4
----- if (pq.size() > k) pq.pop(); -----------------------//3f5
--- } ----------------------------------------------------//3f6
--- int nL = L, nR = M, fL = M + 1, fR = R; --------------//3f7
--- if (delta > 0) {swap(nL, fL); swap(nR, fR);} ---------//3f8
--- dfs(nL, nR, !dvx); ------------------------------------//3f9
--- D = delta * delta; -----------------------------------//3fa
--- if (D<=prune && (pq.size()<k||D<pq.top().first)) -----//3fb
--- dfs(fL, fR, !dvx); ------------------------------------//3fc
- } ------------------------------------------------------//3fd
- // returns k nearest neighbors of (x, y) in tree -------//3fe
- // usage: vector<point> ans = tree.knn(x, y, 2); -------//3ff
- vector<point> knn(double x, double y, -----------------//400
------------------- int k=1, double r=-1) { -------------//401
--- qx=x; qy=y; this->k=k; prune=r<0?HUGE_VAL:r*r; -------//402
--- dfs(0, n, false); vector<point> v; -------------------//403
```

```
--- while (!pq.empty()) { --------------------------------//404
----- v.push_back(*pq.top().second); ---------------------//405
----- pq.pop(); ------------------------------------------//406
--- } reverse(v.begin(), v.end()); -----------------------//407
--- return v; --------------------------------------------//408
- } ------------------------------------------------------//409
}; -------------------------------------------------------//40a
```

8.19. **Line Sweep (Closest Pair).** Get the closest pair distance of a set of points in $O(n \log n)$ by sweeping a line and keeping a bounded rectangle. Modifiable for other metrics such as Minkowski and Manhattan distance. For external point queries, see kD Tree.

```
bool cmpy(const point& a, const point& b) ---------------//369
- {return a.y < b.y;} ------------------------------------//36a
double closest_pair_sweep(point p[], int n) { -----------//36b
- if (n <= 1) return HUGE_VAL; ---------------------------//36c
- sort(p, p + n, cmpy); ----------------------------------//36d
- set<point> box; box.insert(p[0]); ----------------------//36e
- double best = 1e13; // infinity, but not HUGE_VAL ------//36f
- for (int L = 0, i = 1; i < n; ++i) { -------------------//370
--- while(L < i && p[i].y - p[L].y > best) ---------------//371
----- box.erase(p[L++]); ---------------------------------//372
--- point bound(p[i].x - best, p[i].y - best); -----------//373
--- set<point>::iterator it= box.lower_bound(bound); -----//374
--- while (it != box.end() && p[i].x+best >= it->x){ -----//375
----- double dx = p[i].x - it->x; ------------------------//376
----- double dy = p[i].y - it->y; ------------------------//377
----- best = min(best, sqrt(dx*dx + dy*dy)); -------------//378
----- ++it; ----------------------------------------------//379
--- } ----------------------------------------------------//37a
--- box.insert(p[i]); ------------------------------------//37b
- } return best; -----------------------------------------//37c
} --------------------------------------------------------//37d
```

8.20. **Line upper/lower envelope.** To find the upper/lower envelope of a collection of lines $a_i + b_i x$, plot the points (b_i, a_i), add the point $(0, \pm\infty)$ (depending on if upper/lower envelope is desired), and then find the convex hull.

8.21. **Formulas.** Let $a = (a_x, a_y)$ and $b = (b_x, b_y)$ be two-dimensional vectors.

- $a \cdot b = |a||b| \cos\theta$, where θ is the angle between a and b.
- $a \times b = |a||b| \sin\theta$, where θ is the signed angle between a and b.
- $a \times b$ is equal to the area of the parallelogram with two of its sides formed by a and b. Half of that is the area of the triangle formed by a and b.
- The line going through a and b is $Ax+By = C$ where $A = b_y - a_y$, $B = a_x - b_x$, $C = Aa_x + Ba_y$.
- Two lines $A_1x + B_1y = C_1$, $A_2x + B_2y = C_2$ are parallel iff. $D = A_1B_2 - A_2B_1$ is zero. Otherwise their unique intersection is $(B_2C_1 - B_1C_2, A_1C_2 - A_2C_1)/D$.
- **Euler's formula:** $V - E + F = 2$
- Side lengths a, b, c can form a triangle iff. $a + b > c$, $b + c > a$ and $a + c > b$.
- Sum of internal angles of a regular convex n-gon is $(n - 2)\pi$.
- **Law of sines:** $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
- **Law of cosines:** $b^2 = a^2 + c^2 - 2ac \cos B$
- Internal tangents of circles $(c_1, r_1), (c_2, r_2)$ intersect at $(c_1r_2 + c_2r_1)/(r_1 + r_2)$, external intersect at $(c_1r_2 - c_2r_1)/(r_1 + r_2)$.

9. Other Algorithms

9.1. **2SAT.** A fast 2SAT solver.

```
struct { vi adj; int val, num, lo; bool done; } V[2*1000+100];
struct TwoSat { -----------------------------------------//ad0
- int n, at = 0; vi S; -----------------------------------//ad1
- TwoSat(int _n) : n(_n) { -------------------------------//ad2
--- rep(i,0,2*n+1) ---------------------------------------//ad3
----- V[i].adj.clear(), ----------------------------------//ad4
----- V[i].val = V[i].num = -1, V[i].done = false; } -----//ad5
- bool put(int x, int v) { -------------------------------//ad6
--- return (V[n+x].val &= v) != (V[n-x].val &= 1-v); } ---//ad7
- void add_or(int x, int y) { ----------------------------//ad8
--- V[n-x].adj.push_back(n+y), V[n-y].adj.push_back(n+x); }
- int dfs(int u) { ---------------------------------------//ada
--- int br = 2, res; -------------------------------------//adb
--- S.push_back(u), V[u].num = V[u].lo = at++; -----------//adc
--- iter(v,V[u].adj) { -----------------------------------//add
----- if (V[*v].num == -1) { -----------------------------//ade
------- if (!(res = dfs(*v))) return 0; ------------------//adf
------- br |= res, V[u].lo = min(V[u].lo, V[*v].lo); -----//ae0
----- } else if (!V[*v].done) ----------------------------//ae1
------- V[u].lo = min(V[u].lo, V[*v].num); ---------------//ae2
----- br |= !V[*v].val; } --------------------------------//ae3
--- res = br - 3; ----------------------------------------//ae4
--- if (V[u].num == V[u].lo) rep(i,res+1,2) { ------------//ae5
----- for (int j = (int)size(S)-1; ; j--) { --------------//ae6
------- int v = S[j]; ------------------------------------//ae7
------- if (i) { -----------------------------------------//ae8
--------- if (!put(v-n, res)) return 0; ------------------//ae9
--------- V[v].done = true, S.pop_back(); ----------------//aea
------- } else res &= V[v].val; --------------------------//aeb
------- if (v == u) break; } -----------------------------//aec
----- res &= 1; } ----------------------------------------//aed
--- return br | !res; } ----------------------------------//aee
- bool sat() { -------------------------------------------//aef
--- rep(i,0,2*n+1) ---------------------------------------//af0
----- if (i != n && V[i].num == -1 && !dfs(i)) return false;
--- return true; } }; ------------------------------------//af2
```

9.2. **DPLL Algorithm.** A SAT solver that can solve a random 1000-variable SAT instance within a second.

```
#define IDX(x) ((abs(x)-1)*2+((x)>0)) -------------------//975
struct SAT { ---------------------------------------------//976
- int n; -------------------------------------------------//977
- vi cl, head, tail, val; --------------------------------//978
- vii log; vvi w, loc; -----------------------------------//979
- SAT() : n(0) { } ---------------------------------------//97a
- int var() { return ++n; } ------------------------------//97b
- void clause(vi vars) { ---------------------------------//97c
--- set<int> seen; iter(it,vars) { -----------------------//97d
----- if (seen.find(IDX(*it)^1) != seen.end()) return; ---//97e
----- seen.insert(IDX(*it)); } ---------------------------//97f
--- head.push_back(cl.size()); ---------------------------//980
--- iter(it,seen) cl.push_back(*it); ---------------------//981
--- tail.push_back((int)cl.size() - 2); } ----------------//982
- bool assume(int x) { -----------------------------------//983
--- if (val[x^1]) return false; --------------------------//984
--- if (val[x]) return true; -----------------------------//985
--- val[x] = true; log.push_back(ii(-1, x)); -------------//986
--- rep(i,0,w[x^1].size()) { -----------------------------//987
----- int at = w[x^1][i], h = head[at], t = tail[at]; ----//988
----- log.push_back(ii(at, h)); --------------------------//989
----- if (cl[t+1] != (x^1)) swap(cl[t], cl[t+1]); --------//98a
----- while (h < t && val[cl[h]^1]) h++; -----------------//98b
----- if ((head[at] = h) < t) { --------------------------//98c
------- w[cl[h]].push_back(w[x^1][i]); -------------------//98d
------- swap(w[x^1][i--], w[x^1].back()); ----------------//98e
------- w[x^1].pop_back(); -------------------------------//98f
------- swap(cl[head[at]++], cl[t+1]); -------------------//990
----- } else if (!assume(cl[t])) return false; } ---------//991
--- return true; } ---------------------------------------//992
- bool bt() { --------------------------------------------//993
--- int v = log.size(), x; ll b = -1; --------------------//994
--- rep(i,0,n) if (val[2*i] == val[2*i+1]) { -------------//995
----- ll s = 0, t = 0; -----------------------------------//996
----- rep(j,0,2) { iter(it,loc[2*i+j]) -------------------//997
------- s+=1LL<<max(0,40-tail[*it]+head[*it]); swap(s,t); }
----- if (max(s,t) >= b) b = max(s,t), x = 2*i + (t>=s); }//999
--- if (b == -1 || (assume(x) && bt())) return true; -----//99a
--- while (log.size() != v) { ----------------------------//99b
----- int p = log.back().first, q = log.back().second; ---//99c
----- if (p == -1) val[q] = false; else head[p] = q; -----//99d
----- log.pop_back(); } ----------------------------------//99e
--- return assume(x^1) && bt(); } ------------------------//99f
- bool solve() { -----------------------------------------//9a0
--- val.assign(2*n+1, false); ----------------------------//9a1
--- w.assign(2*n+1, vi()); loc.assign(2*n+1, vi()); ------//9a2
--- rep(i,0,head.size()) { -------------------------------//9a3
----- if (head[i] == tail[i]+2) return false; ------------//9a4
----- rep(at,head[i],tail[i]+2) loc[cl[at]].push_back(i); }
--- rep(i,0,head.size()) if (head[i] < tail[i]+1) rep(t,0,2)
----- w[cl[tail[i]+t]].push_back(i); ---------------------//9a7
--- rep(i,0,head.size()) if (head[i] == tail[i]+1) -------//9a8
----- if (!assume(cl[head[i]])) return false; ------------//9a9
--- return bt(); } ---------------------------------------//9aa
- bool get_value(int x) { return val[IDX(x)]; } }; -------//9ab
```

9.3. **Dynamic Convex Hull Trick.**

```
// USAGE: hull.insert_line(m, b); hull.gety(x); ----------//9ac
typedef long long ll; ------------------------------------//9ad
bool UPPER_HULL = true; // you can edit this -------------//9ae
bool IS_QUERY = false, SPECIAL = false; ------------------//9af
struct line { --------------------------------------------//9b0
--- ll m, b; line(ll m=0, ll b=0): m(m), b(b) {} ---------//9b1
--- mutable multiset<line>::iterator it; -----------------//9b2
--- const line *see(multiset<line>::iterator it)const; ---//9b3
--- bool operator < (const line& k) const { --------------//9b4
------- if (!IS_QUERY) return m < k.m; -------------------//9b5
------- if (!SPECIAL) { ----------------------------------//9b6
```

```
----------- ll x = k.m; const line *s = see(it); ---------//9b7
----------- if (!s) return 0; ----------------------------//9b8
----------- return (b - s->b) < (x) * (s->m - m); --------//9b9
------- } else { -----------------------------------------//9ba
----------- ll y = k.m; const line *s = see(it); ---------//9bb
----------- if (!s) return 0; ----------------------------//9bc
----------- ll n1 = y - b, d1 = m; -----------------------//9bd
----------- ll n2 = b - s->b, d2 = s->m - m; -------------//9be
----------- if (d1 < 0) n1 *= -1, d1 *= -1; --------------//9bf
----------- if (d2 < 0) n2 *= -1, d2 *= -1; --------------//9c0
----------- return (n1) * d2 > (n2) * d1; ----------------//9c1
------- }}}; ---------------------------------------------//9c2
struct dynamic_hull : multiset<line> { -------------------//9c3
--- bool bad(iterator y) { -------------------------------//9c4
------- iterator z = next(y); ----------------------------//9c5
------- if (y == begin()) { ------------------------------//9c6
----------- if (z == end()) return 0; --------------------//9c7
----------- return y->m == z->m && y->b <= z->b; ---------//9c8
------- } ------------------------------------------------//9c9
------- iterator x = prev(y); ----------------------------//9ca
------- if (z == end()) ----------------------------------//9cb
----------- return y->m == x->m && y->b <= x->b; ---------//9cc
------- return (x->b - y->b)*(z->m - y->m)>=(y->b - z->b)*(y->m - x->m);
--- } ----------------------------------------------------//9ce
--- iterator next(iterator y) {return ++y;} --------------//9cf
--- iterator prev(iterator y) {return --y;} --------------//9d0
--- void insert_line(ll m, ll b) { -----------------------//9d1
------- IS_QUERY = false; --------------------------------//9d2
------- if (!UPPER_HULL) m *= -1; ------------------------//9d3
------- iterator y = insert(line(m, b)); -----------------//9d4
------- y->it = y; if (bad(y)) {erase(y); return;} -------//9d5
------- while (next(y) != end() && bad(next(y))) ---------//9d6
----------- erase(next(y)); ------------------------------//9d7
------- while (y != begin() && bad(prev(y))) -------------//9d8
----------- erase(prev(y)); ------------------------------//9d9
--- } ----------------------------------------------------//9da
--- ll gety(ll x) { --------------------------------------//9db
------- IS_QUERY = true; SPECIAL = false; ----------------//9dc
------- const line& L = *lower_bound(line(x, 0)); --------//9dd
------- ll y = (L.m) * x + L.b; --------------------------//9de
------- return UPPER_HULL ? y : -y; ----------------------//9df
--- } ----------------------------------------------------//9e0
--- ll getx(ll y) { --------------------------------------//9e1
------- IS_QUERY = true; SPECIAL = true; -----------------//9e2
------- const line& l = *lower_bound(line(y, 0)); --------//9e3
------- return /*floor*/ ((y - l.b + l.m - 1) / l.m); ----//9e4
--- } ----------------------------------------------------//9e5
} hull; --------------------------------------------------//9e6
const line* line::see(multiset<line>::iterator it) -------//9e7
const {return ++it == hull.end() ? NULL : &*it;} ---------//9e8
```

9.4. **Stable Marriage.** The Gale-Shapley algorithm for solving the stable marriage problem.

```
vi stable_marriage(int n, int** m, int** w) { ------------//ac0
- queue<int> q; ------------------------------------------//ac1
- vi at(n, 0), eng(n, -1), res(n, -1); vvi inv(n, vi(n)); //ac2
- rep(i,0,n) rep(j,0,n) inv[i][w[i][j]] = j; -------------//ac3
- rep(i,0,n) q.push(i); ----------------------------------//ac4
- while (!q.empty()) { -----------------------------------//ac5
--- int curm = q.front(); q.pop(); -----------------------//ac6
--- for (int &i = at[curm]; i < n; i++) { ----------------//ac7
----- int curw = m[curm][i]; -----------------------------//ac8
----- if (eng[curw] == -1) { } ---------------------------//ac9
----- else if (inv[curw][curm] < inv[curw][eng[curw]]) ---//aca
------- q.push(eng[curw]); -------------------------------//acb
----- else continue; -------------------------------------//acc
----- res[eng[curw] = curm] = curw, ++i; break; } } ------//acd
- return res; } ------------------------------------------//ace
```

9.5. **Algorithm X.** An implementation of Knuth's Algorithm X, using dancing links. Solves the Exact Cover problem.

```
bool handle_solution(vi rows) { return false; } ----------//90f
struct exact_cover { -------------------------------------//910
- struct node { ------------------------------------------//911
--- node *l, *r, *u, *d, *p; -----------------------------//912
--- int row, col, size; ----------------------------------//913
--- node(int _row, int _col) : row(_row), col(_col) { ----//914
----- size = 0; l = r = u = d = p = NULL; } }; -----------//915
- int rows, cols, *sol; ----------------------------------//916
- bool **arr; --------------------------------------------//917
- node *head; --------------------------------------------//918
- exact_cover(int _rows, int _cols) ----------------------//919
--- : rows(_rows), cols(_cols), head(NULL) { -------------//91a
--- arr = new bool*[rows]; -------------------------------//91b
--- sol = new int[rows]; ---------------------------------//91c
--- rep(i,0,rows) ----------------------------------------//91d
----- arr[i] = new bool[cols], memset(arr[i], 0, cols); } //91e
- void set_value(int row, int col, bool val = true) { ----//91f
--- arr[row][col] = val; } -------------------------------//920
- void setup() { -----------------------------------------//921
--- node ***ptr = new node**[rows + 1]; ------------------//922
--- rep(i,0,rows+1) { ------------------------------------//923
----- ptr[i] = new node*[cols]; --------------------------//924
----- rep(j,0,cols) --------------------------------------//925
------- if (i == rows || arr[i][j]) ptr[i][j] = new node(i,j);
------- else ptr[i][j] = NULL; } -------------------------//927
--- rep(i,0,rows+1) { ------------------------------------//928
----- rep(j,0,cols) { ------------------------------------//929
------- if (!ptr[i][j]) continue; ------------------------//92a
------- int ni = i + 1, nj = j + 1; ----------------------//92b
------- while (true) { -----------------------------------//92c
--------- if (ni == rows + 1) ni = 0; --------------------//92d
--------- if (ni == rows || arr[ni][j]) break; -----------//92e
--------- ++ni; } ----------------------------------------//92f
------- ptr[i][j]->d = ptr[ni][j]; -----------------------//930
------- ptr[ni][j]->u = ptr[i][j]; -----------------------//931
------- while (true) { -----------------------------------//932
--------- if (nj == cols) nj = 0; ------------------------//933
--------- if (i == rows || arr[i][nj]) break; ------------//934
--------- ++nj; } ----------------------------------------//935
------- ptr[i][j]->r = ptr[i][nj]; -----------------------//936
------- ptr[i][nj]->l = ptr[i][j]; } } -------------------//937
--- head = new node(rows, -1); ---------------------------//938
--- head->r = ptr[rows][0]; ------------------------------//939
--- ptr[rows][0]->l = head; ------------------------------//93a
--- head->l = ptr[rows][cols - 1]; -----------------------//93b
--- ptr[rows][cols - 1]->r = head; -----------------------//93c
--- rep(j,0,cols) { --------------------------------------//93d
----- int cnt = -1; --------------------------------------//93e
----- rep(i,0,rows+1) ------------------------------------//93f
------- if (ptr[i][j]) cnt++, ptr[i][j]->p = ptr[rows][j];//940
----- ptr[rows][j]->size = cnt; } ------------------------//941
--- rep(i,0,rows+1) delete[] ptr[i]; ---------------------//942
--- delete[] ptr; } --------------------------------------//943
- #define COVER(c, i, j) \ --------------------------------//944
--- c->r->l = c->l, c->l->r = c->r; \ --------------------//945
--- for (node *i = c->d; i != c; i = i->d) \ -------------//946
----- for (node *j = i->r; j != i; j = j->r) \ -----------//947
------- j->d->u = j->u, j->u->d = j->d, j->p->size--; ----//948
- #define UNCOVER(c, i, j) \ ------------------------------//949
--- for (node *i = c->u; i != c; i = i->u) \ -------------//94a
----- for (node *j = i->l; j != i; j = j->l) \ -----------//94b
------- j->p->size++, j->d->u = j->u->d = j; \ -----------//94c
--- c->r->l = c->l->r = c; -------------------------------//94d
- bool search(int k = 0) { -------------------------------//94e
--- if (head == head->r) { -------------------------------//94f
----- vi res(k); -----------------------------------------//950
----- rep(i,0,k) res[i] = sol[i]; ------------------------//951
----- sort(res.begin(), res.end()); ----------------------//952
----- return handle_solution(res); } ---------------------//953
--- node *c = head->r, *tmp = head->r; -------------------//954
--- for ( ; tmp != head; tmp = tmp->r) -------------------//955
----- if (tmp->size < c->size) c = tmp; ------------------//956
--- if (c == c->d) return false; -------------------------//957
--- COVER(c, i, j); --------------------------------------//958
--- bool found = false; -----------------------------------//959
--- for (node *r = c->d; !found && r != c; r = r->d) { ---//95a
----- sol[k] = r->row; -----------------------------------//95b
----- for (node *j = r->r; j != r; j = j->r) { -----------//95c
------- COVER(j->p, a, b); } -----------------------------//95d
----- found = search(k + 1); -----------------------------//95e
----- for (node *j = r->l; j != r; j = j->l) { -----------//95f
------- UNCOVER(j->p, a, b); } } -------------------------//960
--- UNCOVER(c, i, j); ------------------------------------//961
--- return found; } }; -----------------------------------//962
```

9.6. **Matroid Intersection.** Computes the maximum weight and cardinality intersection of two matroids, specified by implementing the required abstract methods, in $O(n^3(M_1 + M_2))$.

```
struct MatroidIntersection { -----------------------------//a01
- virtual void add(int element) = 0; ---------------------//a02
- virtual void remove(int element) = 0; ------------------//a03
- virtual bool valid1(int element) = 0; ------------------//a04
- virtual bool valid2(int element) = 0; ------------------//a05
- int n, found; vi arr; vector<ll> ws; ll weight; --------//a06
- MatroidIntersection(vector<ll> weights) ----------------//a07
--- : n(weights.size()), found(0), ws(weights), weight(0) {
```

```
----- rep(i,0,n) arr.push_back(i); } ---------------------//a09
- bool increase() { --------------------------------------//a0a
--- vector<tuple<int,int,ll>> es; ------------------------//a0b
--- vector<pair<ll,int>> d(n+1, {1000000000000000000LL,0});
--- vi p(n+1,-1), a, r; bool ch; -------------------------//a0d
--- rep(at,found,n) { ------------------------------------//a0e
----- if (valid1(arr[at])) d[p[at] = at] = {-ws[arr[at]],0};
----- if (valid2(arr[at])) es.emplace_back(at, n, 0); } --//a10
--- rep(cur,0,found) { -----------------------------------//a11
----- remove(arr[cur]); ----------------------------------//a12
----- rep(nxt,found,n) { ---------------------------------//a13
------- if (valid1(arr[nxt])) ----------------------------//a14
--------- es.emplace_back(cur, nxt, -ws[arr[nxt]]); ------//a15
------- if (valid2(arr[nxt])) ----------------------------//a16
--------- es.emplace_back(nxt, cur, ws[arr[cur]]); } -----//a17
----- add(arr[cur]); } -----------------------------------//a18
--- do { ch = false; -------------------------------------//a19
----- for (auto [u,v,c] : es) { --------------------------//a1a
------- pair<ll,int> nd(d[u].first + c, d[u].second + 1); //a1b
------- if (p[u] != -1 && nd < d[v]) ---------------------//a1c
--------- d[v] = nd, p[v] = u, ch = true; } } while (ch); //a1d
--- if (p[n] == -1) return false; ------------------------//a1e
--- int cur = p[n]; --------------------------------------//a1f
--- while(p[cur]!=cur)a.push_back(cur),a.swap(r),cur=p[cur];
--- a.push_back(cur); ------------------------------------//a21
--- sort(a.begin(), a.end()); sort(r.rbegin(), r.rend()); //a22
--- iter(it,r)remove(arr[*it]),swap(arr[--found],arr[*it]);
--- iter(it,a)add(arr[*it]),swap(arr[found++],arr[*it]); -//a24
--- weight -= d[n].first; return true; } }; --------------//a25
```

9.7. **nth Permutation.** A very fast algorithm for computing the nth permutation of the list $\{0, 1, \dots, k-1\}$.

```
vector<int> nth_permutation(int cnt, int n) { ------------//a26
- vector<int> idx(cnt), per(cnt), fac(cnt); --------------//a27
- rep(i,0,cnt) idx[i] = i; -------------------------------//a28
- rep(i,1,cnt+1) fac[i - 1] = n % i, n /= i; -------------//a29
- for (int i = cnt - 1; i >= 0; i--) ---------------------//a2a
--- per[cnt - i - 1] = idx[fac[i]], ----------------------//a2b
--- idx.erase(idx.begin() + fac[i]); ---------------------//a2c
- return per; } ------------------------------------------//a2d
```

9.8. **Cycle-Finding.** An implementation of Floyd's Cycle-Finding algorithm.

```
ii find_cycle(int x0, int (*f)(int)) { -------------------//9e9
- int t = f(x0), h = f(t), mu = 0, lam = 1; --------------//9ea
- while (t != h) t = f(t), h = f(f(h)); ------------------//9eb
- h = x0; ------------------------------------------------//9ec
- while (t != h) t = f(t), h = f(h), mu++; ---------------//9ed
- h = f(t); ----------------------------------------------//9ee
- while (t != h) h = f(h), lam++; ------------------------//9ef
- return ii(mu, lam); } ----------------------------------//9f0
```

9.9. **Longest Increasing Subsequence.**

```
vi lis(vi arr) { -----------------------------------------//9f1
- if (arr.empty()) return vi(); --------------------------//9f2
- vi seq, back(size(arr)), ans; --------------------------//9f3
- rep(i,0,size(arr)) { -----------------------------------//9f4
--- int res = 0, lo = 1, hi = size(seq); -----------------//9f5
--- while (lo <= hi) { -----------------------------------//9f6
----- int mid = (lo+hi)/2; -------------------------------//9f7
----- if (arr[seq[mid-1]] < arr[i]) res = mid, lo = mid + 1;
----- else hi = mid - 1; } -------------------------------//9f9
--- if (res < size(seq)) seq[res] = i; -------------------//9fa
--- else seq.push_back(i); -------------------------------//9fb
--- back[i] = res == 0 ? -1 : seq[res-1]; } --------------//9fc
- int at = seq.back(); -----------------------------------//9fd
- while (at != -1) ans.push_back(at), at = back[at]; -----//9fe
- reverse(ans.begin(), ans.end()); -----------------------//9ff
- return ans; } ------------------------------------------//a00
```

9.10. **Dates.** Functions to simplify date calculations.

```
int intToDay(int jd) { return jd % 7; } ------------------//963
int dateToInt(int y, int m, int d) { ---------------------//964
- return 1461 * (y + 4800 + (m - 14) / 12) / 4 + ---------//965
--- 367 * (m - 2 - (m - 14) / 12 * 12) / 12 - ------------//966
--- 3 * ((y + 4900 + (m - 14) / 12) / 100) / 4 + ---------//967
--- d - 32075; } -----------------------------------------//968
void intToDate(int jd, int &y, int &m, int &d) { ---------//969
- int x, n, i, j; ----------------------------------------//96a
- x = jd + 68569; ----------------------------------------//96b
- n = 4 * x / 146097; ------------------------------------//96c
- x -= (146097 * n + 3) / 4; -----------------------------//96d
- i = (4000 * (x + 1)) / 1461001; ------------------------//96e
- x -= 1461 * i / 4 - 31; --------------------------------//96f
- j = 80 * x / 2447; -------------------------------------//970
- d = x - 2447 * j / 80; ---------------------------------//971
- x = j / 11; --------------------------------------------//972
- m = j + 2 - 12 * x; ------------------------------------//973
- y = 100 * (n - 49) + i + x; } --------------------------//974
```

9.11. **Simulated Annealing.** An example use of Simulated Annealing to find a permutation of length n that maximizes $\sum_{i=1}^{n-1} |p_i - p_{i+1}|$.

```
double curtime() { ---------------------------------------//a9a
- return static_cast<double>(clock()) / CLOCKS_PER_SEC; } //a9b
int simulated_annealing(int n, double seconds) { ---------//a9c
- default_random_engine rng; -----------------------------//a9d
- uniform_real_distribution<double> randfloat(0.0, 1.0); -//a9e
- uniform_int_distribution<int> randint(0, n - 2); -------//a9f
- // random initial solution ----------------------------//aa0
- vi sol(n); ---------------------------------------------//aa1
- rep(i,0,n) sol[i] = i + 1; -----------------------------//aa2
- random_shuffle(sol.begin(), sol.end()); ----------------//aa3
- // initialize score -----------------------------------//aa4
- int score = 0; -----------------------------------------//aa5
- rep(i,1,n) score += abs(sol[i] - sol[i-1]); ------------//aa6
- int iters = 0; -----------------------------------------//aa7
- double T0 = 100.0, T1 = 0.001, -------------------------//aa8
----- progress = 0, temp = T0, ---------------------------//aa9
----- starttime = curtime(); -----------------------------//aaa
- while (true) { -----------------------------------------//aab
--- if (!(iters & ((1 << 4) - 1))) { ---------------------//aac
----- progress = (curtime() - starttime) / seconds; ------//aad
----- temp = T0 * pow(T1 / T0, progress); ----------------//aae
----- if (progress > 1.0) break; } -----------------------//aaf
--- // random mutation ---------------------------------//ab0
--- int a = randint(rng); --------------------------------//ab1
--- // compute delta for mutation ---------------------//ab2
--- int delta = 0; ---------------------------------------//ab3
--- if (a > 0) delta += abs(sol[a+1] - sol[a-1]) ---------//ab4
--------------------- - abs(sol[a] - sol[a-1]); ----------//ab5
--- if (a+2 < n) delta += abs(sol[a] - sol[a+2]) ---------//ab6
---------------------- - abs(sol[a+1] - sol[a+2]); ------//ab7
--- // maybe apply mutation ----------------------------//ab8
--- if (delta >= 0 || randfloat(rng) < exp(delta / temp)) {
----- swap(sol[a], sol[a+1]); ----------------------------//aba
----- score += delta; ------------------------------------//abb
----- // if (score >= target) return; ------------------//abc
--- } ----------------------------------------------------//abd
--- iters++; } -------------------------------------------//abe
- return score; } ----------------------------------------//abf
```

9.12. **Simplex.**

```
typedef long double DOUBLE; ------------------------------//a2e
typedef vector<DOUBLE> VD; -------------------------------//a2f
typedef vector<VD> VVD; ----------------------------------//a30
typedef vector<int> VI; ----------------------------------//a31
const DOUBLE EPS = 1e-9; ---------------------------------//a32
struct LPSolver { ----------------------------------------//a33
 int m, n; -----------------------------------------------//a34
 VI B, N; ------------------------------------------------//a35
 VVD D; --------------------------------------------------//a36
 LPSolver(const VVD &A, const VD &b, const VD &c) : ------//a37
- m(b.size()), n(c.size()), ------------------------------//a38
- N(n + 1), B(m), D(m + 2, VD(n + 2)) { ------------------//a39
- for (int i = 0; i < m; i++) for (int j = 0; j < n; j++) //a3a
--- D[i][j] = A[i][j]; -----------------------------------//a3b
- for (int i = 0; i < m; i++) { B[i] = n + i; D[i][n] = -1;
--- D[i][n + 1] = b[i]; } --------------------------------//a3d
- for (int j = 0; j < n; j++) { N[j] = j; D[m][j] = -c[j]; }
- N[n] = -1; D[m + 1][n] = 1; } --------------------------//a3f
 void Pivot(int r, int s) { ------------------------------//a40
- double inv = 1.0 / D[r][s]; ----------------------------//a41
- for (int i = 0; i < m + 2; i++) if (i != r) -----------//a42
-- for (int j = 0; j < n + 2; j++) if (j != s) ----------//a43
--- D[i][j] -= D[r][j] * D[i][s] * inv; ------------------//a44
- for (int j = 0; j < n + 2; j++) if (j != s) D[r][j] *= inv;
- for (int i = 0; i < m + 2; i++) if (i != r) D[i][s] *= -inv;
- D[r][s] = inv; -----------------------------------------//a47
- swap(B[r], N[s]); } ------------------------------------//a48
 bool Simplex(int phase) { -------------------------------//a49
- int x = phase == 1 ? m + 1 : m; ------------------------//a4a
- while (true) { -----------------------------------------//a4b
-- int s = -1; -------------------------------------------//a4c
-- for (int j = 0; j <= n; j++) { ------------------------//a4d
--- if (phase == 2 && N[j] == -1) continue; --------------//a4e
--- if (s == -1 || D[x][j] < D[x][s] || ------------------//a4f
------- D[x][j] == D[x][s] && N[j] < N[s]) s = j; } ------//a50
-- if (D[x][s] > -EPS) return true; ----------------------//a51
-- int r = -1; -------------------------------------------//a52
-- for (int i = 0; i < m; i++) { -------------------------//a53
```

```
--- if (D[i][s] < EPS) continue; -------------------------//a54
--- if (r == -1 || D[i][n + 1] / D[i][s] < D[r][n + 1] / -//a55
------- D[r][s] || (D[i][n + 1] / D[i][s]) == (D[r][n + 1] /
------- D[r][s]) && B[i] < B[r]) r = i; } ----------------//a57
-- if (r == -1) return false; ----------------------------//a58
-- Pivot(r, s); } } --------------------------------------//a59
 DOUBLE Solve(VD &x) { -----------------------------------//a5a
- int r = 0; ---------------------------------------------//a5b
- for (int i = 1; i < m; i++) if (D[i][n + 1] < D[r][n + 1])
--- r = i; -----------------------------------------------//a5d
- if (D[r][n + 1] < -EPS) { ------------------------------//a5e
-- Pivot(r, n); ------------------------------------------//a5f
-- if (!Simplex(1) || D[m + 1][n + 1] < -EPS) ------------//a60
---- return -numeric_limits<DOUBLE>::infinity(); ---------//a61
-- for (int i = 0; i < m; i++) if (B[i] == -1) { ---------//a62
--- int s = -1; ------------------------------------------//a63
--- for (int j = 0; j <= n; j++) -------------------------//a64
---- if (s == -1 || D[i][j] < D[i][s] || -----------------//a65
------- D[i][j] == D[i][s] && N[j] < N[s]) ---------------//a66
------ s = j; --------------------------------------------//a67
--- Pivot(i, s); } } -------------------------------------//a68
- if (!Simplex(2)) return numeric_limits<DOUBLE>::infinity();
- x = VD(n); ---------------------------------------------//a6a
- for (int i = 0; i < m; i++) if (B[i] < n) --------------//a6b
--- x[B[i]] = D[i][n + 1]; -------------------------------//a6c
- return D[m][n + 1]; } }; -------------------------------//a6d
// Two-phase simplex algorithm for solving linear programs//a6e
// of the form -------------------------------------------//a6f
//     maximize     c^T x --------------------------------//a70
//     subject to   Ax <= b ------------------------------//a71
//                  x >= 0 -------------------------------//a72
// INPUT: A -- an m x n matrix ---------------------------//a73
//        b -- an m-dimensional vector -------------------//a74
//        c -- an n-dimensional vector -------------------//a75
//        x -- a vector where the optimal solution will be//a76
//             stored ------------------------------------//a77
// OUTPUT: value of the optimal solution (infinity if ----//a78
//                  unbounded above, nan if infeasible) -//a79
// To use this code, create an LPSolver object with A, b, //a7a
// and c as arguments.  Then, call Solve(x). -------------//a7b
// #include <iostream> -----------------------------------//a7c
// #include <iomanip> ------------------------------------//a7d
// #include <vector> -------------------------------------//a7e
// #include <cmath> --------------------------------------//a7f
// #include <limits> -------------------------------------//a80
// using namespace std; ----------------------------------//a81
// int main() { ------------------------------------------//a82
//   const int m = 4; ------------------------------------//a83
//   const int n = 3; ------------------------------------//a84
//   DOUBLE _A[m][n] = { ---------------------------------//a85
//     { 6, -1, 0 }, -------------------------------------//a86
//     { -1, -5, 0 }, ------------------------------------//a87
//     { 1, 5, 1 }, --------------------------------------//a88
//     { -1, -5, -1 } ------------------------------------//a89
//   }; --------------------------------------------------//a8a
//   DOUBLE _b[m] = { 10, -4, 5, -5 }; -------------------//a8b
//   DOUBLE _c[n] = { 1, -1, 0 }; ------------------------//a8c
//   VVD A(m); -------------------------------------------//a8d
//   VD b(_b, _b + m); -----------------------------------//a8e
//   VD c(_c, _c + n); -----------------------------------//a8f
//   for (int i = 0; i < m; i++) A[i] = VD(_A[i], _A[i] + n);
//   LPSolver solver(A, b, c); ---------------------------//a91
//   VD x; -----------------------------------------------//a92
//   DOUBLE value = solver.Solve(x); ---------------------//a93
//   cerr << "VALUE: " << value << endl; // VALUE: 1.29032//a94
//   cerr << "SOLUTION:"; // SOLUTION: 1.74194 0.451613 1 //a95
//   for (size_t i = 0; i < x.size(); i++) cerr << " " << x[i];
//   cerr << endl; ---------------------------------------//a97
//   return 0; -------------------------------------------//a98
// } -----------------------------------------------------//a99
```

9.13. **Fast Square Testing.** An optimized test for square integers.

```
long long M; --------------------------------------------//c05
void init_is_square() { ---------------------------------//c06
- rep(i,0,64) M |= 1ULL << (63-(i*i)%64); } -------------//c07
inline bool is_square(ll x) { ---------------------------//c08
- if (x == 0) return true; // XXX -----------------------//c09
- if ((M << x) >= 0) return false; ----------------------//c0a
- int c = __builtin_ctz(x); -----------------------------//c0b
- if (c & 1) return false; ------------------------------//c0c
- x >>= c; ----------------------------------------------//c0d
- if ((x&7) - 1) return false; --------------------------//c0e
- ll r = sqrt(x); ---------------------------------------//c0f
- return r*r == x; } ------------------------------------//c10
```

9.14. **Fast Input Reading.** If input or output is huge, sometimes it is beneficial to optimize the input reading/output writing. This can be achieved by reading all input in at once (using fread), and then parsing it manually. Output can also be stored in an output buffer and then dumped once in the end (using fwrite). A simpler, but still effective, way to achieve speed is to use the following input reading method.

```
void readn(register int *n) { ---------------------------//bf9
- int sign = 1; -----------------------------------------//bfa
- register char c; --------------------------------------//bfb
- *n = 0; -----------------------------------------------//bfc
- while((c = getc_unlocked(stdin)) != '\n') { -----------//bfd
--- switch(c) { -----------------------------------------//bfe
----- case '-': sign = -1; break; -----------------------//bff
----- case ' ': goto hell; ------------------------------//c00
----- case '\n': goto hell; -----------------------------//c01
----- default: *n *= 10; *n += c - '0'; break; } } ------//c02
hell: ---------------------------------------------------//c03
- *n *= sign; } -----------------------------------------//c04
```

9.15. **128-bit Integer.** GCC has a 128-bit integer data type named `__int128`. Useful if doing multiplication of 64-bit integers, or something needing a little more than 64-bits to represent. There's also `__float128`.

9.16. **Bit Hacks.**

```
int snoob(int x) { --------------------------------------//c11
- int y = x & -x, z = x + y; ----------------------------//c12
- return z | ((x ^ z) >> 2) / y; } ----------------------//c13
```

10. Other Combinatorics Stuff

Catalan	$C_0 = 1,\ C_n = \frac{1}{n+1}\binom{2n}{n} = \sum_{i=0}^{n-1} C_i C_{n-i-1} = \frac{4n-2}{n+1} C_{n-1}$	
Stirling 1st kind	$\genfrac{[}{]}{0pt}{}{0}{0} = 1,\ \genfrac{[}{]}{0pt}{}{n}{0} = \genfrac{[}{]}{0pt}{}{0}{n} = 0,\ \genfrac{[}{]}{0pt}{}{n}{k} = (n-1)\genfrac{[}{]}{0pt}{}{n-1}{k} + \genfrac{[}{]}{0pt}{}{n-1}{k-1}$	#perms of n objs with exactly k cycles
Stirling 2nd kind	$\genfrac{\{}{\}}{0pt}{}{n}{1} = \genfrac{\{}{\}}{0pt}{}{n}{n} = 1,\ \genfrac{\{}{\}}{0pt}{}{n}{k} = k\genfrac{\{}{\}}{0pt}{}{n-1}{k} + \genfrac{\{}{\}}{0pt}{}{n-1}{k-1}$	#ways to partition n objs into k nonempty sets
Euler	$\genfrac{\langle}{\rangle}{0pt}{}{n}{0} = \genfrac{\langle}{\rangle}{0pt}{}{n}{n-1} = 1,\ \genfrac{\langle}{\rangle}{0pt}{}{n}{k} = (k+1)\genfrac{\langle}{\rangle}{0pt}{}{n-1}{k} + (n-k)\genfrac{\langle}{\rangle}{0pt}{}{n-1}{k-1}$	#perms of n objs with exactly k ascents
Euler 2nd Order	$\left\langle\!\!\left\langle {n \atop k} \right\rangle\!\!\right\rangle = (k+1)\left\langle\!\!\left\langle {n-1 \atop k} \right\rangle\!\!\right\rangle + (2n-k-1)\left\langle\!\!\left\langle {n-1 \atop k-1} \right\rangle\!\!\right\rangle$	#perms of $1, 1, 2, 2, ..., n, n$ with exactly k ascents
Bell	$B_1 = 1,\ B_n = \sum_{k=0}^{n-1} B_k \binom{n-1}{k} = \sum_{k=0}^{n} \genfrac{\{}{\}}{0pt}{}{n}{k}$	#partitions of $1..n$ (Stirling 2nd, no limit on k)

#labeled rooted trees	n^{n-1}
#labeled unrooted trees	n^{n-2}
#forests of k rooted trees	$\frac{k}{n}\binom{n}{k}n^{n-k}$
$\sum_{i=1}^{n} i^2 = n(n+1)(2n+1)/6$	$\sum_{i=1}^{n} i^3 = n^2(n+1)^2/4$
$!n = n\times!(n-1) + (-1)^n$	$!n = (n-1)(!(n-1)+!(n-2))$
$\sum_{i=1}^{n}\binom{n}{i}F_i = F_{2n}$	$\sum_i \binom{n-i}{i} = F_{n+1}$
$\sum_{k=0}^{n}\binom{k}{m} = \binom{n+1}{m+1}$	$x^k = \sum_{i=0}^{k} i!\genfrac{\{}{\}}{0pt}{}{k}{i}\binom{x}{i} = \sum_{i=0}^{k}\genfrac{\langle}{\rangle}{0pt}{}{k}{i}\binom{x+i}{k}$
$a \equiv b \pmod{x, y} \Rightarrow a \equiv b \pmod{\mathrm{lcm}(x, y)}$	$\sum_{d\mid n}\phi(d) = n$
$ac \equiv bc \pmod{m} \Rightarrow a \equiv b \pmod{\frac{m}{\gcd(c,m)}}$	$(\sum_{d\mid n}\sigma_0(d))^2 = \sum_{d\mid n}\sigma_0(d)^3$
p prime $\Leftrightarrow (p-1)! \equiv -1 \pmod{p}$	$\gcd(n^a - 1, n^b - 1) = n^{\gcd(a,b)} - 1$
$\sigma_x(n) = \prod_{i=0}^{r}\frac{p_i^{(a_i+1)x} - 1}{p_i^x - 1}$	$\sigma_0(n) = \prod_{i=0}^{r}(a_i + 1)$
$\sum_{k=0}^{m}(-1)^k\binom{n}{k} = (-1)^m\binom{n-1}{m}$	
$2^{\omega(n)} = O(\sqrt{n})$	$\sum_{i=1}^{n} 2^{\omega(i)} = O(n\log n)$
$d = v_i t + \frac{1}{2}at^2$	$v_f^2 = v_i^2 + 2ad$
$v_f = v_i + at$	$d = \frac{v_i + v_f}{2}t$

10.1. **The Twelvefold Way.** Putting n balls into k boxes.

Balls Boxes	same same	distinct same	same distinct	distinct distinct	Remarks
-	$\mathrm{p}_k(n)$	$\sum_{i=0}^{k}\genfrac{\{}{\}}{0pt}{}{n}{i}$	$\binom{n+k-1}{k-1}$	k^n	$\mathrm{p}_k(n)$: #partitions of n into $\le k$ positive parts
size ≥ 1	$\mathrm{p}(n, k)$	$\genfrac{\{}{\}}{0pt}{}{n}{k}$	$\binom{n-1}{k-1}$	$k!\genfrac{\{}{\}}{0pt}{}{n}{k}$	$\mathrm{p}(n, k)$: #partitions of n into k positive parts
size ≤ 1	$[n \le k]$	$[n \le k]$	$\binom{k}{n}$	$n!\binom{k}{n}$	$[cond]$: 1 if $cond = true$, else 0

11. Useful Information (CLEAN THIS UP!!)

12. Misc

12.1. Debugging Tips.

- Stack overflow? Recursive DFS on tree that is actually a long path?
- Floating-point numbers
  - Getting `NaN`? Make sure `acos` etc. are not getting values out of their range (perhaps `1+eps`).
  - Rounding negative numbers?
  - Outputting in scientific notation?
- Wrong Answer?
  - Read the problem statement again!
  - Are multiple test cases being handled correctly? Try repeating the same test case many times.
  - Integer overflow?
  - Think very carefully about boundaries of all input parameters
  - Try out possible edge cases:
    * $n = 0, n = -1, n = 1, n = 2^{31} - 1$ or $n = -2^{31}$
    * List is empty, or contains a single element
    * n is even, n is odd
    * Graph is empty, or contains a single vertex
    * Graph is a multigraph (loops or multiple edges)
    * Polygon is concave or non-simple
  - Is initial condition wrong for small cases?
  - Are you sure the algorithm is correct?
  - Explain your solution to someone.
  - Are you using any functions that you don't completely understand? Maybe STL functions?
  - Maybe you (or someone else) should rewrite the solution?
  - Can the input line be empty?
- Run-Time Error?
  - Is it actually Memory Limit Exceeded?

12.2. Solution Ideas.

- Dynamic Programming
  - Parsing CFGs: CYK Algorithm
  - Drop a parameter, recover from others
  - Swap answer and a parameter
  - When grouping: try splitting in two
  - 2^k trick
  - When optimizing
    * Convex hull optimization
      · $\text{dp}[i] = \min_{j<i}\{\text{dp}[j] + b[j] \times a[i]\}$
      · $b[j] \geq b[j+1]$
      · optionally $a[i] \leq a[i+1]$
      · $O(n^2)$ to $O(n)$
    * Divide and conquer optimization
      · $\text{dp}[i][j] = \min_{k<j}\{\text{dp}[i-1][k] + C[k][j]\}$
      · $A[i][j] \leq A[i][j+1]$
      · $O(kn^2)$ to $O(kn \log n)$
      · sufficient: $C[a][c] + C[b][d] \leq C[a][d] + C[b][c]$, $a \leq b \leq c \leq d$ (QI)
    * Knuth optimization
      · $\text{dp}[i][j] = \min_{i<k<j}\{\text{dp}[i][k] + \text{dp}[k][j] + C[i][j]\}$
      · $A[i][j-1] \leq A[i][j] \leq A[i+1][j]$
      · $O(n^3)$ to $O(n^2)$
      · sufficient: QI and $C[b][c] \leq C[a][d]$, $a \leq b \leq c \leq d$
- Greedy
- Randomized
- Optimizations
  - Use bitset (/64)
  - Switch order of loops (cache locality)
- Process queries offline
  - Mo's algorithm
- Square-root decomposition
- Precomputation
- Efficient simulation
  - Mo's algorithm
  - Sqrt decomposition
  - Store 2^k jump pointers
- Data structure techniques
  - Sqrt buckets
  - Store 2^k jump pointers
  - 2^k merging trick
- Counting
  - Inclusion-exclusion principle
  - Generating functions
- Graphs
  - Can we model the problem as a graph?
  - Can we use any properties of the graph?
  - Strongly connected components
  - Cycles (or odd cycles)
  - Bipartite (no odd cycles)
    * Bipartite matching
    * Hall's marriage theorem
    * Stable Marriage
  - Cut vertex/bridge
  - Biconnected components
  - Degrees of vertices (odd/even)
  - Trees
    * Heavy-light decomposition
    * Centroid decomposition
    * Least common ancestor
    * Centers of the tree
  - Eulerian path/circuit
  - Chinese postman problem
  - Topological sort
  - (Min-Cost) Max Flow
  - Min Cut
    * Maximum Density Subgraph
  - Huffman Coding
  - Min-Cost Arborescence
  - Steiner Tree
  - Kirchoff's matrix tree theorem
  - Prüfer sequences
  - Lovász Toggle
  - Look at the DFS tree (which has no cross-edges)
  - Is the graph a DFA or NFA?
    * Is it the Synchronizing word problem?
- Mathematics
  - Is the function multiplicative?
  - Look for a pattern
  - Permutations
    * Consider the cycles of the permutation
  - Functions
    * Sum of piecewise-linear functions is a piecewise-linear function
    * Sum of convex (concave) functions is convex (concave)
  - Modular arithmetic
    * Chinese Remainder Theorem
    * Linear Congruence
  - Sieve
  - System of linear equations
  - Values too big to represent?
    * Compute using the logarithm
    * Divide everything by some large value
  - Linear programming
    * Is the dual problem easier to solve?
  - Can the problem be modeled as a different combinatorial problem? Does that simplify calculations?
- Logic
  - 2-SAT
  - XOR-SAT (Gauss elimination or Bipartite matching)
- Meet in the middle
- Only work with the smaller half ($\log(n)$)
- Strings
  - Trie (maybe over something weird, like bits)
  - Suffix array
  - Suffix automaton (+DP?)
  - Aho-Corasick
  - eerTree
  - Work with $S + S$
- Hashing
- Euler tour, tree to array
- Segment trees
  - Lazy propagation
  - Persistent
  - Implicit
  - Segment tree of X
- Geometry
  - Minkowski sum (of convex sets)
  - Rotating calipers
  - Sweep line (horizontally or vertically?)
  - Sweep angle
  - Convex hull
- Fix a parameter (possibly the answer).
- Are there few distinct values?
- Binary search
- Sliding Window (+ Monotonic Queue)
- Computing a Convolution? Fast Fourier Transform
- Computing a 2D Convolution? FFT on each row, and then on each column
- Exact Cover (+ Algorithm X)
- Cycle-Finding
- What is the smallest set of values that identify the solution? The cycle structure of the permutation? The powers of primes in the factorization?
- Look at the complement problem

- Minimize something instead of maximizing
- Immediately enforce necessary conditions. (All values greater than 0? Initialize them all to 1)
- Add large constant to negative numbers to make them positive
- Counting/Bucket sort

13. Formulas

- **Legendre symbol:** $\left(\frac{a}{b}\right) = a^{(b-1)/2} \pmod b$, b odd prime.
- **Heron's formula:** A triangle with side lengths a, b, c has area $\sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{a+b+c}{2}$.
- **Pick's theorem:** A polygon on an integer grid strictly containing i lattice points and having b lattice points on the boundary has area $i + \frac{b}{2} - 1$. (Nothing similar in higher dimensions)
- **Euler's totient:** The number of integers less than n that are coprime to n are $n\prod_{p|n}\left(1 - \frac{1}{p}\right)$ where each p is a distinct prime factor of n.
- **König's theorem:** In any bipartite graph $G = (L \cup R, E)$, the number of edges in a maximum matching is equal to the number of vertices in a minimum vertex cover. Let U be the set of unmatched vertices in L, and Z be the set of vertices that are either in U or are connected to U by an alternating path. Then $K = (L \setminus Z) \cup (R \cap Z)$ is the minimum vertex cover.
- A minumum Steiner tree for n vertices requires at most $n-2$ additional Steiner vertices.
- The number of vertices of a graph is equal to its minimum vertex cover number plus the size of a maximum independent set.
- **Lagrange polynomial** through points $(x_0, y_0), \ldots, (x_k, y_k)$ is $L(x) = \sum_{j=0}^{k} y_j \prod_{\substack{0 \le m \le k \\ m \ne j}} \frac{x - x_m}{x_j - x_m}$
- **Hook length formula:** If λ is a Young diagram and $h_\lambda(i, j)$ is the hook-length of cell (i, j), then then the number of Young tableux $d_\lambda = n!/\prod h_\lambda(i, j)$.
- **Möbius inversion formula:** If $f(n) = \sum_{d|n} g(d)$, then $g(n) = \sum_{d|n} \mu(d) f(n/d)$. If $f(n) = \sum_{m=1}^{n} g(\lfloor n/m \rfloor)$, then $g(n) = \sum_{m=1}^{n} \mu(m) f(\lfloor \frac{n}{m} \rfloor)$.
- #primitive pythagorean triples with hypotenuse $< n$ approx $n/(2\pi)$.
- **Frobenius Number:** largest number which can't be expressed as a linear combination of numbers $a_1, \ldots, a_n$ with non-negative coefficients. $g(a_1, a_2) = a_1 a_2 - a_1 - a_2$, $N(a_1, a_2) = (a_1 - 1)(a_2 - 1)/2$. $g(d \cdot a_1, d \cdot a_2, a_3) = d \cdot g(a_1, a_2, a_3) + a_3(d-1)$. An integer $x > (\max_i a_i)^2$ can be expressed in such a way iff. $x \mid \gcd(a_1, \ldots, a_n)$.

13.1. Physics.

- **Snell's law:** $\frac{\sin\theta_1}{v_1} = \frac{\sin\theta_2}{v_2}$

13.2. Markov Chains.

A Markov Chain can be represented as a weighted directed graph of states, where the weight of an edge represents the probability of transitioning over that edge in one timestep. Let $P^{(m)} = (p_{ij}^{(m)})$ be the probability matrix of transitioning from state i to state j in m timesteps, and note that $P^{(1)}$ is the adjacency matrix of the graph. **Chapman-Kolmogorov:** $p_{ij}^{(m+n)} = \sum_k p_{ik}^{(m)} p_{kj}^{(n)}$. It follows that $P^{(m+n)} = P^{(m)} P^{(n)}$ and $P^{(m)} = P^m$. If $p^{(0)}$ is the initial probability distribution (a vector), then $p^{(0)} P^{(m)}$ is the probability distribution after m timesteps.

The return times of a state i is $R_i = \{m \mid p_{ii}^{(m)} > 0\}$, and i is *aperiodic* if $\gcd(R_i) = 1$. A MC is aperiodic if any of its vertices is aperiodic. A MC is *irreducible* if the corresponding graph is strongly connected.

A distribution π is stationary if $\pi P = \pi$. If MC is irreducible then $\pi_i = 1/\mathbb{E}[T_i]$, where T_i is the expected time between two visits at i. π_j/π_i is the expected number of visits at j in between two consecutive visits at i. A MC is *ergodic* if $\lim_{m\to\infty} p^{(0)} P^m = \pi$. A MC is ergodic iff. it is irreducible and aperiodic.

A MC for a random walk in an undirected weighted graph (unweighted graph can be made weighted by adding 1-weights) has $p_{uv} = w_{uv}/\sum_x w_{ux}$. If the graph is connected, then $\pi_u = \sum_x w_{ux}/\sum_v \sum_x w_{vx}$. Such a random walk is aperiodic iff. the graph is not bipartite.

An *absorbing* MC is of the form $P = \begin{pmatrix} Q & R \\ 0 & I_r \end{pmatrix}$. Let $N = \sum_{m=0}^{\infty} Q^m = (I_t - Q)^{-1}$. Then, if starting in state i, the expected number of steps till absorption is the i-th entry in $N1$. If starting in state i, the probability of being absorbed in state j is the (i, j)-th entry of NR.

Many problems on MC can be formulated in terms of a system of recurrence relations, and then solved using Gaussian elimination.

13.3. Burnside's Lemma.

Let G be a finite group that acts on a set X. For each g in G let X^g denote the set of elements in X that are fixed by g. Then the number of orbits

$$|X/G| = \frac{1}{|G|} \sum_{g \in G} |X^g|$$

$$Z(S_n) = \frac{1}{n} \sum_{l=1}^{n} a_l Z(S_{n-l})$$

13.4. Bézout's identity.

If (x, y) is any solution to $ax + by = d$ (e.g. found by the Extended Euclidean Algorithm), then all solutions are given by

$$\left(x + k\frac{b}{\gcd(a, b)}, y - k\frac{a}{\gcd(a, b)}\right)$$

13.5. Misc.

13.5.1. *Determinants and PM.*

$$det(A) = \sum_{\sigma \in S_n} \operatorname{sgn}(\sigma) \prod_{i=1}^{n} a_{i,\sigma(i)}$$

$$perm(A) = \sum_{\sigma \in S_n} \prod_{i=1}^{n} a_{i,\sigma(i)}$$

$$pf(A) = \frac{1}{2^n n!} \sum_{\sigma \in S_{2n}} \operatorname{sgn}(\sigma) \prod_{i=1}^{n} a_{\sigma(2i-1),\sigma(2i)}$$

$$= \sum_{M \in \mathrm{PM}(n)} \operatorname{sgn}(M) \prod_{(i,j) \in M} a_{i,j}$$

13.5.2. *BEST Theorem.*

Count directed Eulerian cycles. Number of OST given by Kirchoff's Theorem (remove r/c with root) $\#\mathrm{OST}(G, r) \cdot \prod_v (d_v - 1)!$

13.5.3. *Primitive Roots.*

Only exists when n is $2, 4, p^k, 2p^k$, where p odd prime. Assume n prime. Number of primitive roots $\phi(\phi(n))$ Let g be primitive root. All primitive roots are of the form g^k where $k, \phi(p)$ are coprime.
k-roots: $g^{i \cdot \phi(n)/k}$ for $0 \le i < k$

13.5.4. *Sum of primes.*

For any multiplicative f:

$$S(n, p) = S(n, p-1) - f(p) \cdot (S(n/p, p-1) - S(p-1, p-1))$$

13.5.5. *Floor.*

$$\lfloor \lfloor x/y \rfloor / z \rfloor = \lfloor x/(yz) \rfloor$$

$$x\%y = x - y \lfloor x/y \rfloor$$