Analyzing Stock Market Bubbles with Evolutionary Game Theory

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Introduction

Economic bubbles and subsequent bursts have been defining moments for humans several times in history. Notable examples include the Dutch Tulip Mania in the 17th century, Great depression of the 1920s, the Dot-Com bubble in the 90s, and recently the Covid-time stock market rally. An economic bubble occurs when out of boundedly rational traits like herd mentality, conformance, cognitive biases, etc the value of an asset is driven way above its fundamental value, only to come crashing down later.

As markets are essentially made of multiple agents interacting with one another, all with their own beliefs, expectations about the future and strategies, game theory provides a strong framework to analyze market behavior.

Evolutionary game theory considers an ecosystem of multiple agents, having a demographic of strategies. The agents play games among each other across many generations and the strategy demographics can change. The ecosystem can be invaded by new strategies, and that can change the demographics too. I will be using this framework to look at the different trading strategies different agents can have, how hype and negative news 'invade' the strategy space and individual incentives driving the boom and bust.

Stock Market Background

The stock market is essentially a marketplace where people can buy and sell equity/share of a company. The goal of the traders is eventually to make a profit, i.e to sell a stock at a higher price than it was bought at. The price of a stock is essentially determined by supply and demand. If there are more buyers than sellers at a given instant, then the sellers can sell at a markup from the Current Market Price (CMP) and the increased price will be recorded as the CMP at the next instant. Similarly, excess supply drives down the CMP.

There are many types of strategies that investors employ, like:

Value Investing (Strategy V)

These types of investors believe that the price of a stock is driven by the company's financial performance. The value of a company can be calculated by Discounted Cash Flow analysis (the present value is the total value a company will generate in its lifetime), Book

Value (the value of all the assets the company owns), etc. Fundamental value of the stock is simply calculated as (Company's value/ Number of Shares).

Momentum Investing (Strategy M)

These types of investors believe that the market will continue its current trend and aim to capitalize on that. For example, if there is positive news about a company and the price starts moving up, the MIs will start buying and sell off just when the price starts going down.

Contrarian Investing (Strategy C)

This is an investment style where the investors purposefully go against the prevalent market sentiment to buy when others are selling and vice versa.

Strategy H

Most of the investors however, use a bit of some of these strategies working mainly on herd mentality, emotions and incomplete knowledge. We will be calling this strategy as H (Herd). These are the ones who are mainly responsible for the boom and bust cycles of stock prices driven by hype and panic. These investors can be easily turned into O (Optimists) or P (Pessimists) for a particular stock.

Source: Investopedia, showing a typical bubble

The Model Setup

We will be analyzing a particular stock, let's call it A. To take a specific example, let's say it's the company eToys from the 90s.(Source: Investopedia)

In the beginning, we assume that the stock price is randomly fluctuating around the Fundamental value. In these conditions all types of strategies (V,M,C,H) don't give any major returns or cause any price changes. For simplicity, we will be assuming that there are 1000 agents in total, where 20 are C and the rest are H.

Now suppose company A proclaims to revolutionize the industry. Here, eToys claims it can take the market share of all the toys as people will prefer online shopping over store visits any day.

The Bubble formation

In the model, a small critical number, say 90 Agents with a strategy O (who believe eToys will achieve its goal) invade the space. O's only want to buy, and only the Contrarians will sell to the Os while the rest are passive. The rest will take on O or C based on the replicator dynamics of each strategy we will look at later. In the payoff matrix of a O vs C game, (Buy, Buy) and (Sell,Sell) don't make any sense and hence are assigned (0,0). (Sell,Buy) will never happen as explained, and hence assigned (0,0).

		C's	
		Buy	Sell
O's	Buy	0,0	PV-CMP, CMP - FV
	Sell	0,0	0,0

Here,

FV = Fundamental value of the stock. In the model, taken as 100

PV = Perceived Value of the Stock by O's, here taken as 1000

CMP = Current Market Price

(All currency units taken in $)

Here Optimists believe that CMP will eventually reach PV, hence by buying at CMP and selling at PV, they hope to get a profit of PV - CMP.

Here Cs believe that CMP is higher than the true value of the stock, hence they make a profit of CMP - FV. The Cs can also ‘short sell’ the stock, which means selling the stock first at a higher price and later buying it at a lower price to cash in the difference. This is done through Options contracts or Broker facilities, but we won’t be going in detail about that. Here, we are assuming Cs are actually short-selling when they are selling as this is how they act in the real world when they see a bubble formation.

Replicator Dynamics

In evolutionary game theory, agents play games over generations. After each generation, the population of agents playing any strategy is related to the payoff the strategy gets in the previous generation. The relation is defined by the replicator dynamics. It could be logistic equation, proportionality, etc.

Here, the increase is proportional to U with some normalization factor:

$$O_{n+1} = O_n*(1+0.01*U_O(B))$$

$$C_{n+1} = C_n*(1+0.3*U_C(S))$$

The normalization factors are obtained by experimentally trying to match the outputs to realistic values.

Generation hops

We first define Demand excess,DE as (number of buyers/number of sellers).

Then,

$$markup = \min(1.2, 1+0.2*(DE/10))$$

The markup and price finding is actually subject to the individual decisions of the buyers and sellers and there's no fixed algorithm. However, here we are assuming that a DE of 10 makes a 20% markup, and the upper limit on markup is 20% per trading session (as fixed by many real exchanges).

Now,

$$CMP_{n+1} = CMP_n*(markup)$$

Also,

$$PV_{n+1} = PV_n*(O_n/O_0)$$

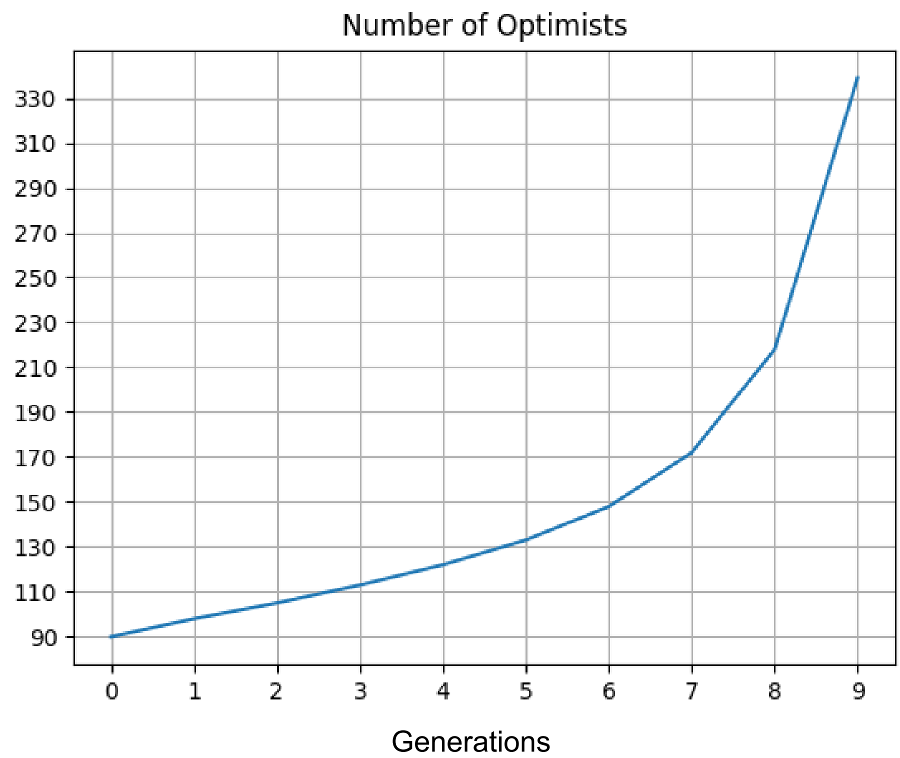
The perceived final value of the stock rises in proportion to the number of traders who believe the price will go up. This creates a positive feedback loop, where increase in PV increases O which inturn increases the PV, thus causing the bubble.

Observations

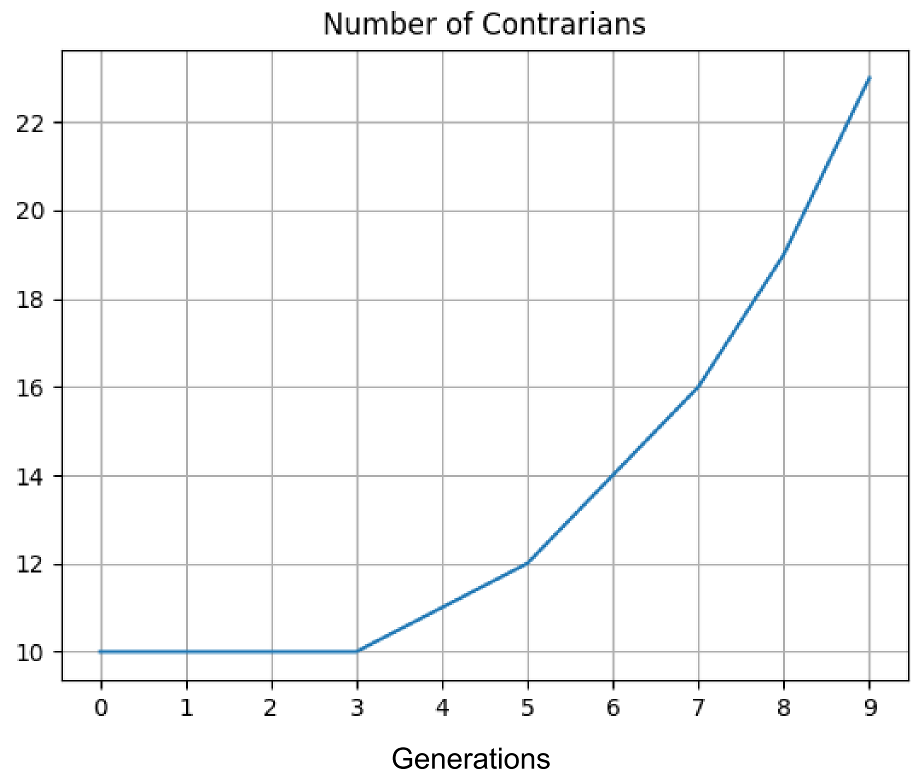
I have built the model in python and have obtained the following results.

Generations (can be roughly taken to be weeks)

The price rises exponentially from the fundamental value of 100 to 500+. We can clearly see an exponential curve and thus bubble formation in play. We also see that the number of agents trading also rapidly rises compared to a near FV market.



The Number of Optimists increases exponentially as expected. O evades the strategy H. However C is resilient and even increases marginally as some traders see the immediate profit of being C



The Busting of the Bubble

When the stock price is very high, there usually occurs one incident that 'busts' the bubble. In the case of eToys it was the expiry of lockup agreements on insider sales. The lockup agreement prevented insiders from selling their shares. But now, a few company insiders who knew the company's poor financials, began to sell off their shares. Their Perceived Value (PV) of the share price was very low and they wanted to cash out before the CMP went that low.

In our model, we are assuming a small number of agents(30 here) with strategy P, i.e Pessimists invade the space.

		C's	
		Buy	Sell
P's	Buy	0,0	0,0
	Sell	CMP-PV, CMP - Peak/2	0,0

Peak = The peak price the stock had reached just before the bust

Here, similar to the boom case, Ps only sell and Cs only buy. The rest of the agents who were initially H and now O, will go onto C or P based on the replicator dynamics. The P's utility comes from trying to sell off now at CMP to avoid selling at it's perceived final price of PV.

For C, as we had mentioned earlier, the C's had short-sold their stocks at prices (F,Peak), approximately at an average price of Peak/2 (F is considered insignificant relative to Peak). C's can make a profit of CMP - Peak/2 from buying the stocks now, thus their assigned utility. Note: UC(B) will be assigned 0 as soon as CMP goes below Peak/2. There will be no more incentive to buy and C's won't replicate further.

Replicator Dynamics

$$P_{n+1} = P_n*(1+0.01*U_P(S))$$

$$C_{n+1} = C_n*(1+0.3*U_C(B))$$

The normalization factors are again, adjusted experimentally.

Generation Steps

We define supply excess, SE as (number of sellers/number of buyers)

Then,

$$\text{markdown} = \max(0.8,1-0.2*(SE/10))$$

Here, again the markdown is subjective to the agents' actions. We are assuming a maximum markdown of 20% (which is the lower circuit in many real exchanges) for a SE of 10. The markdown is proportional to the SE and 20% at most.

Now,

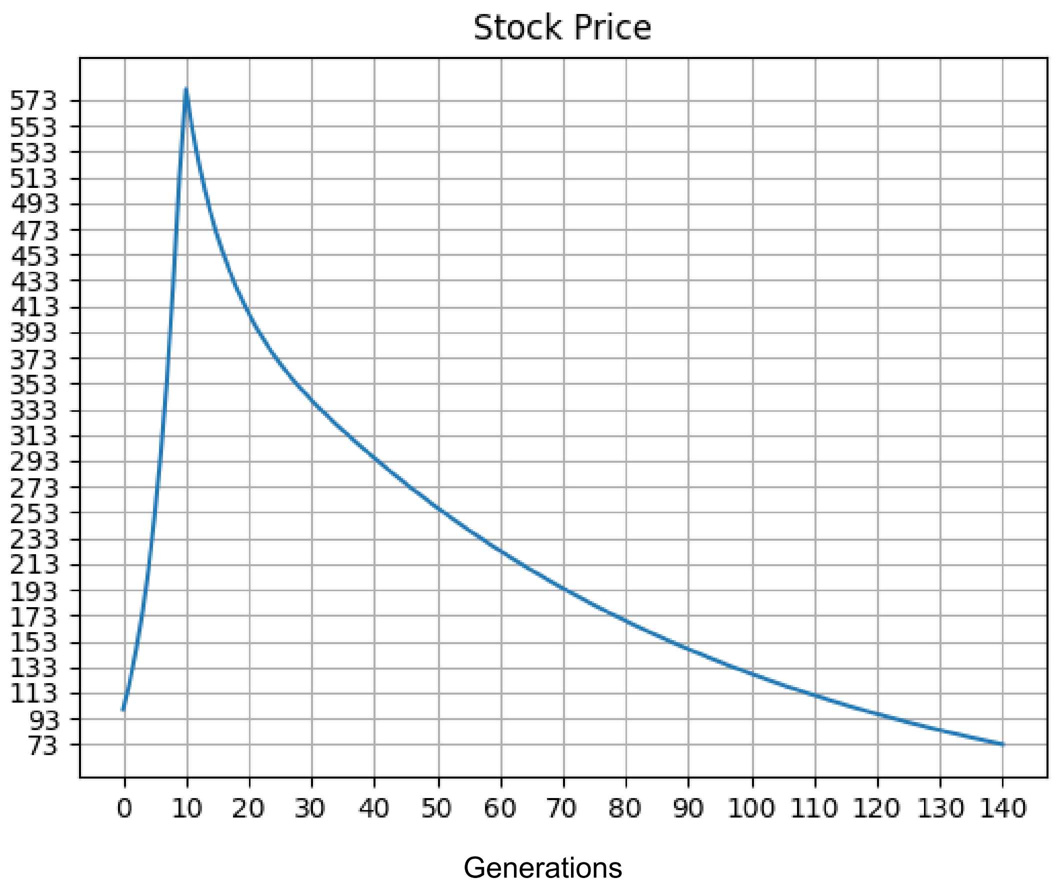
$$CMP_{n+1} = CMP_n*(\text{markdown})$$

Also,

$$PV_{n+1} = PV_n*(P_0/P_n)$$

The perceived value of the final price of the stock goes down in proportion to the number of pessimists. This eventually creates a loop of mania-selling leading to wiping out of the stock's value.

Observations



We see that the stock exponentially falls to a low value (even below the fundamental). This matches our expectations about busts.

Conclusions

"A rapid price rise, high trading volume, and word-of-mouth spread are the hallmarks of typical bubbles," - Timothy R. Burch.

We saw all these factors playing out in our evolutionary game model. Though the model is primitive and ignores much of the details of a trade in a real market, by capturing the behavioral essence of the mass of the traders, and modeling them in a clear framework of evolutionary games, we were able to see the emergent phenomenon expected. A deeper study of markets in the context of evolutionary games can give us useful results about predicting and being able to better handle economic bubbles with large impacts.

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