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Dataflow Analysis

Masters in Informatics and Computing Engineering
(MIEIC), 3rd Year

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Outline

- Dataflow Analysis
- Liveness Analysis
- Exercise
- Dataflow Analysis Issues
- Applications of Dataflow Analysis

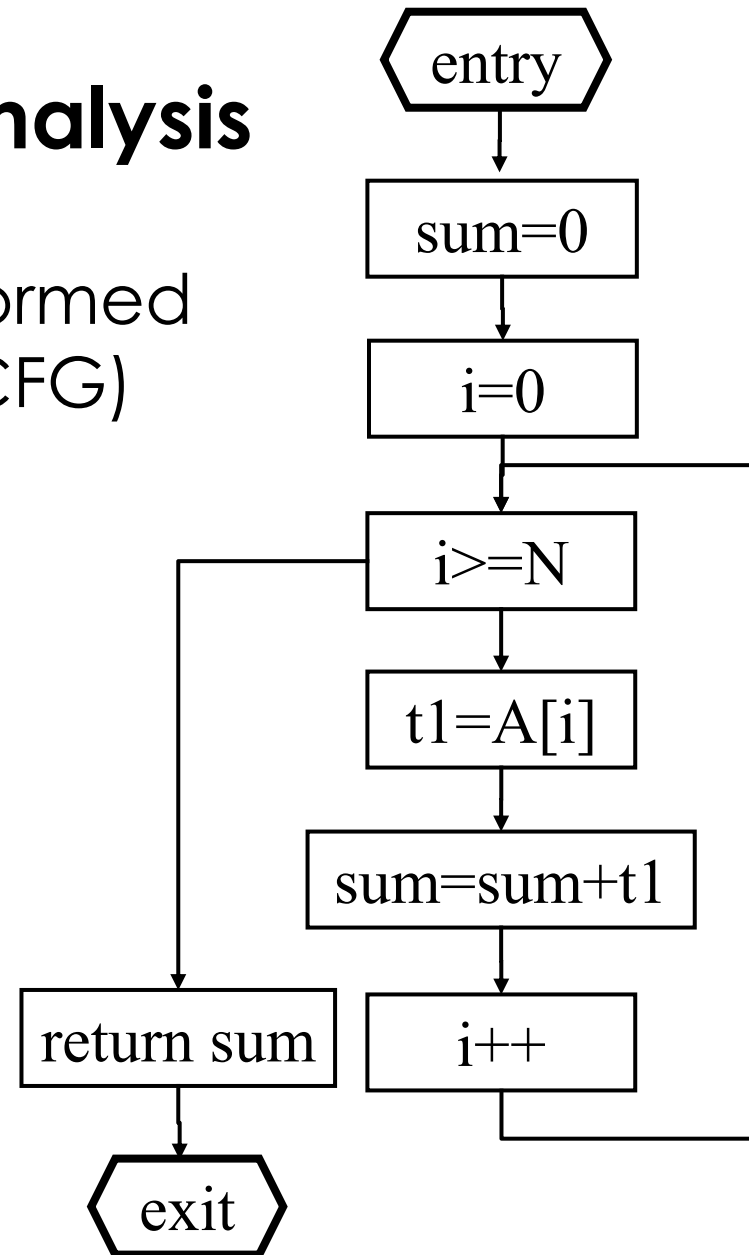
Dataflow Analysis

- Example of a dataflow problem
 - Two variables “a” and “b” can be stored in the same register if their lifetimes do not overlap, but how to determine their lifetimes?
 - We need to analyse the program flow in order to determine the lifetime of each variable
 - A problem known as *liveness analysis*
- *Liveness analysis* is performed using dataflow analysis
- There are many compiler optimizations using dataflow analysis

Dataflow Analysis

- Dataflow analysis is often performed using the control flow graph (CFG)

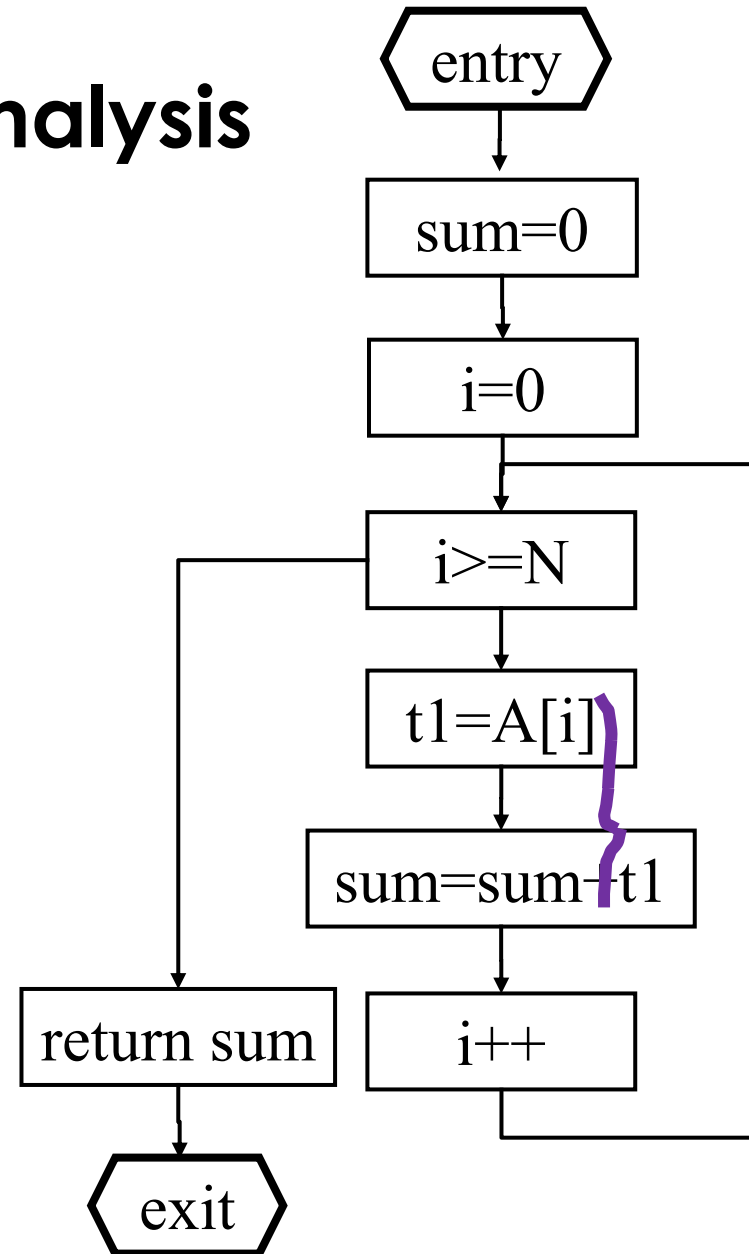
```
sum = 0;  
i=0;  
loop: If(i>=N) goto end:  
      t1=A[i];  
      sum = sum + t1;  
      i++;  
      goto loop;  
end:  return sum;
```



Dataflow Analysis

➤ Live range for variable t1?

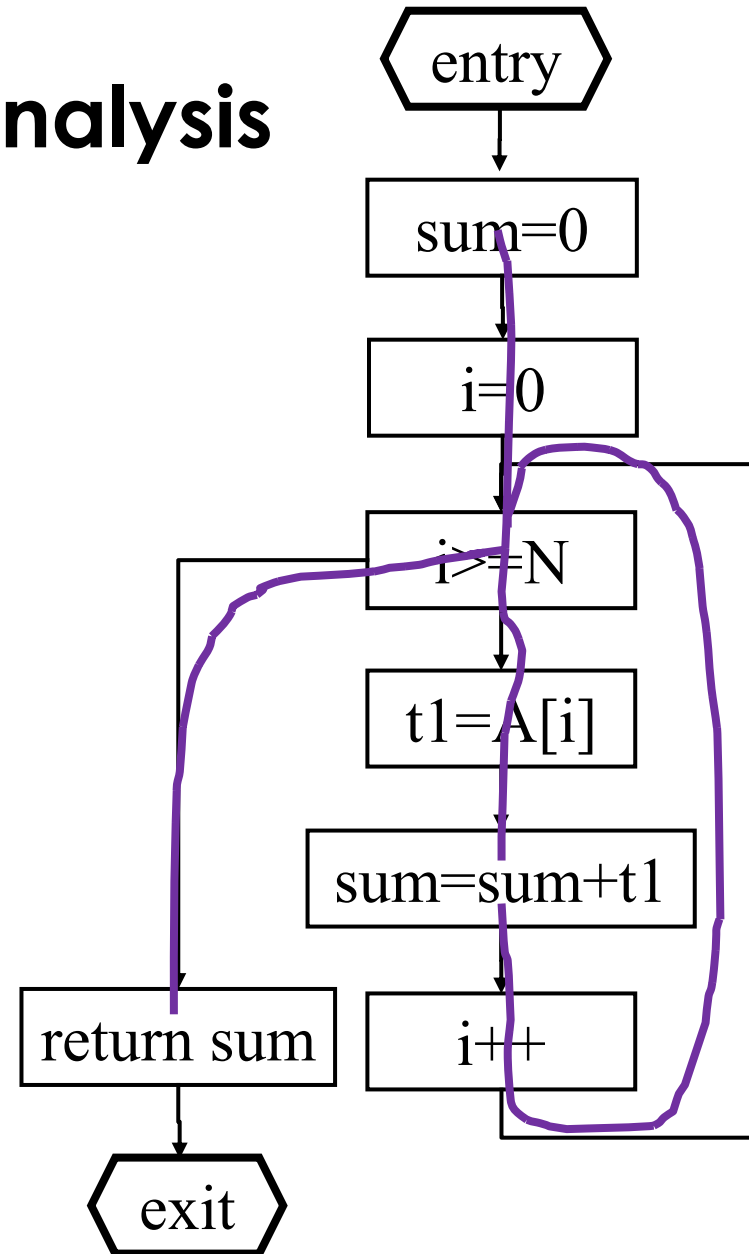
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      sum = sum + t1;
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      goto loop;
end:   return sum;
```



Data Flow Analysis

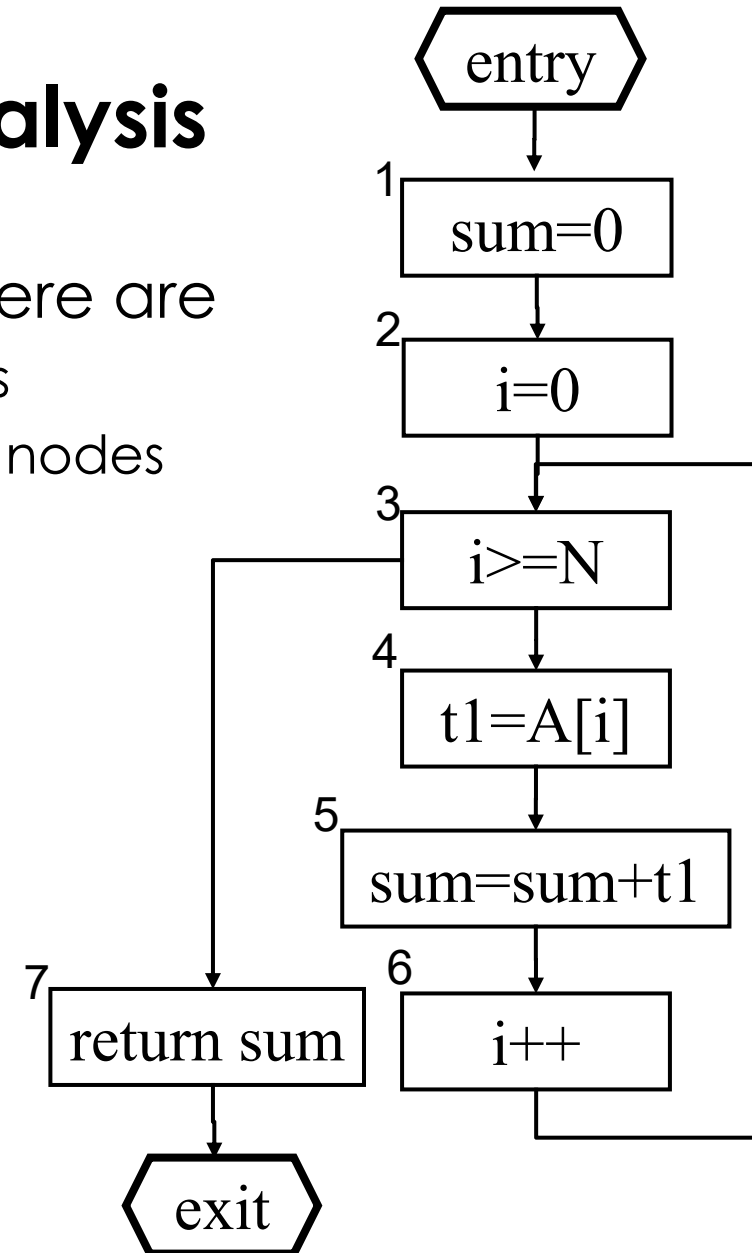
➤ Live range for variable sum?

```
sum = 0;  
i=0;  
loop:  If(i>=N) goto end;  
       t1=A[i];  
       sum = sum + t1;  
       i++;  
       goto loop;  
end:   return sum;
```



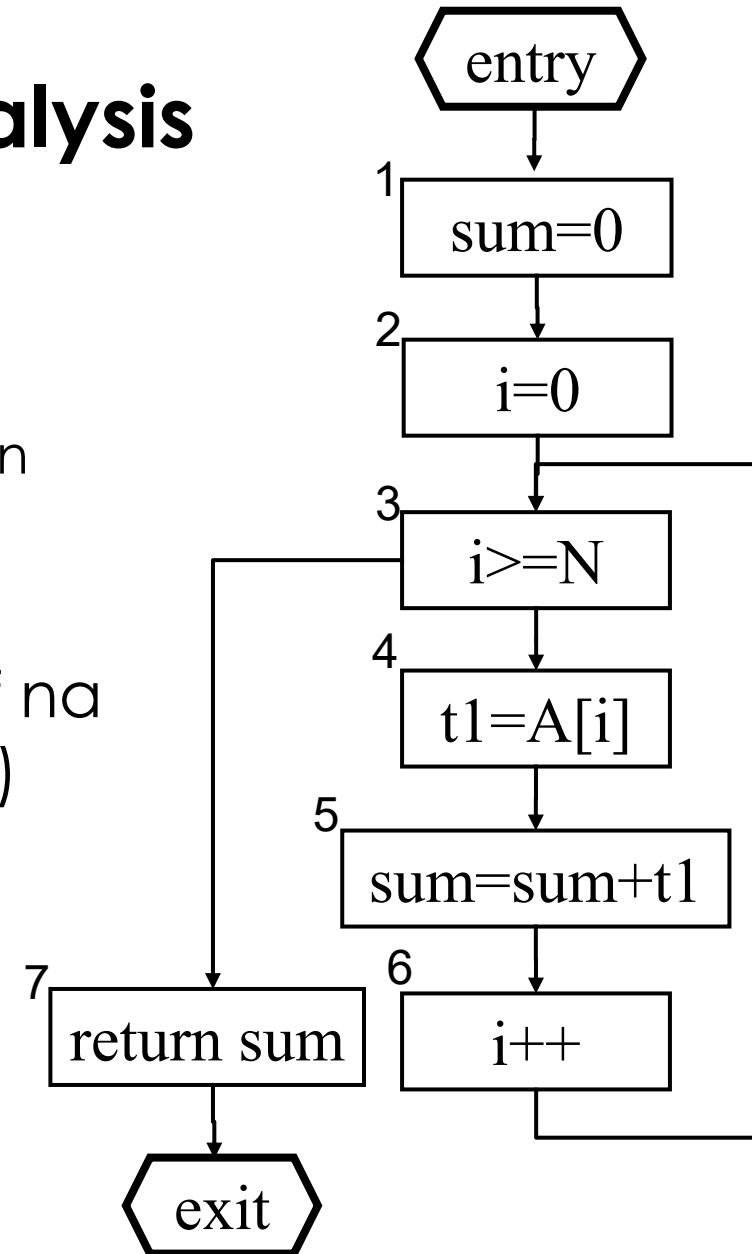
Dataflow Analysis

- Given a node n in a flow graph, there are
 - Out-edges that lead to successor nodes
 - In-edges that come from predecessor nodes
- Sets:
 - **$succ[n]$** is the set of successors
 - **$succ[3] = \{4, 7\}$**
 - **$pred[n]$** is the set of predecessors
 - **$pred[3] = \{2, 6\}$**



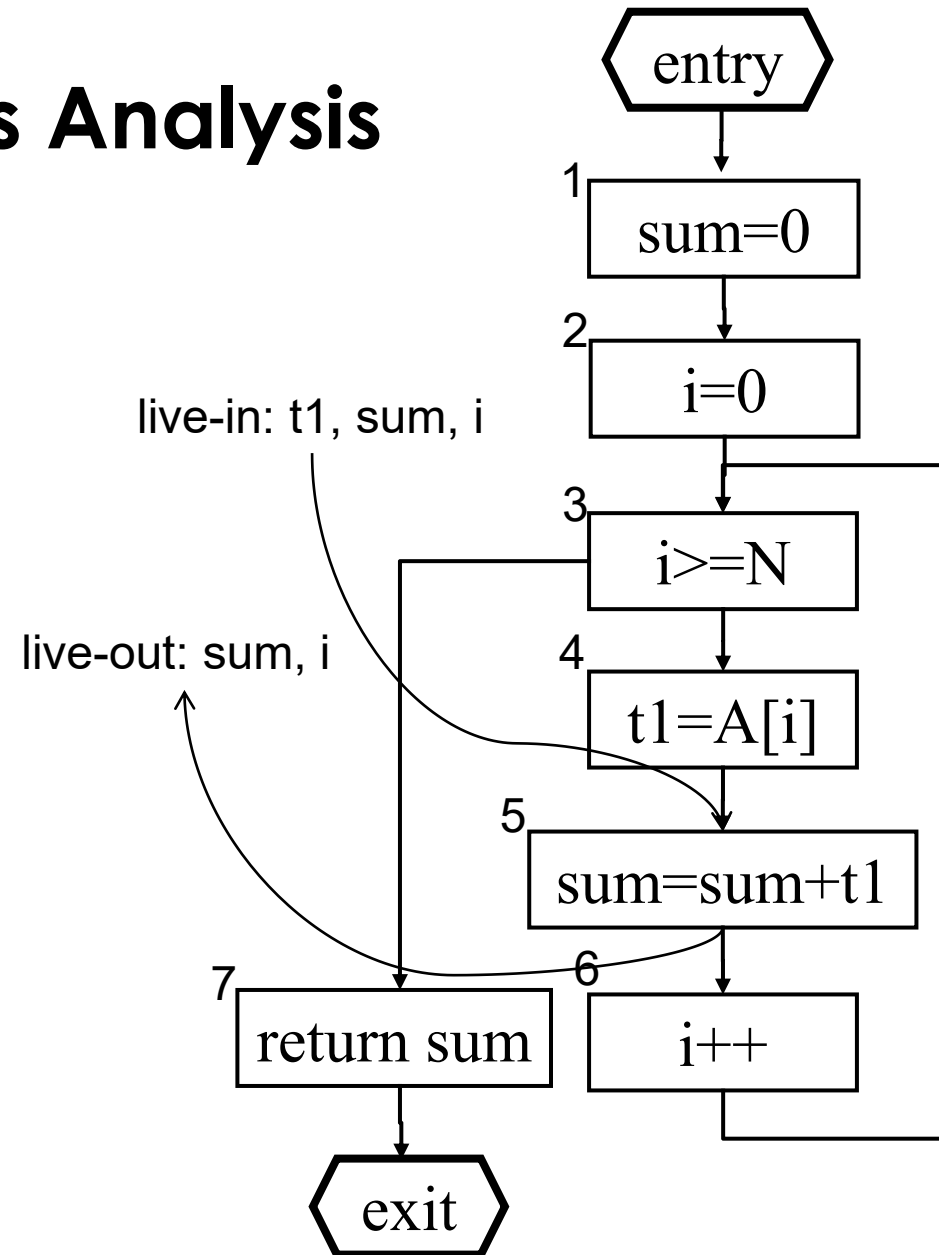
Liveness Analysis

- An assignment to a variable or temporary defines the variable
 - **def[n]** is the set of variables defined in n
 - **def[5]** = {sum}
- An occurrence of a variable or temporary in the right-hand side of an assignment (or in other expressions) uses the variable
 - **use[n]** is the set of variables used in n
 - **use[5]** = {sum, t1}



Liveness Analysis

- A variable is live on an edge if there is a forward path from that edge to a *use* that does not go through any *def* of the same variable
- A variable is *live-in* at a node if it is live in any of the *in-edges* of the node
- A variable is *live-out* at a node if it is live on any of the *out-edges* of the node

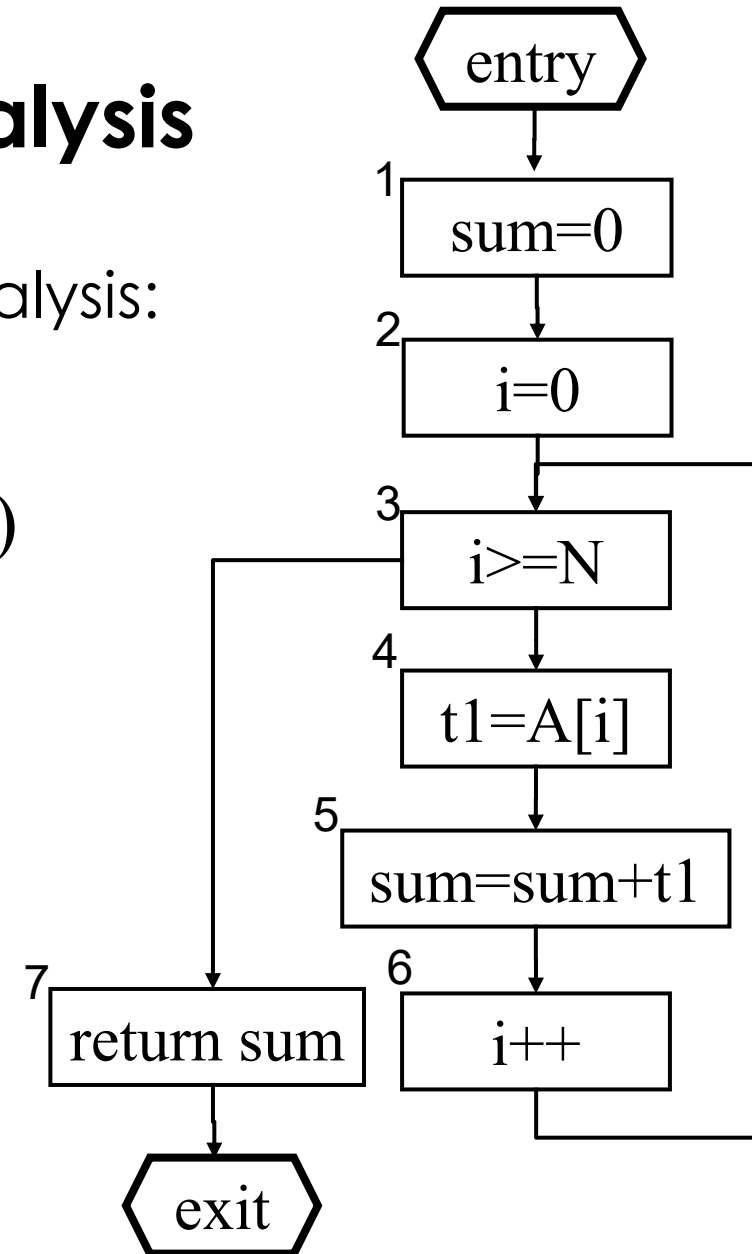


Liveness Analysis

- Dataflow equations for liveness analysis:

$$in[n] = use[n] \cup (out[n] - def[n])$$

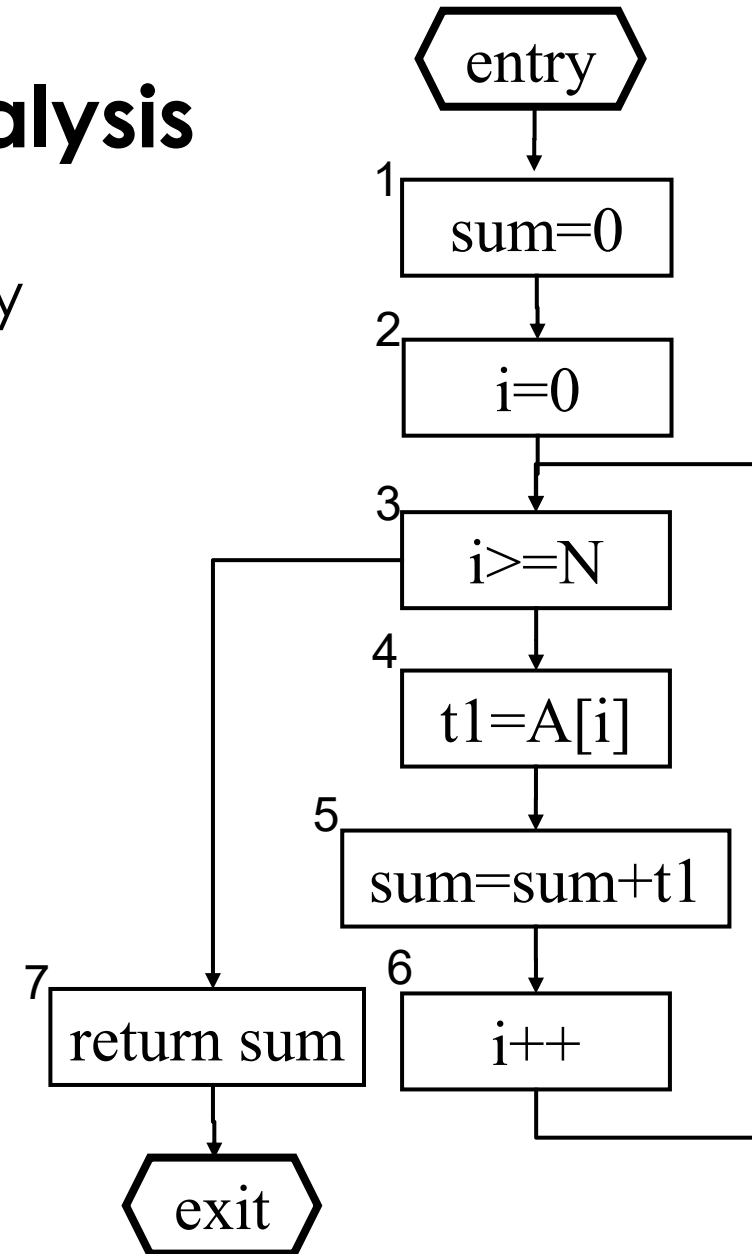
$$out[n] = \bigcup_{s \in succ[n]} in[s]$$



Liveness Analysis

- Computation of liveness analysis by iterative algorithm:

```
for each n
  in[n]  $\leftarrow$  {}; out[n]  $\leftarrow$  {}
repeat
  for each n
    in'[n]  $\leftarrow$  in[n]; out'[n]  $\leftarrow$  out[n]
    in[n]  $\leftarrow$  use[n]  $\cup$  (out[n] - def [n])
    out[n]  $\leftarrow$   $\bigcup_{s \in succ[n]} in[s]$ 
  until in'[n] = in[n] and out'[n] = out[n] for all n
```

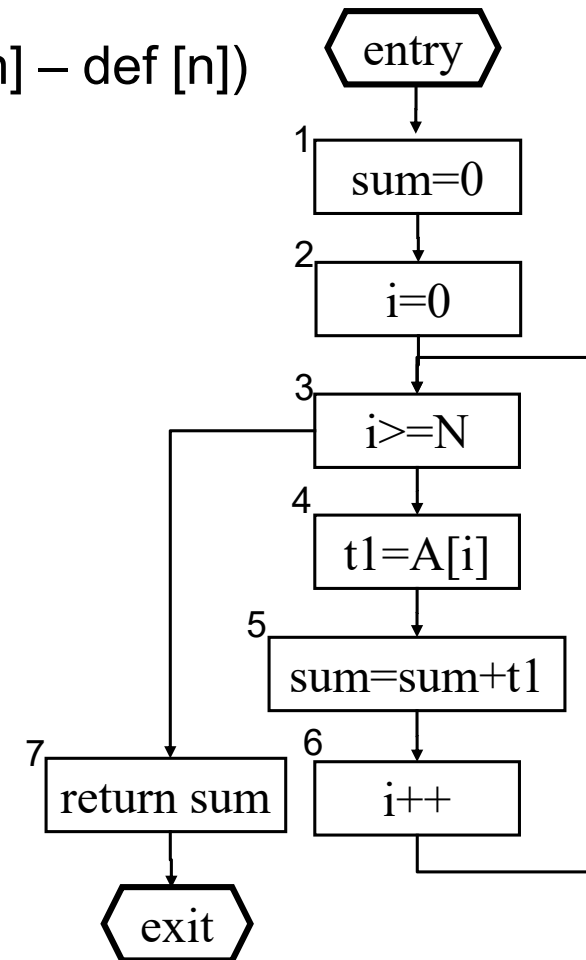


Liveness Analysis

- Computation of liveness analysis (forward):

$$\begin{aligned} \text{in}[n] &\leftarrow \text{use}[n] \cup (\text{out}[n] - \text{def}[n]) \\ \text{out}[n] &\leftarrow \bigcup_{s \in \text{succ}[n]} \text{in}[s] \end{aligned}$$

				1st iteration		2nd iteration		3rd iteration		4th iteration	
node	use	def	succ	in	out	in	out	in	out	in	out
1											
2											
3											
4											
5											
6											
7											

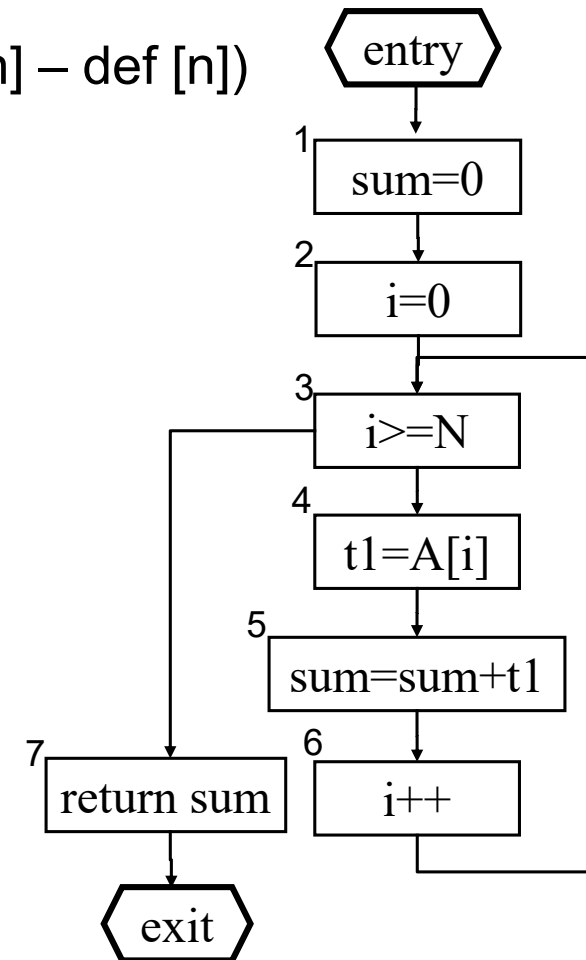


Liveness Analysis

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				1st iteration		2nd iteration		3rd iteration		4th iteration	
node	use	def	succ	in	out	in	out	in	out	in	out
1		s	2								
2		i	3								
3	i		4,7								
4	i	t	5								
5	s,t	s	6								
6	i	i	3								
7	s										



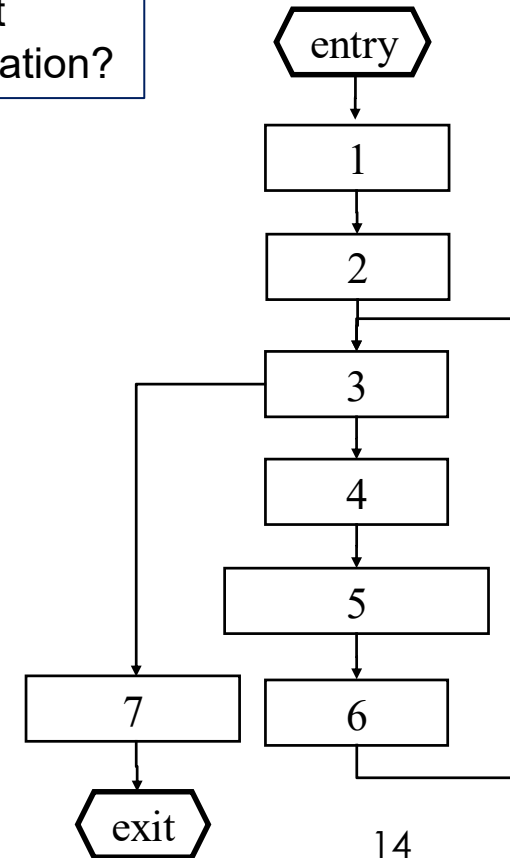
Liveness Analysis

- Computation of liveness analysis (forward):

$$\begin{aligned} \text{in}[n] &\leftarrow \text{use}[n] \cup (\text{out}[n] - \text{def}[n]) \\ \text{out}[n] &\leftarrow \bigcup_{s \in \text{succ}[n]} \text{in}[s] \end{aligned}$$

last iteration?

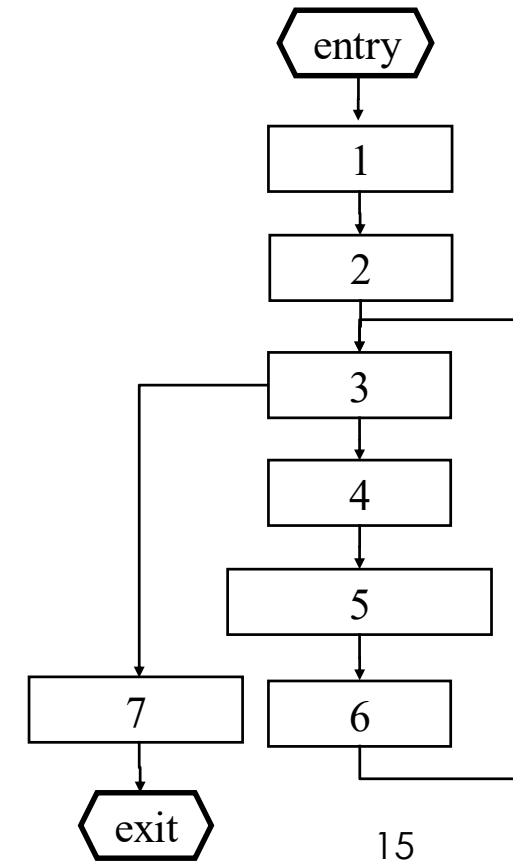
				1st iteration		2nd iteration		3rd iteration		4th iteration		6th iteration	
node	use	def	succ	in	out	in	out	in	out	in	out	in	out
1		s	2										s
2		i	3				i		i		s,i	s	s,i
3	i		4,7	i		i	s,i	s,i	s,i	s,i	s,i	s,i	s,i
4	i	t	5	i		i	s,t	s,i	s,t	s,i	s,t,i	s,i	s,t,i
5	s,t	s	6	s,t		s,t	i	s,t,i	i	s,t,i	i	s,t,i	s,i
6	i	i	3	i	i	i	i	i	s,i	s,i	s,i	s,i	s,i
7	s			s		s		s		s		s	



Liveness Analysis

- Computation of liveness analysis (backward):
- $$\begin{aligned} \text{out}[n] &\leftarrow \bigcup_{s \in \text{succ}[n]} \text{in}[s] \\ \text{in}[n] &\leftarrow \text{use}[n] \cup (\text{out}[n] - \text{def}[n]) \end{aligned}$$

beginning				1st iteration		2nd iteration		3rd iteration	
Node	use	def	succ	out	in	out	in	out	in
7	s								
6	i	i	3						
5	s,t	s	6						
4	i	t	5						
3	i		4,7						
2		i	3						
1		s	2						

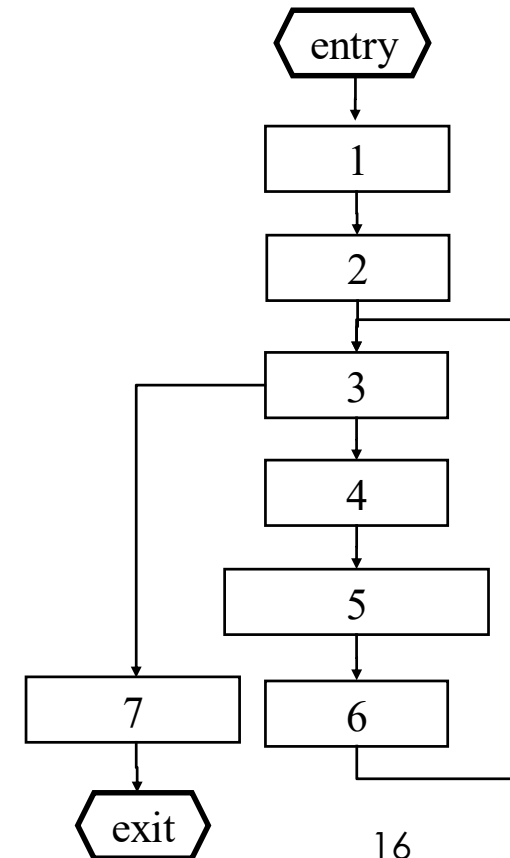


Liveness Analysis

- Computation of liveness analysis (backward):
- $$\text{out}[n] \leftarrow \bigcup_{s \in \text{succ}[n]} \text{in}[s]$$
- $$\text{in}[n] \leftarrow \text{use}[n] \cup (\text{out}[n] - \text{def}[n])$$

beginning				1st iteration		2nd iteration		3rd iteration	
node	use	def	succ	out	in	out	in	out	in
7	s				s		s		s
6	i	i	3		i	s,i	s,i	s,i	s,i
5	s,t	s	6	i	s,t,i	s,i	s,t,i	s,i	s,t,i
4	i	t	5	s,t,i	s,i	s,t,i	s,i	s,t,i	s,i
3	i		4,7	s,i	s,i	s,i	s,i	s,i	s,i
2		i	3	s,i	s	s,i	s	s,i	s
1		s	2	s		s		s	

3 iterations



Liveness Analysis

- It is a backward problem
- Thus we should use a backward dataflow analysis formulation

- Note that the use of

$$out[n] = \bigcup_{s \in succ[n]} in[s]$$

$$in[n] = use[n] \cup (out[n] - def[n])$$

- Instead of:

$$in[n] = use[n] \cup (out[n] - def[n])$$

$$out[n] = \bigcup_{s \in succ[n]} in[s]$$

- Saves iterations

Liveness Analysis

- It is a backward problem
- **Depth-first search** can be used for ordering the nodes of the CFG
- What would be the order of nodes for the CFG of the previous example?

https://en.wikipedia.org/wiki/Depth-first_search

Exercise 1

- Computation of liveness analysis (backward):

			1st iteration		2nd iteration		3rd iteration	
stmt	use	def	out	in	out	in	out	in
6								
5								
4								
3								
2								
1								

Example:

1: $t1 = x * x$;

2: $t2 = a * t1$;

3: $t3 = b * x$;

4: $t4 = t3 + c$;

5: $t5 = t4 + t2$;

6: $y = t5$;

Exercise 2

- Computation of liveness analysis
- Do we need an iterative algorithm for computing liveness analysis in straightline code (as in the example below)?

Example:

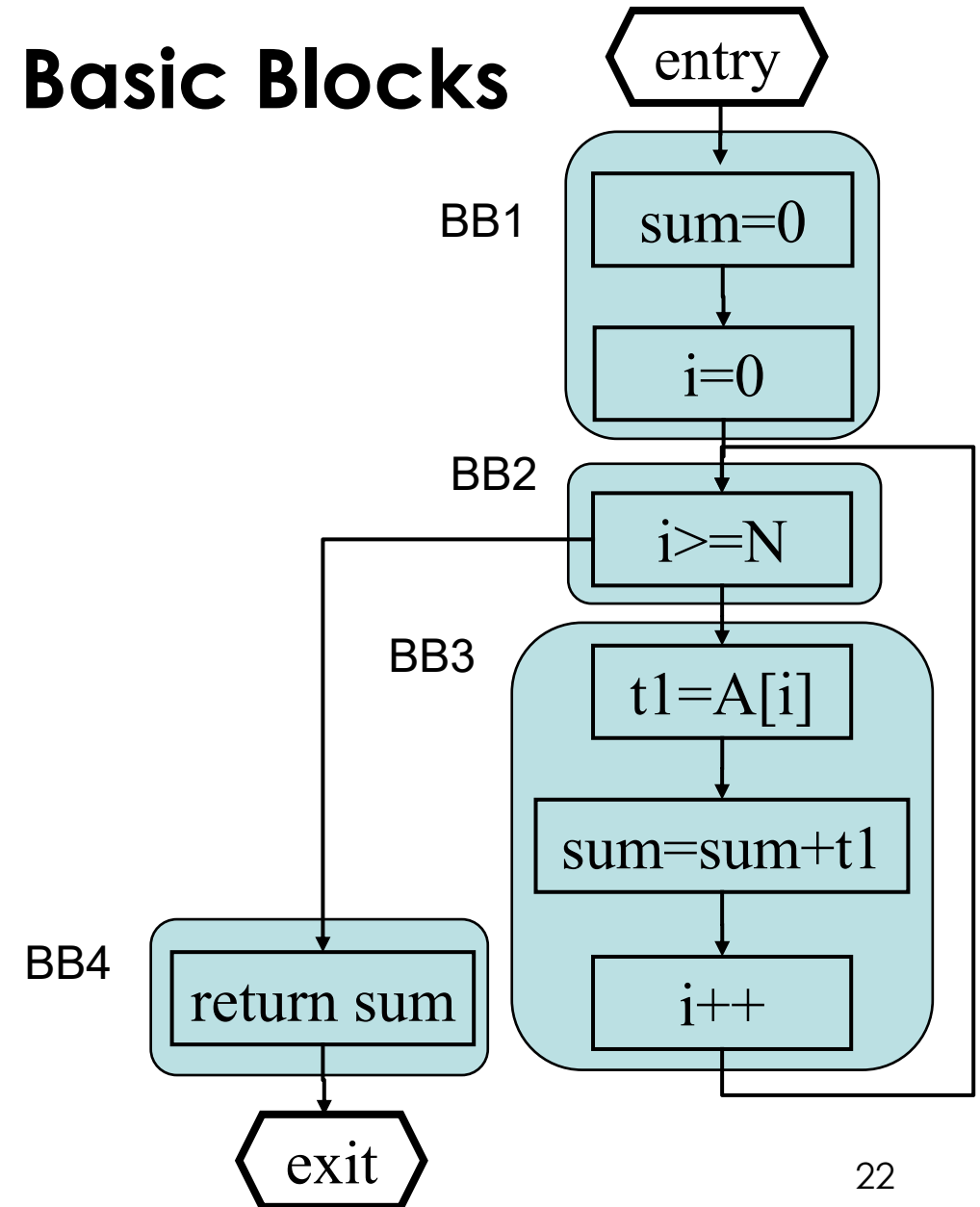
```
1: t1=x*x;  
2: t2=a*t1;  
3: t3=b*x;  
4: t4=t3+c;  
5: t5=t4+t2;  
6: y=t5;
```

Dataflow Analysis Issues

- Speed of the iterative algorithm depends
 - on the number of nodes of the CFG (control flow graph) and
 - on the visiting order of the nodes of the CFG
- It is common to use CFGs consisting of basic blocks (see next slide) instead of one node per instruction
- Order of visiting should be according to the flow direction of the problem (e.g., liveness analysis is a backward problem)

Control Flow Graphs of Basic Blocks

- CFGs consisting of basic blocks (BBs)
- Basic blocks are the largest groups of instructions where
 - the control flow begins at the first instruction and only exit in the last one
 - Once in a basic block, all its instructions are executed



Applications of Dataflow Analysis

- Used for many optimizations
 - liveness analysis
 - def-use and use-def computations
 - constant propagation
 - copy propagation
 - dead-code elimination
 - common subexpression elimination
 - etc.
- Example:
 - How to determine the definitions of a variable that reach a certain use of the variable?