Exame de Época Normal / First Exam (2019/01/18)

Duration: 2h30 **Version A**

No consultation is allowed, other than the supplied document.
No electronic means are allowed (computer, cellphone, …).
Fraud attempts lead to the annulment of the exam for all students involved.

Answer each group in separate sheets!
Write your full name and exam version in all sheets!

Group I: [4.5 Points] Finite Automata and Regular Expressions

Consider the language L = { x ∈ {0,1}* | x has even length or ends with 11}:

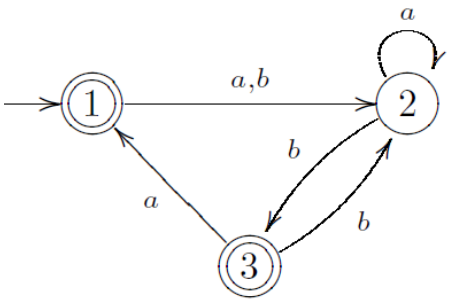
a) Draw a Finite Automata (FA) for L in a methodologically way and present all the necessary steps;

b) Draw a DFA for L;

Given a DFA for L_1 = { x ∈ {0,1}* | 4th bit from left of x is 0 }:

c) How can we automatically obtain a DFA for L_2 = { x ∈ {0,1}* | 4th bit from right of x is 0}?

Consider the DFA below:

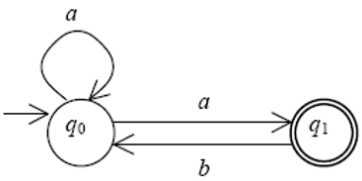


d) Using the state elimination technique, convert the DFA into a regular expression;

Consider the NFA on the right:

e) Convert the NFA to a DFA using the subset construction;

f) Apply the DFA minimization method and to the resultant DFA and draw the DFA obtained after the minimization;



Group II: [2 Pts] Properties of Regular Languages

a) Knowing that L1 = {$a^n b^n$ | n ≥ 0} is a non-regular language, prove, without using the Pumping Lemma for regular languages, that L = {$0^n 1^m 2^{n-m}$ | n ≥ m ≥ 0} is also a non-regular language;

b) The symmetric difference of two sets (commonly denoted by A⊗B) is the set of elements which are in either of the sets and not in their intersection. For example, the symmetric difference of the sets {1,2,3} and {3,4} is {1,2,4}. Are regular languages closed under the symmetric difference operation? Justify your answer, explaining the main reasons that make this operation not closed or showing, in the case it is closed, how one can obtain a regular expression representing the symmetric difference of two languages given by regular expressions.

Group III: [4.5 Pts] Context-Free Grammars (CFG) and Push-Down Automata (PDA)

Consider the language: L = {$a^n w w^R a^j$ | n ≥ 0, n ≤ j ≤ 2n, w∈{a, b}*}

a) Write a CFG for L;
b) Show a leftmost derivation for the string "aabbaa" and draw the respective syntax tree;
c) Is the CFG you provide ambiguous? Justify your answer and eliminate the ambiguity of the CFG in the case it is ambiguous;
d) Draw the PDA, which accepts by empty stack, directly obtained from the CFG;
e) Indicate a sequence of instantaneous descriptions that result in the acceptance of the string: "aabbaa".

Group IV: [4 Pts] Turing Machine

We intend to implement a Turing Machine (TM) that can analyze a sequence of digits and answers the question: "are there more 1's than 0's on the tape?"

a) Describe a strategy to implement a TM to answer this question.

b) Draw a possible TM.

c) Indicate the computing trace when the input to the TM is: 101.

Group V: [5 Pts] Statements about Languages (T/F: 20%, justification: 80%; wrong answer = reduction of 50%)

Indicate, justifying succinctly (with a couple of sentences or a counter example), whether each of the following statements is True (T) or False (F).

a) The regular languages are all those languages which can be generated starting from the finite languages by applying the regular operations;

b) $L=\{a^{2i}b^{3i} \mid i \geq 0\}$ is a CFL.

c) The language represented by the CFG, $S \rightarrow 00S \mid S1 \mid S2 \mid 3$, is a regular language.

d) If we prove with the Pumping Lemma for context-free languages (CFLs) that a subset of a language is not a CFL then we can always conclude that the language is not a CFL.

e) If L is a non-regular language then its complement L^C is also non-regular.

f) If L1 and L2 are two non-regular languages then $L1 \cup L2$ is also non-regular.

g) If the stack of a PDA is finite then the languages that the PDA recognizes can be recognized by DFAs.

h) There is a systematic way to provide a non-deterministic Turing Machine for any given regular expression;

i) There is a systematic way to provide a non-deterministic Turing Machine for any given CFG;