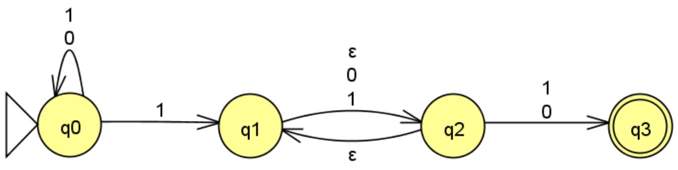
Exame de Época de Recurso / Final Exam (2018/01/30)

Duration: 2h30 **Version A**
No consultation is allowed, other than the supplied document.
No electronic means are allowed (computer, cellphone, …).
Fraud attempts lead to the annulment of the exam for all participants.

Answer each group in separate sheets!
Write your full name and exam version in all sheets!

Group I: [4.5 Points] Finite Automata and Regular Expressions

Consider the following ε-NFA.

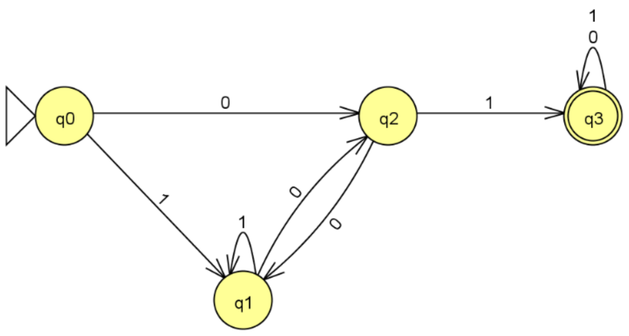


a) Determine the ε-closure for each state of the ε-NFA.

b) Obtain the equivalent DFA for the ε-NFA. Show the transition table and the state diagram of the DFA.

c) Draw a DFA that represents the complement of the language given by the above ε-NFA.

Consider the following DFA:



d) Indicate a regular expression that represents the language of the DFA, obtained by the state elimination technique and following the ordering of eliminations: q2 → q1. Indicate all the intermediate steps needed in the conversion.

In the context of a project, a team intends to eliminate as many as ε-transitions in ε-NFAs without converting them to DFAs. One of the reasons is that the team needs to implement the NFAs, but wants to reduce or if possible to eliminate the number of ε-transitions.

e) Describe a method that allows to eliminate as many as possible ε-transitions and if possible to show an NFA. If necessary, show examples of the application of the method.

Group II: [2 Pts] Properties of Regular Languages

One intends to validate sequences of requests and answers, such that the sequences with a number of answers equal to the number of requests, and in which there cannot exist answers without previous requests, are the only valid sequences. The sequences are represented by chains of symbols in which 'R' is the symbol that identifies a request and 'A' is the symbol that identifies an answer. Ex. of valid chains: RRARAA, RARRARAA, RA. Ex. of invalid chains: AR, RRA, R, A, RAR.

a) Indicate a regular expression and if not possible a CFG that represents the language of the valid sequences.

Pumping lemma for regular languages:

a) Prove using the pumping lemma for regular languages that the language L = {ww | w ∈ {0, 1}*} is not a regular language.

Group III: [4.5 Pts] Context-Free Grammars (CFG) and Push-Down Automata (PDA)

$S \rightarrow aSa \mid aBa$

$B \rightarrow bB \mid b$

Consider the CFG G on the left, in which S is the start variable.

a) Show a syntax tree and the rightmost derivation for the string "aabbaa".

b) The grammar is ambiguous? Justify your answer.

c) Convert the CFG to a PDA that accepts by empty stack and show the state diagram of the resultant PDA.

d) Indicate a CFG to represent the language $L = \{a^m b^k c^n \mid m \neq n \text{ and } m,n,k \geq 1\}$.

Suppose a finite automaton of PDA type, but that instead of a stack it uses a queue of type FIFO (*First-In-First-Out*). The transitions of the automaton use a format similar to the PDAs, but in which the Reading of the queue may or may not pop the first symbol in the queue and the push puts one or more symbols in the end of the queue. For example, the transition "x, A=/AB" between two states q0 and q1 and supposing AZ in the FIFO implies: (q0,x,A) = (q1,AZAB) while "x, A/AB" between two states q0 and q1 and supposing AZ in the FIFO: (q0,x,A) = (q1,ZAB).

e) Indicate is this kind of automaton is able to implement the language $L = \{ww \mid w \in \{0,1\}^+\}$ and in the case of being able show a possible queue-based automaton implementing L;

f) Indicate is this kind of automaton is able to implement the language $L = \{ww^R \mid w \in \{0,1\}^+\}$ and in the case of being able show a possible queue-based automaton implementing L;

Group IV: [4 Pts] Turing Machine

One intends to implement a deterministic Turing Machine (TM) to convert positive integer numbers represented in unary to the binary representation. The TM receives sequences of 1's as input (assume that there exist at least one '1' in the input), and in the end must show the result in the format Binary=Unary, in which Unary represents the original input (in unary) and Binary represents the same number in binary.

Ex.: the string "11111" results in "101=11111"; the string "1111" results in "100=1111".

a) Describe the a strategy to implement a TM able to perform the intended operation.

b) Implement a deterministic TM described in the previous answer.

c) Indicate the computing trace when the string in the tape is 11 (show at least 10 step of the computing trace).

Group V: [5 Pts] Statements about Languages (V/F: 20%, justification: 80%; wrong answer = reduction of 50% of the points for V/F selection)

Indicate, justifying succinctly (with 1 or 2 sentences or a counter example), whether each of the following statements is True or False.

a) When the intersection of two CFLs (Context Free Languages) does not result in the empty state, the resulting language can never be a CFL.

b) Depending on the alphabet, the complement of a regular language can be or not a CFL.

c) It is not possible to have a deterministic PDA to implement the language $L = \{wcw^R \mid w \in \{a,b\}^+\}$.

d) The intersection of a regular language with a CFL is always a regular language.

e) In some cases it is possible to verify with a DFA is the sum of two integers is correct.

f) There exist languages that a non-deterministic PDA is able to recognize and that a deterministic Turing Machine cannot recognize.

g) The language $L = \{wxw \mid w,x \in \{0,1\}^+ \wedge |w| \leq 10\}$ is regular language.

h) The language $L = \{xyz \mid x, z \in \{0,1\}^* \wedge y \in \{a,b\}^* \wedge |x| = |z|\}$ is a CFL.