IMAGE PROCESSING

Topics

- Image Enhancement
  - Brightness mapping
  - Contrast stretching/enhancement
  - Histogram modification
  - Noise Reduction
  - ………
- Mathematical Techniques
  - Convolution
    - Mean filtering
    - Gaussian filtering
- Edge and Line Detection and Extraction
- Contour Extraction
- Corner Detection
- Region Segmentation

Histogram-based transformations

- Thresholding
  - threshold selection (manual & automatic)
- Transformations for contrast enhancement
  - linear
    - linear stretching
  - non-linear
    - power law
    - logarithmic
    - equalization
    - CLAHE

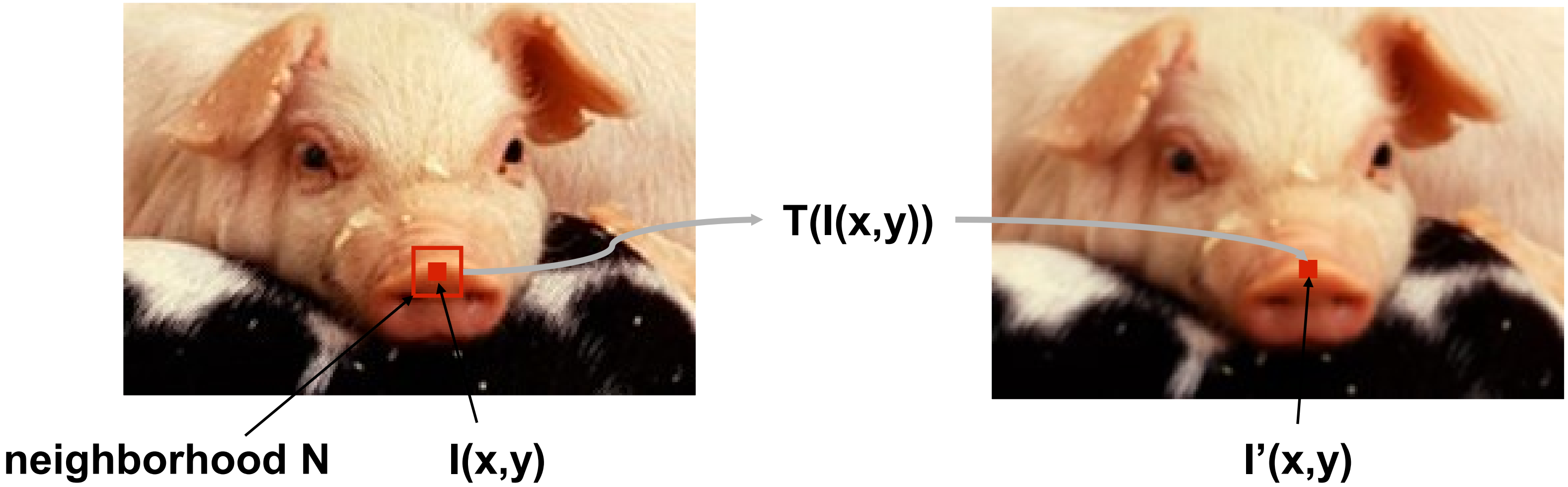
Filtering for noise reduction

- Linear filters: mean and Gaussian
  - convolution operation
- Non-linear filters
  - median
  - anisotropic diffusion filter
  - bilateral filter
  - …
- Frequency domain filters

Image enhancement

- Goal: improve the 'visual quality' of the image
  - for human viewing
  - for subsequent processing
- Two typical methods
  - spatial domain techniques....
    - operate directly on image pixels
  - frequency domain techniques....
    - operate on the Fourier transform of the image
- No general theory of 'visual quality'
  - General assumption: if it looks better, it is better
  - Often not a good assumption

Spatial domain methods



- Transformation T
  - point - pixel to pixel
  - local - local area to pixel
  - global - output value at a specific coordinate depends on all values in the input image. (ex: DFT)
- Local - neighborhoods
  - typically quadrangular
  - typically an odd size: 3x3, 5x5, etc. (why odd size? see later)
  - centered on pixel I(x,y)

$$I'(x,y) = T(I(x,y))$$

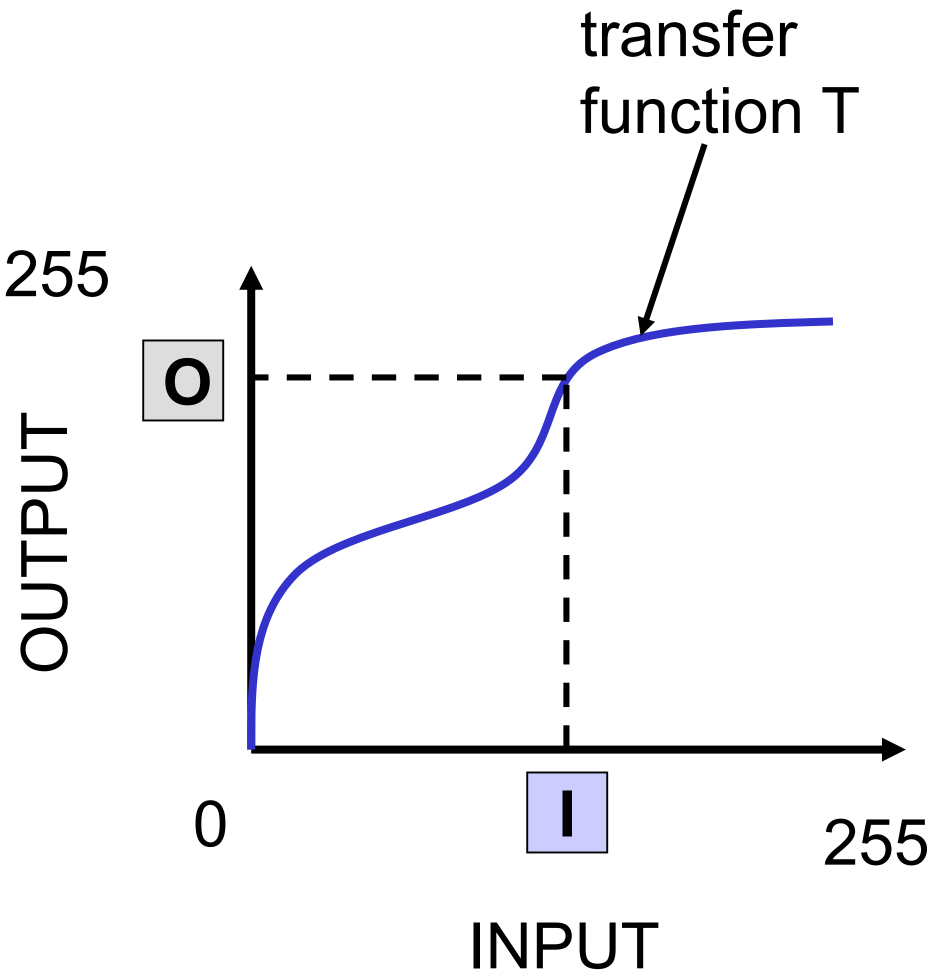
Point transformations

Histogram-based transformations

Point transformations: general idea

O = T(I)

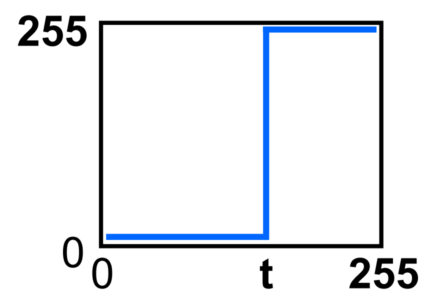
Input pixel value, I, mapped to output pixel value, O, via transfer function T.



Point transformations:thresholding

- T is a point-to-point transformation
  - only information at I(x,y) used to generate I'(x,y)
- Thresholding

$$I'(x,y) = \begin{cases} I_{max} \text{ if } I(x,y) > t \\ I_{min} \text{ if } I(x,y) \leq t \end{cases}$$



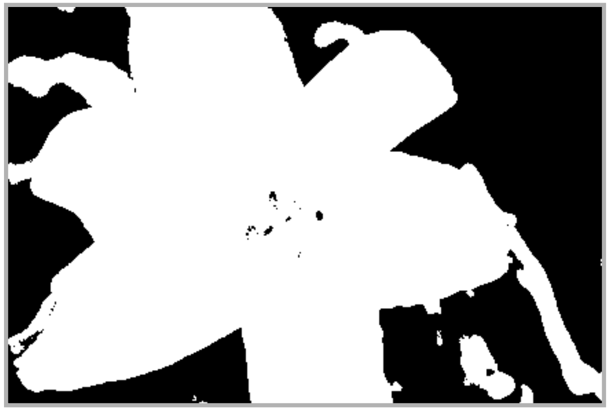
Color image

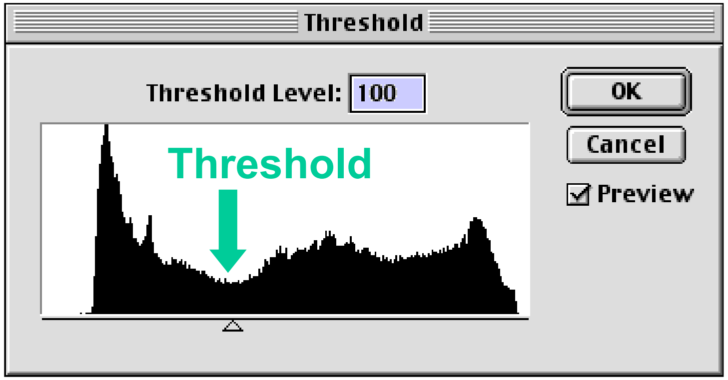
Graylevel image

Thresholded graylevel image (**t=89**)

Threshold selection

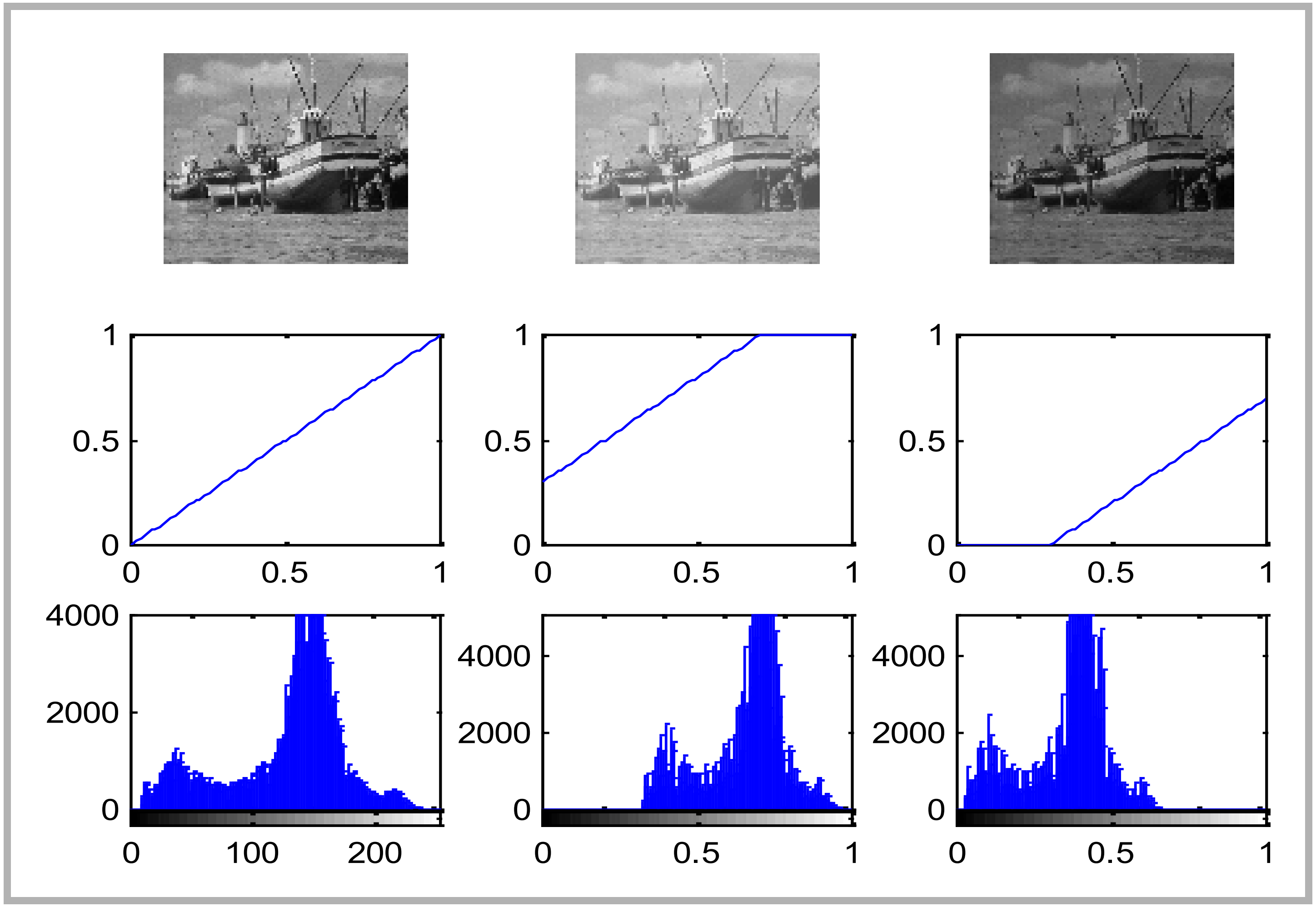
- Arbitrary selection
  - select visually
- Use image histogram



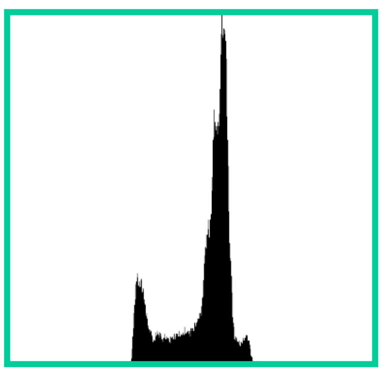


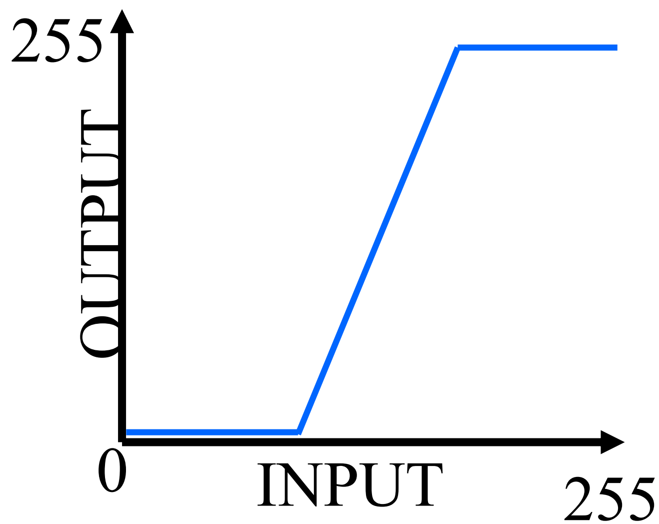
Later, we'll be back to threshold selection methods

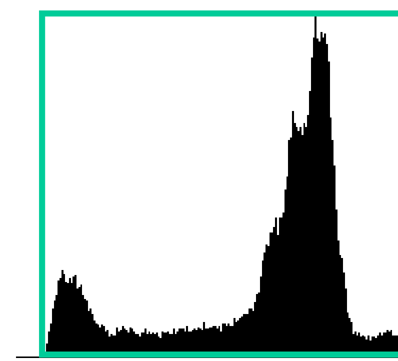
Point transformations: brightness



Point transformations: linear stretch



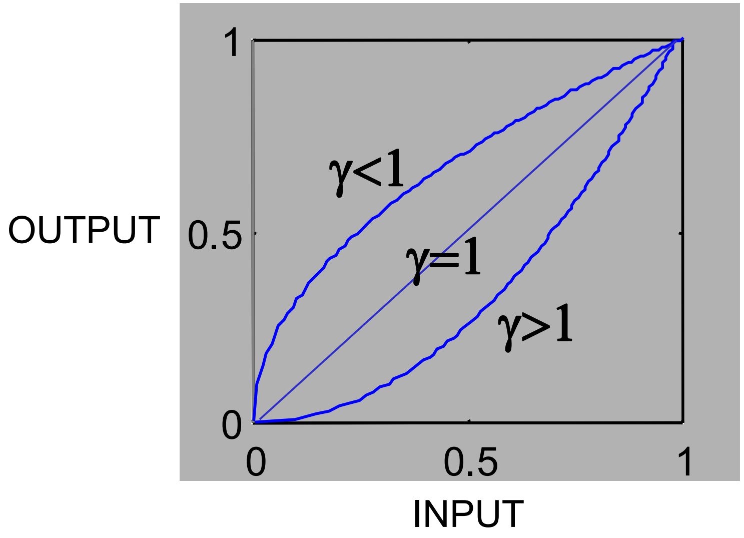




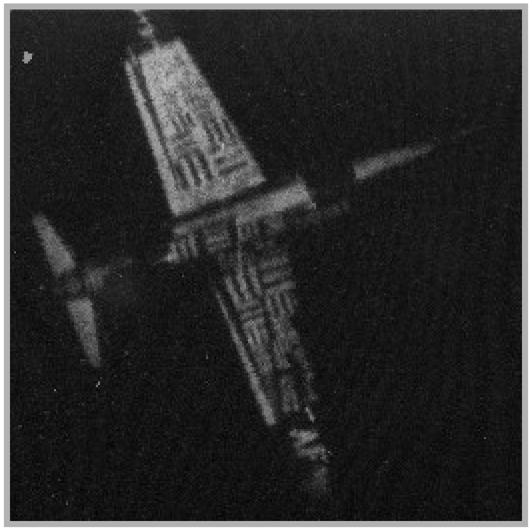
Non-linear scaling: power law

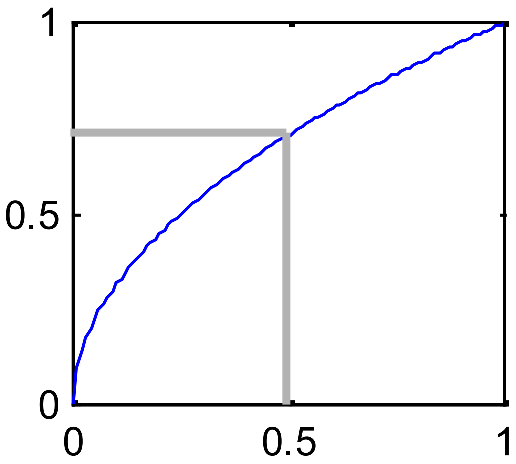
$$O = I^{\gamma}$$

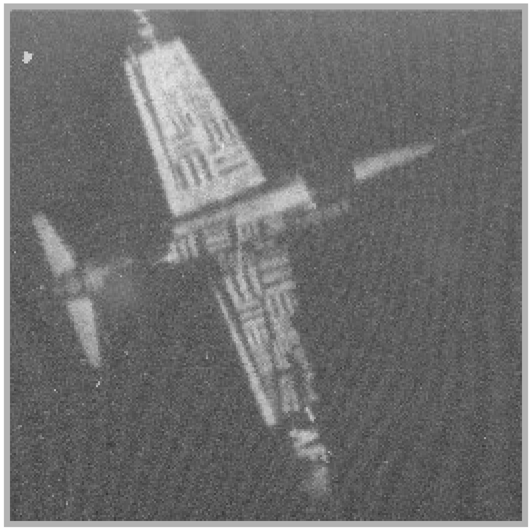
- $\gamma < 1$ to enhance contrast in dark regions
- $\gamma > 1$ to enhance contrast in bright regions.

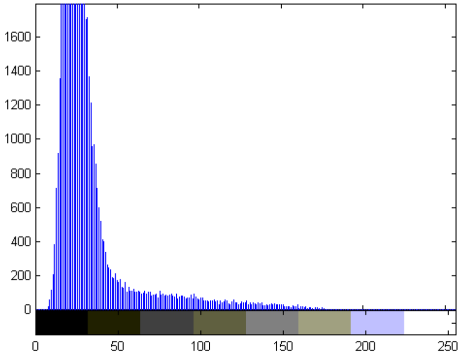


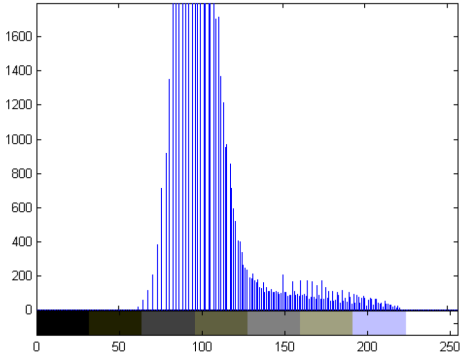
γ=0.5



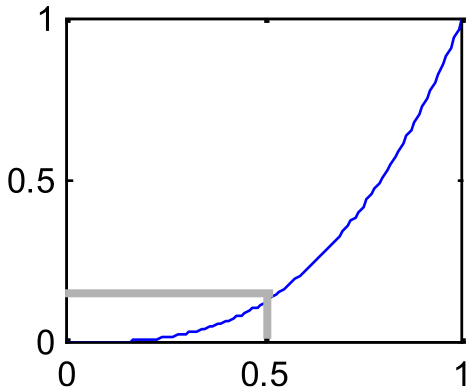

1
0.5
0
0
0.5
1

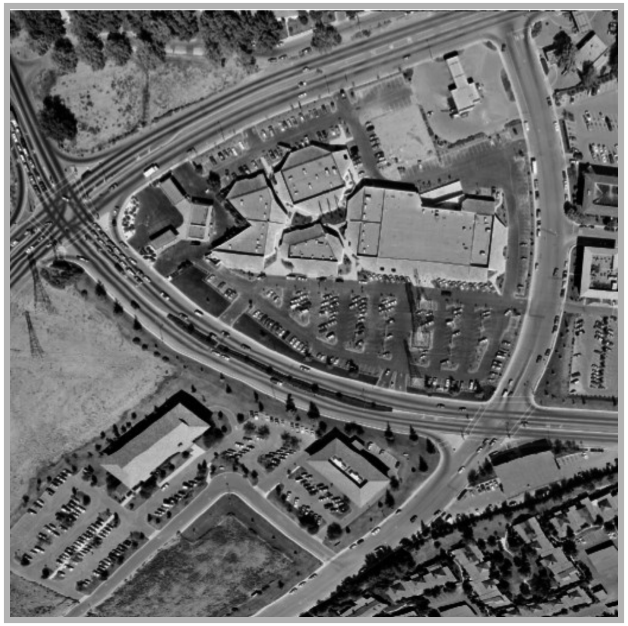


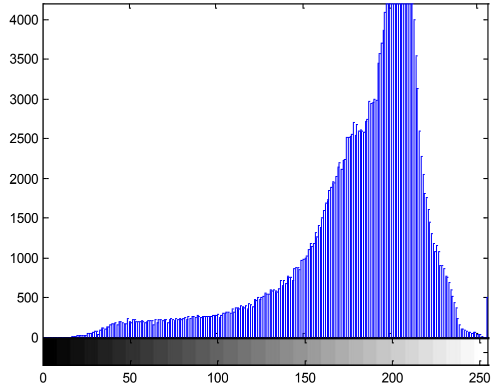

1600
1400
1200
1000
800
600
400
200
0
0
50
100
150
200
250


1600
1400
1200
1000
800
600
400
200
0
0
50
100
150
200
250

γ=3.0


1
0.5
0
0
0.5
1




4000
3500
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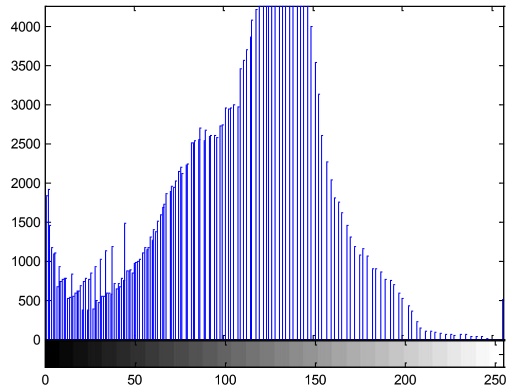
150
200
250


4000
3500
3000
2500
2000
1500
1000
500
0
0
50
100
150
200
250

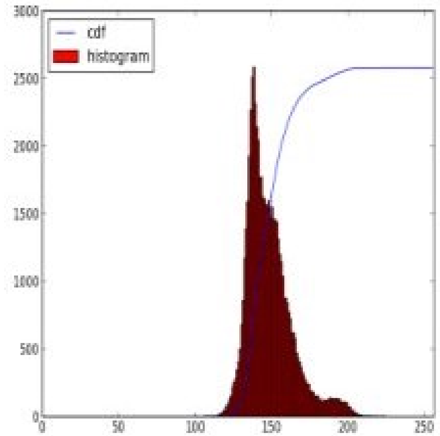
Point transformations: color images

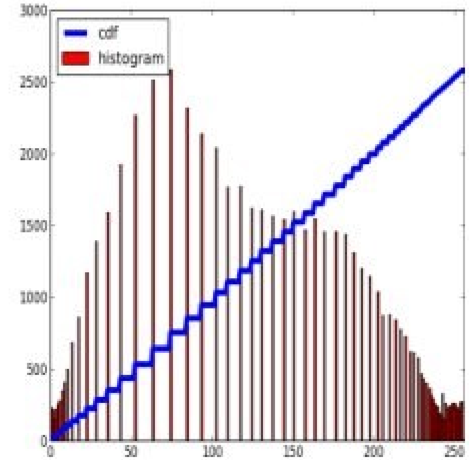
- Technique can be applied to **color images**
  - same curve to all color bands
  - different curves to separate color bands:

  - but ... **be careful**, colors may become distorted...!

- Point transformations are usually applied using **LUT's (Look Up Tables)**

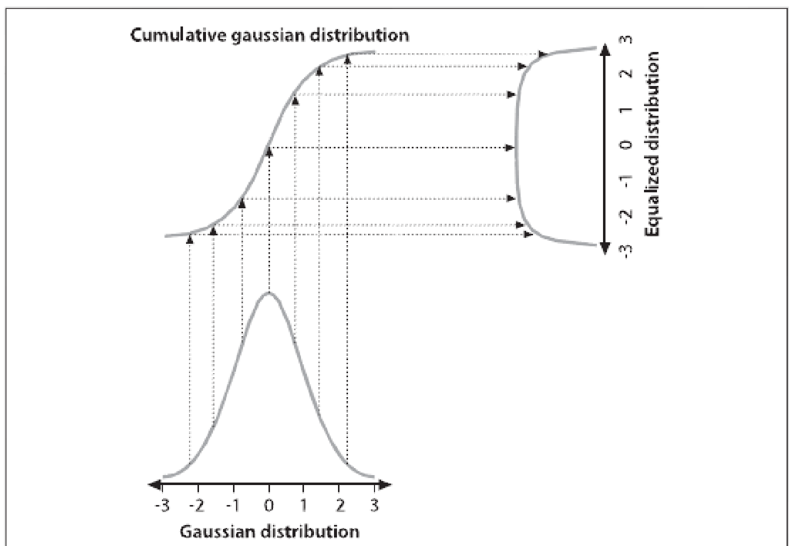
Histogram equalization





source: http://docs.opencv.org/

After equalization the cumulative distribution function is almost linear



Using the cumulative distribution function to equalize a Gaussian distribution

Comparison

Original

Histogram stretching

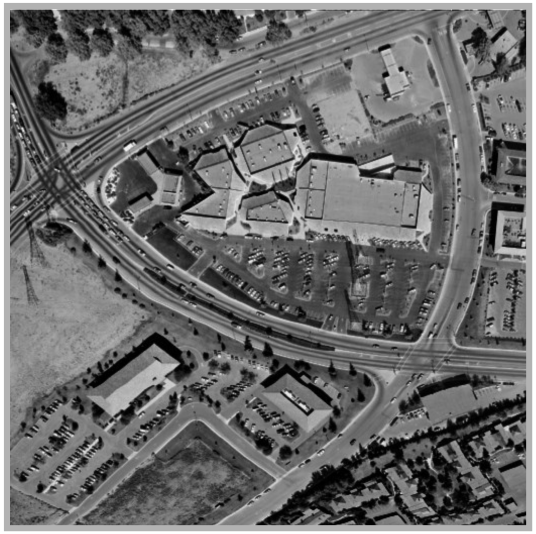
Histogram equalization

- The contrast enhancement is better after the histogram equalization, which more easily detects structures located in the shade.
- In fact any strongly represented gray-level is stretched while any weakly represented graylevel is merged with other close levels

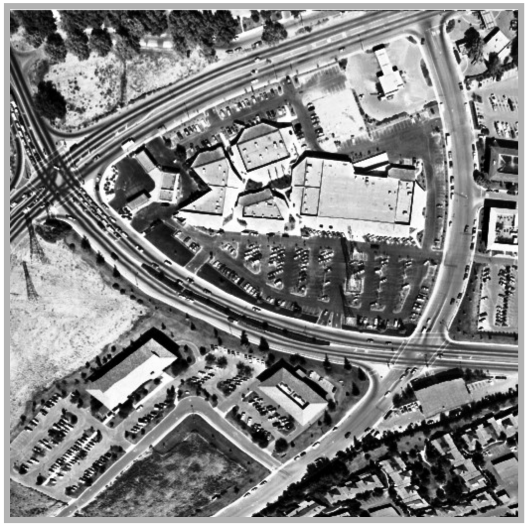
Comparison

Original

$\gamma > 1$

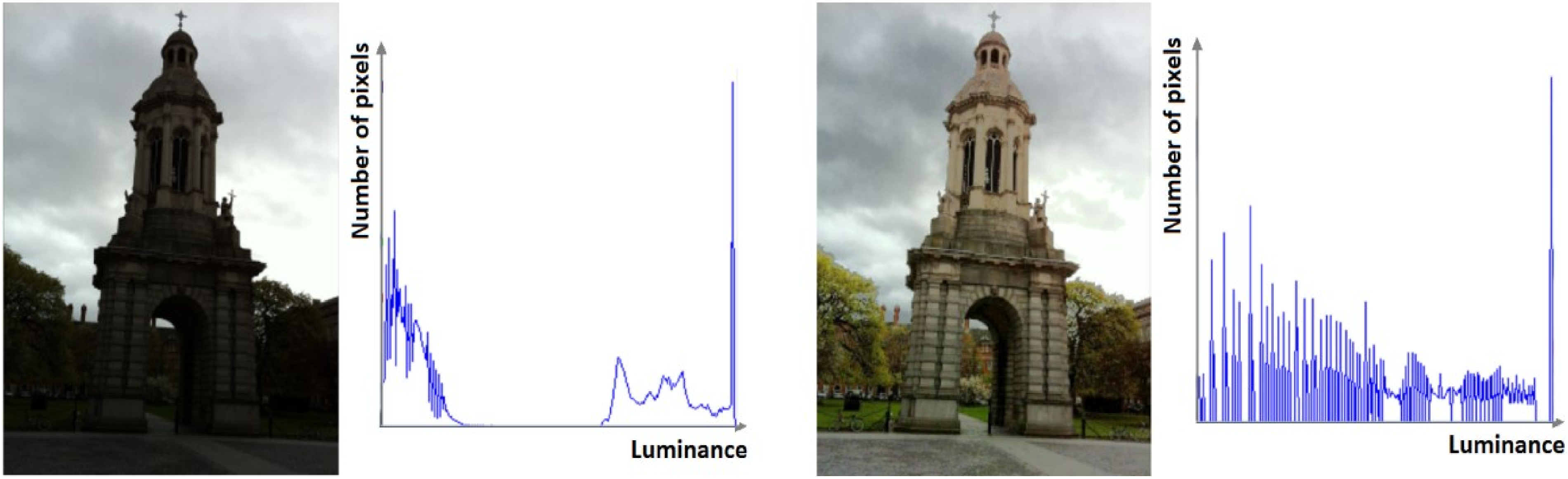


Histogram
equalization



Equalization of color images

- In color images,
  <u>only</u> the <u>luminance</u> channel is usually equalized
  as otherwise the colors can become distorted.

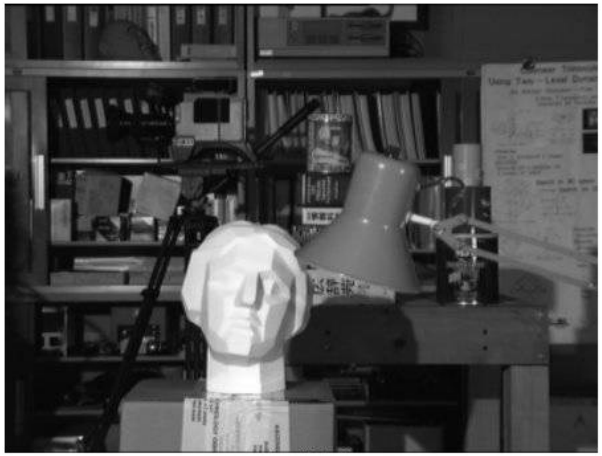


source: *A Practical Introduction to Computer Vision with OpenCV* , by Kenneth Dawson-Howe

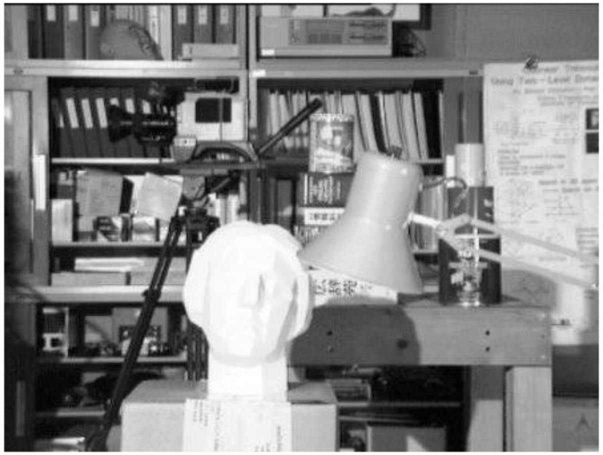
Observations

- Acquisition process degrades image
- Brightness and contrast enhancement implemented by pixel operations
- No one algorithm universally useful
- $\gamma > 1$ enhances contrast in bright images
- $\gamma < 1$ enhances contrast in dark images
- Transfer function for histogram equalization proportional to cumulative histogram
- It is essential a process of trial-and-error to determine whether a particular type of images will benefit from histogram transformation operations.

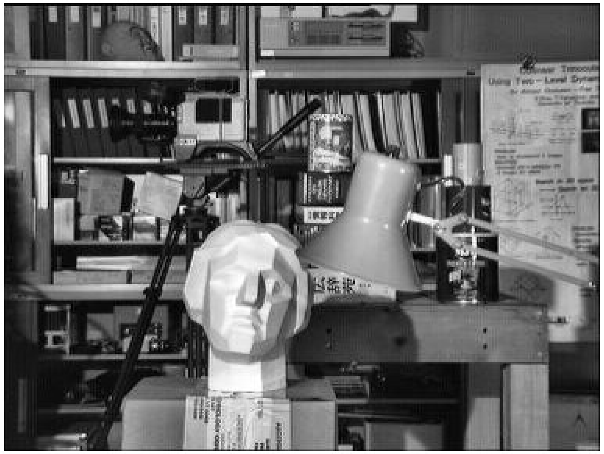
- **CLAHE – Contrast Limited Adaptive Histogram Equalization**
  - Considering the global contrast of the image is not a good idea, in some cases ...
    (look at the face of the statue, in the middle image)
  - For some images,
    it might be preferable to apply different kinds of equalization in different regions.
  - Instead of computing a single curve, the image is divided into
    MxM pixel non-overlapped sub-blocks and
    separate histogram equalization is performed in each sub-block.
  - To avoid blocking artifacts (i.e., intensity discontinuities at block boundaries)
    in the resulting image,
    the equalization functions are smoothly interpolated as we move between blocks.
  - This technique is known as adaptive histogram equalization (AHE) and
    its contrast limited (gain-limited, to avoid noise amplification) version is known as CLAHE.



Original image



After global equalization
(notice the effect on the face of the statue)



After CLAHE

source: Szeliski's book + http://docs.opencv.org/

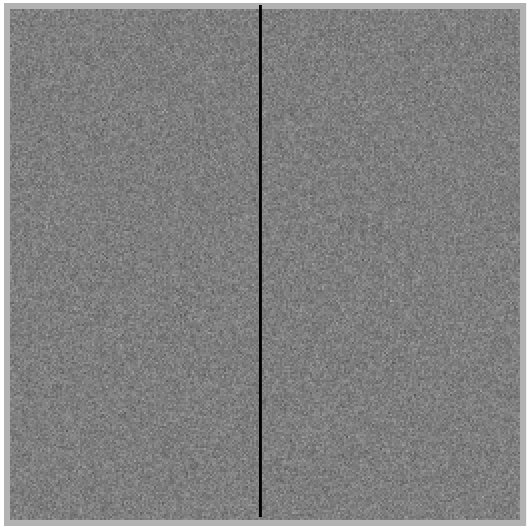
Filtering

Noise Reduction

- What is noise?
- How is noise reduction performed?
  - Noise reduction from first principles
  - Neighbourhood operators
    - linear filters (low pass)
    - non-linear filters (median)

image

+



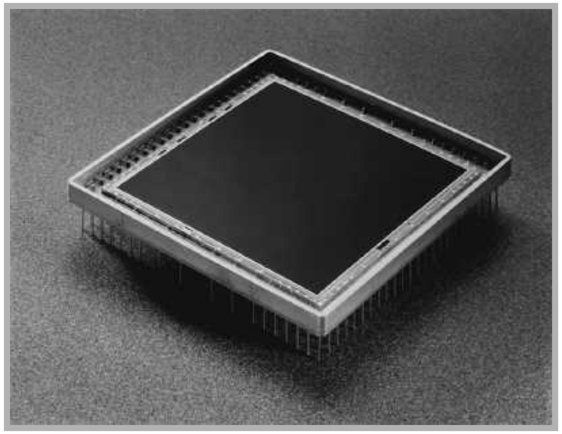
noise

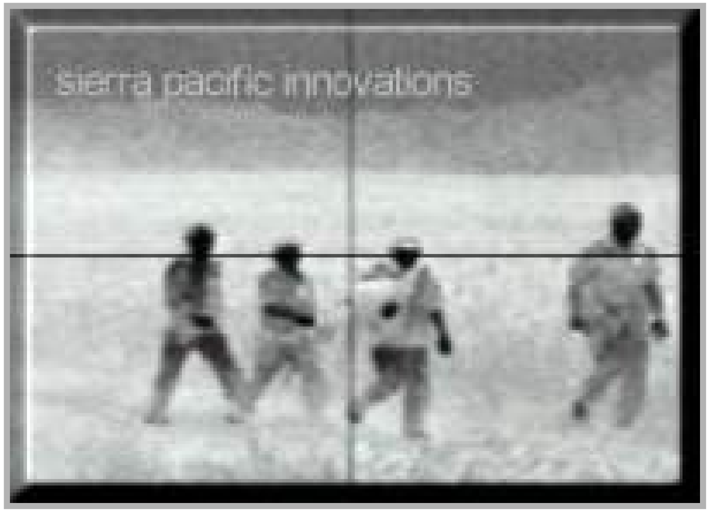
=

'grainy' image

Sources of Noise

- Sources of noise = CCD chip.
- Electronic signal fluctuations in detector.
  - Caused by thermal energy.
  - Worse for infra-red sensors.
- Other electronics
- Transmission (analog)





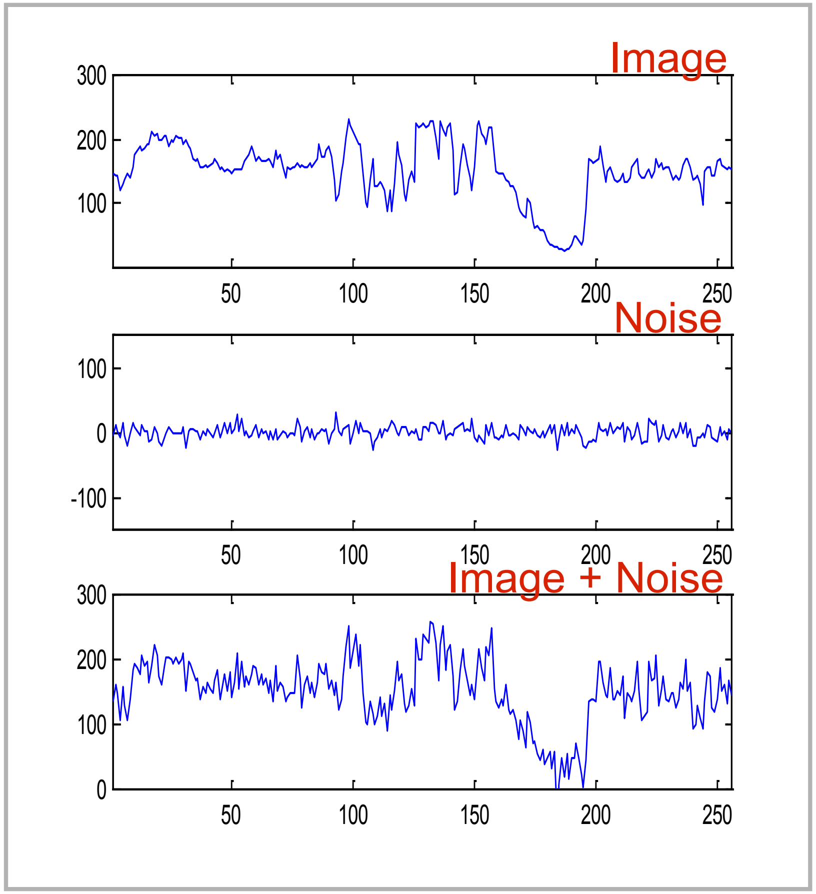
Radiation from the long wavelength IR band is used in most infrared imaging applications

Noise

- Plot of image brightness.
- <u>Noise models</u>:
  - additive
    - $f(i,j) = g(i,j) + v(i,j)$
      - Gaussian
      - salt and pepper
  - multiplicative
    - $f(i,j) = g(i,j) + g(i,j).v(i,j)$

  where

  - *$f(i,j)$ - acquired signal (noisy)*
  - *$g(i,j)$ - uncorrupted signal*
  - *$v(i,j)$ – noise component*



- Noise fluctuations are rapid
  - high frequency.

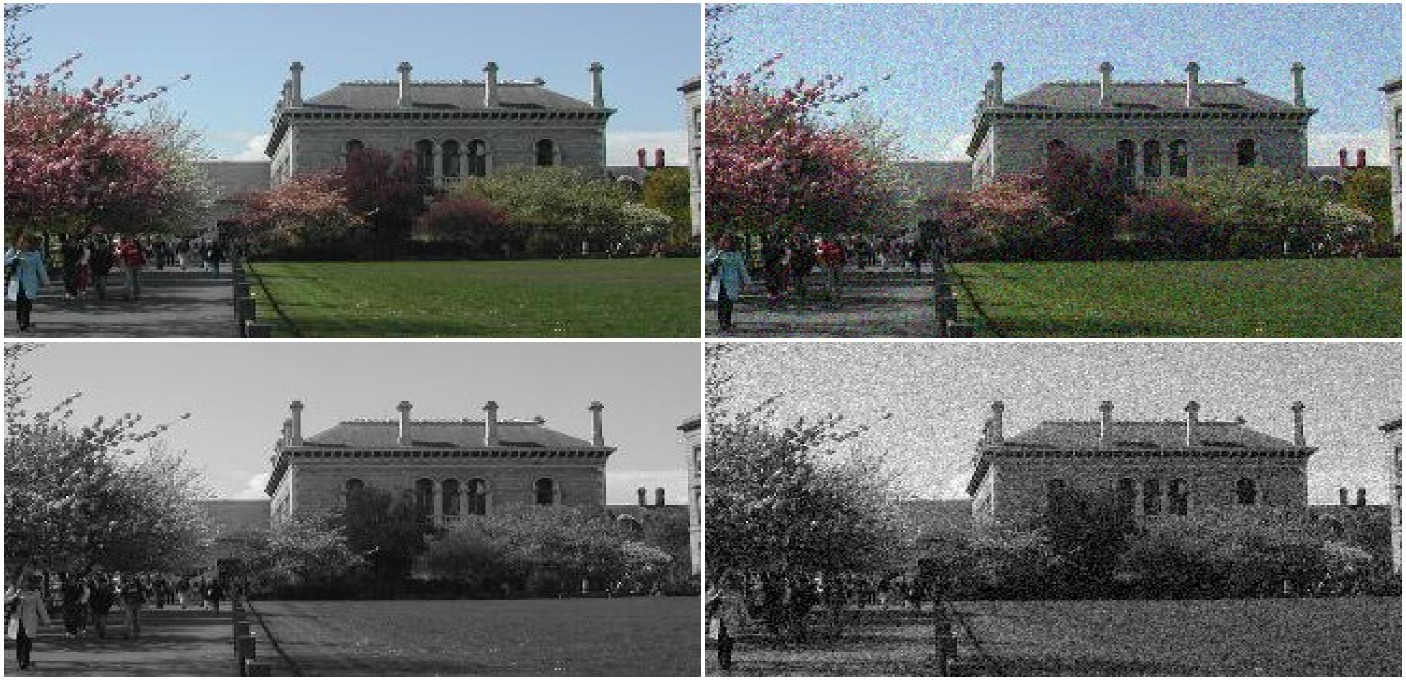
Noise – salt & pepper

- Impulse noise
  - Noise is maximum or minimum values

source: *A Practical Introduction to Computer Vision with OpenCV* , by Kenneth Dawson-Howe

Noise – Gaussian

- Good approximation to real noise
- Distribution is Gaussian (mean & standard deviation)

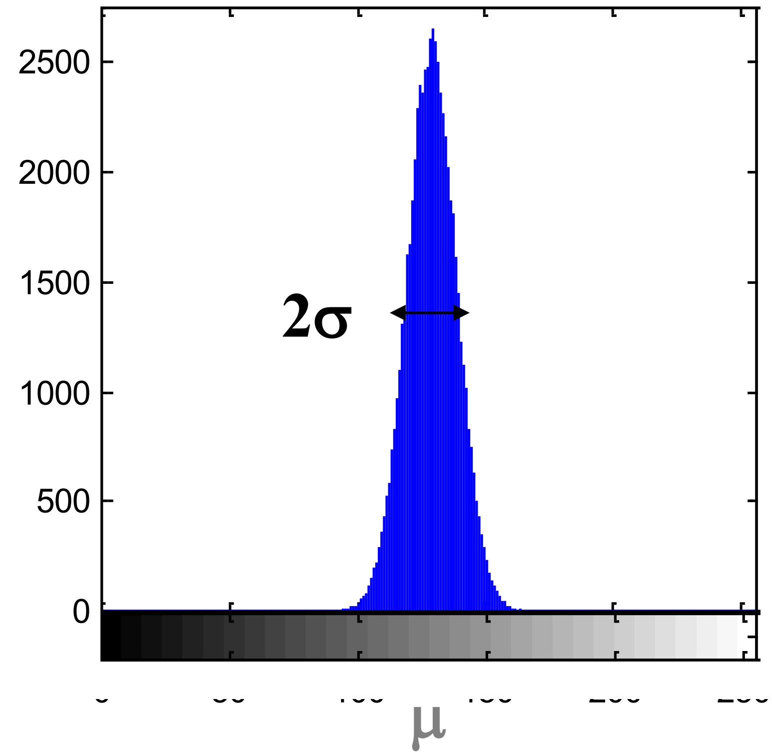


source: *A Practical Introduction to Computer Vision with OpenCV* , by Kenneth Dawson-Howe

Noise Model

- Plot noise histogram
- Typical noise distribution is normal or Gaussian
- Mean(noise) $\mu = 0$
- Standard deviation σ

$$\eta(x) = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{1}{2} \frac{(x-\mu)^2}{\sigma} \right)$$



- Noise varies above and below uncorrupted image.

Image

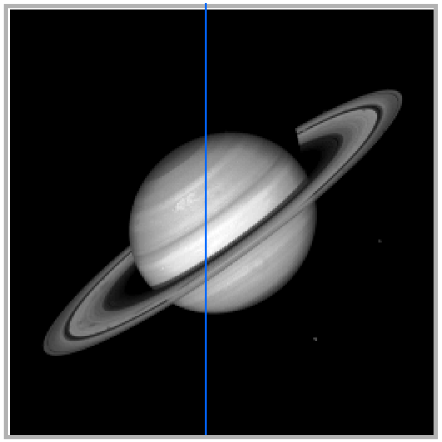
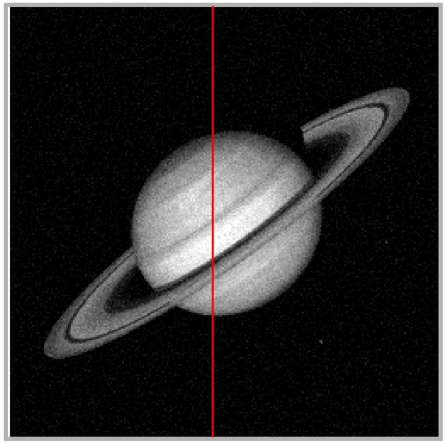
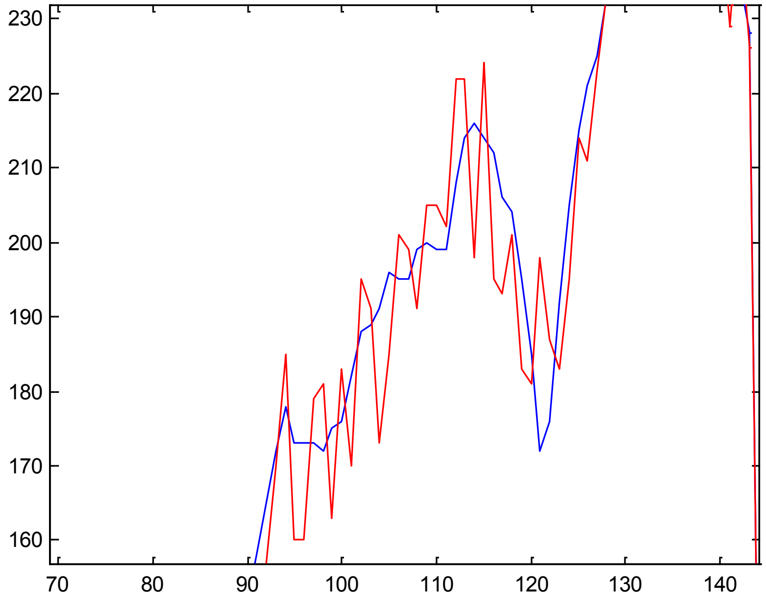


Image
+
Noise





Noise reduction

- Removing (?) or <u>reducing</u> noise…
- <u>Linear smoothing transformations</u>
  - frame averaging (=> acquire several frames of static scene)
  - local averaging
  - Gaussian smoothing
- <u>Non-linear transformations</u>
  - median filter
  - rotating mask

Correlation

Image Data:

10	12	40	16	19	10
14	22	52	10	55	41
10	14	51	21	14	10
32	22	9	**9**	19	14
41	18	9	22	27	11
10	7	8	8	4	5

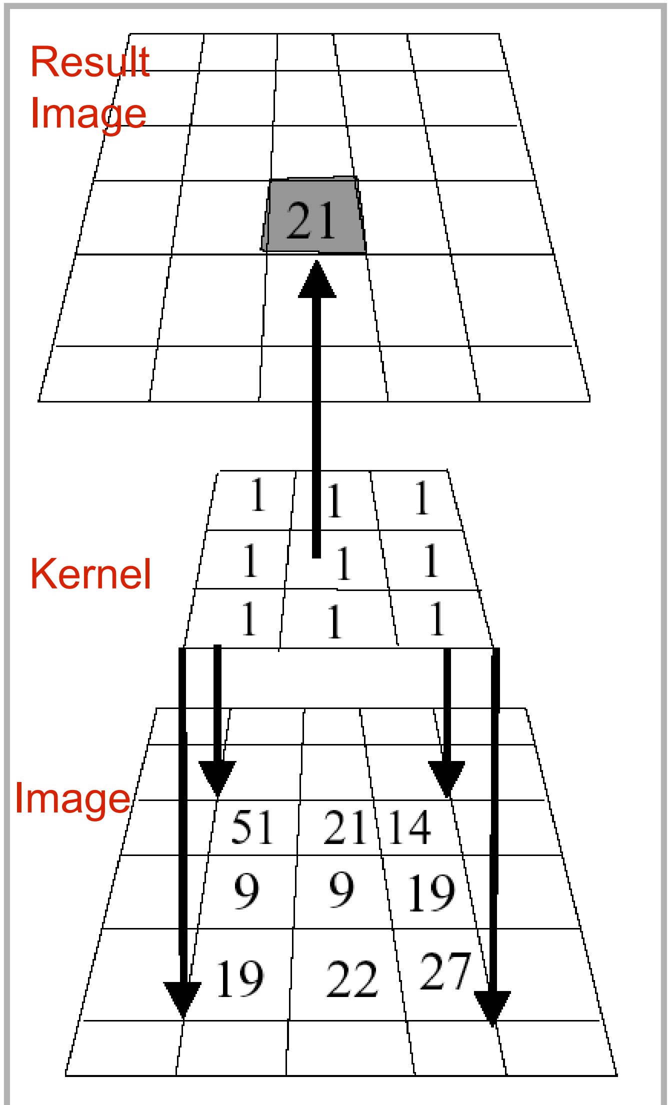
Mask / Filter / Kernel:

1	1	1	1
0	1	1	1
-1	1	1	1
	-1	*0*	*1*

Filter Image

$$F \circ I(x,y) = \sum_{j=-N}^{N} \sum_{i=-N}^{N} F(i,j) I(x+i, y+j)$$

Example



- Kernel is aligned with pixel in image, multiplicative sum is computed, normalized,
- and stored in result image.
- Process is repeated across image.
- What happens when kernel is near edge of input image?

$$\left[\begin{array}{l} 1*51 \; + 1*21 + 1*14 + \\ 1*8 \quad + 1*9 \quad + 1*19 + \\ 1*19 + 1*22 + 1*27 \end{array}\right] 1/9 = 21$$

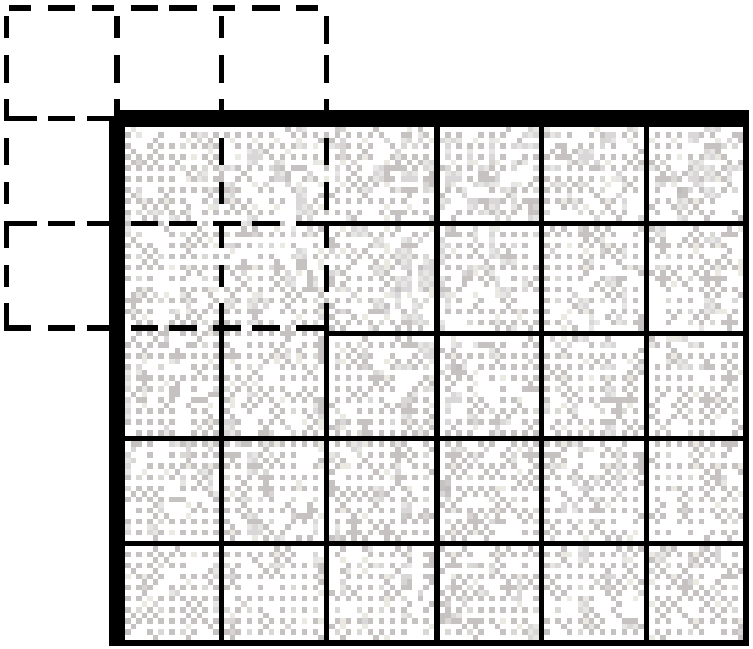
1/9	1/9	1/9
1/9	1/9	1/9
1/9	1/9	1/9

F

note the factor

Border problem

- missing samples are zero
- missing samples are gray
- copying last lines
- reflected indexing (mirror)
- circular indexing (periodic)
- reduce size of resulting image



- OpenCV
  allows some control on how to do this;
  see CopyMakeBorder(): copies the source 2D array into the interior of the destination array and makes a border of the specified type around the copied area

Border problem

1/9	1/9	1/9
1/9	1/9	1/9
1/9	1/9	1/9

F

8	3	4	5
7	6	4	5
4	5	7	8
6	5	5	6

I

8	*8*	*3*	*4*	*5*	*5*
8	8	3	4	5	*5*
7	7	6	4	5	*5*
4	4	5	7	8	*8*
6	6	5	5	6	*6*
6	*6*	*5*	*5*	*6*	*6*

I with padded boundaries

6.44	5.22	4.33	4.67
5.78	5.33	5.22	5.67
5.56	5.44	5.67	6.00
5.22	5.33	5.78	6.33

J = F∘I

Convolution

- A **convolution** operation is a correlation where the filter is flipped both horizontally and vertically before being applied to the image:

$$F * I(x,y) = \sum_{j=-N}^{N} \sum_{i=-N}^{N} F(i,j) I(x-i, y-j)$$

- Suppose F is a Gaussian or mean kernel.
  How does convolution differ from cross-correlation?

Convolution

- Extremely important concept in
  computer vision, image processing, signal processing, etc.
- Lots of related mathematics (we won't do)
- General idea: reduce a filtering operation to the repeated application of a mask (or filter kernel) to the image
  - Kernel can be thought of as an NxN image
  - N is usually odd so kernel has a central pixel
- In practice
  - (flip kernel)
  - Align kernel center pixel with an image pixel
  - Pointwise multiply each kernel pixel value
    with corresponding image pixel value and add results
  - Resulting sum is normalized by kernel weight
  - Result is the value of the pixel centered on the kernel

Correlation vs Convolution

Correlation

Origin f

$$\begin{matrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{matrix} \qquad \begin{matrix} & w & \\ 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{matrix}$$

(a)

Padded f

$$\begin{matrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{matrix}$$

(b)

Initial position for w

$$\begin{matrix} 1 & 2 & 3 & 0 & 0 & 0 & 0 \\ 4 & 5 & 6 & 0 & 0 & 0 & 0 \\ 7 & 8 & 9 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{matrix}$$

(c)

Correlation result

$$\begin{matrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 9 & 8 & 7 & 0 \\ 0 & 6 & 5 & 4 & 0 \\ 0 & 3 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{matrix}$$

(d)

Full correlation result

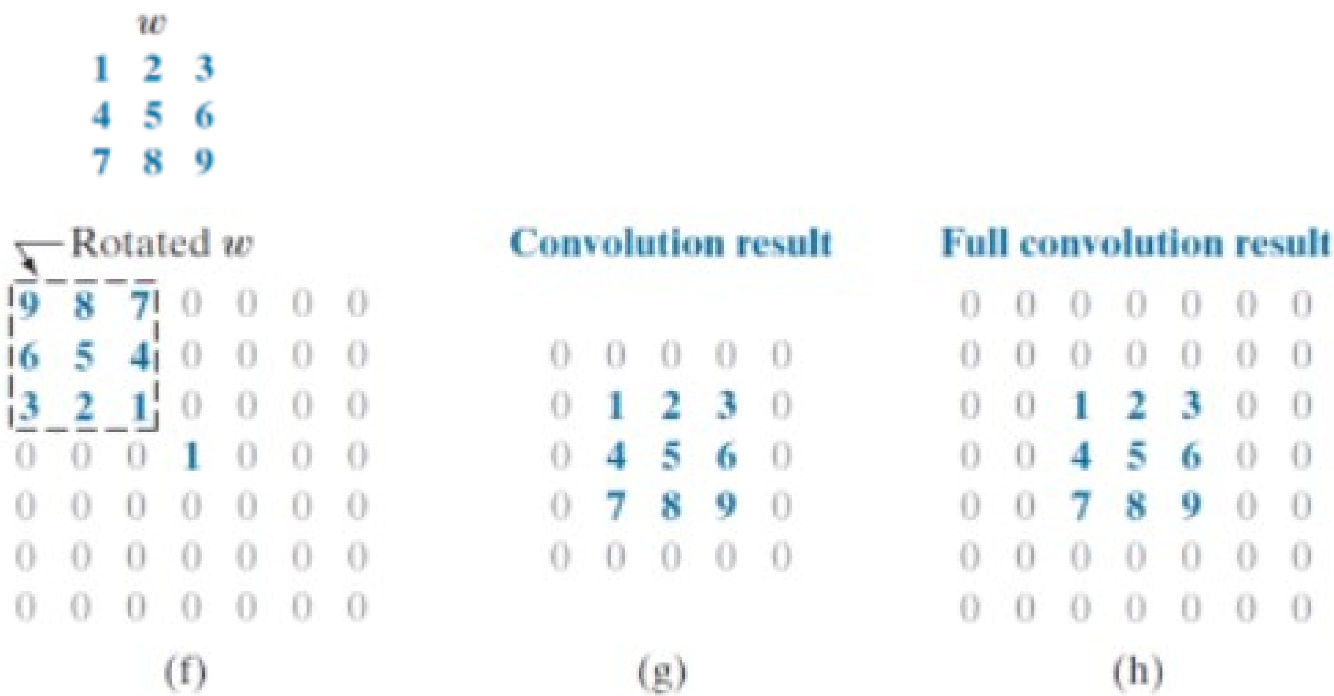
$$\begin{matrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 9 & 8 & 7 & 0 & 0 \\ 0 & 0 & 6 & 5 & 4 & 0 & 0 \\ 0 & 0 & 3 & 2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{matrix}$$

(e)

source: https://towardsdatascience.com/convolution-vs-correlation-af868b6b4fb5

Correlation vs Convolution

Convolution

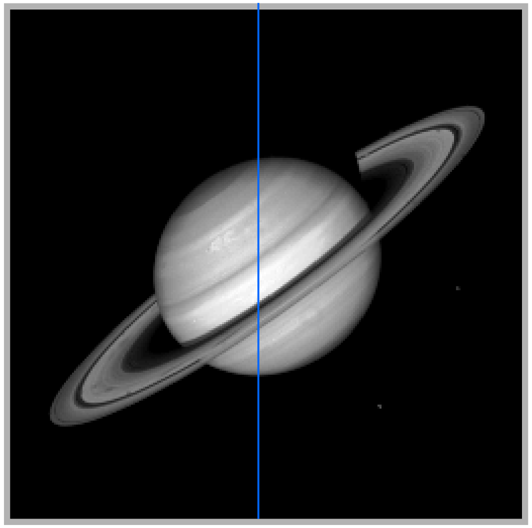


source: https://towardsdatascience.com/convolution-vs-correlation-af868b6b4fb5

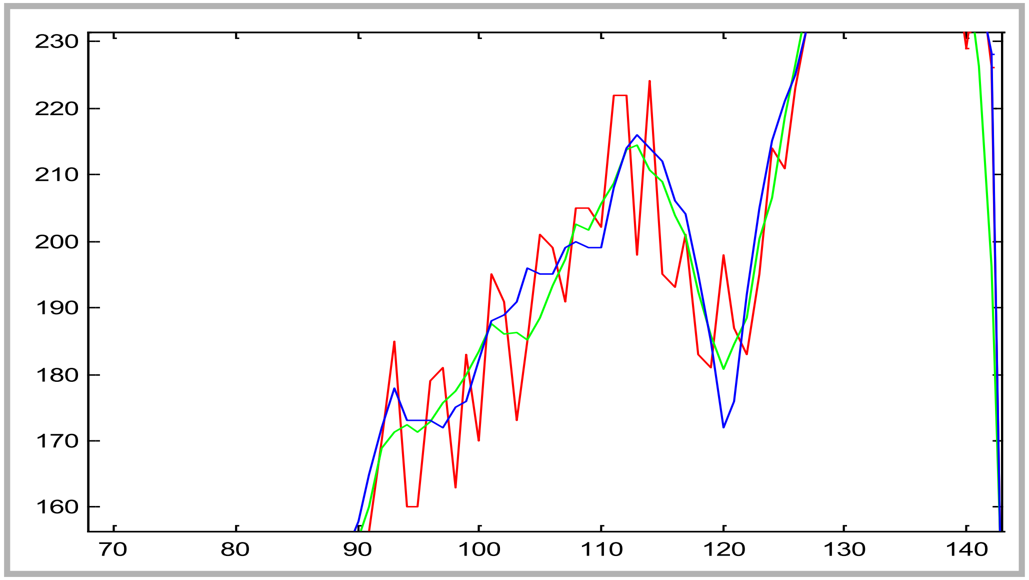
Notes:

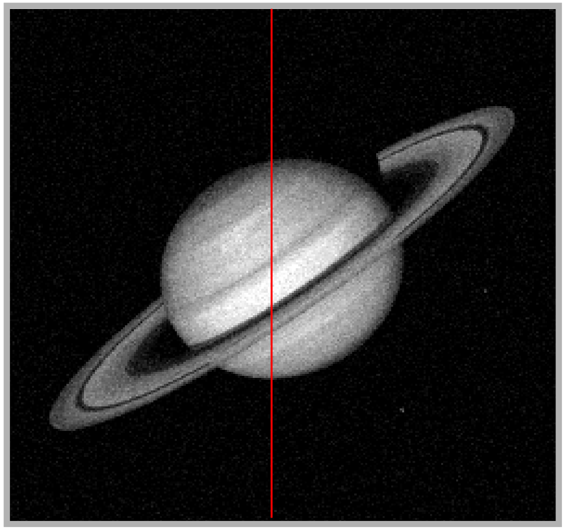
1) Correlation and convolution are identical when the filter is symmetric.
2) The key difference between the two is that convolution is associative, that is, if F and G are filters, then F*(G*Img) = (F*G)*Img
3) In general, people use **convolution** for image processing operations such as smoothing, and they use **correlation** to match a template to an image (*see later*).

Noise reduction - 1



Uncorrupted Image





Uncorrupted Image + Noise

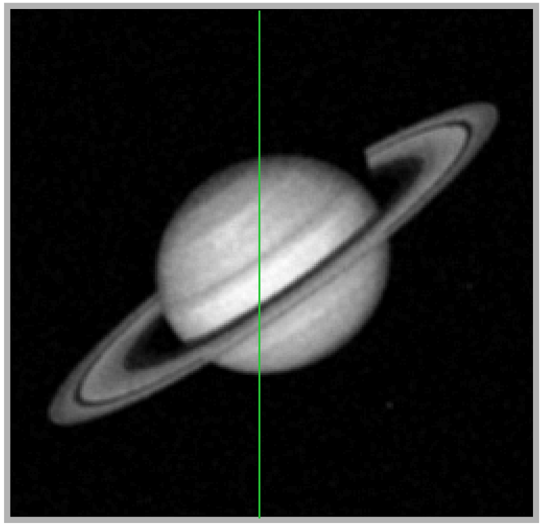


Image + Noise - Blurred

Noise neduction - 1

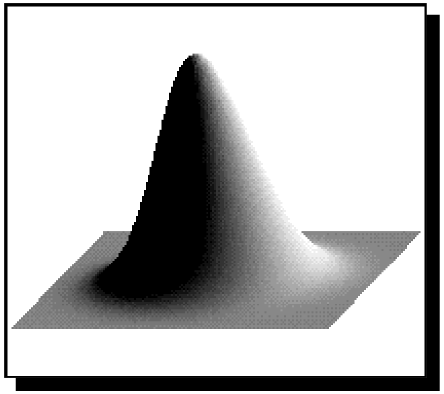
- Technique relies on high frequency noise fluctuations being ‘blocked’ by filter.
  Hence, low-pass filter.
- Fine detail in image may also be smoothed.
- Balance between
  keeping image fine detail and reducing noise.

Noise reduction - 1

- Saturn image
  (*previous slide*)
  has coarse detail
- Boat image
  contains fine detail
- Noise reduced but
  fine detail also smoothed

Gaussian smoothing

- Smoothing operator should be
  - 'tunable' in what it leaves behind
  - smooth and localized in image space.
- One operator which satisfies
  these two constraints is the Gaussian



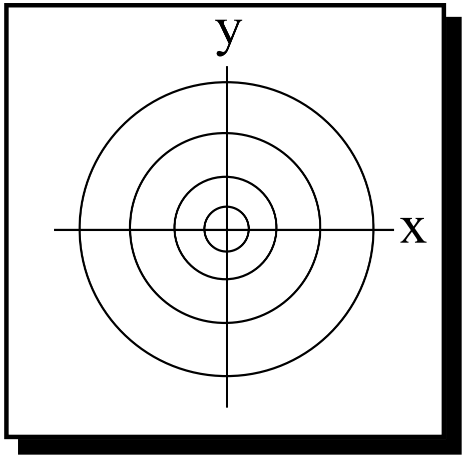
- *OpenCV: Smooth()*

2D Gaussian distribution

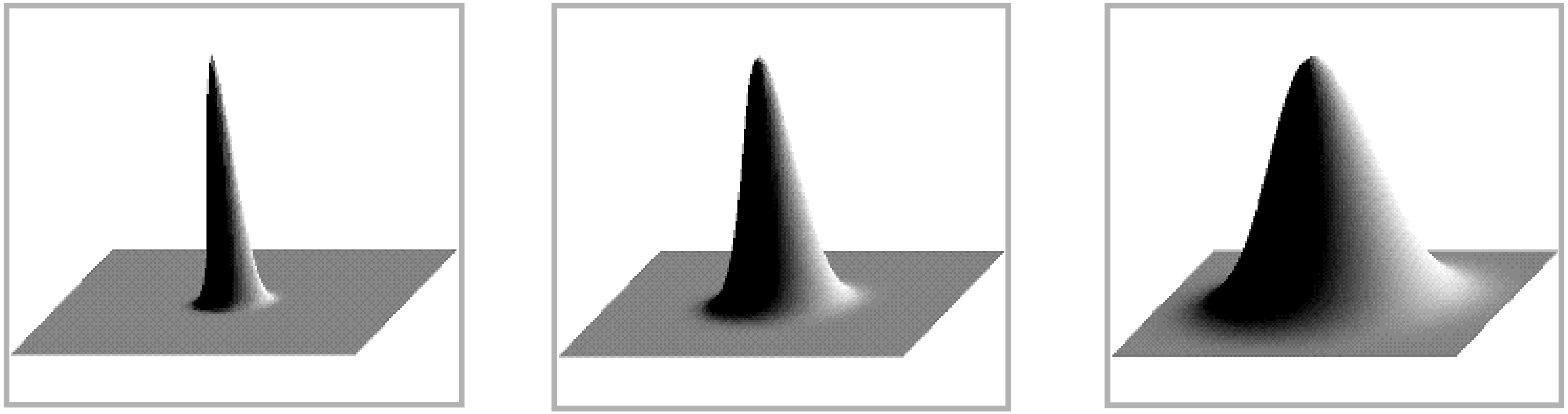
- The two-dimensional Gaussian distribution is defined by:

$$G(x,y) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\left[\frac{(x^2+y^2)}{2\sigma^2}\right]}$$

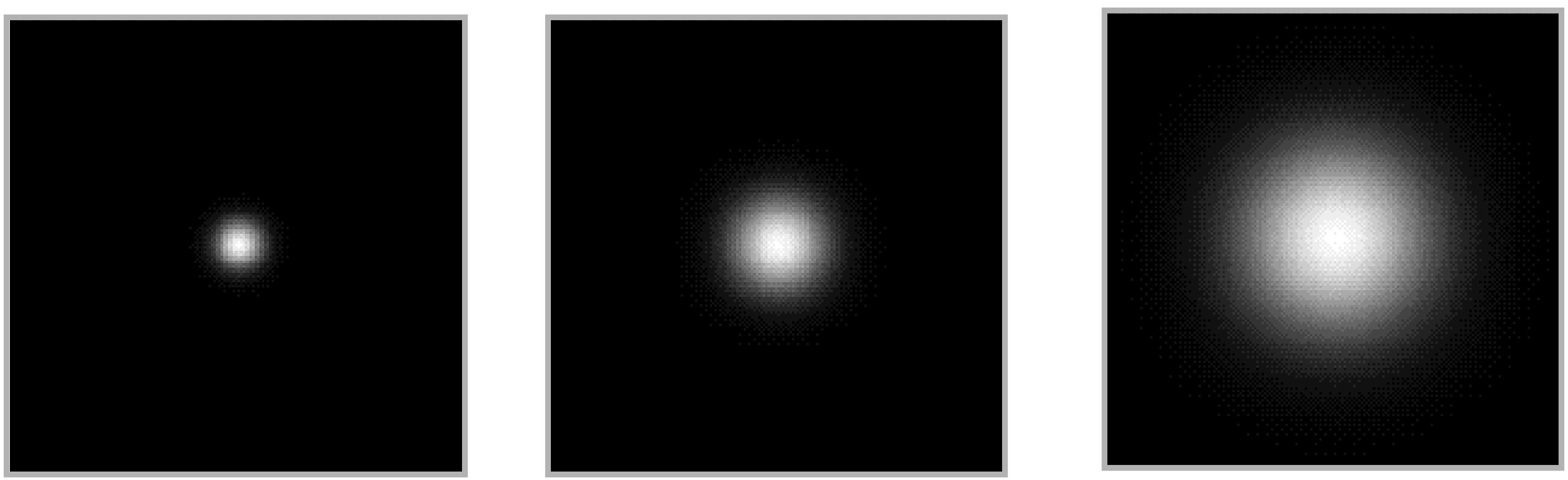
- From this distribution, can generate smoothing masks whose width depends upon σ:



σ defines kernel "width"



$\sigma^2 = .25$ $\sigma^2 = 1.0$ $\sigma^2 = 4.0$



Example

- Choose σ^2 = 2 and n = 7, then:

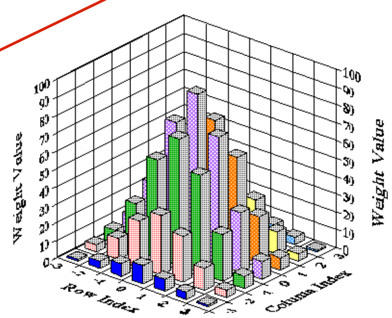
i \ j	-3	-2	-1	0	1	2	3
-3	.011	.039	.082	.105	.082	.039	.011
-2	.039	.135	.287	.368	.287	.135	.039
-1	.082	.287	.606	.779	.606	.287	.082
0	.105	.039	.779	1.000	.779	.368	.105
1	.082	.287	.606	.779	.606	$\boxed{.287}$	.082
2	.039	.135	.287	.368	.287	.135	.039
3	.011	.039	.082	.105	.082	.039	$\boxed{.011}$

$$\boxed{\frac{W(1,2)}{k} = \exp\left(-\frac{1^2 + 2^2}{2*2}\right)}$$

1	4	7	10	7	4	1
4	12	26	33	26	12	4
7	26	55	71	55	26	7
10	33	71	91	71	33	10
7	26	55	71	55	26	7
4	12	26	33	26	12	4
1	4	7	10	7	4	1

7x7 Gaussian filter

To make this value 1,
choose k=91



Kernel Application

7x7 Gaussian kernel

15x15 Gaussian kernel

Why Gaussian for smoothing

- Gaussian is not the only choice, but it has a number of important properties
  - If we convolve a Gaussian with another Gaussian, the result is a Gaussian
    - This is called linear scale space

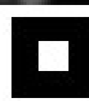
$$G_{\sigma_1} * G_{\sigma_2} = G_{\sqrt{\sigma_1^2+\sigma_2^2}}.$$

  - Efficiency: separable

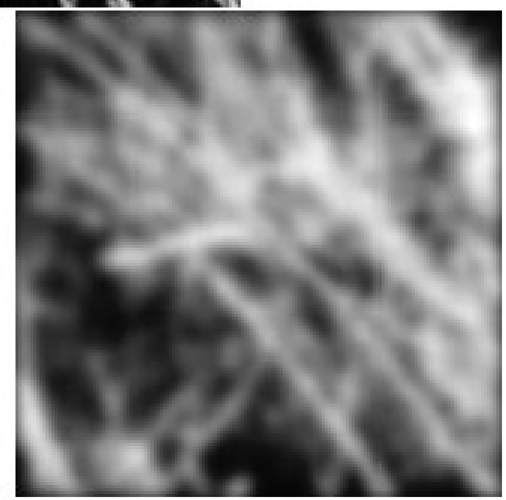
$$G_\sigma(x, y) = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{(x^2+y^2)}{2\sigma^2}\right)$$
$$= \left(\frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(x^2)}{2\sigma^2}\right)\right) \times \left(\frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(y^2)}{2\sigma^2}\right)\right),$$

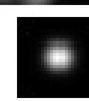
Mean vs. Gaussian filtering

Original image



After mean filtering





After Gaussian filtering

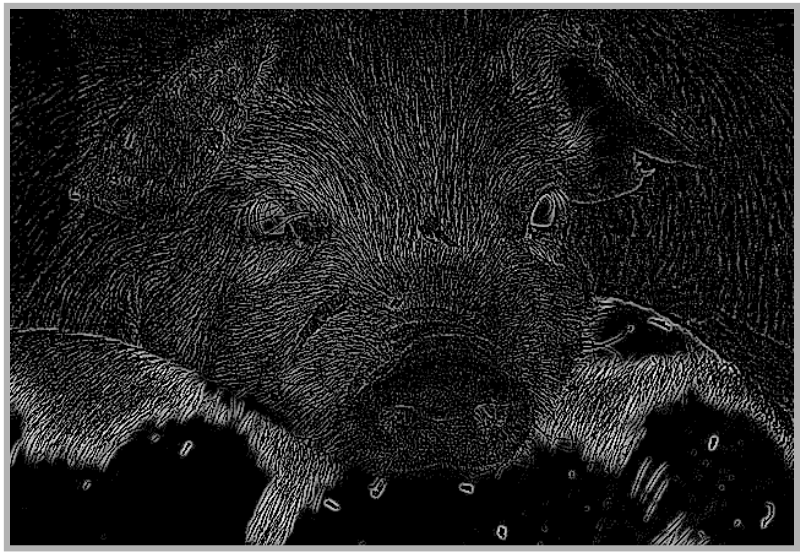
hmmmmm.....

Image

-

Blurred image

=



Noise reduction - 2: Median filter

- Compute median of points in neighborhood
- Nonlinear filter
  - example of a larger class of filters named "rank-filters" (ex: min, median, max)
- Has a tendency to blur detail less than averaging
- Works very well for 'shot' or 'salt and pepper' noise

Linear vs Non-linear

Mean(I1+I2) = =
Mean (I1) + Mean(I2)

Median(I1+I2) !=
Median(I1)+Median(I2)

Original

Low-pass

Median

Linear vs Non-linear filters:
https://dsp.stackexchange.com/questions/14241/what-is-the-difference-between-linear-and-non-linear-filters

Mean vs. Median

- Mean
  - Linear
  - Signal frequencies shared with noise are lost, resulting in blurring.
  - Impulsive noise is diffused but not removed.
  - It spreads the noise, resulting in blurring.
  - Blurs edges.
- Median
  - Non-linear
  - Does not spread the noise.
  - Can remove spike noise.
  - Preserves (some) edges.
  - Small details (small regions, this edges, …) may be lost.
  - Expensive to run.

Edge Preserving Smoothing

Smooth (average, blur) an image without disturbing

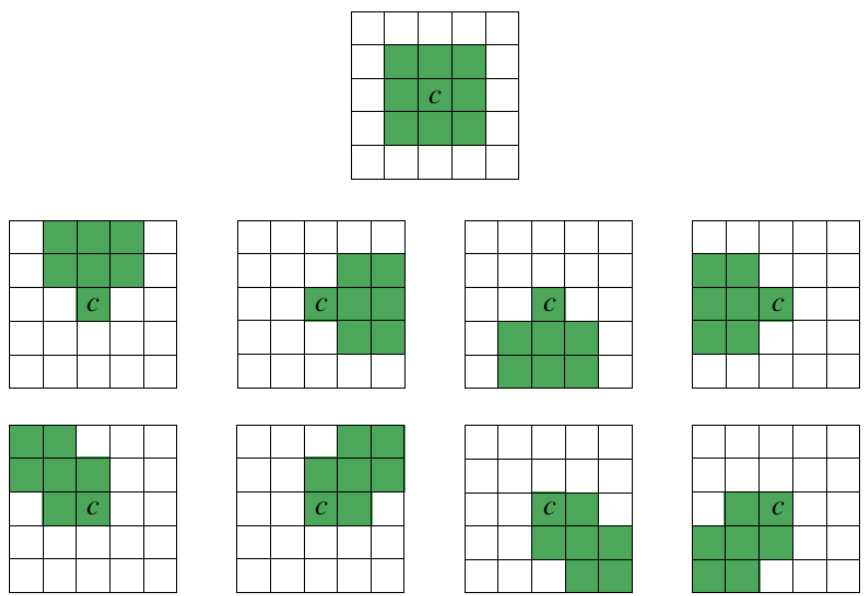
- sharpness or
- position

of the edges

- Nagao-Matsuyama filter
- Kuwahara filter
- Anisotropic diffusion filters (Perona & Malik, ...)
- Bilateral filtering
- ...

Nagao-Matsuyama filter

- Calculate the variance within nine subwindows of a 5x5 moving window
- Output value is the **mean** of the **subwindow with the smallest variance**
- Nine subwindows used:



Kuwahara filter

- Principle:
  - divide filter mask into four regions (a, b, c, d).
  - in each compute the mean brightness and the variance
  - the output value of the center pixel (abcd) in the window is the **mean** value of that **region** that has the **smallest variance**.

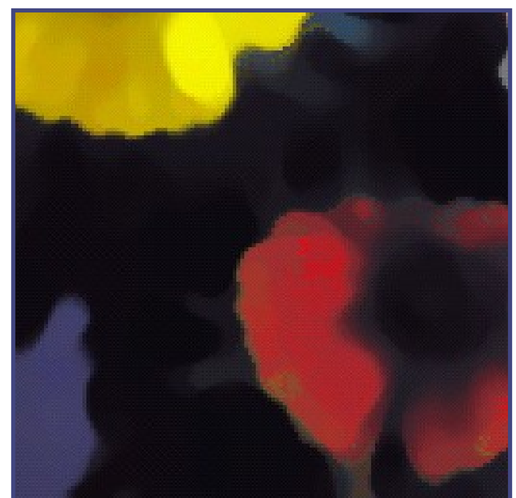
a	a	ab	b	b
a	a	ab	b	b
ac	ac	abcd	bd	bd
c	c	cd	d	d
c	c	cd	d	d

Kuwahara filter example

Original

Median (1 iteration)

Kuwahara



Median (10 iterations)

Anisotropic diffusion filtering

- The anisotropic diffusion is a nonlinear filter that smoothes the intraregions of an image without blurring the strong edges
  - Encourages the **smoothing in homogeneous regions in preference to** smoothing across the boundaries.
- Based on the solution of a partial differential equation, inspired in heat diffusion equation
  - *The algorithm results from the discretization of the non-linear diffusion equation.*
  - *The derivatives are approximated by differences.*
- Diffusion methods average over extended regions by solving partial differential equations, and are therefore inherently iterative. Iteration may raise issues of stability and, depending on the computational architecture, efficiency.
- This algorithm is also used to detect the edges

Anisotropic diffusion filtering

Original

Perona, Malik, 1987

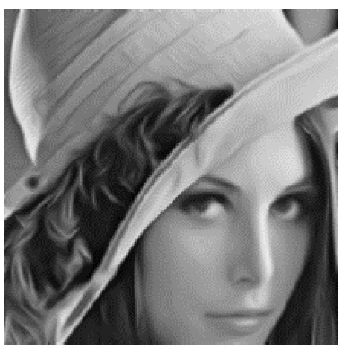
$$\frac{\partial u}{\partial t} = div\left(\frac{Du}{|Du|^2 + \lambda^2}\right)$$

Rudin, Osher, Fatemi, 1992

$$\frac{\partial u}{\partial t} = div\left(\frac{Du}{|Du|}\right)$$

Alvarez, Lions, 1992

$$\frac{\partial u}{\partial t} = \frac{|Du|}{|k * Du|} div\left(\frac{Du}{|Du|}\right)$$



Weickert, 1994

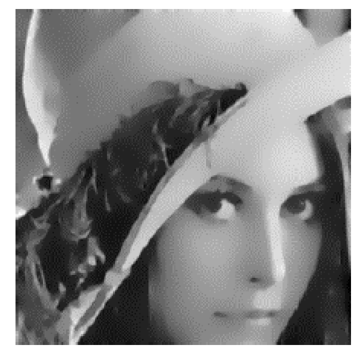
$$\frac{\partial u}{\partial t} = D^2 u(d,d)$$
$$d = SEigen(k * (Du \otimes Du))$$

Caselles, Sbert, 1997

$$\frac{\partial u}{\partial t} = \frac{1}{|Du|^2} D^2 u(Du, Du)$$

Zhong Carmona, 1998

$$\frac{\partial u}{\partial t} = D^2 u(d,d)$$
$$d = SEigen(D^2 u)$$

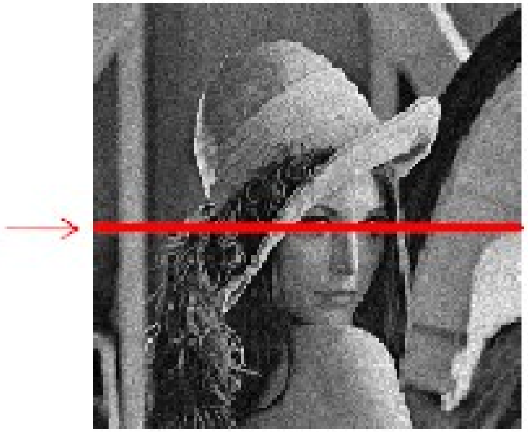


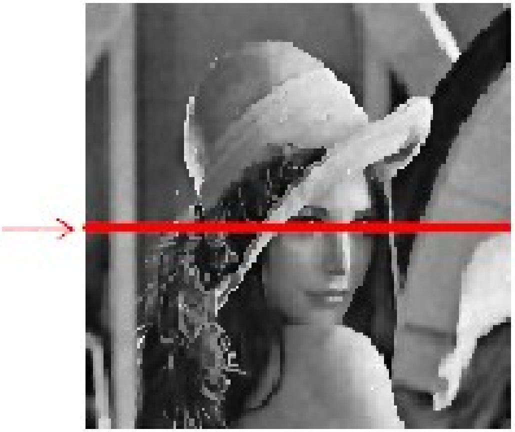
Sochen, Kimmel, Malladi,1998

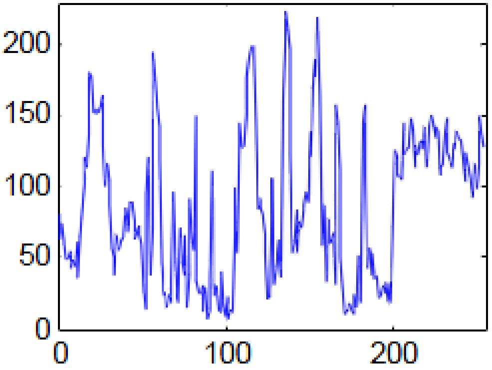
$$\frac{\partial u}{\partial t} = div\left(\frac{Du}{\sqrt{|Du|^2 + 1}}\right)$$

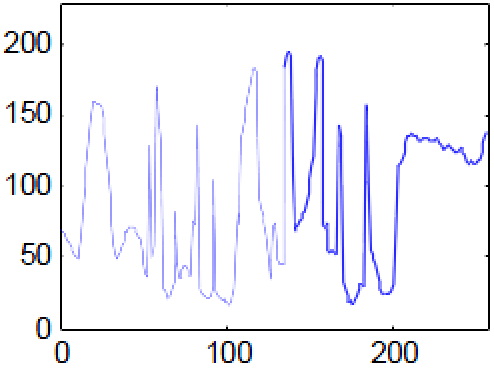
Note: Original Perona-Malik diffusion process is NOT anisotropic, although they erroneously said it was.

Anisotropic diffusion filtering





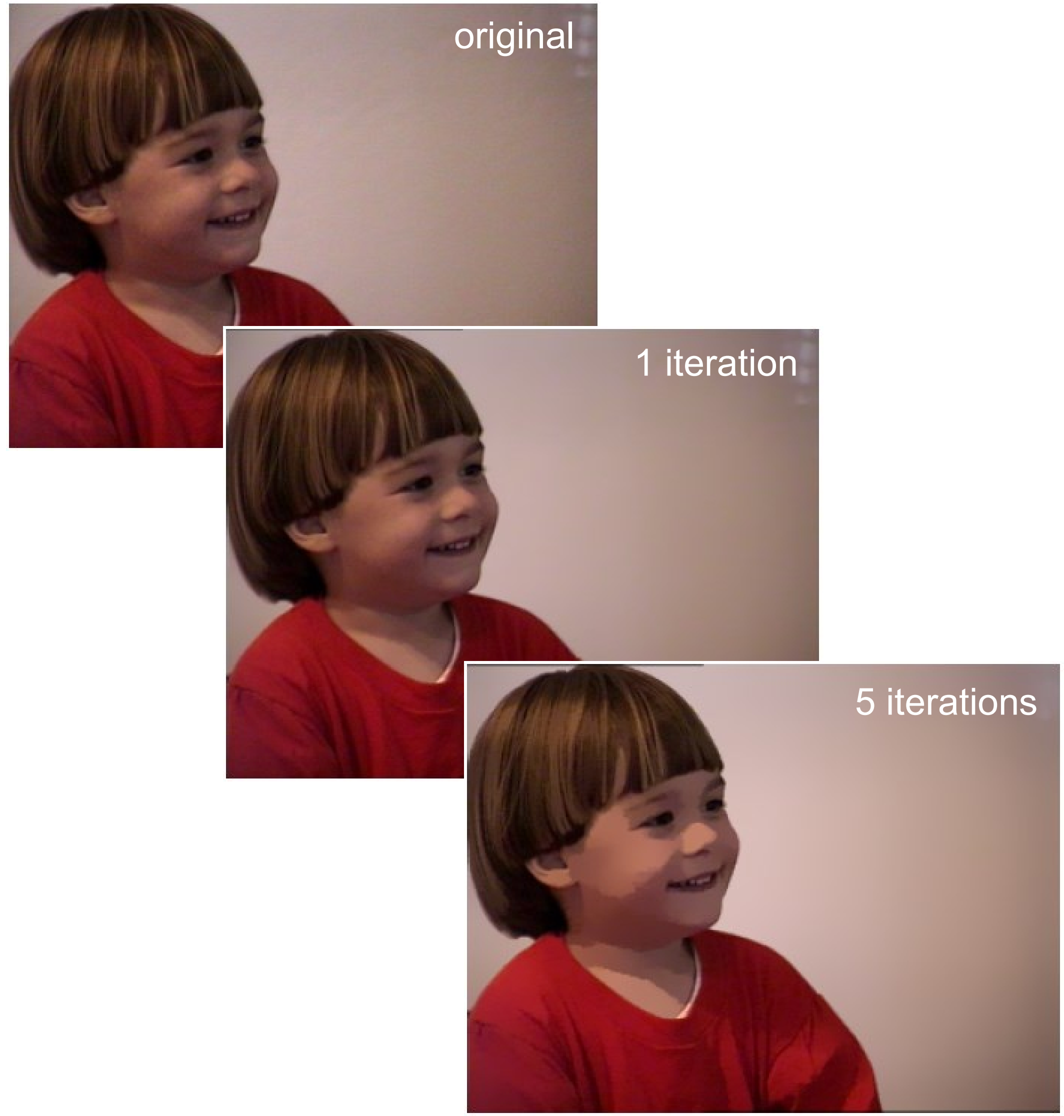




Bilateral filtering

- A bilateral filter is an edge-preserving and noise reducing smoothing filter.
- Traditional filtering is **domain filtering**, and enforces closeness by weighing pixel values with weights that fall off with distance.
- Similarly, we define **range filtering**, which averages image values with weights that decay with dissimilarity.
- **Range filters**
  - are nonlinear because their weights depend on image intensity or color
  - preserve edges
  - by themselves, my distort an image's color map
    (see C.Tomasi at al., Bilateral Filtering for Gray and Color, ICCV 1998)
- **Bilateral filtering** combines range and domain filtering.
- The intensity value at each pixel in an image is replaced by a **weighted average of intensity values from nearby pixels.**
- This weight is based on a Gaussian distribution.
  The **weights** depend not only on Euclidean distance but also on the radiometric differences (e.g. color intensity).
- It preserves sharp edges by systematically looping through each pixel and attributing weights to the adjacent pixels accordingly.
- *OpenCV: Smooth()*

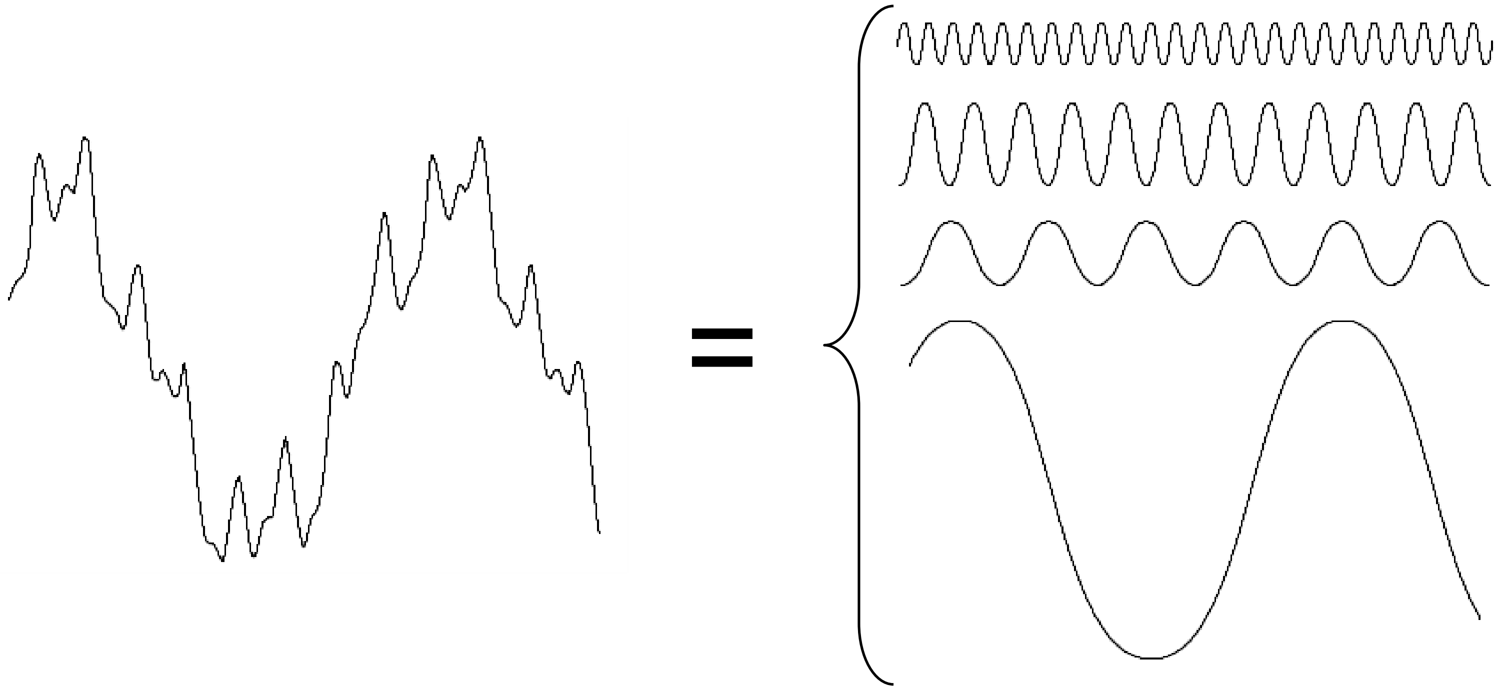
Bilateral filtering



Frequency domain filtering

- An image can be represented in the frequency domain, using the Fourier Transform (FT).
- The FT encodes the amplitude and phase of each frequency component.
- The values near the origin of the transformed space are called low-frequency components of the FT, and those distant from the origin are the high-frequency components.
- Convolution in the image domain corresponds to multiplication in the spatial frequency domain.
- Therefore, convolution with large filters, which would normally be expensive in the image domain, can be implemented efficiently using the Fast Fourier Transform (FFT).
- This is an important technique in many image processing applications.

Fourier transform



- Any function that periodically repeats itself
  can be expressed as a sum of sines and cosines of different frequencies
  each multiplied by a different coefficient
  – a *Fourier series*

Discrete Fourier Transform - DFT

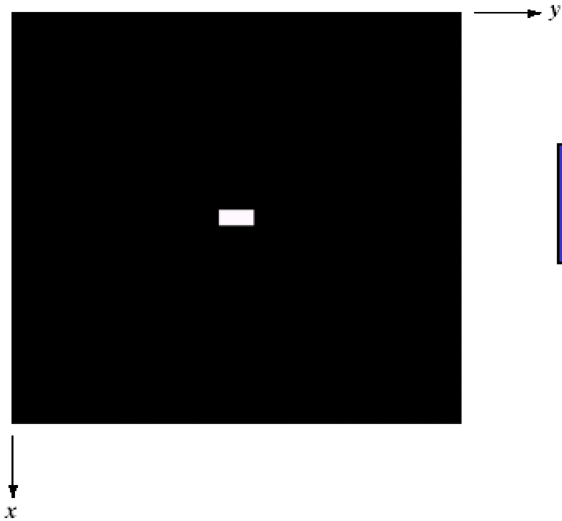
The *Discrete Fourier Transform* of $f(x, y)$,
for x = 0, 1, 2…M-1 and y = 0,1,2…N-1,
denoted by $F(u, v)$, is given by the equation:

$$F(u,v) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y) e^{-j2\pi(ux/M + vy/N)}$$

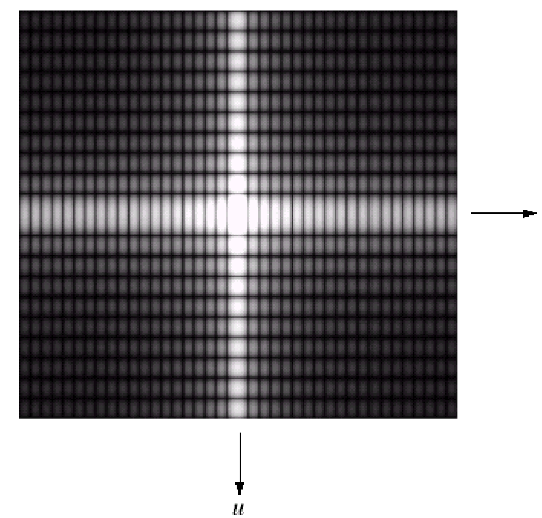
for u = 0, 1, 2…M-1 and v = 0, 1, 2…N-1.

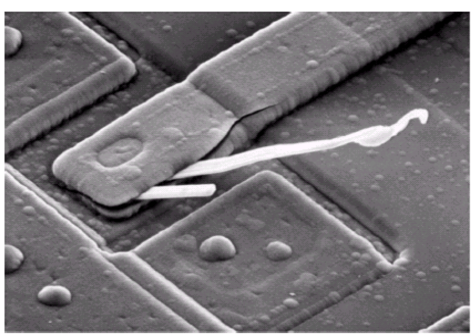
DFT of an image

- The DFT of a 2D image can be visualised by showing the spectrum of the images component frequencies

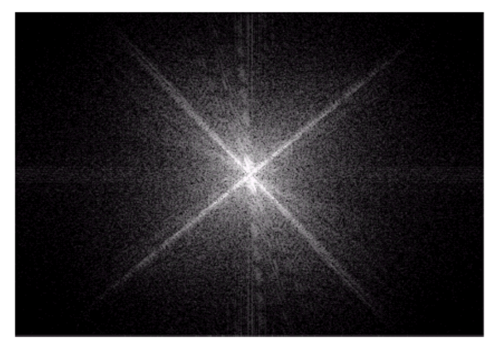


DFT





DFT

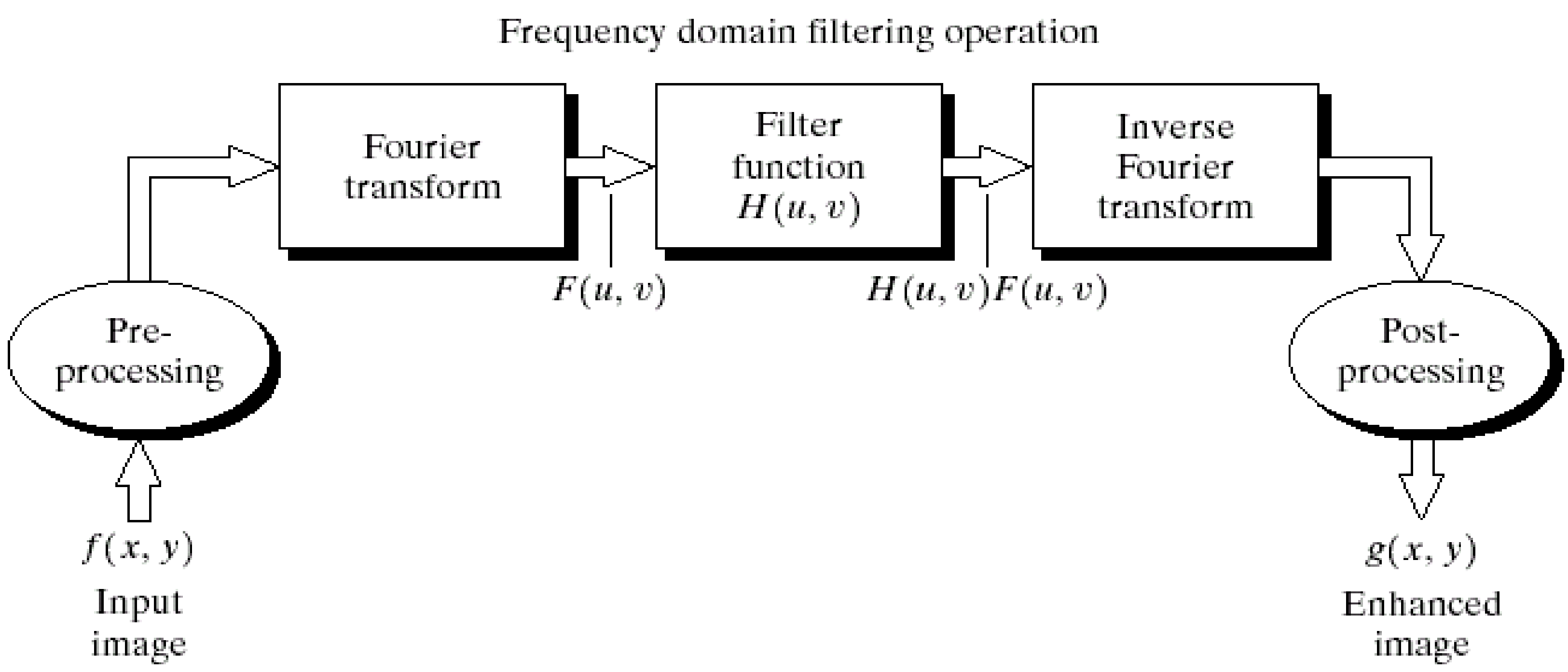


Fourier spectrum of the image

Frequency domain filtering

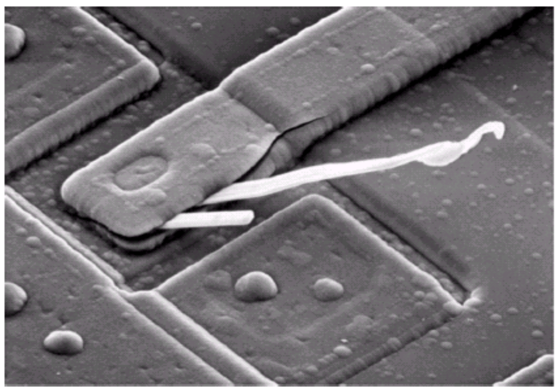
To filter an image in the frequency domain:

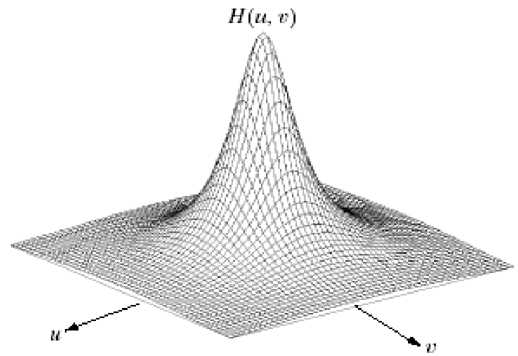
1. Compute $F(u,v)$ the DFT of the image
2. Multiply $F(u,v)$ by a filter function $H(u,v)$
3. Compute the inverse DFT of the result

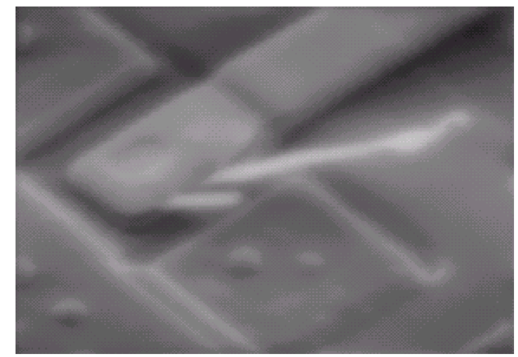


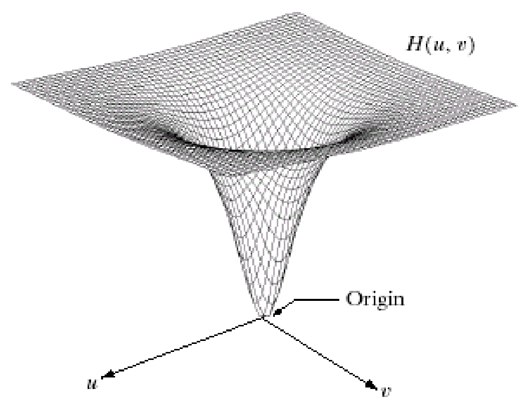
Frequency domain filtering

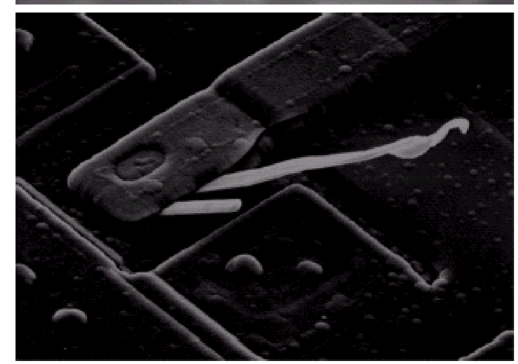
Low Pass Filter





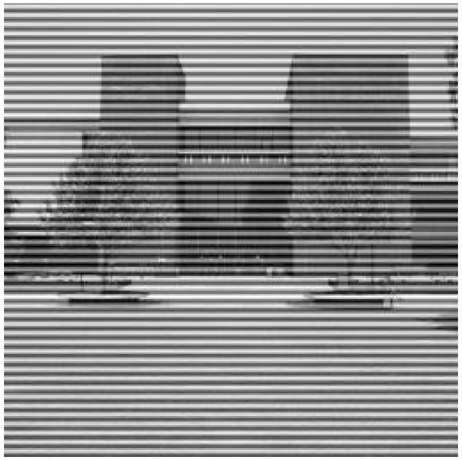


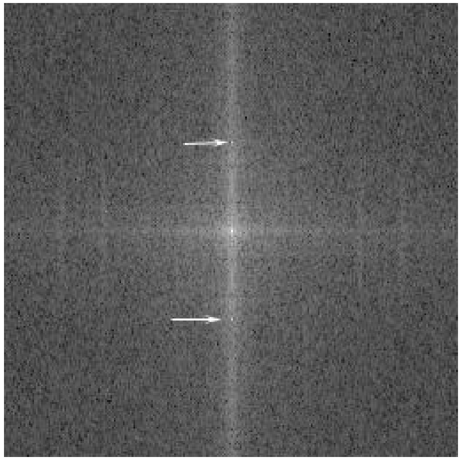




High Pass Filter

Frequency domain filtering





2D Fourier transform

After filtering