IMAGE PROCESSING

Topics

- Image Enhancement
  - Brightness mapping
  - Contrast stretching/enhancement
  - Histogram modification
  - Noise Reduction
  - ………
- Mathematical Techniques
  - Convolution
    - Mean filtering
    - Gaussian filtering
- Edge and Line Detection and Extraction
- Contour Extraction
- Corner Detection
- Region Segmentation

Histogram-based transformations

- Thresholding
  - threshold selection (manual & automatic)
- Transformations for contrast enhancement
  - linear
    - linear stretching
  - non-linear
    - power law
    - logarithmic
    - equalization
    - CLAHE

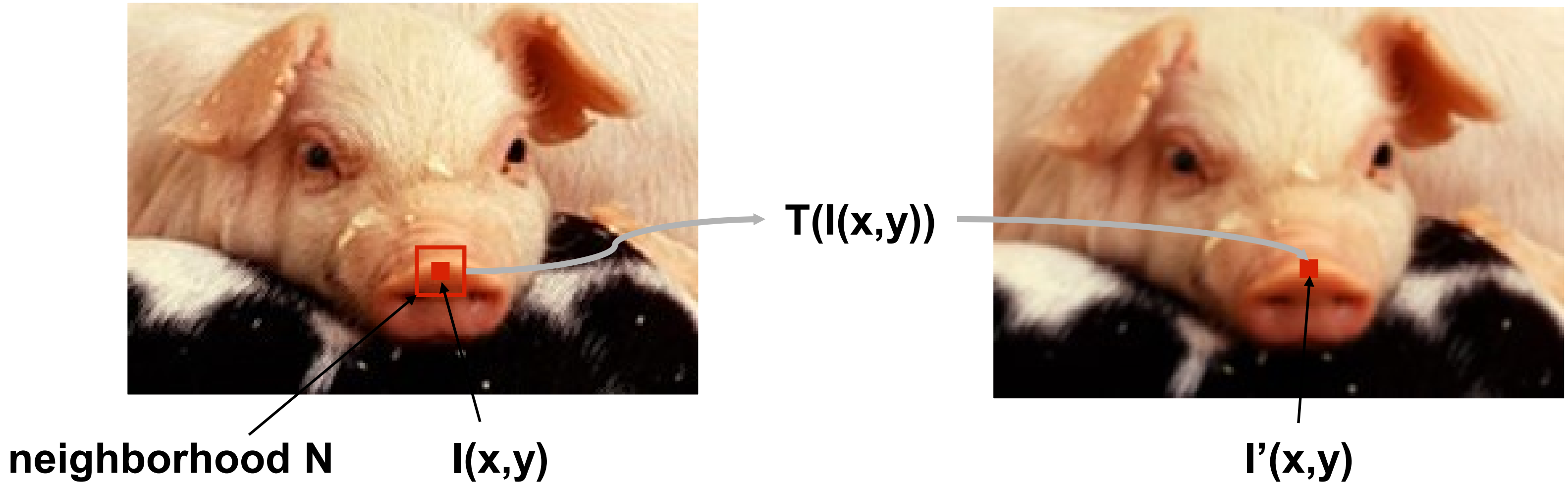
Filtering for noise reduction

- Linear filters: mean and Gaussian
  - convolution operation
- Non-linear filters
  - median
  - anisotropic diffusion filter
  - bilateral filter
  - …
- Frequency domain filters

Image enhancement

- Goal: improve the 'visual quality' of the image
  - for human viewing
  - for subsequent processing
- Two typical methods
  - spatial domain techniques....
    - operate directly on image pixels
  - frequency domain techniques....
    - operate on the Fourier transform of the image
- No general theory of 'visual quality'
  - General assumption: if it looks better, it is better
  - Often not a good assumption

Spatial domain methods



- Transformation T
  - point - pixel to pixel
  - local - local area to pixel
  - global - output value at a specific coordinate depends on all values in the input image. (ex: DFT)
- Local - neighborhoods
  - typically quadrangular
  - typically an odd size: 3x3, 5x5, etc. (why odd size? see later)
  - centered on pixel I(x,y)

$$I'(x,y) = T(I(x,y))$$

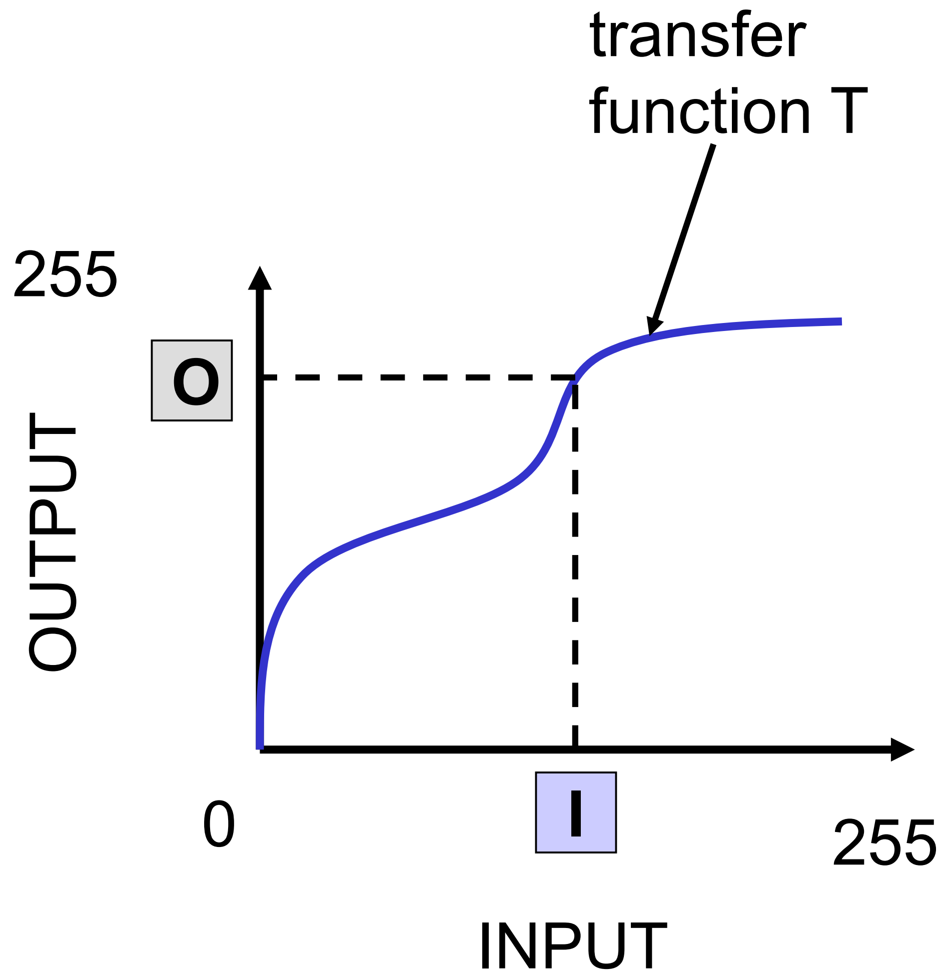
Point transformations

Histogram-based transformations

Point transformations: general idea

$$O = T(I)$$

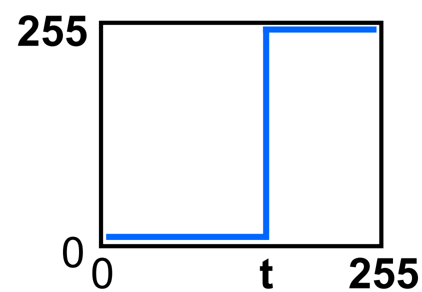
Input pixel value, I, mapped to output pixel value, O, via transfer function T.

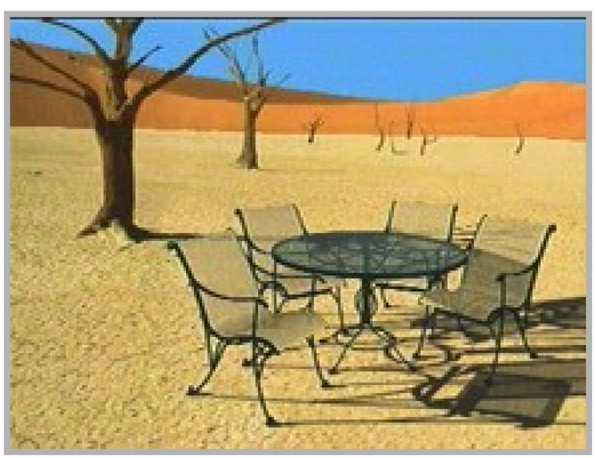


Point transformations:thresholding

- T is a point-to-point transformation
  - only information at I(x,y) used to generate I'(x,y)
- Thresholding

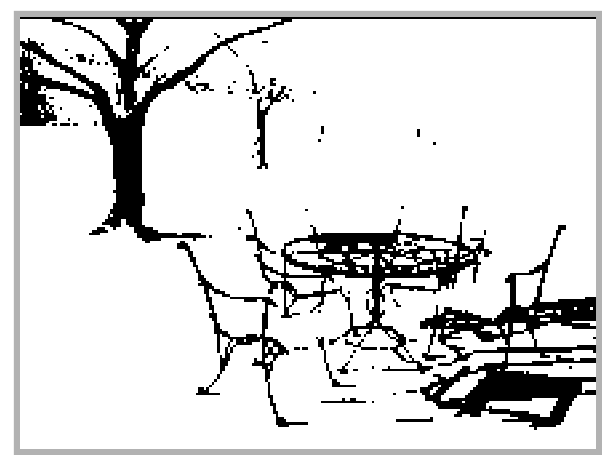
$$I'(x,y) = \begin{cases} I_{max} \text{ if } I(x,y) > t \\ I_{min} \text{ if } I(x,y) \leq t \end{cases}$$





Color image

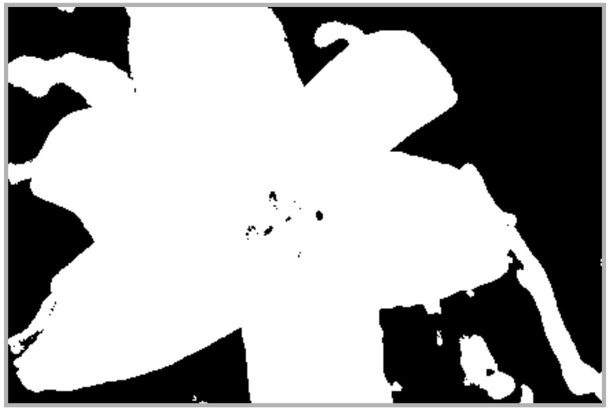
Graylevel image

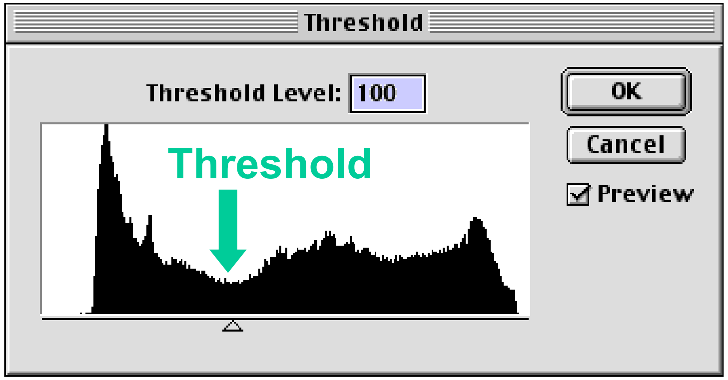


Thresholded graylevel image (**t=89**)

Threshold selection

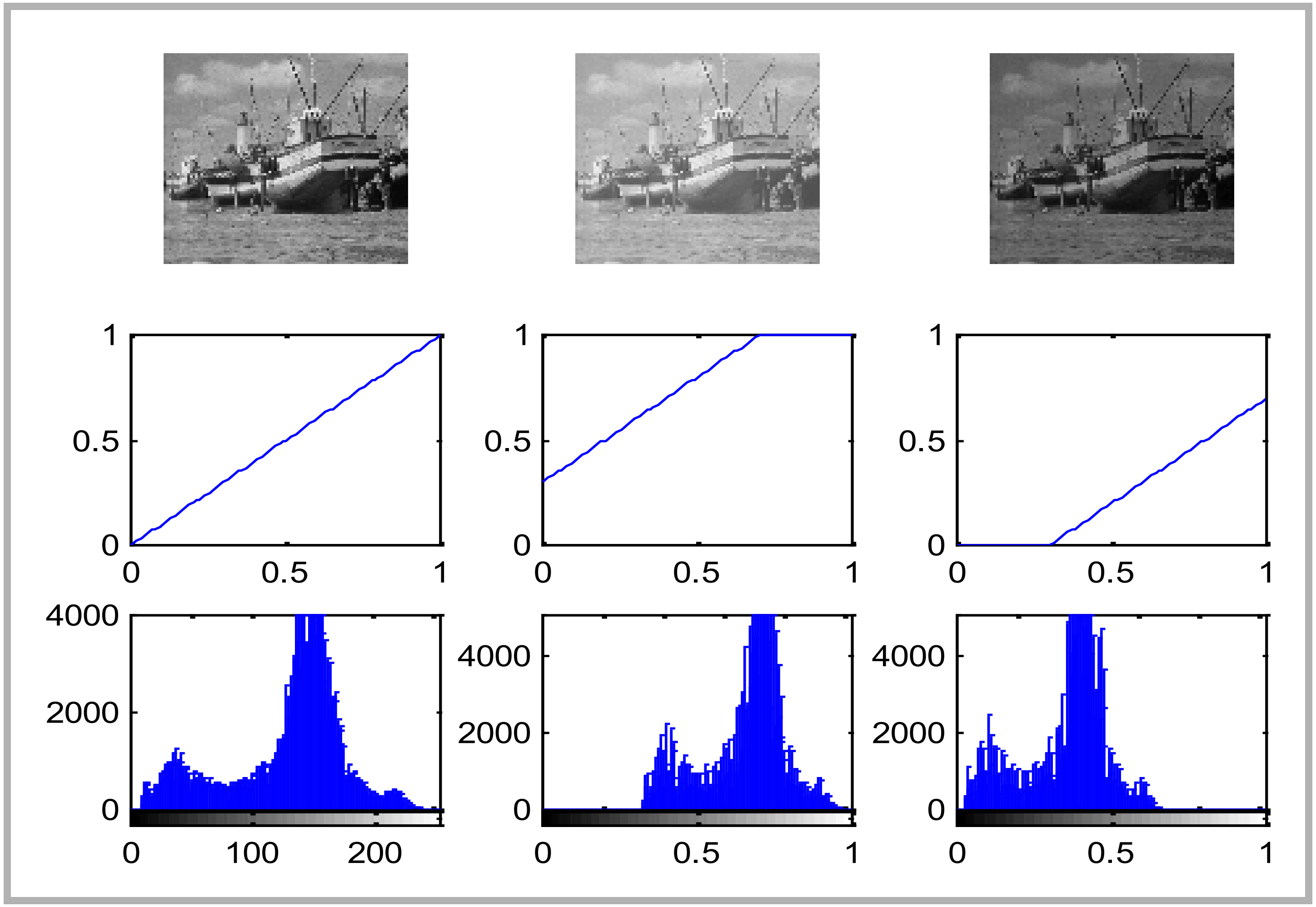
- Arbitrary selection
  - select visually
- Use image histogram



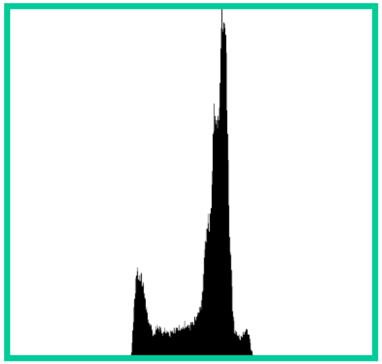


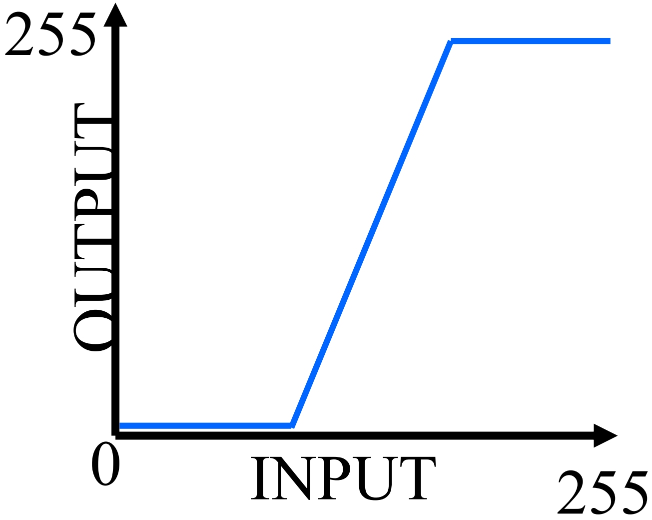
Later, we'll be back to threshold selection methods

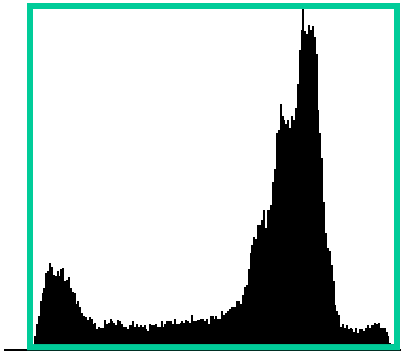
Point transformations: brightness



Point transformations: linear stretch



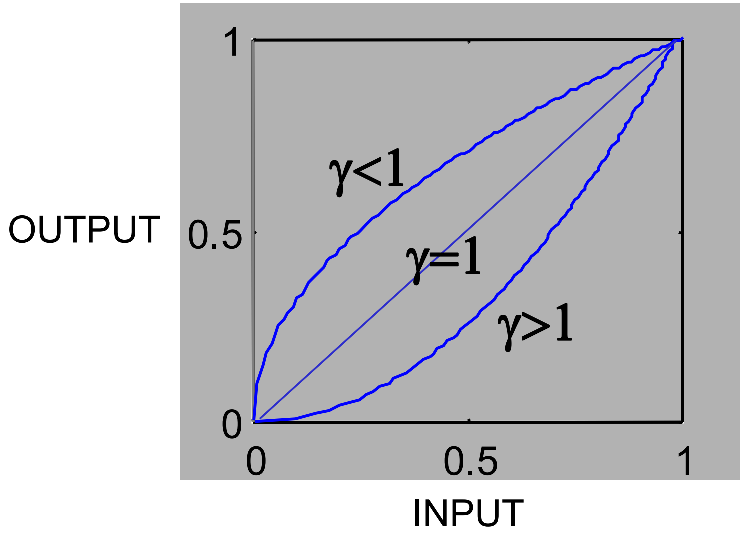




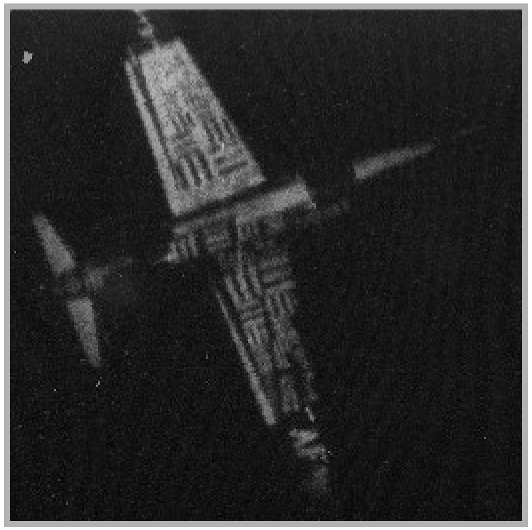
Non-linear scaling: power law

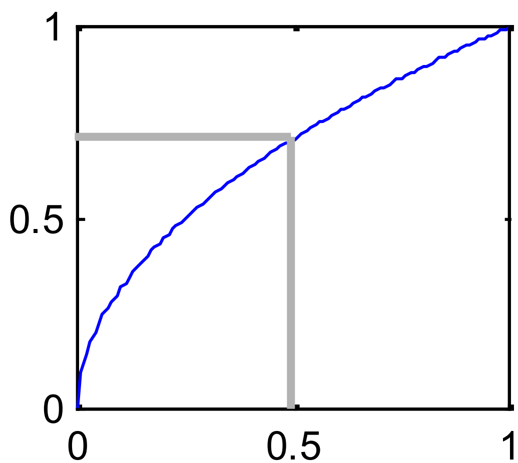
$$O = I^{\gamma}$$

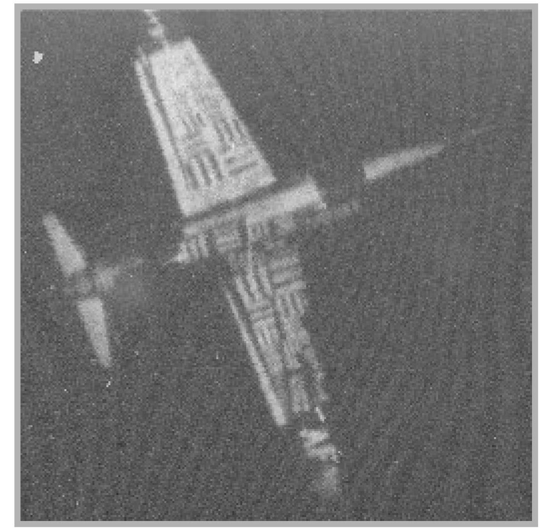
- $\gamma < 1$ to enhance contrast in dark regions
- $\gamma > 1$ to enhance contrast in bright regions.

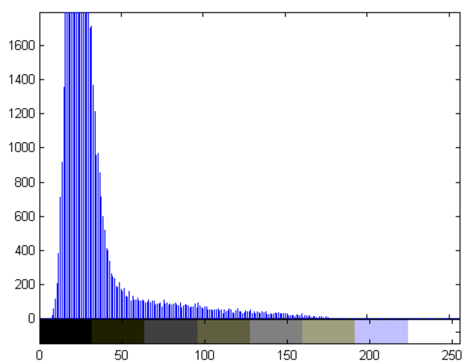


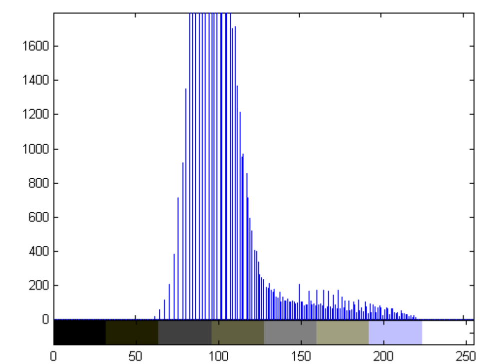
γ=0.5



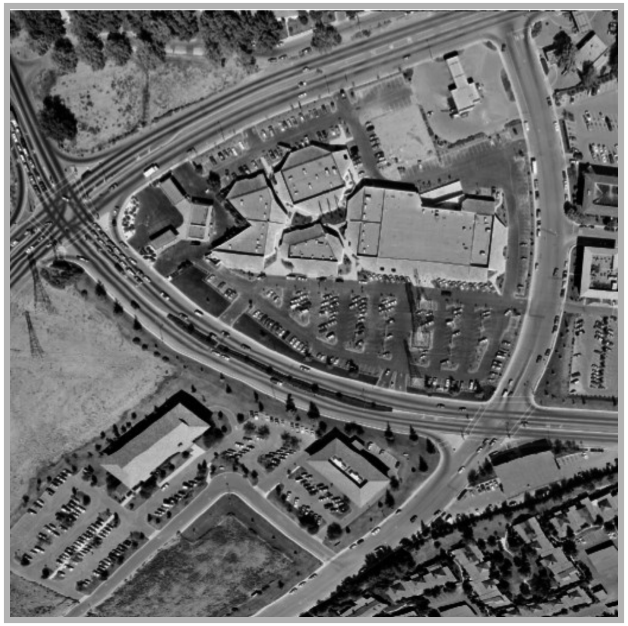

1
0.5
0
0
0.5
1

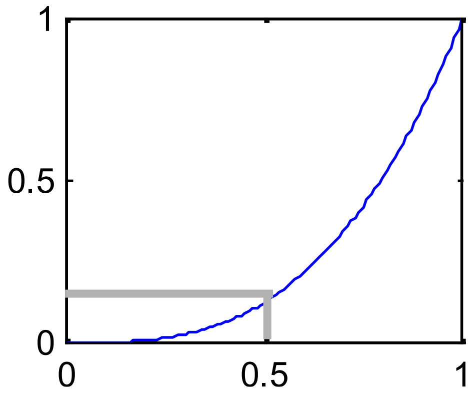


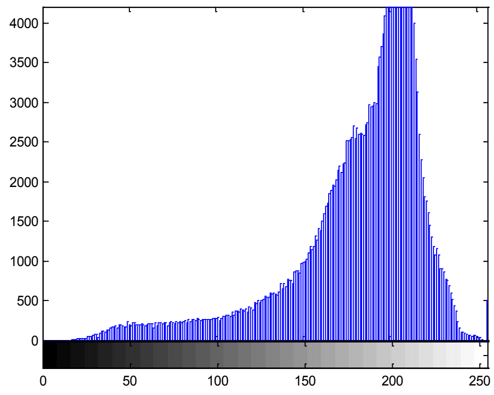

1600
1400
1200
1000
800
600
400
200
0
0
50
100
150
200
250

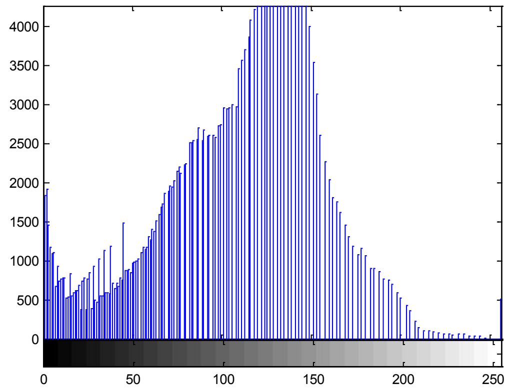

1600
1400
1200
1000
800
600
400
200
0
0
50
100
150
200
250

γ=3.0




1
0.5
0
0
0.5
1


4000
3500
3000
2500
2000
1500
1000
500
0
0
50
100
150
200
250


4000
3500
3000
2500
2000
1500
1000
500
0
0
50
100
150
200
250

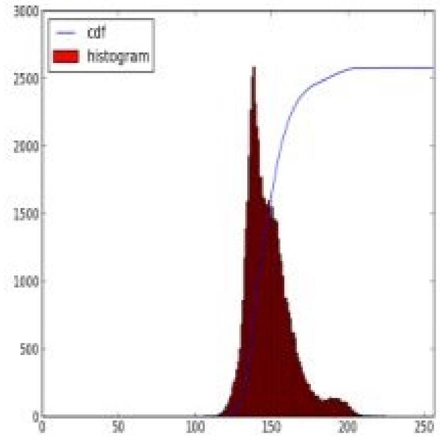
Point transformations: color images

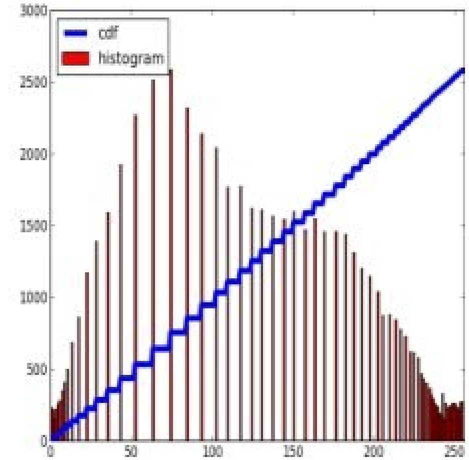
- Technique can be applied to **color images**
  - same curve to all color bands
  - different curves to separate color bands:

  - but ... **be careful**, colors may become distorted...!

- Point transformations are usually applied using **LUT's (Look Up Tables)**

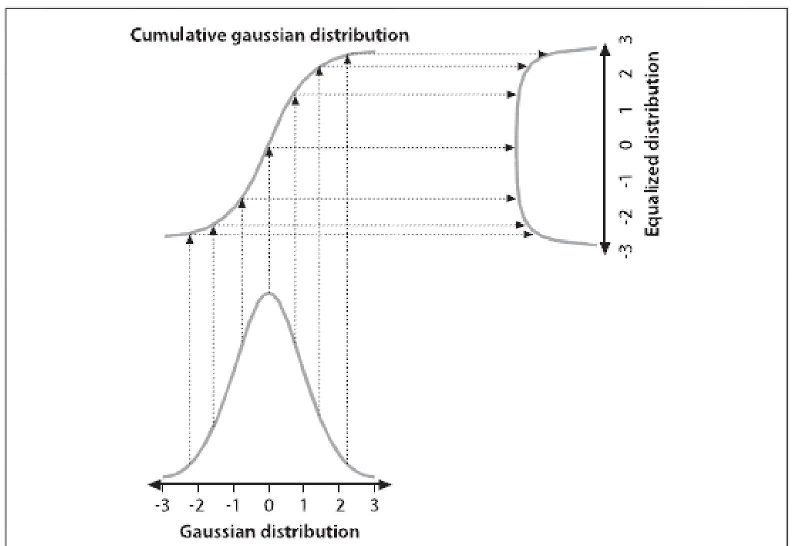
Histogram equalization





source: http://docs.opencv.org/

After equalization the cumulative distribution function is almost linear



Using the cumulative distribution function to equalize a Gaussian distribution

Comparison


Original


Histogram stretching

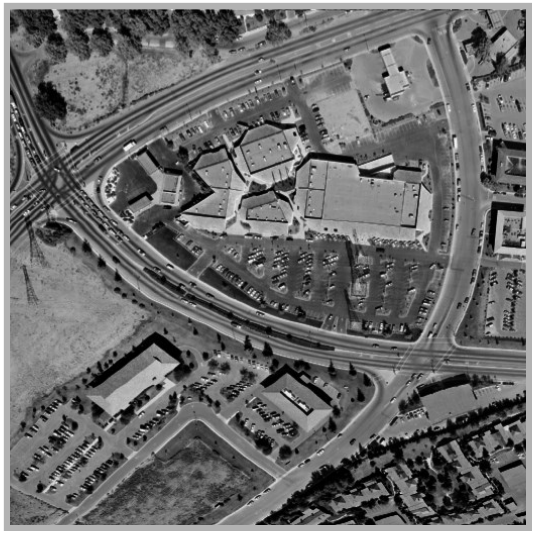

Histogram equalization

- The contrast enhancement is better after the histogram equalization, which more easily detects structures located in the shade.
- In fact any strongly represented gray-level is stretched while any <u>weakly represented graylevel is merged with other close levels</u>

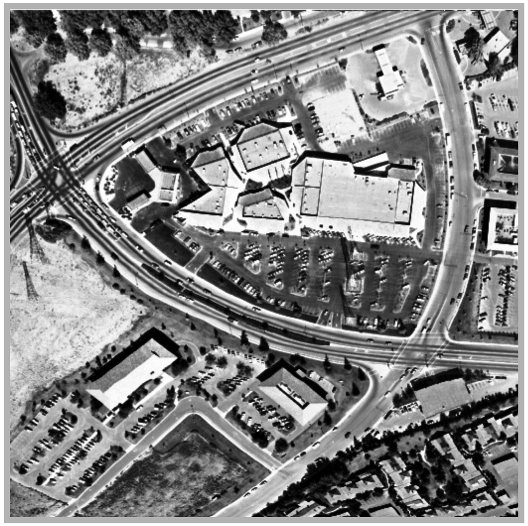
Comparison

Original

$\gamma > 1$

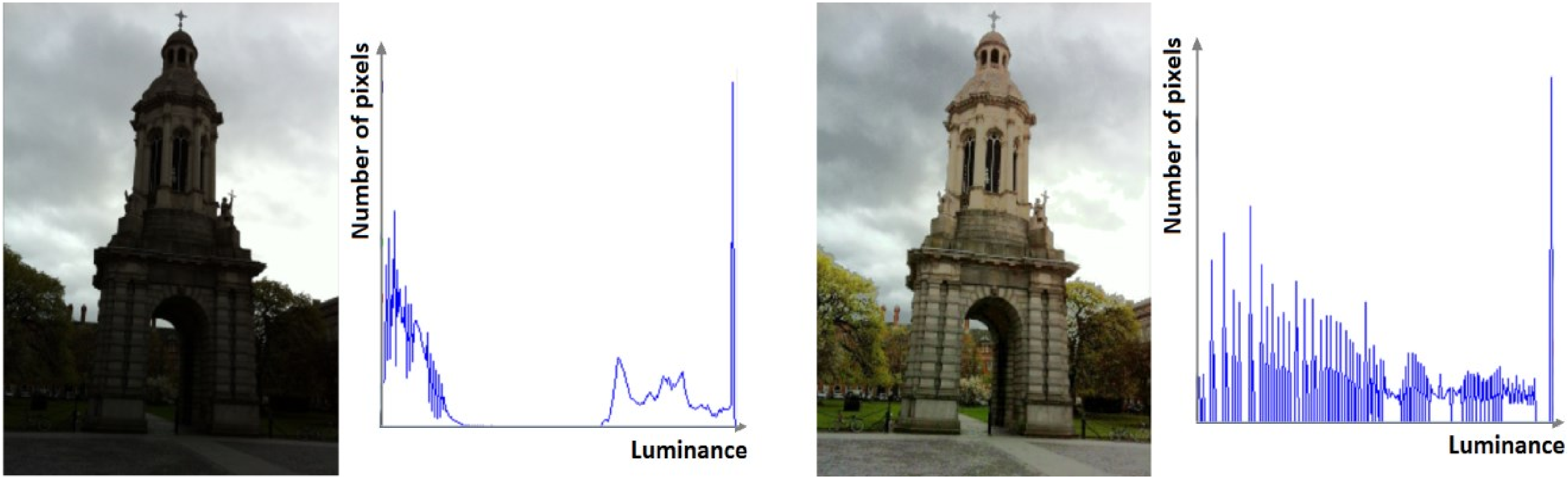


Histogram equalization



Equalization of color images

- In color images,
  <u>only</u> the <u>luminance</u> channel is usually equalized
  as otherwise the colors can become distorted.

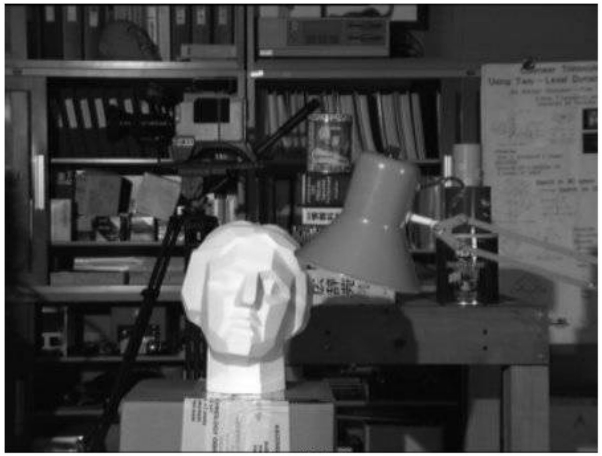


source: *A Practical Introduction to Computer Vision with OpenCV* , by Kenneth Dawson-Howe

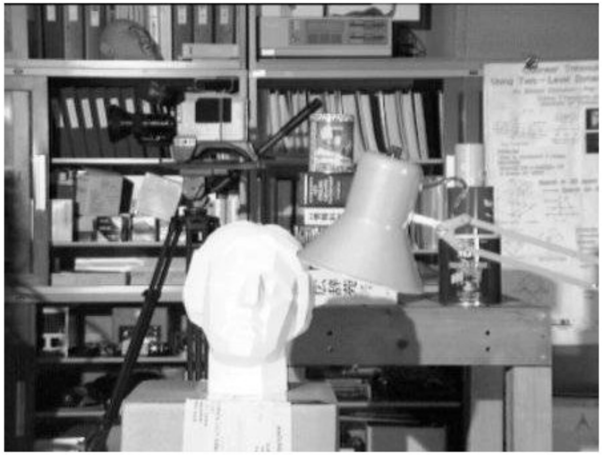
Observations

- Acquisition process degrades image
- Brightness and contrast enhancement implemented by pixel operations
- No one algorithm universally useful
- $\gamma > 1$ enhances contrast in bright images
- $\gamma < 1$ enhances contrast in dark images
- Transfer function for histogram equalization proportional to cumulative histogram
- It is essential a process of trial-and-error to determine whether a particular type of images will benefit from histogram transformation operations.

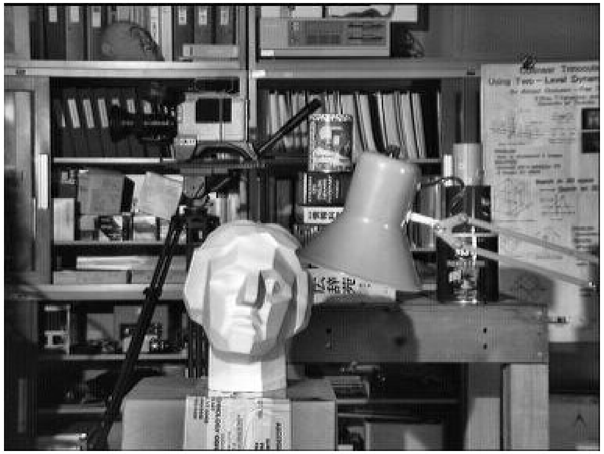
- **CLAHE – Contrast Limited Adaptive Histogram Equalization**
  - Considering the global contrast of the image is not a good idea, in some cases ...
    (look at the face of the statue, in the middle image)
  - For some images,
    it might be preferable to apply different kinds of equalization in different regions.
  - Instead of computing a single curve, the image is divided into
    MxM pixel non-overlapped sub-blocks and
    separate histogram equalization is performed in each sub-block.
  - To avoid blocking artifacts (i.e., intensity discontinuities at block boundaries)
    in the resulting image,
    the equalization functions are smoothly interpolated as we move between blocks.
  - This technique is known as adaptive histogram equalization (AHE) and
    its contrast limited (gain-limited, to avoid noise amplification) version is known as CLAHE.



Original image



After global equalization
(notice the effect on the face of the statue)



After CLAHE

source: Szeliski's book + http://docs.opencv.org/

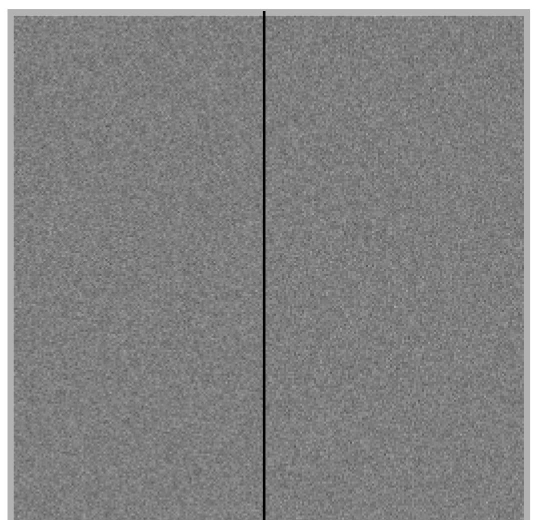
Filtering

Noise Reduction

- What is noise?
- How is noise reduction performed?
  - Noise reduction from first principles
  - Neighbourhood operators
    - linear filters (low pass)
    - non-linear filters (median)

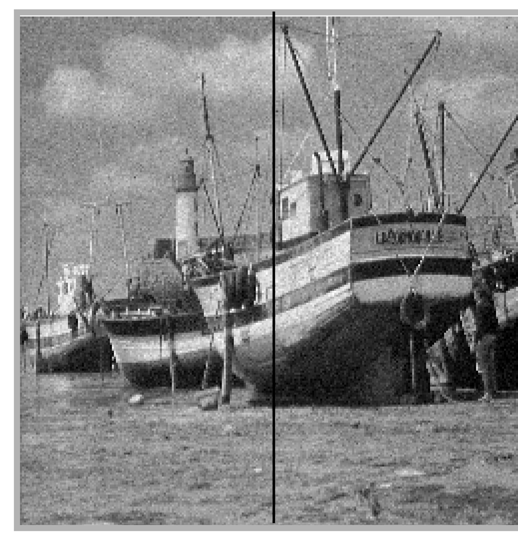
image

+



noise

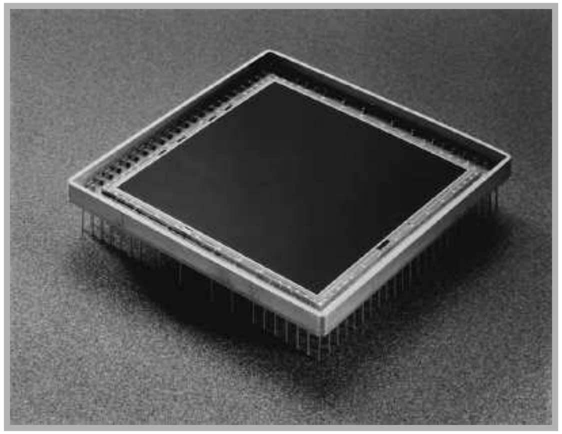
=

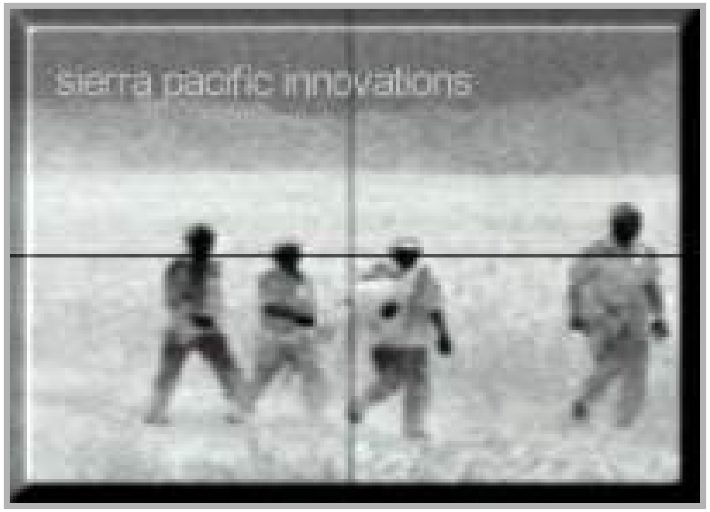


‘grainy’ image

Sources of Noise

- Sources of noise = CCD chip.
- Electronic signal fluctuations in detector.
  - Caused by thermal energy.
  - Worse for infra-red sensors.
- Other electronics
- Transmission (analog)





Radiation from the long wavelength IR band is used in most infrared imaging applications

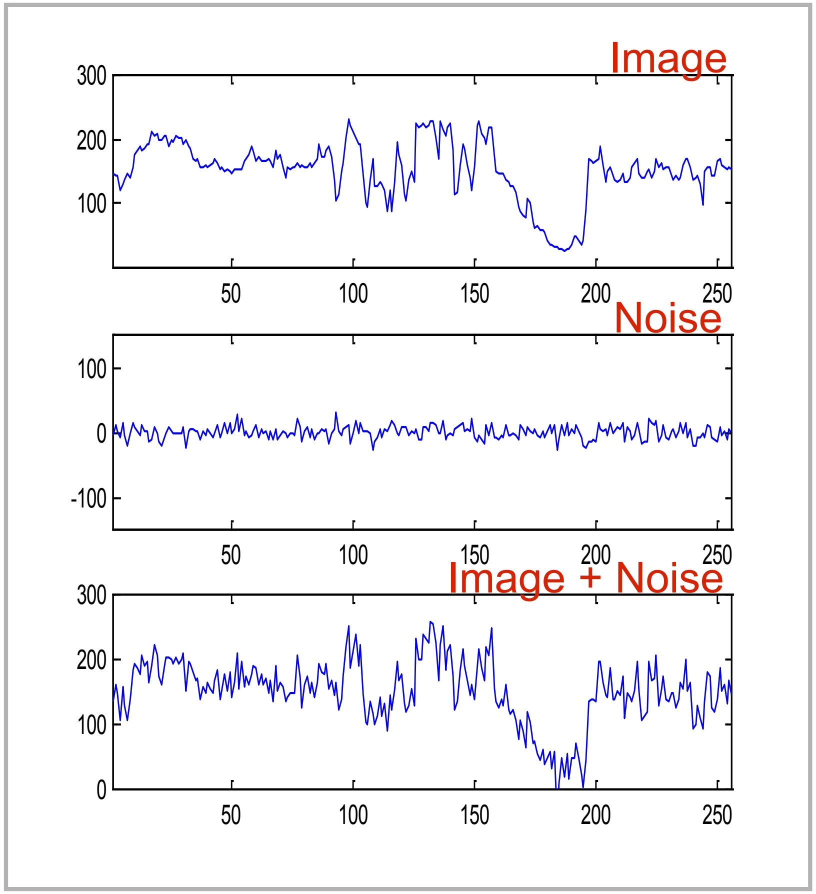
Noise

- Plot of image brightness.
- Noise models:
  - additive
    - $f(i,j) = g(i,j) + v(i,j)$
      - Gaussian
      - salt and pepper
  - multiplicative
    - $f(i,j) = g(i,j) + g(i,j).v(i,j)$

  where

  - $f(i,j)$ - *acquired signal (noisy)*
  - $g(i,j)$ - *uncorrupted signal*
  - $v(i,j)$ – *noise component*

- Noise fluctuations are rapid
  - high frequency.



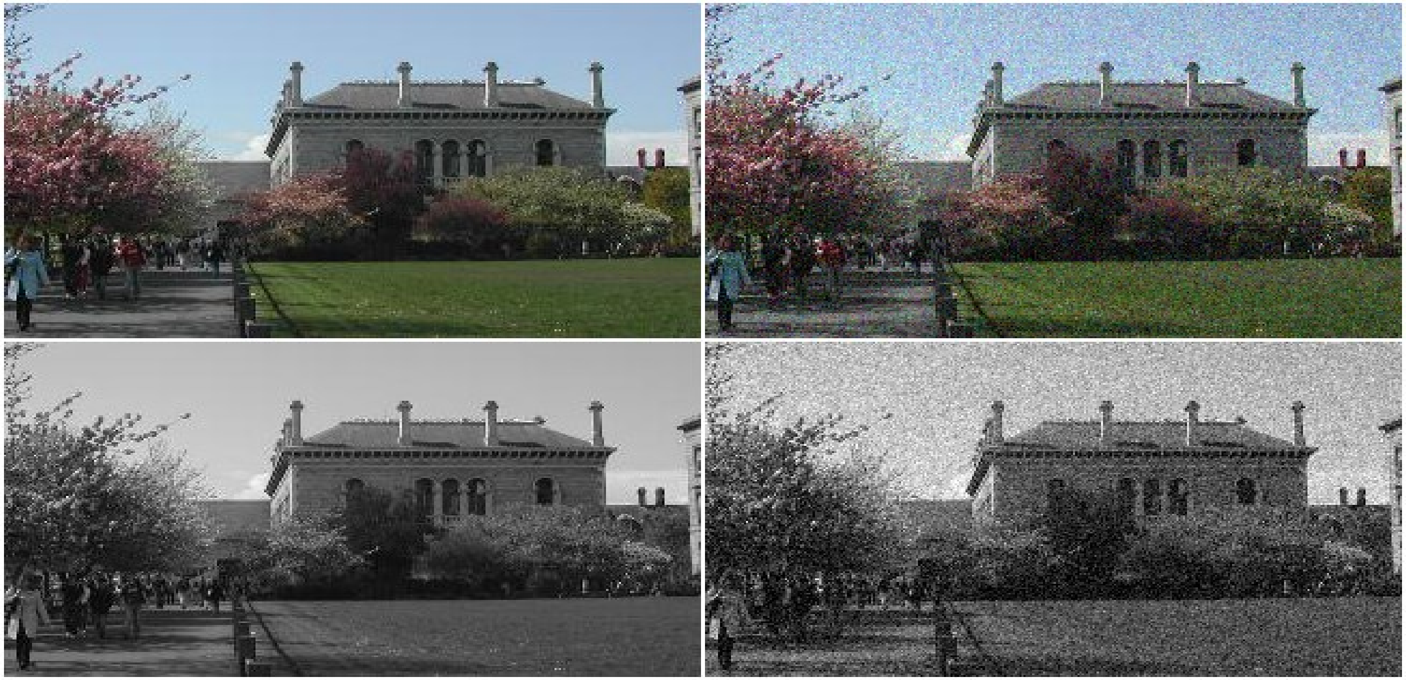
Noise – salt & pepper

- Impulse noise
  - Noise is maximum or minimum values

source: *A Practical Introduction to Computer Vision with OpenCV* , by Kenneth Dawson-Howe

Noise – Gaussian

- Good approximation to real noise
- Distribution is Gaussian (mean & standard deviation)

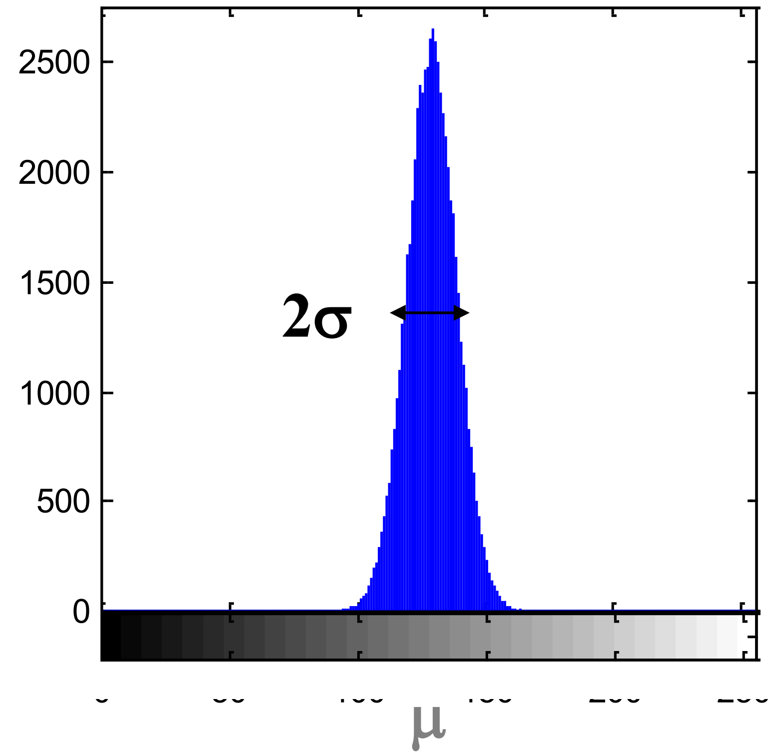


source: *A Practical Introduction to Computer Vision with OpenCV* , by Kenneth Dawson-Howe

Noise Model

- Plot noise histogram
- Typical noise distribution is normal or Gaussian
- Mean(noise) $\mu = 0$
- Standard deviation σ

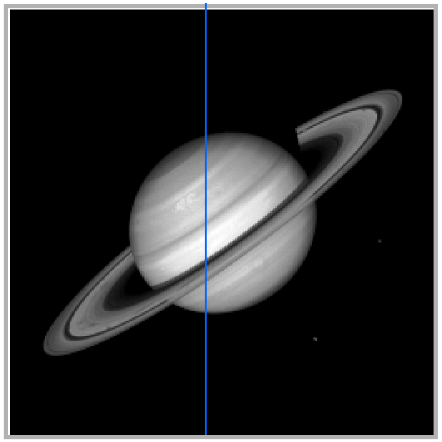
$$\eta(x) = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{1}{2} \frac{(x-\mu)^2}{\sigma} \right)$$



Noise

- Noise varies above and below uncorrupted image.

Image



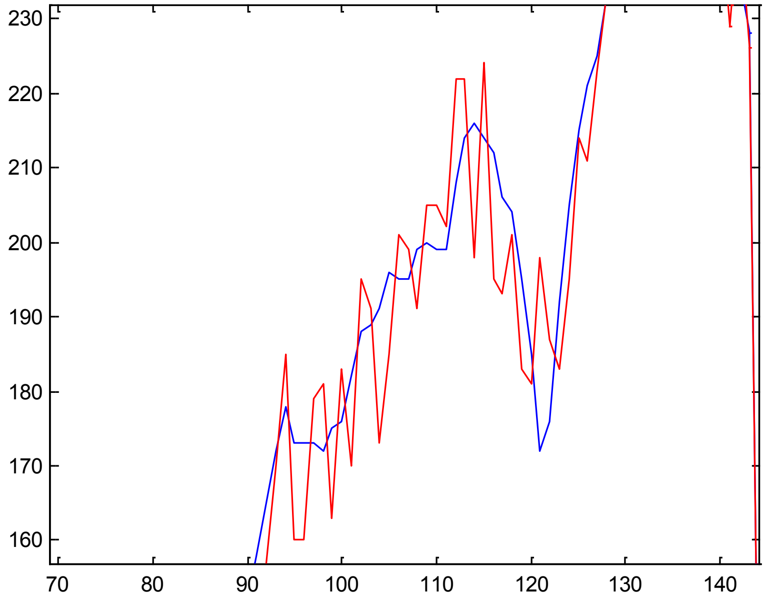
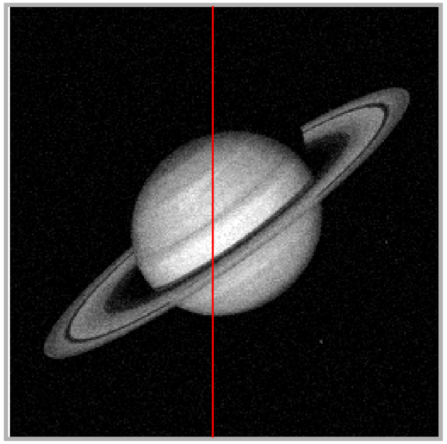


Image
+
Noise



Noise reduction

- Removing (?) or reducing noise…
- Linear smoothing transformations
  - frame averaging (=> acquire several frames of static scene)
  - local averaging
  - Gaussian smoothing
- Non-linear transformations
  - median filter
  - rotating mask

Correlation

Image Data:

10	12	40	16	19	10
14	22	52	10	55	41
10	14	51	21	14	10
32	22	9	**9**	19	14
41	18	9	22	27	11
10	7	8	8	4	5

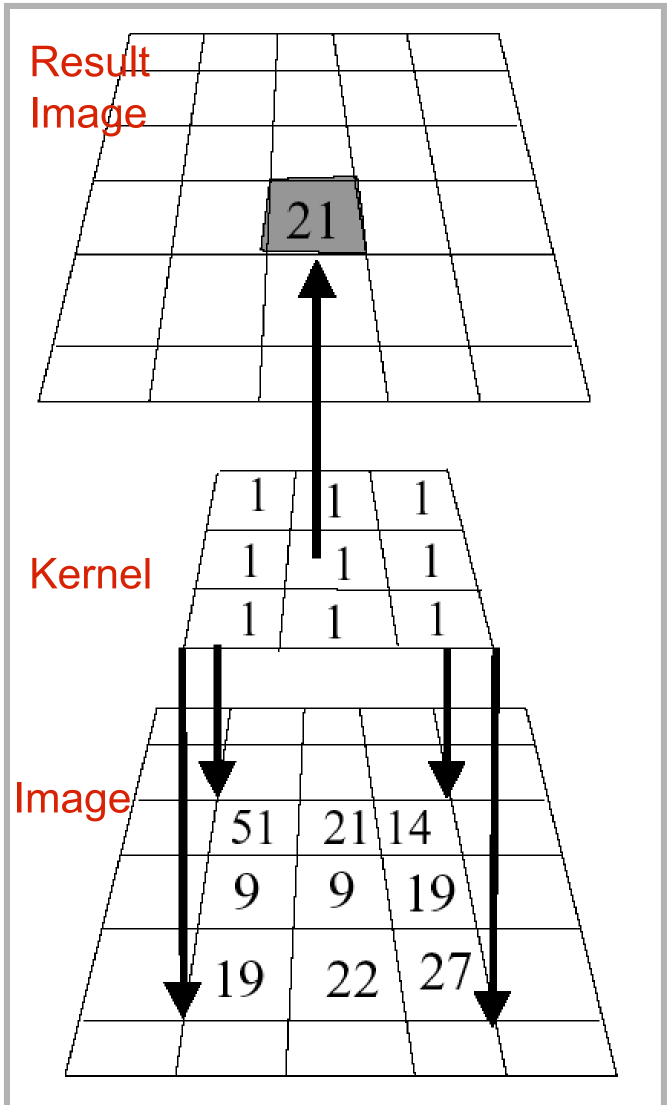
Mask / Filter / Kernel:

1	1	1	1
0	1	1	1
-1	1	1	1
	-1	*0*	*1*

Filter Image

$$F \circ I(x,y) = \sum_{j=-N}^{N} \sum_{i=-N}^{N} F(i,j) I(x+i, y+j)$$

Example



- Kernel is aligned with pixel in image, multiplicative sum is computed, normalized,
- and stored in result image.
- Process is repeated across image.
- What happens when kernel is near edge of input image?

$$\left[\begin{array}{l} 1*51 + 1*21 + 1*14 + \\ 1*8 \;\; + 1*9 \;\; + 1*19 + \\ 1*19 + 1*22 + 1*27 \end{array}\right] 1/9 = 21$$

1/9	1/9	1/9
1/9	1/9	1/9
1/9	1/9	1/9

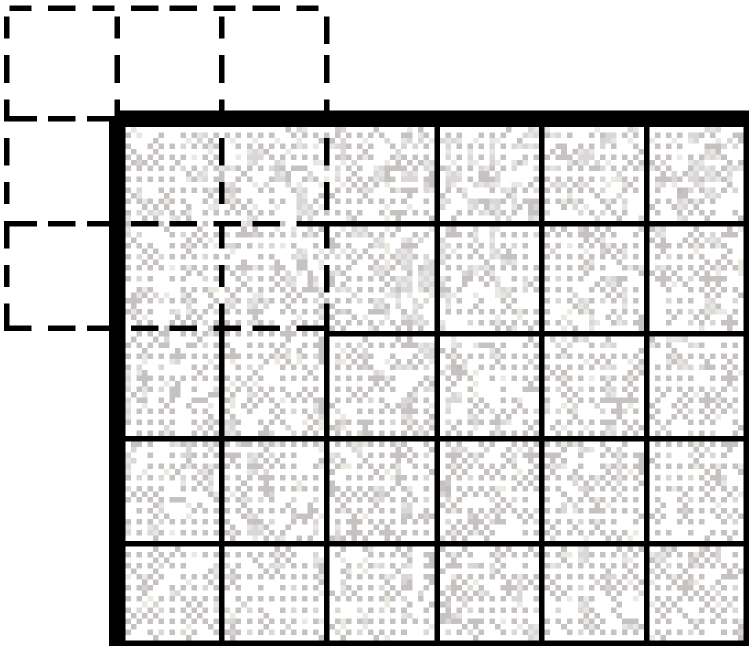
F

note the factor

Border problem

- missing samples are zero
- missing samples are gray
- copying last lines
- reflected indexing (mirror)
- circular indexing (periodic)
- reduce size of resulting image

- OpenCV
  allows some control on how to do this;
  see CopyMakeBorder(): copies the source 2D array into the interior of the destination array and makes a border of the specified type around the copied area



Border problem

1/9	1/9	1/9
1/9	1/9	1/9
1/9	1/9	1/9

F

8	3	4	5
7	6	4	5
4	5	7	8
6	5	5	6

I

8	*8*	*3*	*4*	*5*	*5*
8	8	3	4	5	*5*
7	7	6	4	5	*5*
4	4	5	7	8	*8*
6	6	5	5	6	*6*
6	*6*	*5*	*5*	*6*	*6*

I with padded boundaries

6.44	5.22	4.33	4.67
5.78	5.33	5.22	5.67
5.56	5.44	5.67	6.00
5.22	5.33	5.78	6.33

$J = F \circ I$

Convolution

- A **convolution** operation is a correlation where the filter is flipped both horizontally and vertically before being applied to the image:

$$F * I(x,y) = \sum_{j=-N}^{N} \sum_{i=-N}^{N} F(i,j) I(x-i, y-j)$$

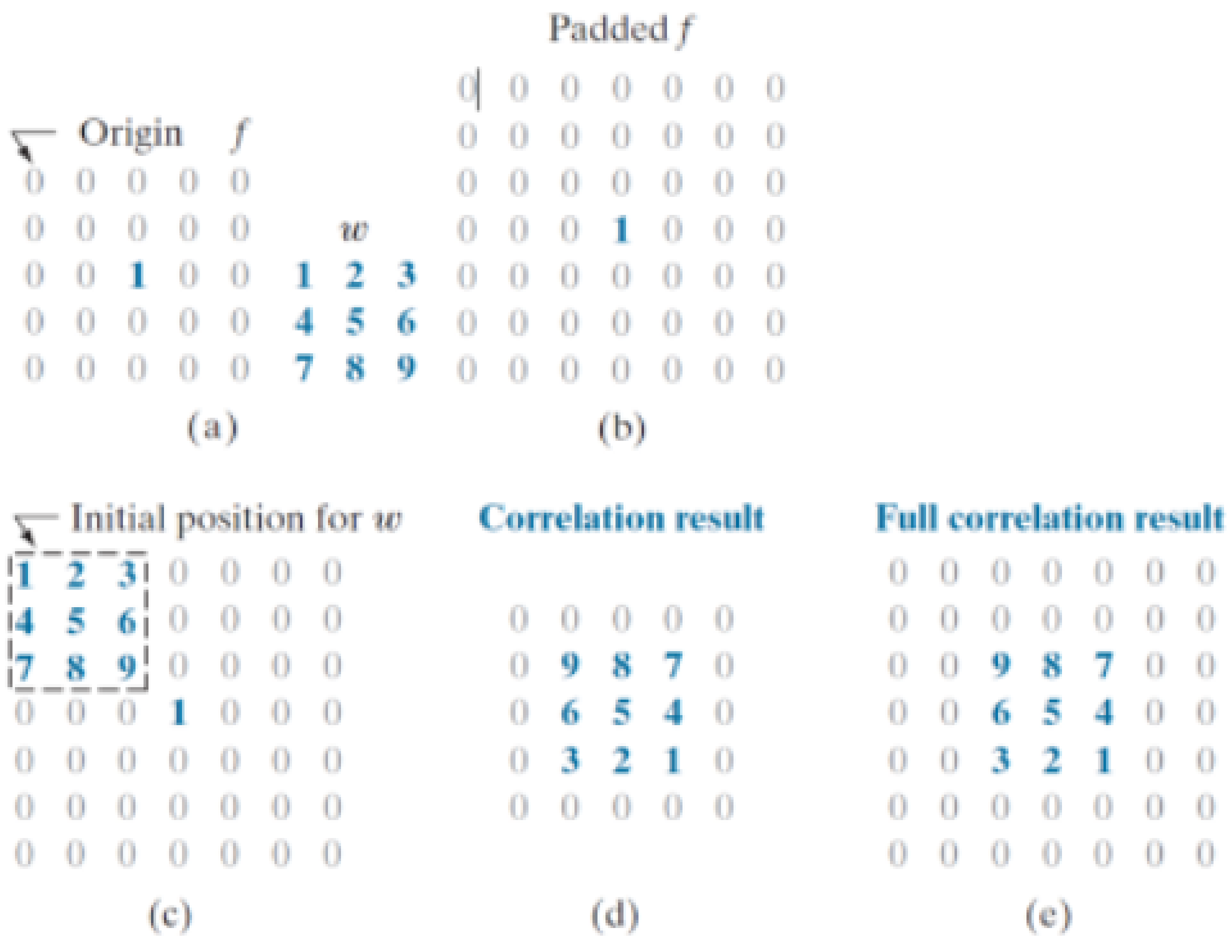
- Suppose F is a Gaussian or mean kernel.
How does convolution differ from cross-correlation?

Convolution

- Extremely important concept in computer vision, image processing, signal processing, etc.
- Lots of related mathematics (we won't do)
- General idea: reduce a filtering operation to the repeated application of a mask (or filter kernel) to the image
  - Kernel can be thought of as an NxN image
  - N is usually odd so kernel has a central pixel
- In practice
  - (flip kernel)
  - Align kernel center pixel with an image pixel
  - Pointwise multiply each kernel pixel value with corresponding image pixel value and add results
  - Resulting sum is normalized by kernel weight
  - Result is the value of the pixel centered on the kernel

Correlation vs Convolution

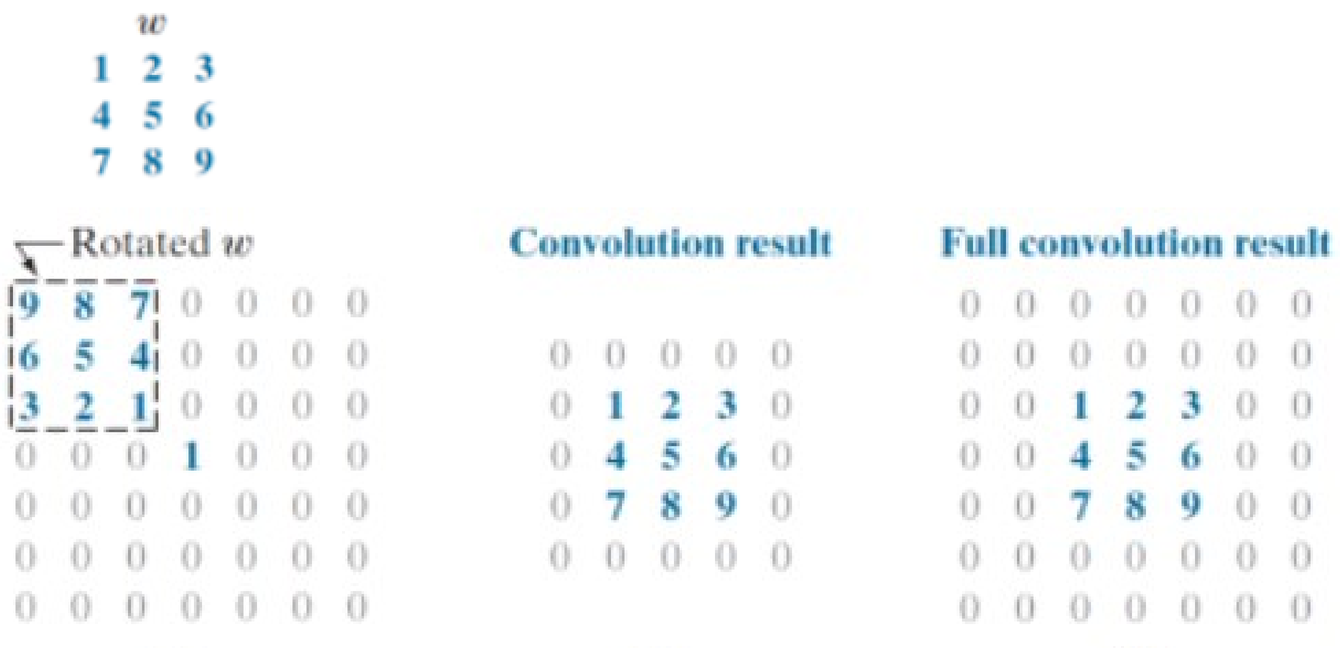
Correlation



source: https://towardsdatascience.com/convolution-vs-correlation-af868b6b4fb5

Correlation vs Convolution

Convolution



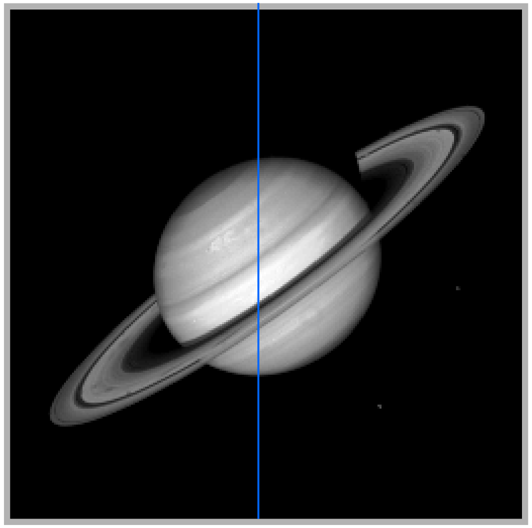
(f) (g) (h)

source: https://towardsdatascience.com/convolution-vs-correlation-af868b6b4fb5

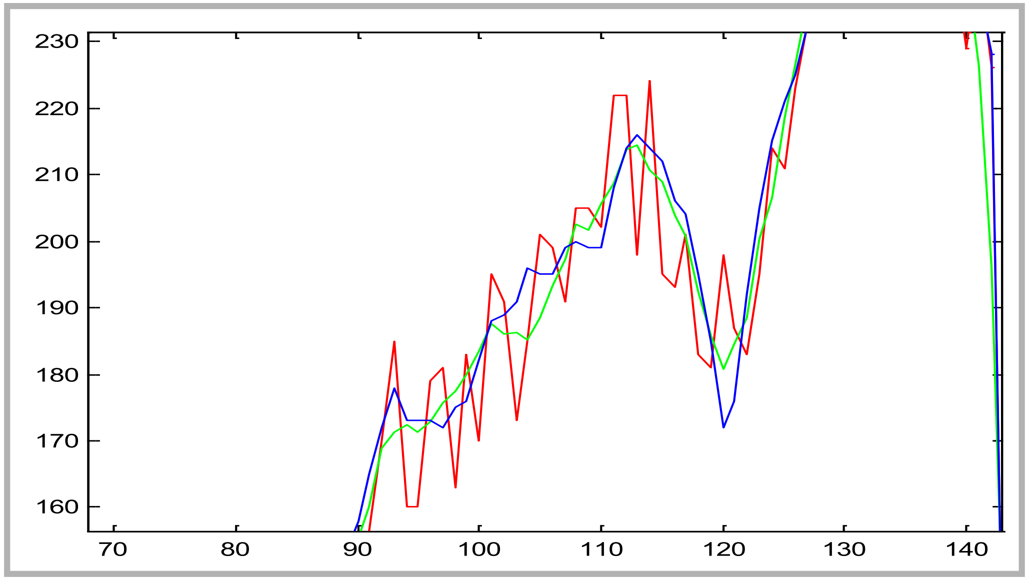
Notes:

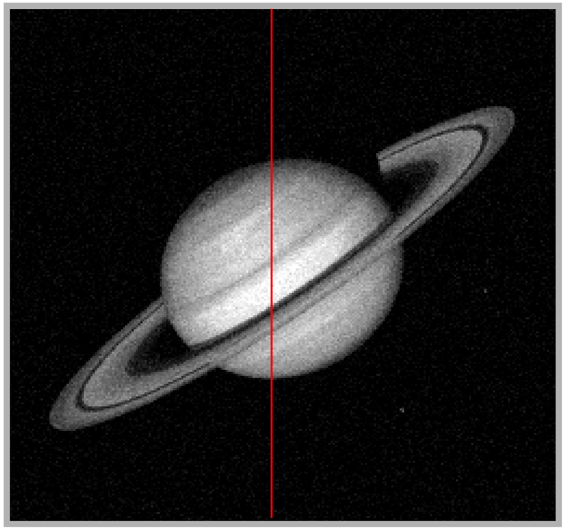
1) Correlation and convolution are identical when the filter is symmetric.
2) The key difference between the two is that convolution is associative, that is, if F and G are filters, then F*(G*Img) = (F*G)*Img
3) In general, people use **convolution** for image processing operations such as smoothing, and they use **correlation** to match a template to an image (*see later*).

Noise reduction - 1



Uncorrupted Image





Uncorrupted Image + Noise

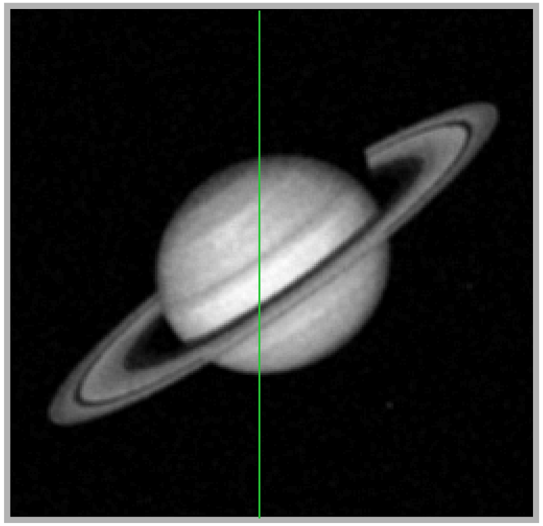


Image + Noise - Blurred

Noise neduction - 1

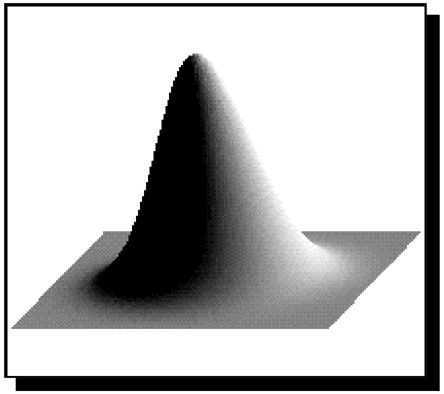
- Technique relies on high frequency noise fluctuations being ‘blocked’ by filter.
  Hence, low-pass filter.
- Fine detail in image may also be smoothed.
- Balance between
  keeping image fine detail and reducing noise.

Noise reduction - 1

- Saturn image
  (*previous slide*)
  has coarse detail
- Boat image
  contains fine detail
- Noise reduced but
  fine detail also smoothed

Gaussian smoothing

- Smoothing operator should be
  - 'tunable' in what it leaves behind
  - smooth and localized in image space.
- One operator which satisfies these two constraints is the Gaussian



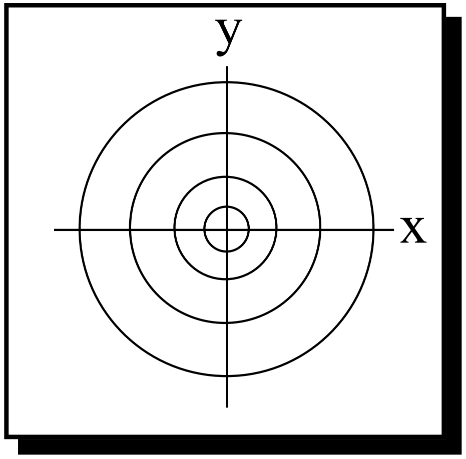
- *OpenCV: Smooth()*

2D Gaussian distribution

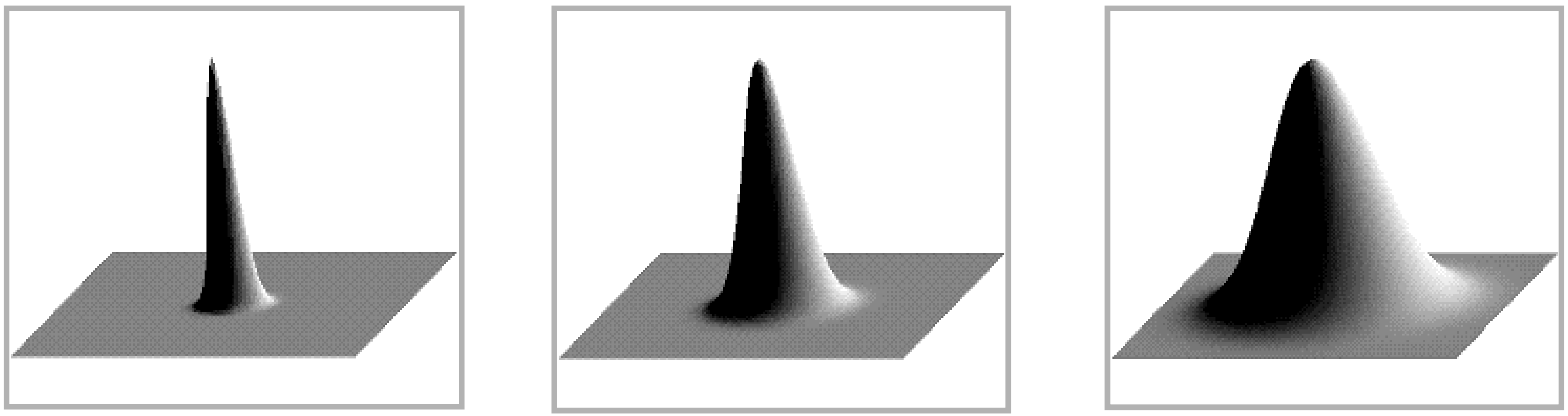
- The two-dimensional Gaussian distribution is defined by:

$$G(x,y) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\left[\frac{(x^2+y^2)}{2\sigma^2}\right]}$$

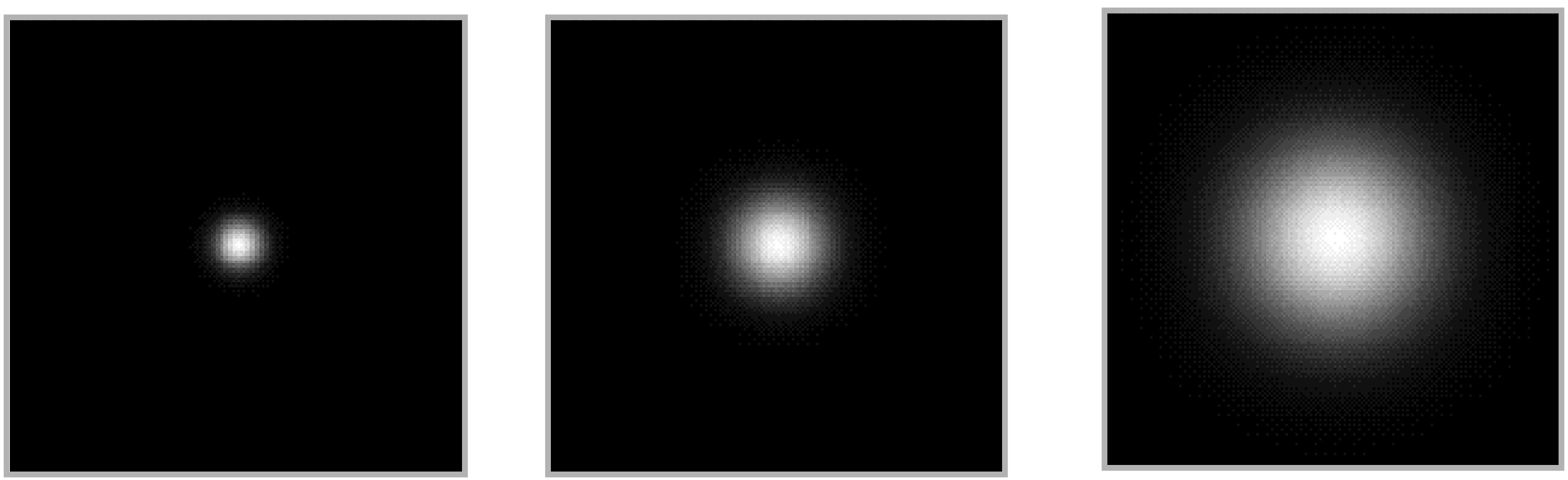
- From this distribution, can generate smoothing masks whose width depends upon σ:



σ defines kernel "width"



$\sigma^2 = .25$ $\sigma^2 = 1.0$ $\sigma^2 = 4.0$



Example

- Choose σ^2 = 2 and n = 7, then:

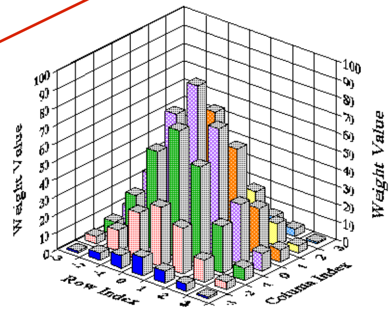
i \ j	-3	-2	-1	0	1	2	3
-3	.011	.039	.082	.105	.082	.039	.011
-2	.039	.135	.287	.368	.287	.135	.039
-1	.082	.287	.606	.779	.606	.287	.082
0	.105	.039	.779	1.000	.779	.368	.105
1	.082	.287	.606	.779	.606	$\boxed{.287}$	.082
2	.039	.135	.287	.368	.287	.135	.039
3	.011	.039	.082	.105	.082	.039	$\boxed{.011}$

$$\frac{W(1,2)}{k} = \exp\left(-\frac{1^2+2^2}{2*2}\right)$$

1	4	7	10	7	4	1
4	12	26	33	26	12	4
7	26	55	71	55	26	7
10	33	71	91	71	33	10
7	26	55	71	55	26	7
4	12	26	33	26	12	4
1	4	7	10	7	4	1

7x7 Gaussian filter

To make this value 1,
choose k=91



Kernel Application

7x7 Gaussian kernel

15x15 Gaussian kernel

Why Gaussian for smoothing

- Gaussian is not the only choice, but
  it has a number of important properties
  - If we <u>convolve a Gaussian with another Gaussian</u>, the <u>result is a Gaussian</u>
    - This is called linear scale space

$$G_{\sigma_1} * G_{\sigma_2} = G_{\sqrt{\sigma_1^2+\sigma_2^2}}.$$

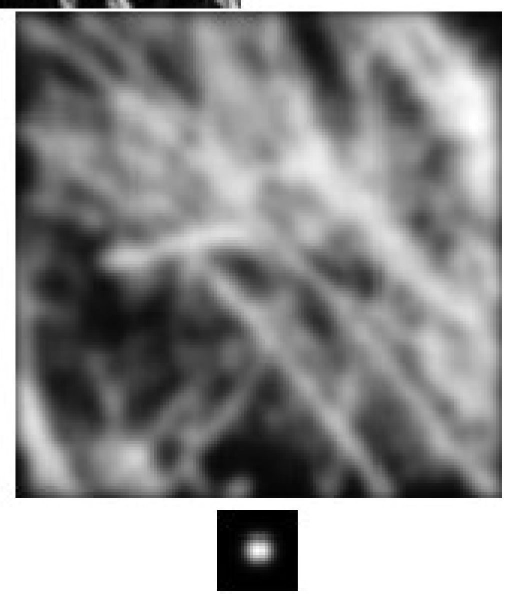
  - Efficiency: <u>separable</u>

$$G_\sigma(x,y) = \frac{1}{2\pi\sigma^2}\exp\left(-\frac{(x^2+y^2)}{2\sigma^2}\right)$$
$$= \left(\frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{(x^2)}{2\sigma^2}\right)\right) \times \left(\frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{(y^2)}{2\sigma^2}\right)\right).$$

Mean vs. Gaussian filtering

Original image

After mean filtering



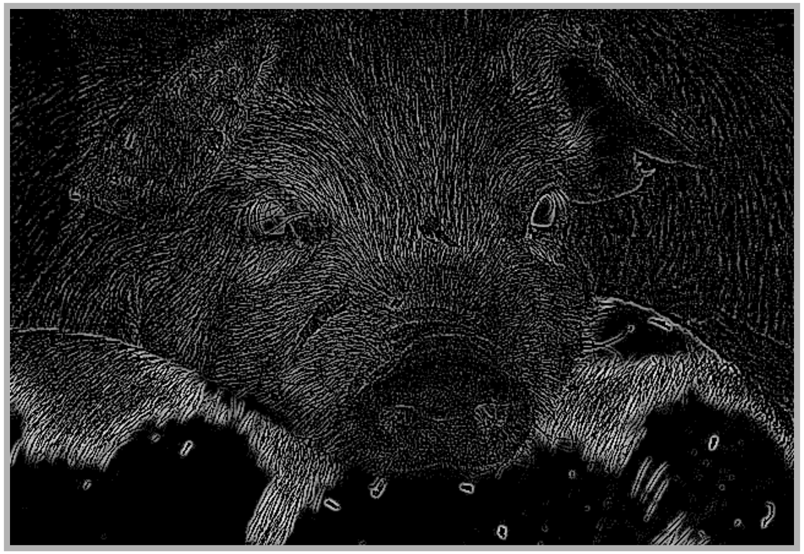
After Gaussian filtering

hmmmmm.....

Image

Blurred image

\- =



Noise reduction - 2: Median filter

- Compute median of points in neighborhood
- Nonlinear filter
  - example of a larger class of filters named "rank-filters" (ex: min, median, max)
- Has a tendency to blur detail less than averaging
- Works very well for ‘shot’ or ‘salt and pepper’ noise

Linear vs Non-linear

Mean(I1+I2) = =
Mean (I1) + Mean(I2)

Median(I1+I2) !=
Median(I1)+Median(I2)

Original

Low-pass

Median

Linear vs Non-linear filters:
https://dsp.stackexchange.com/questions/14241/what-is-the-difference-between-linear-and-non-linear-filters

Mean vs. Median

- Mean
  - Linear
  - Signal frequencies shared with noise are lost, resulting in blurring.
  - Impulsive noise is diffused but not removed.
  - It spreads the noise, resulting in blurring.
  - Blurs edges.
- Median
  - Non-linear
  - Does not spread the noise.
  - Can remove spike noise.
  - Preserves (some) edges.
  - Small details (small regions, this edges, …) may be lost.
  - Expensive to run.

Edge Preserving Smoothing

Smooth (average, blur) an image without disturbing

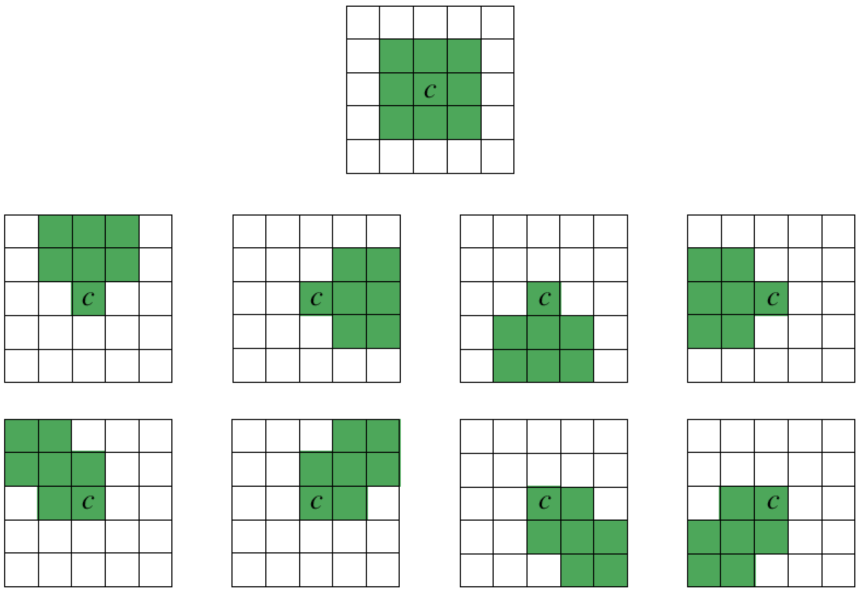
- sharpness or
- position

of the edges

- Nagao-Matsuyama filter
- Kuwahara filter
- Anisotropic diffusion filters (Perona & Malik, ...)
- Bilateral filtering
- ...

Nagao-Matsuyama filter

- Calculate the variance within
  nine subwindows of a 5x5 moving window
- Output value is
  the **mean** of the **subwindow with the smallest variance**
- Nine subwindows used:



Kuwahara filter

- Principle:
  - divide filter mask into four regions (a, b, c, d).
  - in each compute the mean brightness and the variance
  - the output value of the center pixel (abcd) in the window is the **mean** value of that **region** that has the **smallest variance**.

a	a	ab	b	b
a	a	ab	b	b
ac	ac	abcd	bd	bd
c	c	cd	d	d
c	c	cd	d	d

Kuwahara filter example

Original

Median (1 iteration)

Kuwahara

Median (10 iterations)

Anisotropic diffusion filtering

- The anisotropic diffusion is a nonlinear filter that smoothes the intraregions of an image without blurring the strong edges
  - Encourages the **smoothing in homogeneous regions in preference to** smoothing across the boundaries.
- Based on the solution of a partial differential equation, inspired in heat diffusion equation
  - *The algorithm results from the discretization of the non-linear diffusion equation.*
  - *The derivatives are approximated by differences.*
- Diffusion methods average over extended regions by solving partial differential equations, and are therefore inherently iterative. Iteration may raise issues of stability and, depending on the computational architecture, efficiency.
- This algorithm is also used to detect the edges

Anisotropic diffusion filtering

Original

Perona, Malik, 1987

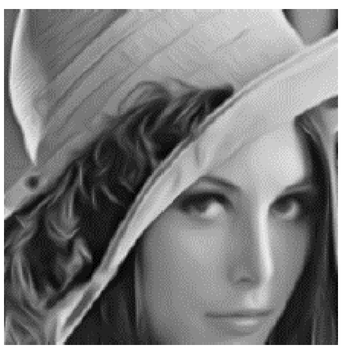
$$\frac{\partial u}{\partial t} = div\left(\frac{Du}{|Du|^2 + \lambda^2}\right)$$

Rudin, Osher, Fatemi, 1992

$$\frac{\partial u}{\partial t} = div\left(\frac{Du}{|Du|}\right)$$

Alvarez, Lions, 1992

$$\frac{\partial u}{\partial t} = \frac{|Du|}{|k * Du|} div\left(\frac{Du}{|Du|}\right)$$



Weickert, 1994

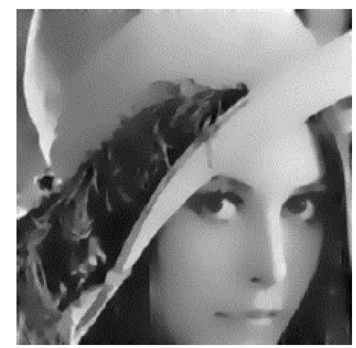
$$\frac{\partial u}{\partial t} = D^2 u(d,d)$$
$$d = SEigen(k * (Du \otimes Du))$$

Caselles, Sbert, 1997

$$\frac{\partial u}{\partial t} = \frac{1}{|Du|^2} D^2 u(Du, Du)$$

Zhong Carmona, 1998

$$\frac{\partial u}{\partial t} = D^2 u(d,d)$$
$$d = SEigen(D^2 u)$$

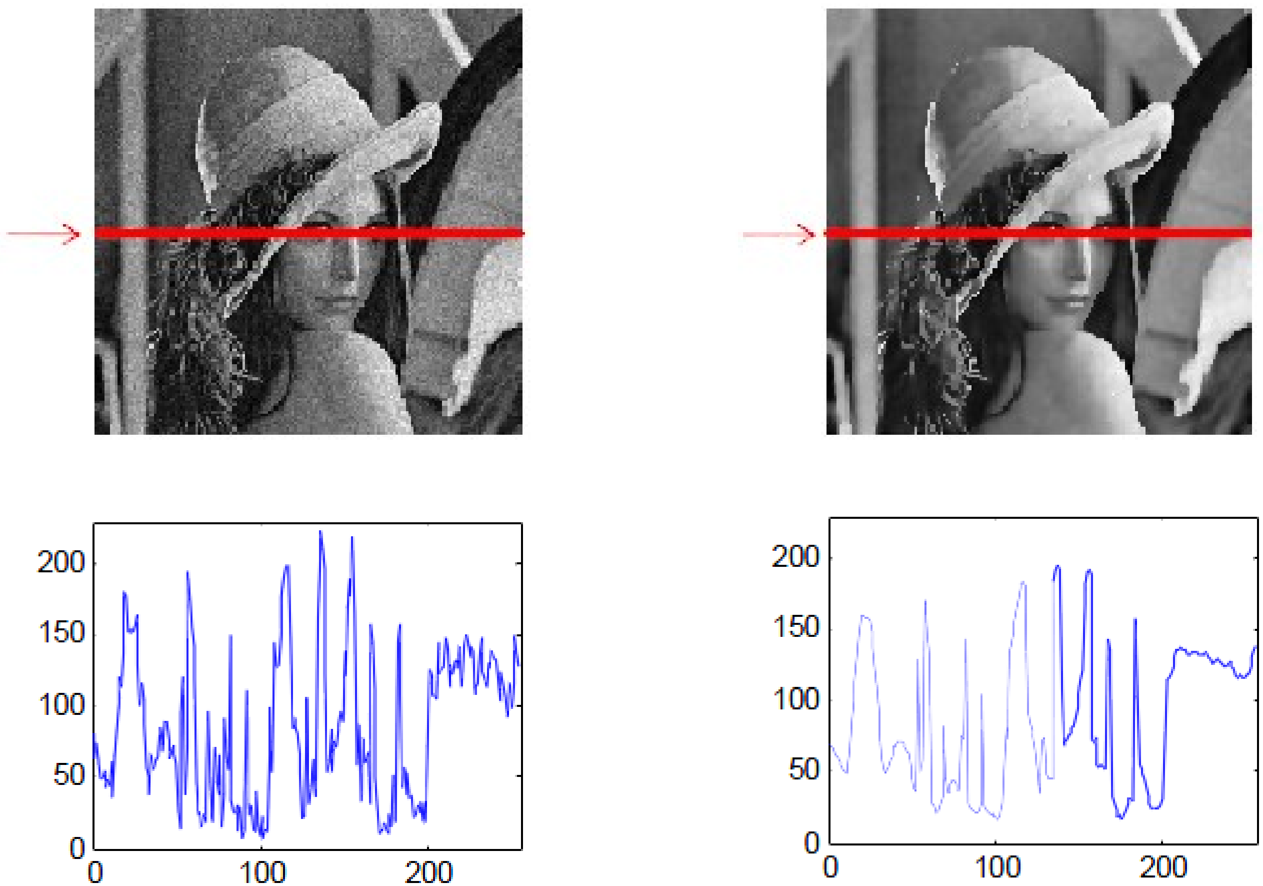


Sochen, Kimmel, Malladi, 1998

$$\frac{\partial u}{\partial t} = div\left(\frac{Du}{\sqrt{|Du|^2 + 1}}\right)$$

Note: Original Perona-Malik diffusion process is NOT anisotropic, although they erroneously said it was.

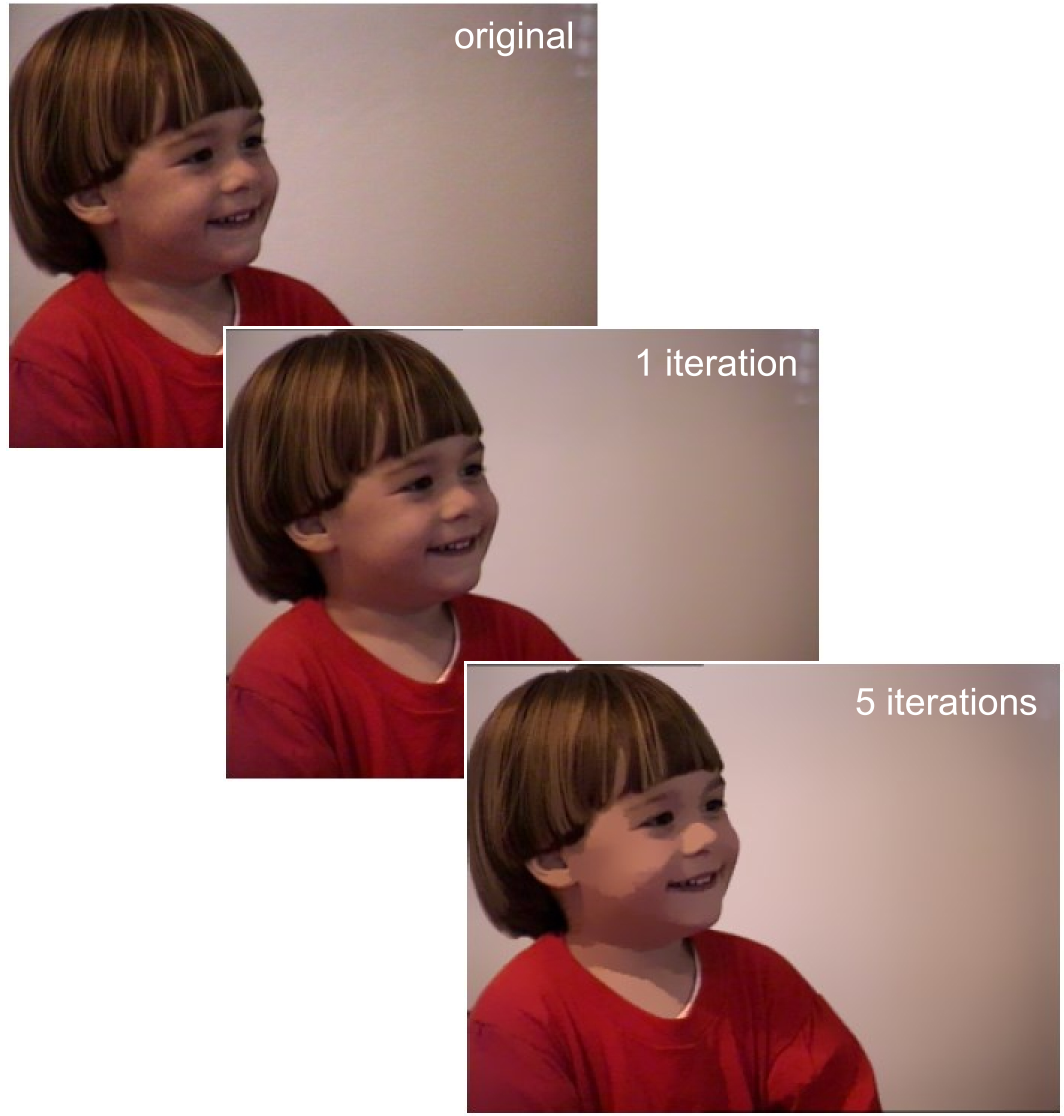
Anisotropic diffusion filtering



Bilateral filtering

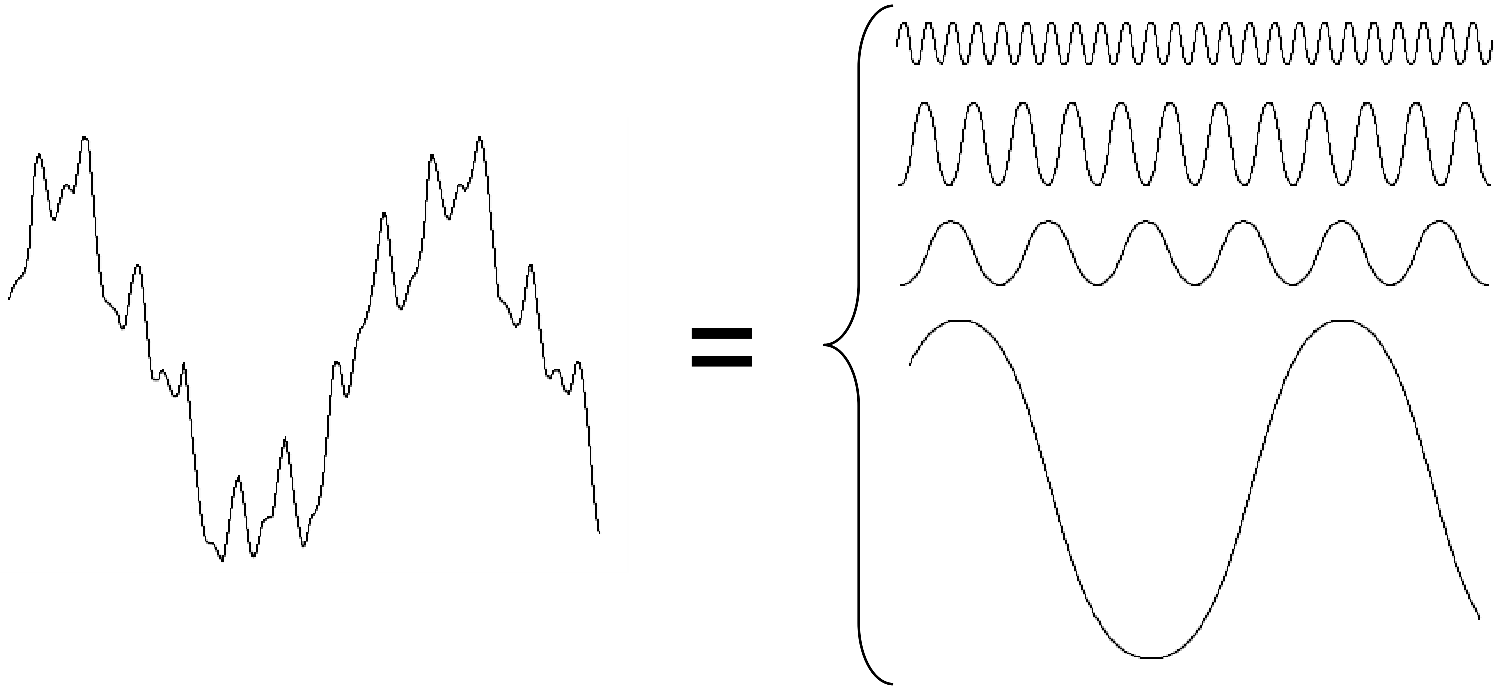
- A bilateral filter is an edge-preserving and noise reducing smoothing filter.
- Traditional filtering is **domain filtering**, and enforces closeness by weighing pixel values with weights that fall off with distance.
- Similarly, we define **range filtering**, which averages image values with weights that decay with dissimilarity.
- **Range filters**
  - are nonlinear because their weights depend on image intensity or color
  - preserve edges
  - by themselves, my distort an image's color map
    (see C.Tomasi at al., Bilateral Filtering for Gray and Color, ICCV 1998)
- **Bilateral filtering** combines range and domain filtering.
- The intensity value at each pixel in an image is replaced by a **weighted average of intensity values from nearby pixels.**
- This weight is based on a Gaussian distribution.
  The **weights** depend not only on Euclidean distance but also on the radiometric differences (e.g. color intensity).
- It preserves sharp edges by systematically looping through each pixel and attributing weights to the adjacent pixels accordingly.
- *OpenCV: Smooth()*

Bilateral filtering



Frequency domain filtering

- An image can be represented in the frequency domain, using the Fourier Transform (FT).
- The FT encodes the amplitude and phase of each frequency component.
- The values near the origin of the transformed space are called low-frequency components of the FT, and those distant from the origin are the high-frequency components.
- Convolution in the image domain corresponds to multiplication in the spatial frequency domain.
- Therefore, convolution with large filters, which would normally be expensive in the image domain, can be implemented efficiently using the Fast Fourier Transform (FFT).
- This is an important technique in many image processing applications.



- Any function that periodically repeats itself can be expressed as a sum of sines and cosines of different frequencies each multiplied by a different coefficient – a *Fourier series*

Discrete Fourier Transform - DFT

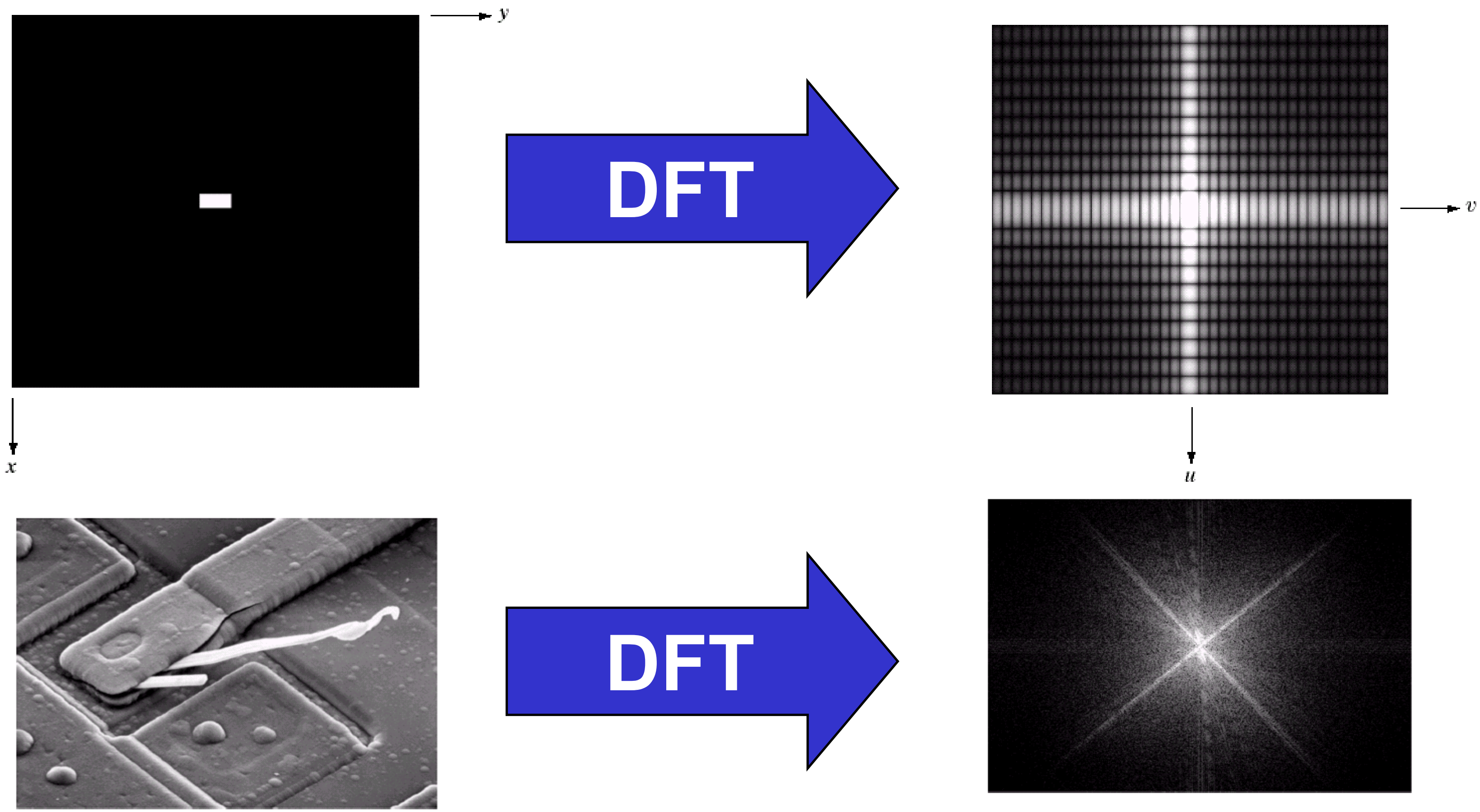
The *Discrete Fourier Transform* of $f(x, y)$,
for x = 0, 1, 2…M-1 and y = 0,1,2…N-1,
denoted by $F(u, v)$, is given by the equation:

$$F(u,v) = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)e^{-j2\pi(ux/M+vy/N)}$$

for u = 0, 1, 2…M-1 and v = 0, 1, 2…N-1.

DFT of an image

- The DFT of a 2D image can be visualised by showing the spectrum of the images component frequencies

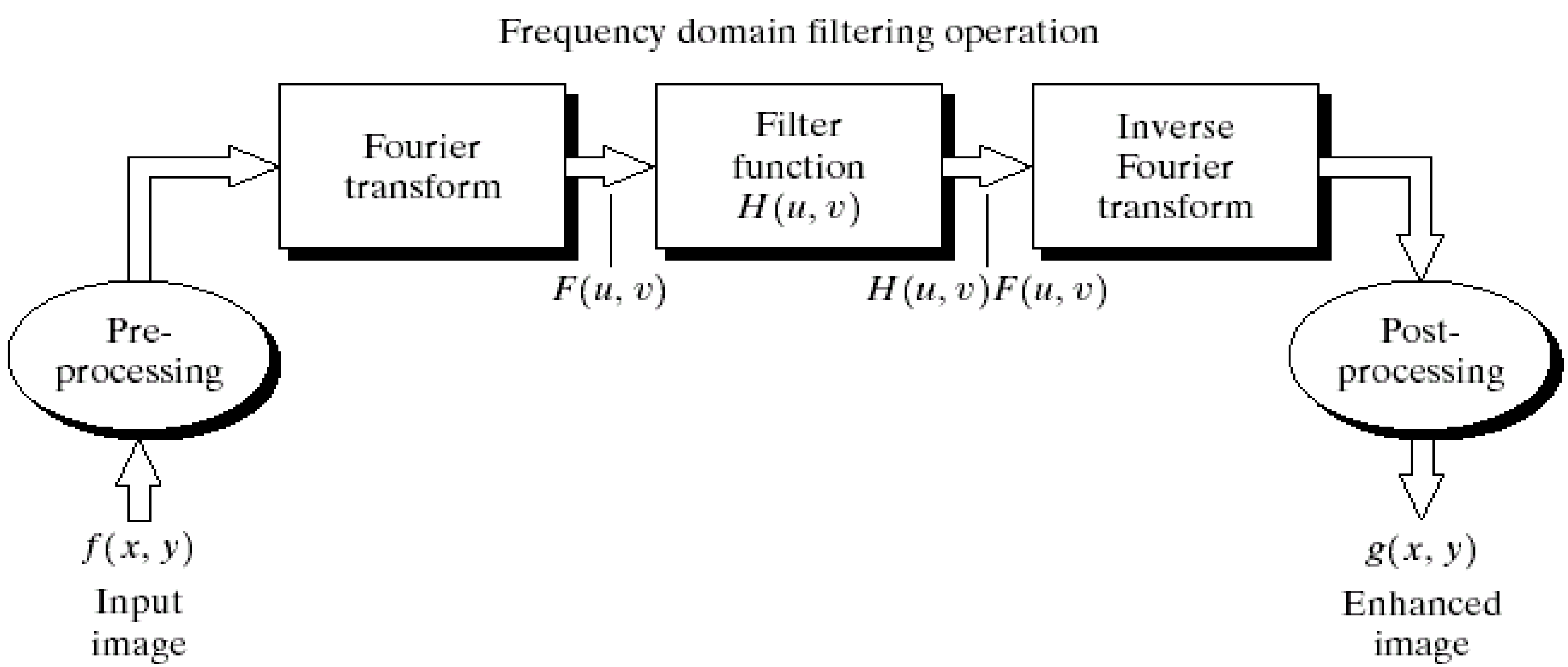


Fourier spectrum of the image

Frequency domain filtering

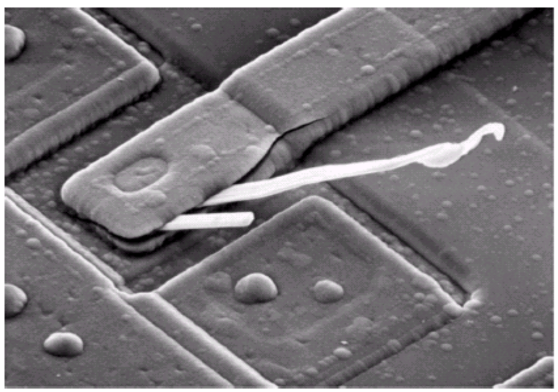
To filter an image in the frequency domain:

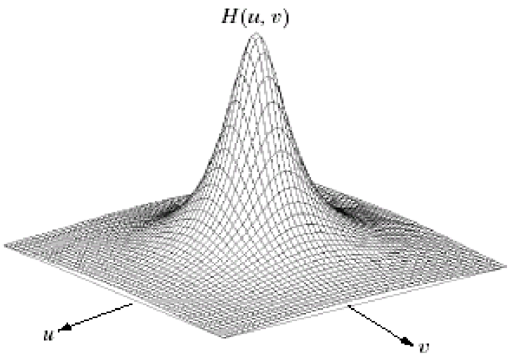
1. Compute $F(u,v)$ the DFT of the image
2. Multiply $F(u,v)$ by a filter function $H(u,v)$
3. Compute the inverse DFT of the result

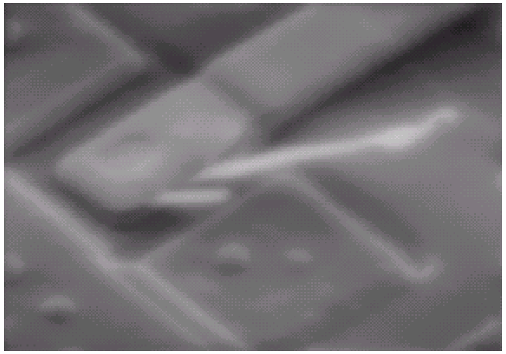


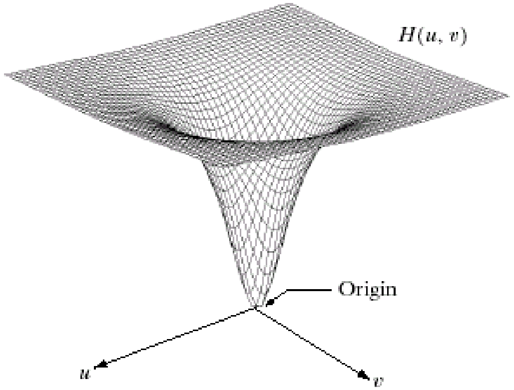
Frequency domain filtering

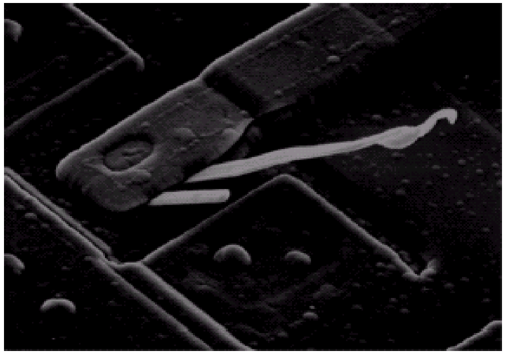
Low Pass Filter





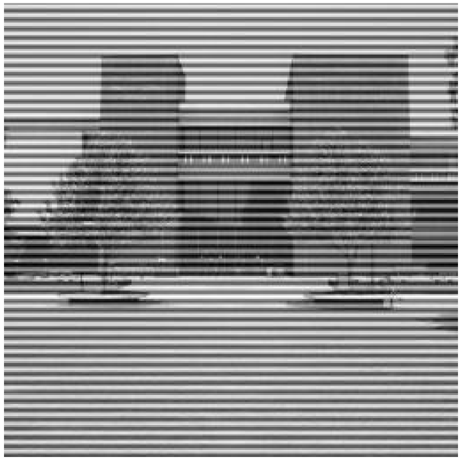


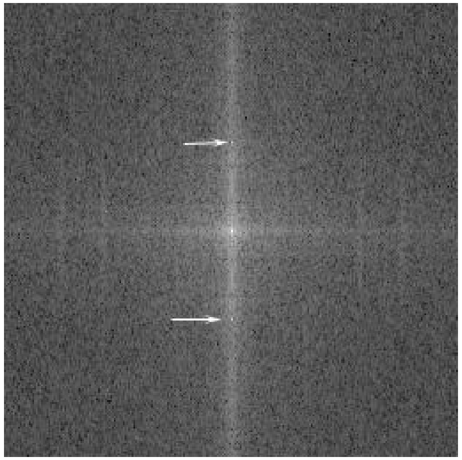




High Pass Filter

Frequency domain filtering





2D Fourier transform

After filtering