MATHEMATICS PROJECT REPORT

Project on Mathematics-III

Paper Code:- M351

A project report about the virtual application of Fourier Series

Semester:- *3 rd semester*

2019-2020

Group member's name:-

Name

Soujit Das

Titas Saha

Subhankar Pal

Sweety Mandal

Tapabrata Das

Sourav Mandal

-:CONTENTS:-

-:ACKNOWLEDGEMENT:-

We would like to express our special thanks of gratitude to my teacher ***Dr. Debashis Majumder*** *, who gave us the golden opportunity to do this wonderful project of virtual application of 'Fourier Series', who also helped us in completing our project. we came to know about so many new things, we are really thankful to them. Secondly I would also like thankful to my respected principal* ***Dr. Malay R. Dave*** *who also helped us lot in finalizing this project within the limited time frame.*

DATE:
20/11/2019

Titas Saha
Soujit Das
Sweety mandal
Tapabrata Das
Subhankar pal
Sourav mandal
(2nd year ECE department,ECE2A)

-:INTRODUCTION:-

Jean-Baptiste Joseph Fourier (1768-1830)

Fourier was a French mathematician, who was taught by Lagrange and Laplace.

Fourier was a buddy of Napoleon and worked as scientific adviser for Napoleon's army.

He worked on theories of heat and expansions of functions as trigonometric series... but these were controversial at the time. Like many scientists, he had to battle to get his ideas accepted.

Fourier's idea was to model a complicated heat source as a linear combination of simple sine and cosine waves, and to write the solution as a superposition of the corresponding eigensolutions. This superposition or linear combination is called the Fourier series.

-:**FOURIER SERIES**:-

A Fourier series is an expansion of a periodic function $f(x)$ in terms of an infinite sum of sines and cosines. Fourier series make use of the orthogonality relationships of the sine and cosine functions.

Let $f(x)$ be defined and integrable in the interval (-T,T).

Extend the Function to a periodic function of period 2T by defining $f(x+2T) = f(x)$ for all values of x of period 2T. The fourier Series of $f(x)$ is given by

$$\frac{a_\circ}{2}+\sum_{n=1}^{\infty}\left(a_n \operatorname{Cos}\frac{\boldsymbol{n\pi x}}{\boldsymbol{T}} + b_n \operatorname{Sin}\frac{\boldsymbol{n\pi x}}{\boldsymbol{T}}\right)$$

Where a_n , b_n are called Fourier Co-efficients and these are according to Euler,

$$a_0 = \frac{1}{T}\int_{-T}^{T} f(x)dx$$

$$a_n = \frac{1}{T}\int_{-T}^{T} f(x)cos\frac{\pi nx}{T}dx$$

$$b_n = \frac{1}{T}\int_{-T}^{T} f(x)sin\frac{\pi nx}{T}dx$$

where n=1,2,3,….s

-:FOURIER SERIES FACTS:-

- *Fourier discovered that a periodic signal could be expressed mathematically as a sum of sines and cosines. Each sine or cosine is multiplied by a coefficient, and then everything is added together.*

- *Fourier not only discovered that the signal could be expressed mathematically as a sum of sines and cosines.*

- *The fourier series expression can be used to represent any periodic signal.*

- *The most usable form of a fourier series is amplitude phase form.*

-:APPLICATION OF FOURIER SERIES:-

The Fourier transform has many applications, in fact any field of physical science that uses sinusoidal signals, such as engineering, physics, applied mathematics, and chemistry, will make use of Fourier series and Fourier transforms. It would be impossible to give examples of all the areas where the Fourier transform is involved, but here are some examples from physics, engineering, and signal processing.

- ***Communications***
- ***Astronomy***
- ***Geology***
- ***Optics***

Communications

In communications theory the signal is usually a voltage, and Fourier theory is essential to understanding how a signal behaves when it passes through filters, amplifiers and communications channels. Even discrete digital communications which use 0's or 1's to send information still have frequency contents.

Astronomy

Some times it isn't possible to get all the information we need from a normal telescope and you need to use radio waves or radar instead of light. These radar signals are treated just like any other ordinary time varying voltage signal and can be processed digitally.

Geology

The major exploration tool used by the oil and gas industry for locating new hydrocarbon reserves is the SEISMIC REFLECTION METHOD. This technique enables sub-surface sedimentary rock layers to be mapped, from measurements of the amplitude and reflection times of events obtained by the reflection of seismic (sound) waves from the layers.

Optics

In electromagnetic theory, the intensity of light is proportional to the square of the oscillating electric field which exists at any point in space. The Fourier transform of this signal is the equivalent of breaking the light into it's component parts of the spectrum, a mathematical spectrometer.

-:INTRODUCTION OF A VIRTUAL ASTRONOMICALAPPLICATION OF FOURIER SERIES:-

The most common virtual application of Fourier series is to calculate the elliptical orbit of any planet. In Ancient Greek astronomy this theory is very much valuable because first time ever this theory reveals that those orbits are ellipse and earth is not the center of solar system.

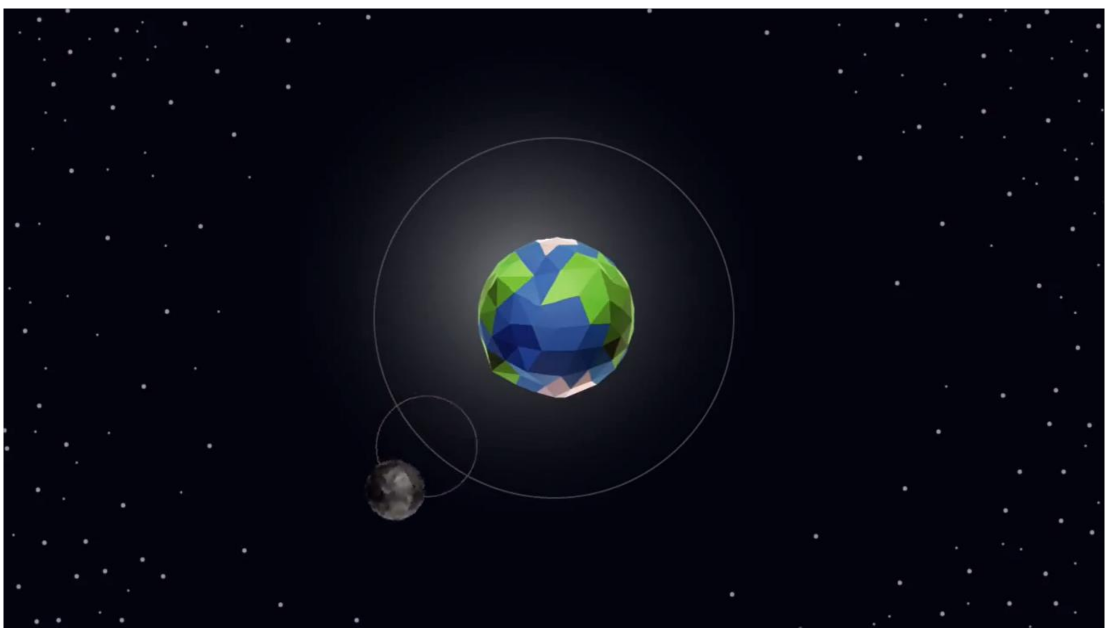
Ancient Greek astronomers theorized that the earth lay at the centre of the universe and moon,sun and other planet's rotating around earth in perfect circles. But when they looked into the night sky and observed how the moon and planets were really moving they realized that the motion was inconsistent with their theory.

Taking the moon for example. If the moon was moving in a circle like according to the Ancient Greeks theory it should always be the same distance away from earth. So why did it appear darker on some nights and brighter on other nights? It was also bigger on some nights and smaller on other nights.

-:CALCULATE A SIMPLE ELLIPTICAL ORBIT OF A PLANET USING FOURIER SERIES METHOD:-

The Greek Astronomers had to change their hypothesis for those faults which occurs between theoretical calculations and practical observations.

They thought what if for each heavenly body there was a smaller circle rotating around a bigger circle and the body was rotating on that smaller circle.



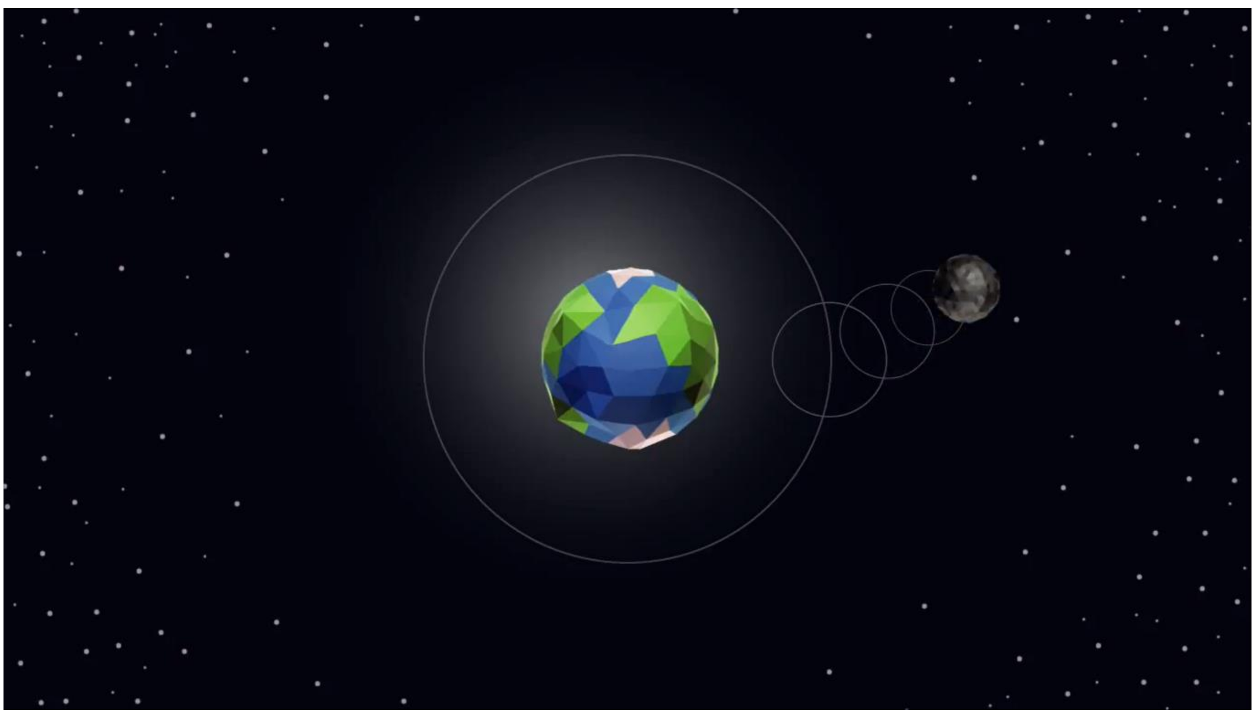
i.e. According to the ancient Greek's astronomy the orbit of moon and earth is circle.

so in this picture we draw those orbit as a circular shape.

In that theory they observed that some faults or problems can be solved.

But again when Astronomers saw in the night sky, they failed to create errorless balance condition between theory and practical observation.

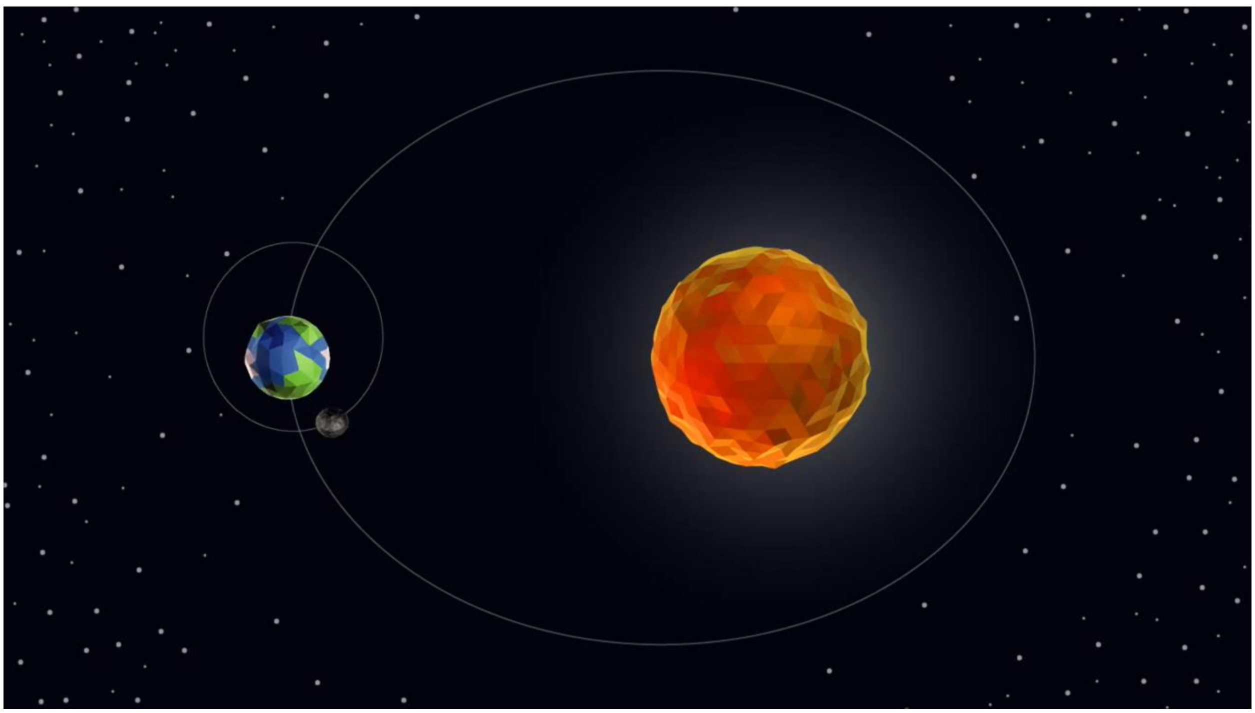
So the Astronomers started adding circles on circles on circles until they had a model of celestial motion that matched what they saw on their observations. It's known today that orbits are ellipses not circles on circles.



i.e. According to the ancient Greek's astronomy the orbit of moon and earth is circle.

So in this picture we draw those orbit as a circular shape.

Greek Astronomers corrected their theory and match what they observed in the sky using Fourier series adding up the many circles the parts allowed.



Elliptical orbit of planets constructed by Fourier series

In mathematics orbiting around in a circle can be represented in a manner of equation which we can transform to Fourier series.

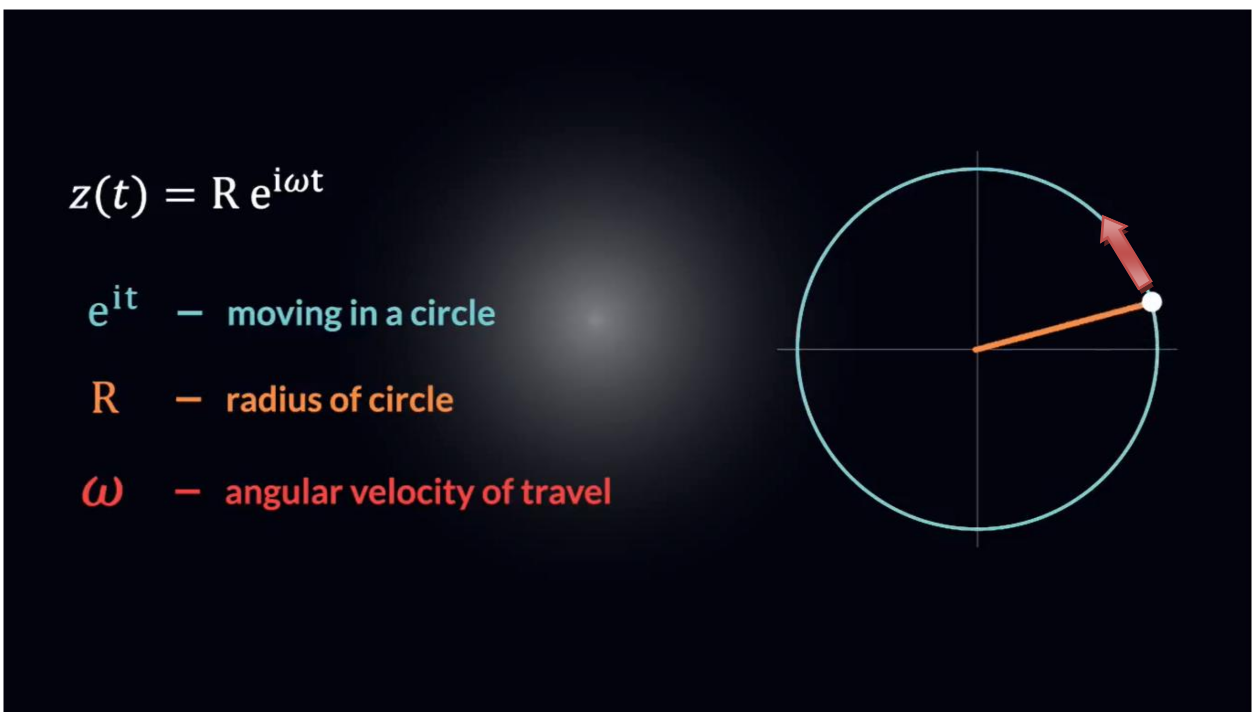
The orbiting around in a circle representing the orbit **Z(t)** is equal to **R** times $e^{i\omega t}$ **.**

$$Z(t) = Re^{i\omega t}$$

Where **R** is radius of a circle or radius of orbit and controls how big the circle or orbit is.

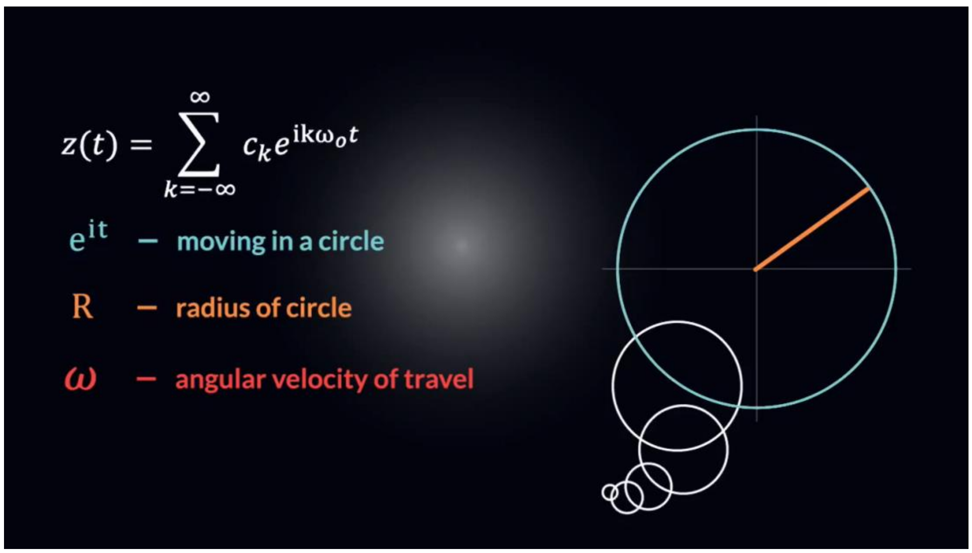
e^{it} represents moving in a circle or orbit.

ω is angular velocity of travel or velocity of planets. It means controls how fast the circle rotates around.



When Astronomers adding more and more circles rotating on circles and simply adding of those terms up to reaching an infinite number of circles they got a mathematical series equation. That is known as the Fourier series of the orbit.

$$z(t) = R_1 e^{i\omega_1 t} + R_2 e^{i\omega_2 t} + R_3 e^{i\omega_3 t}$$



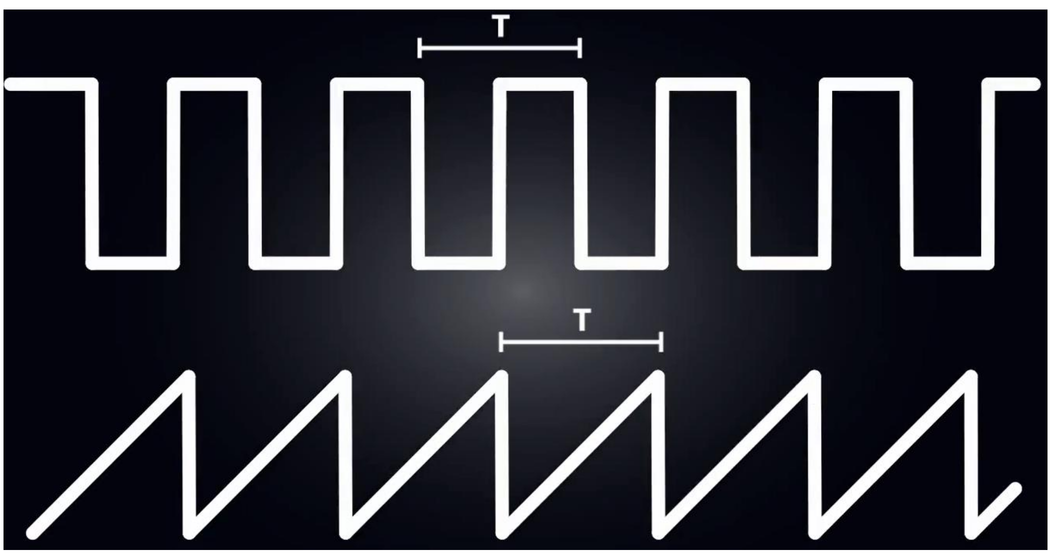
Ultimately the Greek Astronomers were not right about how orbits worked. So they ended up doing was constructing a Fourier series approximation of the real elliptical orbits.

It is sure that Fourier analysis can create an ellipse approximation but as per the characteristic of Fourier series, it can create any shape. So, the real problem is that to construct correct arrangement of circles that can make an orbit in the shape of Homer Simpson space.

Fourier Series as a periodic wave Equation

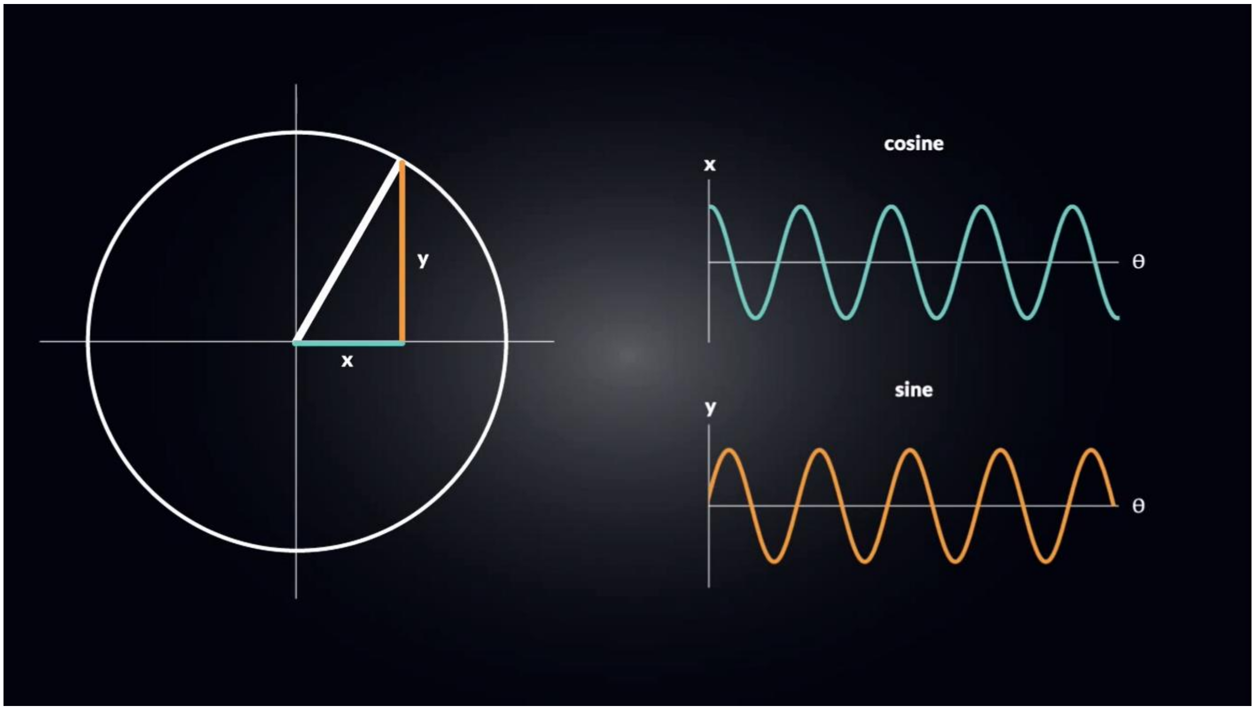
For solving the shape problem of orbit Astronomers noticed periodicity nature of a orbit equation and Fourier series. They noticed that in an orbit planets or objects repeats itself over time, so that every day sun rises and sunset happened, year after year position of any star or moon or planet remain same depending on specific time and date.

Astronomers knows that Fourier series can take anything periodic like a waveform and they can analyze it as sums of circles.



This picture shows us that two waves present and they are time dependent. If we look very carefully, then we can observed that after some specific time gaps those pics repeats itself with same amplitude but different time. That means periodicity occurs.

In mathematics from the unit circle it is also known that circles are basically sines and cosines, and sines – cosines waves are periodic in nature.



So Astronomers takes these two ideas together and puts Fourier series in it's more familiar form as sums of sines and cosines representing waveforms.

They calculate for every single point of the circular orbit up to the correct balanced postion of planet and then they join all those points. After all the calculations Astronomers finally join all those points and they predict a suitable elliptical orbit.

-:CONCLUSION:-

We concluded from the project was that theory and practical life is something different. We need to apply that theory in our practical life for better and effective results. If we abale to balanced between them, then also our job will be successful. We also learn that Fourier series is a very imporatant part of our virtual life. Fourier series can explain all those unsolved phenomena easily using simple series equation acting like a periodic wave. We also noticed that how circles join with sine and cosine functions in mathematical terms.