COMPUTER VISION ASSIGNMENT

Titash Mandal (tm2761@nyu.edu, N13592626)

1.THEORETICAL PROBLEMS:

a. "A straight line in the world space is projected onto a straight line at the image plane". Prove by geometric consideration (qualitative explanation via reasoning). Assume perspective projection.

Answer:
Assumption: Let us consider a fronto-parallel plane (π_0) in the world space. There is a line on the plane $\pi 0$ passing through points P and Q as shown in figure. The 3-d points on the world plane are projected on to the 2d image plane (π') as P′ and Q′. They undergo Perspective Projection hence the image is inverted and we need to prove that they are also forming a straight line.

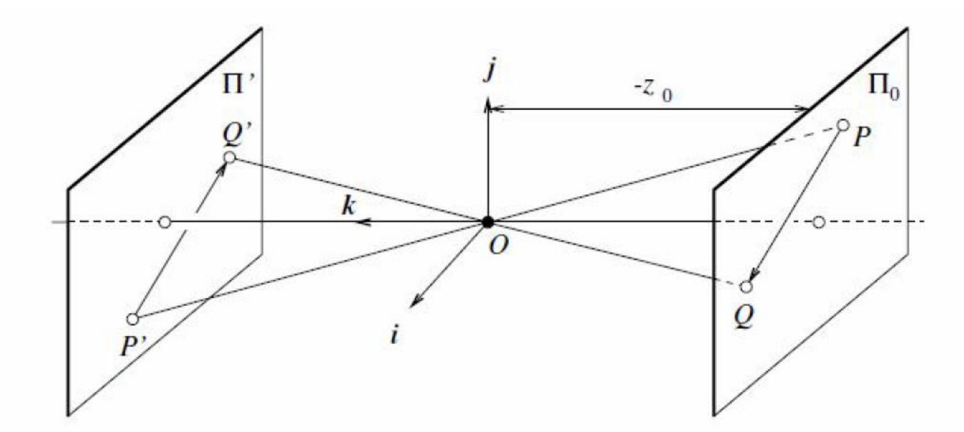


Figure 1.1: Weak perspective projection: A straight line in the world space is projected onto a straight line at the image plane

Proof: Let us consider the triangle P'OQ' and POQ. P', O, P are co-linear points as the pin hole camera keeps the object point and image point in a straight line. We know that P (X,Y,Z) in world space is P′ (x, y) in image space with relation

$$x = -mX$$
$$y_0 = -mY$$
$$\text{where } m = -f'/z$$

Using this we can say that |OP'|=m|OP| and |OQ'|=m|OQ| and |P'Q'|=m|PQ|. Hence the triangles POQ and P'OQ' are similar hence, P' and Q' also lies on a straight line.

Also, we know that light rays emitted from a straight-line pass through a pinhole. Consequently, they all lie on a plane which is spanned by the straight line and the pinhole. This plane intersects the image plane in a straight line.

b. Show that, in the pinhole camera model, three collinear points in 3-D space are imaged into three collinear points on the image plane (show via a formal solution).

Answer: Let us consider 3 co-linear points in the 3-d world space all lying on a plane with coordinates P(x1, y1,z1), Q (x2, y2 ,z2) and R somewhere on the line joining P and Q, So R = (1- t)P+ tQ for any tϵ[0,1]. Their corresponding image coordinates are P(x1', y1'), Q(x2', y2') and R.
From the equations of perspective projection, we have,

$$P' = \frac{-f'}{z_1}P$$

$$Q' = \frac{-f'}{z_2}Q$$

$$R' = \frac{-f'}{z_3}R$$

We can write z3 = (1 - t) z1 + tz2 ,
Substituting the values of z3 and R in the last equation we get,

$$R' = \frac{-f'}{(1-t)z_1 + tz_2}(1-t)P + tQ$$

$$R' = \frac{(1-t)z_1}{(1-t)z_1 + tz_2}\frac{-f'}{z_1}P + \frac{tz_2}{(1-t)z_1 + tz_2}\frac{-f'}{z_2}Q$$

$$R' = \frac{(1-t)z_1}{(1-t)z_1 + tz_2}P' + \frac{tz_2}{(1-t)z_1 + tz_2}Q'$$

Substituting $s = \frac{tz_2}{(1-t)z_1+tz_2}$

$$R' = (1-s)P' + sQ'$$

Thus, we see that 3 co-linear points in the 3-d world plane project to three co-linear points in the 2-d plane.

Also, we also know that 3 collinear points in world coordinates X1, X2, X3 form a matrix whose determinant is 0. We can apply the perspective projection to each point to get the three points x1, x2, x3 and their determinant in image space:

$$\text{World coordinates:} \quad Det\begin{vmatrix} X_1 & Y_1 & Z_1 \\ X_2 & Y_2 & Z_2 \\ X_3 & Y_3 & Z_3 \end{vmatrix} = 0\text{, via collinearity.}$$

$$\text{Image coordinates:} \quad Det\begin{vmatrix} \frac{X_1}{Z_1} & \frac{Y_1}{Z_1} & 1 \\ \frac{X_2}{Z_2} & \frac{Y_2}{Z_2} & 1 \\ \frac{X_3}{Z_3} & \frac{Y_3}{Z_3} & 1 \end{vmatrix} = \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0\,.$$

For points on a line: $\overline{x}_3 = (k)\overline{x}_2 + (1-k)\overline{x}_1$ are the same for all components. Substituting this into the equation above shows that the Det is zero and that the projected points are collinear.

PROBLEM 2: PERSPECTIVE PROJECTION

a) Prove geometrically that the projections of two parallel lines lying in some plane Q appear to converge on a horizon line H formed by the intersection of the image plane with the plane parallel to Q and passing through the pinhole.

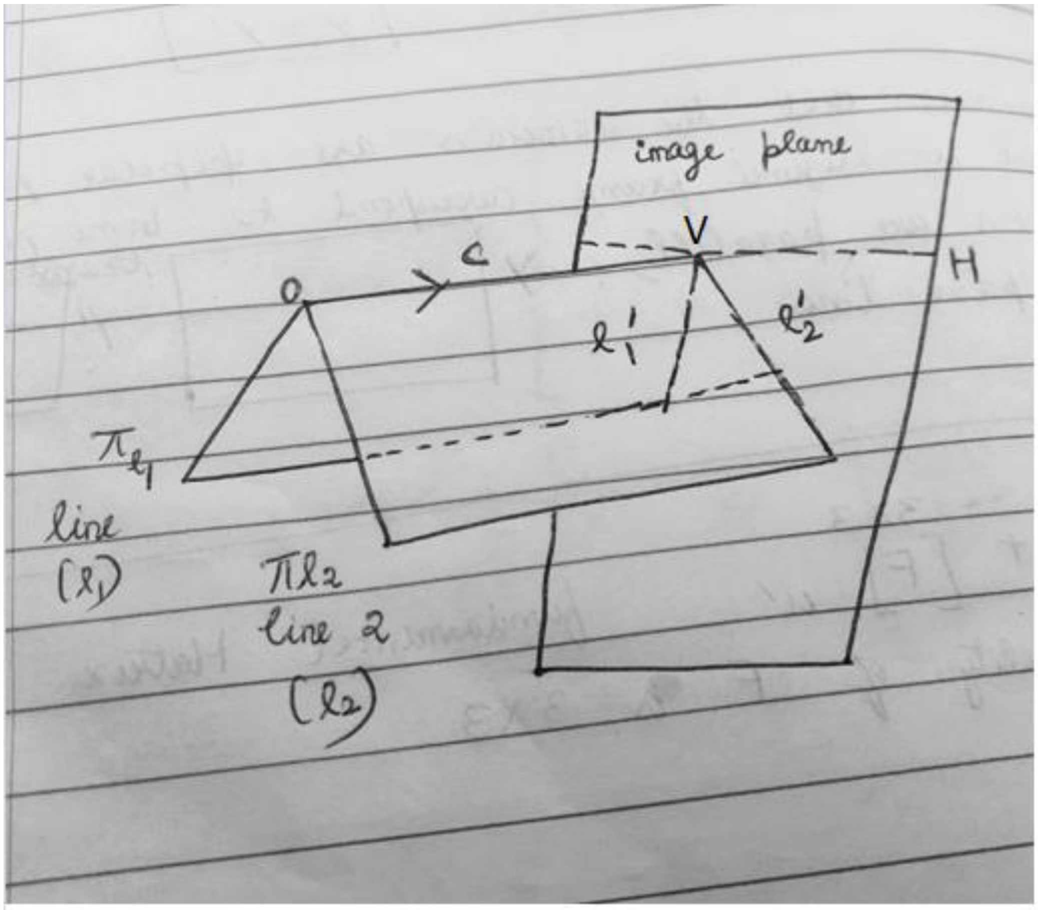


Figure 1:

Answer: Let us consider Figure 1 having two parallel lines l1 and l2. Let the plane πl1 pass through O and the line l1 and a second plane πl2 pass through O and line l2. We can show that the line of intersection of the two planes has the same direction d as lines l1 and l2 and that the line OV coincides with the line of intersection of planes l1 and l2.

As, we see the line of intersection of πl1 and πl2 has a direction d as the basis vector. The plane πl1 intersects image plane along line l1'. Similarly, plane πl2 intersects the image plane along the line l2'. Also, we see that line l1' and l2' intersect on the image plane at the vanishing point V. This point is the vanishing point associated with the family of lines parallel to l1 and l2, and the projection of any line in the family appears to converge on it.

b) Prove the same result algebraically using the perspective projection equation. You can assume for simplicity that the plane Q is orthogonal to the image plane (as you might see in an image of railway tracks, e.g.).

Answer: The equations of perspective projection for relating world coordinate system P(X,Y,Z) to image coordinate p(x, y) are as follows:

x=f(X/Z)
y=f(Y/Z)

Since its given the image plane say I is orthogonal to the plane Q where the two parallel lines lie, we can consider the equation for plane to be y = c.
If we consider a line l in this plane, the equation of the line in this plane is given by.
l: ax + bz = d

The point P on this line projects onto the image point p defined by:

x=f(X/Z) =f(d-bz/az)
y=f(Y/Z) =f(c/z)

This is the parametric representation of the image of the line l with z as the parameter.
Now consider the situation as z->∞. The image points x and y are:
x=-f(b/a)
y=0
This is the point on the x axis of the image plane and is actually the vanishing point for all parallel lines with the slope -b/a in the plane Q. If we repeat this process for another line in the plane Q, and connect the two vanishing points with a line, we will have the horizon line H in the image. Thus, the projections of two parallel lines lying in some plane Q appear to converge on a horizon line H.

PROBLEM 3: COORDINATES OF OPTICAL CENTER

Let O denote the homogeneous coordinate vector of the optical center of a camera in some reference frame, and let M denote the corresponding perspective projection matrix. Show that MO = 0. a

Answer: As per the extrinsic parameters of a camera, we know the equation,

$$\boldsymbol{p} = \frac{1}{Z}\mathcal{M}\,{}^{C}\boldsymbol{P}$$

Where ${}^{C}\boldsymbol{P}$ denotes the vector of homogeneous coordinates of the point P expressed in (C). The change of coordinates between (C)camera and (W)World is a rigid transformation, and it can be written as,

$${}^{C}\boldsymbol{P} = \begin{pmatrix} \mathcal{R} & \boldsymbol{t} \\ 0^T & 1 \end{pmatrix} {}^{W}\boldsymbol{P},$$

where ${}^{W}\boldsymbol{P}$ is the vector of homogeneous coordinates of the point P in the coordinate frame (W). The final equation of the extrinsic parameters is ,

$$\mathcal{M} = \mathcal{K}(\mathcal{R}t)$$

$$\mathcal{M} = \mathcal{K}\begin{pmatrix} {}^{C}_{W}\mathcal{R} & {}^{C}\mathcal{O}_W \end{pmatrix}$$

The coordinate vector O of the cameras optical center in the world coordinate system

$$\mathcal{O} = \binom{^W\mathcal{O}_C}{1}$$

Calculating MO,

$$\mathcal{MO} = \mathcal{K}\left({}^C_W\mathcal{R} \quad {}^C\mathcal{O}_W \right) \binom{^W\mathcal{O}_C}{1}$$
$$= \mathcal{K}[{}^C_W\mathcal{R}^W\mathcal{O}_C +^C \mathcal{O}_W]$$

Now if we rotate the camera coordinate(C) to world coordinate (W) , ${}^C_W\mathcal{R}$ vanishes and we are left with,

$$\mathcal{MO} = \mathcal{K}[-^C\mathcal{O}_W +^C \mathcal{O}_W]$$
$$\mathcal{MO} = 0$$

HW 1: Assignment Report (Camera Calibration)

Titash Mandal (tm2761@nyu.edu, N13592626)

1 Scope of the Experiment

The problem is to calibrate a camera with a fixed focal length using two orthogonal checkerboard planes, and to find intrinsic and extrinsic parameters. We also must reconstruct the image coordinates p from the world coordinates P using the calibration matrix and compare the calculated pixel locations to the measured locations.

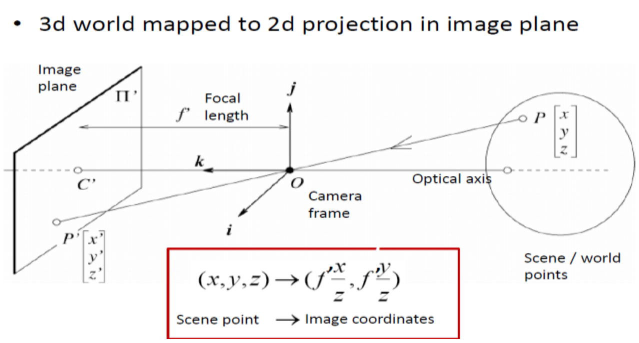


Figure 1: The diagram of perspective projection.

2 Experimental Procedure

2.1 Data Capture

Two checkerboard patterns are pasted on a wall corner at an angle of 90 degree to each other, as shown in the following figure.
The world frame axes are chosen as follows:

- The origin is at the lower end where two images meet on the corner.
- X axis is pointed along the right wall from the origin.
- Y axis is pointed upwards from the right wall and goes through the origin.
- Z axis points towards the left wall from the origin.
- The origin and the three axes are shown in the following figure.

Figure 2: Checkerboard pattern on the wall corner and the world frame coordinate axes

The points to be measured are shown with blue dots in the above image. Total 6(A,B,C,D,E,F) are chosen as shown in figure for the experiment, with 3 on one checkerboard and 3 on the other checkerboard. The real-world coordinates for these 6 points are measured using a ruler (in the reference of the world frames axes shown in Figure 1). All world frame coordinates are measured in millimeters.
The picture of the checkerboard pattern is captured using a camera with a focal length 4.15mm kept at 522mm from the origin of the world co-ordinate. The coordinates of these 6 points are measured in the world coordinate as follows

1) A-(X=121mm, Y=124mm, Z=0mm)
2) B-(X=65mm, Y=68mm, Z=0mm)
3) C- (X=121mm, Y=209mm, Z=0mm)
4) D-(X=0mm, Y=97mm, Z=66mm)
5) E- (X=0mm, Y=154mm, Z=121mm)
6) F- (X=0mm, Y=210mm, Z=150mm)

The image pixel resolution is 4023×3024.

This step gives us the coordinates of 3 pairs of points with each pair having one point in world frame and the corresponding point in image frame. We need to estimate 11 free parameters for camera calibration so this number of points is more than sufficient. We choose 6 points so that human measurement errors (in marking points in image and world frame) are averaged out and we get a robust estimate of the camera parameters.

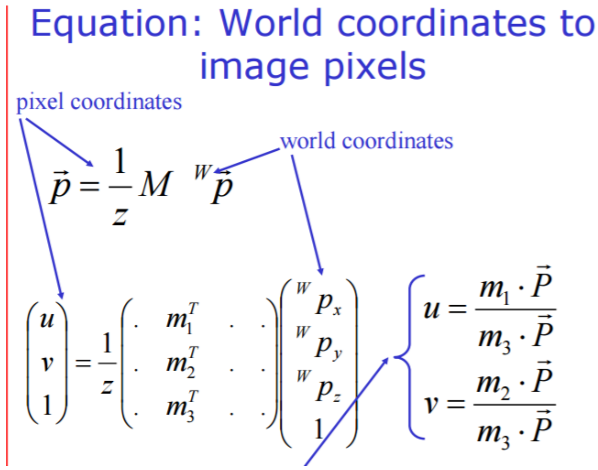
2.2 Calculating the pixel co-ordinates corresponding to the co-ordinates in the 3d World space.

After taking the photograph of the checkerboard and measuring the distances, using MATLAB function ginput() the pixel co-ordinates corresponding to the 3-D World co-ordinates were calculated. Thus, we have 6 sets of pixel co-ordinates also available to us. The pixel values calculated are as follows:

1) A[x,y] =2181.5, 1833.5
2) B[x,y]=1809.5, 2223.5
3) C[x,y]= 2187.5, 1179.5
4) D[x,y]=1173.5, 1179.5
5) E[x,y]= 891.5 , 1593.5
6) F[x,y]= 711.5, 1083.5

2.3 Calculation of the P Matrix

We know the world Co-ordinates and the pixel co-ordinates are related by the following equation.



Using the already estimated values of the pixel co-ordinates and the 3d world co-ordinates, the P matrix was calculated. This would help us in performing the SVD calculation.

$$\begin{pmatrix} P_{1x} & P_{1y} & P_{1z} & 1 & 0 & 0 & 0 & 0 & -u_1P_{1x} & -u_1P_{1y} & -u_1P_{1z} & -u_1 \\ 0 & 0 & 0 & 0 & P_{1x} & P_{1y} & P_{1z} & 1 & -v_1P_{1x} & -v_1P_{1y} & -v_1P_{1z} & -v_1 \\ & & & & & & \cdots & \cdots & \cdots & & & \\ P_{nx} & P_{ny} & P_{nz} & 1 & 0 & 0 & 0 & 0 & -u_nP_{nx} & -u_nP_{ny} & -u_nP_{nz} & -u_n \\ 0 & 0 & 0 & 0 & P_{nx} & P_{ny} & P_{nz} & 1 & -v_nP_{nx} & -v_nP_{ny} & -v_nP_{nz} & -v_n \end{pmatrix}$$

The P matrix with the values of the world co-ordinates and the pixel co-ordinates.

2.4 Estimation of the Calibration Matrix

Least squares method is used to estimate the calibration matrix. There are 120 homogeneous linear equations in twelve variables, which are the coefficients of the calibration matrix M. Lets denote this system of linear equations as

$$P\mathbf{m} = 0, \qquad \mathbf{m} := [\mathbf{m}_1\, \mathbf{m}_2\, \mathbf{m}_3]^T, \tag{1}$$

where, $\mathbf{m}_1$,$\mathbf{m}_2$,$\mathbf{m}_3$ are first, second and third rows of the matrix M respectively. **m** is a 12×1 vector, and P is a 120 × 12 matrix. The problem of least square estimation of P is defined as

$$\text{minkP}\mathbf{m}\text{k}^2, \qquad \text{subject to k}\mathbf{m}\text{k}^2 = 1. \tag{2}$$

As it turns out, the solution of above problem is given by the eigenvector of matrix P^TP having the least eigenvalue. The eigenvectors of matrix P^TP can also be computed by performing the singular value decomposition (SVD) of P. The following method was used to perform the SVD of P in MATLAB.

```
%Perform SVD of P
[U S V] = svd(P);
[min_val, min_index] = min(diag(S(1:12,1:12)));

%m is given by right singular vector of min. singular value
m = V(1:12, min_index);
```

The 12 right singular vectors of P are also the eigenvectors of P^TP. The SVD method is used here to get the eigenvector corresponding to the least eigenvalue. This eigenvector is the solution to the above problem.

2.5 Computation of Intrinsic and Extrinsic Parameters

The intrinsic and extrinsic parameters are computed from the calibration matrix M as follows. The matrix M is written as M =$(A \quad b)$, with a_1^T, a_2^T, and a_3^T denoting the rows of A.

ρ = ε/||a3||
r3 = ρa3
x0 = ρ2(a1 · a3)
y0 = ρ2(a2 · a3), where ε = ∓1.

Here ρ is an unknown scale factor, introduced here to account for the fact that the recovered matrix M has unit Frobenius form since ||M||F = ||m|| = 1. These methods use the various properties of the rotation vectors, namely the norm of them being one and the dot product of two rotation matrices being zero. Once the calibration matrix is estimated, the points calculated are then transformed to image frame using the calibration matrix obtained using least squares estimation. These coordinates are then compared with measured coordinates of these points in the image frame. We give the errors obtained in the following section.

3 Results

The **3*4 Perspective projection matrix M calculated** is :

$$\begin{pmatrix} 0.00131375411071148 & -8.58401814210905898 & -0.00192004255346801 & 0.47807685185540 \\ -0.00059810112521762 & -0.00225810239291039 & -0.000947561260722514 & 0.878311242471122 \\ -3.23387371816901e-07 & -4.97707496930475e-08 & -5.32868804208901e-07 & 0.0003325100636127 \end{pmatrix}$$

The estimated **calibration matrix K** is given as below:

$$\begin{pmatrix} 2007.40457341012 & 0.0003473864376 & 549.57291150785 \\ 0 & 2004.0457437868 & 345.19537742255 \\ 0 & 0 & 1 \end{pmatrix}$$

The intrinsic parameters are tabulated below.

Table 1: Intrinsic Parameters

Parameter	Value
θ	89.725
u_0	2073.45
v_0	1541.03
α	3389.15
β	3457.24

The extrinsic parameters consist of rotation matrix and the translation vector. The translation vector **t** is estimated as K^{-1}**b**, where K is the intrinsic parameter matrix and **b** is the last column of the calibration matrix M.

The **rotation matrix** in the form of [r1, r2,r3] is as follows:

$$\begin{pmatrix} 0.855228041647086 & 0.0335023489051963 & -0.517167854181139 \\ -0.009084135669263 & -0.996785937919216 & -0.0795944247146224 \\ -0.518172244779312 & 0.0727694069258763 & -0.852174945745312 \end{pmatrix}$$

The **translation matrix** is as follows:

$$\begin{pmatrix} -32.7342843282373 \\ 85.785754582578 \\ 531.757053858002 \end{pmatrix}$$

4 Discussion

4.1 Intrinsic Parameters

The intrinsic parameters relate the camera's co-ordinate system to the idealized co-ordinate system. In our experiment the value of θ found is almost equal to *90 degrees* (Table 1), which means that the camera coordinates are not skewed much and the X and Y axes in the image frame are almost at 90° to each other. As mentioned in the Sec. 2, the image resolution is 4023×3024. This means that the center of the image lies at (2011.5,1512). We obtained u_0 and v_0 equal to 2073.45 and 1541.03 respectively. The image center does not coincide with the principle point C_0 and is offset by (61.95,29.03), as estimated in the experiment. The α and β are magnifications equal to kf and lf respectively. The terms k and l denote the number of pixels per centimeter, and f is the distance of physical image frame from the pinhole or equivalent lens. The magnitude of α and β are reasonable from this point of view.

4.2 Extrinsic Parameters

The extrinsic parameters relate the camera's coordinate system to the fixed world coordinate and specify its position and orientation in space. Three rows of rotation matrix have norm equal to 1 and their dot product with each other is a very low number. This is justified because of possible errors in the measurement process which is purely manual. The translation vector also approximately matches the real values. We reconstructed the pixel coordinates and measured the difference between the calculated values and the values measured in the three coordinates and we find that values match approximately but there still is some error.

1) For point **A**: the error value calculated is 2.35.
2) For point **B**: the error value calculated is 2.66.
3) For point **C**: the error value calculated is 0.85.
4) For point **D**: the error value calculated is 0.53.
5) For point **E**: the error value calculated is 4.19.
6) For point **F**: the error value calculated is 1.79.

4.3 Would we want compare the pixel density estimates α and β in terms of sensor size, number of pixels per axis and focal length?

Yes, knowing the sensor size, the pixel density and the focal length has an impact on the photograph taken and used for the experiment. The camera used for the experiment has the following specification of 29mm, f2.2 lens, with a 12 megapixel, 1/3", 1.22μm sensor.

Pixel density is the number of pixels per mm of an electronic image device which helps determine the resolution of the image. The pixel density is a measure of how close the pixels are packed on the display and higher the pixel density, more details in the image are clearly visible. Hence it is an important gfactor to consider while taking care of image calibration.

The size of sensor that a camera has ultimately determines how much light it uses to create an image. Image sensors consist of millions of light-sensitive spots called photo sites which are used to record information about what is seen through the lens. Therefore, we can assume that that a bigger sensor can gain more information than a smaller one and produce better images.

MATLAB CODE

%read the image from the location of the computer

```
img1=imread('C:\Users\ Computer Vision\CV Assignment\original.jpg');
imshow(img1);
```

%estimation of the pixel co-ordinates from the selected points on the image

```
[xa,ya]=ginput();
[xb,yb]=ginput();
[xc,yc]=ginput();
[xd,yd]=ginput();
[xe,ye]=ginput();
[xf,yf]=ginput();
```

%estimation of the P matrix and storing it in a excel sheet and copying it to MATLAB by extracting it.

```
P=xlsread('C: \Desktop\CV Assignment\pmatrix.xlsx');
Format long g
```

%estimating camera calibration

```
[U,S,V]=svd(P);
[min_val,min_index]=min(diag(S(1:12,1:12)));
m=V(1:12,min_index);
```

% converting the matrix to a 3×4 matrix

```
M=[m(1:4)';m(5:8)';m(9:12)';]
tz=M(4,3);
r3t=[M(3,1:3)];
a1=[M(1,1:3)]';
a2=[M(2,1:3)]';
a3=[M(3,1:3)]';
```

% estimating the intrinsic parameters

```
roh=1/(norm(a3));
u_0=(roh*roh)*(dot(a1,a3));
v_0=(roh*roh)*(dot(a2,a3));
r3=roh*a3;
a1crossa3=cross(a1,a3);
a2crossa3=cross(a2,a3);
costheta=-(dot(a1crossa3,a2crossa3))/(norm(a1crossa3)*norm(a2crossa3));
theta=acosd(costheta);
alpha=(roh*roh)*(norm(a1crossa3))*sind(theta);
beta=(roh*roh)*(norm(a2crossa3))*sind(theta);
```

% estimating the extrinsic parameters

```
r1=a2crossa3/norm(a2crossa3);
r2=cross(r3,r1);
K=[alpha, -1*alpha*cot(theta),u_0;0,beta/sin(theta),v_0;0,0,1];
```

```
b=M(1:3,4);
t=roh*(inv(K)*b);
Intrinsic=[alpha,beta,theta,u_0,v_0];
Extrinsic=[r1,r2,r3,t];
my_X=[xa,xb,xc,xd,xe,xf];
my_Y=[ya,yb,yc,yc,ye,yf];
worldX=[121,65,121,0,0,0];
worldY=[124,68,209,97,154,210];
worldZ=[0,0,0,66,121,150];
%reconstruction of the image coordinates and estimation of error
for i=1:6
temporary(1:4)=[worldX(i) worldY(i) worldZ(i) 1];
recon=M*temporary';
reconsX(i)=recon(1)/recon(3);
reconsY(i)=recon(2)/recon(3);
error(i)=norm([reconsX(i)-my_X(i) reconsY(i)-my_Y(i)]);
end
```

BONUS QUESTION

1. Scope of the Experiment

Two pictures of the checkerboard calibration pattern were taken with slightly different camera locations. The calibration method was rerun for the second image to compare the intrinsic and the extrinsic parameters of this image with respect to the previous one taken.

2. Experimental Procedure

2.1 Data Capture

Two checkerboard patterns are pasted on a wall corner at an angle of 90 degree to each other, as shown in the following figure. In this experiment, two images are taken to understand a relationship between the extrinsic and the intrinsic parameters.
The world frame axes are chosen as follows:

- The origin is at the lower end where two images meet on the corner.
- X axis is pointed along the right wall from the origin.
- Y axis is pointed upwards from the right wall and goes through the origin.
- Z axis points towards the left wall from the origin.
- The origin and the three axes are shown in the following figure.

Figure1

Original Image

Figure 2

Shifted Image

The points to be measured are shown with blue dots in the above image. Total 6(A, B, C,D,E,F) are chosen as shown in figure for the experiment, with 3 on one checkerboard and 3 on the other checkerboard. The real-world coordinates for these 6 points are measured using a ruler (in the reference of the world frames axes shown in Figure. 2). All world frame coordinates are measured in millimeters.
The picture of the checkerboard pattern is captured using a camera with a focal length 4.15mm kept at 538mm from the origin of the world co-ordinate. This is the second photograph of the experimental setup that is taken by shifting the distance of the camera from the image.

The image pixel resolution is 4023×3024. This step gives us the coordinates of 3 pairs of points with each pair having one point in world frame and the corresponding point in image frame. We choose 6 points so that human measurement errors (in marking points in image and world frame) are averaged out and we get a robust estimate of the camera parameters.

3. Computation of Intrinsic and Extrinsic Parameters

After calculating the pixel co-ordinates corresponding to the points chosen, we perform a similar experiment to estimate the intrinsic and the extrinsic parameters of the image

4. Discussion

4.1 Intrinsic Parameters

We observe that the intrinsic parameters remain unchanged with very little difference between the newly estimated value calculated from the shifted image with respect to the image in the previous experiment. We know that the intrinsic parameters are the camera parameters that are internal and fixed to a definite camera/digitization setup. As we have not changed the camera or the focal length, neither have we changed the image and the world coordinates, hence the intrinsic parameters remain almost unchanged though there are slight differences. The newly calculated values of the intrinsic parameters are as follows:

The **perspective projection matrix M is:**

$$\begin{pmatrix} -0.000683050059701974 & -8.79489513548157 & 0.002317059923823 & -0.483628614756835 \\ 0.00103107183163076 & 0.002077162814258 & 0.0010191817091865 & -0.875266243660246 \\ 0.28109794032842e\text{-}07 & -2.50807501100118 & 5.283803138040064 & -0.000325870804637 \end{pmatrix}$$

Parameter	Value
θ	89.435
u_0	2069.45
v_0	1531.03
α	3371.36
β	3425.77

Table 3: Intrinsic Parameters

4.2 Extrinsic Parameters

The extrinsic parameters relate the camera's coordinate system to the fixed world coordinate and specify its position and orientation in space. The position of the camera in the world is recovered by a rotational component and a translation component. Because we change the angle in which we take the image, we get different values of the extrinsic parameters.

The new matrix containing the **extrinsic parameters of rotation matrix** is as follows:

$$\begin{pmatrix} 0.899672251396103 & 0.99972295139610 & 0.021706094056956 \\ -0.754760942991753 & -0.00836969579768 & -0.079594424714622 \\ 0.655958540243245 & 0.655936775155612 & -0.852174945745312 \end{pmatrix}$$

The **translation matrix** is as follows.

$$\begin{pmatrix} -32.7342843282373 \\ 150.102438280196 \\ 492.75090287981 \end{pmatrix}$$

Thus, there is a lot of difference between the previously calculated values and the newly estimated one which is due to the change in the direction of taking the image for the experiment.

CONCLUSION

Through this experiment we have addressed the problem of estimating the intrinsic and extrinsic parameters of a camera from the image positions of scene features such as points of lines, whose positions are known in a fixed world coordinate system was known. In this context, camera calibration can be modeled as an optimization process, where the discrepancy between the observed image features and their theoretical positions is determined with respect to the camera's intrinsic and extrinsic parameters.