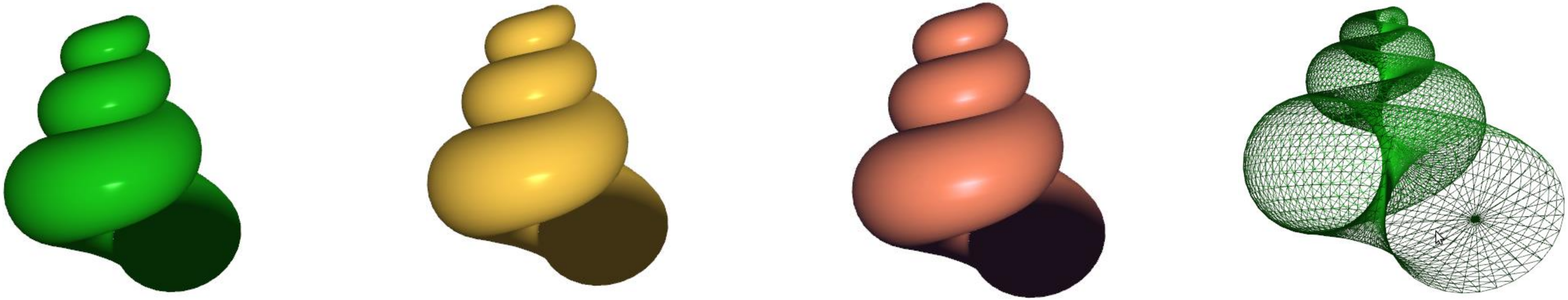
សាកលវិទ្យាល័យភូមិន្ទភ្នំពេញ

អប់រំ ស្រាវជ្រាវ និង សេវាសង្គម

មហាវិទ្យាល័យវិស្វកម្ម

Vector

Department of IT Engineering



Administrivia

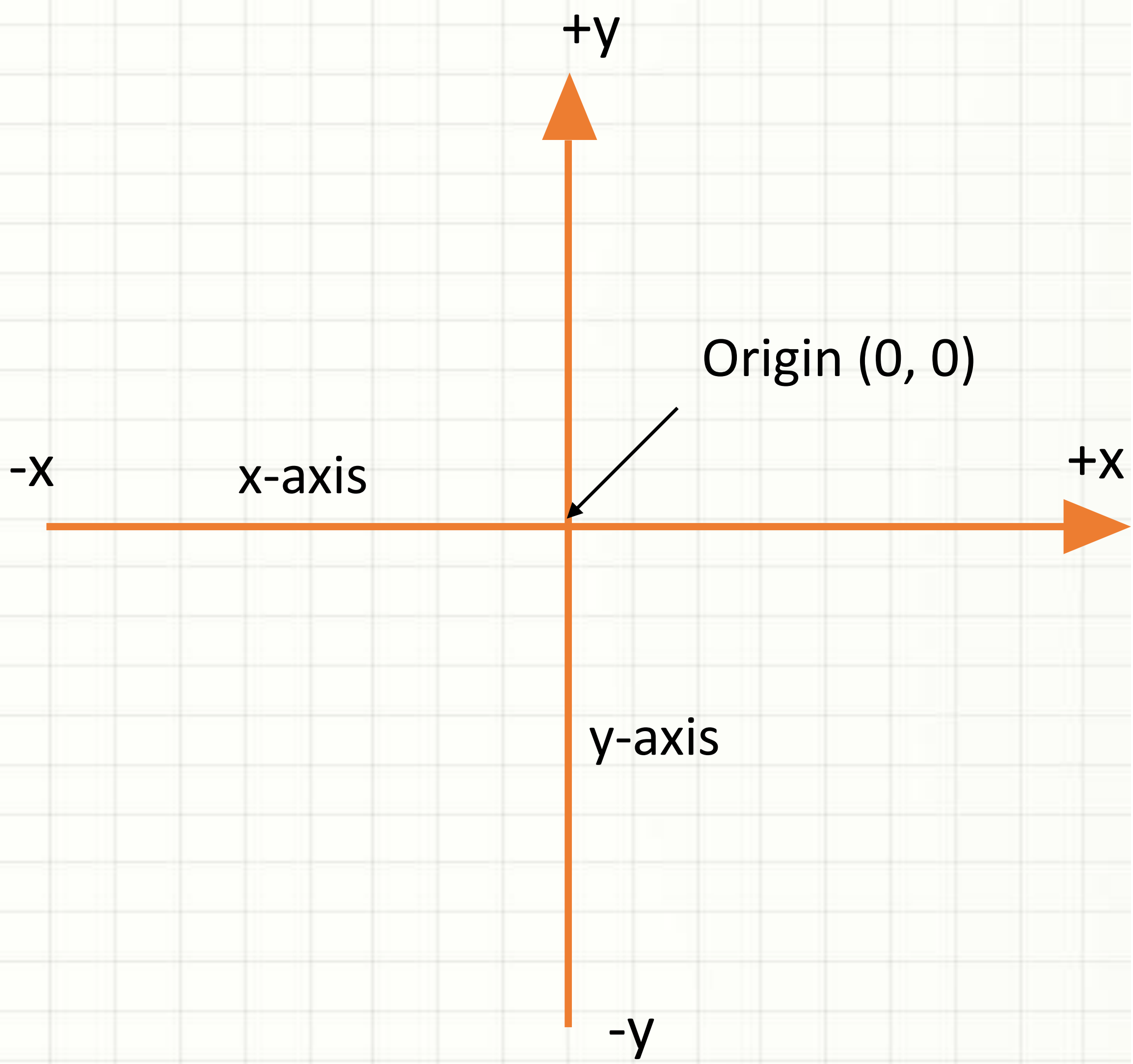
- Class
  - Theory: T002
    - Tuesday: 1:00pm – 2:30pm
  - Lab
    - Monday: 1:00pm – 2:30pm (G1)
    - Monday: 2:30pm – 4:00pm (G2)

- Exams
  - Final Exams: 60%
  - Assignment: 20%
  - Homework: 10%
  - Attendance: 10%

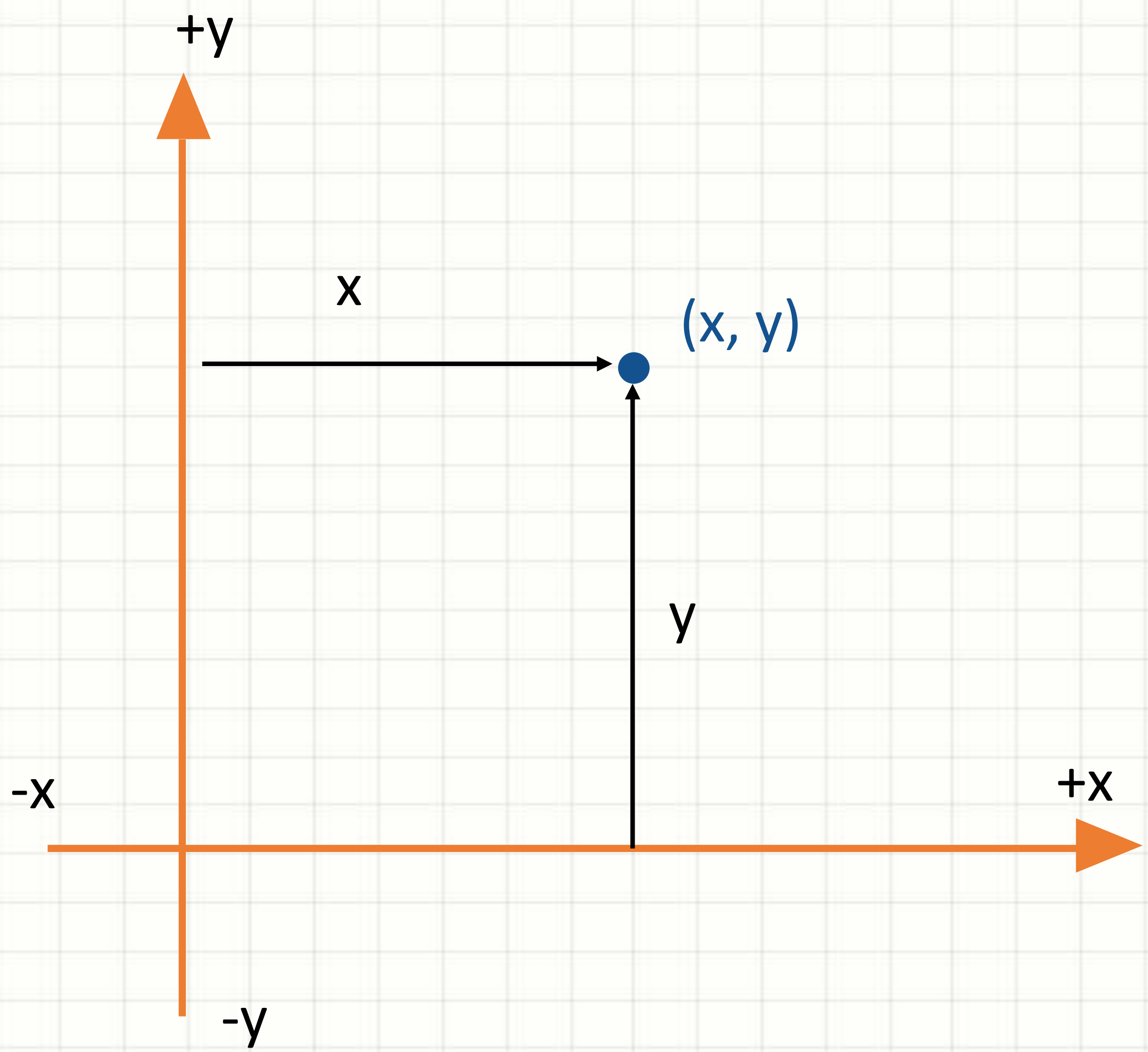
Contents

2D Coordinate Spaces

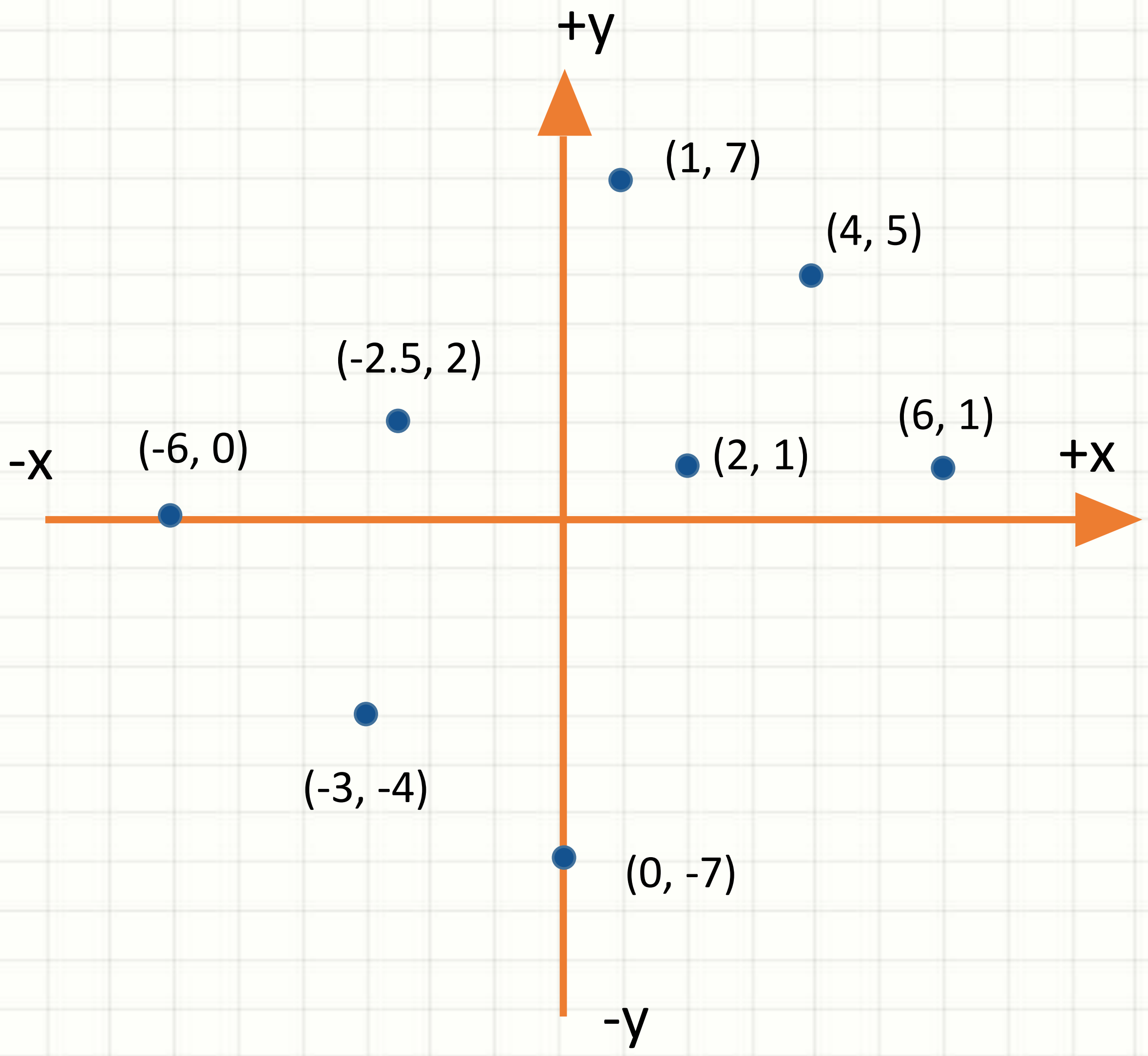
- Consist of a special location called the origin
- Exist two straight lines that pass through the origin
- Two axes are perpendicular to each other



Specify a point in 2D Cartesian

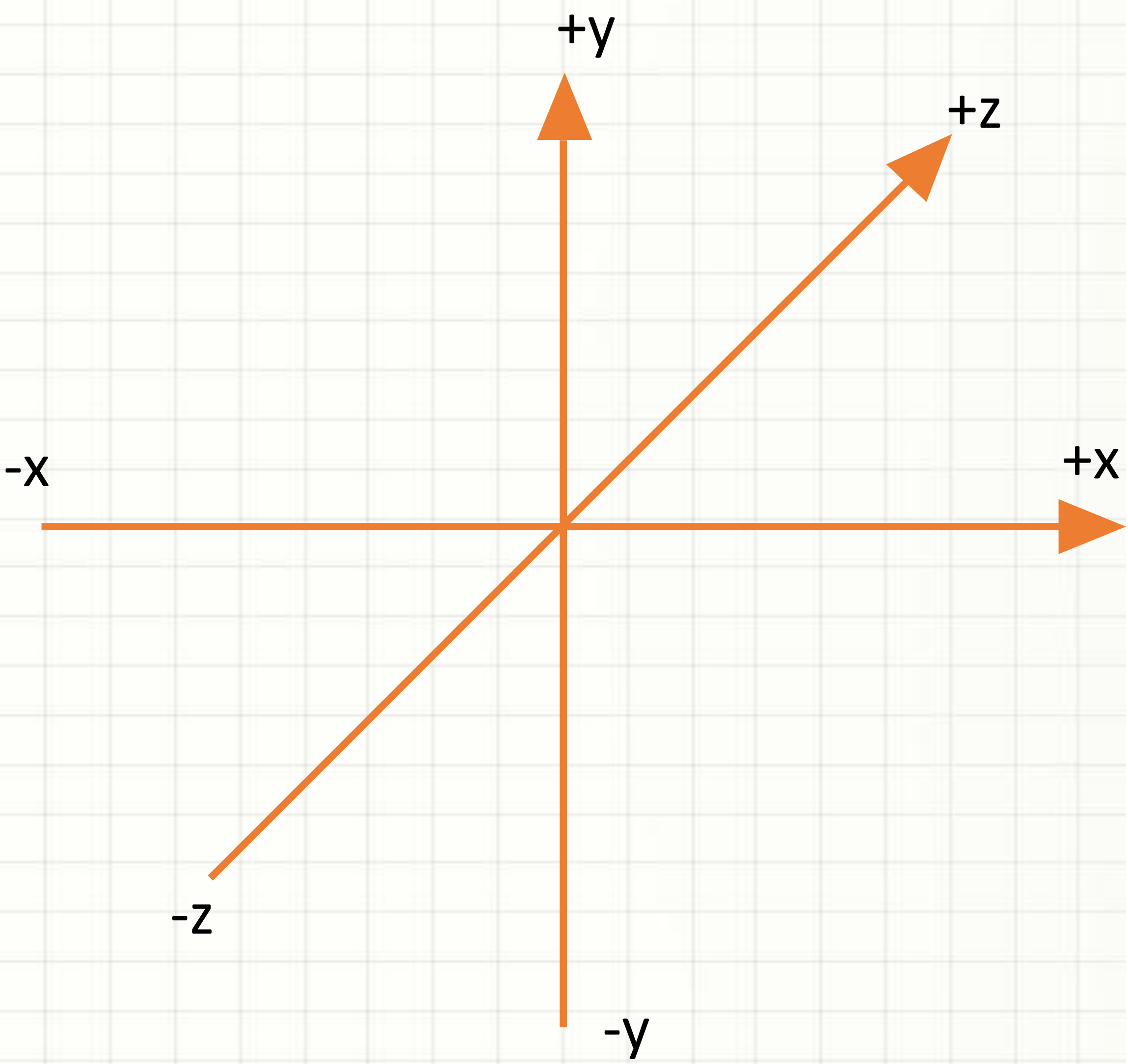


Example points labeled with 2D Cartesian Coordinates

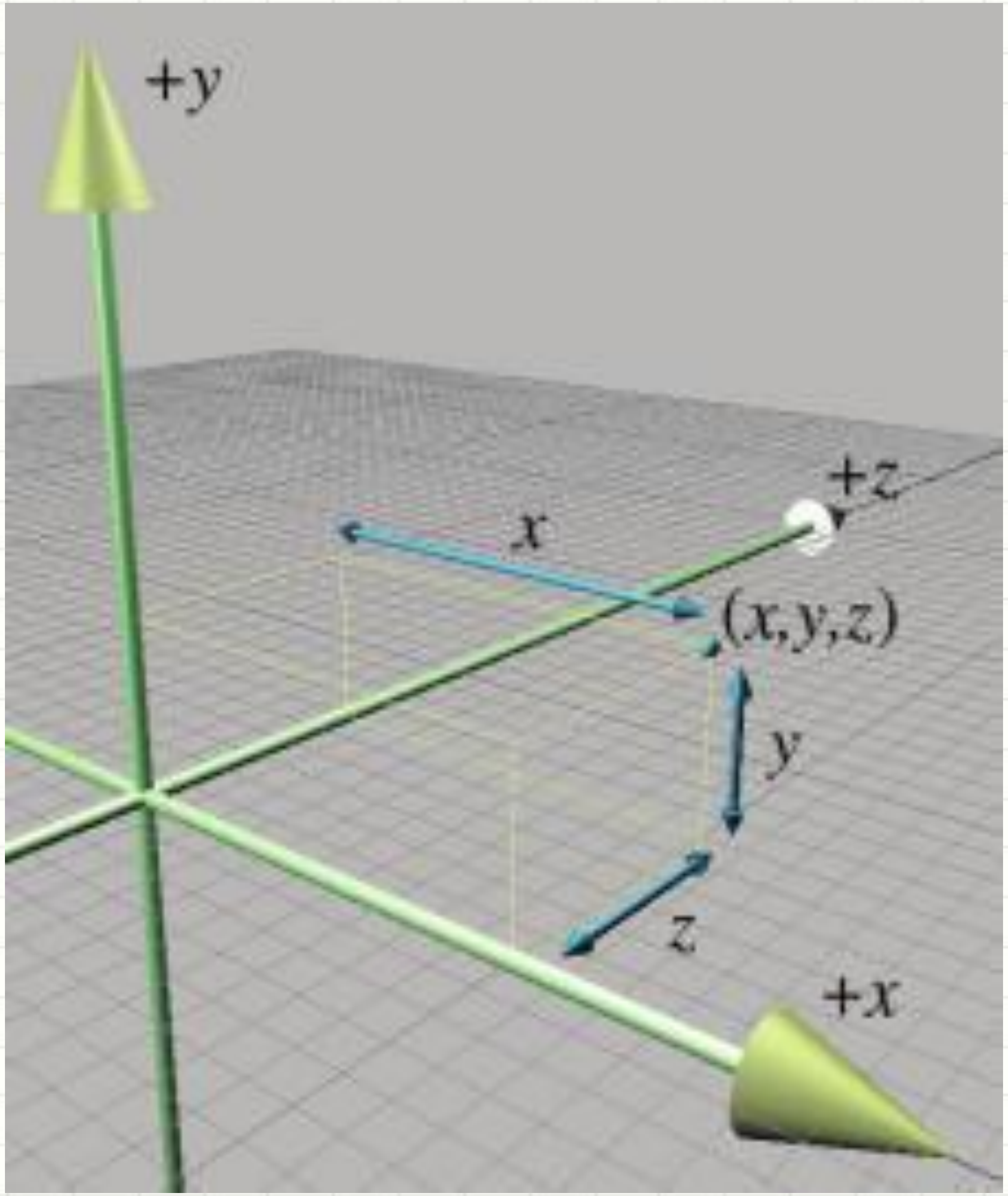


3D Cartesian Space

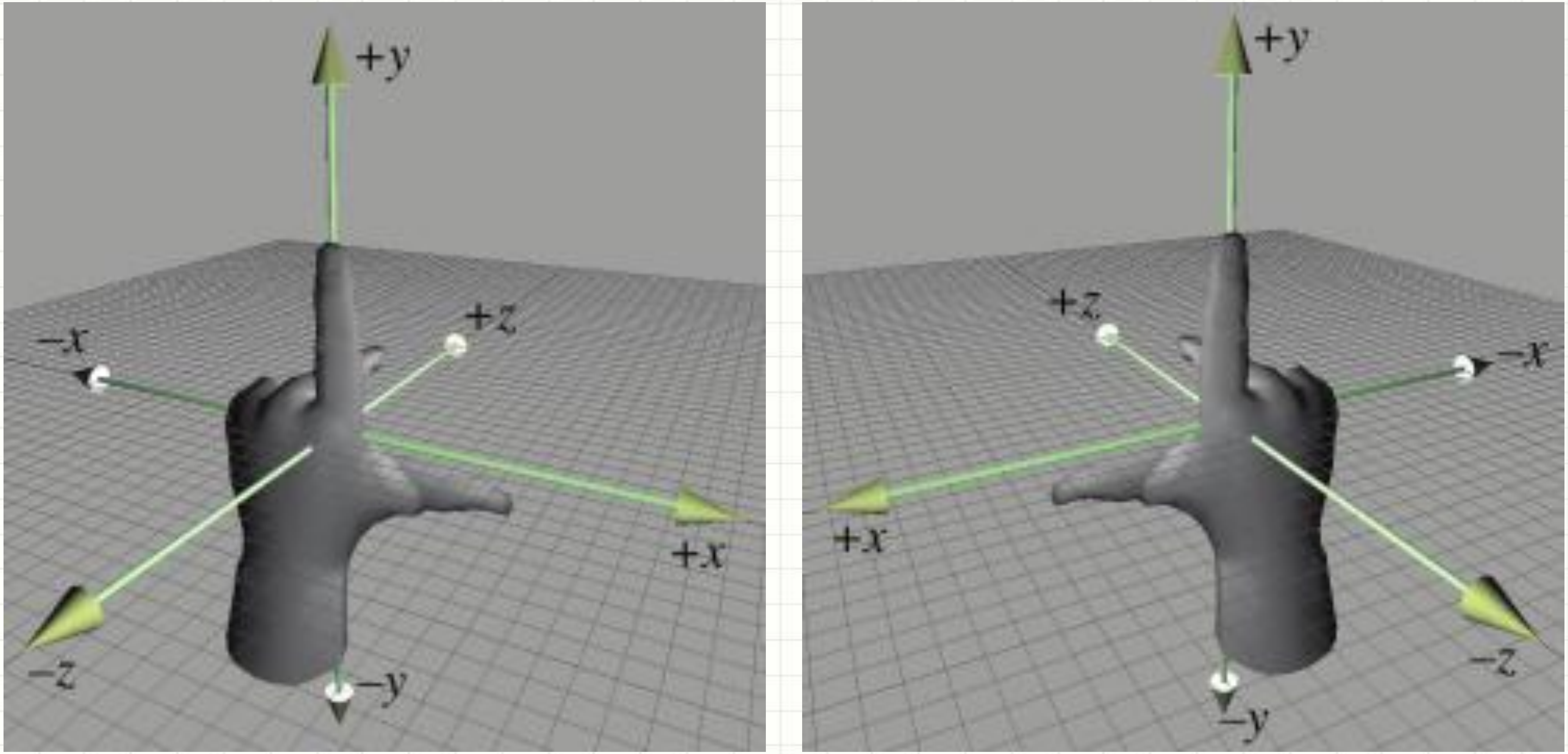
- Extend 2D into 3D by adding a 3rd axis
- **xy**, **xz**, and **yz** planes



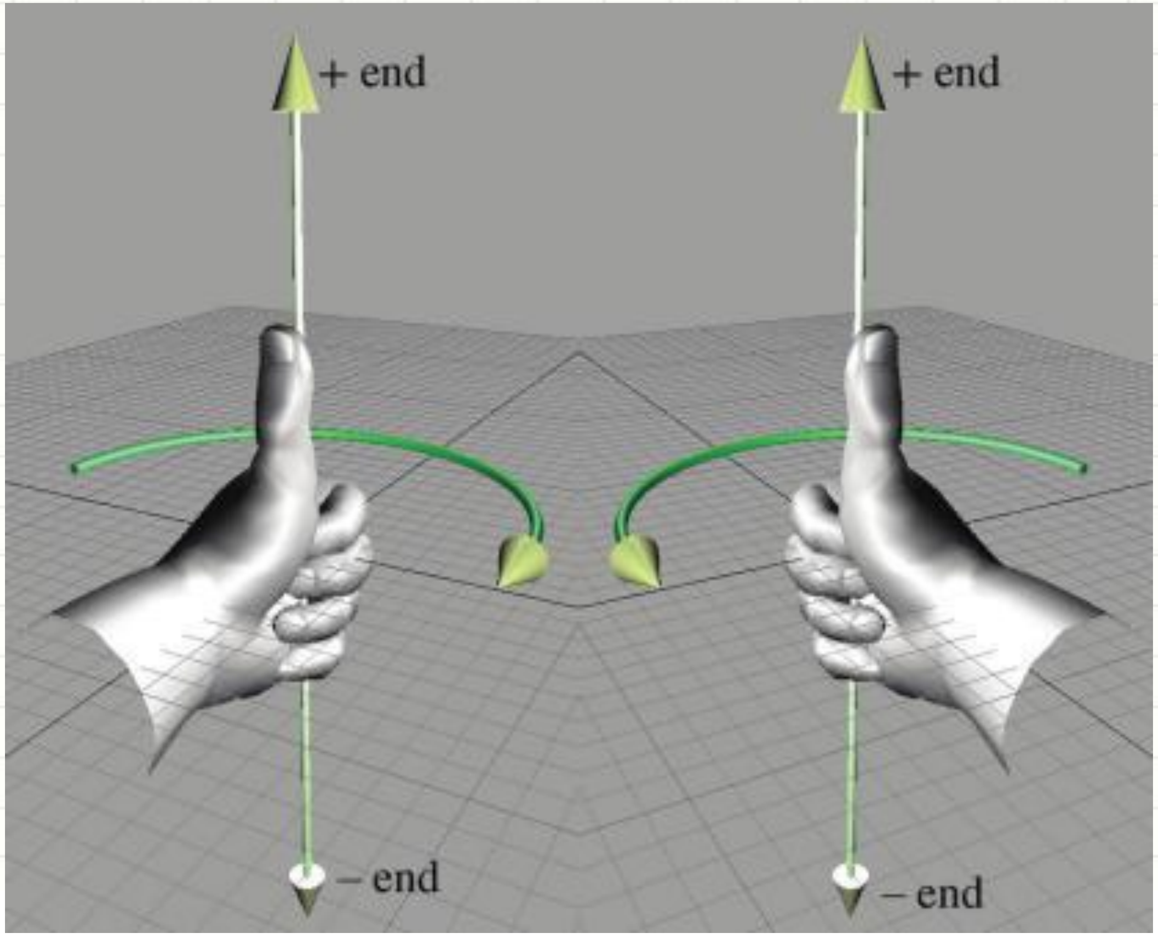
Specify a point in 3D Cartesian



Left-handed & Right-handed Rule



Positive Rotation for Left-Right Hand



When looking towards the origin from...	**Positive rotation** Left-handed: Clockwise Right-handed: Counterclockwise	**Negative rotation** Left-handed: Counterclockwise Right-handed: Clockwise
$+x$	$+y \rightarrow +z \rightarrow -y \rightarrow -z \rightarrow +y$	$+y \rightarrow -z \rightarrow -y \rightarrow +z \rightarrow +y$
$+y$	$+z \rightarrow +x \rightarrow -z \rightarrow -x \rightarrow +z$	$+z \rightarrow -x \rightarrow -z \rightarrow +x \rightarrow +z$
$+z$	$+x \rightarrow +y \rightarrow -x \rightarrow -y \rightarrow +x$	$+x \rightarrow -y \rightarrow -x \rightarrow +y \rightarrow +x$

Mathematical Definition of Vector

- Vectors are the formal mathematical entities we use to do 2D and 3D math
- Mathematically, a vector is nothing more than an array of numbers
- The dimension of a vector tells how many numbers the vector contains
- Vectors may be of any positive dimension (1D, 2D, 3D, 4D)

Vector Notations

- There are two ways to write vector:
- Column vector

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

- Row vector

$$\begin{bmatrix} x & y & z \end{bmatrix}$$

- What is the difference between them?

Vector Notations

- In math, integer indices are used to access the elements of vector

$$a = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$a_1 = a_x = 1$$
$$a_2 = a_y = 2$$

$$b = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$$

$$b_1 = b_x = 3$$
$$b_2 = b_y = 4$$
$$b_3 = b_z = 5$$

$$c = \begin{bmatrix} 6 \\ 7 \\ 8 \\ 9 \end{bmatrix}$$

$$c_1 = c_x = 6$$
$$c_2 = c_y = 7$$
$$c_3 = c_z = 8$$
$$c_4 = c_w = 9$$

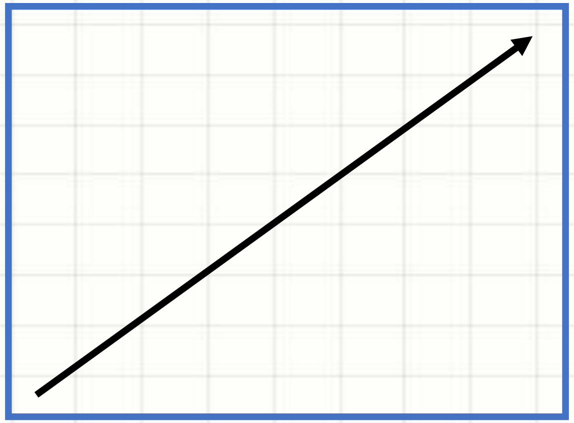
Geometric Definition of Vector

- The branch of mathematics that deals primarily with vectors and matrices is called *Linear Algebra*

- For 3D math, we are mostly concerned with the geometric interpretations of vectors and vector operations

- Geometrically speaking, a vector is a directed line segment that has **magnitude** and **direction**

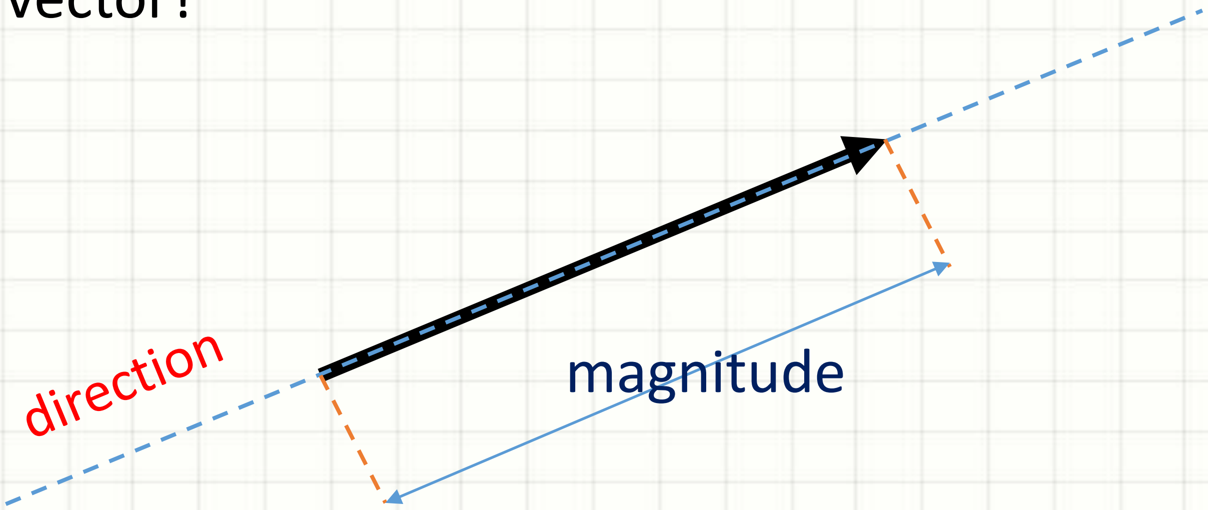
- What is **magnitude** and **direction**?



A 2D vector

Geometric Definition of Vector

- The **magnitude** of a vector is the length of the vector
- A vector may have any nonnegative length
- **Direction** of a vector describes which way the vector is pointing in space
- Where is this vector?



Geometric Definition of Vector

- Vectors do not have position, only ***magnitude*** and ***direction***
- Sound Impossible?

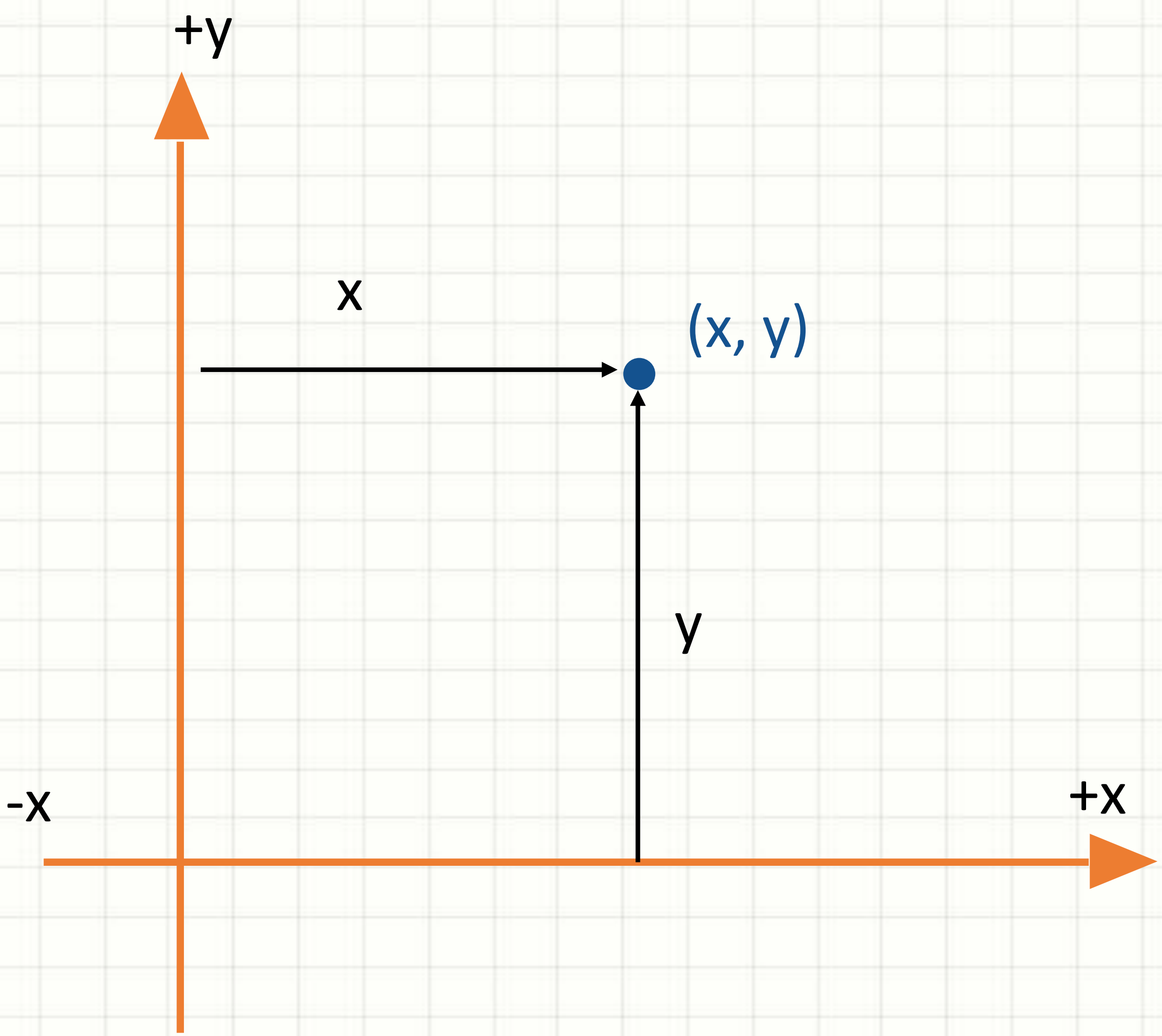
Geometric Definition of Vector

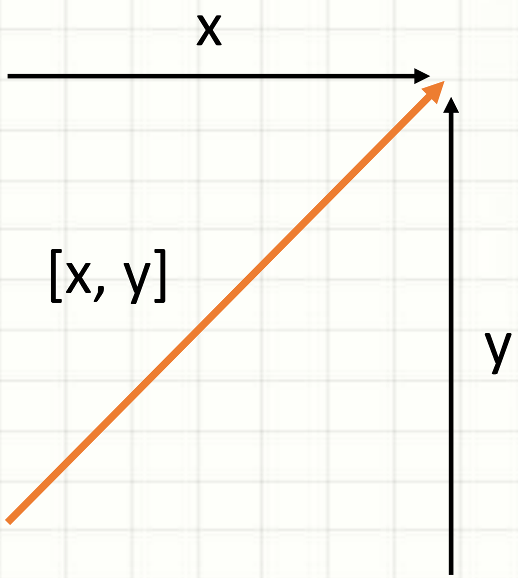
- Many quantities we deal with on a daily basis have magnitude and direction

- Vector Quantities
  - **Displacement** “Take three step forward”
  - **Velocity** “ I am traveling northeast at 50 mph”

- Scalar Quantities
  - **Distance**
  - **Speed**

Specifying Vectors with Cartesian Coordinates





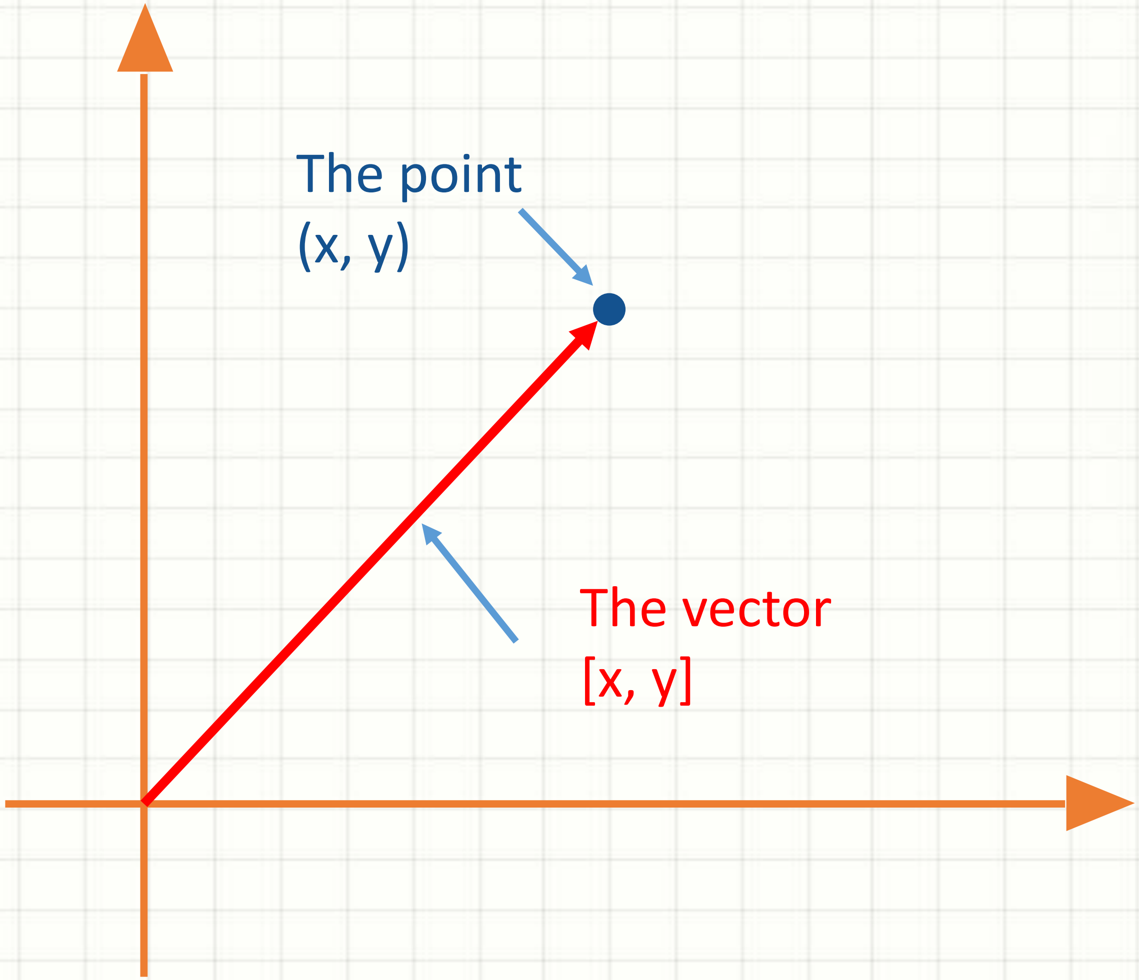
Zero Vector

- There is a special vector, known as the *zero vector* for any given vector dimension
- The zero vector notation

$$0 = \begin{pmatrix} 0 \\ 0 \\ \cdots \\ \cdots \\ 0 \end{pmatrix}$$

- The zero vector consists of a magnitude of zero
- The zero vector has no direction
- The zero vector of a given dimension is the *additive identity* for the set of vectors of that dimension

The Relationship between Points and Vectors



Negating a Vector

- The negation operation can be applied to vectors

- Definition
  - Every vector $\mathbf{v}$ has an additive inverse $-\mathrm{v}$ of the same dimension such that

$$\boldsymbol{v} + (-\boldsymbol{v}) = \mathbf{0}$$

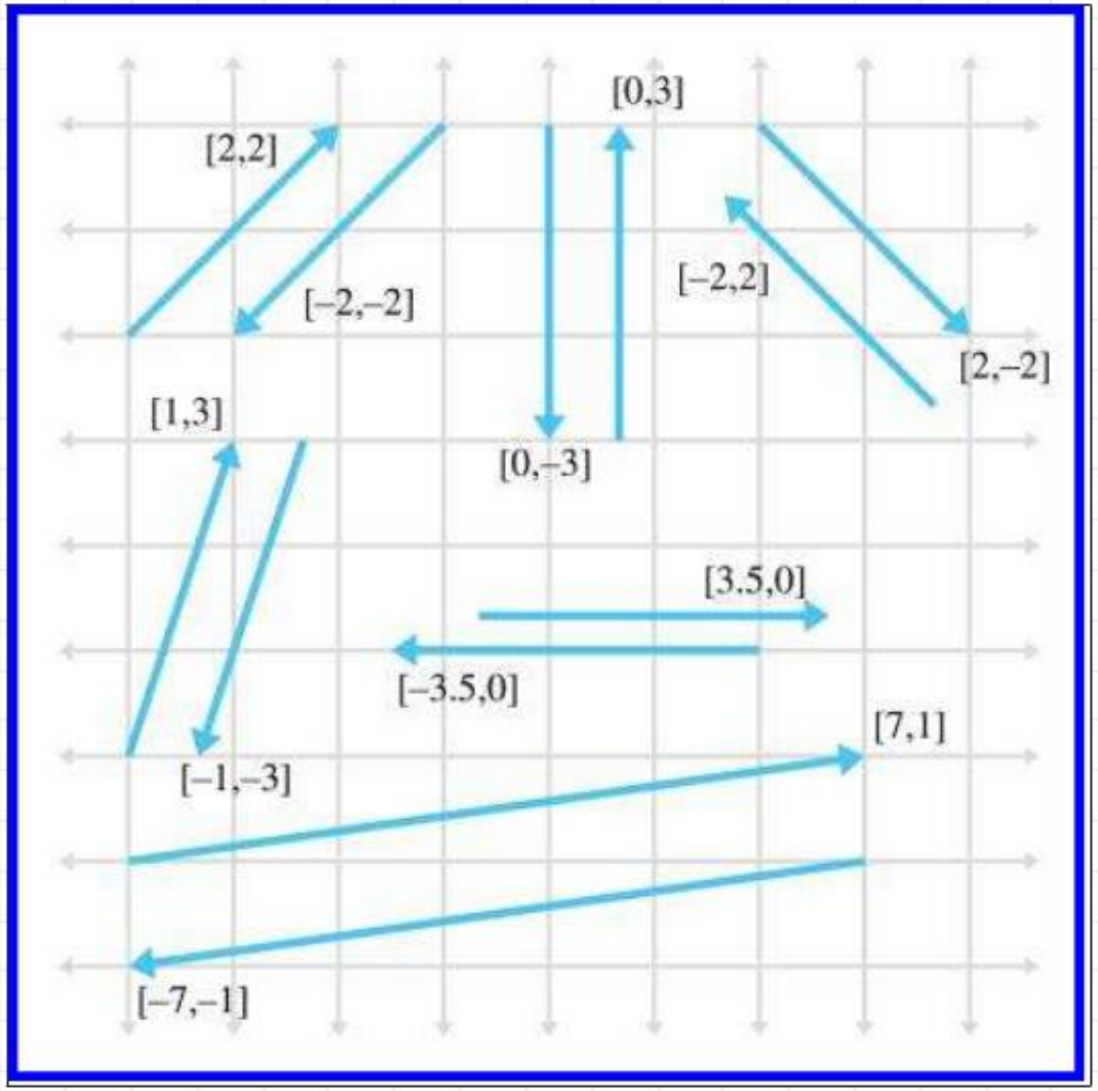
Negating a Vector

- Official Linear Algebra Rules
  - To negate a vector of any dimension, we simply negate each component of the vector as follows:

$$-\begin{bmatrix} a_1 \\ a_2 \\ \cdot \\ \cdot \\ a_{n-1} \\ a_n \end{bmatrix} = \begin{bmatrix} -a_1 \\ -a_2 \\ \cdot \\ \cdot \\ -a_{n-1} \\ -a_n \end{bmatrix}$$

Negating Vector: Geometric Interpretation

- Negating a vector results in a vector of the same magnitude but opposite direction



Vector Multiplication by a Scalar

- Official Linear Algebra Rules

$$k\begin{bmatrix} a_1 \\ a_2 \\ . \\ . \\ . \\ a_{n-1} \\ a_n \end{bmatrix} = \begin{bmatrix} a_1 \\ a_2 \\ . \\ . \\ . \\ a_{n-1} \\ a_n \end{bmatrix} k = \begin{bmatrix} ka_1 \\ ka_2 \\ . \\ . \\ . \\ ka_{n-1} \\ ka_n \end{bmatrix}$$

Vector Multiplication by a Scalar

- Example: Multiplying a 3D Vector by scalar

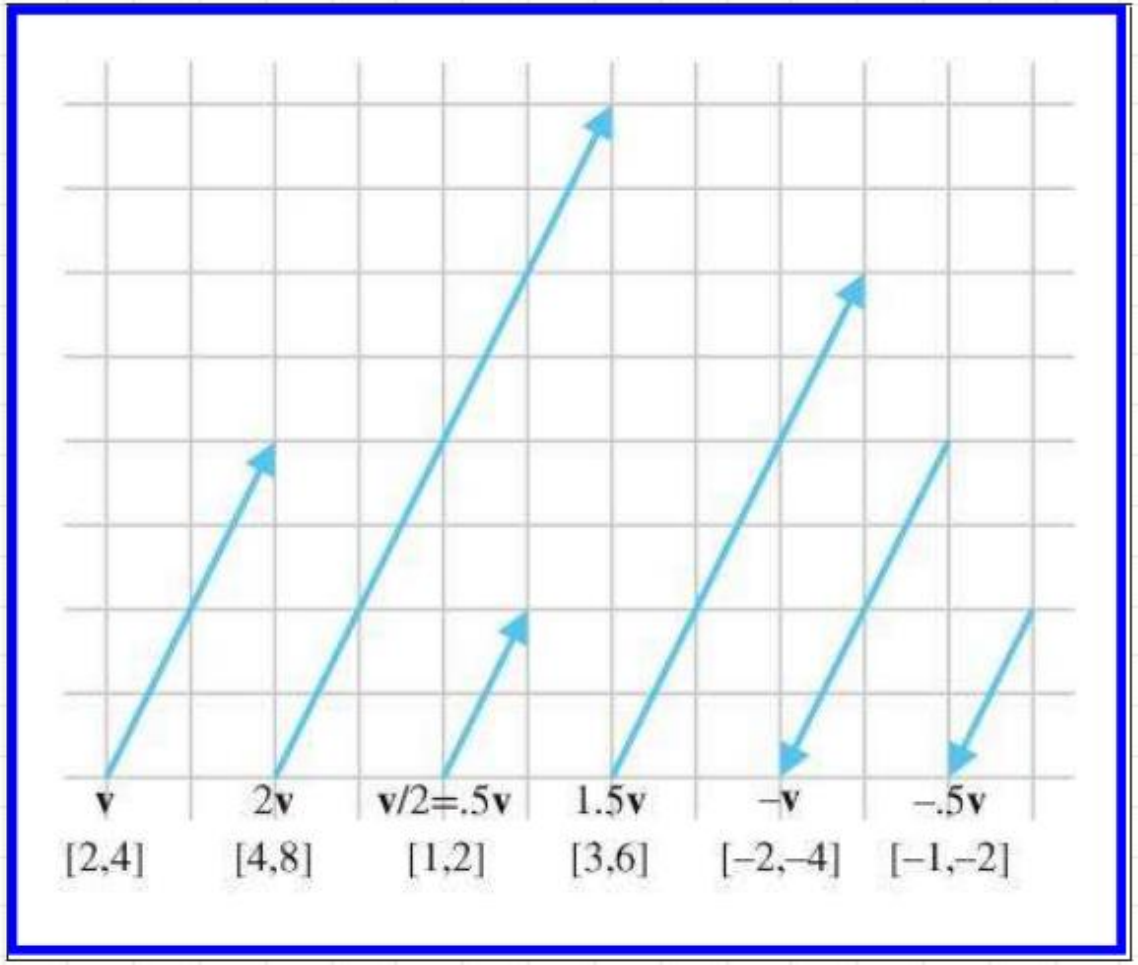
$$k \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} k = \begin{bmatrix} kx \\ ky \\ kz \end{bmatrix}$$

- Example: Dividing a 3D Vector by a nonzero scalar

$$\frac{v}{k} = \left(\frac{1}{k}\right) v = \begin{bmatrix} v_x/k \\ v_y/k \\ v_z/k \end{bmatrix}$$

Vector Multiplication by a Scalar

- Geometrically, multiplying a vector by a scalar k has the effect of scaling the length by a factor of k



Vector Addition

- Two vectors can be added if they are of the same dimension

- Official Linear Algebra Rules

$$\begin{bmatrix} a_1 \\ a_2 \\ . \\ . \\ . \\ a_{n-1} \\ a_n \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \\ . \\ . \\ . \\ b_{n-1} \\ b_n \end{bmatrix} = \begin{bmatrix} a_1 + b_1 \\ a_2 + b_2 \\ . \\ . \\ . \\ a_{n-1} + b_{n-1} \\ a_n + b_n \end{bmatrix}$$

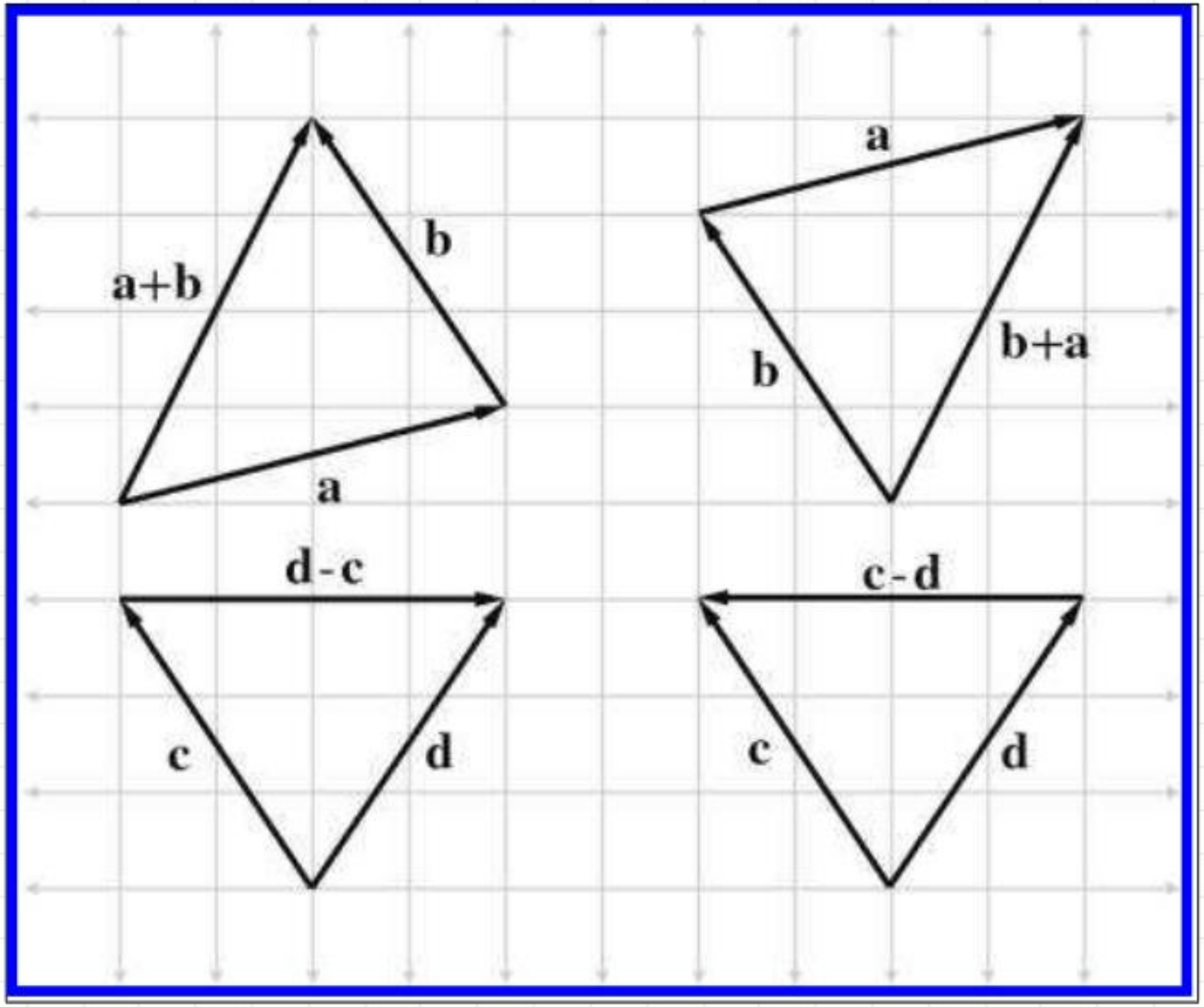
Vector Subtraction

- Two vectors can be subtracted if they are of the same dimension

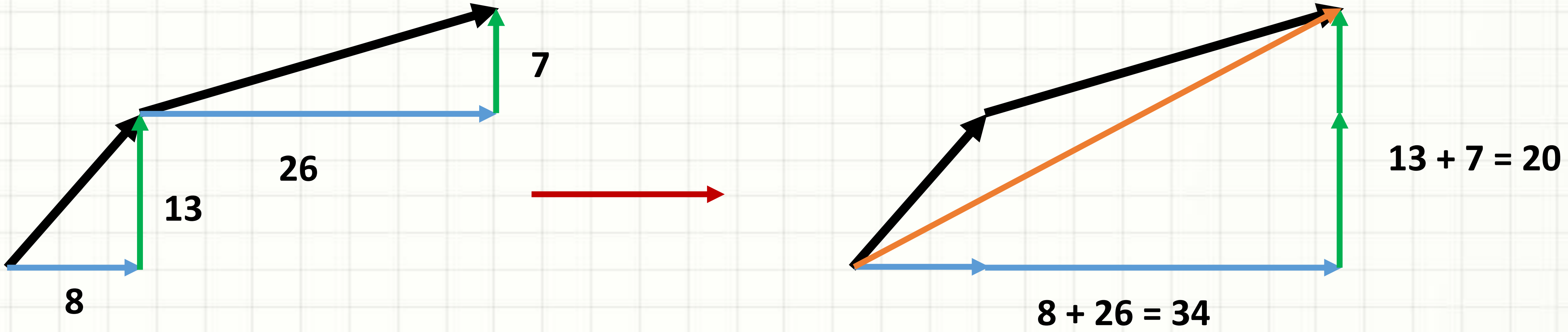
- Official Linear Algebra Rules

$$\begin{bmatrix} a_1 \\ a_2 \\ . \\ . \\ . \\ a_{n-1} \\ a_n \end{bmatrix} - \begin{bmatrix} b_1 \\ b_2 \\ . \\ . \\ . \\ b_{n-1} \\ b_n \end{bmatrix} = \begin{bmatrix} a_1 \\ a_2 \\ . \\ . \\ . \\ a_{n-1} \\ a_n \end{bmatrix} + \left(- \begin{bmatrix} b_1 \\ b_2 \\ . \\ . \\ . \\ b_{n-1} \\ b_n \end{bmatrix} \right) = \begin{bmatrix} a_1 - b_1 \\ a_2 - b_2 \\ . \\ . \\ . \\ a_{n-1} - b_{n-1} \\ a_n - b_n \end{bmatrix}$$

Vector Addition & Subtraction: Geometric Interpretation



Adding Vector Example



- Example

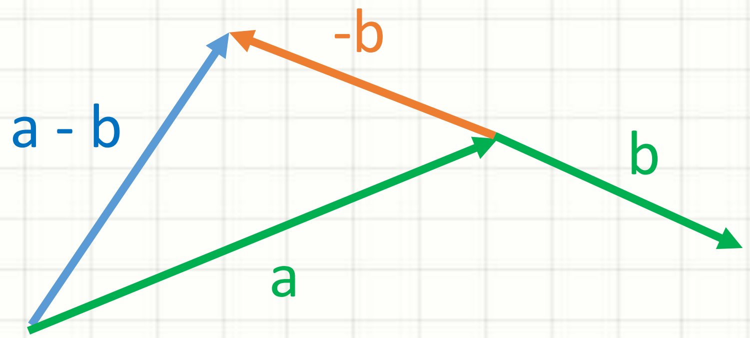
  Add the vectors **a** = (8, 13) and **b** = (26, 7)

  c = **a** + **b**

  c = (8, 13) + (26, 7) = (8 + 26, 13 + 7) = (34, 20)

Subtracting Vector Example

- Reversing the vector we want to subtract by adding negative value, then add



- Example

Subtract the vector **k** = (6, 5) from **v** = (8, 3)

a = **v** - **k**

a = (8, 3) + (6, 5) = (8 - 6, 3 - 5) = (2, -2)

Vector Magnitude (Length)

$$\|\boldsymbol{v}\| = \sqrt{\sum_{i=1}^{n} \boldsymbol{v}_i^2} = \sqrt{\boldsymbol{v}_1^2 + \boldsymbol{v}_2^2 + \ldots + \boldsymbol{v}_{n-1}^2 + \boldsymbol{v}_n^2}$$

- Example

$$\|v\| = \sqrt{v_x^2 + v_y^2} \quad (\,for\ 2D\ vector\ v)$$

$$\|v\| = \sqrt{v_x^2 + v_y^2 + v_z^2} \quad (\,for\ 3D\ vector\ v)$$

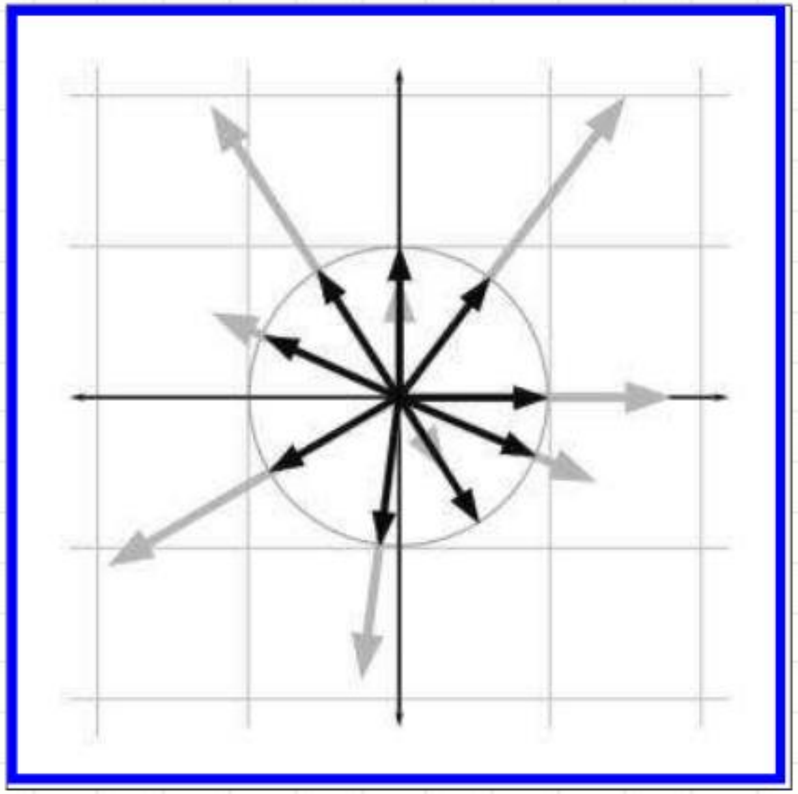
Vector Magnitude (Length)

- A *unit vectors* or *normalized vectors* are a vector that has a magnitude of one

- Some vectors become shorter after normalization *if their length was greater than 1*

- Some vectors become longer after normalization *if their length was less than 1*

$$\hat{v} = \frac{v}{\|v\|}$$



Vector Dot Product

❖ Official Linear Algebra Rules

$$\begin{bmatrix} \boldsymbol{a_1} \\ \boldsymbol{a_2} \\ \cdots \\ \cdots \\ \boldsymbol{a_{n-1}} \\ \boldsymbol{a_n} \end{bmatrix} . \begin{bmatrix} \boldsymbol{b_1} \\ \boldsymbol{b_2} \\ \cdots \\ \cdots \\ \boldsymbol{b_{n-1}} \\ \boldsymbol{b_n} \end{bmatrix} = \boldsymbol{a_1 b_1 + a_2 b_2 + \ldots + a_{n-1} b_{n-1} + a_n b_n}$$

$$a \,.\, b = \sum_{i=1}^{n} a_i b_i$$

Vector Cross Product

- Official Linear Algebra Rules

$$\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} \times \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} = \begin{bmatrix} y_1 z_2 - z_1 y_2 \\ z_1 x_2 - x_1 z_2 \\ x_1 y_2 - y_1 x_2 \end{bmatrix}$$

Literature

- Foley, J. D., Van Dam, A., Feiner, S.K., Hughes, J. F., & Phillips R. L. (1996). *Introduction to Computer Graphics*.
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