Answer Script

Question No. 01
How many edges does a tree with **n** nodes have?
Answer No. 01
In graph theory, a tree is defined as a connected acyclic graph. In simpler terms, a tree is a graph that consists of nodes connected by edges, where there are no loops (i.e., an edge connecting a node to itself) and no cycles (i.e., a sequence of edges that loop back to the starting node). Now, let's consider a tree with n nodes. To determine how many edges it has, we can use the fact that a tree with n nodes always has exactly n-1 edges. This is known as the "handshaking lemma" or "tree formula" and can be proven using mathematical induction, as follows: Base case: A tree with one node has no edges, so the formula holds. Induction step: Assume that a tree with k nodes has k-1 edges. Now we want to show that a tree with k+1 nodes has k edges. To do this, we can add the (k+1)th node to any of the k existing nodes in the tree to create a new edge. No matter where we add the new node, we will add exactly one edge to the tree. This is because the new node can only be connected to one existing node, and each edge connects exactly two nodes. Thus, we have increased the number of edges in the tree by 1. Since we started with a tree of k nodes and k-1 edges, and adding a new node and edge gives us a tree with k+1 nodes and k edges, we have shown that the formula holds for all trees with n nodes. So, to answer the original question, a tree with n nodes has n-1 edges.

Question No. 02
How many edges does a complete graph with **n** nodes have?
Answer No. 02
A complete graph with n nodes has n*(n-1)/2 edges. To see why, note that each vertex in a complete graph is connected to every other vertex, except itself. So the first vertex has n-1 edges, the second vertex has n-2 edges, and so on, until the last vertex has 0 edges. The total number of edges in the complete graph is the sum of the number of edges for each vertex, which is: (n-1) + (n-2) + ... + 2 + 1 This sum can be simplified using the formula for the sum of an arithmetic series: (n-1) + (n-2) + ... + 2 + 1 = n*(n-1)/2 Therefore, a complete graph with n nodes has n*(n-1)/2 edges. For example, a complete graph with 4 nodes has 4*(4-1)/2 = 6 edges, and a complete graph with 5 nodes has 5*(5-1)/2 = 10 edges.

Question No. 03

Convert the following **Adjacency Matrix** into an **Adjacency List** and draw the graph**.**
(no need to code)
 	
`1 0 0 1`
`1 0 1 0`
`0 1 0 1`
`0 0 1 1`

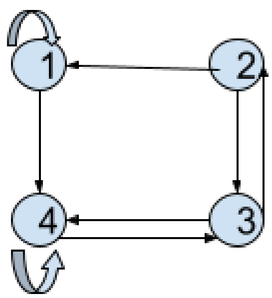
Answer No. 03
The given adjacency matrix represents a graph with 4 vertices labeled as 1, 2, 3, and 4. To convert the adjacency matrix into an adjacency list, we can iterate over the matrix row by row and create a list of adjacent vertices for each vertex. For example:

Vertex 1: 1, 4
Vertex 2: 1, 3
Vertex 3: 2, 4
Vertex 4: 3, 4

This can be represented as the following adjacency list:

1: 1, 4
2: 1, 3
3: 2, 4
4: 3, 4

Graph:

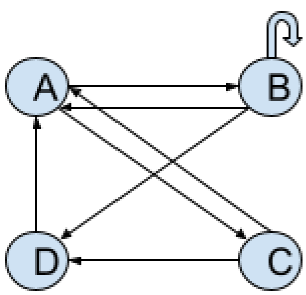


Question No. 04
Convert the following **Adjacency List** into an **Adjacency Matrix** and draw the graph. (no need to code) **`A -> B, C`** **`B -> B, A, D`** **`C -> D, A`** **`D -> A`**
Answer No. 04

The following Adjacency Matrix:

	A	B	C	D
A	0	1	1	0
B	1	1	0	1
C	1	0	0	1
D	1	0	0	0

The Following graph is—



Question No. 05

Convert the following edge list of an **undirected graph** to its respective **Adjacency List** representation.

```
[
  [A, B, 1]
  [B, C, 3]
  [C, A, 2]
  [E, F, 9]
```

```
  [C, D, 1]
  [E, F, 7]
]
```

Mention the properties of this graph

a. Is the graph weighted or unweighted?
b. Does the graph have cycles?
c. If the graph has cycles, remove some edges to make it acyclic (i.e. with no cycle)
d. Has the graph become a tree? Why or why not?
e. If not, then add edges to make it a tree.

Answer No. 05

a. The graph is weighted. Here from the above graph, [A, B, 1] in this statement, A and B are two nodes connected with each other and they have 1 weight. So the graph is weighted.

b. Yes, the graph has two cycles. Here,
   [A, B, 1]
   [B, C, 3]
   [C, A, 2] in this A->B->C there has a cyclic order.
   Also, [E, F, 9] and [E, F, 7] there has multiple edges between node E -> F.
So, there has another cycle in this graph.

c. Removing C -> A and any edge between E -> F would make the graph acyclic.
d. No, it does not become a tree because the graph is not connected. To make the graph tree, it should be connected.
e. Add an edge between nodes D and E that would connect the graph and make it a tree.