

$$\begin{array}{c}
[I_1] \frac{}{\vdash . : a[x_1], \cancel{b^\perp[x_2x_4]} \Downarrow a^\perp[x_2x_3]} \\
[D_1] \frac{}{\vdash . : a[x_1], a^\perp[x_2x_3], \cancel{b^\perp[x_2x_4]} \Uparrow .} \\
[R \Uparrow] \frac{}{\vdash . : a^\perp[x_2x_3], \cancel{b^\perp[x_2x_4]} \Uparrow a[x_1]} \\
[R \Downarrow] \frac{}{\vdash . : a^\perp[x_2x_3], \cancel{b^\perp[x_2x_4]} \Downarrow a[x_1]} \\
[\otimes] \frac{}{}
\end{array}$$

$$\begin{array}{c}
[I_1] \frac{}{\vdash . : b[x_1], \cancel{a^\perp[x_2\bar{x}_3]} \Downarrow b^\perp[x_2\bar{x}_4]} \\
[D_1] \frac{}{\vdash . : b[x_1], \cancel{a^\perp[x_2\bar{x}_3]}, b^\perp[x_2\bar{x}_4] \Uparrow .} \\
[R \Uparrow] \frac{}{\vdash . : \cancel{a^\perp[x_2\bar{x}_3]}, b^\perp[x_2\bar{x}_4] \Uparrow b[x_1]} \\
[R \Downarrow] \frac{}{\vdash . : \cancel{a^\perp[x_2\bar{x}_3]}, b^\perp[x_2\bar{x}_4] \Downarrow b[x_1]}
\end{array}$$

$$\begin{array}{c}
\vdash . : a^\perp[x_2], b^\perp[x_2] \Downarrow (a \otimes b)[x_1] \\
[D_1] \frac{}{\vdash . : a^\perp[x_2], b^\perp[x_2], (a \otimes b)[x_1] \Uparrow .} \\
[R \Uparrow] \frac{}{\vdash . : b^\perp[x_2], (a \otimes b)[x_1] \Uparrow a^\perp[x_2]} \\
[R \Uparrow] \frac{}{\vdash . : (a \otimes b)[x_1] \Uparrow b^\perp[x_2], a^\perp[x_2]} \\
[\wp] \frac{}{\vdash . : (a \otimes b)[x_1] \Uparrow (b^\perp \wp a^\perp)[x_2]} \\
[R \Uparrow] \frac{}{\vdash . : . \Uparrow (a \otimes b)[x_1], (b^\perp \wp a^\perp)[x_2]}
\end{array}$$