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EXERCISE I want to perform a transformation, on an ideal gas, such that pressure and volume are proportional, but I want to keep the temperature fixed. Can I do that?

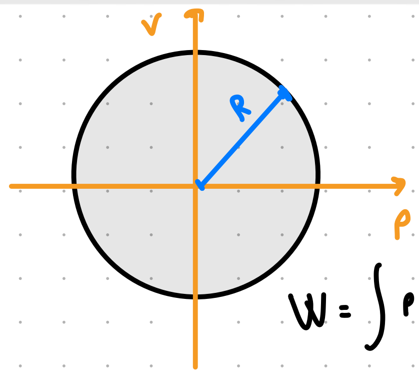
$$T = PV \qquad k_B = 1.38 \cdot 10^{-23}$$

$$k_B = \frac{PV}{TN}$$

$$P = \alpha V$$

$$N = \frac{PV}{Tk_B} = \frac{\alpha V^2}{Tk_B}$$

EXERCISE What is the work done by a gas in a cyclic transformation whose diagram in the PV plane (with the standard units of measure) is a circle of radius R?



$$W_{A \to B} = 2\int_{V_A}^{V_B} P(V)\,dV = 2\int_{V_A}^{V_B} x^2 + y^2 - R^2\,dV = 2(x^2+y^2-R^2)(V_B - V_A)$$

$$W = \int P(V)\,dV = \text{AREA} = \pi R^2$$

TEMP. CONSTANT

EXERCISE Compute the "isothermal compressibility" k_T and the "thermal expansion coefficient" α for the ideal gas. They are defined as follows:

$$k_T = -\frac{1}{V}\frac{\partial V}{\partial P}\bigg|_T$$

$$\alpha = \frac{1}{V}\frac{\partial V}{\partial T}\bigg|_P$$

NOTE: The notation

$$\frac{\partial f}{\partial x}\bigg|_y$$

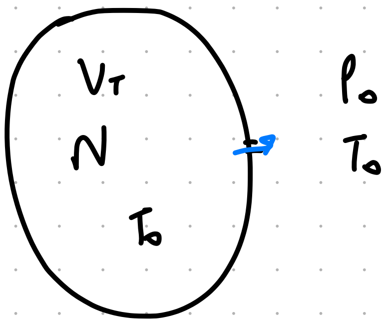
refers to the partial derivative of the function f with respect to the variable x by keeping y constant. In other words, it is equal to the partial derivative of $f(x, y)$ with respect to x. This notation is commonly used in thermodynamics because sometimes it is not immediately clear what variables a given quantity depends on, as one may write simply V or P, instead of, say, $V(P, T)$ and $P(V, T)$.

$$\frac{PV}{TN} = k_B; \qquad V = \frac{k_B TN}{P} \qquad P = \frac{k_B TN}{V}$$

$$k_T = -\frac{1}{V}\frac{\partial V}{\partial P}\bigg|_T = -\frac{1}{V}\left(\frac{k_B TN}{P}\frac{1}{\partial P}\right) = +\frac{k_B TN}{VP^2} \xrightarrow{} P = \boxed{\frac{1}{P}}$$

$$\alpha = \frac{1}{V}\frac{\partial V}{\partial T}\bigg|_P = \frac{1}{V}\left(\frac{k_B TN}{P}\frac{1}{\partial T}\right) = \frac{k_B N}{VP} \xrightarrow{} T = \boxed{\frac{1}{T}}$$

EXERCISE [difficult] A tank of volume V_t is filled with N molecules of gas at high pressure, at the same temperature T_0 as the environment. A small hole in the tank is opened, and some gas goes out into the atmosphere (at pressure P_0), until thermodynamic equilibrium. What is the work done on the mass of gas that has escaped?



THERMODYNAMIC EQ. $W = ?$

↳ SAME T