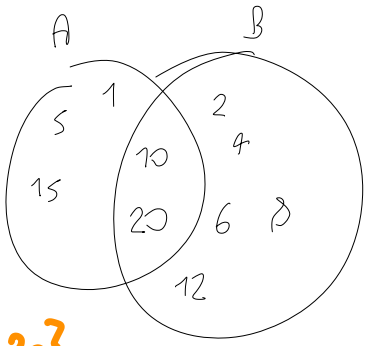
Exercises - Calculus

Academic Year 2021-2022

Sheet 0



1. Let us consider the following sets

$$A := \{\text{natural numbers divisible by } 5\}$$
$$B := \{\text{even natural numbers}\}$$
$$C := \{5, 10, 9, 20\}$$

Determine the set

$$(A \cap B) \cap C.$$

{10}, {20}

2. Let us consider the following sets

$$A := \{\sqrt{2},\ \pi,\ 42\}$$
$$B := \{\text{natural numbers multiple of } 2\}$$
$$C := \{99\}$$

Determine the set

$$(A \cap B) \cup C.$$

{42}, {99}

3. Let us consider the following sets

$$A := \{\sqrt{2},\ \pi,\ 33\}$$
$$B := \{\text{natural numbers multiple of } 3\}$$
$$C := \{99\}$$

Determine the set

$$(A \cup C) \cap B.$$

{33}, {99}

4. Let us consider the following sets

$$A := \{\sqrt{2},\ \pi,\ 77\}$$
$$B := \{\text{natural numbers multiple of } 7\}$$
$$C := \{77\}$$

Determine the set

$$(A \cap B) \cap C.$$

{77}

5. Let us consider the following sets

$$A := \{\sqrt{2},\ \pi,\ 77,\ 21\}$$
$$B := \{\text{natural numbers multiple of } 7\}$$
$$C := \{77\}$$

Determine the set

$$(A \cap B) \setminus C.$$

{21}

6. Let us consider the following sets

$$A := \{\sqrt{11},\ \pi,\ 55\}$$
$$B := \{\text{natural numbers multiple of } 5\}$$
$$C := \{99\}$$

Determine the set

$$(A \cup C) \cap B.$$

7. Let us consider the sets $A := \{a, b, c, d\}$ and $B := \{c, d, e, f\}$.
Determine the following sets

$$A \cap B; \quad A \cup B; \quad A \setminus B; \quad A \times B.$$

8. Let us consider the following intervals

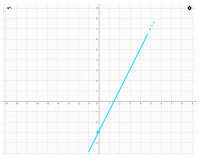
$$A := (-\infty, 7)$$
$$B := [-2, 9)$$
$$C := [7, 10)$$

Determine the set

$$(A \cap B) \cup C.$$

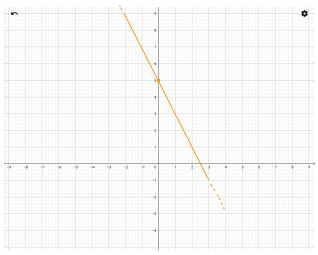
9. Draw the subset of points (x, y) in the plane satisfying

$$y = 2x - 3$$



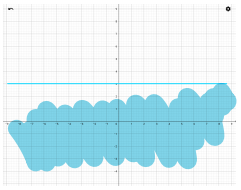
10. Draw the subset of points (x, y) in the plane satisfying

$$y < -2x + 5$$



11. Draw the subset of points (x, y) in the plane satisfying

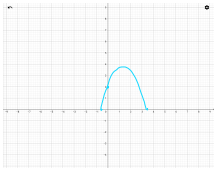
$$y \geq 3$$



12. Determine the equation of the line passing through the points $(1, 0)$ and $(2, 3)$ and draw it.

13. Draw the subset of points (x, y) in the plane satisfying

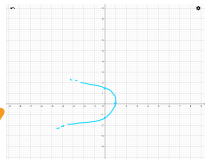
$$x^2 - 3x + y = 2.$$



Which kind of geometric objects is it? Find its intersections, if any, with the x- and y-axis.

14. Draw the subset of points (x, y) in the plane satisfying

$$x + y^2 - 2y = 1.$$



Which kind of geometric objects is it? Find its intersections, if any, with the x- and y-axis.

15. Draw the subset of points (x, y) in the plane satisfying

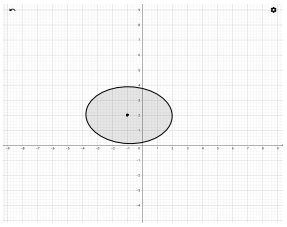
$$x^2 + 4x + 3 + y < 0.$$

16. Determine the equation of the circle centred at $(2, 1)$ with radius 2 and draw it.

17. Draw the subset of points (x, y) in the plane satisfying

$$x^2 + 2x + y^2 - 4y = 0.$$

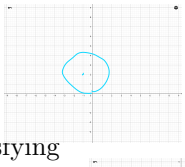
Which kind of geometric objects is it?



18. Draw the subset of points (x, y) in the plane satisfying

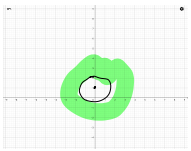
$$x^2 + 2x + 2y^2 - 8y = 0.$$

Which kind of geometric objects is it?



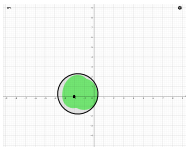
19. Draw the subset of points (x, y) in the plane satisfying

$$x^2 + y^2 - 2y \geq 1.$$



20. Draw the subset of points (x, y) in the plane satisfying

$$x^2 + 4x + y^2 < 2.$$



21. Solve the following inequalities

$$x^2 + 3x - 1 \geq 5; \quad x^3 + 8 < 0; \quad x^3 - x^2 - x + 1 > 0;$$

$$x^4 - 2x^2 - 3 < 0; \quad \frac{(x^2+5)(3-x)}{x-2} < 0; \quad \frac{(x^2+\pi)(x^4+1)(7-x)}{x-1} < 0;$$

$$\frac{x-3}{x+3} > \frac{x+3}{x-3}; \quad \frac{x^2+x-6}{x^3-1} < 0; \quad \frac{(x-\pi)(25+x^2)}{15-x} \geq 0;$$

$$\frac{7x+1}{x^2-4x+1} > 0; \quad \frac{3x+5}{2x+2} \geq 2.$$

22. For any real number a, solve the following inequality

$$2x^2 - \alpha x < x^3.$$

23. Solve the following inequalities

$$|x-2| \geq x+3; \quad x|1+x| - 3|4x+1| \geq 2; \quad x + |3x+2| \geq |2-x|.$$

24. Solve the following inequalities

$$\sqrt{x-1} > -2; \quad \sqrt{2x+1} \leq x; \quad \sqrt{x+8} < 12 - x;$$

$$\sqrt{2x+1} \geq 5x+3; \quad x+1 \geq \sqrt{x^2-2x}; \quad \sqrt{x^2-2x} \geq |2x-1|.$$

25. For any real number a, solve the following inequality

$$\frac{1}{2} < \sqrt{\frac{x-a}{x}} < 1.$$

26. Solve the following inequalities

$$\sqrt{x+1} \leq 5 - \sqrt{x+6}; \quad \frac{1}{2x} + |2x-1| < 2; \quad \frac{|x|-3}{\sqrt{x-2}} > \sqrt{x};$$

$$\frac{|x^2-9|}{|3x+2|} \geq 1; \qquad \frac{2}{x} + \frac{2}{|x-1|} \geq 3; \qquad \frac{1}{x} + |2x-1| \leq 1.$$

$x^2+3x-1\geq 5;$ $x^3+8<0;$ $x^3-x^2-x+1>0;$

$x^4-2x^2-3<0;$ $\dfrac{(x^2+5)(3-x)}{x-2}<0;$ $\dfrac{(x^2+\pi)(x^4+1)(7-x)}{x-1}<0;$

$\dfrac{x-3}{x+3}>\dfrac{x+3}{x-3};$ $\dfrac{x^2+x-6}{x^3-1}<0;$ $\dfrac{(x-\pi)(25+x^2)}{15-x}\geq 0;$

$\dfrac{7x+1}{x^2-4x+1}>0;$ $\dfrac{3x+5}{2x+2}\geq 2.$

I. $x^2+3x-6\geq 0$

$D: 9-4(-6)$

$-3\pm\sqrt{33}$

II. $x^3+8<0$

$x<-2$

III. $x^3-x^2-x+1>0$

$-x^2(-x+1)-x+1>0$

$(-x+1)(-x^2+1)>0$

$(1-x)(1-x^2)>0$

$1-x>0;\ x<1$

$1-x^2>0;\ x^2<1;\ -1<x<1$

$x^2+3x-1\geq 5;\quad x^3+8<0;\quad x^3-x^2-x+1>0;$

$x^4-2x^2-3<0;\quad \dfrac{(x^2+5)(3-x)}{x-2}<0;\quad \dfrac{(x^2+\pi)(x^4+1)(7-x)}{x-1}<0;$

$\dfrac{x-3}{x+3}>\dfrac{x+3}{x-3};\quad \dfrac{x^2+x-6}{x^3-1}<0;\quad \dfrac{(x-\pi)(25+x^2)}{15-x}\geq 0;$

$\dfrac{7x+1}{x^2-4x+1}>0;\quad \dfrac{3x+5}{2x+2}\geq 2.$

IV. $x^4-2x^2-3<0 \quad t=x^2$

$t^2-2t-3<0$

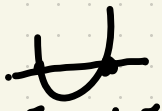
$\Delta=4-4(-3)=16$

$t=\dfrac{2\pm 4}{2}<\begin{matrix}6\\-2\end{matrix}$

$t=6 \vee t=-2$

$x^2=t$

$x=\pm\sqrt{6}$



$-\sqrt{6} \quad +\sqrt{6}$

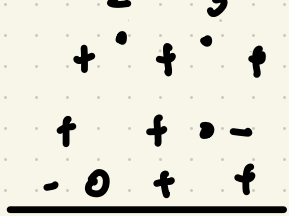
$-\sqrt{6}<x<\sqrt{6}$

V. $\dfrac{(x^2+5)(3-x)}{x-2}\leq 0 \quad x\neq 2$

$N_1\ x^2+5\geq 0 \quad \forall x\in\mathbb{R}$

$N_2\ 3-x\geq 0;\ x\leq 3$

$D\ x-2>0;\ x>2$



$(-\infty,2)\cup(3,+\infty)$

$x^2+3x-1\geq 5;$ $x^3+8<0;$ $x^3-x^2-x+1>0;$

$x^4-2x^2-3<0;$ $\dfrac{(x^2+5)(3-x)}{x-2}<0;$ $\dfrac{(x^2+\pi)(x^4+1)(7-x)}{x-1}<0;$

$\dfrac{x-3}{x+3}>\dfrac{x+3}{x-3};$ $\dfrac{x^2+x-6}{x^3-1}<0;$ $\dfrac{(x-\pi)(25+x^2)}{15-x}\geq 0;$

$\dfrac{7x+1}{x^2-4x+1}>0;$ $\dfrac{3x+5}{2x+2}\geq 2.$

VI. $\dfrac{(x^2+\pi)(x^4+1)(7-x)}{x-1}<0$

$x-1$

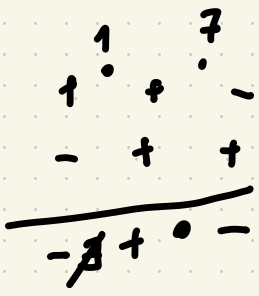
$x\neq 1$

$N_1\geq 0;\ x^2+\pi\geq 0;\ \forall x\in\mathbb{R}$

$N_2\geq 0\ x^4+1\geq 0;\ \forall x\in\mathbb{R}$

$N_3\geq 0\ \ 7-x\geq 0;\ x\leq 7$

$D>0\ \ x-1>0;\ x>1$



$(-\infty;1)\cup(7;+\infty)$

$$\frac{x-3}{x+3} > \frac{x+3}{x-3};$$ $x \neq \pm 3$

$$\frac{(x-3)^2-(x+3)^2}{(x+3)(x-3)} > 0$$

$x^2+9-6x-x^2-9-6x$

$$\frac{-12x}{(x+3)(x-7)} > 0$$

N. $-12x > 0$; $x < 0$

D1 $x+3 > 0$; $x > -3$

D2 $x-3 > 0$; $x > 3$

$-3 \quad 0 \quad 3$

$- \quad -\bullet- \quad +$

$-\circ+ \quad + \quad +$

$- \quad \backslash \quad -\circ+$

$-\not{\infty}+\bullet+\not{\infty}+$

$(-3, 3) \cup (3, +\infty)$

$$\frac{x^2+x-6}{x^3-1}<0;$$

$x \neq 1$

$$\frac{(x-\pi)(25+x^2)}{15-x}\geq 0;$$

$$\frac{(x+3)(x-2)}{x^3-1}<0$$

N1 $x+3>0$; $x>-3$

N2 $x-2>0$; $x>2$

D3 $x^3-1>0$; $x>1$

	-3	1	2
	$-$	$+$	$+$
	$-$	$-$	$+$
	$-$	$+$	$+$
	$-$	$+$	$\times$

$(-\infty,-3)\cup(1,2)$

$$\frac{(x-\pi)(25+x^2)}{15-x}\geq 0$$

$x \neq 15$

N1 $x-\pi \geq 0$; $x \geq \pi$

N2 $25+x^2 \geq 0$; $\forall x \in \mathbb{R}$

D $15-x>0$; $x<15$

	π	15
	$-$	$+$
	$+$	$+$
	$+$	$\times$

$(-\infty, 15)$

$$\frac{7x+1}{x^2-4x+1} > 0;$$

$7x+1>0;\ x>-\frac{1}{7}$

$\frac{4\pm 2\sqrt{3}}{2} = 2\pm\sqrt{3} < \begin{matrix}2+\sqrt{3}\\ 2-\sqrt{3}\end{matrix}$ ~~+~~ $2-\sqrt{3}$ $2+\sqrt{3}$

$-\frac{1}{7}$ $2-\sqrt{3}$ $2+\sqrt{3}$

−	+	+	|	+
+	+	0	− |	+
−	•	+	−	+

$\left(-\frac{1}{7}, 2-\sqrt{3}\right) \cup (2+\sqrt{3}; +\infty)$

$$\frac{3x+5}{2x+2} \geq 2.$$

$x \neq -1$

$$\frac{3x+5-2(2x+2)}{2x+2} \geq 0$$

$$\frac{3x+5-4x-4}{2x+2} \geq 0$$

$$\frac{-x+1}{2x+2} \geq 0$$

$-x+1 \geq 0;\ x \leq 1$

$2x+2 \geq 0;\ x \geq -1$

-1 1 $(-1, 1]$

+	+	•	−
−	0	+	−

22. For any real number a, solve the following inequality

$$2x^2 - \alpha x < x^3.$$

23. Solve the following inequalities

$$|x-2| \geq x+3; \quad x|1+x| - 3|4x+1| \geq 2; \quad x + |3x+2| \geq |2-x|$$

24. Solve the following inequalities

$$\sqrt{x-1} > -2; \qquad \sqrt{2x+1} \leq x; \qquad \sqrt{x+8} < 12 - x;$$

$$\sqrt{2x+1} \geq 5x+3; \quad x+1 \geq \sqrt{x^2-2x}; \quad \sqrt{x^2-2x} \geq |2x-1|.$$

$x^3 - 2x^2 + Ax > 0$

$x(x^2 - 2x + A) > 0$

$\frac{2 \pm \sqrt{4 - 4A}}{2}$ $\alpha < 2 + \sqrt{4-4A}$

$|x-2| \geq x+3$

$x-2 \geq 0 ; x \geq 2$ $x-2 \geq x+3 ; \quad -2 \geq 3$ $\emptyset$

$x-2 < 0 \quad x < 2$ $-x+2 \geq x+3 ; \quad 2x \leq -1 ; \quad x \leq -\frac{1}{2}$

24. Solve the following inequalities

$\sqrt{x-1} > -2$; $\sqrt{2x+1} \leq x$; $\sqrt{x+8} < 12-x$;

$\sqrt{2x+1} \geq 5x+3$; $x+1 \geq \sqrt{x^2-2x}$; $\sqrt{x^2-2x} \geq |2x-1|$.

$\sqrt{x-1} > -2$

$x-1 \geq 0$; $x \geq 1$

$\sqrt{2x+1} \leq x$

$2x+1 \geq 0$; $x \geq -\frac{1}{2}$

$2x+1 \leq x^2$; $x^2-2x-1 \geq 0$

$\frac{-2 \pm 2\sqrt{2}}{2} = -1 \pm \sqrt{2}$ $-1-\sqrt{2}$ NON ACCEP $-1+\sqrt{2}$

$\sqrt{x+8} < 12-x$

$x+8 \geq 0$; $x \geq -8$

$x+8 < 144 + x^2 - 24x$

$x^2 - 25x + 136 > 0$

$625 - 544$

$\frac{25 \pm 9}{2} < 17$ 8

$\sqrt{2x+1} \geq 5x+3$

$2x+1 \geq 0$; $x \geq -\frac{1}{2}$

$2x+1 \geq 25x^2 + 9 + 30x$

$25x^2 + 28x + 8 \leq 0$

$784 - 800$

$$x+1 \geq \sqrt{x^2-2x}; \quad \sqrt{x^2-2x} \geq |2x-1|.$$

$x+1 \geq \sqrt{x^2-2x}$

$x(x-2) \geq 0;$

$x \geq 0$

$x-2 \geq 0; \; x \geq 2$

$-\;0\;+\;2\;+$

$-\;-\;+$

$+\;-\;0$

$x \leq 0 \vee x \geq 2$

$x^2+1+2x \geq x^2-2x$

$4x \geq -1$

$x \geq -\frac{1}{4}$

$\left[-\frac{1}{4}, 0\right] \cup [2, +\infty)$

$\sqrt{x^2-2x} \geq |2x-1|$

$x \leq 0 \vee x \geq 2$

For any real number a, solve the following inequality

$$\frac{1}{2} < \sqrt{\frac{x-a}{x}} < 1.$$

$x \neq 0$

$\frac{x-A}{x} \geq 0$ $\quad x - A \geq 0;\ x \geq A$

$x > 0$

A 0 A

− − + +

26. Solve the following inequalities

$$\sqrt{x+1} \leq 5 - \sqrt{x+6}; \quad \frac{1}{2x} + |2x-1| < 2; \quad \frac{|x|-3}{\sqrt{x-2}} > \sqrt{x};$$

$$\frac{|x^2-9|}{|3x+2|} \geq 1; \quad \frac{2}{x} + \frac{2}{|x-1|} \geq 3; \quad \frac{1}{x} + |2x-1| \leq 1.$$