

# Knowledge Representation and Reasoning

## Exercise Session 6

### Exercise 1. Probabilistic Reasoning

(\*)

Consider the KB  $K = K_P \cup K_C$  where

$$K_P := \{0.5 :: x, \quad 0.5 :: y, \quad 0.5 :: y \leftarrow z\}$$

$$K_C := \{z \leftarrow x, \quad w \leftarrow x, y\}$$

Compute the probability of the following consequences assuming **probabilistic independence** between clauses:

1.  $w$
2.  $y$
3.  $w \leftarrow x$

### Exercise 2. Probability Distribution

(\*\*)

Consider again the KB from Exercise 1. Suppose that  $0.5 :: y$  is **not** a probabilistic clause, but rather a constraint on the joint distribution of the clauses. That is, we want  $P(y) = 0.5$ .

1. Find an adequate probability distribution and compute  $P(w)$
2. How would the result change if we required  $P(y) = p$  for some  $p > 0.5$ ? And for  $p < 0.5$ ?

### Exercise 3. Extreme Probabilities

(\*)

Following the **open world approach** compute the extreme probabilities for  $w$  from the KB of Exercise 1.

**Exercise 4. DLs**

(\*\*)

Consider the probabilistic  $\mathcal{EL}_\perp$  TBox

$$\mathcal{T} := \{ 0.5 :: \top \sqsubseteq Male, \quad 0.5 :: \top \sqsubseteq Female, \\ Male \sqcap Female \sqsubseteq \perp, \quad \top \sqsubseteq \exists.hasParent.\top \}$$

1. Explain in words what the GCIs in this TBox mean
2. Compute the probabilities of:
  - $Male \sqsubseteq \exists.hasParent.Female$  and
  - $Male \sqsubseteq \exists.hasParent.Male$

**Exercise 5. Semantics**

(\*\*\*)

Suppose that you want to represent the (uncertain) knowledge about the spread, consequences, and cost of a recently discovered disease.

1. Which probabilistic semantics do you think is more adequate? why?
2. Identify constructors necessary to express all relevant notions

**Exercise 6. Probabilities**

(\*\*\*)

A *uniform* probability distribution is one that assigns the same probability to events of the same size (e.g., assigning 1/6 to each face in a die). Suppose that we define a uniform distribution over all the natural numbers.

1. what is the probability of finding an *even* number? And a multiple of 5?
2. how do you define the probability of a set  $N$ ?
3. what is the probability of observing the number 42?
4. is it impossible to observe 42?

### Exercise 1. Probabilistic Reasoning

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Consider the KB  $K = K_P \cup K_C$  where

$$K_P := \{0.5 :: x, \quad 0.5 :: y, \quad 0.5 :: y \leftarrow z\}$$

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Compute the probability of the following consequences assuming **probabilistic independence** between clauses:

1.  $w$
2.  $y$
3.  $w \leftarrow x$

### • 3 PROBABILISTIC CLAUSES

- $K_P := \{0.5 :: x, \quad 0.5 :: y, \quad 0.5 :: y \leftarrow z\}$

$$K_C := \{z \leftarrow x, \quad w \leftarrow x, y\}$$

8 WORDS

$0.5 :: x$	$0.5 :: y$	$0.5 :: y \leftarrow z$	$w$	$y$	$w \leftarrow x$
0	0	0	0	0	0
0	0	1	0	0	1
0	1	0	0	1	1
1	0	0	0	0	
0	1	1	0	1	1
1	0	1	1	1	1
1	1	0	1	1	1
1	1	1	1	1	1

$$P(w) = 3/8 \quad P(y) = 3/8 \quad P(y \leftarrow z) = 6/8$$

## Exercise 2. Probability Distribution

(\*\*)

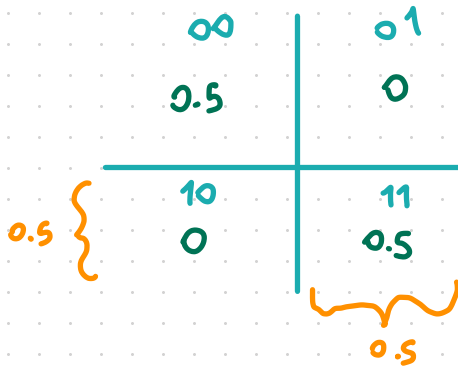
Consider again the KB from Exercise 1. Suppose that  $0.5 :: y$  is **not** a probabilistic clause, but rather a constraint on the joint distribution of the clauses. That is, we want  $P(y) = 0.5$ .

1. Find an adequate probability distribution and compute  $P(w)$
2. How would the result change if we required  $P(y) = p$  for some  $p > 0.5$ ? And for  $p < 0.5$ ?

$$K_P := \{0.5 :: x, \text{ ~~0.5 :: y~~, } 0.5 :: y \leftarrow z\}$$

$$K_C := \{z \leftarrow x, \quad w \leftarrow x, y\}$$

**0.5::y IS NOT A PROBABILISTIC CLAUSE ANYMORE  
BUT RATHER A CONSTRAINT**



### Exercise 3. Extreme Probabilities

(\*)

Following the **open world approach** compute the extreme probabilities for  $w$  from the KB of Exercise 1.

#### FIND THE EXTREMES

FIND WHICH DISTRIBUTION MAXIMIZE OR MINIMIZE THE PROBABILITY OF THE THREE WORDS IN WHICH  $w$  IS 1

$w$  IS ENTAILED IN WORDS 101, 110, 111

↙ LOWER PROBABILITY OF  $w$

$$\underline{P}(w) = \min_{P \in \mathcal{CH}(K)} P(101) + P(110) + P(111)$$

$$\bar{P}(w) = \max_{P \in \mathcal{CH}(K)} P(101) + P(110) + P(111)$$

$$(**)$$

Consider the probabilistic  $\mathcal{EL}_{\perp}$  TBox

$$\mathcal{T} := \{ 0.5 :: \top \sqsubseteq Male, \quad 0.5 :: \top \sqsubseteq Female, \\ Male \sqcap Female \sqsubseteq \perp, \quad \top \sqsubseteq \exists.hasParent.\top \}$$

2. Compute the probabilities of:

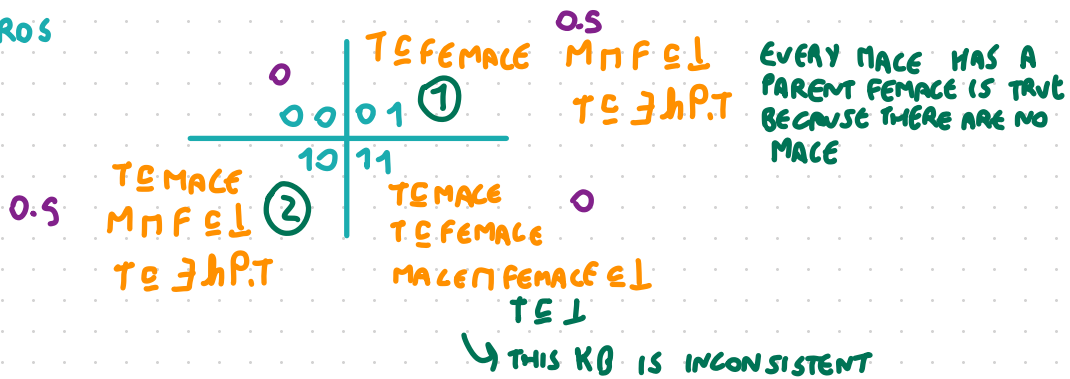
- ① •  $Male \sqsubseteq \exists hasParent.Female$  and
- ② •  $Male \sqsubseteq \exists hasParent.Male$

MALE  $\in$  3 WAS PARENT. FEMALE

### PROBABILITY THAT A MALE HAS A PARENT THAT IS FEMALE

THE PROBABILITY IS 0. EITHER EVERYONE IS MALE,  
EITHER EVERYONE IS FEMALE

## 4 words



## Exercise 5. Semantics

(\*\*\*)

Suppose that you want to represent the (uncertain) knowledge about the spread, consequences, and cost of a recently discovered disease.

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BORING

**Exercise 6. Probabilities**

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A *uniform* probability distribution is one that assigns the same probability to events of the same size (e.g., assigning  $1/6$  to each face in a die). Suppose that we define a uniform distribution over all the natural numbers.

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