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DIFFERENTIABILITY OF A FUNCTION OF MORE THAN ONE VARIABLE
JACOBIAN MATRIX
GRADIENT

LET $f : A \subseteq \mathbb{R}^N \rightarrow \mathbb{R}^M$, $x^0 \in A$, A OPEN.

f IS DIFFERENTIABLE IN x^0 IF IT EXISTS A MATRIX $L \in M^{M \times N}(\mathbb{R})$ SUCH THAT

$$\lim_{\mathbb{R}^N \ni h \to 0} \frac{f(x^0 + h) - f(x^0) - L[h]}{\|h\|} = 0 \in \mathbb{R}^M$$

IN THIS CASE WE CALL

$L = Jf(x^0)$ JACOBIAN MATRIX OF f AT x^0

IF $M = 1$, $Jf(x^0) \in M^{1 \times N}(\mathbb{R})$

$Jf(x^0) = \nabla f(x^0) \in \mathbb{R}^N$ ROW VECTOR IN $\mathbb{R}^N$ WHERE $\nabla f(x^0)$ IS CALLED GRADIENT OF f AT x^0

$Jf(x^0)h = \nabla f(x^0)h = \langle \nabla f(x^0), h \rangle \qquad \forall h \in \mathbb{R}^N$

EXTRA (SKIP)

REMARKING THAT

- $g : (a, b) \rightarrow \mathbb{R}$ $\quad x_0 \in (a, b)$ $\quad g$ IS DIFFERENTIABLE IN x_0 WITH DERIVATIVE $g'(x_0)$ IFF $\lim_{\mathbb{R} \ni h \to 0} \frac{g(x_0 + h) - g(x_0) - g'(x_0)h}{h} = 0 \in \mathbb{R}$??

- f IS DIFFERENTIABLE IN x^0 IFF ANY OF ITS COMPONENT $f_i : A \subseteq \mathbb{R}^N \rightarrow \mathbb{R}$ IS DIFFERENTIABLE IN x^0. IN THIS CASE $Jf(x^0) = \begin{bmatrix} \nabla f_1(x^0) \\ \nabla f_i(x^0) \\ \nabla f_M(x^0) \end{bmatrix}$

$i = 1, \dots, M$