$+$ $\times$

$-$ $\div$

1. Solve the following minimum problems: determine, if it exists, $\min_C f$ where

(a) $f(x) = x + \dfrac{1}{x+2}$ and $C = [-3/2, 5]$. What can you say when $C = (-2, +\infty)$?

(b) $f(x) = \dfrac{|x|-1}{|x|+1}$ and $C = [-1, 2]$

(c) $f(x) = -e^x\sqrt{1-x}$ and $C = [-1, 1]$

(d) $f(x) = \dfrac{x^3}{4x^2+1}$ and $C = [-3, 5]$

(e) $f(x) = -\dfrac{x^2}{4x^3+1}$ and $C = [0, 5]$. What can you say about $\max_C f$?

When the minimum exists, determine also all the absolute minimizers.

(a) $f(x) = x + \dfrac{1}{x+2}$ and $C = [-3/2, 5]$. What can you say when $C = (-2, +\infty)$?

$$f(x) = \frac{x^2+2x+1}{x+2} \qquad C = \left[-\frac{3}{2}, 5\right]$$

THE MIN CAN BE AT THE BOUNDRAIRES OR IN $f'(0)$

THAT IS

$$f\left(-\frac{3}{2}\right) = \frac{\frac{9}{4} - 3 + 1}{-\frac{3}{2}+2} = \frac{\frac{5}{4}}{\frac{1}{2}} = \frac{5}{\cancel{4}} \cdot \cancel{2} = 5$$

$$f(5) = \frac{36}{7}$$

$$f'(x) = \frac{(2x+2)(x+2) - (x^2+2x+1)}{(x+2)^2} = \frac{2x^2+4x+\cancel{2x}+4-x^2-\cancel{2x}-1}{(x+2)^2}$$

$$= \frac{x^2+4x+3}{(x+2)^2}$$

N: $16 - 4(3)$ $\quad \dfrac{-4 \pm 2}{2}$ → -3, -1

		-3		-2		-1	
N	+	•	−		−	•	+
D	+		+	•	+		+
	+	•	−	∄	−	•	+
	↗		↘		↘	•	↗

MIN: $f(-1) = -1 + \dfrac{1}{1} = 0$

SO THE MIN IS $(-1, 0)$

6)

$$f(x) = \frac{|x|-1}{|x|+1} \qquad C = [-1, 2]$$

$f(-1) = 0$

$f(2) = \frac{1}{3}$

$x < 0$

$$f_1(x) \frac{-x-1}{-x+1}$$

$$f_1'(x) = \frac{-1(-x+1) - (-x-1)(-1)}{(-x+1)^2}$$

$$f_1'(x) = \frac{x-1 -x-1}{(1-x)^2}$$

$$f_1'(x) = \frac{-2}{(1-x)^2}$$

$x > 0$

$$f_2(x) = \frac{x-1}{x+1}$$

$$f_2'(x) = \frac{2}{(x+1)^2}$$

$- \quad 1 \quad -$

$\searrow \quad \searrow$

$- \quad (-1) \quad -$

$\nearrow \quad \nearrow$

x

$- \quad 0 \quad -$

$\searrow \quad \searrow$

$f(0)$ MIN

(e) $f(x) = -\dfrac{x^2}{4x^3+1}$ and $C = [0,5]$. What can you say about $\max\limits_C f$?

$$f(x) = \frac{-x^2}{4x^3+1} \qquad C[0,5]$$

$$f(0) = 0$$

$$f(5) = \frac{-25}{501}$$

$$f'(x) = \frac{-2x(4x^3+1) - x^2(12x^2)}{(4x^3+1)^2}$$

$$f'(x) = \frac{-8x^4 - 2x - 12x^4}{(4x^3+1)^2}$$

$$f'(x) = \frac{-20x^4 - 2x}{(4x^3+1)^2} = \frac{-2x(10x^3+1)}{(4x^3+1)^2}$$

$f'(x) = 0$

		$-1/\sqrt[3]{10}$		0	
N1	+		+		−
N2	−		+		+
	↘		↗		↘

$10x^3 + 1 > 0 \quad x > \dfrac{-1}{\sqrt[3]{10}}$

f(0) MAX

f(5) MIN