Exercises - Calculus
Academic Year 2021-2022

Sheet 3

1. Compute

$$\log_2(1/8); \quad \sqrt[3]{1/8}; \quad \tan(-\pi/3); \quad \arctan(-1/\sqrt{3}); \quad \arccos(\sqrt{2}/2);$$
$$\arcsin(1/2); \quad \arctan\left(2\sin(\pi/3)\right); \quad \log(e^5); \quad e^{-\log 10}.$$

2. Let

$$A := \left\{ \log(1/e^3), \frac{\sqrt{2}}{3}, \sin(-\pi/2), \cos(-\pi/6) \right\}$$

Determine the set

$$A \backslash \mathbb{Q}$$

3. Let

$$A := \left\{ e^{-\log 3}, \sin(\pi/3), 2\pi, 2 \right\}$$
$$C := \left\{ \frac{1}{3}, 5 \right\}$$

Determine the set

$$(A \cap \mathbb{Q}) \backslash C$$

4. Let

$$A := \left\{ \cos(\pi), e^{\log(2\pi)}, \sqrt{2}, 7 \right\}$$
$$C := \{3, 2e\}$$

Determine the set

$$(A \backslash \mathbb{Q}) \cup C$$

5. Let

$$A = \{\cos(\pi/6), 2, \sqrt{2}+2, \pi\}$$
$$B = \{2(\cos(\pi/4)+1), \sin(\pi/3), \log(e^2), \arctan(1)\}$$

Determine the set

$$(A \backslash \mathbb{Q}) \cap B$$

6. Let

$$A := \{\log(e^2), e, 2\pi + 1\}$$

Determine the set

$$(A \backslash \mathbb{Q})$$

7. Let

$$A := \{3\pi, \cos(\pi), 2e\},$$
$$B := \{2\sqrt{2}, 1/2\}.$$

Determine the set

$$(A \backslash \mathbb{Q}) \cup B.$$

8. Let
$$A := \{\pi/4, \sin(\pi/4), 1/4\},$$
$$B := \{\sqrt{3}, 1\}.$$
Determine the set
$$(A \cap \mathbb{Q}) \cup B.$$

9. Let
$$A := \{\log(e^{-3}),\ 1+\pi/2,\ 3/4,\ \sin(\pi/4),\ \cos(\pi/3),\ \cos(\pi)\}$$
$$B := \{\text{irrational numbers}\}$$
$$C := \{2,\ -\sqrt{2}\}$$
Determine the set
$$(A \cap B) \cup C.$$

10. Is the function
$$f(x) = \log\left(\exp(3x^3) + 1\right)$$
injective?

11. Is the function $f(x) = e^{x^2-1}$ injective?

12. Is the function $f(x) = \sin(x^3)$ injective?

13. Is the function $f(x) = \sin(x^3)$ injective on $[1, 1]$?

14. Let $f(x) = e^{2x+1} - 2$ and $g(x) = \log(x+1)$. Determine the domain of definition of $g \circ f$ and compute its value.

15. Let $f(x) = x^2$ and $g(x) = \log(x-5)$. Determine the domain of definition of $g \circ f$ and compute its value.

16. Solve the following inequalities

$\log(x-1) \le \sin(\pi/2)$; $\log(x-2) \le \cos(0)$; $e^{(x+1)}-1 < 0$; $\log(x+1) \le 2$

17. Solve the following inequalities
$$\frac{\log(1+x^2)}{\cos(x)-2} \le 0; \quad \frac{\log(x)}{x^3-1} < 0$$

18. Solve the following inequalities

$|\cos(x)| < \frac{1}{\sqrt{2}}$; $\cos|x| \ge \frac{1}{2}$; $2^{x-\sqrt{1-x^2}} \le 1$; $|3^x - 1| \le 2$.

19. Solve the following inequalities

(a) $\dfrac{(x^2-3x+2)\log(x)}{2-\sin(x)} > 0$

(b) $\dfrac{(e^x+5)(x^2-2x-3)}{e^{2x}-1} < 0$

(c) $\dfrac{(\sin(x)+2)(x^2-x+4)}{\log(3x)-1} > 0$

(d) $\dfrac{(e^{-x^2}+1)(x^2+2x-3)}{\log(x+3)-1} \leq 0$

(e) $\dfrac{(2+\sin(x^3))(x^2-2x-3)}{(x-4)^3} \geq 0$

(f) $\dfrac{\arctan(x-1)(e^{\sin(x)}+5)}{e^{x-2}-1} > 0$

(g) $\dfrac{\log(1+2x^4)(x^3-1)}{x^2-4} \geq 0$

(h) $\dfrac{e^{-(x^2-x-6)}(x^2+x-6)}{(x-1)^3} \geq 0$

(i) $\dfrac{\log(1/e)}{\cos(-\pi/3)}\dfrac{\log(x)-1}{(x-1)x} > 0.$

(j) $\dfrac{\log(1/e)}{\sin(-\pi/3)}\dfrac{\log(x)-1}{(x-2)x} > 0.$

(k) $\dfrac{x^2+x+1}{(e^x+1)(x+3)} \geq 0$

20. Solve the following inequalities

    (a) $2(\log x)^2 + 3\log x - 2 > 0$;

    (b) $\log x \geq 3(\log x)^2 + |\log|x| - 1|$;

    (c) $\log(3+2x) - 2\log x > 0$.

    (d) $\sqrt{e^{2x}-1} \geq e^x + 5$.

21. We recall the definition of the following functions

$$\cosh(x) := \frac{e^x + e^{-x}}{2} \quad \text{hyperbolic cosine}$$

and

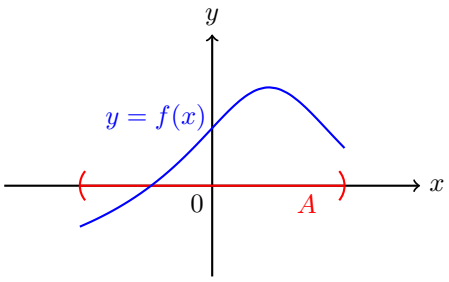
$$\sinh(x) := \frac{e^x - e^{-x}}{2} \quad \text{hyperbolic sine}$$

Prove that for any real number x we have $\cosh^2(x) - \sin^2(x) = 1$. Determine whether these functions are odd or even.

For any $y \geq 1$, solve the equation

$$\cosh(x) = y.$$

22. Let us consider the function $f : A \subseteq \mathbb{R} \to \mathbb{R}$ whose graph is shown in the next picture.



For any the following functions, determine what is its domain and draw its graph

$x \mapsto f(x)+c$, for some given $c \in \mathbb{R}$; $\quad x \mapsto f(x+c)$, for some given $c \in \mathbb{R}$;

$$x \mapsto -f(x); \quad x \mapsto f(-x); \quad x \mapsto -f(-x);$$
$$x \mapsto |f(x)|; \quad x \mapsto f(|x|); \quad x \mapsto f(-|x|);$$
$$x \mapsto f(2x); \quad x \mapsto f(x/2).$$

1. Compute

$$\log_2(1/8); \quad \sqrt[3]{1/8}; \quad \tan(-\pi/3); \quad \arctan(-1/\sqrt{3}); \quad \arccos(\sqrt{2}/2);$$
$$\arcsin(1/2); \quad \arctan\left(2\sin(\pi/3)\right); \quad \log(e^5); \quad e^{-\log 10}.$$

$\log_2\left(2^{-3}\right) = -3$

$\sqrt[3]{\frac{1}{8}} = \frac{1}{2}$

$\tan\left(-\frac{\pi}{3}\right) = \frac{\sin\left(-\frac{\pi}{3}\right)}{\cos\left(-\frac{\pi}{3}\right)} = \frac{-\frac{\sqrt{3}}{2}}{\frac{1}{2}} = -\sqrt{3}$

$\arctan\left(-\frac{1}{\sqrt{3}}\right) = \frac{-1/2}{\frac{\sqrt{3}}{2}}$ NO $-\frac{\pi}{6}$

$\arccos\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$

$\arcsin\left(\frac{1}{2}\right) = \frac{\pi}{6}$

$\arctan\left(2\sin\left(\frac{\pi}{3}\right)\right) = \arctan\left(\sqrt{3}\right) = \frac{\pi}{3}$

$\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$

$\log\left(e^5\right) = 5$

$e^{-\log 10} = \frac{1}{10}$

2) $A := \left\{ \log\left(\frac{1}{e^3}\right), \frac{\sqrt{2}}{3}, \sin\left(-\frac{\pi}{2}\right), \cos\left(-\frac{\pi}{6}\right) \right\}$

$A := \left\{ -3, \frac{\sqrt{2}}{3}, -1, \frac{\sqrt{3}}{2} \right\}$

$A \setminus \mathbb{Q} := \left\{ \frac{\sqrt{2}}{3}, \frac{\sqrt{3}}{2} \right\}$

3) $A := \left\{ e^{-\log 3}, \sin\left(\frac{\pi}{3}\right), 2\pi, 2 \right\}$

$A := \left\{ \frac{1}{3}, \frac{\sqrt{3}}{2}, 2\pi, 2 \right\}$

$C := \left\{ \frac{1}{3}, 5 \right\}$

$\left\{ \frac{1}{3}, 2 \right\}$

$A \cap \mathbb{Q} \setminus C := \{2\}$

4) $A := \{ \cos(\pi), e^{\log(2\pi)}, \sqrt{2}, 7 \}$

$C := \{ 3, 2e \}$

$A := \{ -1, 2\pi, \sqrt{2}, 7 \}$

$\{2\pi, \sqrt{2}\} \cup \{3, 2e\}$

$\{ 2\pi, \sqrt{2}, 3, 2e \}$

5) $A = \{\cos(\frac{\pi}{6}), 2, \sqrt{2}+2, \pi\}$

$B = \{2(\cos(\frac{\pi}{2})+1), \sin(\frac{\pi}{3}), \log \epsilon^2, \arctan(1)\}$

$A = \{\frac{\sqrt{3}}{2}, 2, \sqrt{2}+2, \pi\}$

$B = \{\sqrt{2}+2, \frac{\sqrt{3}}{2}, 2, \frac{\pi}{4}\}$

$\{\frac{\sqrt{3}}{2}, \sqrt{2}+2, \pi\}$

$\{\sqrt{2}+2, \frac{\sqrt{3}}{2}\}$

6) $A := \{\log(\epsilon^2), e, 2\pi+1\}$

$A := \{2, \epsilon, 2\pi+1\}$

$\{\epsilon, 2\pi+1\}$

7) $A := \{3\pi, -1, 2\epsilon\}$

$B := \{2\sqrt{2}, \frac{1}{2}\}$

$A \setminus B = \{3\pi, 2\epsilon\}$

$A \cup B := \{3\pi, 2\epsilon, 2\sqrt{2}, \frac{1}{2}\}$

8) $A := \left\{ \frac{\pi}{4}, \frac{\sqrt{2}}{2}, \frac{1}{4} \right\}$

$B := \{\sqrt{3}, 1\}$

$A \cap B = \left\{\frac{1}{4}\right\}$

$(A \cap B) \cup B = \left\{ \frac{1}{4}, \sqrt{3}, 1 \right\}$

9) $A: \left\{ \frac{1}{3}, 1 + \frac{\pi}{2}, \frac{3}{4}, \frac{\sqrt{2}}{2}, \frac{1}{2}, -1 \right\}$

$A \cap B = \left\{ 1 + \frac{\pi}{2}, \frac{\sqrt{2}}{2} \right\}$ $\quad (A \cap B) \cup C: \left\{ 1 + \frac{\pi}{2}, \frac{\sqrt{2}}{2}, 2, -\sqrt{2} \right\}$

10) $f(x) = \log\left(e^{3x^3}\right) + 1$

$f(a) = f(b)$

$\log\left(e^{3a^3}\right) + 1 = \log\left(e^{3b^3}\right) + 1$

$e^{3a^3} = e^{3b^3}$

$3a^3 = 3b^3$

$a = b$

11) $f(x) = e^{x^2} - 1$ INJECTIVE?

$f(a) = f(b)$

$e^{a^2} - 1 = e^{b^2} - 1$

$e^{a^2} = e^{b^2}$

$a^2 = b^2$ $\quad a = \pm b$ NO

12) $f(x) = \sin(x^3)$ INJECTIVE?

$f(a) = f(b)$

$\sin(a^3) = \sin(b^3)$

$\arcsin(\sin(a^3)) = \arcsin(\sin(b^3))$

$a^3 = b^3$

$a = b$

13) $f(x) = \sin(x^3)$ INJECTIVE ON $[1,1]$

YES

$f(a) = f(b)$

14) $f(x) = e^{2x+1} - 2$ $g(x) = \log(x+1)$

g of

$D: x+1 > 0;$ $x > -1$

$\log(e^{2x+1} - 2 + 1) = \log(e^{2x+1} - 1)$

$e^{2x+1} - 1 > 0$

$e^{2x+1} > 1$

$2x+1 > 0$

$x > -\frac{1}{2}$

15) $f(x) = x^2$ $g(x) = \log(x-5)$

g of

$x > 5$

$\log(x^2 - 5)$ $x^2 - 5 > 0$ $-\sqrt{5}$ $\sqrt{5}$

$x < -\sqrt{5} \vee x > \sqrt{5}$

16)

a) $\log(x-1) \leq \sin\left(\frac{\pi}{2}\right)$ $\quad x > 1$

$\log(x-1) \leq 1$

$\log(x-1) \leq \log(e)$

$x-1 \leq e$

$x \leq e+1$

$(1, e+1]$

b) $\log(x-2) \leq \cos(0)$ $\quad x-2 > 0;\ x > 2$

$\log(x-2) \leq 1$

$x-2 \leq e$

$x \leq e+2$

$(2, e+2]$

c) $e^{x+1} - 1 < 0$

$e^{x+1} < 1$

$e^{x+1} < e^0$

$x+1 < 0$

$x < -1$ $\quad (-\infty, -1)$

d) $\log(x+1) \leq 2$ $\quad x+1 > 0;\ x > -1$

$x+1 \leq e^2$

$x \leq e^2 - 1$

$(-1, e^2-1]$

17) a) $\frac{\log(1+x^2)}{\cos(x)-2} \leq 0$

D: $1+x^2>0 \quad \forall x \in \mathbb{R}$

N $>0 \quad \forall x \in \mathbb{R}$

$\cos(x)-2>0;$

$\cos(x)>2 \quad \emptyset$

N	+	
D	−	
	−	

$\forall x \in \mathbb{R}$

b) $\frac{\log(x)}{x^3-1} < 0 \quad x>0$

N: $\log(x)>0; \quad x>1$

D: $x^3-1>0; \quad x^3>1; \quad x>1$

	1	
N	−	+
D	−	+
	+ ∘	+

$\emptyset$

18) a) $|\cos(x)| < \frac{1}{\sqrt{2}}$

$|\cos(x)| < \frac{\sqrt{2}}{2}$

$|\cos(x)| < \cos\frac{\pi}{4} \qquad |\cos(x)| < \cos -\frac{\pi}{4}$

$x < \frac{\pi}{4}$

b) $\cos|x| \geq \frac{1}{2}$

$\cos|x| \geq \cos\left(\frac{\pi}{6}\right) \wedge \cos|x| \geq \cos\left(-\frac{\pi}{6}\right)$

$x \geq \frac{\pi}{6}$

c) $2^{x-\sqrt{1-x^2}} \leq 1$

$1-x^2 \geq 0 \; / \; \cdot(-1) \quad \boxed{-1 \leq x \leq 1}$

$2^{x-\sqrt{1-x^2}} \leq 2^0$

$x - \sqrt{1-x^2} \leq 0$

$x \leq \sqrt{1-x^2}$ $\quad -1 \leq x < 0 \quad \forall x$

$x^2 \leq 1-x^2$ $\quad \wedge x \leq 1 \quad x^2 \leq 1-x^2$

$2x^2 \leq 1$

$x^2 \leq \frac{1}{2}$

d) $|3^x - 1| \leq 2$

$-2 \leq 3^x - 1 \leq 2$

$-1 \leq 3^x \leq 3$

$-1 \leq 3^x \quad \vee \quad 3^x \leq 3$

$\forall x \in \mathbb{R} \quad \vee \quad x \leq 1$

$(-\infty, 1]$

19) a) $\dfrac{(x^2 - 3x + 2)\log(x)}{2 - \sin(x)} \geq 0$ (N1, N2 / D3)

N1 $x^2 - 3x + 2 \geq 0$

$\dfrac{3 \pm 1}{2} \begin{cases} 2 \\ 1 \end{cases}$ x_1, x_2

$x \leq 1 \vee x \geq 2$

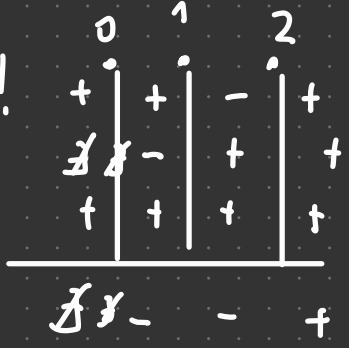
N3 $2 - \sin(x) \geq 0$

$2 \geq \sin(x)$

$\forall x \in \mathbb{R}$

N2 $\log(x)$ $x > 0$!!

$\log(x) \geq 0$ $x \geq 1$



$x \geq 2$

b) $\dfrac{(e^x+5)(x^2-2x-3)}{e^{2x}-1} < 0$

(N_1, N_2, D_1)

N_1: $e^x+5>0$; $\forall x \in \mathbb{R}$

N_2: $x^2-2x-3>0$

$\Delta < 0$ $\quad \emptyset$

D_1: $e^{2x}-1 \neq 0$; $e^{2x} \neq 1$

$e^{2x} > e^0$

$2x > 0$; $\boxed{x>0}$

		0	
N_1	+	|	+
N_2	−	|	−
D_3	−	|	+
	+		−

$(0, +\infty)$

c) $\dfrac{(\sin(x)+2)(x^2-x+4)}{\log(3x)-1} > 0$

(N_1, N_2, D_1)

N_1: $\sin(x)+2>0$ $\quad \forall x \in \mathbb{R}$

N_2: $x^2-x+4>0$ $\quad \emptyset$

D_1: $\log(3x)-1 \neq 0$; $\log(3x) \neq 1$

$3x \neq e$; $\quad x \neq \frac{e}{3}$

		$e/3$	
	+	|	+
	−	|	−
	−	|	+
	+		−

$\left(-\infty, \frac{e}{3}\right)$

d) $\dfrac{(e^{-x^2}+1)(x^2+2x-3)}{\log(x+3)-1} \leq 0$

N_1 N_2 D_1

$N_1 \geq 0 \quad e^{-x^2}+1 \geq 0 \quad \forall x \in \mathbb{R}$

$N_2 \geq 0 \quad x^2+2x-3 \geq 0 \quad x \leq -3 \vee x \geq 1$

$\dfrac{-2 \pm 4}{2} < \begin{matrix} -3 \\ 1 \end{matrix} \quad -3, 1$

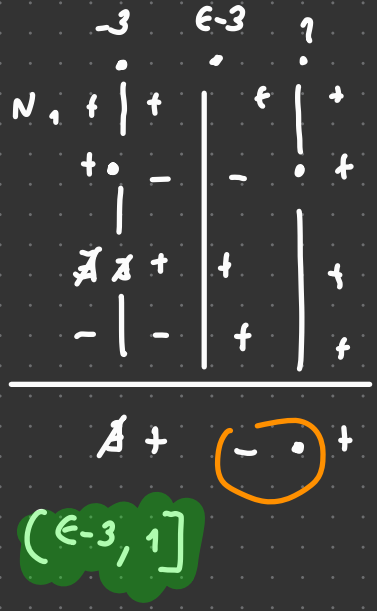
$D_1 > 0 \quad \log(x+3)-1 > 0$

$x+3 > 0; \quad \boxed{x > -3}$

$\log(x+3) > 1$

$x+3 > e$

$x > e-3$



$(e-3, 1]$

e) $\dfrac{\overbrace{(2+\sin(x^3))}^{N_1}\overbrace{(x^2-2x-3)}^{N_2}}{\underbrace{(x-4)^3}_{D_1}} \geq 0$

N_1: $2+\sin(x^3) \geq 0 \quad \forall x \in \mathbb{R}$

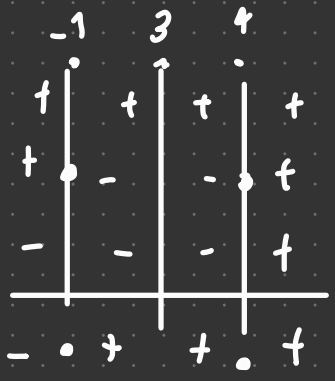
N_2: $x^2-2x-3 \geq 0$ $\qquad x \leq -1 \vee x \geq 3$

$\dfrac{2 \pm 4}{2} \begin{matrix} 3 \\ -1 \end{matrix}$ $\qquad -1 \quad 3$

D_1 $(x-4)^3 > 0$

$(x-4) > 0$

$x > 4$



$[-1, 4]$

5) $\dfrac{\overset{N_1}{\text{ARCTAN}(x-1)}\left(\overset{N_2}{e^{\sin(x)}+5}\right)}{\underset{D_1}{e^{x-2}-1}} \geq 0$

N1 $\text{ARCTAN}(x-1) \geq 0$

$\boxed{x \geq 1}$

N2 $e^{\sin(x)} + 5 > 0$

$\forall x \in \mathbb{R}$

D1 $e^{x-2} - 1 > 0$

$e^{x-2} > 1$

$e^{x-2} > e^0$

$x - 2 > 0$

$\boxed{x > 2}$

	1	2	
	−	+	+
	−	−	+
	+	−	+

$(-\infty, 1) \cup (2, +\infty)$

8) $\dfrac{\overset{N_1}{\log(1+2x^4)}\overset{N_2}{(x^3-1)}}{\underset{D_1}{x^2-4}} \geq 0$

N1: $\log(1+2x^4) \geq 0 \quad 1+2x^4 > 0 \quad \forall x \in \mathbb{R}$

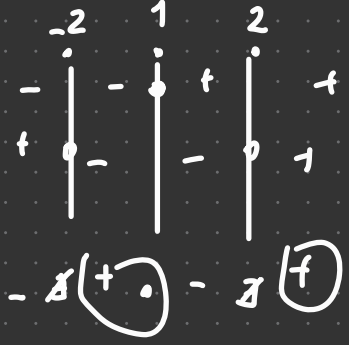
$1 + 2x^4 \geq e^0$

$1 + 2x^4 \geq 1; \quad 2x^4 \geq 0$ $\boxed{\forall x \in \mathbb{R}}$

N_2: $x^3 - 1 \geq 0$ $x^3 \geq 1$; $\boxed{x \geq 1}$

D_1: $x^2 + x > 0$; $x^2 > 4$ $\boxed{x < -2 \ \vee \ x > 2}$

-2 (−) 2 (+)



$(-2, 1] \cup (2, +\infty)$

H) $\dfrac{\epsilon^{-(x^2-x-6)}\,(x^2+x-6)}{(x-1)^2} \geq 0$ (N_1, N_2)

D_1

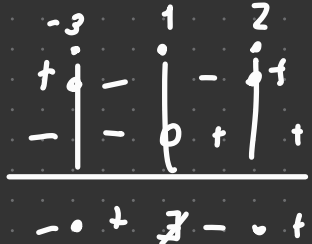
$N_1 \geq 0$ $\epsilon^{-x^2-x-6} \geq 0$; $\forall x \in \mathbb{R}$

$N_2 \geq 0$ $x^2 + x - 6 \geq 0$ $x \leq -3 \ \vee \ x \geq 2$

$\dfrac{-1 \pm 5}{2}$ -3 2 (+) (−) (+)

D_1 $(x-1)^3 > 0$; $x > 1$

-3 (+) 1 (+) 2 (+)



$[-3, 1) \cup [2, +\infty)$

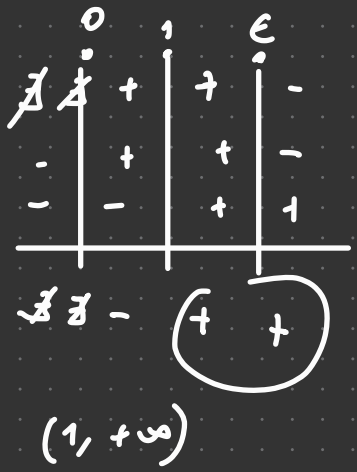
1) $\dfrac{\overset{N_1}{\log\left(\frac{1}{\epsilon}\right)} \cdot \overset{N_2}{(\log(x)-1)}}{\underset{D_1}{\cos\left(-\frac{\pi}{3}\right)} \quad \underset{D_2\ D_3}{(x-1)x}} > 0$

$N_1: \log\left(\frac{1}{\epsilon}\right) = -1$

$N_{1,2}: -\log(x)+1 > 0$ $\boxed{x>0}$!

$\log(x) < 1$

$x < \epsilon$



$(1, +\infty)$

$D_1: \cos\left(-\frac{\pi}{3}\right) = \frac{1}{2}$

$D_{1,3}: \frac{1}{2}x > 0 \ ; \quad x > 0$

$D_2 \quad x-1 > 0 \ ; \quad x > 1$

2) $\dfrac{\log\left(\frac{1}{\epsilon}\right) \quad \log(x)-1}{\sin\left(-\frac{\pi}{3}\right) \quad (x-2)x} > 0$

$x > 0$

$N_1 > 0 \quad \log(x) > 1$

$x > \epsilon$

$D_1 \quad x > 2$

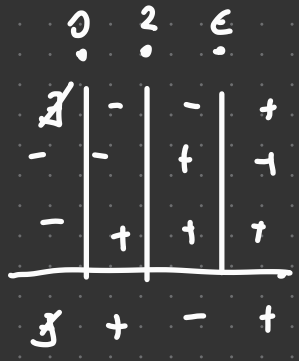
$D_2 \quad x > 0$

$(0,2)\ (3,+\infty)$

$\dfrac{-\log(x)+1}{-\frac{\sqrt{3}}{2} \cdot (x-2)x} > 0$

$\dfrac{\overset{N_1}{\log(x)-1}}{(x-2)x} > 0$

$D_1 \ D_2$



k) $\frac{x^2+x+1}{(e^y+1)(x+3)} \geq 0$

N_1 D_1 D_2

$N_1 \geq 0 \quad x^2+x+1 \geq 0 \quad \forall x \in \mathbb{R}$

$D_1 \geq 0 \quad e^x+1 \geq 0 \quad \forall x \in \mathbb{R}$

$D_2 \geq 0 \quad x+3 \geq 0; \quad x \geq -3$

-3

$- \; \cdot \; +$

$(-3, +\infty)$

20 ω) $2(\log x)^2 + 3\log(x) - 2 > 0$

$x > 0$

$\epsilon = \log(x)$

$2\epsilon^2 + 3\epsilon - 2 > 0$

$\Delta: 9 - 4(2)(-2)$

$\frac{-3 \pm 5}{4} < \begin{matrix} -2 \\ \frac{1}{2} \end{matrix}$ $\quad -2 \cup \frac{1}{2}$

$\epsilon < -2 \quad \vee \quad \epsilon > \frac{1}{2}$

$\log(x) < -2 \quad \vee \quad \log(x) > \frac{1}{2}$

$x < e^{-2}$

$0 < x < \frac{1}{e^2} \quad \vee \quad x > \sqrt{e}$

$(0, \frac{1}{e^2}) \cup (\sqrt{e}, +\infty)$

c) $\log(x) \geq 3(\log x)^2 + |\log(x)| - 1$

$x > 0$

$\log(x) = t$

$t \geq 3t^2 + |t| - 1$

$3t^2 + |t| - t - 1 \leq 0$

$t > 0$ $\quad 3t^2 - 1 \leq 0 \quad t^2 \leq \frac{1}{3}$

$-\frac{1}{\sqrt{3}} \quad \frac{1}{\sqrt{3}}$

$t < -\frac{1}{\sqrt{3}} \vee t > \frac{1}{\sqrt{3}}$

$t < 0 \quad 3t^2 - 2t - 1 \leq 0$

$\Delta: 4 - 4(-1)(3)$

$\frac{2 \pm 4}{6} < 1 \quad -\frac{1}{3}$

$-\frac{1}{3} \quad 1$

$t < -\frac{1}{3} \vee t > 1$

$t > 0 \quad \log(x) < -\frac{1}{\sqrt{3}} \vee \log(x) > \frac{1}{\sqrt{3}}$

$e^{1/\sqrt{3}}, \; e^{\sqrt{3}/3}$

$\log(x) > 0$

$x > 1 \quad \emptyset \quad x > e^{\sqrt{3}/3}$

$t < 0 \quad \log(x) < -\frac{1}{3} \quad \log(x) > 1$

$\emptyset \quad x > e$

$\log(x) < 0$

$x < 1 \quad (1/e) \cup (e^{\sqrt{3}/3}; +\infty)$

$(0; 0.76)$

20c) $\log(3+2x) - 2\log(x) > 0$

$\boxed{x > 0}$

$3+2x > 0; \quad x > -\frac{3}{2}$

$\log(3+2x) > \log(x)^2$

$3+2x > x^2$

$x^2 - 2x - 3 < 0$

$\frac{2 \pm 4}{2} \quad 3 \quad -1$

$-1 \quad + \quad 3$

$(3, +\infty)$

20d) $\sqrt{e^{2x} - 1} \geq e^x + 5$

$e^{2x} - 1 \geq 0; \quad e^{2x} \geq 1; \quad 2x \geq 0 \quad \boxed{x \geq 0}$

$e^{2x} - 1 \geq (e^x + 5)^2$

$e^{2x} - 1 \geq e^{2x} + 25 + 10e^x$

$-26 \geq 10e^x$

$-\frac{13}{5} \geq e^x$

$x \geq 0$

21)

$$\cosh(x) = \frac{e^x + e^{-x}}{2}$$

$$\sinh(x) = \frac{e^x - e^{-x}}{2}$$

$$\cosh^2(x) - \sinh^2(x) = 1$$

$$\frac{e^{2x} + e^{-2x} + 2e^x \cdot e^{-x}}{4} - \frac{e^{2x} + e^{-2x} - 2e^x \cdot e^{-x}}{4} = 1$$

$$\frac{e^{2x} + e^{-2x} + 2e^x \cdot \frac{1}{e^x} - e^{2x} - e^{-2x} + 2e^x \cdot \frac{1}{e^x}}{4} = 1$$

$$0 = 1$$

EVEN $f(x) = f(-x)$

ODD $-f(x) = f(-x)$

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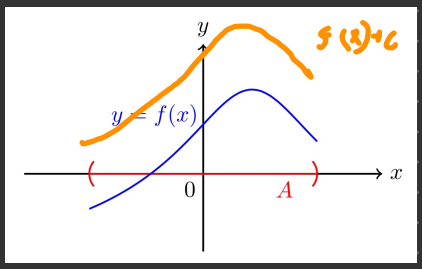
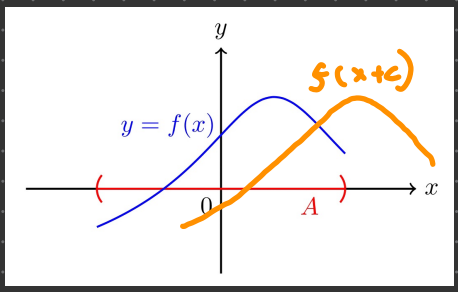
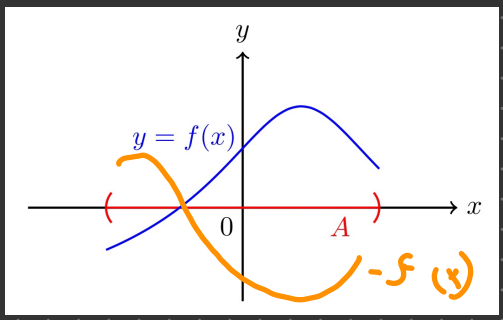
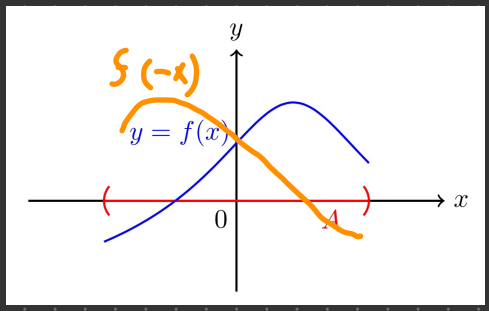
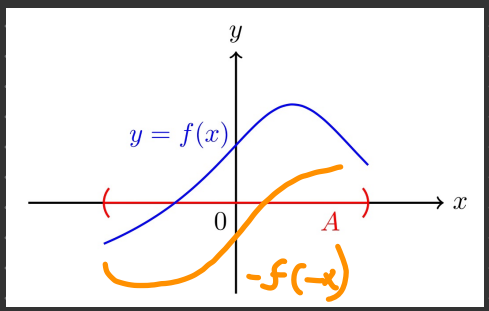
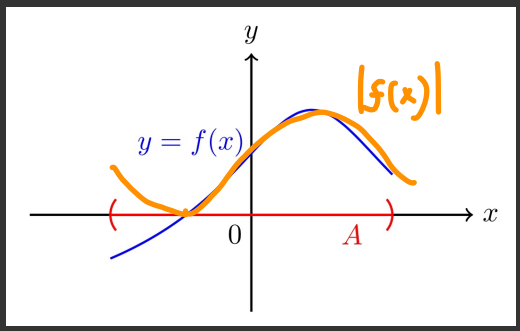

y
f(x)+c
y = f(x)
0
A
x

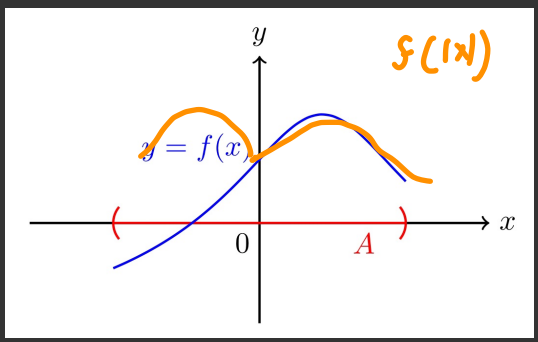

y
f(x+c)
y = f(x)
0
A
x

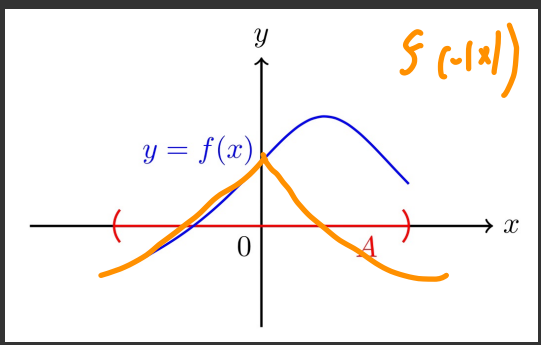

y
y = f(x)
0
A
x
-f(x)

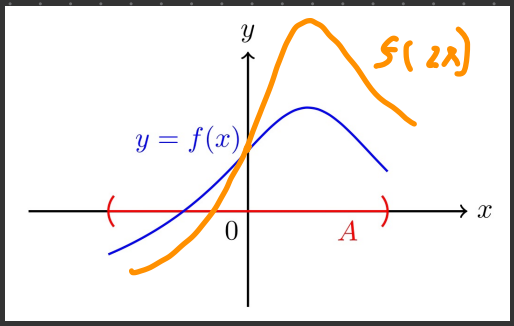

y
f(-x)
y = f(x)
0
A
x

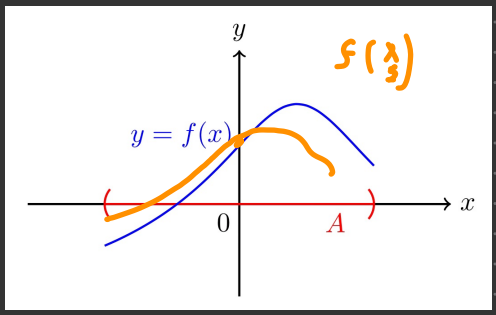

y
y = f(x)
0
A
x
-f(-x)


y
|f(x)|
y = f(x)
0
A
x


y
f(|x|)
y = f(x)
0
A
x


y
f(-|x|)
y = f(x)
0
A
x


y
x
0
A
y = f(x)
f(2x)


y
x
0
A
y = f(x)
f(x/3)