+	×
−	÷

22-05-16
MIN 17:00

SOLUTION OF A LINEAR SCALAR FIRST ORDER ORDINARY DIFFERENTIAL EQUATION

EXTRA (SKIP)

- LET $I \subseteq \mathbb{R}$ OPEN INTERVAL
- LET $a: I \to \mathbb{R}$, $b: I \to \mathbb{R}$ CONTINOUS
- LET $g: I \times \mathbb{R} \to \mathbb{R}$ $\quad g(t, y) = a(t)y + b(t)$

THE EQUATION (*) $y' = a(t)y + b(t)$ IS A LINEAR SCALAR EQUATION OF FIRST ORDER

TERMINOLOGY

- IF $b(t) = 0$ THE EQUATION IS HOMOGENEOUS
- IF $b(t) \neq 0$ THE EQUATION IS NOT HOMOGENEOUS
- $a(t)$ COEFFICENT
- $b(t)$ "CONSTANT TERM" (TERMINE NOTO)

THEOREM

- LET $A(t) \in \int a(t)\,dt$ BE A PRIMITIVE OF a ON I
- LET $B(t) \in \int e^{-A(t)} b(t)\,dt$ BE A PRIMITIVE OF $e^{-A} b$ ON I

THEN THE GENERAL SOLUTION OF (*) $y' = a(t)y + b(t)$ IS GIVEN BY (**) $y(t) = e^{A(t)}\left[B(t) + c\right]$, $c \in \mathbb{R}$ $t \in I$

PROOF 1) WE NEED TO SHOW THAT y LIKE IN (**) SOLVES (*)

IF $y(t) = e^{A(t)}\left[B(t) + c\right]$, $c \in \mathbb{R}$

$$y'(t) = e^{A(t)} \underbrace{A'(t)}_{= a(t)} \left[B(t) + c\right] + e^{A(t)} \underbrace{B'(t)}_{= e^{-A(t)} b(t)} = a(t) \underbrace{e^{A(t)}\left[B(t) + c\right]}_{= y(t)} + \underbrace{e^{A(t)} e^{-A(t)}}_{= 1} b(t) = a(t) y(t) + b(t)$$

2) WE NEED TO SHOW THAT IF y SOLVES (*) THEN y IS OF THE FORM (**) FOR SOME $c \in \mathbb{R}$

IF $y'(t) = a(t)y(t) + b(t)$ THEN $y'(t) - a(t)y(t) = b(t)$; $\underbrace{e^{-A(t)} y'(t) - e^{-A(t)} a(t) y(t)}_{= \frac{d}{dt}\left(e^{-A(t)} y(t)\right)} = e^{-A(t)} b(t)$;

$\frac{d}{dt}\left(e^{-A(t)} y(t)\right) = e^{-A(t)} b(t)$; $\quad e^{-A(t)} y(t) = B(t) + c$ $c \in \mathbb{R}$; $\quad y(t) = e^{A(t)}\left[B(t) + c\right]$, $c \in \mathbb{R}$ $t \in I$