$+ \quad \times$

$- \quad \div$

DINAMICAL SYSTEM → ORDINARY DIFF. EQUATION

THE BOUNDARIES ARE DIFFERENT

CONTINOUS TIME $\frac{d}{dx} x(t) = f(x(t))$ $f(\bar{x}) = 0$

DISCRETE TIME $x_{n+1} = f(x_n)$ $f(\bar{x}) = \bar{x}$

$f'(\bar{x}) \gtrless 0$

$|f'(\bar{x})| \gtrless 1$ 1 IS THE CRYTICAL POINT

GREATER OR LOWER

IN 1-DIMENSION

SPIRAL

NO SPIRAL FOR CONTINOUS TIME

PROPERTY OF BIFORCATION DIAGRAM OF LOGISTIC MAP $x_{n+1} = r x_n(1-x_n)$

RECURRENT STATE

WHAT ARE THE STATES VISITED INFINITELY MANY TIMES BY THE DYNAMICS?

- FIXED POINTS
- ALSO THIS: NOT FIXED POINT BUT RECURRENT

SPLIT IN MORE BRANCHES

BEYOND THIS POINT IS CHAOS

CRITICAL POINT OF r

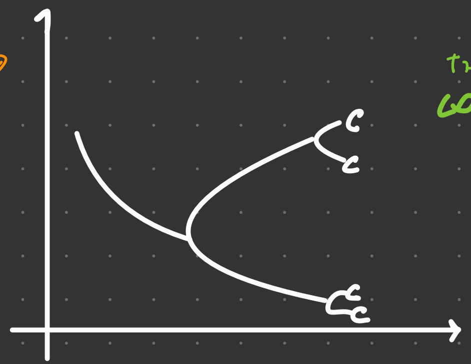
PARAMETER THAT SPECIFIES THE DYNAMICS (r) IS A NUMBER

$\lim_{k \to \infty} r_k = r^*$

BEFORE VALUE OF r^* I FIND ANY POSSIBLE PERIOD OF 2^K AFTER THAT IS CHAOS

ZOOM IN

BEFORE r^* IF I GO CLOSE TO A BIFORCATION POINT IF I ZOOM IN



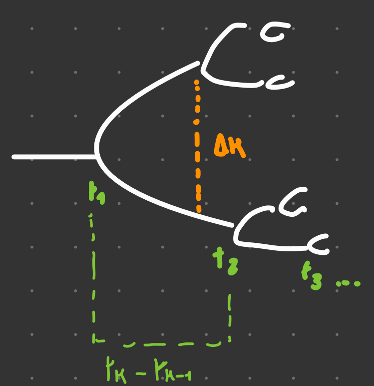
THE ZOOM IN PLOT LOOK LIKE A CUT THE WHOLE GRAPH

ZOOMING IN AMOUNTS TO A SMALL DEFORMATION OF THE WHOLE PLOT

↓

SELF SIMILARITY PROPERTY. SMALL PART OF A SYSTEM (BIFORCATION DIAGRAM) LOOK LIKE THE WHOLE SYSTEM

FEIGENBAUM 1975



MEASURING GAPS BETWEEN BRANCHES r_1 HAVE Δ_1, r_2 WITH Δ_2, r_3 WITH Δ_3, ...

$\frac{\Delta_K}{\Delta_{K+1}} \xrightarrow{K \to \infty} \alpha \cong 2.50$

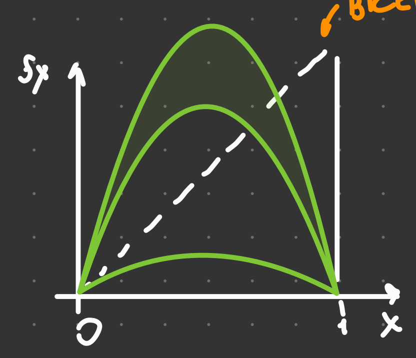
DELTA ON A GIVEN LEVEL AND ON THE FOLLOWING LEVEL WITH K TO ∞ CONVERGES TO ALPHA

$\frac{r_K - r_{K-1}}{r_{K+1} - r_K} \longrightarrow \delta \cong 4.67$

DISTANCE BETWEEN TWO BIFORCATION POINT

- THE FRACTAL HAVE WELL DEFINED LIMIT. THE DISTANCES THAT ARE INVOLVED CONVERGES TO NUMBERS

"SCALE INVARIANT OBJECT"



$\boxed{\alpha \text{ AND } \delta \text{ ARE UNIVERSAL}}$

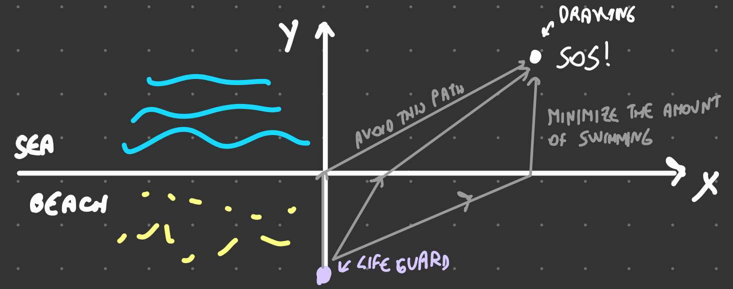
THE QUANTITIES ARE THE SAME. NOT DEPENDS ON THE SPECIFIC SYSTEM.

DESCRIPTION OF A SYSTEM → $\boxed{\text{(LAGRANGE) MECHANICS}}$ → ORDINARY DIFFERENTIAL EQUATION (ODE)

↓

OTHER VERSION OF THE BOX
$F = ma$ (NEWTONIAN MECHANICS)

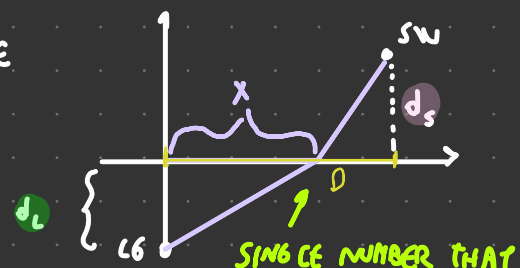
OPTIMIZATION PRINCIPLES (VARIATIONAL)



GET TO THE SWIMMER ASAP
RUNNING SAND > SWIMMING IN WATER

WE CARE ABOUT TIME AND NOT DISTANCE
I WANT TO MINIMIZE TIME

CAN BE REPRESENTED BY A SINGLE VALUE



SINGLE NUMBER THAT PARAMETRIZE EVERY POSSIBLE SOLUTION

- FIND EXPRESSION TOTAL TIME AS A FUNCTION OF X $T(x)$
- MINIMIZE $T(x)$ ($\bar{x} = \text{ARG} \underset{(\text{MIN})}{\text{INF}}\ T(x)$) ← BEST TRAJECTORY

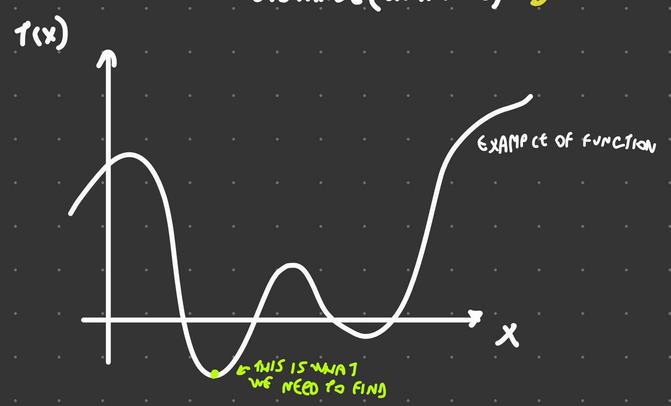
$V_{\text{RUNNING}} = v_r$

$V_{\text{SWIMMING}} = v_s$

DISTANCE (ON X-AXIS) = D

TIME SPLIT IN 2 PARTS: ON BEACH AND SWIMMING

$$T(x) = \underbrace{\frac{1}{v_r}\sqrt{x^2 + d_L^2}}_{\text{RUNNING}} + \underbrace{\frac{1}{v_s}\sqrt{(D-x)^2 + d_s^2}}_{\text{SWIMMING}}$$



DERIVATIVE TO FIND THE MINIMUM

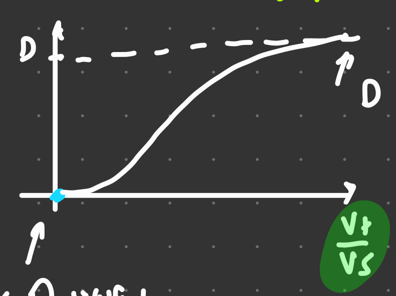
$$\frac{d}{dx}T(x) = 0$$

$$\frac{1}{v_r}\frac{x}{\sqrt{d_L^2 + x^2}} - \frac{1}{v_s}\frac{D-x}{\sqrt{d_s^2 + (D-x)^2}} = 0$$

MESS TO FIND THE RESULT

$$\frac{x}{\sqrt{d_L^2 + x^2}} - \frac{v_r}{v_s}\frac{D-x}{\sqrt{d_s^2 + (D-x)^2}} = 0$$

ONLY MOMENT WHERE VELOCITY APPEAR IS THIS RATIO



D WHEN $\frac{V_r}{V_S}$ GOES TO ∞

$\frac{V_r}{V_s}$

IS 0 WHEN

$\frac{V_r}{V_s} = 0$