$$f(x) = \begin{cases} x^3 & \text{if } x \geq 0 \\ (-x)^3 & x < 0 \end{cases}$$

$$f'(x) = \begin{cases} 3x^2 & x \geq 0 \\ -3x^2 & x < 0 \end{cases}$$

$$\lim_{x \to 0^+} 3x^2 = \lim_{x \to 0^-} -3x^2 = f(0)$$

$$\downarrow \qquad \downarrow \qquad \downarrow$$

$$0^+ \qquad -0^+ \qquad 0$$

$$\frac{f'(x)}{1!} x^1$$

2) $2 + x + 3x^2 - x^3$

$$f(0) + \frac{f'(x}{1} \; 1$$

$$\frac{e^0}{2!} x^2 \qquad \frac{e^0}{3!} x^3$$

e^{-x} $a=1$

$f(1) + \frac{f'(1)}{1!}(x-A)^1 + \frac{f''(1)}{2!}(x-A)^2 + \frac{f'''(1)}{3!}(x-A)^3$

$\downarrow$

$e + e(x-1) + \frac{e}{2!}(x-1)^2$

$f'(x) = e^x + xe^x$

$f''(x)\ e^x + e^x + xe^x$

$f'''(x) = 3e^x + xe^x$

$f(1) + \frac{f'(1)}{1!} \cdot (x-1) +$

9

$\frac{1}{e} + -\frac{1}{e}(x-1)$

xe^x $3+0$

e^{-x}

$-e^{-x}$

$-e^{-1}$

$\frac{f'''(0)x^3}{3!}$

$\frac{1}{2}x^3$ $\frac{3x^3}{3!}$

$f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3$

$\downarrow$ $\downarrow$

0

$$f(1) + \frac{f'(1)}{1!}x + \frac{f''(1)}{2!}x^2 + \frac{f'''(1)}{3!}x^3$$

$\frac{1}{x}$

x^{-1}

$-1x^{-2}$

$$1 + (-1x^{-2}x) + \left(\frac{+2x^{-3}x^2}{2!}\right) + \left(\frac{-6x^{-4}}{3!}(x-1)^3\right)$$

$$-1(x-1)^3$$

$$\cancel{x} - x^3 + 1 + 3x^2 - 3x$$

$$1 - x^3 + 3x^2 - 3x$$

$$f(a) + \frac{f'(a)}{1!}x^1 + \frac{f''(a)}{2!}x^2 + \frac{f'''(a)}{3!}x^3$$

$f(x) = \ln(x)$

$f'(x) = \frac{1}{x}$

$f''(x) = -x^{-2}$

$f'''(x) = \boxed{2x^{-3}}$

$$\frac{f'''(2)(x-2)^3}{3!}$$

$$\frac{2 \cdot 2^{-3}(x-2)^3}{3!}$$

$$\frac{\frac{1}{2^3}}{3!}(x-2)^3$$

$\frac{2}{x^3}$ $\frac{1}{3!}$

$\cancel{2} \cdot 2^{-3} \cdot \frac{1}{3 \cdot \cancel{2}}$

$\frac{1}{8 \cdot 3} (x-2)^3$

$\frac{1}{x^3}$

$f(x) = x^{-2}$

$f'(x) = -2x^{-3}$

$f''(x) = 6x^{-4}$

$f'''(x) - 24x^{-5}$

$$\frac{f'''(-1)\ (x-1)^3}{3!}$$

$$\frac{-24(-1)^{-5}(x-1)^3}{3!}$$

$\downarrow$

$$\frac{24(x-1)^3}{3 \cdot 2}$$

$\boxed{4(x+1)^3}$

$$f(x) = \cos(x)$$

$$f'(x) = -\sin(x)$$

$$f''(x) = -\cos(x)$$

$$f'''(x) = \sin(x)$$

f' $\quad$ f'' $\quad$ f'''

$$f(\pi) - \frac{\sin(x)(x-\pi)}{1!} - \frac{\cos(x)(x-\pi)^2}{2!} + \frac{\sin(x)(x-\pi)^3}{3!}$$

$$-1 - 0 \qquad \frac{1\ (x-\pi)^2}{2}$$

$$f(\pi) + \frac{f'(\pi)(x-\pi)}{1!} + \frac{f''(\pi)(x-\pi)^2}{2!}$$

$$f(x) = \sin(2x)$$

$$f'(x) = 2\cos(2x)$$

$$f''(x) = -4\sin(2x)$$

$$0 \qquad 2(x-\pi) \qquad \frac{-\cancel{4}^{\,2}(x-\pi)^2}{\cancel{2}}$$

$$2(x-\pi) - 0$$

$x \sin(x)$

$f(x) = x\sin(x)$

$f'(x) = x\cos(x) + \sin(x)$

$f''(x) = -\sin(x)x + \cos(x) + \cos(x)$

$2\cos(x) - x\sin(x)$

$$\frac{f''(x)\left(x-\frac{\pi}{2}\right)^2}{2!}$$

$$\frac{\left(0-\frac{\pi}{2}\right)\left(x-\frac{\pi}{2}\right)^2}{2!}$$

$$-\frac{\pi}{4}\left(x-\frac{\pi}{2}\right)^2$$

$f(x) = \dfrac{\cos(x)}{\sin(x)}$

$f'(x) = \dfrac{-\sin(x)\sin(x) - \cos(x)\cos(x)}{\sin^2(x)}$

$$\frac{f''(x)(x-1)^2}{2!}$$

$f'(x) = \dfrac{-\sin^2(x) - \cos^2(x)}{\sin^2(x)}$

$f'(x) = -1 - \dfrac{\cos^2(x)}{\sin^2(x)}$

$f'(x)\ -1 - \dfrac{\cos^2}{\sin^2(x)}(x)$

$f''(x) =$

$$\frac{-}{\sin^4(x)}$$

3A

$$f(x) + \frac{f'(x)(x-1)}{1!} + \frac{f''(x)(x-1)^2}{2!} + \frac{f'''(x)(x-1)^3}{3!} + \frac{f''''(x)(x-1)^4}{4!}$$

$$\log(1) + \frac{3(x-1)}{1+3x} + \frac{-9 \quad (x-1)^2}{(9x^2+6x+1)\,2!} + \frac{18(9x^2+6x+1)(18x+6)(x-1)^3}{3!\,(9x^2+6x+1)^2}$$

$$+ \frac{\qquad}{4!\,(9x^2+6x+1)^4}(x-1)^4$$

$$(324x+108)(9x^2+6x+1)$$

$$2916x^3+1944x^2+324x+972x^2+648x+108$$

$$2916x^3+2916x^2+972x+108$$

$$\frac{(8748x^2+5832x+972)(9x^2+6x+1)-(2916x^3+2916x^2+972x+108)}{(4(9x^2+6x+1)(18x+6))}$$

b) $f(x)\cos(x^2)$

$f'(x) \quad -2x\sin(x^2)$

$f''(x) = -4x^2\cos(x^2)$

$f'''(x) = 8x^3\sin(x^2)$

$1 + 0 +$

3c)- $\cos(1+\sin(x)) \quad x_0 = 0 \quad N=3$

$f'(0) = \dfrac{\cos(x)}{1+\sin(x)} = 1$

$f''(0) = \dfrac{\cancel{-\sin(x)(1+\sin(x))} - \cos(x)(\cos(x))}{1+\cancel{\sin^2}(0) + 2\cancel{\sin}(0)} = -\cos^2(x)$

$f'''(0) = 2\sin(x)$

$0 - x + \dfrac{-2\cos(x)(-1)^2}{3!} + \dfrac{0}{4!}$

e^{x-1} $x_0 = 2$ $N = 3$

$f(2) = e^{x-1}$

$f'(2) = e^{x-1}$

$f''(2) = e^{x-1}$

$f'''(2) = e^{x-1}$

$e + \frac{e}{2!}^{(x-2)} + \frac{e}{3!}^{(x-2)^2} + \frac{e}{4!}(x-3)^3$

5a)

$$\lim_{x \to 0} \frac{\sin(x) + \log(1-x)}{x^2} \qquad \frac{0+0}{0}$$

TAYLOR

$$f(0) = \sin(x) + \log(1-x) = 0+0$$

$$f'(0) = -\cos(x) + \frac{-1}{1-x} = -2$$

$$f''(0) = -\sin(x) - 1(1-x)^{-2} \cdot (-1) = 0+1 = 1$$

TAYLOR

SIN

TAYLOR LOG

$$T_n(x) = f(0) + f'(0)(x-0) + \frac{f''(0)(x-0)^2}{2}$$

$$0 + -2x + \frac{x^2}{2}$$

$$\frac{-2x + \frac{x^2}{2}}{x^2} = \quad -\frac{2}{x} + \frac{1}{2} = -\infty$$

e) $$\lim_{x \to 0} \frac{\log(1-x^4)}{e^{x^2} - 1 - x^2}$$

$$\frac{\frac{-4x^3}{1-x^4}}{2x \cdot e^{x^2} - 2x} = \frac{-4x^3}{(1-x^4)(2x e^{x^2} - 2x)}$$

$$\frac{-2x^2}{(1-x^4)(e^{x^2}-1)} \quad \left[\frac{0}{0}\right]$$

HOPITAL

$$\lim_{x \to 0} \frac{-\overset{2}{\cancel{4}}\cancel{x}}{-\overset{2}{\cancel{4}}x^{\overset{2}{\cancel{3}}}(e^{x^2}-1) + (1-x^4)(\cancel{2}\cancel{x}e^{x^2})} = -2$$

f) $\lim_{x\to 0} \dfrac{\cos(1-x^3)}{e^{x^2}-1-x^2}$

HOPITAL $\left[\frac{0}{0}\right]$

$\lim_{x\to 0} \dfrac{f(x)}{g(x)} = \lim_{x\to 0} \dfrac{f'(x)}{g'(x)}$

$$\dfrac{\dfrac{-3x^2}{1-x^3}}{2xe^{x^2}-2x}$$

$$\lim_{x\to 0} \dfrac{-3x^{\cancel{2}}}{(1-x^3)(2\cancel{x}e^{x^2}-2\cancel{x})}$$

$$\lim_{x\to 0} \dfrac{-3x}{(1-x^3)(2e^{x^2}-2)}$$

$\left[\frac{0}{0}\right]$ HOPITAL

$\lim_{x\to 0} \dfrac{f(x)}{g(x)} = \lim_{x\to 0} \dfrac{f'(x)}{g'(x)}$

$$\lim_{x\to 0} \dfrac{-3}{-3x^2(2e^{x^2}-2)+(1-x^3)(2e^{x^2}\cdot 2x-2)}$$

$$\lim_{x\to 0} \dfrac{-3}{-6x^2(e^{x^2}-1)+2(1-x^3)(xe^{x^2}-1)} =$$

$$\dfrac{-3}{2\cdot(-1)} = \boxed{\dfrac{3}{2}}$$

g) $\lim_{x\to 0} \dfrac{\cos(x^2-\sin^2 x+1)}{e^{x^2}-1-x^2}$ $\left[\frac{0}{0}\right]$

HOPITAL

$\lim_{x\to 0} \dfrac{f(x)}{g(x)} = \lim_{x\to 0} \dfrac{f'(x)}{g'(x)}$

$$\lim_{x\to 0} \dfrac{\dfrac{2x-2\sin(x)\cos(x)}{x^2-\sin^2 x+1}}{2xe^{x^2}-2x}$$

$$\lim_{x\to 0} \dfrac{\cancel{2}x-\cancel{2}\sin(x)\cos(x)}{(x^2-\sin^2 x+1)(\cancel{2}xe^{x^2}-\cancel{2}x)}$$

$$\lim_{x\to 0} \dfrac{x-\sin(x)\cos(x)}{(x^2-\sin^2(x)+1)(xe^{x^2}-x)} \quad \left[\frac{0}{0}\right]$$

HOPITAL

$\sin^2(x)+\cos^2(x)=1$

$$\lim_{x\to 0} \dfrac{-\cos^2(x)-\sin^2(x)}{(2x-2\sin(x)\cos(x))(xe^{x^2}-x)+(x^2-\sin^2(x)+1)(2x^2e^{x^2}+e^{x^2}-1)}$$

$$\lim_{x\to 0} \dfrac{-1}{0+0} = -\infty$$

H) $\lim_{x \to 0} \frac{x\left(2e^{-x} - 2 + 2x - x^2\right)}{(\cos(x) - 1)^2} \quad \left[\frac{0}{0}\right]$

$$\lim_{x \to 0} \frac{2xe^{-x} - 2x + 2x^2 - x^3}{\cos^2(x) + 1 - 2\cos(x)}$$

HOPITAL 1°

$$\lim_{x \to 0} \frac{2e^{-x} - 2xe^{-x} - 2 + 4x - 3x^2}{-2\cos(x)\sin(x) + 2\sin(x)}$$

HOPITAL 2°

$$\lim_{x \to 0} \frac{-4e^{-x} + 2xe^{-x} + 4 - 6x}{2\sin^2(x) - 2\cos^2(x) + 2\cos(x)}$$

$$\lim_{x \to 0} \frac{-\overset{2}{\cancel{4}}e^{-x} + \cancel{2}xe^{-x} + \overset{2}{\cancel{4}} - \overset{3}{\cancel{6}}x}{\overset{2}{\cancel{4}}\sin^2(x) - \overset{1}{\cancel{2}} + \cancel{2}\cos(x)}$$

$2\sin^2(x) + 2\cos^2(x) = 2$

$$\lim_{x \to 0} \frac{-2e^{-x} + xe^{-x} + 2 - 3x}{2\sin^2(x) - 1 + \cos(x)}$$

HOPITAL 3°

$$\lim_{x \to 0} \frac{2e^{-x} + e^{-x} - xe^{-x} - 3}{4\sin(x)\cos(x) - \sin(x)}$$

$$\lim_{x \to 0} \frac{3e^{-x} - xe^{-x} - 3}{4\sin(x)\cos(x) - \sin(x)}$$

HOPITAL 4°

$$\frac{-3e^{-x} - e^{-x} + xe^{-x}}{-4\sin^2(x) + 4\cos^2(x) - \cos(x)}$$

$$\lim_{x \to 0} \frac{-4e^{-x} + xe^{-x}}{-4\sin^2(x) + 4\cos^2(x) - \cos(x)}$$

$$\frac{-4 + 0}{0 + 4 - 1} = \boxed{-\frac{4}{3}}$$

(i) $\lim\limits_{x\to 0} \dfrac{\log(1+x^2)-x^2}{2x^3\sin(x)}$

$$\frac{\log(1+x^2)}{2x^3\sin(x)} - \frac{1}{2x\sin(x)}$$

$$\lim_{x\to 0}\frac{\log(1+x)}{x} = 1$$

$$\frac{1}{2x\sin(x)} - \frac{1}{2x\sin(x)} = 0$$

$$\frac{\frac{2x}{1+x^2} - 2x}{6x^2\sin(x)+2x^3\cos(x)}$$

$$\frac{\cancel{2x}-\cancel{2x}-2x^3}{1+x^2}\cdot\frac{1}{6x^2\sin(x)+2x^3\cos(x)}$$

$$\frac{-2x^3}{(1+x^2)(6x^2\sin(x)+2x^3\cos(x))}$$

$$\lim_{x\to 0}\frac{-2x^3}{6x^2\sin(x)+2x^3\cos(x)+6x^4\sin(x)+2x^5\cos(x)}$$

$$\frac{-2x}{6\sin(x)+2x\cos(x)+6x^2\sin(x)+2x^3\cos(x)}$$

J

$$\lim_{x \to 0} \frac{\log(1+x^2) - x^2}{2x^3 \sin(x)} \qquad \frac{0}{0}$$

HOPITAL

$$\lim_{x \to 0} \frac{\frac{2x}{1+x^2} - 2x}{6x^2 \sin(x) + 2x^3 \cos(x)}$$

$$\lim_{x \to 0} \frac{\cancel{2x} - \cancel{2x} - 2x^3}{(1+x^2)(6x^2 \sin(x) + 2x^3 \cos(x))}$$

$$\lim_{x \to 0} \frac{-x}{(1+x^2)(3\sin(x) + x\cos(x))} \qquad \left[\frac{0}{0}\right]$$

$$\lim_{x \to 0} \frac{-x}{3\sin(x) + x\cos(x) + 3x^2 \sin(x) + x^3 \cos(x)}$$

HOPITAL

$$\lim_{x \to 0} \frac{-1}{3\cos(x) + \cos(x) - x\sin(x) + 6x\sin(x) + 3x^2\cos(x) + 3x^2\cos(x) - x^3 \sin(x)}$$

$$\lim_{x \to 0} \frac{1}{3+1-0+0+0+0-0} = \boxed{-\frac{1}{4}}$$

H

$$\lim_{x \to 0} \frac{e^{x^2} - 1 - x^2 + 3x^4}{\cos(2x^2) - 1} \qquad \frac{0}{0}$$

HOPITAL

$$\lim_{x \to 0} \frac{\cancel{2x} \cdot e^{x^2} - \cancel{2x} + \cancel{12}^{\,6} \cancel{x^3}^{\,2}}{-\cancel{4}_{\,2}\cancel{x}\,\text{sen}(2x^2)}$$

$$\lim_{x \to 0} \frac{e^{x^2} - 1 + 6x^2}{-2\,\text{sen}(2x^2)} \qquad \left[\frac{0}{0}\right]$$

HOPITAL

$$\lim_{x \to 0} \frac{\cancel{2x}\, e^{x^2} + \cancel{12}^{\,6}\cancel{x}}{-\cancel{8}_{\,4}\cancel{x}\cos(2x^2)}$$

$$\lim_{x \to 0} \frac{e^{x^2} + 6}{-4\cos(2x^2)} = -\frac{7}{4}$$

(L)

$$\lim_{x \to 0^+} \frac{2\cos(\sqrt{x}) - 2 + x}{\arctan(3x))^2} \quad \left[\frac{0}{0}\right]$$

HOPTAL

$$\lim_{x \to 0^+} \frac{-\cancel{2}\sin(\sqrt{x}) \cdot \frac{1}{2\sqrt{x}} + 1}{3\,\cancel{6}\arctan(3x) \cdot \frac{1}{9x^2+1}}$$

*

$$\lim_{x \to 0} \frac{\frac{-\sin\sqrt{x} + 2\sqrt{x}}{2\sqrt{x}}}{\frac{3\arctan(3x)}{9x^2+1}}$$

$$\lim_{x \to 0} \frac{-\sin\sqrt{x} + 2\sqrt{x}}{2\sqrt{x}} \cdot \frac{\overset{3}{\cancel{9}}x^2+1}{\underset{1}{\cancel{3}}\arctan(3x)} \quad \left[\frac{0}{0}\right]$$

HOPITAL

$$\frac{-2x\sin\sqrt{x} - x^2\cos\sqrt{x} \cdot \frac{1}{2\sqrt{x}} + \frac{1}{\sqrt{x}} \cdot x^2}{2\sqrt{x}\,\frac{3}{9x^2+1} + \frac{\cancel{2}}{\cancel{2}\sqrt{x}}\arctan(3x)}$$

$$\frac{\qquad}{\frac{\qquad}{9x^2+1}}$$

$$f(x)=\begin{cases}\dfrac{\sin(4x)}{e^{2x}-1} & \text{IF } x<0\\ 2b & \text{IF } x=0\\ b^2-2b\,\dfrac{\log(1+x)-x}{x^2} & \text{IF } x>0\end{cases}$$

$$f(0)=\lim_{x\to0^+} f(x)=\lim_{x\to0^-} f(x)$$

$$f(0)=2b$$

$$\lim_{x\to0^-}\frac{\sin(4x)}{e^{2x}-1}\quad\frac{4x}{4x}\ ;\quad \lim_{x\to0^-}\frac{4x}{e^{2x}-1}\quad\left[\frac{0}{0}\right]$$

HOPITAL

$$\lim_{x\to0^-}\frac{\cancel{4}\,2}{\cancel{2}_{1}\,e^{2x}}=\frac{2}{e^{2x}}=2$$

$$2b=2;\quad b=1$$

$$\lim_{x\to0^+} b^2-2b\,\frac{\log(1+x)-x}{x^2}=2$$

$$\lim_{x\to0^+} 1+\frac{-2\log(1+x)+2x}{x^2}\quad\left[\frac{0}{0}\right]$$

HOPITAL

$$\lim_{x\to0}\frac{\frac{-2}{1+x}+2}{2x}=\lim_{x\to0^+}\frac{\frac{\cancel{-2}+\cancel{2}+2x}{1+x}}{2x}$$

$$\lim_{x\to0^+}\frac{2x}{1+x}\cdot\frac{1}{2x}\quad\boxed{1}$$

$$1+\lim_{x\to0}\frac{-2\log(1+x)+2x}{x^2}=1+1=2$$

$$b=2$$

RIFA

l) $\lim_{x\to 0^+} \dfrac{2\cos(\sqrt{x})-2+x}{(\arctan(3x))^2}$ $\left[\frac{0}{0}\right]$

HOPITAL

$$\lim_{x\to 0^+} \frac{\dfrac{-\cancel{2}\,\text{sen}(\sqrt{x})}{\cancel{2}\sqrt{x}}+1}{2\arctan(3x)\cdot 3\cdot \dfrac{1}{1+9x^2}} = \frac{\dfrac{\text{sen}(\sqrt{x})+\sqrt{x}}{\sqrt{x}}}{\dfrac{6\arctan(3x)}{1+9x^2}}$$

1

$$\lim_{x\to 0^+} \frac{-\text{sen}(\sqrt{x})+\sqrt{x}}{\sqrt{x}}\cdot\frac{1+9x^2}{6\arctan(3x)} \quad \left[\frac{0}{0}\right]$$

$$\lim_{x\to 0} \frac{-\text{sen}(\sqrt{x})+\sqrt{x}-9x^2\text{sen}(\sqrt{x})+9x^2\sqrt{x}}{6\sqrt{x}\arctan(3x)} \quad \left[\frac{0}{0}\right]$$

HOPITAL

$$\lim_{x\to 0} \frac{\dfrac{-\cos(\sqrt{x})}{2\sqrt{x}}+\dfrac{1}{2\sqrt{x}}-18x\,\text{sen}(\sqrt{x})-\dfrac{9x^2\cos(\sqrt{x})}{2\sqrt{x}}+18x\sqrt{x}+\dfrac{9x^2}{2\sqrt{x}}}{\dfrac{6}{2\sqrt{x}}\arctan(3x)+\dfrac{18\sqrt{x}}{1+9x^2}}$$

$$\lim_{x\to 0} \frac{\dfrac{-\cos(\sqrt{x})+1-36x\sqrt{x}\,\text{sen}(\sqrt{x})-9x^2\cos(\sqrt{x})++36x^2+9x^2}{2\sqrt{x}}}{\dfrac{3}{\sqrt{x}}\arctan(3x)+\dfrac{18\sqrt{x}}{1+9x^2}}$$