$+$ $\times$

$-$ $\div$

POWER LAWS

IF I CHANGE BOTH Y AND X APPROPRIATELY GET BACK EXACTLY THE SAME FUNCTION

- $y = c\, x^{\alpha}$ ← SCALE INVARIANT

- EXPONENTIAL ARE NOT SCALE INVARIANT

$$c \exp(\beta x)$$

THE CHARACTERISTIC SCALE

$$x_0 = \frac{1}{\beta}$$

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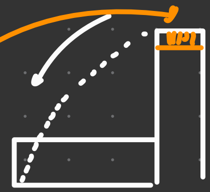
STILL SCALE INVARIANT

- $\lambda = 100$ SCALE DOWN of 100 (100m → 1m)

SKYSCRAPER OF IT FALLING FOR RECORD THE MOVIE. HOW SLOW SHOULD I RECORD THE MINIATURE TO MAKE IT REALISTIC



DON'T CARE ABOUT THE BUILD, JUST FOCUS TOP PART (SIMPLIFIED ASSUMPTION)

$V_0 = 0$ m/s

$\Delta s = -\frac{1}{2} g t^2$

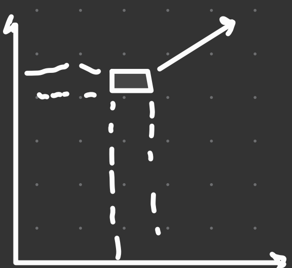
$t \propto \sqrt{\Delta s}$

$s' = \lambda s$ $\quad \Delta s = \frac{\Delta s'}{\lambda}$

$\Delta s' = -\frac{1}{2}\underbrace{\lambda g}\, t^2 \longrightarrow$ DEFINE IT S.T. $(t')^2 = \lambda t^2 \rightarrow \Delta s' = -\frac{1}{2} g (t')^2$

INSTEAD OF g. g DOES NOT CHANGE BETWEEN THE MODEL IN THE STUDIO AND THE SKYSCRAPER

$t' = \sqrt{\lambda}\, t$ ∎



CASE 2 PIECE EXPLODE W/ HIGH VELOCITY

$\Delta S = V_0 t$

$\Delta s' = \lambda V_0 t$

CAN NOT REPEAT SAME ARGUMENT AS BEFORE. THE ONLY CONSTANT IS V_0 THAT IS NOT UNIVERSAL →

IF IT WAS A UNIV. CONSTANT (LIKE c FOR LIGTH) I SHOULD SCALE THE TIME LINEARLY

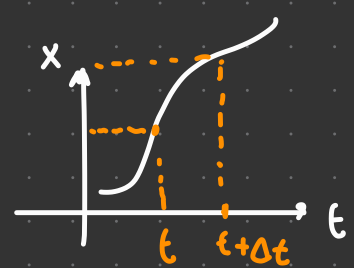
IN THIS NEW COORDINATE SYSTEM.

NOTE: $\left[\frac{dA}{dB}\right] = \frac{[A]}{[B]}$

X AS A FUNCTION OF t

$$\frac{dx}{dt} = \lim_{\Delta t \to 0} \frac{x(t-\Delta t) - x(t)}{\underbrace{\cancel{t} + \Delta t - \cancel{t}}}$$

SAME DIMENSION OF $\frac{dx}{dt}$: X ON TOP AND Δt DOWN



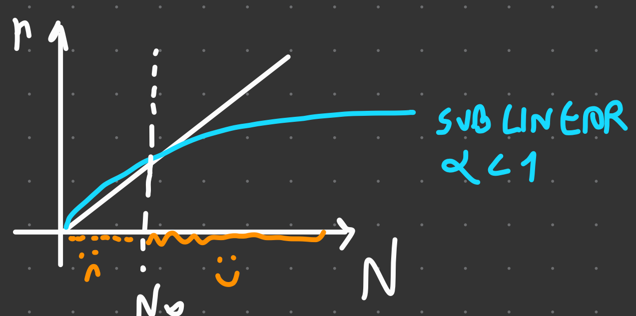
THE DIMENSION DOES NOT CHANGE W/ THE LIMIT

$$\left[\frac{dx}{dt}\right] = \frac{[x]}{[t]}$$

SCALING ARGUMENT

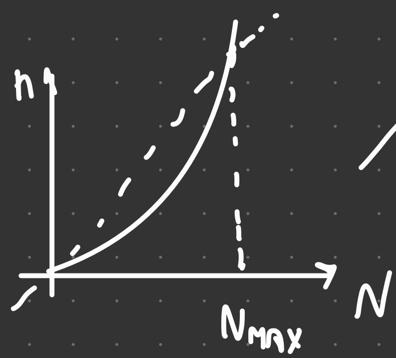
1. PREDICTING $C = AB$
2. PREDICTING THE EXISTENCE OF BOUNDS
3. USEFUL FOR MODELING

$$N \overset{?}{=} n \propto N^{\alpha > 1}$$



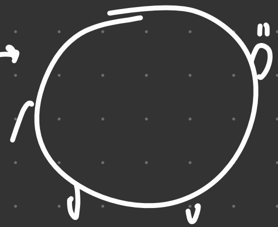
THE PROPORTIONALITY ARE GOOD IN "FAR" CASES

BOUND ABOVE WHICH IT DOES NOT HOLD

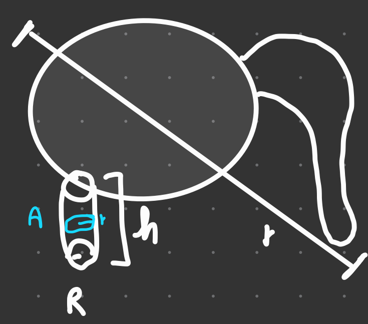


"SPHERICAL COW" MODELS

DESCRIBE A COW AS A SPHERE
FIRST APPROXIMATION



ELEPHANT



WEIGHT FORCE

$F_W = c_1 r^3 \rho g$

$F_{MAX} = c_2 A$

SURFACE AREA ON A SECTION OF A CYLINDER

FORCE OF LEG BEFORE IT BREAKS

$" C_3 R^2 = c_4 r^2$

$F_{MAX} = c_3 R^2$

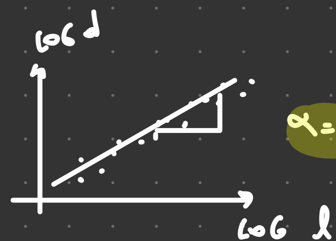
$F_W \leq F_{MAX}$

$c_1 r^3 \rho g = c_4 r^2 \Rightarrow r = \dfrac{c_4}{c_1 \rho g}$

IT BREAKS IN THIS CASE

CONSTANT

A NAIL



$\alpha = 2/3$

F

$F < F_b$ IF $F > F_B$ IT IS FAVORABLE TO BEND

F

F_B

BUCKLING (AKA BENDING)

MINIMIZE THE FORCE F NEED TO THE NAIL

$F \propto d$ → F DEPENDS ON d MINIMIZE d TO Ø

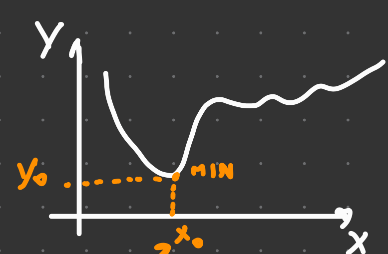
$\mathrm{ARG\,MIN}_d\, f(d)$

LARGER DIAMETER → LARGE FORCE CAN BE SUSTAINED

$$F_B \propto \frac{d^4}{l^2}$$

$Y_0 = \mathrm{MIN}\ y(x)$

$x_0 = \mathrm{ARG\ MIN}\ y(x)$

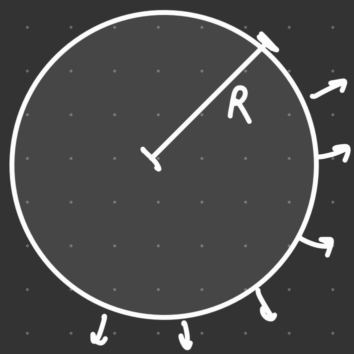


$d = \text{ARG MIN } f(d)$

$d \propto \frac{d^4}{\ell^2} \Rightarrow d^3 \propto \ell^2 \Rightarrow d \propto \ell^{2/3}$ IS WHAT WE EXPECTED

CONSIDERING SPHERICAL COW

$$\left.\begin{array}{l} G \propto A \propto R^2 \\ M = \rho V \propto R^3 \end{array}\right\} \Rightarrow R \propto M^{1/3}, \quad G \propto M^{2/3} \quad 2/3 = \alpha$$



ASSUMPTION: ρ IS CONSTANT

ρ = CONSTANT

IF $A \propto B$ THEN $B \propto A$

ALSO IF $A \propto \frac{C}{B}$ THEN $AB \propto C$