$+$ $\times$

$-$ $\div$

1 The Reyleigh algorithm allows one to

1. check whether a given physical quantity, as a function of other quantities, has the correct units of measure
- find a formula for a given physical quantity, as a function of other quantities, such that it is dimensionally correct
3. find how two quantities scale with each other, starting from their scaling relations with a third one
4. check whether a scaling argument yields a result that has the same dimensions on both sides of the equation

2 What are the dimensions of pressure, in a system where the fundamental dimensions are energy, time, length?

PRESSURE $= \frac{F}{A}$

1. force/length^2 N/M^2
2. NO energy*time/length $\frac{Fm}{M}$
3. mass/(length*time^2) $\frac{Kg}{M\,s^2}$
4. energy/length^3 $\frac{Fm}{M^3} = \frac{F}{M^2}$

$F = Kg\,\frac{M}{s^2}$

3 The power P (energy per unit time) consumed by an electric heater, of linear size r, scales as $P \propto r^{\alpha}$. To reach a certain temperature in a room, the heater must be turned on for a time t that scales as $t \propto r^{-\beta}$. The total energy consumed during this time is E. How does E scale with the size of the heater?

$P = \frac{E}{T}$ $\quad r \quad$ $P \propto r^{\alpha}$ $\quad T \propto r^{-\beta}$ $\quad E = PT = r^{\alpha} r^{-\beta}$

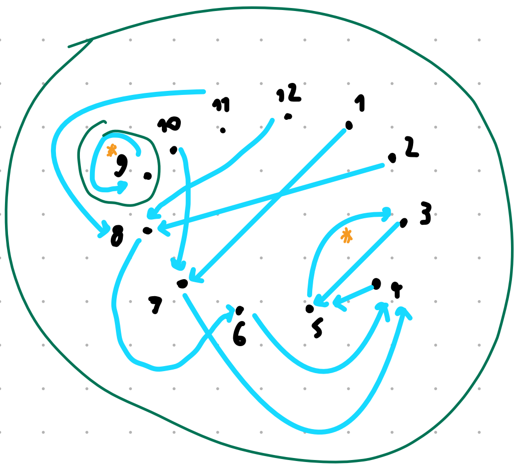
1. ● $E \propto r^{\alpha-\beta}$
2. $E \propto r^{\beta-\alpha}$
3. $E \propto r^{-\alpha/\beta}$
4. $E \propto r^{\beta/\alpha}$

4 Consider the dynamical system $x_{n+1} = f(x_n)$, with discrete time n and discrete state $x \in \{1,2,3,4,5,6,7,8,9,10,11,12\}$. The function $f(x)$ is the number of letters in the English name for the xth month ($1 =$"January", $2 =$"February", $3 =$"March", $4 =$"April", $5 =$"May", $6 =$"June", $7 =$"July", $8 =$"August", $9 =$"September", $10 =$"October", $11 =$"November", $12 =$"December"). The dynamical system has

N (TIME) DISCRETE

$x_{N+1} = f(x_N)$ x (STATE) DISCRETE $x \in \{1, \dots 12\}$

1. 1 cycle and 1 connected component
2. 2 cycles and 1 connected component
3. 1 cycle and 2 connected components
4. → 2 cycles and 2 connected components



2 CYCLE

2 CONNECTED COMPONENT

5 Consider the dynamical system (with continuous time and continuous state) specified by the following ordinary differential equation:

$$\dot{x}(t) = \cos\left(\frac{\pi}{2}x(t)\right)$$

Which one of the following statements is correct?

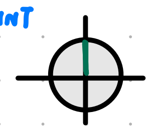
1. The point $x = 0$ is a fixed point and it is stable
2. The point $x = 0$ is a fixed point and it is unstable
3. ● The point $x = 1$ is a fixed point and it is stable
4. The point $x = 1$ is a fixed point and it is unstable

$$\dot{x}(t) = \cos\left(\frac{\pi}{2}x(t)\right)$$

1° FIND FIXED POINT

$x = \cos\left(\frac{\pi}{2}x\right)$

$1 = \cos(\pi/2)$



2° DERIVATE

$f'(x) = -\frac{\pi}{2}\sin\left(\frac{\pi}{2}x(t)\right)$

3° SUBSTITUTE

$f'(1) = -\frac{\pi}{2}\sin(\pi/2) = -\pi/2$

SINCE $-\pi/2 < 0$ THEN $\boxed{\text{X=1 IS A FIXED POINT AND STABLE}}$

6 The function $f(x)$ is the solution to a variational principle with respect to the action $S[f(\cdot)]$. This means that

???

1. the function $f(x)$ can be obtained by inverting the function S
2. the function $f(x)$ is a solution to the equation $S[f(\cdot)] = 0$
3. the action S, when evaluated on the derivative of $f(x)$ with respect to x, is 0
4. the action S, when evaluated on $f(x)$, is stationary

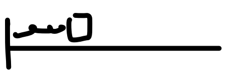
???

7 What is the Lagrangian of a mass attached to a spring with spring constant k, in 1 dimension? (The coordinate x here is the position of the mass.)

1. $L = \frac{m}{2}\dot{x}^2 + \frac{k}{2}x^2$
2. $L = \frac{m}{2}\dot{x}^2 - \frac{k}{2}x^2$
3. $L = -\frac{m}{2}\dot{x}^2 + \frac{k}{2}x^2$
4. $L = -\frac{m}{2}\dot{x}^2 - \frac{k}{2}x^2$

$K = \frac{1}{2}mv^2 = \frac{1}{2}m\dot{x}^2$

$U = \frac{1}{2}kx^2$



$L = K - U = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}kx^2$

EXTRA FIND $\ddot{x}$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) = \frac{\partial L}{\partial x} \Rightarrow m\ddot{x} = -kx \Rightarrow \ddot{x} = \frac{-kx}{m}$$

8 The equation of state of a thermodynamic system

1. is a relation that allows to compute the thermodynamic variables from the microscopic degrees of freedom
2. is a relation specifying how a specific thermodynamic transformation is performed
3. is a relation allowing to express temperature in terms of the other variables $T = N$
4. is the definition of each state variable that specifies the thermodynamic state of a system

9 Two samples of an ideal gas have thermodynamic variables (P, V, T) (pressure, volume and temperature of system A) and $(2P, 3V, 4T)$ (pressure, volume and temperature of system B). What is the ratio N_B/N_A? Here N_A is the number of molecules of system A and N_B that of system B.

1. 1/2
2. 2/3
3. 3/2
4. 2

P, V, T

$2P, 3V, 4T$

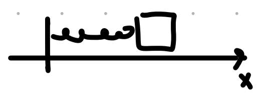
$k_B = \frac{PV}{TN_A} \Rightarrow N_A = \frac{PV}{Tk_B}$

$k_B = \frac{6PV}{4TN_B} \Rightarrow N_B = \frac{6}{4}\frac{PV}{Tk_B}$

$\frac{N_B}{N_A} = \frac{6}{4} = 3/2$

7 What is the Lagrangian of a mass attached to a spring with spring constant k, in 1 dimension? (The coordinate x here is the position of the mass.)

1. $L = \frac{m}{2}\dot{x}^2 + \frac{k}{2}x^2$
2. $L = \frac{m}{2}\dot{x}^2 - \frac{k}{2}x^2$
3. $L = -\frac{m}{2}\dot{x}^2 + \frac{k}{2}x^2$
4. $L = -\frac{m}{2}\dot{x}^2 - \frac{k}{2}x^2$



$U = \frac{1}{2}kx^2$

$K = \frac{1}{2}mv^2 = \frac{1}{2}m\dot{x}^2$

$\ddot{x}$

$L = K - U = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}kx^2$

$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) = \frac{\partial L}{\partial x} \Rightarrow m\ddot{x} = -kx$

$\ddot{x} = \frac{-kx}{m}$

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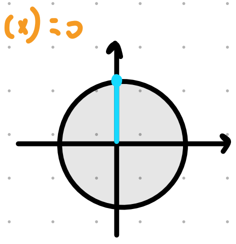
5 $\dot{x} = \cos\left(\frac{\pi}{2}x(t)\right)$

1 FIND FIXED POINT $f(x) = 0$

$0 = \cos\left(\frac{\pi}{2}x(t)\right)$

$\Downarrow$

$\boxed{x=1}$, 3, 5,



2 DERIVE $\dot{x} \Rightarrow \ddot{x}$

$\ddot{x} = -\frac{\pi}{2}\sin\left(\frac{\pi}{2}x(t)\right)$

3 CHECK WITH THE FIXED POINT

$\ddot{x} = -\frac{\pi}{2}\sin(\pi/2) = -\pi/2$

IF $f(\bar{x}) > 0$ UNSTABLE

IF $f(\bar{x}) < 0$ STABLE

SO $x = 1$ STABLE