PERMANENCE OF SIGN THEOREM

LET $\{a_n\}_{n\in\mathbb{N}} \subseteq \mathbb{R}$ BE A SEQUENCE OF REAL NUMBERS SUCH THAT $\exists \lim_n a_n = l \in [-\infty, +\infty]$

a) IF $l>0$ (INCLUDING $l=+\infty$) THEN $a_n>0$ DEFINETLY, THAN IS, $\exists n_0\in\mathbb{N}$ SUCH THAT $\forall n\geq n_0$ WE HAVE $a_n>0$

b) IF $l<0$ (INCLUDING $l=-\infty$) THEN $a_n<0$, DEFINETLY, THAN IS, $\exists n_0\in\mathbb{N}$ S.T. $\forall n\geq n_0$ WE HAVE $a_n<0$

PROOF)

a) $l\in\mathbb{R}$, $l>0$

HYP: $\forall \varepsilon>0$ $\exists n_0\in\mathbb{N}$ S.T. $\forall n\geq n_0$ WE HAVE $l-\varepsilon<a_n<l+\varepsilon$

PICK $\varepsilon>0$ S.T. $l-\varepsilon>0$ $\left(\varepsilon=\frac{l}{2}\right)$, THEN $\forall n\geq n_0$ WE HAVE $0<l-\varepsilon<a_n$ SO $a_n>0$, IN PARTICULAR $\exists n_0\in\mathbb{N}$ S.T. $\forall n\geq n_0$ $a_n>l/2$

THEN IF l IS POSITIVE ALSO a_n IS POSITIVE