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DIFFERENTIABILITY IMPLIES CONTINUITY

$f : A \subseteq \mathbb{R}^N \longrightarrow \mathbb{R}^m$ $\quad A$ open, $x^0 \in A$

f is differentiable in x^0, then f is continous in x^0

PROOF: we want to prove that $\lim_{x \to x^0} \left(f(x) - f(x^0) \right) = 0$;

$f(x) - f(x^0) = Jf(x^0)\left[x - x^0\right] + R_1(x)$

- $\lim_{x \to x^0} Jf(x^0)\left[x - x^0\right] = 0 = Jf(x^0)[0]$ by continuity

- $\lim_{x \to x^0} R_1(x) = \lim_{x \to x^0} \underbrace{\frac{R_1(x)}{\|x - x^0\|}}_{\to 0} \cdot \underbrace{\|x - x^0\|}_{\to 0} = 0$

$\Rightarrow f$ continous in x^0