Knowledge Representation and Reasoning

Exercise Session 1

Exercise 1. Truth Tables

(*)

Build the **truth tables** of the following formulas, remembering the precedence of the operators.

- 1. $\neg(x \land y) \lor z$
- 2. $(x \land y \lor \neg x \land \neg w) \land z$
- 3. $(x \lor y) \land x$
- 4. $\neg (p \land (\neg p \lor q \lor s)) \lor q \lor s$

Exercise 2. Boolean Functions

(*)

For each of the following truth tables, build a formula expressing the same Boolean function.

x	y	z	φ_1	_	\boldsymbol{x}	z	φ_2		x	y	z	w	φ_3
0	0	0	0		0	0	1		0	0	0	0	0
0	0	1	1		0	1	0		0	0	0	1	1
0	1	0	0		1	0	0		0	0	1	0	0
0	1	1	1		1	1	1		0	0	1	1	0
1	0	0	1				•		0	1	0	0	1
1	0	1	0						0	1	0	1	0
1	1	0	0						0	1	1	0	1
1	1	1	1						0	1	1	1	1
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									1	0	0	1	1
									1	0	1	0	0
									1	0	1	1	0
									1	1	0	0	1
									1	1	0	1	0
									1	1	1	0	0
									1	1	1	1	0

If the tables represent Boolean functions, what is their arity?

Exercise 3. Types of Formulas

(*)

Use truth tables (and, if needed the equivalences seen during the lecture) to verify whether the following formulas are tautologies, contradictions, or non-tautological satisfiable formulas.

- 1. $x \rightarrow \neg x$
- 2. $(x \to y) \land (\neg y \to \neg x)$
- 3. $(x \to y) \to (\neg y \to \neg x)$
- 4. $x \wedge y \wedge (\neg x \vee \neg y) \wedge z$

Exercise 4. NNF (**)

A formula is in **negation normal form** (NNF). For example, the second and third formulas in Exercise 1 are in NNF, while the other two formulas are not.

- 1. Using the DeMorgan equivalences, transform the formulas of Exercise 1 to NNF
- 2. Do you notice something from the constructions in Exercise 2?

Exercise 5. Sheffer Functions

(* * *)

We have seen that \neg, \land, \lor form a **functionally complete** set of logical connectives in the sense that every Boolean function can be described using those connectives. In fact, we \neg, \land and \neg, \lor are functionally complete by themselves.

A logical connective is called a **Sheffer function** or **sole sufficient operator** if it is functionally complete by itself.

- 1. show that the NAND connective (denoted as \uparrow) is a Sheffer function
- 2. are there other Sheffer functions?
- 3. could a unary connective be a Sheffer function?

Exercise 6. Knowledge Bases

(**)

For the following truth table, construct a knowledge base (expressed as a set of propositional rules) which represents the same knowledge.

\boldsymbol{x}	y	z	K
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

Exercise 7. Expressivity

(**)

Can any propositional formula be expressed as a knowledge base?

If the answer is **yes**, argue why; if it is **no**, give at least two examples of formulas which cannot be expressed.

Exercise 8. Reasoning

(*)

Consider the following knowledge base K:

$$\begin{array}{l} x \leftarrow \\ y \leftarrow x, \ z, \ w \\ x \leftarrow v \\ w \leftarrow y, \ z \\ z \leftarrow v, \ x \\ z \leftarrow y, \ w \\ z \leftarrow u, \ x \\ u \leftarrow \\ p \leftarrow \\ t \leftarrow w, \ u \\ r \leftarrow s, \ t \end{array}$$

- 1. Compute the redux \hat{K}
- 2. Find all the facts that are entailed by K
- 3. Decide whether the following clauses are consequences of K
 - a) $v \leftarrow u$
 - b) $t \leftarrow y$
 - c) $q \leftarrow q$
 - d) $r \leftarrow w$

Exercise 9. Revision

(**)

In the knowledge base from Exercise 8, substitute the fact $x \leftarrow$ with $u \leftarrow$. Call this new knowledge base K'.

- 1. Do your answers from Exercise 8 change?
- 2. Which fact(s) should you remove to ensure that z is **not** a consequence of K'?
- 3. If facts cannot be removed, which rules would you remove to ensure that z is not entailed?

Exercise 10. Tautologies

(* * *)

The soundness of the fact extraction algorithm seen during the lecture depends on the following property:

if $\varphi \to \psi$ and $\psi \to \xi$ are both tautologies, then $\varphi \to \xi$ is also a tautology.

Show that this property holds always in propositional logic.

Exercise 1. Truth Tables

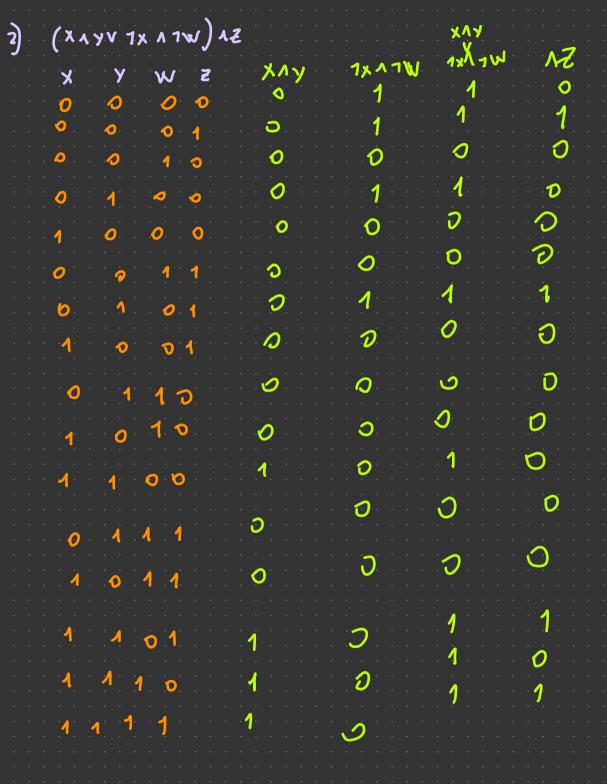
(*)

Build the **truth tables** of the following formulas, remembering the precedence of the operators.

- 1. $\neg(x \land y) \lor z$
- $2. \ (x \wedge y \vee \neg x \wedge \neg w) \wedge z$
- 3. $(x \lor y) \land x$
- 4. $\neg (p \land (\neg p \lor q \lor s)) \lor q \lor s$

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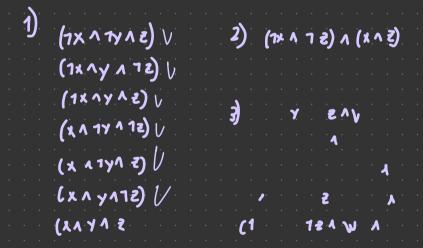
Exercise 2. Boolean Functions

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0	0	1	1		0	1	0		0	0	0	1	1
0	1	0	0		1	0	0		0	0	1	0	0
0	1	1	1		1	1	1		0	0	1	1	0
1	0	0	1						0	1	0	0	1
1	0	1	0						0	1	0	1	0
1	1	0	0						0	1	1	0	1
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- 4. $x \wedge y \wedge (\neg x \vee \neg y) \wedge z$
- 1 1 X V 1X > 1X NON TAUTOLOGICAL SATISFIABLE FORMULA
- (1x y) A (y y 1x) Not TAUTOCO GICAL SATISFIABLE FORMULA
- 3) 1(x2 y) (14 = 13)

(x 117) V y V1 X NOW - TAVIOCOGILAR SATISFIABLE FORTIGE
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(**)

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- 1 1 1 1 1 1 7 7 7
- (1PV 1 (1PV qV S)) Vq V S (1PV P 179 175) Vq V S

Exercise 8. Reasoning

Consider the following knowledge base K:

$$y \leftarrow y, z, w$$

$$x \leftarrow v$$

$$w \leftarrow y, z$$

$$z \leftarrow v, x$$

$$z \leftarrow y, w$$

$$z \leftarrow y, x$$

$$w \leftarrow x$$

$$t \leftarrow w, x$$

$$r \leftarrow s, t$$

- 1. Compute the redux \hat{K}
- 2. Find all the facts that are entailed by $K \wedge \mathcal{I}$
- 3. Decide whether the following clauses are consequences of ${\cal K}$
 - a) $v \leftarrow u$
 - b) $t \leftarrow y$
 - c) $q \leftarrow q$
 - d) $r \leftarrow w$

Exercise 9. Revision (**)

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Tautologies Exercise 10.

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