Knowledge Representation and Reasoning

Exercise Session 6

Exercise 1. Probabilistic Reasoning (∗)

Consider the KB $K = K_P \cup K_C$ where

$$K_P := \{0.5 :: x, \quad 0.5 :: y, \quad 0.5 :: y \leftarrow z\}$$
$$K_C := \{z \leftarrow x, \quad w \leftarrow x, y\}$$

Compute the probability of the following consequences assuming **probabilistic independence** between clauses:

1. w
2. y
3. $w \leftarrow x$

Exercise 2. Probability Distribution (∗∗)

Consider again the KB from Exercise 1. Suppose that $0.5 :: y$ is **not** a probabilistic clause, but rather a constraint on the joint distribution of the clauses. That is, we want $P(y) = 0.5$.

1. Find an adequate probability distribution and compute $P(w)$
2. How would the result change if we required $P(y) = p$ for some $p > 0.5$? And for $p < 0.5$?

Exercise 3. Extreme Probabilities (∗)

Following the **open world approach** compute the extreme probabilities for w from the KB of Exercise 1.

Exercise 4. DLs (∗∗)

Consider the probabilistic $\mathcal{EL}_\perp$ TBox

$$\begin{aligned}\mathcal{T} := \{\ & 0.5 :: \top \sqsubseteq Male, \quad 0.5 :: \top \sqsubseteq Female, \\ & Male \sqcap Female \sqsubseteq \perp, \quad \top \sqsubseteq \exists.hasParent.\top\ \}\end{aligned}$$

1. Explain in words what the GCIs in this TBox mean
2. Compute the probabilities of:
   - $Male \sqsubseteq \exists hasParent.Female$ and
   - $Male \sqsubseteq \exists hasParent.Male$

Exercise 5. Semantics (∗∗∗)

Suppose that you want to represent the (uncertain) knowledge about the spread, consequences, and cost of a recently discovered disease.

1. Which probabilistic semantics do you think is more adequate? why?
2. Identify constructors necessary to express all relevant notions

Exercise 6. Probabilities (∗∗∗)

A *uniform* probability distribution is one that assigns the same probability to events of the same size (e.g., assigning 1/6 to each face in a die). Suppose that we define a uniform distribution over all the natural numbers.

1. what is the probability of finding an *even* number? And a multiple of 5?
2. how do you define the probability of a set N?
3. what is the probability of observing the number 42?
4. is it impossible to observe 42?

Exercise 1. Probabilistic Reasoning (*)

Consider the KB $K = K_P \cup K_C$ where

$$K_P := \{0.5 :: x, \quad 0.5 :: y, \quad 0.5 :: y \leftarrow z\}$$
$$K_C := \{z \leftarrow x, \quad w \leftarrow x, y\}$$

Compute the probability of the following consequences assuming **probabilistic independence** between clauses:

1. w
2. y
3. $w \leftarrow x$

$P(w) = 3/8 \quad P(y) = 5/8 \quad P(w \leftarrow z) =$

w:

$x \mid y \mid y \leftarrow z$	000	001	010	100	011	110	101	111
	$P(w)=0$	$P(w)=0$	$P(w)=0$	$P(w)=0$	$P(w)=0$	$P(w)=1$	$P(w)=1$	$P(w)=1$
	$P(y)=0$	$P(y)=0$	$P(y)=1$	$P(y)=0$	$P(y)=1$	$P(y)=1$	$P(y)=1$	$P(y)=1$
	0	0	$P(w \leftarrow x)=1$	$P(w \leftarrow x)=1$	$P(w \leftarrow x)=1$	$P(w \leftarrow x)=1$	$P(w \leftarrow x)=1$	$P(w \leftarrow x)=1$

Exercise 2. Probability Distribution (**)

Consider again the KB from Exercise 1. Suppose that $0.5 :: y$ is **not** a probabilistic clause, but rather a constraint on the joint distribution of the clauses. That is, we want $P(y) = 0.5$.

1. Find an adequate probability distribution and compute $P(w)$
2. How would the result change if we required $P(y) = p$ for some $p > 0.5$? And for $p < 0.5$?

$K_p = \{0.5 :: x \ , \ 0.5 :: y \leftarrow z\}$

$K_c = \{z \leftarrow x, \ w \leftarrow x, y\}$

	00	01	10	11	
$P(w)$	0	0	1	1	$P(w) = 0.5$

Exercise 3. Extreme Probabilities (*)

Following the **open world approach** compute the extreme probabilities for w from the KB of Exercise 1.

$$K_P := \{0.5 :: x, \quad 0.5 :: y, \quad 0.5 :: y \leftarrow z\}$$

111, 101, 110

$x, y, y \leftarrow z$ $\quad 0.5 \cdot 0.5 \cdot 0.5 = 0.125$

$x, \quad y \leftarrow z$ $\quad 0.5 \cdot (1-0.5) \cdot 0.5 = 0.125$

Exercise 4. DLs (**)

Consider the probabilistic $\mathcal{EL}_\bot$ TBox

$$\mathcal{T} := \{\ 0.5 :: \top \sqsubseteq Male, \quad 0.5 :: \top \sqsubseteq Female, \quad Male \sqcap Female \sqsubseteq \bot, \quad \top \sqsubseteq \exists hasParent.\top\ \}$$

1. Explain in words what the GCIs in this TBox mean
2. Compute the probabilities of:
   - $Male \sqsubseteq \exists hasParent.Female$ and
   - $Male \sqsubseteq \exists hasParent.Male$

2. $MALE \sqsubseteq \exists hasParent.FEMALE$

$\downarrow$

0.5

2. Compute the probabilities of: 0.5 P no males is 0.5
   - $Male \sqsubseteq \exists hasParent.Female$ and
   - $Male \sqsubseteq \exists hasParent.Male$ 0.5

00 $\boxed{0}$

01 $\boxed{0.5}$

1

$\top \sqsubseteq Female$

$Male \sqcap Female \sqsubseteq \bot$

$\top \sqsubseteq \exists hP.\top$

KC

10 $\boxed{0.5}$

$\top \sqsubseteq Male$

2

11 $\boxed{0}$

$\top \sqsubseteq Male$

$\top \sqsubseteq Female$

$Male \sqcap Female \sqsubseteq \bot$

$\top \sqsubseteq \bot$

Female

$\cup$

$Male \sqsubseteq \top \sqsubseteq \exists hP.\top \sqsubseteq \exists hP.F$

$Male \sqsubseteq \bot$

Exercise 6. Probabilities (***)

A *uniform* probability distribution is one that assigns the same probability to events of the same size (e.g., assigning 1/6 to each face in a die). Suppose that we define a uniform distribution over all the natural numbers.

1. what is the probability of finding an *even* number? And a multiple of 5?
2. how do you define the probability of a set N?
3. what is the probability of observing the number 42? $P(\{42\}) = 0$
4. is it impossible to observe 42? NO

$$\lim_{k \to \infty} \frac{|\{n \in N \mid n \leq k\}|}{k} = P(N)$$