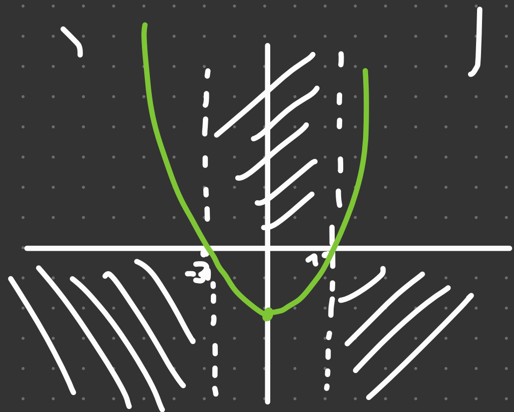
$+$ $\times$

$-$ $\div$

$f(x) = e^{x^2+2x-2} - e$



D: $\mathbb{R}$

SIGN

$f(0) = e^{-2} - e$

$e^{x^2+2x-2} - e > 0$

$x^2+2x-2 > 1$

$x^2+2x-3 > 0$

$4-4(-3)$

$\frac{-2 \pm 4}{2} < \begin{matrix} -3 \\ 1 \end{matrix}$

$\lim_{x \to \pm\infty} e^{x^2+2x-2} - e = +\infty$

ASYNDOTES

$m = \lim_{x \to \pm\infty} \frac{e^{x^2+2x-2} - e}{x} = +\infty$ NO ASYNDOTES ORC[illegible]

DERIVATES

$f'(x) = (2x+2)e^{x^2}$

SIGN $e^{x^2} > 0 \quad \forall x \in \mathbb{R}$

$2x+2 > 0; \quad x > -1$

-1

$- \mid +$

$\searrow \nearrow$

MIN -1

$f(-1) =$

$e^{1-2-2} - e$

$f(-1) = e^{-3} - e$

$$f(x) = \begin{cases} a(x-1) & \text{if } x \leq 0 \\ \dfrac{(\cos(1+\sqrt{x}))^2(\cos(\sqrt{x})-1)}{\sin(x^2)} & 0 \leq x \leq 1 \end{cases}$$

$$f(0) = \lim_{x \to 0^+} f(x) = \lim_{x \to 0^-} f(x)$$

$$\lim_{x \to 0^+} \frac{(\cos(1+\sqrt{x}))^2}{x} \frac{(\cos(\sqrt{x})-1)}{\sin(x^2)} \; x$$

$\to 1$

$$\lim_{x \to 0^+} \frac{x^2(\cos(\sqrt{x})-1)}{x \sin(x^2)}$$

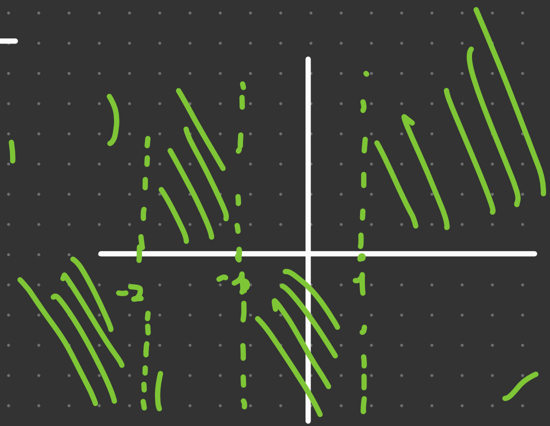
$\to 1$

$$\lim_{x \to 0^+} \frac{\cos(\sqrt{x})-1}{x} = -\frac{1}{2}$$

$$f(0) = a \qquad a = -\frac{1}{2}$$

$$\lim_{x \to 0^-} -\frac{1}{2}(x-1) = -\frac{1}{2}$$

$$f(x) = \frac{1-x^2}{x+2}$$



D: $x \neq -2$

SIGN

$1-x^2 \geq 0$ $\quad x^2 \geq 1$

$x+2>0$; $\quad x>-2$

	-2		-1		1	
−		−		+		−
−		+		+		+
+	∄	−	•	+	•	−

$$\lim_{x \to -\infty} f(x) = +\infty$$

$$\lim_{x \to +\infty} f(x) = -\infty$$

$$\lim_{x \to 2^+} \frac{1/x - x}{1 + 2/x} = -\infty$$

$$\lim_{x \to 2^-} f(x) = +\infty$$

ASYNDOTEI

$$m = \lim_{x \to +\infty} \frac{f(x)}{x} = \lim_{x \to +\infty} \frac{x^2\left(1/x^2 - 1\right)}{x^2\left(1 + 2/x\right)} = -1 \qquad q = \lim_{x \to +\infty} f(x) + x$$

$$q = \lim_{x \to +\infty} \frac{1-x^2}{x+2} + x = \lim_{x \to +\infty} \frac{1-x^2+x^2+2x}{x+2} =$$

$$\frac{2x+1}{x+2} = 2$$

$y = -x+2$

$$n_2 = \lim_{x \to -\infty} \frac{x^2(1/x^2 - 1)}{x^2(1 + 2/x)} = -1$$

q_2: $y = -x + 2$

DERIVATES

$$f'(x) = \frac{-2x(x+2) - (1 - x^2)}{x^2 + 2x + 4}$$

$$f'(x) = \frac{-2x^2 - 4x - 1 + x^2}{x^2 + 2x + 4}$$

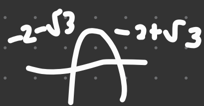
$$f'(x) = \frac{-x^2 - 4x - 1}{x^2 + 2x + 4}$$

FINIRE

$16 - 4(-1)(-1)$

12

$$\frac{4 \pm 2\sqrt{3}}{-2} = -2 \mp \sqrt{3}$$



$$\lim_{x\to 0} \frac{x(2e^{-x}-2+2x-x^2)}{(\cos(x)-1)^2} \quad \left(\frac{0}{0}\right)$$

RNEDERE

$$\boxed{-\frac{4}{3}}$$

$$\lim_{x\to 0} \frac{2xe^{-x}-2x+2x^2-x^3}{(\cos(x)-1)(\cos(x)-1)}$$

APPLY HOPITAL

$$\lim_{x\to 0} \frac{2e^{-x}-2xe^{-x}-2+4x-3x^2}{-\sin(x)(\cos(x)-1)-\sin(x)(\cos(x)-1)}$$

$$\lim_{x\to 0} \frac{2e^{-x}-2xe^{-x}-2+4x-3x^2}{-2\sin(x)(\cos(x)-1)} \quad \left(\frac{0}{0}\right)$$

APPLY HOPITAL

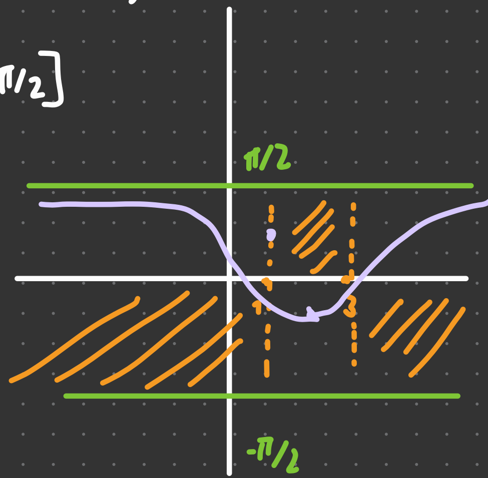
$$\lim_{x\to 0} \frac{-2e^{-x}-2e^{-x}+2xe^{-x}+4-6x}{-2\cos(x)(\cos(x)-1)+2\sin(x)^2}$$

$$\lim_{x\to 0} \frac{-e^{-x}-e^{-x}+xe^{-x}+2-3x}{-\cos(x)(\cos(x)-1)+\sin(x)^2} =$$

$$\lim_{x\to 0} \frac{-2e^{-x}+xe^{-x}+2-3x}{-\cos(x)^2+\cos(x)+\sin(x)^2} \quad [$$

$$f(x) = \arctan(x^2-4x+3)$$

$D: \mathbb{R} \qquad C\,[-\pi/2, \pi/2]$



$x^2-4x+3>0 \qquad 16-4(3)=$

$$\frac{4\pm 2}{2} \begin{cases} 3 \\ 1 \end{cases}$$

$+ \quad 1 \quad - \quad 3 \quad +$

$$\lim_{x\to+\infty} f(x) = \pi/2$$

$$\lim_{x\to+\infty} f(x) = \pi/2$$

$$f'(x) = \frac{2x-4}{1+(x^2-4x+3)^2}$$

$$f'(x) = \frac{2x-4}{1+(x^2-4x+3)^2}$$

$N>0 \quad 2x-4>0;\ x>2$

$D>0 \quad \forall x\in\mathbb{R}$

$- \ \overset{2}{|} \ +$

$\searrow \ \nearrow$

MIN

$f(2) = \arctan(-1) = -\frac{\pi}{4}$

$$x = e^{\ln x}$$

$$\lim_{n \to +\infty} \left(n^2 + 2n - \log(n)\right)^{1/\log(n)}$$

$$\lim_{n \to \infty} e^{\frac{1}{\log(n)} \log(n^2 + 2n - \log(n))}$$

$$\lim_{n \to +\infty} e^{\frac{\log(n^2 \cancel{+2n} - \cancel{\log(n)})}{\log(n)}} = e^{\frac{\log(n^2)}{\log(n)}} = e^{\frac{2\cancel{\log(n)}}{\cancel{\log(n)}}} = e^2$$

ESERCIZIO 2.3 (punti 8)
Sia

$$f(x) = \begin{cases} \dfrac{x-1}{2x+1} & \text{se } x \leq 0 \\ \\ \dfrac{e^x - 1}{x(x-2)} & \text{se } x > 0 \end{cases}$$

Determinare il dominio di definizione di f. Enunciare la definizione di funzione continua in un punto del suo dominio di definizione e stabilire in quali punti la funzione f è continua.

$$f(0) = \lim_{x \to 0^+} f(x) = \lim_{x \to 0^-} f(x)$$

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} \frac{x-1}{2x+1} = -1$$

$$f(0) = -1$$

HOPITAL

$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} \frac{e^x - 1}{x^2 - 2x} \left[\frac{0}{0}\right] = \lim_{x \to 0^+} \frac{e^x}{2x-2} = -\frac{1}{2}$$

ESERCIZIO 2.4 (punti 8)
Enunciare e dimostrare il Teorema della permanenza del segno per le successioni.

ENUNCIATO: GIVEN A SEQUENCE a_n a_n IS POSITIVE DEFINETLY IIF l IS POSITIVE

DIMOSTRAZIONE:

LET $\{a_n\}_{n \in \mathbb{N}} \in \mathbb{R}$ BE A SEQUENCE S.T.

$$\lim_n a_n = l \in [-\infty, +\infty]$$

a_n IS POSITIVE DEFINETLY, THAT IS $\exists n_0 \in \mathbb{N}\ \forall n > n_0\ \ a_n > 0$

LET $l > 0$ $\exists n_0 \in \mathbb{N}\ \forall n > n_0$

$l - \varepsilon < a_n$ $\varepsilon > 0$

$\varepsilon = \frac{l}{2}$

$0 < l - \varepsilon$ SO $\exists n_0 \in \mathbb{N}\ \forall n > n_0$

$l - \varepsilon < a_n$, FURTHERMORE $\exists M \in \mathbb{N}\ \forall n > n_0\ a_n < \frac{1}{\varepsilon}$

ESERCIZIO 1.1 Determinare tutte le soluzioni di

$$\sqrt{x+1} > -2x - 3.$$

$x+1 \geq 0; \quad x \geq -1$

$\sqrt{x+1} \geq 0 \quad \forall x \geq -1$

$(-1, +\infty)$

ESERCIZIO 1.2 Siano $f(x) = \log(x-1)$ e $g(x) = e^x$. Calcolare $g \circ f$ e stabilire il suo dominio di definizione.

RISULTATO:

$f(x) = \log(x-1)$

$g(x) = e^x$

$g \circ f \qquad g(f(x)) = e^{\log(x-1)}$

$D: x > 1$

ESERCIZIO 1.3 Quanti sono i numeri naturali di 3 cifre la cui prima cifra risulta essere un numero dispari e l'ultima un numero positivo divisibile per 3?

RISULTATO:

_	_	_
1	0	3
3	1	6
5	2	9
7	3	
9	4	
	5	
	6	
	7	
	8	
	9	

$\boxed{150}$

ESERCIZIO 2.2 (punti 7)
Enunciare la definizione di limite di una successione a $+\infty$ e calcolare il seguente limite

$$\lim_{n\to+\infty} -n^2\left(\log(n+1)-\log(n)\right).$$

$$\lim_{N\to+\infty} -N^2\left(\mathrm{LOG}(N+1)-\mathrm{LOG}(N)\right) \qquad \lim_{x\to+\infty}\frac{\mathrm{LOG}(x)}{x}=1$$

$$\lim_{N\to+\infty} -N^2\left(\mathrm{LOG}\left(\frac{N+1}{N}\right)\right)$$

$$\lim_{N\to+\infty} \frac{-\mathrm{LOG}\left(1+\frac{1}{N}\right)}{\frac{1}{N^2}} \quad \left[\frac{0}{0}\right] \quad \text{APPLY HOPITAL}$$

$$\lim_{N\to+\infty} \frac{\dfrac{\cancel{-}\dfrac{N+1+N}{N^2+1+2N}}{\dfrac{N}{N+1}}}{\cancel{-}2N^{-3}}$$

$$\lim_{N\to+\infty} \frac{\dfrac{2N+1}{N^2+2N+1}\cdot\dfrac{N+1}{N}}{2N^{-3}}$$

$$\lim_{N\to+\infty} \frac{\dfrac{2N^2+N+2N+1}{N^3+2N^2+N}}{2N^{-3}}$$

$$\lim_{N\to+\infty} \frac{2N^2+3N+1}{N^3+2N^2+N}\cdot\frac{N^3}{2}$$

$$\lim_{N\to+\infty} \frac{2N^5+3N^4+N^3}{2N^3+4N^2+2N}$$

$$\lim_{N\to+\infty} \frac{\dfrac{N^{-2}}{1+1/N}}{-2N^{-3}}$$

$$\lim_{N\to+\infty} \frac{\dfrac{\frac{1}{N^2}}{1+1/N}}{-2N^{-3}}$$

$$\lim_{N\to+\infty} \frac{1}{N^2}\cdot\frac{1}{1+1/N}\cdot\frac{1}{-2N^{-3}}$$

$$\lim_{N\to+\infty} \frac{1}{N^2}\cdot\frac{1}{\frac{N+1}{N}}\cdot\frac{1}{\frac{-2}{N^3}}$$

$$\lim_{N\to+\infty} \frac{1}{\cancel{N^2}}\cdot\frac{\cancel{N}}{N+1}\cdot\frac{-N^{\cancel{3}\,2}}{2} = \frac{-N^2}{2N+2}$$

$$f(x) = \begin{cases} \dfrac{2\omega(x+1)^2}{x-1} & x \le 0 \\ \dfrac{\sin(x/2)}{e^x - 1} & x > 0 \end{cases}$$

$$f(0) = \lim_{x \to 0^+} f(x) = \lim_{x \to 0^-} f(x)$$

$$f(0) = -2\omega$$

$$\lim_{x \to 0^+} \frac{\sin\left(\frac{x}{2}\right)}{e^x - 1} \quad \left[\frac{0}{0}\right] \text{ APPLY HOPITAL}$$

$$\lim_{x \to 0^+} \frac{\frac{1}{2}\cos\left(\frac{x}{2}\right)}{e^x} = \frac{1}{2}$$

$$\frac{1}{2} = -2\omega; \quad \omega = -\frac{1}{4}$$

$$\lim_{x \to 0^-} \frac{2\omega(x+1)^2}{x-1} = \lim_{x \to 0^-} \frac{-\frac{1}{2}(x+1)^2}{x-1} = \frac{1}{2}$$

$$f(x) = \begin{cases} x^2 - x + 2 & x \le 0 \\ \dfrac{(1-\cos(2x))^2}{\log(1+2x^4)} & x > 0 \end{cases}$$

$x^2 - x + 2$ IS CONTINUE IN $(-\infty, 0]$

$$\frac{(1-\cos(2x))^2}{\log(1+2x^4)}$$ CONTINUE IN $(0, +\infty)$

$$f(x) = \lim_{x \to 0^+} f(x) = \lim_{x \to 0^-} f(x)$$

$f(0) = 2$

$$\lim_{x \to 0^-} f(x) = 2$$

$$\lim_{x \to 0^+} \frac{1 + \cos(2x)^2 - 2\cos(2x)}{\log(1+2x^4)}$$ $\left[\frac{0}{0}\right]$ APPLY HOPITAL

$$\lim_{x \to 0^+} \frac{-4\cos(2x)\sin(2x) + 4\sin(2x)}{\frac{8x^3}{1+2x^4}}$$

$$\lim_{x \to 0} \frac{-4\cos(2x)\sin(2x) + 4\sin(2x) - 8x^4\cos(x)\sin(x) + 8x^4\sin(2x)}{8x^3}$$

$$(2-i)\,(2+i)z = 2-i\,(2-i)$$

$$(4-i^2)z = 4+i^2-4i$$

$$5z = 3-4i$$

$$z = \frac{3}{5} - \frac{4}{5}i$$

$$-\frac{4}{5}$$

$z = 1 - 3i \qquad w = 4 + i$

$z = 1 - 3i \qquad \bar{w} = 4 - i$

$z\bar{w} = (1 - 3i)(4 - i)$

$z\bar{w} = 4 - i - 12i + 3i^2$

$z\bar{w} = 1 - 13i$

TAYLOR

d) $f(x) = \cos(x^2) + x^2 + x$ $\quad x_0 = 1$ AND $N = 3$

$f(1) = 2$

$f'(x) = \dfrac{2x}{x^2} + 2x + 1 \qquad f'(1) = 5$

$f''(x) = \dfrac{2x^2 - 4x^2}{x^4} + 2 \qquad f''(1) = 0$

$\dfrac{-2}{x^2}$

$f'''(x) = \dfrac{4}{x^3} \qquad f'''(1) = 4$

$2 + 5(x-1) + 0 + \dfrac{4(x-1)^3}{3!}$

$2 + 5(x-1) + \dfrac{2(x-1)^3}{3}$

H) $$\lim_{n\to+\infty} \frac{\sqrt{n^2+2n}}{n+1}\left(\sqrt{n^4+n^2+1}-n^2\right)$$

1. $$\lim_{n\to+\infty} \sqrt{n^4+n^2+1}-n^2$$

$$\lim_{n\to+\infty} \frac{\left(\sqrt{n^4+n^2+1}-n^2\right)\left(\sqrt{n^4+n^2+1}+n^2\right)}{\sqrt{n^4+n^2+1}+n^2)}$$

$$\lim_{n\to+\infty} \frac{n^4+n^2+1-n^4}{\sqrt{n^4+n^2+1}+n^2}$$

$$\lim_{n\to+\infty} \frac{n^2+1}{\sqrt{n^4\left(1+\frac{1}{n^2}+1/n^4\right)}+n^2} = \frac{1}{2}$$

I) $$\lim_{n} \sqrt{n^2+n}-n$$

$$\lim_{n} \frac{\cancel{n^2}+n-\cancel{n^2}}{\sqrt{n^2+n}+n} = \frac{n}{2n} = \frac{1}{2}$$

$$\frac{N!}{K!\,(N-K)!}$$

$$\frac{5!}{2!\,3!}$$

$$\frac{5\cdot\cancel{4}\cdot\cancel{3}\cdot 2}{\cancel{2}\cdot\cancel{3}\cdot\cancel{2}}$$

10

10

$$\lim_{x \to 0^+} \frac{2\cos(\sqrt{x}) - 2 + x}{(\arctan(3x))^2} \quad \left[\frac{0}{0}\right]$$

APPLY DE L'HÔPITAL

$$\lim_{x \to 0^+} \frac{\dfrac{-\cancel{2}\sin(\sqrt{x})}{\cancel{2}\sqrt{x}} + 1}{\dfrac{6\arctan(3x)}{1+9x^2}}$$

$$\lim_{x \to 0^+} \frac{-\sin(\sqrt{x}) + \sqrt{x}}{\sqrt{x}} \cdot \frac{1+9x^2}{6\arctan(3x)}$$

$$\lim_{x \to 0^+} \frac{-\sin(\sqrt{x}) + \sqrt{x} - 9x^2\sin(\sqrt{x}) + 9x^2\sqrt{x}}{6\sqrt{x}\arctan(3x)} \quad \left[\frac{0}{0}\right] \text{APPLY HOPITAL}$$

$$\lim_{x \to 0^+} \frac{-\dfrac{\cos(\sqrt{x})}{2\sqrt{x}} + \dfrac{1}{2\sqrt{x}} - 18x\sin(\sqrt{x}) - \dfrac{9x^2\cos(\sqrt{x})}{2\sqrt{x}} + 18x\sqrt{x} + \dfrac{9x^2}{2\sqrt{x}}}{\dfrac{6}{2\sqrt{x}}\arctan(3x) + \dfrac{18\sqrt{x}}{1+9x^2}}$$

$$\lim_{x \to 0^+} \frac{\dfrac{-\cos(\sqrt{x}) + 1 - 32x\sqrt{x}\sin(\sqrt{x}) - 9x^2\cos(\sqrt{x}) + 32x^2 + 9x^2}{2\sqrt{x}}}{\dfrac{6\arctan(3x) + 54x^2\arctan(3x) + 32x}{2\sqrt{x} + 18x^2\sqrt{x}}}$$

$$\lim_{x \to 0} \frac{-\cos(\sqrt{x}) + 1 - 32x\sqrt{x}\sin(\sqrt{x}) - 9x^2\cos(\sqrt{x}) + 32x^2 + 9x^2}{2\sqrt{x}} \cdot \frac{2\sqrt{x} + 18x^2\sqrt{x}}{6\arctan(3x) + 54x^2\arctan(3x) + 32x}$$

$$\lim_{x\to 0^+} \frac{2\cos(\sqrt{x}) - 2 + x}{(\arctan(3x))^2}$$

$t=\sqrt{x}$

$$\lim_{t\to 0^+} \frac{2\cos(t)-2+t^2}{(\arctan(3t^2))^2} \quad \left[\frac{0}{0}\right]$$

APPLY DE L'HÔPITAL

$$\lim_{t\to 0^+} \frac{-2\sin(t)+2t}{\frac{12t\arctan(3t^2)}{1+9t^4}}$$

$$\lim_{t\to 0^+} \frac{-\cancel{2}\sin(t)+\cancel{2}t}{} \cdot \frac{1+9t^4}{\cancelto{6}{12}\, t\arctan(3t^2)}$$

$$\lim_{t\to 0^+} \frac{-\sin(t)+t-9t^4\sin(t)+9t^5}{6t\arctan(3t^2)} \quad \left[\frac{0}{0}\right]$$

HÔPITAL

$$\lim_{t\to 0^+} \frac{-\cos(t)+1-36t^3\sin(t)-9t^4\cos(t)+45t^4}{6\arctan(3t^2)+\frac{6t\arctan(3t^2)}{1+9t^4}}$$

$$\lim_{t\to 0^+} \frac{45t^4-9t^4\cos(t)-36t^3\sin(t)-\cos(t)+1}{\frac{6\arctan(3t^2)+54t^4\arctan(3t^2)+6t\arctan(3t^2)}{1+9t^4}}$$

$$\lim_{t\to 0} \frac{(45t^4-9t^4\cos(t)-36t^3\sin(t)-\cos(t)+1)(1+9x^4)}{6\arctan(3t^2)+54t^4\arctan(3t^2)+6t\arctan(3t^2)}$$