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1) DIMENSIONAL ANALYSIS

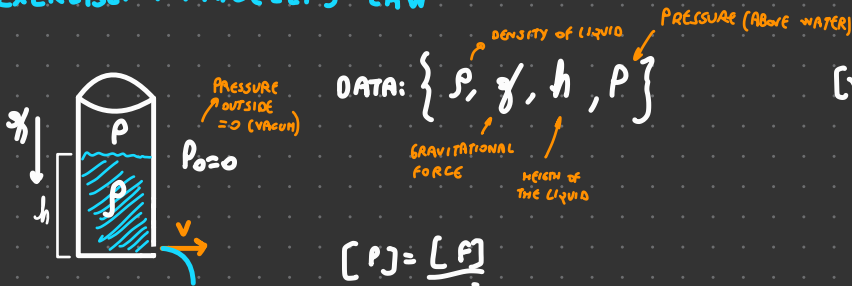
- CHECKING RESULTS
- GUESSING RESULTS \rightarrow REYLEIGH ALGO.

$$\{M, L, T\}$$

↑
MASS
↑
LENGTH
↑
TIME

- PHYSICAL QUANTITIES $\{a, b, \dots\}$
 - $x \propto a^\alpha b^\beta c^\gamma \dots \Rightarrow [x] = [a]^\alpha [b]^\beta \dots$
- ↑
PROPORTIONAL

EXERCISE: TORRICELLI'S LAW



$$[v] = \frac{L}{T}$$

$$[p] = \frac{[F]}{L^2}$$

$$[v] = [h]^\alpha [\gamma]^\beta [p]^\gamma [p]^\delta$$

$$\frac{L}{T} = L^\alpha \left(\frac{L}{T^2}\right)^\beta \left(\frac{ML}{T^2 L^2}\right)^\gamma \left(\frac{M}{L^3}\right)^\delta$$

$$\begin{cases} \delta + \gamma = 0 \\ \gamma = \frac{1}{2} - \beta \\ \alpha = \beta \end{cases}$$

CONSIDER 2 LIMITS

$$\bullet p \rightarrow 0: \gamma = 0 \quad \alpha = \beta = 1/2 \quad \delta = 0 \quad \Rightarrow v \propto \sqrt{h\gamma}$$

$$\bullet h \rightarrow 0: \alpha = 0 \quad v \propto \sqrt{\frac{p}{\rho}}$$

CONNECT THEM

$$\textcircled{A} \quad v \propto \sqrt{hg} + \sqrt{\frac{p}{\rho}}$$

$$\textcircled{B} \quad v \propto \sqrt{hg + \frac{p}{\rho}}$$

PROBLEM 1: CENTRIPETAL FORCE



COMPUTE F IN TERMS OF $\{v, r, m\}$

(AMPLITUDE OF) VELOCITY
RADIUS
MASS

$F?$

$$F = [M]^{\alpha} [\omega]^{\beta} = [M]^{\alpha} \left[\frac{v}{r} \right]^{\beta} = [M]^{\alpha} \left[\frac{L}{T^2} \right]^{\beta} =$$

$M=0$

$$M \cdot \frac{L}{T^2} = [v]^{\alpha} [r]^{\beta} [M]^{\gamma}$$

\downarrow
 $\left[\frac{L}{T} \right]^{\alpha}$

$$M \frac{L}{T^2} = \left[\frac{L}{T} \right]^{\alpha}$$

$$M \frac{L}{T^2} = \left[\frac{L}{T} \right]^{\alpha} [L]^{\beta} [M]^{\gamma}$$

$\left\{ \begin{array}{l} M=1 \\ L=1 \\ T=-2 \end{array} \right. \quad \left\{ \begin{array}{l} \gamma=1 \\ \alpha+\beta=1 \\ \alpha=2 \end{array} \right. \quad \left\{ \begin{array}{l} \gamma=1 \\ \beta=-1 \\ \alpha=2 \end{array} \right.$

$$F = \frac{L^2}{T^2} \frac{1}{L} M \Rightarrow F = \left(\frac{L}{T^2} \right) M \Rightarrow F = M a$$

\downarrow
 a

$$F \propto m \frac{v^2}{R}$$

PROBLEM 2: ESCAPE VELOCITY

$$\{G, M, R\}$$



$v \geq v_e$ THEN THE OBJECT WILL GET DOWN
THE ORBIT OF THE PLANET
↑
ESCAPE VELOCITY

$$v_e = ?$$

$$v_e = \frac{L}{T}$$

$$v_e \propto \sqrt{\frac{GM}{R}}$$

$$F = G \frac{M^2}{L^2}$$

$$G = F \frac{L^2}{M^2}$$

$$\frac{L}{T} = [G]^{\alpha} [M]^{\beta} [R]^{\gamma} \Rightarrow \frac{L}{T} = \left[F \frac{L^2}{M^2} \right]^{\alpha} [M]^{\beta} [R]^{\gamma}$$

$$\begin{cases} L=1 \\ T=-1 \end{cases}$$