Knowledge Representation and Reasoning

Exercise Session 6

Exercise 1. Probabilistic Reasoning (∗)

Consider the KB $K = K_P \cup K_C$ where

$$K_P := \{0.5 :: x, \quad 0.5 :: y, \quad 0.5 :: y \leftarrow z\}$$
$$K_C := \{z \leftarrow x, \quad w \leftarrow x, y\}$$

Compute the probability of the following consequences assuming **probabilistic independence** between clauses:

1. w
2. y
3. $w \leftarrow x$

Exercise 2. Probability Distribution (∗∗)

Consider again the KB from Exercise 1. Suppose that $0.5 :: y$ is **not** a probabilistic clause, but rather a constraint on the joint distribution of the clauses. That is, we want $P(y) = 0.5$.

1. Find an adequate probability distribution and compute $P(w)$
2. How would the result change if we required $P(y) = p$ for some $p > 0.5$? And for $p < 0.5$?

Exercise 3. Extreme Probabilities (∗)

Following the **open world approach** compute the extreme probabilities for w from the KB of Exercise 1.

Exercise 4. DLs (**)

Consider the probabilistic $\mathcal{EL}_\perp$ TBox

$$\mathcal{T} := \{\ 0.5 :: \top \sqsubseteq Male, \quad 0.5 :: \top \sqsubseteq Female,$$
$$Male \sqcap Female \sqsubseteq \bot, \quad \top \sqsubseteq \exists.hasParent.\top\ \}$$

1. Explain in words what the GCIs in this TBox mean
2. Compute the probabilities of:
   - $Male \sqsubseteq \exists hasParent.Female$ and
   - $Male \sqsubseteq \exists hasParent.Male$

Exercise 5. Semantics (***)

Suppose that you want to represent the (uncertain) knowledge about the spread, consequences, and cost of a recently discovered disease.

1. Which probabilistic semantics do you think is more adequate? why?
2. Identify constructors necessary to express all relevant notions

Exercise 6. Probabilities (***)

A *uniform* probability distribution is one that assigns the same probability to events of the same size (e.g., assigning 1/6 to each face in a die). Suppose that we define a uniform distribution over all the natural numbers.

1. what is the probability of finding an *even* number? And a multiple of 5?
2. how do you define the probability of a set N?
3. what is the probability of observing the number 42?
4. is it impossible to observe 42?

Exercise 1. Probabilistic Reasoning (∗)

Consider the KB $K = K_P \cup K_C$ where

$$K_P := \{0.5 :: x, \quad 0.5 :: y, \quad 0.5 :: y \leftarrow z\}$$
$$K_C := \{z \leftarrow x, \quad w \leftarrow x, y\}$$

Compute the probability of the following consequences assuming **probabilistic independence** between clauses:

1. w
2. y
3. $w \leftarrow x$

- 3 PROBABILISTIC CLAUSES
- $$K_P := \{0.5 :: x, \quad 0.5 :: y, \quad 0.5 :: y \leftarrow z\}$$
  $$K_C := \{z \leftarrow x, \quad w \leftarrow x, y\}$$

8 WORDS

$0.5::x$	$0.5::y$	$0.5::y \leftarrow z$	w	y	$w \leftarrow x$
0	0	0	0	0	0
0	0	1	0	0	1
0	1	0	0	1	1
1	0	0	0	0	
0	1	1	0	1	1
1	0	1	1	1	1
1	1	0	1	1	1
1	1	1	1	1	1

$P(w) = 3/8$ $P(y) = \frac{5}{8}$ $P(y \leftarrow z) 6/8$

Exercise 2. Probability Distribution (**)

Consider again the KB from Exercise 1. Suppose that $0.5 :: y$ is **not** a probabilistic clause, but rather a constraint on the joint distribution of the clauses. That is, we want $P(y) = 0.5$.

1. Find an adequate probability distribution and compute $P(w)$
2. How would the result change if we required $P(y) = p$ for some $p > 0.5$? And for $p < 0.5$?

$$K_P := \{0.5 :: x, \quad \cancel{0.5 :: y}, \quad 0.5 :: y \leftarrow z\}$$
$$K_C := \{z \leftarrow x, \quad w \leftarrow x, y\}$$

0.5::y IS NOT A PROBABILISTIC CLAUSE ANYMORE

BUT RATHER A CONSTRAINT

	00 0.5	01 0
0.5	10 0	11 0.5
		0.5

Exercise 3. Extreme Probabilities (*)

Following the **open world approach** compute the extreme probabilities for w from the KB of Exercise 1.

FIND THE EXTREMES

FIND WHICH DISTRIBUTION MAXIMIZE OR MINIMIZE THE PROBABILITY OF THE THREE WORDS IN WHICH W IS 1

W IS ENTAILED IN WORDS 101, 110, 111

LOWER PROBABILITY OF W

$$\underline{P}(w) = \min_{P \in COH(K)} P(101) + P(110) + P(111)$$

$$\overline{P}(w) = \max_{P \in COH(K)} P(101) + P(110) + P(111)$$

Exercise 4. DLs (**)

Consider the probabilistic $\mathcal{EL}_\bot$ TBox

$$\mathcal{T} := \{\ 0.5 :: \top \sqsubseteq Male, \quad 0.5 :: \top \sqsubseteq Female,$$
$$Male \sqcap Female \sqsubseteq \bot, \quad \top \sqsubseteq \exists.hasParent.\top\ \}$$

1. Explain in words what the GCIs in this TBox mean
2. Compute the probabilities of:
   - 1 • $Male \sqsubseteq \exists hasParent.Female$ and
   - 2 • $Male \sqsubseteq \exists hasParent.Male$

MALE $\sqsubseteq$ ∃ HAS PARENT. FEMALE

PROBABILITY THAT A MALE HAS A PARENT THAT IS FEMALE

THE PROBABILITY IS 0. EITHER EVERYONE IS MALE, EITHER EVERYONE IS FEMALE

4 WORDS

	0	1
0	00: 0	01: 0.5 — $\top \sqsubseteq$ FEMALE, $M \sqcap F \sqsubseteq \bot$, $\top \sqsubseteq \exists hP.\top$ — 1
1	10: 0.5 — $\top \sqsubseteq$ MALE, $M \sqcap F \sqsubseteq \bot$, $\top \sqsubseteq \exists hP.\top$ — 2	11: 0 — $\top \sqsubseteq$ MALE, $\top \sqsubseteq$ FEMALE, MALE $\sqcap$ FEMALE $\sqsubseteq \bot$, $\top \sqsubseteq \bot$

EVERY MALE HAS A PARENT FEMALE IS TRUE BECAUSE THERE ARE NO MALE

↳ THIS KB IS INCONSISTENT

Exercise 5. Semantics (***)

Suppose that you want to represent the (uncertain) knowledge about the spread, consequences, and cost of a recently discovered disease.

1. Which probabilistic semantics do you think is more adequate? why?

2. Identify constructors necessary to express all relevant notions

BORING

Exercise 6. Probabilities (***)

A *uniform* probability distribution is one that assigns the same probability to events of the same size (e.g., assigning 1/6 to each face in a die). Suppose that we define a uniform distribution over all the natural numbers.

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