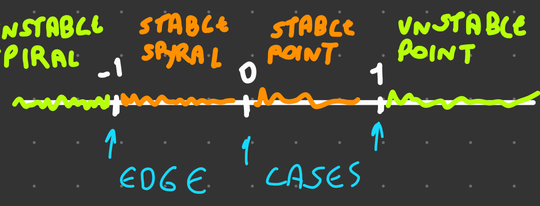
$+$	$\times$
$-$	$\div$

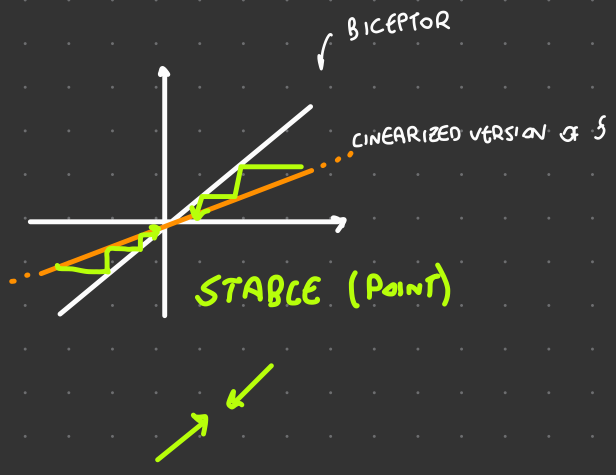
COBWEB PLOT

4 CASES $\quad m = f'(\bar{x}), \quad \bar{x} = f(\bar{x})$

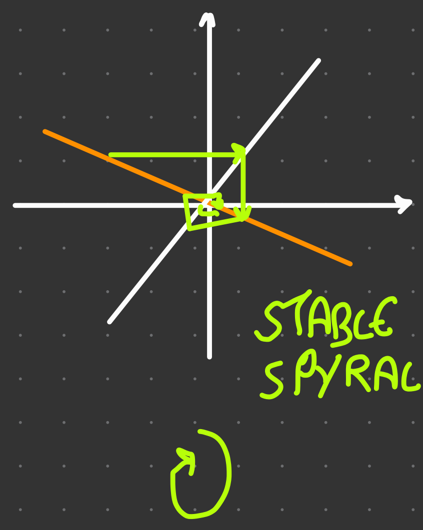


AXIS
X - START
Y - LINEARISED VERSION OF $f(x)$

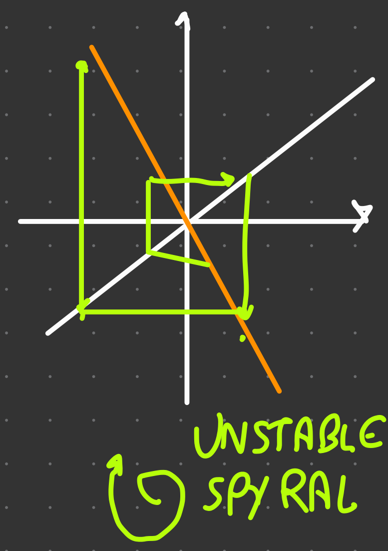
1. $0 < m < 1$



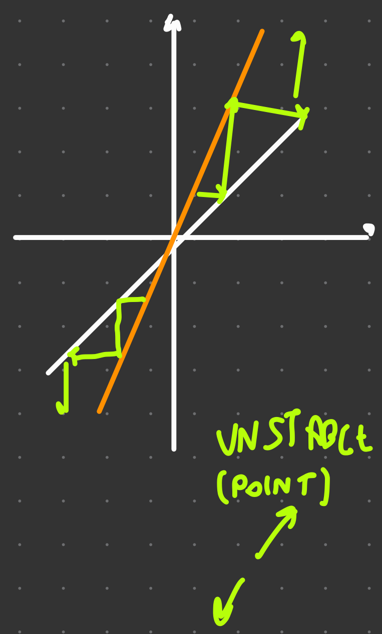
2. $-1 < m < 0$



3. $m < -1$

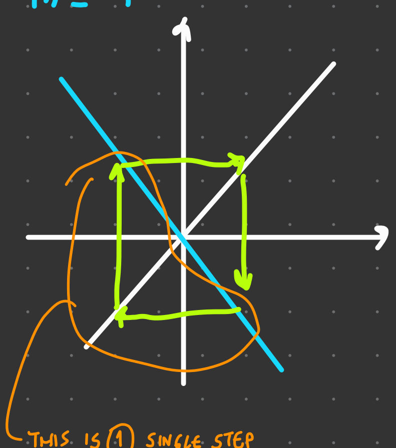


4. $m > 1$



EDGE CASES $(m = -1 \vee m = 0 \vee m = 1)$

1. $m = -1$



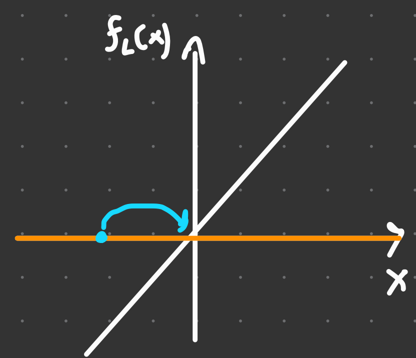
THIS IS 1 SINGLE STEP

$y = -x$ GET BACK TO WHERE I STARED IN 2 JUMPS

$x_{m+1} = f(x_m)$

WE HAVE A CYCLE: $x_{m+1} = -x_m$

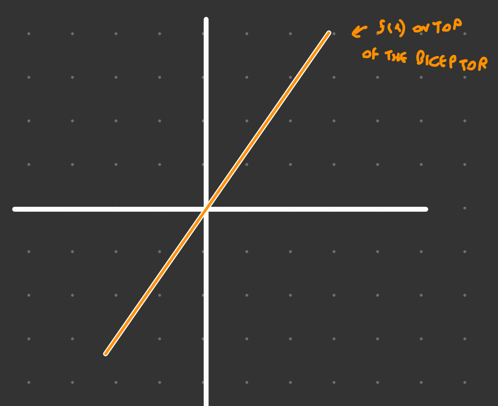
2. $m = 0$



TO THE FIXED POINT IMMEDIATELY IN ONE STEP

$$(x_{m+1} - \bar{x}) = \underbrace{f'(\bar{x})}_{m} (x_m - \bar{x})$$

3. $m = 1$ BETWEEN STABILITY AND UNSTABILITY



WHEREVER YOU START YOU STAY THERE

$x_{m+1} - \bar{x} = x_m - \bar{x}$

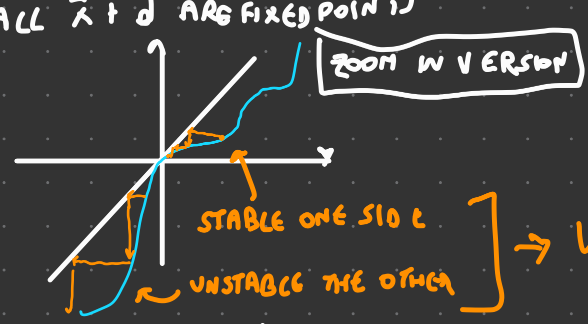
YOU ARE ALREADY ON THE BICEPTOR, YOU DON'T MOVE

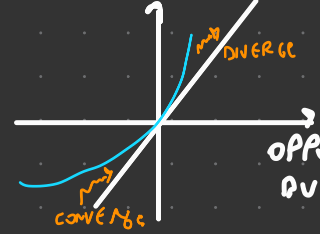
IS MARGINALLY STABLE

IS AN EDGE CASE, AT BOUNDARY BETWEEN STABILITY AND UNSTABILITY

ALL $\bar{x}$ + f ARE FIXED POINTS

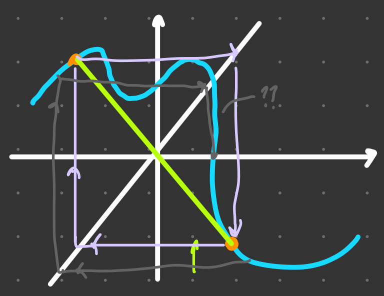
INFORMALLY
IF x_0 IS CLOSE TO $\bar{x}$
x_n IS CLOSE TO $\bar{x}$





EXTRA:

NON LINEAR



TWO POINTS CONNECTED ON "OPPOSITE" BISECTOR

• TEST IF STABLE/UNSTABLE

THIS CYCLE CAN BE STABLE OR UNSTABLE

↓

LIMIT CYCLE

(ONLY IN NON-LINEAR SYSTEM)

"NON PERTURBATIVE"

↓

YOU SEE IT WHEN YOU CONSIDER THE FULL FUNCTION AND NOT WHEN YOU LOOK AT TAYLOR EXPANSION

EXERCISE:

$f(x) = \tanh(\beta x)$ DISCUSS STABILITY OF $\bar{x} = 0$

→ $\beta > 0$

a) STABILITY

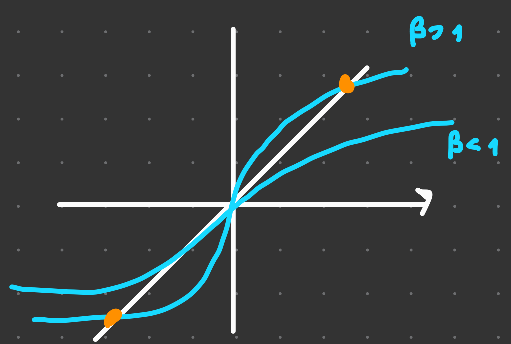
$$\tanh(z) = z - \frac{z^3}{3} + o(z^3) \Rightarrow \tanh(\beta x) = \beta x - \frac{(\beta x)^3}{3} + o(\beta x)^5$$

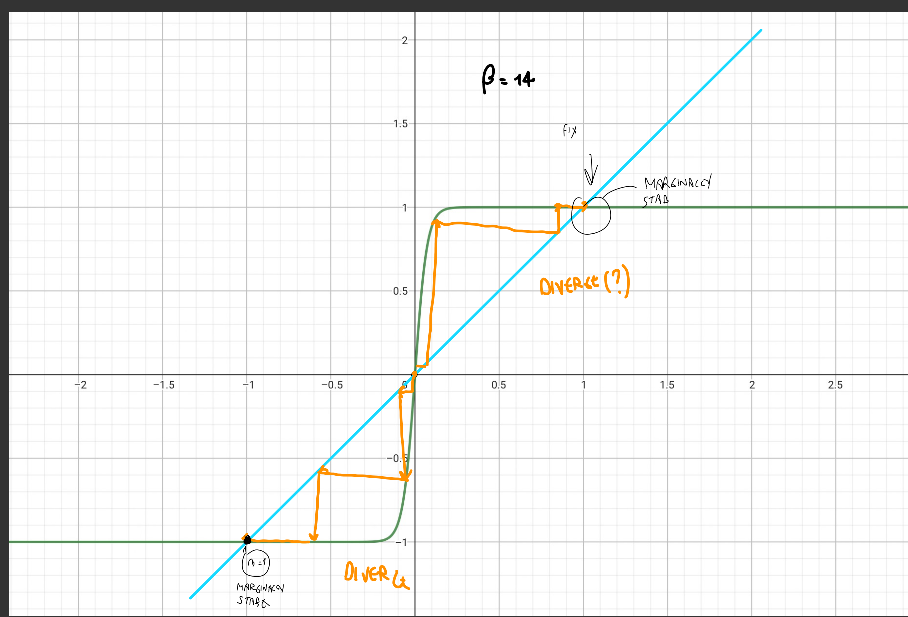
b) OTHER FIXED POINT

$$f(x) = \beta x - \frac{\beta^3 x^3}{3}$$

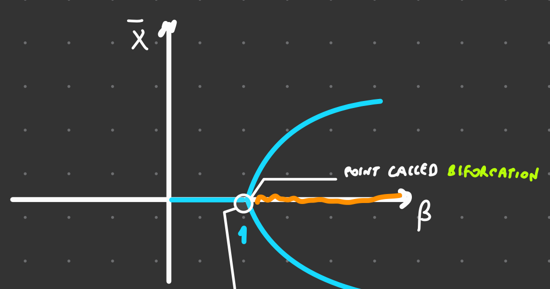
↓

$f'(x=0)$

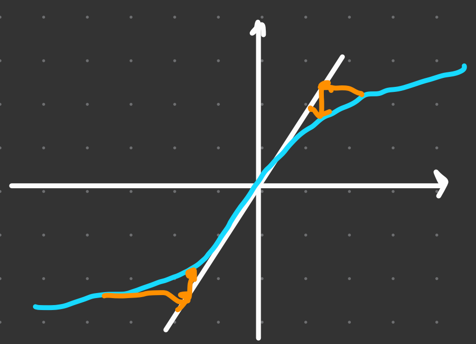




BIFURCATION DIAGRAM



DERIVATIVE AT (1) IS INFINITE (SO IS VERTICAL)



$f(x) = r x (1 - x)$

$0 < r < 4$

LOGISTIC MAP (MALTUS 1798)

RABBITS REPRODUCING

$x_{n+1} = x_n r$

REPRODUCTING RATE

RABBITS GENERATION BEFORE

RABBITS AT NEXT GENERATION

$r > 1$: RABBITS POPOLATION KEEPS GROWING TO ∞

↓

VERHULST 1838 STATES THAT THEY WOULD NEED INFINITE AMOUNT OF FOOD

$$\underbrace{x_{n+1} = x_n r}_{\text{MALTUS}} \underbrace{\left(1 - \frac{x_n}{K}\right)}$$

TAKE INTO ACCOUNT THE FOOD

↓

IF $K = x_n$ ALL DIES