# Exercises - Calculus
Academic Year 2021-2022

Sheet 10

1. Solve the following minimum problems: determine, if it exists, $\min_C f$ where

   (a) $f(x) = x + \dfrac{1}{x+2}$ and $C = [-3/2, 5]$. What can you say when $C = (-2, +\infty)$?

   (b) $f(x) = \dfrac{|x|-1}{|x|+1}$ and $C = [-1, 2]$

   (c) $f(x) = -e^x\sqrt{1-x}$ and $C = [-1, 1]$

   (d) $f(x) = \dfrac{x^3}{4x^2+1}$ and $C = [-3, 5]$

   (e) $f(x) = -\dfrac{x^2}{4x^3+1}$ and $C = [0, 5]$. What can you say about $\max_C f$?

   When the minimum exists, determine also all the absolute minimizers.

2. Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function. Suppose that $\lim_{x\to+\infty} f(x) = +\infty$ and $\lim_{x\to-\infty} f(x) = +\infty$. Prove that there exists $\min_{\mathbb{R}} f$.

   Hint: this, as the next three exercises, is a variant of Weierstrass Theorem. The idea is to show that there exists a closed and bounded interval $[a, b]$ such that $\inf_{\mathbb{R}} f = \inf_{[a,b]} f$ and then use Weierstrass to conclude that there exists $x_0 \in [a, b]$ such that $f(x_0) = \min_{[a,b]} f$ and conclude that $f(x_0) = \min_{\mathbb{R}} f$. Use the assumptions to find a closed and bounded interval $[a, b]$ such that for any $x \notin [a, b]$ we have $f(x) > f(0) + 1$.

3. Let $f : [0, +\infty) \to \mathbb{R}$ be a continuous function. Suppose that $\lim_{x\to+\infty} f(x) = +\infty$. Prove that there exists $\min_{[0,+\infty)} f$.

4. Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function such that $f(x) > 0$ for any $x \in \mathbb{R}$. Suppose that $\lim_{x\to+\infty} f(x) = 0$ and $\lim_{x\to-\infty} f(x) = 0$. Prove that there exists $\max_{\mathbb{R}} f$.

   Hint: use the assumptions to find a closed and bounded interval $[a, b]$ such that for any $x \notin [a, b]$ we have $f(x) < f(0)/2$.

5. Let $f : [0, +\infty) \to \mathbb{R}$ be a continuous function such that $f(x) > 0$ for any $x \in [0, +\infty)$. Suppose that $\lim_{x\to+\infty} f(x) = 0$. Prove that there exists $\max_{[0,+\infty)} f$.

6. Find the right cone, with volume equal to $\frac{\pi}{3}m^3$, with minimal lateral surface area.

   Hint: in this and the next exercise, use one of the variants of Weierstrass above to prove existence of a minimizer. Recall that the following formulas holds for a right cone with base a disk of radius $r > 0$ and height $h > 0$

   $$\begin{array}{ll} \text{Volume} = \frac{1}{3}\pi r^2 h; & \text{Are of the lateral surface} = \pi r\sqrt{r^2+h^2}; \\ \text{Area of the base} = \pi r^2; & \text{Area of the total surface} = \pi r^2 + \pi r\sqrt{r^2+h^2} \end{array}$$

7. Find the right pyramid, with base a square and volume equal to $\frac{1}{6}m^3$, with minimal lateral surface area.

   Hint: recall that the following formulas holds for a right pyramid with base a square of side $l > 0$ and height $h > 0$

   $$\begin{array}{ll} \text{Volume} = \frac{1}{3}l^2 h; & \text{Area of the lateral surface} = 2l\sqrt{(l/2)^2+h^2}; \\ \text{Area of the base} = l^2; & \text{Area of the total surface} = l^2 + 2l\sqrt{(l/2)^2+h^2} \end{array}$$

8. For the following functions, determine whether they have asymptotes as $x \to +\infty$ and as $x \to -\infty$.

   (a) $f(x) = 3|x| - \dfrac{\sin(x)}{x}$

   (b) $f(x) = \sqrt{2+4x^2}$

   (c) $f(x) = (x+1)\exp\left(\dfrac{x+1}{x}\right)$

   Hint: note that

   $$\exp\left(\frac{x+1}{x}\right) - e = e\left(e^{1/x} - 1\right)$$

   (d) $f(x) = \log(2^{x-1})$

9. Study the following functions and draw their graphs

   (a) $f(x) = x^5 - x^3$

   (b) $f(x) = |x^2 - 1| - \frac{1}{2}x^3$

   (c) $f(x) = \dfrac{x^2}{4x^3+1}$.

   For any $y_0 \in \mathbb{R}$, how many solutions does the equation $f(x) = y_0$ have?

   (d) $f(x) = \dfrac{2+x^2}{x-2}$

   (e) $f(x) = \dfrac{2-x^2}{x-2}$

(f) $f(x) = \dfrac{x-3}{(x-2)^2}$

For any $y_0 \in \mathbb{R}$, how many solutions does the equation $f(x) = y_0$ have?

(g) $f(x) = \log(-(x^2 + 3x + 2))$

(h) $f(x) = e^{-(x^2+3x+2)}$

(i) $f(x) = \log(1 + x^2)$

(j) $f(x) = e^x - \log(x + 1)$

Hint: in order to determine the sign of $f'$, call $g = f'$, compute $g(0) = f'(0)$ and study the function $g$, in particular the behaviour of its derivative $g'$.

(k) $f(x) = (\log(x))^2 + 2\log(x)$

(l) $f(x) = \sqrt{x^2 - x - 2}$

(m) $f(x) = 3x + 4\sqrt{1 - x^2}$

Hint: be careful on solving correctly the equation $f'(x) = 0$.

(n) $f(x) = \log(x + 1) + \dfrac{x}{x-1}$

(o) $f(x) = -2\arctan\left(\dfrac{1}{x-1}\right) + x$

Hint: be careful to the behaviour of $f$ for $x$ close to 1. Does it exists $\lim_{x\to 1} f(x)$?

(p) $f(x) = -\log(|\sin(x)|)$

(q) $f(x) = \sqrt[3]{\dfrac{x^3 - 1}{x}}$

Hint: be careful on finding the points where $f$ differentiable. Is $f$ differentiable in $x = 1$?

1A) $f(x) = x + \frac{1}{x+2}$ $\qquad C = [-3/2, 5]$

$\downarrow$

$$f'(x) = 1 + \frac{0-1}{(x+2)^2} = 1 - \frac{1}{(x+2)^2} =$$

$$= \frac{x^2+4x+3}{(x+2)^2}$$

$$\boxed{x \neq -2}$$


$x<-3 \vee x>1$

N: $\boxed{16-12} = 4$ $\quad \frac{-4 \pm 2}{2} < \begin{matrix} -3 \\ 1 \end{matrix}$

D: $x^2+4x+4$ $\quad 0$

$\forall x \in \mathbb{R}$

$\quad -3 \quad 1$

$+ \quad - \quad +$

$\nearrow \; \bullet \searrow \; \bullet \nearrow$

MAX $\quad$ MIN

1B)

$$f(x) = \frac{|x| - 1}{|x| + 1} \qquad c = [-1, 2]$$

$$\downarrow$$

$$\frac{|x|}{(|x|+1)^2}$$

8A)*

$$f(x) = \frac{3|x| - \sin(x)}{x}$$

$$\lim_{x \to +\infty} \frac{3|x| - \sin(x)}{x} = +\infty$$

$$\lim_{x \to -\infty} \frac{3|x| - \sin(x)}{x} = -\infty$$

8B)

$$f(x) = \sqrt{2 + 4x^2}$$

$$\lim_{x \to +\infty} \frac{f(x)}{x} = \frac{\cancel{x}\sqrt{2/x^2 + 4}}{\cancel{x}} = 2$$

$$\lim_{x \to -\infty} \frac{f(x)}{x} = -2$$

$$q_1 = \lim_{x \to +\infty} f(x) - 2x = x\sqrt{2/x^2 + 4} - 2x = 0$$

$$y = 2x$$

$$q_2 = \lim_{x \to -\infty} f(x) + 2x = x\sqrt{2/x^2 + 4} + 2x = 0$$

$$y = -2x$$

8c) $(x+1)\, e^{\frac{x+1}{x}}$

$$m = \lim_{x \to \pm\infty} \frac{(x+1)\, e^{1+1/x}}{x}$$

$$\lim_{x \to \pm\infty} \frac{\cancel{x} e^{1+1/x}}{\cancel{x}} + \frac{e^{1+1/x}}{x}$$

$$\lim_{x \to \pm\infty} e$$

$$\lim_{x \to \pm\infty} e^{1+1/x} + \frac{e^{1+1/x}}{x} - e = e + 0 - e = 0$$

$$y = ex$$

8D) $\log\left(2^{x-1}\right)$

$$m = \lim_{x \to \pm\infty} \log\left(2^{x-1}\right) = \lim_{x \to \pm\infty} \log\left(2^{x} \cdot 2^{-1}\right) \begin{cases} \lim_{x \to +\infty} f(x) = +\infty \\ \lim_{x \to -\infty} f(x) = -\infty \end{cases}$$

NO ASYMPTOTES

9A) $f(x) = x^5 - x^3$

DOMAIN $\mathbb{R}$

SYMMETRIES NOT EVIDENT

SIGN OF $f$ AND INTERSECTION WITH X-AXIS AND Y-AXIS

$f(0) = 0$

$x^5 - x^3 = 0 \Rightarrow x^3(x^2 - 1) = 0$

$x = 0 \vee x^2 - 1 = 0;\quad x = \pm 1$

INT. X-AXIS: $(0,0)\ (1,0)\ (-1,0)$

INT. Y-AXIS: $(0,0)$

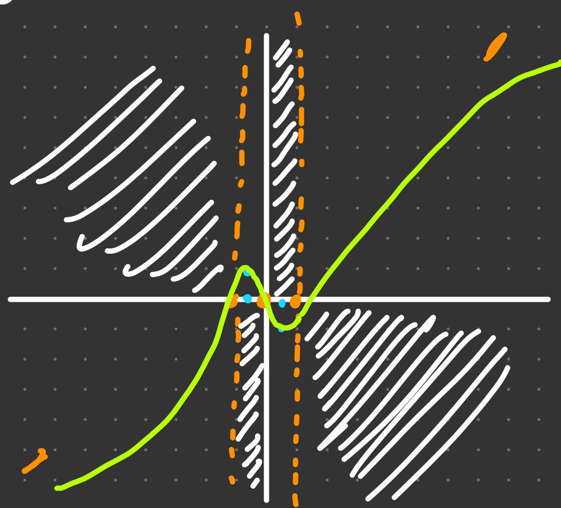
| | -1 | | 0 | | 1 | |
|---|---|---|---|---|---|---|
| N1 | − | | − | | + | + |
| N2 | + | | − | | − | + |
| | − | • | + | • | − • | + |

- CONTINUITY, LIMITS, AND EXTREMES

$$\lim_{x \to +\infty} x^5 - x^3 = +\infty$$

$$\lim_{x \to -\infty} x^5 - x^3 = -\infty$$

- DOMAIN
- SYMMETRIES
- SIGN OF $f$ AN INTERSECTION WITH X-AXIS AND Y-AXIS
- CONTINUITY, LIMITS, EXTREMES
- ASYMPTOTES
- DIFFERENTIABILITY AND DERIVATES



- **ASYMPTOTES** $y = mx + q$ $\quad m, q \in \mathbb{R}$

$$\lim_{x \to \pm\infty} \frac{x^5 - x^3}{x} = \lim_{x \to \pm\infty} \frac{x^{\cancel{3}\,2}(x^2 - 1)}{\cancel{x}} = +\infty \qquad \text{NO ASYMPTOTES}$$

- **DIFFERENTIABICITY AND DERIVATES**

$$f(x) = x^5 - x^3$$

$$f'(x) = 5x^4 - 3x^2 = x^2(5x^2 - 3)$$

$\nexists f'(x)$ NONE $\qquad 5x^2 - 3 = 0 \quad \pm\sqrt{\frac{3}{5}}$

$\exists f'(x) = 0 \qquad x = 0,\ x = +\sqrt{\frac{3}{5}}$ AND $x = -\sqrt{\frac{3}{5}}$

- SIGN OF $f'$

| | $-\sqrt{\frac{3}{5}}$ | $0$ | $+\sqrt{\frac{3}{5}}$ | |
|---|---|---|---|---|
| $-$ | $-$ | $+$ | | $+$ |
| $+$ | $-$ | $-$ | | $+$ |
| $-$ 0 | $+$ 0 | $-$ 0 | | $+$ |

$$f\left(-\sqrt{\tfrac{3}{5}}\right) = -\frac{3}{5}^{5/2} + \frac{3}{5}^{3/2} \cong 0.18$$

$$f\left(\sqrt{\tfrac{3}{5}}\right) = \frac{3}{5}^{5/2} - \frac{3}{5}^{3/2} \cong -0{,}18$$

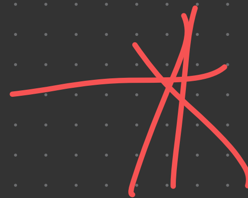
$f$ STRICTLY DEC ON $\left(-\infty, -\sqrt{\frac{3}{5}}\right]$

$f$ STRICTLY INC. ON $\left[-\sqrt{\frac{3}{5}}, 0\right]$

$f$ STRICTLY DEC ON $\left[0, \sqrt{\frac{3}{5}}\right]$

$f$ STRICTLY INC ON $\left[\sqrt{\frac{3}{5}}, +\infty\right)$

93) $f(x) = |x^2 - 1| - \frac{1}{2}x^3$



DOMAIN $\mathbb{R}$

SYMMETRIES NON-EVIDENT

SIGN OF $f$ AND INTERSECTION WITH X-AXIS AND Y-AXIS

$f(0) = 1$

$|x^2-1| - \frac{1}{2}x^3 = 0$ ; $|x^2 - 1| = \frac{1}{2}x^3$

$x^2 - 1 \geq 0$ ;


$x \leq -1 \lor x \geq 1$ $\quad x^2 - 1 = \frac{1}{2}x^3$ ; $\frac{1}{2}x^3 - x^2 + 1 = 0$

- $-1 \leq x \leq 1$ $\quad -x^2 + 1 = \frac{1}{2}x^3$ ; $\frac{1}{2}x^3 + x^2 - 1 = 0$

$\frac{1}{2}x^3 - x^2 + 1$ $\quad$ $\frac{1}{2}x^3 + x^2 - 1$

$-1$ $\quad$ $\frac{1}{2}x^3 - x^2 + 1$

$1$

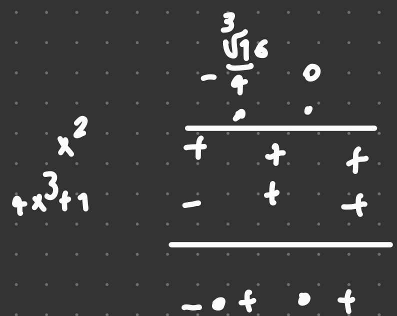
9c) $f(x) = \dfrac{x^2}{4x^3+1}$ $\quad 4x^3+1 \neq 0$ ; $x \neq -\dfrac{1}{\sqrt[3]{4}}$ ; $x \neq -\dfrac{\sqrt[3]{16}}{4} \cong 0.6$

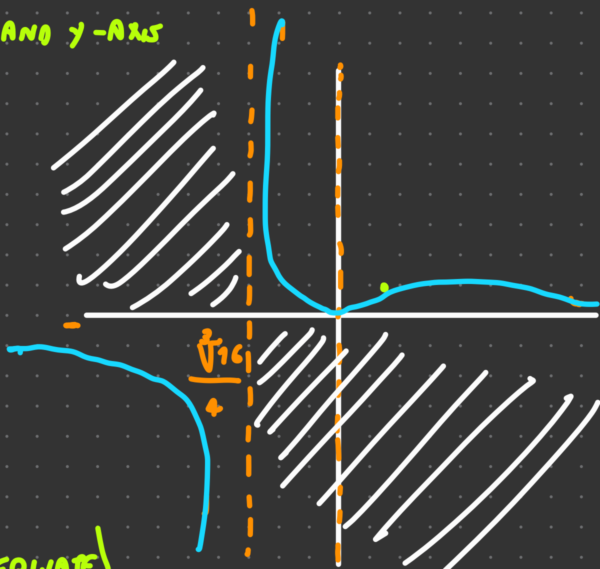
D: $\mathbb{R} - \left\{ -\dfrac{\sqrt[3]{16}}{4} \right\}$

SYMMETRIES NON EVIDENT

INTERSECTION X-AXIS AND Y-AXIS AND SIGN OF $f$

$f(0) = 0 \quad (0,0)$






CONTINUITY, LIMITS, AND DERIVATES

- $\lim_{x \to +\infty} \dfrac{x^2}{4x^3+1} = \dfrac{\cancel{x^2}}{\cancel{x^2}\left(4x+\frac{1}{x^2}\right)} = 0^+$
- $\lim_{x \to -\infty} \dfrac{x^2}{4x^3+1} = 0^-$
- $\lim_{x \to -\frac{\sqrt[3]{16}}{4}^-}$ $\quad -\infty$

$\bullet \lim_{x \to -\frac{\sqrt[3]{16}}{4}^+}$ $+\infty$

ASYMPTOTE

$y = mx + q \quad m, q \in \mathbb{R}$

$$m = \lim_{x \to +\infty} \frac{x^2}{4x^4 + x} = \lim_{x \to +\infty} \frac{\cancel{x^2}}{\cancel{x^2}\left(4x^2 + \frac{1}{x}\right)} = 0$$

$$q = \lim_{x \to +\infty} f(x) - mx = \frac{x^2}{4x^3 + 1} = 0$$

$\boxed{y = 0}$ ASYMPTOTE

DIFFERENTIABILITY AND DERIVATIVES

$$f(x) = \frac{x^2}{4x^3 + 1}$$

$$f'(x) = \frac{2x(4x^3+1) - x^2(12x^2)}{(4x^3+1)^2} = \frac{8x^4 + 2x - 12x^4}{4x^3+1} = \frac{-4x^4 + 2x}{(4x^3+1)^2}$$

$$= \frac{2x - 4x^4}{(4x^3+1)^2} \qquad x = \sqrt[3]{\frac{1}{2}}$$

$$\nexists f'(x) \qquad x = \frac{\sqrt[3]{16}}{4}$$

$$\exists f'(x) = 0 \qquad 2x(-2x^3 + 1) = 0$$

$$x = 0 \text{ AND } x = \frac{1}{\sqrt[3]{2}}$$

| | | $-\sqrt[3]{1/4}$ | | $0$ | | $\sqrt[3]{1/2}$ | |
|---|---|---|---|---|---|---|---|
| N1 | $-$ | | $-$ | | $+$ | | $+$ |
| N2 | $-$ | | $-$ | | $-$ | | $+$ |
| D1 | $-$ | | $+$ | | $+$ | | $+$ |
| | $-$ | • | $+$ | • | $-$ | • | $+$ |
| | $\searrow$ | | $\nearrow$ | | $\searrow$ | | $\nearrow$ |

$-0.629$ $\qquad$ $0.793$

9D) $\dfrac{2+x^2}{x-2}$ • $x \in \mathbb{R} \mid x \neq 2$

• NO EVIDENT SYMMETRIES

• $f(0) = -1$

NO INTERSECTION W/ X-AXIS

• SIGN

$$\begin{array}{ccc} & 2 & \\ - & \vdots & + \end{array}$$

• $\lim\limits_{x \to 2^+} \dfrac{2/x + x}{1 - \frac{2}{x}} = \dfrac{3}{0^+} = +\infty$

$\lim\limits_{x \to 2^-} \dfrac{2/x + x}{1 - \frac{2}{x}} = \dfrac{3}{0^-} = -\infty$

$\lim\limits_{x \to +\infty} \dfrac{+\infty}{1} = +\infty$

$\lim\limits_{x \to -\infty} \dfrac{-\infty}{1} = -\infty$

(d) $f(x) = \dfrac{2+x^2}{x-2}$

(e) $f(x) = \dfrac{2-x^2}{x-2}$

2

(f) $f(x) = \dfrac{x-3}{(x-2)^2}$

For any $y_0 \in \mathbb{R}$, how many solutions does the equation $f(x) = y_0$ have?

(g) $f(x) = \log(-(x^2+3x+2))$

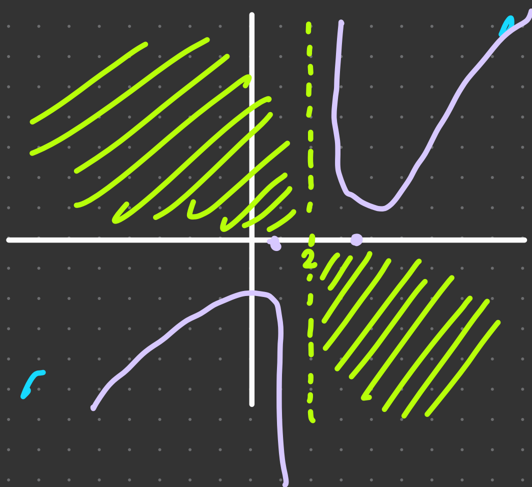
(h) $f(x) = e^{-(x^2+3x+2)}$

(i) $f(x) = \log(1+x^2)$

(j) $f(x) = e^x - \log(x+1)$

Hint: in order to determine the sign of $f'$, call $g = f'$, compute $g(0) = f'(0)$ and study the function $g$, in particular the behaviour of its derivative $g'$.

(k) $f(x) = (\log(x))^2 + 2\log(x)$



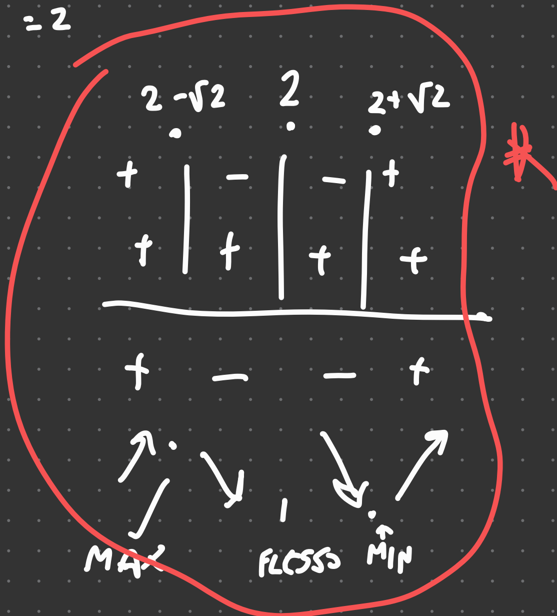

DERIVATES

$f(x) = \dfrac{2+x^2}{x-2}$

$f'(x) = \dfrac{(2x)(x-2) - (2+x^2)}{(x-2)^2} = \dfrac{2x^2-4x-2-x^2}{(x-2)^2} = \dfrac{x^2-4x-2}{x^2-4x+4}$

$16-4(-2)=$ $\quad \dfrac{4 \pm 2\sqrt{2}}{2} = 2 \pm \sqrt{2}$

$16-4(4) \quad = 2$



(g) $f(x) = \log(-(x^2 + 3x + 2))$

• D: $-x^2 - 3x - 2 > 0$

$9 + 4(-2)$ $\qquad \dfrac{3 \pm 1}{-2} \begin{cases} -2 \\ -1 \end{cases}$

$-2 \quad + \quad -1$

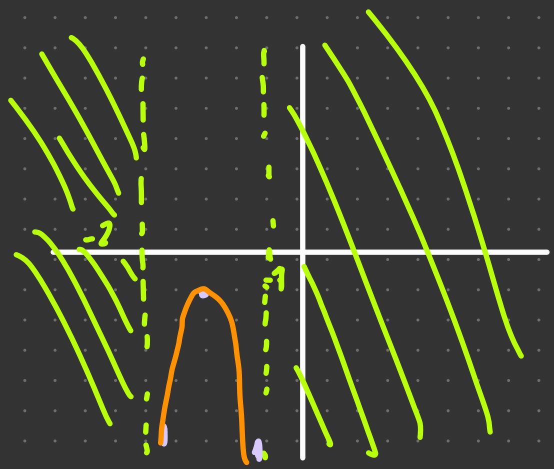
$-2 < x < -1$

D: $(-2, -1)$

$\lim_{x \to -2^+} \log(-x^2 - 3x - 2) = -\infty$

$-4 + 6 - 2$

$\lim_{x \to -1^-} -\infty$



$f(x) = \log(-x^2 - 3x - 2)$

$$\frac{-2x-3}{-x^2-3x-2}$$

$-2x - 3 \geq 0; \quad x \leq -\frac{3}{2}$

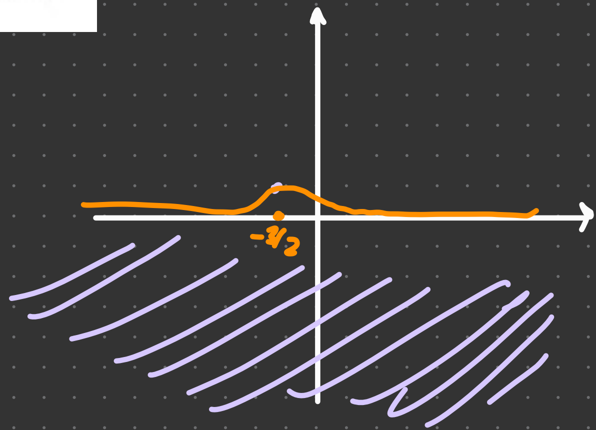
| $-2$ | $-3/2$ | $-1$ |
|---|---|---|

| | | | |
|---|---|---|---|
| + | + | − | − |
| − | + | + | − |
| − | + | − | + |
| ↘ | ↗ | ↘ | ↗ |

$-3/2$ MAX

$(-3/2, -1{,}38)$

(h) $f(x) = e^{-(x^2+3x+2)}$




D: $\mathbb{R}$

NO EVIDENT SIMMETRY

$f(0) = \frac{1}{e^2}$

NO INTERSECTION X AXIS

SIGN $f(x) > 0 \quad \forall x \in \mathbb{R}$

$\lim_{x \to +\infty} \quad 0^+$

$\lim_{x \to -\infty} \quad 0^+$

DERIVATE

$f(x) = e^{-x^2-3x-2}$

$f'(x) = \left(e^{-x^2-3x-2}\right)(-2x-3)$

$f'(x) = 0 \qquad (-2x-3) = 0; \quad x = -\frac{3}{2}$

$-3/2$

$+ \quad \bullet \quad -$

$\nearrow \quad \searrow$

$f(-3/2) =$

(q) $f(x)=\sqrt[3]{\frac{x^3-1}{x}}$

Hint: be careful on finding the points where $f$ differentiable. Is $f$ differentiable in $x=1$?

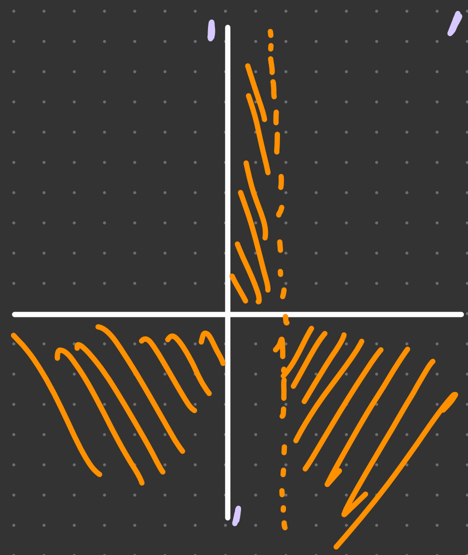
D: $x \neq 0$

SIGN

$x^3-1 \geq 0$; $x \geq 1$

$x > 0$

| | 0 | | 1 | |
|---|---|---|---|---|
| | − | − | | + |
| | − | + | | + |
| | + | − | | + |



$$\lim_{x\to 0^+} \sqrt[3]{x^2-\frac{1}{x}} = -\infty$$

$$\lim_{x\to 0^-} = +\infty$$

$$\lim_{x\to +\infty} = +\infty$$

$$\lim_{x\to -\infty} = +\infty$$

DERIVATE

$$f(x)=\sqrt[3]{x^2-\frac{1}{x}}=\left(x^2-\frac{1}{x}\right)^{1/3}$$

$$f'(x)=\frac{1}{3}\left(x^2-\frac{1}{x}\right)^{-2/3}\left(2x+\frac{1}{x^2}\right)$$

$$f'(x)=\left(\frac{2}{3}x+\frac{1}{3x^2}\right)\frac{1}{\sqrt[3]{\left(x^2-\frac{1}{x}\right)^2}}$$

$$\frac{2x^3+1}{3x^2}$$

$2x^3+1 \geq 0;\quad x \geq \sqrt[3]{-\frac{1}{2}}$

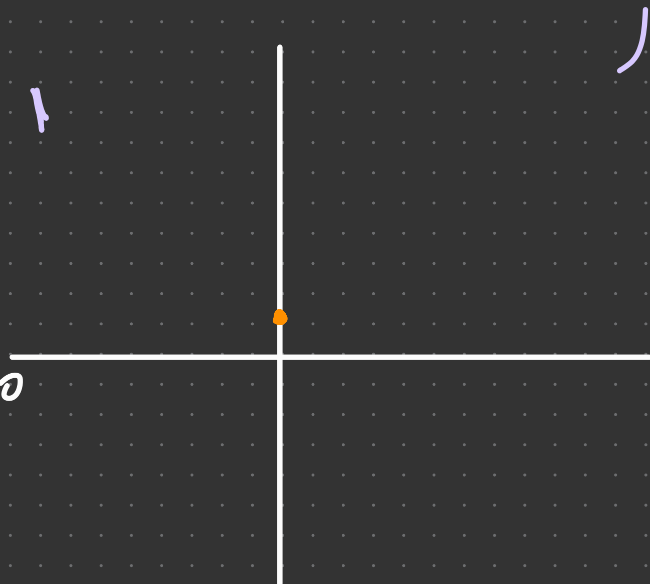
$-1/\sqrt[3]{2}$

$-\quad \bullet \quad +$

(j) $f(x) = e^x - \log(x+1)$

D: $x+1>0$; $x>-1$

SYMMETRIES: NOT EVIDENT



SIGN $e^x - \log(x+1)$

$e^x > 0 \ \forall x \in \mathbb{R}$

$\log(x+1) > 0$; $x+1>1$; $x>0$

$f(0) = 1$

$e^x - \log(x+1) > 0$

$e^x > \log(x+1)$

$x > e^{\log(x+1)}$

$\lim_{x \to +\infty} e^x - \log(x+1) = +\infty$

$\lim_{x \to -1^+} \frac{1}{e} - \log(0) = +\infty$

$f(x)\ e^x - \log(x+1)$ $\qquad e^x - \frac{1}{x+1}$ $\boxed{x=0}$

(k) $f(x) = (\log(x))^2 + 2\log(x)$

D: $x > 0$

SIGN

$x > 1$

$t^2 + 2t = 0$

$t(t+2) = 0$

$t > 0 \vee t+2 > 0$

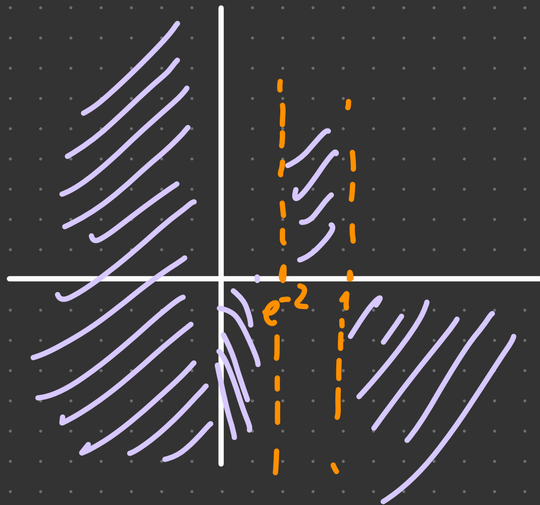
$\log(x) > 0 \quad \log(x) + 2 > 0$

$x > 1 \quad \Downarrow$

$x > e^{-2}$

| 0 | $e^{-2}$ | 1 | |
|---|---|---|---|
| $\not\exists$ | $-$ | $-$ | $+$ |
| $\not\exists$ | $-$ | $+$ | $+$ |

$\not\exists\ \not\exists \; + \cdot - \cdot +$



(m) $f(x) = 3x + 4\sqrt{1-x^2}$

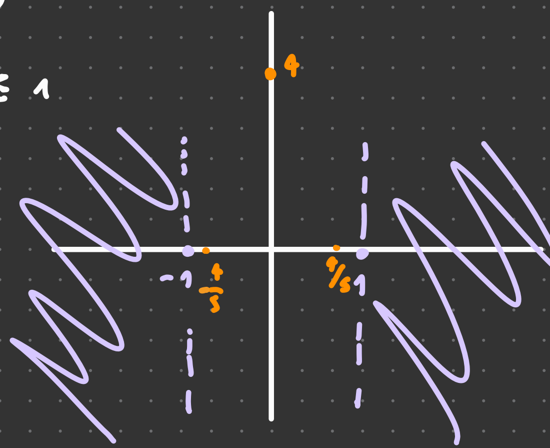
D: $1 - x^2 \ge 0$


$-x^2 + 1$

$x^2 \le 1$

$-1 \le x \le 1$

$-1 \le x \le 1$




• $3x + 4\sqrt{1-x^2}$

$f(0) = 0 + 4$

$(0, 4)$

$0 = 3x + 4\sqrt{1-x^2}$

$-3x = 4\sqrt{1-x^2}$

$9x^2 = 16 - 16x^2$

$9x^2 + 16x^2 = 16$

$25x^2 = 16$

$x^2 = \frac{16}{25}$

$x = \pm\frac{4}{5}$

$3x + 4\sqrt{1-x^2} \ge 0$

$4\sqrt{1-x^2} \ge -3x$

$16(1-x^2) \ge 9x^2$

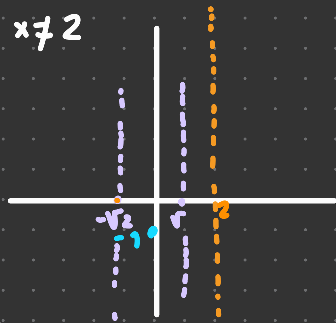
$16 - 16x^2 \ge 9x^2$

$-25x^2 \ge -16$

$x^2 \le \frac{16}{25}$


−

$$\frac{2-x^2}{x-2} \qquad x \neq 2$$

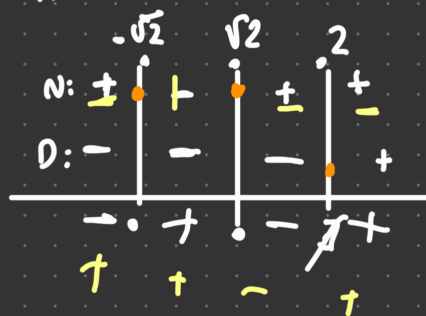



$f(0)=\frac{2}{-2}=-1 \quad (0,-1)$

$\frac{2-x^2}{x-2}=0 \qquad x^2=2 \qquad x=\pm\sqrt{2}$


$N \geq 0 \quad 2-x^2 \geq 0 \qquad x^2 \leq 2 \qquad x \leq -\sqrt{2} \quad x \geq \sqrt{2}$

$D > 0 \quad x-2>0 \qquad x>2 \qquad x>2$




$$f(x)=\frac{2-x^2}{x-2}$$

$$\lim_{x\to+\infty}\frac{2-x^2}{x-2}=\frac{\cancel{x}\left(\frac{2}{x}-x\right)}{\cancel{x}\left(1-\frac{2}{x}\right)}=\frac{0-\infty}{1-0}=-\infty$$

$$\lim_{x\to-\infty} = +\infty$$

$$\lim_{x\to 2^+}$$

$$\lim_{x\to 2^-}$$

g) $\log(-x^2-3x-2)$

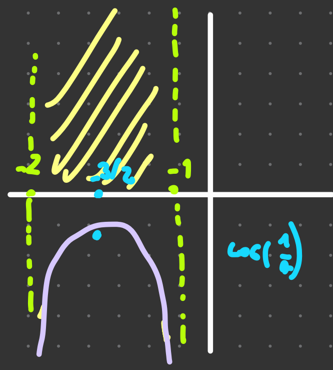
D: $-x^2-3x-2>0$

$9-4(-1)(-2)$

$\frac{3\pm 1}{-2} \lessgtr {-2 \atop -1}$

$-1 \; + \; -2$

$\boxed{-1<x<-2}$




SIGNI:

$-x^2-3x-2>1$ $\quad \log(1)=0$

$-x^2-3x-3>0$

$9-4(-1)(-3)$ $\quad \Delta<0 \quad \emptyset$

$- \; -$

LIMITS

$\lim_{x\to -2^+} \log(-x^2-3x-2) = -\infty$

$\lim_{x\to -1^-} \log(-x^2-3x-2) = -\infty$

$f'(x) = \frac{-2x-3}{-x^2-3x-2} = \frac{2x+3}{x^2+3x+2}$

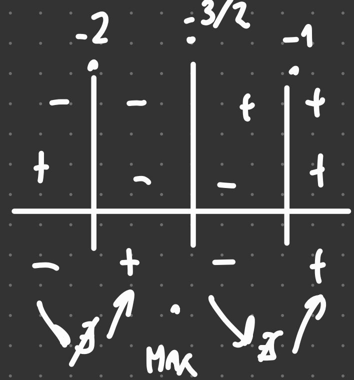
$9-4(2)$

$\frac{-3\pm 1}{2} \lessgtr {-2 \atop -1}$

$-2 \quad -1$

$2x+3>0 \quad x>-3/2$

$\left(-\frac{3}{2}, \log\left(\frac{1}{4}\right)\right)$




$$f'(x) = \frac{2x+3}{x^2+3x+2}$$

$$f''(x) = \frac{2x^2+6x+4-(2x+3)(2x+3)}{(x^2+3x+2)^2}$$

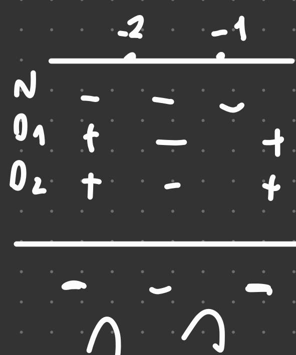
$$f''(x) = \frac{2x^2+6x+4-(4x^2+9+12x)}{(x^2+3x+2)^2}$$

$$f''(x) = \frac{2x^2+6x+4-4x^2-9-12x}{(x^2+3x+2)^2} = \frac{-2x^2-6x-5}{(x^2+3x+2)(x^2+3x+2)}$$

N: $-2x^2-6x-5 \geq 0$

$36-4(-2)(-5)$

$\Delta<0$




D1 

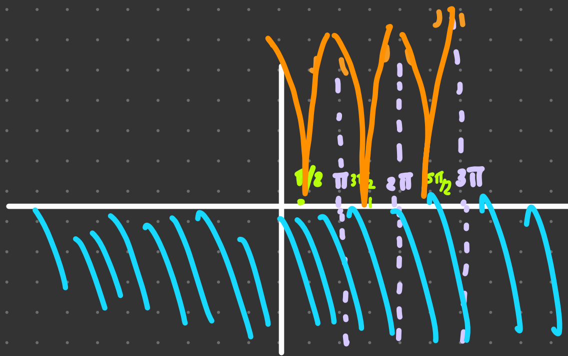

e) $f(x) = -\log(|\sin(x)|)$ $D: \mathbb{R} - \{k\pi\}$ $k \in \mathbb{N}$

SEGNO

$f(x) > 0$ $0 < |\sin(x)| < 1$

$x \neq k\pi$

$f(x) < 0$ $|\sin(x)| > 1$ $\emptyset$




$f(0) = -\log(|\sin(0)|)$ $\emptyset$

$-\log(|\sin(x)|) = 0$

$\log(|\sin(x)|) = 0$

$|\sin(x)| = 1$

$x = \pm \frac{\pi}{2} k$

DERIVATA

$f(x) = -\log(|\sin(x)|)$

$\boxed{\sin(x) > 0}$

$f'(x) = -\frac{\cos(x)}{\sin(x)} = -\tan^{-1}(x)$

$\boxed{\sin(x) < 0}$

$f'(x) = -\frac{-\cos(x)}{-\sin(x)} = -\tan^{-1}(x)$

$\lim_{x \to +\infty}$ $\emptyset$

$\lim_{x \to -\infty}$ $\emptyset$

$-\frac{\cos(x)}{\sin(x)}$

$-k\pi/2$ $0$ $k\pi/2$ $k\pi$

| | | | | | |
|---|---|---|---|---|---|
| | − | + | + | − | − |
| $-\cos(x) > 0$ $\quad -\frac{k\pi}{2} < x < \frac{k\pi}{2}$ | − | − | + | + | − |
| $\sin(x)$ $\quad 0 < x < k\pi$ | + | − | + | − | + |
| | ↗ | ↘ | ↗ | ↘ | ↗ |

$\lim_{x \to k\pi^+} -\log(|\sin(x)|) = +\infty$

$\lim_{x \to k\pi^-} -\log(|\sin(x)|) = +\infty$

$$\frac{d}{dx} \text{ARCTAN} = \frac{1}{1+x^2}$$

$$f''(x) = -\frac{1}{1+x^2} \qquad f''(x) < 0 \quad \forall x \in \mathbb{R}$$