$+ \quad \times$

$- \quad \div$

DEGREES OF FREEDOM

$$L(\underbrace{q_1, q_2, \dots q_N}_{\text{GENERALIZED COORDS}}, \underbrace{\dot q_1, \dot q_2, \dots, \dot q_N}_{\text{GENERALIZED VELOCITIES}})$$

2N DEGREE OF FREEDOM

MISSING   ALSO $\dot q_k$

- $L = L(q_1, \dots \cancel{q_k}, \dots q_N, \dot q_1, \dots, \dot q_N)$

$L(x, \dot x) = \frac{1}{2} m \dot x^2$

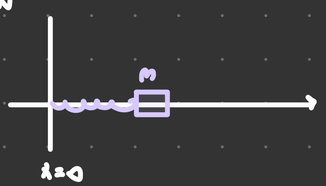
$\rightarrow m\ddot x = 0 \qquad \frac{d}{dt}(m\dot x) = 0 \longrightarrow \dot p = 0$

$\partial_{\dot x} L = p \qquad p$ CONSTANT

$$\frac{d}{dt} \partial_{\dot x} L = \partial_x L$$

$\frac{d}{dt} p = 0 \Rightarrow p_k = \partial_{\dot q_k} L$ IS A CONSERVED QUANTITY

EXERCISE OF MOTION



KINETIC $= \frac{1}{2} m \dot x^2$ v^2

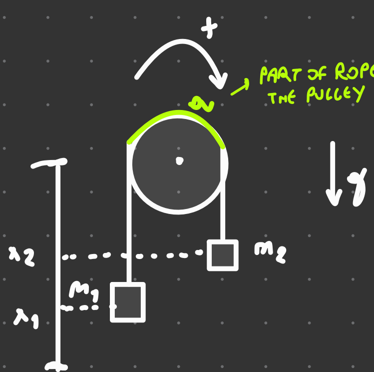
POTENTIAL $= \frac{1}{2} k x^2$

KINETIC   POTENTIAL

$$L(x, \dot x) = \frac{1}{2} m \dot x^2 - \frac{1}{2} k x^2$$

$$\frac{d}{dt} \partial_{\dot x} L = \partial_x L \qquad \begin{cases} \partial_{\dot x} L = m \dot x \\ \partial_x L = -kx \end{cases} \qquad \boxed{m\ddot x = -kx}$$

EXERCISE ATWOOD MACHINE



① $L(x_1, x_2, \dot x_1, \dot x_2)$

② $L(x_1, \dot x_1)$

~~KINETIC m_1: $\frac{1}{2} m_1 \dot x_1^2 - \frac{1}{2} m_2 \dot x_2^2$~~

~~POTENTIAL m_1: $m_1 g x_1 - m_2 g x_2$~~

~~KINETIC m_2: $\frac{1}{2} m_1 \dot x_1^2 - \frac{1}{2} m_1 \dot x_1^2$~~

~~POTENTIAL m_2: $m_2 g x_2 - m_1 g x_1$~~

$K = \frac{1}{2} m_1 \dot x_1^2 + \frac{1}{2} m_2 \dot x_2^2$

$U = -m_1 g x_1 - m_2 g x_2$

$L = \frac{1}{2}(m_1 \dot x_1^2 + m_2 \dot x_2^2) + g(m_1 x_1 + m_2 x_2)$

$l = x_1 + x_2 + a \rightarrow x_1 + x_2 = b$ $l - a$

THERMODYNAMICS

A THERMODYNAMIC SYSTEM IS A MACROSCOPIC SYSTEM WITH A LARGE NUMBER OF DEGREES OF FREEDOM

NOT A SINGLE ATOM

DoF OF SINGLE MOLECULES OF GAS

GAS

- MADE OF SMALL ELEMENTS THAT TRAVEL IN A SPACE
- EACH ONE HAVE ITS OWN VELOCITY
- THEY CAN INTERACT WITH EACHOTHER

- A LIQUID IS A THERMODYNAMIC SYSTEM TOO
- A SOLID IS A THERMODYNAMIC SYSTEM TOO

USE DYNAMICAL SYSTEMS AND ORDINARY DIFFERENTIAL EQUATIONS

LAGRANGIAN MECHANICS TO GET ODE STARTING FROM ITS ENERGY

LAGRANGIAN

SUM OF KINETIC ENERGY OF ALL THE PARTICLES MINUS THE POTENTIAL ENERGY DUE TO INTERACTION

PROBLEMS

- SMALL NUMBER OF DoF. THE EQUATION OF MOTION FOR LARGE SYSTEM WITH MANY DoF IT WOULD BE INTRACTABLE

EVEN IF I CAN WRITE IT DOWN NOBODY CAN SOLVE THEM

- EVEN IF THE MOTION EQUATION WERE TRACTABLE. TO BE SOLVE IT NEEDS THE INITIAL CONDITION

TO KNOW $\ddot{x}=0$ NEEDS
- $x(0)$
- $\dot{x}(0)$

⇒ USE REDUCTION APPROACH

- GAS
- BOILING WATER
- CHEMICAL REACTION
- RESISTORS

DON'T CARE ABOUT THE SINGLE MOLECULE BUT THE GENERAL APPROACH

- USE MACROSCOPIC DoF INSTED OF MICROSCOPIC
- LOOK FAR AWAY THE SYSTEM

- FOR A GAS: TEMPERATURE, PRESSURE, VOLUME, NUMBER OF PARTICLES
- RESISTOR: CURRENT, POTENTIAL DIFFERENCE, TEMPERATURE, RESISTANCE

CURRENT INSIDE

- SPRING/RUBBER BAND: TENSION, $\underset{\Delta x}{\text{ELONGATION}}$, TEMPERATURE, ELASTIC CONSTANT

GOAL OF TERMODYNAMICS: DESCRIBE SYSTEM IN TERMS OF THEIR MACROSCOPIC DEGREES OF FREEDOM (AKA TERMODYNAMICS VARIABLE)