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FROM SHEET 11

5. Compite, if it exists, the following limit

(a) $\lim_{x\to 0} \dfrac{\sin(x)+\log(1-x)}{x^2}$

(b) $\lim_{x\to 0} \dfrac{x\cos x - \sin x}{x^2}$

(c) $\lim_{x\to 0} \dfrac{1-e^{-\sin x}}{1+x-\cos x}$

5a) $\lim_{x\to 0} \dfrac{\sin(x)+\log(1-x)}{x^2}$ $\left[\frac{0}{0}\right]$

APPLY HOPITAL

$$\lim_{x\to 0} \frac{\cos(x)-\frac{1}{1-x}}{2x} = \lim_{x\to 0} \frac{\frac{(1-x)\cos(x)-1}{1-x}}{2x}$$

$$\lim_{x\to 0} \frac{(1-x)\cos(x)-1}{(1-x)(2x)} = \lim_{x\to 0} \frac{\cos(x)-x\cos(x)-1}{2x-2x^2}$$ APPLY HOPITAL

$$= \lim_{x\to 0} \frac{-\operatorname{sen}(x)+x\operatorname{sen}(x)-\cos(x)}{2-4x} = \frac{-0+0-1}{2} = \boxed{-\frac{1}{2}}$$

(b) $\lim_{x\to 0} \frac{x\cos x - \sin x}{x^2}$

$$\lim_{x\to 0} \frac{x\cos x - \sin x}{x^2} \quad \left[\frac{0}{0}\right]$$ APPLY HOPITAL

$$\lim_{x\to 0} \frac{\cancel{\cos x} - x\operatorname{sen} x - \cancel{\cos x}}{2x} = \lim_{x\to 0} \frac{-\cancel{x}\operatorname{sen} x}{2\cancel{x}} = \boxed{0}$$

(c) $\lim_{x\to 0} \frac{1 - e^{-\sin x}}{1 + x - \cos x}$

$$\lim_{x\to 0} \frac{1 - e^{-\sin x}}{1 + x - \cos x} \quad \left[\frac{0}{0}\right]$$ APPLY HOPITAL

$$\lim_{x\to 0} \frac{-e^{-\sin x}\cdot(-\cos x)}{1 + \sin x} = \lim_{x\to 0} \frac{\cos(x)\cdot e^{-\sin x}}{1 + \sin x} = \boxed{1}$$

(d) $\lim_{x\to 0} \frac{e^{\sin x} - 1 - x}{\log(\cos x)}$

$$\lim_{x\to 0} \frac{e^{\sin x} - 1 - x}{\log(\cos(x))} \quad \left[\frac{0}{0}\right]$$ APPLY HOPITAL

$$= \lim_{x\to 0} \frac{\cos(x)\cdot e^{\sin x} - 1}{\frac{-\sin(x)}{\cos(x)}} = \lim_{x\to 0} \left(\cos(x)\cdot e^{\sin(x)} - 1\right)\cdot \frac{-\cos(x)}{\sin(x)}$$

$$= \lim_{x \to 0} \frac{-\cos^2(x) \cdot e^{\sin(x)} + \cos(x)}{\sin(x)}$$ $\left[\frac{0}{0}\right]$ HOPITAL

$$= \lim_{x \to 0} \frac{2\cos(x)\,\mathrm{sen}(x) \cdot e^{\sin(x)} - \cos^3(x) \cdot e^{\sin(x)} - \sin(x)}{\cos(x)} = \boxed{-1}$$

$$\text{(e)} \quad \lim_{x \to 0} \frac{\log(1 - x^4)}{e^{x^2} - 1 - x^2}$$

$$\lim_{x \to 0} \frac{\log(1 - x^4)}{e^{x^2} - 1 - x^2}$$ $\left[\frac{0}{0}\right]$ APPLY HOPITAL

$$= \lim_{x \to 0} \frac{\frac{-4x^3}{1 - x^4}}{2xe^{x^2} - 2x} = \lim_{x \to 0} \frac{-\cancel{4}x^{\cancel{3}\,2}}{1 - x^4} \cdot \frac{1}{\cancel{2x}\,e^{x^2} - \cancel{2x}} =$$

$$= \lim_{x \to 0} \frac{-2x^2}{(1 - x^4)(e^{x^2} - 1)} = \lim_{x \to 0} \frac{-2x^2}{e^{x^2} - 1 - x^4 e^{x^2} + x^4}$$ $\left[\frac{0}{0}\right]$ APPLY HOPITAL

$$= \lim_{x \to 0} \frac{-\cancel{4x}\;2}{\cancel{2x}\, e^{x^2} - \cancel{4}^{\,2} x^{3} e^{x^2} - \cancel{2x}\, x^{\cancel{5}\,4} e^{x^2} + \cancel{4}^{\,2} x^{\cancel{3}\,2}}$$

$$= \lim_{x \to 0} \frac{-2}{e^{x^2} - 2x^2 e^{x^2} - x^4 e^{x^2} + 2x^2} = \frac{-2}{1 - 0 - 0 - 0} = \boxed{-2}$$

(f) $\lim\limits_{x \to 0} \dfrac{\log(1 - x^3)}{e^{x^2} - 1 - x^2}$

$$\lim_{x \to 0} \frac{\log(1-x^3)}{e^{x^2}-1-x^2} \quad \left[\frac{0}{0}\right] \text{ APPLY HOPITAL}$$

$$\lim_{x \to 0} \frac{\dfrac{-3x^2}{1-x^3}}{2xe^{x^2}-2x} = \lim_{x \to 0} \frac{-3x^{\cancel{2}}}{1-x^3} \cdot \frac{1}{2\cancel{x}e^{x^2}-2\cancel{x}}$$

$$= \lim_{x \to 0} \frac{-3x^2}{(1-x^3)(2e^{x^2}-2)} = \lim_{x \to 0} \frac{-3x^2}{2e^{x^2}-2-2x^3e^{x^2}+2x^3} \quad \left[\frac{0}{0}\right]$$

APPLY HOPITAL

$$= \lim_{x \to 0} \frac{-\cancel{6x}\,3}{2\cancel{4x}e^{x^2}-\cancel{6}3x^{\cancel{4}2}e^{x^2}-2x^3\cdot\cancel{2x}e^{x^2}+\cancel{6}3x^{\cancel{4}2}} = \lim_{x \to 0} \frac{-3}{2e^{x^2}-3xe^{x^2}-2x^3e^{x^2}+3x} = \boxed{-\frac{3}{2}}$$

* DOES NOT EXISTS

$$\text{(g)} \lim_{x \to 0} \frac{\log(x^2 - \sin^2 x + 1)}{e^{x^2} - 1 - x^2}$$

$$\lim_{x \to 0} \frac{\cos(x^2 - \sin^2 x + 1)}{e^{x^2} - 1 - x^2} \quad \left[\frac{0}{0}\right]$$

$\boxed{\frac{2}{3}}$

APPLY HOPITAL

$$\lim_{x \to 0} \frac{\frac{2x - 2\sin(x)\cos(x)}{x^2 - \sin^2 x + 1}}{2xe^{x^2} - 2x} = \lim_{x \to 0} \frac{\cancel{2}x - \cancel{2}\sin(x)\cos(x)}{x^2 - \sin^2(x) + 1} \cdot \frac{1}{\cancel{2}xe^{x^2} - \cancel{2}x}$$

$$\lim_{x \to 0} \frac{x - \sin(x)\cos(x)}{x^3 e^{x^2} - x\sin^2(x)\, e^{x^2} + xe^{x^2} - x^3 - x\sin^2(x) + x}$$

(h) $\lim_{x\to 0} \dfrac{x\left(2e^{-x} - 2 + 2x - x^2\right)}{(\cos(x) - 1)^2}$

$$\lim_{x\to 0} \frac{2xe^{-x} - 2x + 2x^2 - x^3}{\cos(x)^2 + 1 - 2\cos(x)} \quad \left[\frac{0}{0}\right]$$

APPLY HOPITAL

$$\lim_{x\to 0} \frac{2e^{-x} - 2xe^{-x} - 2 + 4x - 3x^2}{-2\cos(x)\sin(x) + 2\sin(x)} \quad \left[\frac{0}{0}\right]$$ APPLY HOPITAL

$$\lim_{x\to 0} \frac{-2e^{-x} - 2e^{-x} + 2xe^{-x} + 4 - 6x}{2\sin^2(x) - 2\cos^2(x) + 2\cos(x)} = \lim_{x\to 0} \frac{xe^{-x} - 2e^{-x} - 3x + 2}{\sin^2(x) - \cos^2(x) + \cos(x)} \quad \left[\frac{0}{0}\right]$$

APPLY HOPITAL

$$\lim_{x\to 0} \frac{e^{-x} - xe^{-x} + 2e^{-x} - 3}{2\sin(x)\cos(x) + 2\sin(x)\cos(x) - \sin(x)} = \lim_{x\to 0} \frac{3e^{-x} - xe^{-x} - 3}{4\sin(x)\cos(x) - \sin(x)} \quad \left[\frac{0}{0}\right]$$

APPLY HOPITAL

$$\lim_{x\to 0} \frac{-3e^{-x} - e^{-x} + xe^{-x}}{4\cos^2(x) - 4\sin^2(x) - \cos(x)} = \frac{-3-1}{3} = \boxed{-\frac{4}{3}}$$

(i) $\lim\limits_{x\to 0} \dfrac{\log(1+x^2)-x^2}{2x^3\sin(x)}$

$$\lim_{x\to 0} \frac{\log(1+x^2)}{x^2} = 1$$

$$\lim_{x\to 0} \frac{\dfrac{2x}{1+x^2} - 2x}{6x^2\sin(x) + 2x^3\cos(x)} \quad \left[\frac{0}{0}\right] \text{ USE HOPITAL}$$

$$\lim_{x\to 0} \frac{\dfrac{\cancel{2x} - \cancel{2x} - 2x^3}{1+x^2}}{6x^2\sin(x) + 2x^3\cos(x)}$$

$$\lim_{x\to 0} \frac{-\cancel{2}x^{\cancel{3}}}{1+x^2} \cdot \frac{1}{\underset{3}{\cancel{6x^2}}\sin(x) + \underset{1}{\cancel{2}}x^3\cos(x)} = \lim_{x\to 0} \frac{-x}{(1+x^2)(3\sin(x) + x\cos(x))}$$

$$\lim_{x\to 0} \frac{-x}{3\sin(x) + x\cos(x) + 3x^2\sin(x) + x^3\cos(x)} \quad \left[\frac{0}{0}\right] \text{ HOPITAL}$$

$$\lim_{x\to 0} \frac{-1}{3\cos(x) + \cos(x) - x\sin(x) + 6x\sin(x) + 3x^2\cos(x) + 3x^2\cos(x) - - x^3\sin(x)} = \boxed{-\frac{1}{4}}$$

7)

$$\lim_{x \to 0} \frac{e^x - 1 - \sin(x) - x^2/2}{\cos(1+x^3)} \quad \left[\frac{0}{0}\right] \text{ APPLY HOPITAL}$$

$$\lim_{x \to 0} \frac{e^x - \cos(x) - x}{\frac{3x^2}{1+x^3}} = \lim_{x \to 0} \frac{(e^x - \cos(x) - x)(1+x^3)}{3x}$$

$$\lim_{x \to 0} \frac{e^x - \cos(x) - x + x^3 e^x - x^3 \cos(x) - x^4}{3x} \quad \left[\frac{0}{0}\right] \text{ APPLY HOPITAL}$$

$$\lim_{x \to 0} \frac{e^x + \sin(x) - 1 + 3x^2 e^x + x^3 e^x - 3x^2 \cos(x) + x^3 \sin(x) - 4x^3}{3} = 0$$

k) $\lim_{x \to 0} \dfrac{e^{x^2} - 1 - x^2 + 3x^4}{\cos(2x^2) - 1}$ $\left[\frac{0}{0}\right]$ HOPITAL

$$\lim_{x \to 0} \frac{2xe^{x^2} - 2x + 12x^3}{-4x\sin(2x^2)} \quad \left[\frac{0}{0}\right] \text{ APPLY HOPITAL}$$

$$\lim_{x \to 0} \frac{2e^{x^2} + 4x^2e^{x^2} - 2 + 36x^2}{-4\sin(2x^2) - 16x^2\cos(2x^2)} \quad \left[\frac{0}{0}\right] \text{ APPLY HOPITAL}$$

$$\lim_{x \to 0} \frac{4xe^{x^2} + 8xe^{x^2} + 8x^3e^{x^2} + 72x}{-16x\cos(2x^2) - 32x\cos(2x^2) + 64x^3\sin(2x^2)} =$$

$$= \lim_{x \to 0} \frac{4e^{x^2} + 8e^{x^2} + 8x^2e^{x^2} + 72}{-16\cos(2x^2) - 32\cos(2x^2) + 64x^2\sin(2x^2)} = \frac{12 + 72}{-16 - 32} =$$

$$= \frac{84}{-48} = -\frac{42}{24} = -\frac{21}{12} = -\frac{7}{4}$$

d)

$$\lim_{x\to 0^+} \frac{2\cos(\sqrt{x}) - 2 + x}{(\arctan(3x))^2} \quad \left[\frac{0}{0}\right] \text{ APPLY HOPITAL}$$

$$\lim_{x\to 0^+} \frac{\frac{-2\sin(\sqrt{x})}{2\sqrt{x}} + 1}{\frac{3\arctan(3x)}{1+9x^2} + \frac{3\arctan(3x)}{1+9x^2}} =$$

$$= \lim_{x\to 0^+} \frac{\frac{\sqrt{x} - \sin(\sqrt{x})}{\sqrt{x}}}{\frac{6\arctan(3x)}{1+9x^2}} = \lim_{x\to 0^+} \frac{\sqrt{x} - \sin(\sqrt{x})}{\sqrt{x}} \cdot \frac{1+9x^2}{\arctan(3x)}$$

$$\lim_{x\to 0^+} \frac{\sqrt{x} - \sin(\sqrt{x}) + 9x^2\sqrt{x} - 9x^2\sin(\sqrt{x})}{\sqrt{x}\arctan(3x)} \quad \left[\frac{0}{0}\right] \text{ APPLY HOPITAL}$$

$$\lim_{x\to 0^+} \frac{\frac{1}{2\sqrt{x}} - \frac{1}{2\sqrt{x}}\cos(\sqrt{x}) + 18x\sqrt{x} + \frac{9x^2}{2\sqrt{x}} - 18x\sin(\sqrt{x}) - \frac{9x^2}{2\sqrt{x}}\cos(\sqrt{x})}{\frac{1}{2\sqrt{x}}\arctan(3x) + \frac{3\sqrt{x}}{1+9x^2}}$$

$$\lim_{x\to 0} \frac{\frac{1 - \cos(\sqrt{x}) + 32x^2 + 9x^2 - 32x\sqrt{x}\sin(\sqrt{x}) - 9x^2\cos(\sqrt{x})}{2\sqrt{x}}}{}$$

$$\lim_{n\to+\infty} \left(n^3+1\right)^{1/(2n)}$$

$$\lim_{x\to+\infty} \left(n^3+1\right)^{1/2n} =$$

$$= \lim_{x\to+\infty} e^{\ln\left(n^3+1\right)^{1/2n}} =$$

$$= \lim_{x\to+\infty} e^{\frac{\ln(n^3+1)}{2n}} = \lim_{x\to+\infty} e^{1/2n} = e^0 = 1$$