THE SUM OF DERIVATES IS THE DERIVATES OF A SUM

IF $f(x) = g(x) + h(x)$ PROVE THAT $f'(x) = g'(x) + h'(x)$

SINCE $f'(x_0) = \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0}$, THEN

$$f'(x_0) = \lim_{x \to x_0} \frac{g(x) + h(x) - g(x_0) - h(x_0)}{x - x_0}$$ THAT IS

$$f'(x_0) = \lim_{x \to 0} \frac{g(x) - g(x_0)}{x - x_0} + \frac{h(x) - h(x_0)}{x - x_0}$$ BUT SINCE

$g'(x_0) = \lim_{x \to 0} \frac{g(x) - g(x_0)}{x - x_0}$ AND $h'(x_0) = \lim_{x \to x_0} \frac{h(x) - h(x_0)}{x - x_0}$

SO WE PROVED $f'(x_0) = g'(x_0) + h'(x_0)$