Nome: .... Cognome: .... Matricola: ....

ESERCIZIO A2.4 (punti 8)
Enunciare e dimostrare il Teorema di Lagrange.

ENUNCIATO: LET $f(x)$ BE A FUNCTION CONTINUE IN $[a,b]$ AND DERIVABLE IN (a,b)

SO IT MUST EXIST AT LEAST 1 POINT $c \in (a,b)$ SUCH THAT

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

DIMOSTRAZIONE:

TO DO SO USE ROLLE THEOREM, WE NEED TO HAVE A EQUALITY OF THE FUNCTION AT THE EXTREMIS OF $[a,b]$

CREATE A FUNCTION $\varphi(x)$ THAT DEPENDS FROM $f(x)$ AND WHERE THE ROLLE THEOREM IS VALID

$$\varphi(x) = f(x) - K \cdot x$$

$\varphi(x)$ IS CONTINUE AND IS DERIVABLE THE SAME WAY AS THE FUNCTION $f(x)$

TO HAVE THE LAST THESIS OF THE ROLLE THEOREM WE NEED TO HAVE $\varphi(a) = \varphi(b)$

$$\left.\begin{array}{l}\varphi(a) = f(a) - Ka \\ \varphi(b) = f(b) - Kb\end{array}\right\} \Rightarrow f(a) - Ka = f(b) - Kb$$

WE NEED TO FIND K, ISOLATE K

$$Kb - Ka = f(b) - f(a);$$

$$K = \frac{f(b)-f(a)}{b-a}$$

$$\varphi(x) = f(x) - \frac{f(b)-f(a)}{b-a} x$$

$\varphi(x)$ NOW SATISFY THE THREE ROLLE THESIS

- CONTINUITY
- DERIVABILITY

$\}$ SAME AS LAGRANGE

- $\varphi(a) = \varphi(b)$

APPLY AT THE FUNCTION $\varphi(x)$ THE ROLLE THEOREM WHICH TELLS THAT EXIST A POINT c IN (a,b) THAT IS $\varphi'(c) = 0$

$$\exists c \in (a,b) \mid \varphi'(c) = 0$$

$$\varphi'(x) = f'(x) - \frac{d}{dx}[K \cdot x]$$

$$\varphi(x) = f'(x) - K = f'(x) - \frac{f(b)-f(a)}{b-a}$$

SUBSITUTING x WITH THE VALUE c WE HAVE: THANKS TO ROLLE $\varphi'(c) = 0$

$$0 = f'(c) - \frac{f(b)-f(a)}{b-a}$$

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$