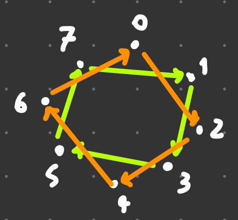
$+$ $\times$

$-$ $\div$

$x_{N+1} = (x_N + 1) \bmod N$

$x_{N+1} = (x_N + 2) \bmod N$



$Q(x) =$ CONST TRIVIAL CONSERVED QUANTITY

$q(x) = x \bmod 2$

$x_{N+1} = x_N^2 \bmod 7 \qquad x \in \{0, 1, 2, \dots 6\}$

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1 DIMENSION

$x \in \mathbb{R}^1 \qquad x_{N+1} = f(x_N)$

STABILITY OF A FIXED POINT

- IMPORTANT CONCEPT IN PHYSIC

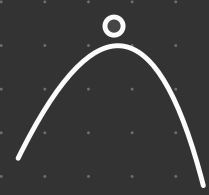
FIXED POINT: $\bar{x}$ S.T. $\bar{x} = f(\bar{x})$

POINT THAT IS SENT TO ITSELF EVERY TIME

DON'T CARE HOW IS MADE

$\downarrow g$

STABLE POINT



UNSTABLE POINT
(BUT STILL A FIXED POINT)

TAYLOR EXPAND f

$f(\bar{x} + dx) = f(\bar{x}) + dx\, f'(\bar{x})$

VERY SMALL $(dx)^2 = 0$

HOW FAST $f(\bar{x} + dx)$ CHANGES?

WE LINEARIZED THE SYSTEM AROUND THE FIXED POINT

LOT OF PHYSIC NOT LINEAR BUT WE PRETEND IT TO BE LINEAR

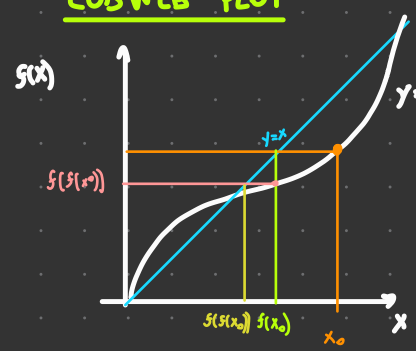
$x_{N+1} = \bar{x} + (x_N - \bar{x})\, f'(\bar{x})$

$\underbrace{x_{N+1} - \bar{x}}_{dx_{N+1}} = \underbrace{(x_N - \bar{x})}_{dx_N}\, f'(\bar{x})$

$dx_{N+1} = dx_N\, f'(\bar{x})$

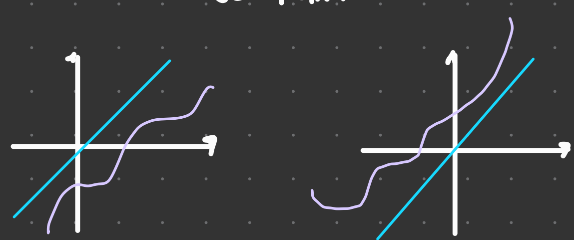
$f'(\bar{x})$ FINITE

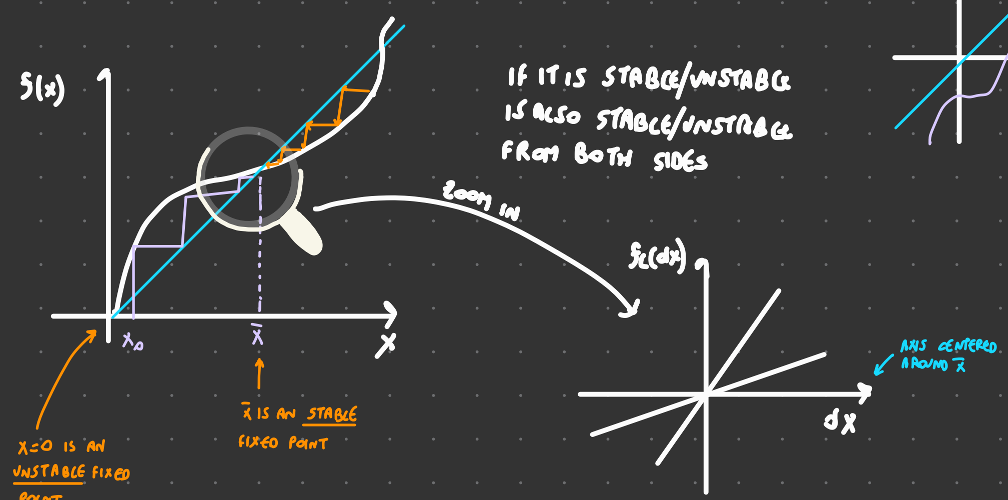
COBWEB PLOT



2 AXIS BUT I AM IN 1D $x \in \mathbb{R}^1$ - 1 REAL DEGREE OF FREEDOM

$\bar{x} = f(\bar{x})$ EVERY POINT ABOVE/BELOW THE BICEPTRON WILL NOT SATISFY THIS CONDITION. NO FIXED POINT





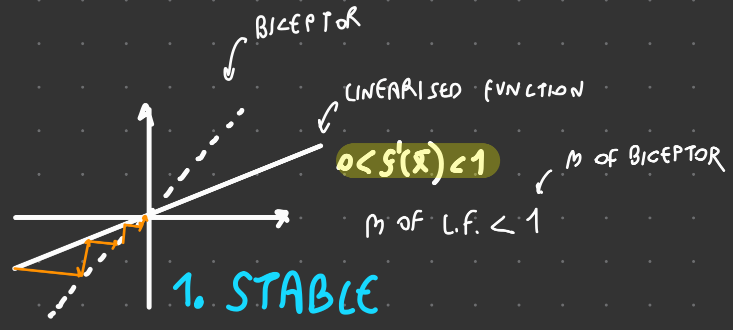
X=0 IS AN UNSTABLE FIXED POINT

$\bar{x}$ IS AN STABLE FIXED POINT

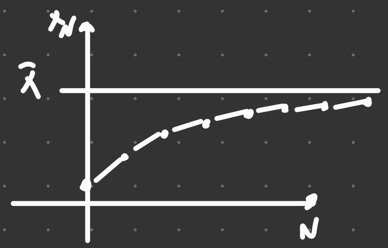
IF IT IS STABLE/UNSTABLE IS ALSO STABLE/UNSTABLE FROM BOTH SIDES

4 CASES

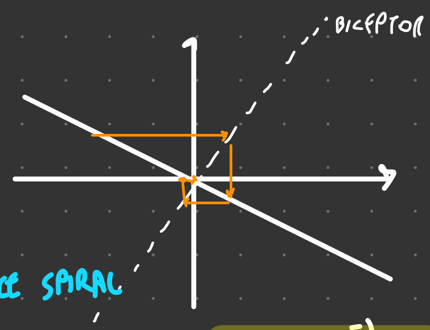
⚠ A STEP IS TWO PIECES. VERTICAL AND THEN HORIZONTAL



1. STABLE

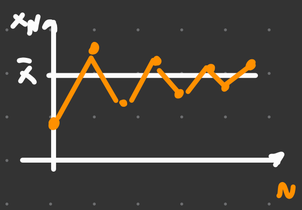


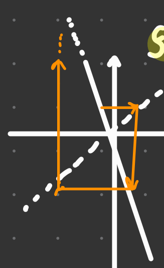
CONVERGE ASYNTOTICALLY TO $\bar{x}$



2. STABLE SPIRAL

$-1 < f'(\bar{x}) < 0$



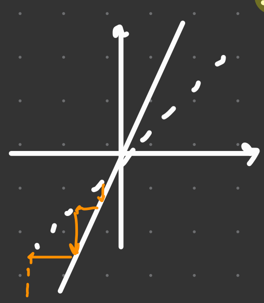


$f'(\bar{x}) < -1$

IT DIVERGES

3. UNSTABLE SPIRAL

$f'(\bar{x}) > 1$



4. UNSTABLE