$+$ $\times$

$-$ $\div$

7) EXERCISE Consider the dynamical system $x_{n+1} = f(x_n)$, with

$$f(x) = \sin(\pi x)$$

The dynamics has 2 fixed points. Linearize around the smaller fixed point $\bar{x}$. Regarding the stability of $\bar{x}$, which one of the following 4 possibilities is realized?

(a) $\bar{x}$ is stable (not a spiral)

(b) $\bar{x}$ is a stable spiral

(c) $\bar{x}$ is unstable (not a spiral)

(d) $\bar{x}$ is an unstable spiral

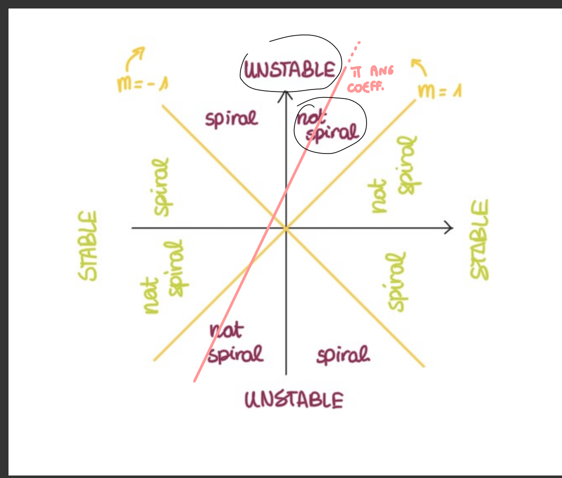
$f(x) = \sin(\pi x)$ $\qquad x_{n+1} = f(x_n)$

$x = f(x)$

$X = 0$ FIXED POINT

IS THE SMALLER FIXED POINT

$X = 0 \Rightarrow \sin(\pi x) = 0$



LINEARIZE AROUND THE SMALLER FIXED POINT

$f(x) = \sin(\pi x)$

$f'(x) = \pi \cos(\pi x)$ SUBSTITUTE x WITH 0

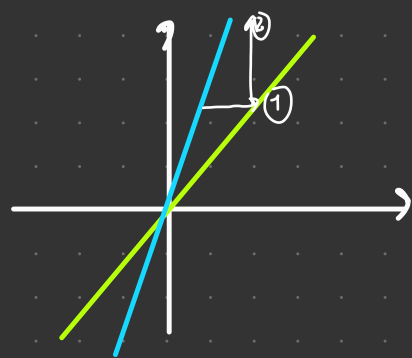
$f'(0) = \pi\cos(0) = \pi$

ANGULAR COEFFICIENT OF THE LINEARIZE FUNCTION

SOLUTION: ↓↓

UNSTABLE NOT SPIRAL

PRIMA TOCCO IN ORIZZONTALE LA BISETTRICE, POI IN VERTICALE TOCCO LA RETTA DNOO



EXERCISE Consider the map

$$f(x) = \frac{1}{\alpha x + 1}$$

with state $x \geq 0$ and parameter $\alpha > 0$. Show that the dynamical system $x_{n+1} = f(x_n)$ has a single fixed point, which is always a stable spiral.

$$f(x) = \frac{1}{\alpha x + 1}$$

$x_{n+1} = f(x_n)$ ONLY 1 SINGLE POINT

$x \geq 0 \qquad \alpha > 0$

TO FIXED POINT

$x = \frac{1}{\alpha x + 1} \Rightarrow \alpha x^2 + x = 1$

$\alpha x^2 + x - 1 = 0$

$\frac{-1 \pm \sqrt{1+4\alpha}}{2\alpha} \Rightarrow \frac{-1 + \sqrt{1+4\alpha}}{2\alpha}$

$f(x) = \frac{1}{\alpha x + 1} = \frac{-\alpha}{(\alpha x+1)^2} = -\alpha(\alpha x + 1)^{-2} = -\alpha\left[\cancel{\alpha}\left(\frac{-1+\sqrt{1+4\alpha}}{2\cancel{\alpha}}\right) + 1\right]^{-2}$

SUBSTITUTE x WITH

$= -\alpha\left(\frac{-1+\sqrt{1+4\alpha}}{2} + \frac{2}{2}\right)^{-2}$

$= -\alpha\left(\frac{\cancel{-1} + \sqrt{1+4\alpha} + \cancel{2}^{\,1}}{2}\right)^{-2}$

$= -\alpha\left(\frac{\sqrt{1+4\alpha}+1}{2}\right)^{-2}$

$= -\alpha \frac{(2)^2}{(\sqrt{1+4\alpha}+1)^2} = \frac{-4\alpha}{(1+\sqrt{1+4\alpha})^2}$