Linear system: a system of linear equations (or a linear system) is a collection of one or more linear equations involving the same variables.

Solution set of a linear system: a solution of the system is a list of numbers that makes each equation a true statement when the values are substituted for any x.
The set of all possible solutions is called the solution set of the linear system.
Two linear systems are called equivalent if they have the same solution set.

Elementary row operations: these are operations that keep the row equivalence and they are 3:

- (Replacement) Subtract one equation multiplied by a real number to any other equation.
- (Interchange) Interchange two rows.
- (Scaling) Multiply all entries in a row by a nonzero constant.

Pivot position: a pivot position in a matrix is the location of a leading entry in the row-echelon form of a matrix.

Reduced echelon form: a matrix is in reduced echelon form if it satisfies these conditions:

- All nonzero rows are above any rows of all zeros.
- Each leading entry of a row is in a column to the right of the leading entry of the row above it.
- All entries in a column below a leading entry are zeros.
- The leading entry in each nonzero row is 1.
- Each leading 1 is the only nonzero entry in its column.

Linear combinations of vectors: a linear combination of vectors {v1, v2, …, vn} each in R^n is any vector that can be written as w = c1*v1 + c2*v2 + … + c3*vn where c1, c2, …, cn are real numbers.

Span of vectors: the span of a set of vectors is the set of all linear combinations of such vectors.

Matrix-vector multiplication: given a matrix R^(n*m) and a vector R^n the multiplication is performed by summing the result of the multiplication of the column n times the nth component of the vector (so first column of the matrix * first element of the vector + the second and so on).

Homogeneous linear system: a system of linear equations is said to be homogeneous if it can be written in the form Ax = 0, where A is a n*m matrix and 0 is the zero vector in R^m. Such a system Ax = 0 always has at least one solution, namely, x = 0 (the zero vector in R^n). This zero solution is usually called the trivial solution. For a given equation Ax = 0 the important question is whether there exists a non trivial solution.

Linear dependence: a set of vectors {v1, v2, …, vn} in R^n is said to be linear dependent iff at least one of the vectors can be written as the linear combination of the others.

Invertible matrixes: an n*n matrix A is said to be invertible if there is an n*n matrix C such that C*A = I and A*C = I where I is the n*n identity matrix.

Elementary matrixes: an elementary matrixis one that is obtained by performing a single elementary row operation on an identity matrix.

Triangular matrixes: a matrix which has only zeros above the dyagonal or under it.

Subspaces of R^n:

Col space of a matrix: is the set of all linear combinations of the columns of a matrix.

Nul space of a matrix: is the set of all solutions of the homogeneous equation A*x = 0.

Basis: a basis for a subspace H is a linerly independent set in H that spans H.

Dimension: the dimension of a nonzero subspace H is the number of vectors in any basis for H.

Rank: the rank of a matrix is the dimension of the column space of it.

Coordinates of a vector with respect to a basis:

Determinant of a square matrix:

Norm of vector or matrix: the norm of a vector is its length and its equal to the square root of the sum of the values to the power of two; for a matrix is the same, but inserting all the values.

Induced norm of a matrix:

Condition number of a square invertible matrix: the larger the condition number, the closer the matrix is to being singular. The condition number of the identity matrix is 1.

Eigenvalues: a scalar value λ is called eigenvalue of A if there is a nontrivial solution of x where A*x = λ*x.

Eigenvectors and eigenspaces of a square matrix: an eigenvector of an n*n matrix A is a nonzero vector x such that A*x = λ*x for some scalar λ. The eigenspace is the null space of the matrix A - λ*I.

Characteristic equation of a square matrix: det(A - λ*I) = 0, used to find the eigenvalues of a matrix.

Similarity of square matrixes: two matrixes A and B are similar if there is an invertible matrix P such that 1/P*A*P = B; if they are similar they have the same characteristic equation so the same eigenvalues.

Diagonal matrixes: a matrix such that only the diagonal has nonzero elements.

Transpose of a matrix: the transpose of a matrix is just a matrix that has as rows the columns of the original one.

Identity matrix: is the square matrix with ones on the diagonal and all the other values are zeros.

Scalar product of vectors: having 2 vectors of the same dimension, the scalar product is the sum of the multiplication of the elements at the same position $(v, w) = \Sigma\, v[i] * w[i]$

Orthogonal and orthonormal vectors: 2 vectors are said to be orthogonal if their scalar product is 0 so they form a 90° angle; if in addition their norm is equal to 1 then they are also an orthonormal set.

Orthogonal and orthonormal basis: an orthogonal basis for a subspace W, is a basis for W that is also an orthogonal set

Orthogonal projection: is the projection of a vector on the other, with the 2 vectors orthogonal to each other.

Least square problem: taken A an $n*m$ ($m >> n$) matrix and b an n vector, and supposing there are no solutions for $A*x = b$, the least sqaure problem is the finding of the nearest value to the solution, so the norm of (A*solution - b) should be the lowest as possible.