PROOF OF QUOTIENT RULE

PROVE $$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$$

USE THE DEFINITION OF DERIVATES

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \lim_{h\to 0} \frac{\frac{f(x+h)}{g(x+h)} - \frac{f(x)}{g(x)}}{h}$$ THAT IS

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \lim_{h\to 0} \frac{f(x+h)g(x) - f(x)g(x+h)}{g(x+h)\,g(x)\cdot h}$$ THAN CAN BE WRITE AS

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \lim_{h\to 0} \frac{1}{\underbrace{g(x+h)\cdot g(x)}_{\text{IF X APPROACH 0 THIS IS } g(x)^2}} \cdot \lim_{h\to 0} \frac{f(x+h)g(x) - f(x)g(x+h)}{h}$$

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{1}{g(x)^2} \cdot \lim_{h\to 0} \frac{f(x+h)g(x) - f(x)g(x+h)}{h}$$ HERE IS JUST THE PRODUCT RULE!

ADD AND SUBTRACT $f(x)g(x)$

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{1}{g(x)^2} \cdot \lim_{h\to 0} \frac{f(x+h)g(x) - f(x)g(x) - f(x)g(x+h) + f(x)g(x)}{h}$$

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{1}{g(x)^2} \cdot \lim_{h\to 0} g(x)\frac{f(x+h) - f(x)}{h} - f(x)\frac{g(x+h) - g(x)}{h}$$

IS A CONSTANT, CAN BE FACTOR OUT

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$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{1}{g(x)^2} \cdot g(x) \cdot \underbrace{\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}}_{f'(x)} - f(x) \underbrace{\lim_{h \to 0} \frac{g(x+h)-g(x)}{h}}_{g'(x)}$$

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{1}{g(x)^2} \cdot g(x) f'(x) - f(x) g'(x)$$

THAT IS

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x) f'(x) - f(x) g'(x)}{g(x)^2}$$