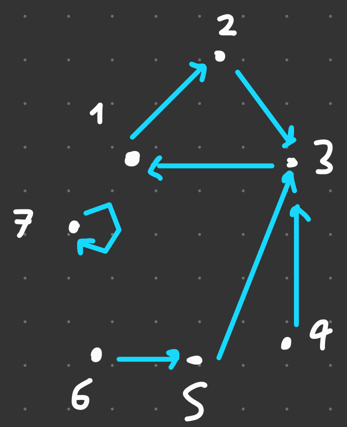
$+ \quad \times$

$- \quad \div$



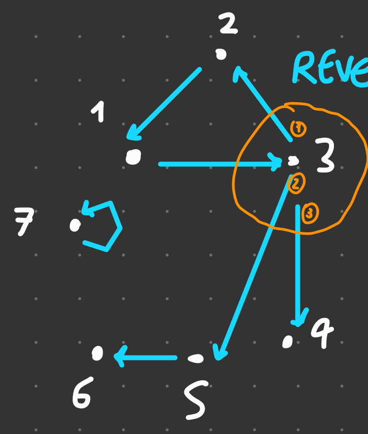
$T = (x_1, x_2, \dots, x_m)$ ← USE ( ) WHEN ORDER IS IMPORTANT — ORDERED SET

IF THIS IS ALLOWED IS THE OPPOSITE ALLOWED?

$\bar{T} = (x_m, x_{m-1}, \dots x_1)$

YES BY THE REVERSE GRAPH → REVERSE EVERY SINGLE ARROW

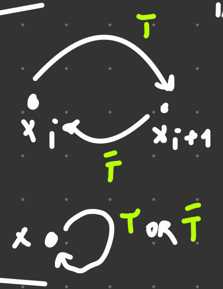
REVERSE GRAPH



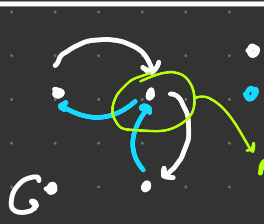
⚠ VIOLATE RULE

GRAPH WITH MORE THAN 1 ARROW GOING TO A POINT THAN IT CAN NOT BE REVERSED

WHEN I HAVE A TRANSITION (TICK OF TIME FROM i TO $i+1$) IN T I HAVE A TRANSITION IN $\bar{T}$



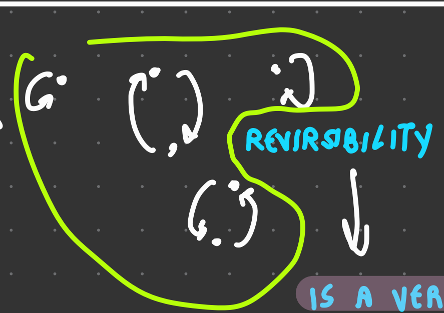
BUILDING BLOCK
ONLY POSSIBLE TO BUILD A GRAPH WHERE EVERY POSSIBLE TRAJECTORY IS REVERSEBLE



- WHAT I HAVE
- WHAT I NEED TO HAVE

AGAINST THE RULE

⇒



REVIRSIBILITY ↓ IS A VERY STRONG CONSTRAINT

ONLY THOSE KIND OF GRAPH ARE ALLOWED, EVERYTHING ELSE $\boxed{\text{NO}}$

⚠ EACH NODE HAVE EXACTLY $\boxed{1}$ OUTGOING ARROW

MAIN PRINCIPLE

DIFFERENT CONNECTED COMPONENTS IN A GRAPH

→ CONSERVED QUANTITIES $Q(x) \in \mathbb{R}/\in\mathbb{N}$

FUNCTION OF STATES

$T = (x_0, f(x_0), f(f(x_0)), \dots)$

$q(\underset{6}{x_0}) = q(\underset{5}{f(x_0)}) = q(\underset{3}{f(f(x_0))}) = \overset{2}{\dots}$

STARTING FROM 6 $\quad q(6) = q(5) = q(3) = q(2) = q(1)$

STARTING FROM 4 $\quad q(4) = q(3)$

GIVEN A CONNECTED COMPONENT $\mathcal{C}$

$\forall x \in \mathcal{C},\ q(x) = Q_{\mathcal{C}}$

DEPENDENT ON THE CONNECTED COMPONENT, NOT FROM THE STATE

RECAP
- FIXED POINT ("ALONE", GO THROUGH ITSELF)
- CONSERVED QUANTITIES
- CYCLE (TRAJECTORY THAT REPEATS ITSELF PERIODICALLY)
- TRANSIENT/RECURRENT STATE

⓪ $x_{N+1} = x_N^2 \bmod 4$ $\quad x_N \in \{0,1,2,3\}$

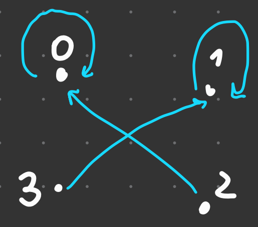
$N=0 \Rightarrow 1 = 0^2 \bmod 4$

$0 \bmod 4 = 0$

$N=1 \Rightarrow 2 = \underbrace{1^2 \bmod 4}_{1}$

$N=2 \Rightarrow 3 = \underbrace{2^2 \bmod 4}_{}= 0$

$N=3 \Rightarrow 4 = 3^2 \bmod 4 = 1$



- CYCLES P=1 (0) (1)
- RECURRENT STATES {0,1}
- TRANSIENT STATES {2,3} → ALL THE OTHERS

$q(0) = q(2)$
$q(1) = q(3)$

① $x_{N+1} = (x_N + 1) \bmod N$, $\quad x_N \in \{0,1,\dots,N-1\}$

$\frac{(x_N+1)}{N}$ → AND TAKE THE REMINDER

$9 \bmod 2 = 1$
$9 \bmod 3 = 0$

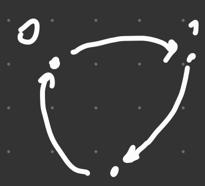
$N=0 \Rightarrow 1 = 1 \bmod 1 \Rightarrow x_1 = 0$ $\qquad x_{N+1} = x_N + 1$

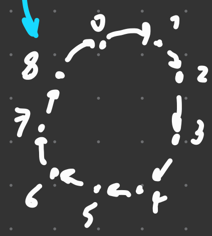
$N=1 \Rightarrow x_2 = (x_1+1) \bmod 1 \Rightarrow x_2 = 1 \bmod 1 \Rightarrow x_2 = 0$

$N=2 \Rightarrow x_3 = (x_2+1) \bmod 2 \Rightarrow x_3 = 1 \bmod 2 \Rightarrow x_3 = 1$

LAST POSSIBLE STATE GET BACK TO 0

N=3

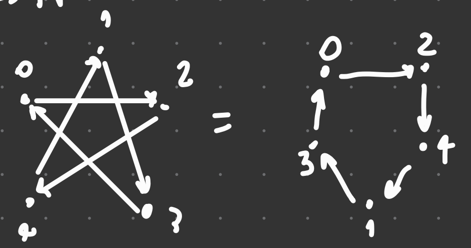




1 CYCLE OF PERIOD P=N
- IF N=1 FIXED POINT
- IF N>0 NO FIXED POINTS
- NO TRANSIENT STATE
- CONNECTED COMPONENT: 1 SINGLE CONNECTED

② $x_{N+2} = (x_N + 2) \bmod N$,

N=5



$x_{N+1} = x_N^2 \bmod 4$ $\quad x_N \in \{0,1,2,3\}$

N=0: $\quad 1 = 0^2 \bmod 4$; $\quad 1 = 0 \bmod 4$; $\quad 1 = 0$

N=1: $\quad 2 = 1^2 \bmod 4$; $\quad 2 = 1 \bmod 4$; $\quad 2 = 1$

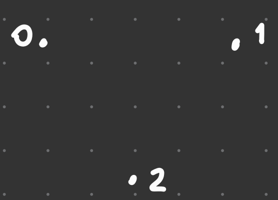
N=2: $\quad 3 = 2^2 \bmod 4$; $\quad 3 = 4 \bmod 4$; $\quad 3 = 0$

N=3: $\quad 4 = 3^2 \bmod 4$; $\quad 4 = 9 \bmod 4$; $\quad 4 = 1$

$x_{N+1} = (x_N + 1) \bmod N$, $\quad x_N \in \{0,1,\dots,N-1\}$

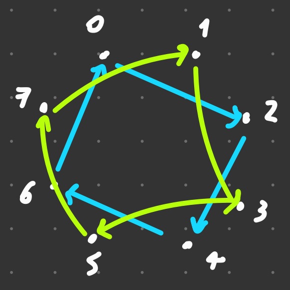
TEST WITH N=3

N=0 $\quad x_1 = (x_0 + 1) \bmod 3$



$X_{N+2} = (X_N + 2) \bmod N$

$N = 8$



$N=0$: $X_{N+2} = (0+2) \bmod 8 = 2 \bmod 8 = 2$

$N=2$ $X_{N+2} = (2+2) \bmod 8 = 4 \bmod 8 = 4$

$N=4$ $X_{N+2} = (4+2) \bmod 8 = 6 \bmod 8 = 6$

$N=6$ $X_{N+2} = (6+2) \bmod 8 = 8 \bmod 8 = 0$

$N=1$ $X_{N+2} = (1+2) \bmod 8 = 3$

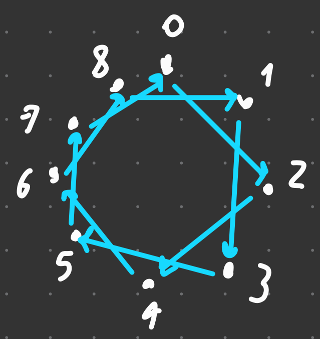
$N=3$ $X_{N+2} = (3+2) \bmod 8 = 5$

$N=5$ $X_{N+2} = (5+2) \bmod 8 = 7$

$N=7$ $X_{N+2} = (7+2) \bmod 8 = 1$

$X_{N+2} = (X_N + 2) \bmod N$

$N = 9$



$N=0$ $X_{N+2} = 2 \bmod 9 = 2$

$N=2$ $X_{N+2} = (2+2) \bmod 9 = 4$

$N=4$ $X_{N+2} = (4+2) \bmod 9 = 6$

$N=6$ $X_{N+2} = (6+2) \bmod 9 = 8$

$N=8$ $X_{N+2} = (8+2) \bmod 9 = 1$

$N=1$ $X_{N+2} = (1+2) \bmod 9 = 3$

$N=3$ $X_{N+2} = (3+2) \bmod 9 = 5$

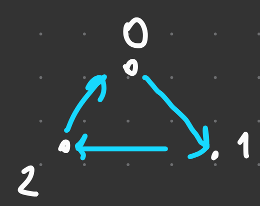
$N=5$ $X_{N+2} = (5+2) \bmod 9 = 7$

$N=7$ $X_{N+2} = (7+2) \bmod 9 = 0$

$x_{N+1} = (x_N + 1) \bmod N, \quad x_N \in \{0, 1, \dots, N-1\}$

$N = 3$

$N = 0; \quad (x_0 + 1) \bmod 3 = 1 \bmod 3 = 1$



$N = 1 \quad (x_1 + 1) \bmod 3 = 2 \bmod 3 = 2$

$N = 2 \quad (x_2 + 1) \bmod 3 = 3 \bmod 3 = 0$