Knowledge Representation and Reasoning

Exercise Session 4

Exercise 1. NNF (*)

Transform the following \mathcal{ALC} concepts to negation normal form.

- 1. $\neg (A \sqcup \exists r.A) \sqcup \forall r. \neg B$
- 2. $\exists r. \neg (\forall s. B \sqcap B) \sqcap (\neg B \sqcup A)$
- 3. $\neg(\exists r. \neg A \sqcup \forall s. \neg(\neg A \sqcup B) \sqcup \neg A)$

Exercise 2. Satisfiability

(*)

Decide whether the following concepts are satisfiable or not. If they are, provide a satisfying interpretation.

- 1. $A \sqcap \neg \forall r. (A \sqcup B) \sqcap \neg \exists r. \neg B$
- 2. $B \sqcap (\neg B \sqcup \exists r.\bot) \sqcup \forall r.\bot$

Exercise 3. Disjunctions

(**)

Let \mathcal{ELU}_{\perp} be the extension of \mathcal{EL}_{\perp} which allows also for the disjunction constructor (\sqcup). Show that \mathcal{ELU}_{\perp} and \mathcal{ALC} are equivalent.

Exercise 4. Domain size

(**)

Construct concepts C and D such that for any interpretation \mathcal{I} it holds that:

- 1. if $C^{\mathcal{I}} \neq \emptyset$, then $\Delta^{\mathcal{I}}$ must have at least two elements
- 2. if $D^{\mathcal{I}} \neq \emptyset$, then $\Delta^{\mathcal{I}}$ must have at least 7 elements

Exercise 5. Disjoint Unions

(**)

Let \mathcal{T} be a consistent \mathcal{ALC} TBox. Show that the disjoint union of two models of \mathcal{T} is also a model of \mathcal{T} .

Exercise 6. TBox Consistency

Check whether the following TBoxes are consistent. If they are, provide a model.

- 1. $\mathcal{T}_1 = \{ A \sqsubseteq \exists r.A, A \sqsubseteq \forall r. \neg A \}$
- 2. $\mathcal{T}_2 = \{ A \sqsubseteq \exists r.A, \ \forall r. \neg A \sqsubseteq A \}$
- 3. $\mathcal{T}_3 = \{A \sqsubseteq \exists r. \neg A, \ \forall s. \neg A \sqsubseteq A, \top \sqsubseteq \forall r. \forall s. A\}$

Exercise 7. Satisfiability

(*)

(*)

Decide whether the following concepts are satisfiable w.r.t. the TBox \mathcal{T}_1 from Exercise 6.

- 1. $A \sqcup \forall r.A$
- $2. \neg A \sqcup \exists r. \bot$
- 3. $\forall r. \exists r. A$

Decide whether $\forall r.\bot$ is satisfiable w.r.t. the TBox \mathcal{T}_2 from Exercise 6.

Exercise 8. Subsumption

(*)

Check whether the following subsumption relations hold w.r.t. the TBox \mathcal{T}_1 from Exercise 6.

- 1. $\exists r.(A \sqcap B) \sqsubseteq \forall r.B$
- 2. $B \sqcup C \sqsubseteq \forall r.A$
- 3. $\exists s. \neg A \sqsubseteq \exists r. \neg A$

Exercise 9. Knowledge Base Consistency

(*)

Check whether the following ABoxes are consistent w.r.t. the TBox \mathcal{T}_1 from Exercise 6.

- 1. $\{r(a,b), \forall r.A(a)\}$
- 2. $\{\exists r.(B \sqcup A)(a), s(b, a), \forall s. \forall r. \neg B(b)\}$
- 3. $\{r(a,b), r(b,c), r(c,a)\}$

Exercise 10. Number Restrictions

(***)

Let \mathcal{ALCQ} be the logic that extends \mathcal{ALC} with qualified number restrictions $\geq n$ r.C expressing the class of objects that have at least n r-successors belonging to the class C. For example,

$$Person \square \geq 2hasChild.Female$$

is the class of people having at least two daughters.

Device adequate tableau rules to handle number restrictions.

Exercise 1.

Transform the following \mathcal{ALC} concepts to negation normal form.

1. $\neg (A \sqcup \exists r.A) \sqcup \forall r. \neg B$

2. $\exists r. \neg (\forall s. B \sqcap B) \sqcap (\neg B \sqcup A)$

3. $\neg(\exists r. \neg A \sqcup \forall s. \neg(\neg A \sqcup B) \sqcup \neg A)$

1. (1ATT V+ 7A) U V+.7B

2. 3 r. (3s.18 u 18) n (16 u

Exercise 2. Satisfiability (*) Decide whether the following concepts are satisfiable or not. If they are, provide a satisfying interpretation. 1. $A \sqcap \neg \forall r. (A \sqcup B) \sqcap \neg \exists r. \neg B$ 2. $B \sqcap (\neg B \sqcup \exists r.\bot) \sqcup \forall r.\bot$ CHECK SATISFIABILITY Anty. (aub) n 131.18 (a) FIRST IT NEED TO BE W NNF THAT IS An At. (TANTB) n Yt. B (a) · ∃ t.(1A∏1B)(w) · لالا ، ١٩٠ (ع) 31(7A113)(a) CLASH B(v) 17 (1843 F.L) (w) 4 7 F (w) . Y r. L (a) WE CAN NOT APPLY THE Y RULE REZVIRES AN Y. SUGGESSOR THAT QUIDES CLASH A CLASH .5A7

Exercise 3. Disjunctions

(**)

Let \mathcal{ELU}_{\perp} be the extension of \mathcal{EL}_{\perp} which allows also for the disjunction constructor (\sqcup). Show that \mathcal{ELU}_{\perp} and \mathcal{ALC} are equivalent.

ALC: C:= Alone | 3 % el 10

ELUL: C: = ALTT CHC 3 .C CUC

THAT EVERYTHING THAT WE CAN EXPRESS IN ECU, CAN BE ALSO

L= ATTA

T= AUTA

SO EVERYTHING THAT IS EXPRESSED IN GULL CAN BE EXPRESSED IN

ALC. NOW CHECK THE OPPOSITE

16: ATT X & C. | A CONCEPT NAME AND ITS NEGATION SHOULD BE DISTOINT (NO DETECT BELONG

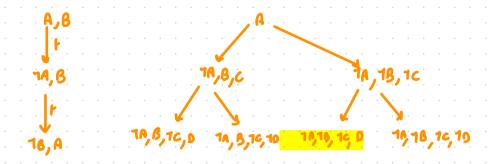
TO BO TH)

ALSO THERE SHOULD NOT BE A CONCEPT THAT BECOMES TO MONTH

So 24 = (14)

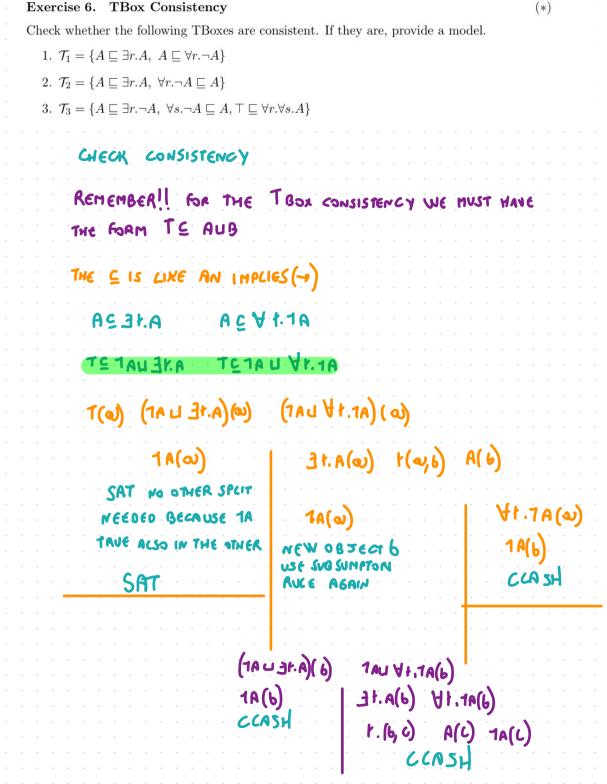
Construct concepts C and D such that for any interpretation $\mathcal I$ it holds that:

- 1. if $C^{\mathcal{I}} \neq \emptyset$, then $\Delta^{\mathcal{I}}$ must have at least two elements
- 2. if $D^{\mathcal{I}} \neq \emptyset$, then $\Delta^{\mathcal{I}}$ must have at least 7 elements



Exercise 5. Disjoint Unions (**)

Let \mathcal{T} be a consistent \mathcal{ALC} TBox. Show that the disjoint union of two models of \mathcal{T} is also a model of \mathcal{T} .



Exercise 7. Satisfiability Decide whether the following concepts are satisfiable w.r.t. the TBox \mathcal{T}_1 from Exercise 6. 1. $A \sqcup \forall r.A$ $2. \neg A \sqcup \exists r. \bot$ $3. \ \forall r. \exists r. A$ Decide whether $\forall r.\bot$ is satisfiable w.r.t. the TBox \mathcal{T}_2 from Exercise 6. (Au V r. A)(w) (1 AU \(1.1A) (2) (W) (A. 1 ELIAT) ٧٢.A(ع) OPEN SET OF ASSERTION

Exercise 8. Subsumption

Check whether the following subsumption relations hold w.r.t. the TBox \mathcal{T}_1 from Exer-

1. $\exists r.(A \sqcap B) \sqsubseteq \forall r.B$

cise 6.

2. $B \sqcup C \sqsubseteq \forall r.A$

3. $\exists s. \neg A \sqsubseteq \exists r. \neg A$

REMEMBER! TO CHECK SUBSUMPTION WE HAVE TO CHECK

THAT BELONGS TO THE LEFT

WE HAVE TO CHECK THAT THE NEGATION IS UNSATISFIABLE

DEMONS ON BIT WEAR) TEL SONGSON BITY 2 (BUR) TE

1 3t. (ANB) N 3t. 7B

r(a,6) A(6) B(6) r(a,6) 1B(6)

CLASH - THE CLASH WAS FINDED BEFORE MY CHOI

SO THERE IS NO OTHER POSSIBILITY

IT IS UNSAT THEN THE SUBSUMPTION HOLOS

Exercise 9. Knowledge Base Consistency

(*)

Check whether the following ABoxes are consistent w.r.t. the TBox \mathcal{T}_1 from Exercise 6.

- Check whether the following Abox
- 2. $\{\exists r.(B \sqcup A)(a), s(b, a), \forall s. \forall r. \neg B(b)\}$

1. $\{r(a,b), \forall r.A(a)\}$

- 3. $\{r(a,b), r(b,c), r(c,a)\}$

WE ACREADY KNOW THAT A(b) WILL CEAD TO A CRASH IT

SO THE ABOX IS NON CONSISTEN WITH THE TOOK

Exercise 10. Number Restrictions

(***)

Let \mathcal{ALCQ} be the logic that extends \mathcal{ALC} with qualified number restrictions $\geq n$ r.C expressing the class of objects that have at least n r-successors belonging to the class C. For example,

 $Person \square \ge 2hasChild.Female$

is the class of people having at least two daughters.

Device adequate tableau rules to handle number restrictions.