$+ \quad \times$

$- \quad \div$

REDUCTION THEOREM (FUBINI THEOREM)

LET $Q = [a,b] \times [c,d]$. LET $f \in R(Q)$

a) IF $\forall y [c,d]$ THERE EXISTS THE INTEGRAL $G(y) = \int_a^b f(x,y)\,dx$, THEN THE FUNCTION $y \longrightarrow G(y)$ IS (RIEMANN) INTEGRABLE IN $[c,d]$ AND WE HAVE

(A) $$\iint_Q f(x,y)\,dx\,dy = \int_c^d G(y)\,dy = \int_c^d \left(\int_a^b f(x,y)\,dx \right) dy$$

b) IF $\forall x \in [a,b]$ THERE EXISTS THE INTEGRAL $H(x) = \int_c^d f(x,y)\,dy$ THEN THE FUNCTION $x \longrightarrow H(x)$ IS RIEMANN INTEGRABLE IN $[a,b]$ AND WE HAVE

(B) $$\iint_Q f(x,y)\,dx\,dy = \int_a^b H(x)\,dx = \int_a^b \left(\int_c^d f(x,y)\,dy \right) dx$$