T_{0100}	T_{0001}
T_{0110}	T_{1001}
T_{1100}	T_{0011}
T_{1110}	T_{1111}

FINAL

WHEN THE INPUT FORMULA IS 1

Knowledge Representation and Reasoning

Exercise Session 5

$\neg(x\,\mathcal{U}\,\neg y)$	$x\,\mathcal{U}\,\neg y$	x	y
0/1	1	0/1	0
0/1	0	0	1
0	0	1	1
1	1	1	1

Exercise 1. Type Graph (∗)

Let $\varphi = x\,\mathcal{U}\neg y$

1. Find the types of φ
2. Construct the type graph
3. Identify initial and final types

Exercise 2. Model Counting (∗)

How many temporal models **of length 2** satisfy the formula φ from Exercise 1?

Exercise 3. KR 1 (∗)

1. Construct an LTL$_f$ formula describing the following specification of a (simplified) traffic light; you can use the abbreviations seen during the lecture.
   - the light is either green or red, but never both
   - whenever the light is red, it will eventually turn green

   Hint: use the propositional variables green and red.

2. What characterises the **last** timepoint of all models satisfying this specification?

Exercise 4. KR 2 (∗∗)

1. Extend the specification from Exercise 3 to include two traffic lights (with variables green$_i$ and red$_i$ ($i = 1, 2$) such that the two green lights are never simultaneously on.
2. Is this specification satisfiable? If yes, give a temporal model satisfying it; if not, envision a way to fix it

Exercise 5. Model Size 1 (∗∗)

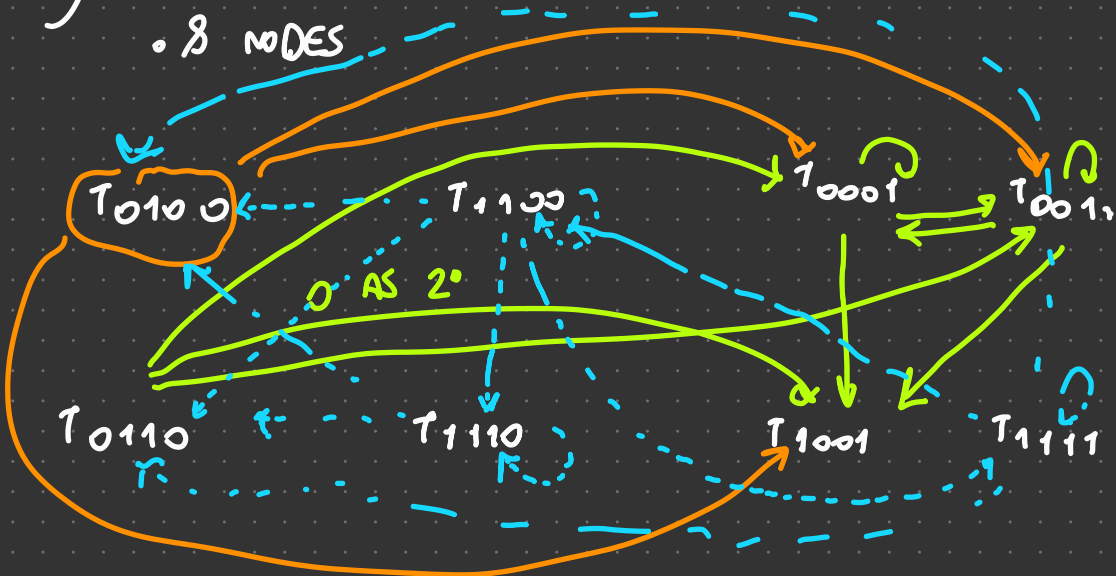
Build a formula that is satisfied by models of **even** length only, or argue why it cannot exist.

Exercise 6. Model Size 2 (∗∗∗)

Build a formula that is satisfied by models of **prime** length only, or argue why it cannot exist.

1) BUILD THE GRAPH

- 8 NODES



0 IN (1°) ARE FINAL STATE — PUT THE ◎ AROUND

1 IN (2°) ARE INITIAL STATE — ADD ARROW ↓ TO INITIAL

Exercise 2. Model Counting (*)

How many temporal models **of length 2** satisfy the formula φ from Exercise 1?

$$x \,\mathcal{U}\, \neg y$$

LENGTH OF A TEMP. MODEL: HAS N + 1 VALUATION

THE LENGTH IS N

SEQUENCE OF PROPOSITIONAL VALUATION

0 OR 1

$$v_0, v_1, v_2 \dots v_N$$

TOT. VALUATION IS $v_N + 1$ (BECAUSE OF v_0)

$$x \,\mathcal{U}\, \neg y$$

$\text{LENGTH}(v_0, v_1, v_2 \dots v_N) = N$

$v\{x, y\} \to \{0, 1\}$

2 CASES	v_0	v_1	v_2
x, $\neg x$	$\neg y$	$x, \neg x$ $y, \neg y$ 4 CASES	x $\neg x$ $y, \neg y$ 4 CASES
	x, y	$\neg y$ $x, \neg y$ 2 CASES	4 CASES
	$x\ y$	x, y	$\neg y$ $x, \neg x$ 2 CASES

DON'T CARE WHAT HAPPEN NEXT

SAT REGARDLESS OF X TRUE OR NOT

TOT 32 CASES WHERE $v_0 \models \neg y$

8 CASES OF TEMPORAL MODEL

$32 + 8 + 2 = \boxed{42}$ TEMP. MODEL

OF LENGTH 3 $= \boxed{170}$

$42 \cdot 4 + 2$

LENGTH 2

4 CHOICES IN THE END

IN THE LAST X OR Y ~~X~~

Exercise 3. KR 1 (*)

1. Construct an LTL_f formula describing the following specification of a (simplified) traffic light; you can use the abbreviations seen during the lecture.
   1) • the light is either green or red, but never both
   2) • whenever the light is red, it will eventually turn green

   Hint: use the propositional variables green and red.

2. What characterises the **last** timepoint of all models satisfying this specification?

GREEN XOR RED

$\mathrm{GREEN} \leftrightarrow \neg \mathrm{RED}$

$\square$ = ALWAYS

$\diamond$ = EVENTUALLY

$(\mathrm{GREEN} \vee \mathrm{RED}) \wedge \neg(\mathrm{GREEN} \wedge \mathrm{RED})$

1) $\square(\mathrm{GREEN} \text{ XOR } \mathrm{RED})$

2) $\square(\mathrm{RED} \rightarrow (\mathrm{RED}\ \mathcal{W}\ \mathrm{GREEN}))$ $\square(\mathrm{RED} \rightarrow \diamond \mathrm{GREEN})$

3) $\square(\mathrm{GREEN}_1 \text{ XOR } \mathrm{GREEN}_2)$ $\cancel{\diamond(\neg(\mathrm{GREEN}_1 \wedge \mathrm{GREEN}_2))}$

Exercise 4. KR 2 (**)

1. Extend the specification from Exercise 3 to include two traffic lights (with variables green_i and red_i ($i = 1, 2$) such that the two green lights are never simultaneously on.

2. Is this specification satisfiable? If yes, give a temporal model satisfying it; if not, envision a way to fix it

UNSAT

WHEN 1 IS GREEN
THE 2 IS RED
↓
NEED TO HAVE
ANOTHER POINT
IN TIME WHEN
IS GREEN
↓
AND SO ON
W/ 1

IT HAS NO FINITE MODEL

HOW TO FIX IT

- NONE IS ON
- BOTH YELLOW??
- BOUNDED TIME
- $\Box(\text{RED} \to \Diamond(\text{GREEN} \lor \underbrace{\bigcirc\bigcirc\bigcirc\ldots\bigcirc}_{k\text{-TIMES}}\bot))$

↓

NO NEXT POINT IN TIME

Exercise 5. Model Size 1 (**)

Build a formula that is satisfied by models of **even** length only, or argue why it cannot exist.

$\bigcirc\bot$

$\circ$

0 LENGTH

$x \quad \neg x \quad x \quad \neg x \quad x$

CAN'T ENDS WITH ¬X

$$x \wedge \Box(\neg x \to \bigcirc x) \wedge \Box(x \to (\bigcirc \neg x \vee \bigcirc \bot))$$

FIRST ELEMENT IS X

FORALL

$$= \Box((x \wedge \bigcirc\top) \to \bigcirc \neg x)$$

FOR ODD PUT

¬X INSTEAD OF X

Exercise 6. Model Size 2 (***)

Build a formula that is satisfied by φ models of **prime** length only, or argue why it cannot exist.

1, 2, 3, 5, 7, 11, 13

NO EXIST

NOT UNIFORMALLY DISTRIBUTED

ALSO THERE ARE INFINETLY MANY PRIME NUMBERS] BUT THE FORMULA IS FINITE

BY CONTRADICTION:

φ IS FINITE $\rightarrow$ HAS N TYPES

$P > N$

PRIME NUMBER

THERE IS A PATH OF LENGTH P

$T_0, T_i, T_j, T_l \quad i < j$

$T_i = T_j$

$r_0 \ldots r_i \quad s_1 \quad s_{i+1} \ldots s_j \quad s_p \quad 1+(j-i)$