# Knowledge Representation and Reasoning

Exercise Session 5

**Exercise 1. Type Graph** (∗)

Let $\varphi = x\, \mathcal{U} \neg y$

1. Find the types of $\varphi$
2. Construct the type graph
3. Identify initial and final types

**Exercise 2. Model Counting** (∗)

How many temporal models **of length 2** satisfy the formula $\varphi$ from Exercise 1?

**Exercise 3. KR 1** (∗)

1. Construct an $\mathrm{LTL_f}$ formula describing the following specification of a (simplified) traffic light; you can use the abbreviations seen during the lecture.
   - the light is either green or red, but never both
   - whenever the light is red, it will eventually turn green

   **Hint**: use the propositional variables $\mathsf{green}$ and $\mathsf{red}$.
2. What characterises the **last** timepoint of all models satisfying this specification?

**Exercise 4. KR 2** (∗∗)

1. Extend the specification from Exercise 3 to include two traffic lights (with variables $\mathsf{green}_i$ and $\mathsf{red}_i$ ($i = 1, 2$) such that the two green lights are never simultaneously on.
2. Is this specification satisfiable? If yes, give a temporal model satisfying it; if not, envision a way to fix it

**Exercise 5. Model Size 1** (∗∗)

Build a formula that is satisfied by models of **even** length only, or argue why it cannot exist.

**Exercise 6. Model Size 2** (∗∗∗)

Build a formula that is satisfied by models of **prime** length only, or argue why it cannot exist.

**Exercise 1.** **Type Graph** (*)

Let $\varphi = x \,\mathcal{U} \neg y$

1. Find the types of $\varphi$
2. Construct the type graph
3. Identify initial and final types

WE HAVE A TEMPORAL FORMULA WANT TO FIND ALL THE MODELS THAT SATISFY THIS FORMULA

$x \,\mathcal{U} \neg y$

- PROPRIETY $x$ UNTIL $y$ BECOMES FALSE
- AFTER WE HAVE $\neg y$ WE CAN DO WHATEVER WE WANT

CHECK THE SUBFORMULAS

- $x \,\mathcal{U} \neg y$
- $x$
- $y$
- $\bigcirc(x \,\mathcal{U} \neg y)$

THOSE ARE THE SUBFORMULAS
WE SHOULD ALSO HAVE THEIR NEGATION
BUT IT CAN BE "ALREADY INSIDE" DEPENDING IF THE SUBFORMULAS ARE TRUE OR NOT

| $\bigcirc(x \,\mathcal{U} \neg y)$ | $x \,\mathcal{U} \neg y$ | $x$ | $y$ |
|---|---|---|---|
| 0/1 | 1 ← BECAUSE $\neg y$ IS ALREADY SAT | 0/1 | 0 ← CHECK WHAT HAPPEN WHEN $y$ IS FALSE |
| 0/1 | 0 X AND $\neg y$ BOTH FALSE | 0 | 1 |
| 0 | 0 | 1 | 1 |
| 1 | 1 | 1 | 1 |

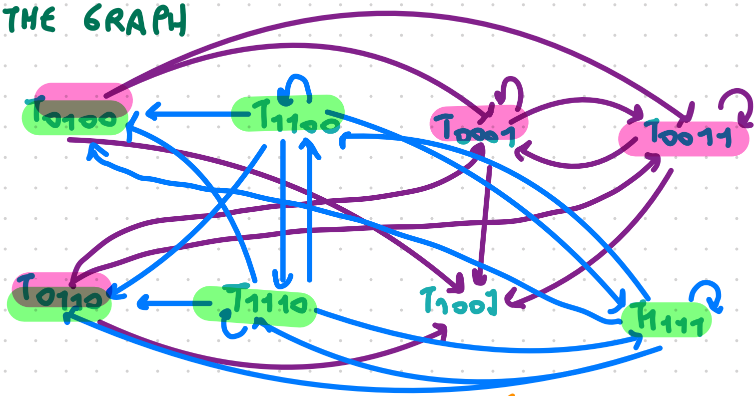
THERE ARE 8 TYPES

- $T_{0100}$
- $T_{0110}$
- $T_{1100}$
- $T_{1110}$
- $T_{0001}$
- $T_{1001}$
- $T_{0011}$
- $T_{1111}$

INITIAL: HAVE THE FORMAL THAT WE ARE INTERESTED IN $(X \, W \, \neg Y)$ TRUE SO ALL THE ONE WITH 1 AS THE SECOND NUMBER

FINAL: NO NEXT FORMULA POSITIVE, SO 0 AS THE FIRST NUMBER

BUILD THE GRAPH




- IF A TYPE HAVE A NOT NEXT FORMULA (0 AS FIRST NUMBER) IT MUST BE CONNECTED TO ALL THE TYPES THAT HAVE THE SECOND NUMBER 0
- IF A TYPE HAVE A NEXT FORMULA TRUE (1 AS FIRST NUMBER) IT MUST BE CONNECTED TO ALL THE TYPES THAT HAVE THE SECOND NUMBER 1

**Exercise 2. Model Counting** (*)

How many temporal models **of length 2** satisfy the formula $\varphi$ from Exercise 1?

TEMPORAL MODEL: SEQUENCE OF PROPOSITIONAL VALUATION

THE LENGTH OF A TEMPORAL MODEL IS THE NUMBER OF VALUATIONS +1

$$\text{LENGTH}(V_0, V_1, V_2 \ldots V_N) = N+1$$

| $V_0$ | $V_1$ | $V_2$ | |
|---|---|---|---|
| $x$ / $\neg x$ $\neg y$ | | | 32 TEMP. MODELS |
| $x\ y$ | $x$ / $\neg x$ $\neg y$ | | 8 TEMP. MODELS |
| $x\ y$ | $x\ y$ | $x$ / $\neg x$ $\neg y$ | 2 TEMP. MODELS |

42 TEMPORAL MODELS

$\neg Y$ MUST BE FOUND AMONG $V_0$ $V_1$ OR $V_2$

THE UNTIL FORMULA IS ALREADY SAT
SO CAN HAVE $x$ OR $\neg x$

DON'T CARE WHAT HAPPEN NEXT, CAN HAVE 4 CASES

$y$ MUST BE BEFORE (OTHERWISE
$V_1$ WILL NOT BE THE FIRST PLACE
IN WITCH $\neg y$ APPEARS

MUST HAVE $X$ TO SATISFY THE UNTIL FORMU

**Exercise 3. KR 1** (*)

1. Construct an $\mathrm{LTL_f}$ formula describing the following specification of a (simplified) traffic light; you can use the abbreviations seen during the lecture.
   - the light is either green or red, but never both
   - whenever the light is red, it will eventually turn green

   **Hint**: use the propositional variables green and red.

2. What characterises the **last** timepoint of all models satisfying this specification?

$\Box$ ALWAYS

$\Diamond$ EVENTUALLY

$\Box$ (GREEN XOR RED)

$\Box$ (RED $\rightarrow \Diamond$ GREEN)

**Exercise 4. KR 2** (**)

1. Extend the specification from Exercise 3 to include two traffic lights (with variables $\mathsf{green}_i$ and $\mathsf{red}_i$ ($i = 1, 2$) such that the two green lights are never simultaneously on.

2. Is this specification satisfiable? If yes, give a temporal model satisfying it; if not, envision a way to fix it

$\Box(\neg(\text{GREEN}_1 \sqcap \text{GREEN}_2))$

$\Box(\text{GREEN}_1 \text{ XOR } \text{RED}_1)$ $\Box(\text{RED}_1 \rightarrow \Diamond \text{GREEN}_1)$

$\Box(\text{GREEN}_2 \text{ XOR } \text{RED}_2)$ $\Box(\text{RED}_2 \rightarrow \Diamond \text{GREEN}_2)$

**Exercise 5. Model Size 1** (∗∗)

Build a formula that is satisfied by models of **even** length only, or argue why it cannot exist.

$x \quad \neg x \quad x \quad \neg x$

CAN NEVER END WITH A $\neg x$ $\Rightarrow$ $\square(\neg x \rightarrow \bigcirc x)$

$\square(x \rightarrow (\bigcirc\neg x \vee \bigcirc\bot))$

**Exercise 6. Model Size 2** (***)

Build a formula that is satisfied by models of **prime** length only, or argue why it cannot exist.

NOT EXISTS

THE DISTANCE BETWEEN TWO PRIME NUMBERS IS NOT CONSTANT