Knowledge Representation and Reasoning

Exercise Session 2

Exercise 1. Knowledge Representation (*)

Assume that all facts in a knowledge base will have the forms

$$parent(a, b) \leftarrow$$
$$female(a) \leftarrow$$
$$male(a) \leftarrow$$

meaning that "a is a parent of b," "a is female," and "a is male" respectively.

1. Using predicate rules, create a knowledge base which describes family relations including **at least**: `aunt`, `uncle`, `grandmother`, `sibling`, and `ancestor`.

2. If, in addition, facts of the form $married(a, b)$ are allowed, extend the knowledge base to allow legal family within the notions of `aunt` and `uncle`.

PREDICATE RULES: $P(\bar{t}) \leftarrow Q_1(\bar{t}_1), \dots$

UNARY

AUNT (X) ← FEMALE (X), SIBLING (X, Y), PARENT (Y, Z)

BINARY

CAN BE TRANSFORMED

AUNT (X, Z) ← FEMALE (X), SIBLIN (X, Y), PARENT (Y, Z)

GRANDMOTHER (X, Y) ← FEMALE (X), PARENT (X, Z), PARENT (Z, Y)

SIBLIN (X, Y) ← PARENT (Z, X), PARENT (Z, Y)

EVERY PARENT IS AN ANCESTOR

ANCESTOR (X, Y) ← PARENT (X, Y)

ANCESTOR (X, Y) ← PARENT (X, Z), PARENT (Z, Y)

2) AUNT (X, Y) ← FEMALE (X), MARRIED (X, Z), SIBLING (Z, W), PARENT (W, Y)

Exercise 2. Canonical Model (∗)

1. Add the following facts to the KB from Exercise 1 and build the **canonical model**:

$$
\begin{array}{rr}
parent(efraim, ana) \leftarrow & parent(ana, ingrid) \leftarrow \\
parent(ingrid, denis) \leftarrow & parent(ana, claudia) \leftarrow \\
parent(denis, hans) \leftarrow & parent(claudia, bob) \leftarrow \\
parent(francis, greta) \leftarrow & married(claudia, francis) \leftarrow \\
\\
female(ana) \leftarrow & male(bob) \leftarrow \\
female(claudia) \leftarrow & male(denis) \leftarrow \\
male(efraim) \leftarrow & male(francis) \leftarrow \\
female(greta) \leftarrow & male(hans) \leftarrow \\
female(ingrid) \leftarrow &
\end{array}
$$

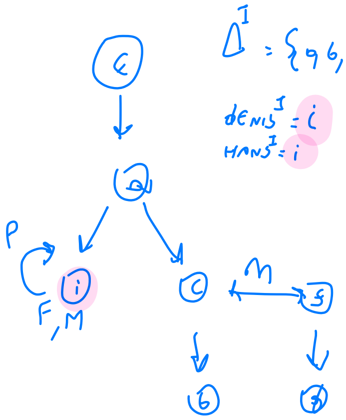
2. Answer the following queries using this canonical model:
   - $ancestor(efraim, denis)$ YES
   - $ancestor(efraim, great)$ ta NO
   - $uncle(francis, bob)$ YES (!)
   - $uncle(francis, denis)$ YES
   - $grandmother(X)$ INSIGN, ANA
   - $sibling(X, Y)$ (INGRID, CLAUDIA), (CLAUDIA, INGRID) (CLAUDIA, CLAUDIA) ...

     + EVERONE W/ THESELF EXCEPT (EFRAIN EFRAIN) (FRANCIS, FRANCIS)

Exercise 3. Models (∗)

1. Build a model of the KB from the previous exercises, whose domain has only **7** elements.

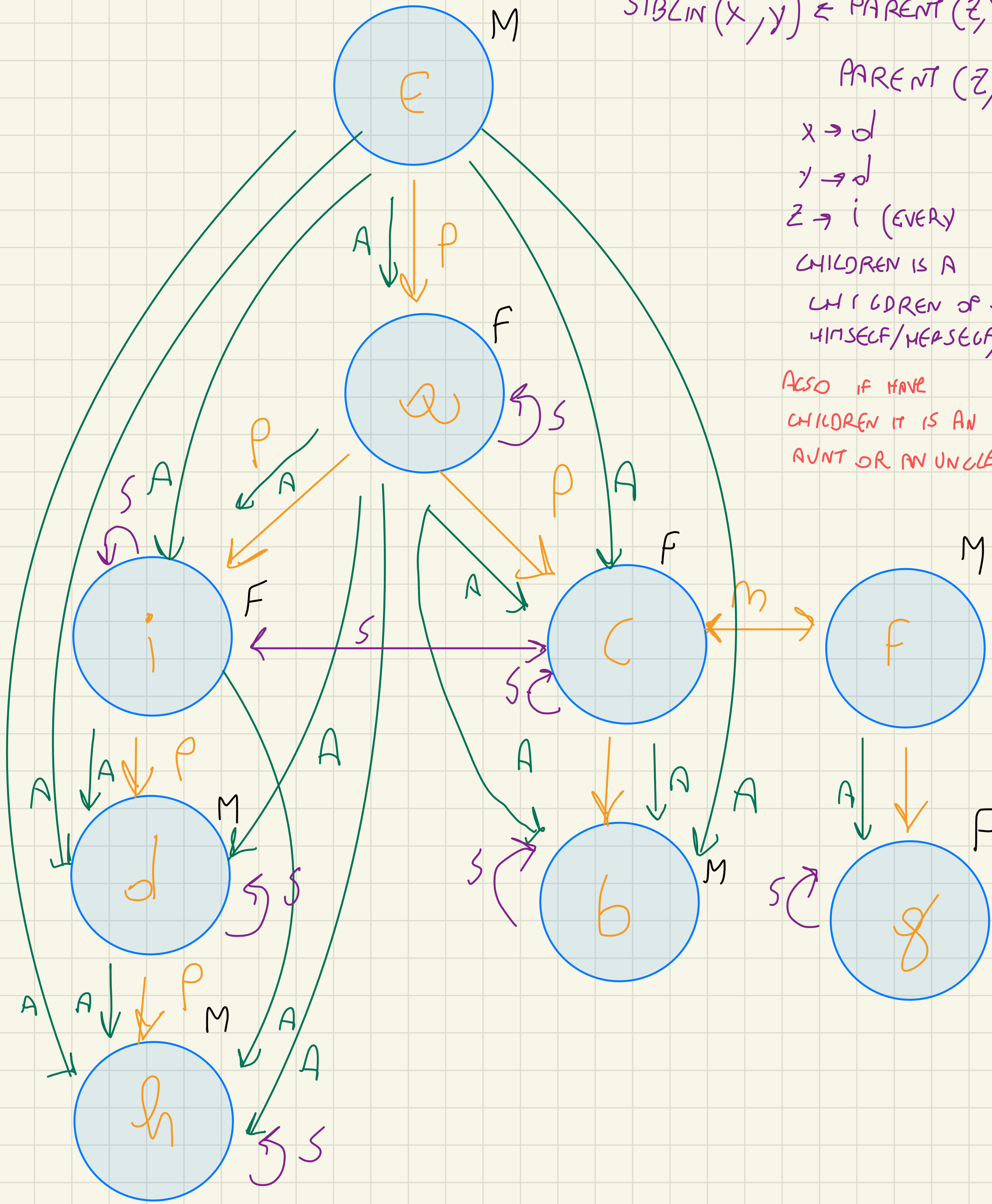
2. Build a model of the KB from the previous exercises, whose domain has only **4** elements.



$\Delta^I = \{a\, b, c, i, e, f, g\}$

$denis^I = i$

$hans^I = i$



SIBLIN(X, Y) ⇐ PARENT(Z,X)
PARENT(Z,Y)

X → d

Y → d

Z → i (EVERY CHILDREN IS A CHILDREN OF HIMSELF/HERSELF)

ALSO IF HAVE CHILDREN IT IS AN AUNT OR AN UNCLE

Exercise 4. Consistency (**)

A knowledge base is *consistent* if it has a model. Tell whether the following statements are true or false, justifying your answer.

1. Every set of predicate rules is consistent TRUE, THE CANONIC MODEL IS A MODEL

2. Every set of predicate rules has a model with one element

   $I = (\{\varepsilon\}, \cdot^I)$ $\quad a^I = \varepsilon$ FOR ALL CONSTANT a

   $P^I = \{\varepsilon\}$ FOR ALL PREDICATES

   $Q^I = (\{\varepsilon, \varepsilon\})$ FOR ALL BINARY PREDICATES

Exercise 5. Canonical model size (***)

1. Construct a KB with 4 facts and 1 rule such that its canonical model construction must add $4^2 = 16$ facts.

2. Generalise the construction to work for any number n of facts in the KB (and n^2 facts in the canonical model)

Exercise 6. Query Expressivity (***)

Suppose that we are interested in deducing whether a rule $p(x) \leftarrow q(x)$ is entailed by a KB K; that is, whether every model of K also satisfies this rule.

Devise a reasoning method that can derive this consequence using the tools that we have seen in the lecture.

Exercise 7. Disjoint unions (**)

Consider two interpretations $\mathcal{I} = (\Delta^\mathcal{I}, \cdot^\mathcal{I})$ and $\mathcal{J} = (\Delta^\mathcal{J}, \cdot^\mathcal{J})$ such that $\Delta^\mathcal{I} \cap \Delta^\mathcal{J}$. The *disjoint union* of $\mathcal{I}$ and $\mathcal{J}$ is the interpretation $\mathcal{I} \oplus \mathcal{J} = (\Delta^\mathcal{I} \cup \Delta^\mathcal{J}, \cdot^{\mathcal{I}\mathcal{J}})$ where for every predicate P, $P^{\mathcal{I}\mathcal{J}} = P^\mathcal{I} \cup P^\mathcal{J}$.

In other words, the disjoint union of $\mathcal{I}$ and $\mathcal{J}$ is the graph obtained by putting together the two graphs defined by $\mathcal{I}$ and $\mathcal{J}$.

Is it true that if $\mathcal{I}$ and $\mathcal{J}$ are both models of a knowledge base K, then $\mathcal{I} \oplus \mathcal{J}$ is also a model of K? **Justify.**

Exercise 8. Representing Constraints (*)

1. Add constraints to your knowledge base from Exercise 1 to remove any unexpected consequences you have observed.

2. Do your answers to Exercise 3 change?

Exercise 9. Model sizes with constraints (∗∗∗)

1. Using constraints, build a knowledge base K such that all models have at least 3 elements
2. Generalise the construction to models with n elements, for any arbitrary n
3. How many constraints are needed?