

MINIMIZER $f(x) = \frac{2x^2 - x + 1}{1-x}$

E' NELL'ESAME

MIN
 $[-2, 1/2]$

D: $x \neq 1$

$\{ [-2, 1/2] \text{ CLOSED, BOUNDED} \}$
 f CONTINUOUS ON $\mathbb{R} \setminus \{1\}$
 \rightarrow WEIERSTRASS

$f'(x) = \frac{4x - 1(1-x) - (-1)(2x^2 - x + 1)}{(1-x)^2}$

$\exists \text{ MIN } [-2, 1/2]$
 $\exists \text{ MAX } [-2, 1/2]$

$f'(x) = \frac{4x - 1 + x + 2x^2 - x + 1}{(1-x)^2} = \frac{2x^2 + 4x}{(1-x)^2}$

$2x^2 + 4x = 0 \quad x=0 \quad \vee \quad x = -2$

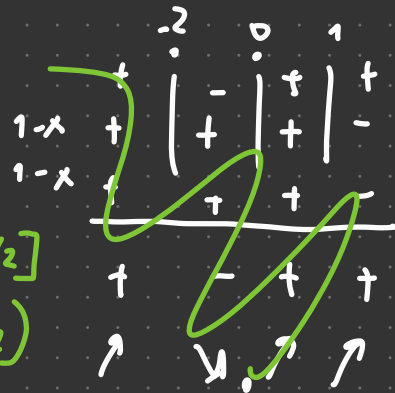
$(1-x)^2 = 0$

$1-x=0, x=1$

CANDIDATES

• ON THE BOUNDARY $[-2, 1/2]$

• IN THE INTERIOR $(-2, 1/2)$



$\frac{d}{dx} = \frac{-2x^2 + 4x}{(1-x)^2} \quad x \neq 1$

$x=0 \quad \vee \quad x=2$

$2 \notin (-2, 1/2)$ DISCARD

$0 \in (-2, 1/2)$ IT IS A CANDIDATE

CANDIDATES

$$0, -2, \frac{1}{2}$$

$$f(0) = 1 \quad f(-2) = \frac{11}{3} \quad f\left(\frac{1}{2}\right) = 2$$

$$\text{MIN} = 1$$

0 ABSOLUTE MINIMUM

$$\text{MAX} = \frac{11}{3}$$

-2 ABSOLUTE MAXIMIZER

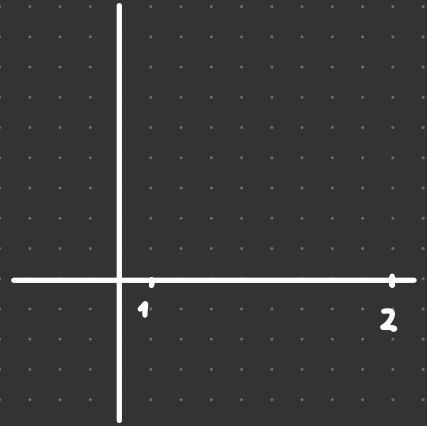
$$f(x) = \cos(x^2 - 3x + 2)$$

$$x^2 - 3x + 2 > 0$$

$$g = \frac{3 \pm 1}{2} \begin{matrix} 1 \\ 2 \end{matrix}$$

$$\frac{1}{2}$$

$$(-\infty, 1) \cup (2, +\infty)$$



Sketch of f

$$f(0) = \cos(2)$$

$$\frac{3 - \sqrt{5}}{2}$$

$$+ \text{ or } -$$

$$x^2 - 3x + 2 > 1$$

$$x^2 - 3x + 1 > 0$$

$$+ \text{ or } -$$

$$\frac{3 \pm \sqrt{5}}{2}$$

f CONTINUOUS

$$\lim_{x \rightarrow +\infty} f(x) = +\infty$$

$$\lim_{x \rightarrow 1^-} f(x) = -\infty$$

$$\lim_{x \rightarrow 2^+} f(x) = -\infty$$

ASYMPTOTES

$$\begin{aligned} m &= \lim_{x \rightarrow \pm\infty} \frac{f(x)}{x} = 0 \\ q &= \lim_{x \rightarrow \pm\infty} f(x) - mx = \pm\infty \end{aligned} \quad \left. \vphantom{\lim_{x \rightarrow \pm\infty} \frac{f(x)}{x} = 0} \right\} \text{NO ASYMPTOTES}$$

DERIVATE

$$f(x) = \cos(x^2 - 3x + 2)$$

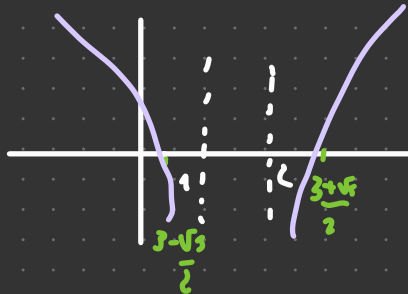
$$f'(x) = \frac{2x - 3}{x^2 - 3x + 2} \quad x \in (-\infty, 1) \cup (2, +\infty)$$

SIGN OF $f'(x)$

$$2x - 3 = 0, \quad x = \left(\frac{3}{2}\right) \quad \text{NOT IN THE DOMAIN}$$

$$x^2 - 3x + 2 > 0 \quad \forall x \in \mathbb{R}$$

$$\begin{aligned} f'(x) &> 0 \quad \forall x > 2 \\ f'(x) &< 0 \quad \forall x < 1 \end{aligned} \quad \left. \vphantom{f'(x) > 0} \right\} \text{NO EXTREMUM POINT}$$



$$f(x) = |x^2 - 1| - \frac{1}{2}x^3 \quad 0: \mathbb{R}$$

$$x^2 - 1 > 0; \quad -1 \cup_1^+; \quad (-\infty, -1) \cup (1, +\infty)$$

$$x^2 - 1 < 0 \quad (-1, 1)$$

f CONT ON \mathbb{R}

$$\lim_{x \rightarrow \pm \infty} f(x) = \mp \infty$$

DERIVATES

$$f(x) = \begin{cases} x^2 - 1 - \frac{1}{2}x^3 & (-\infty, -1) \cup (1, +\infty) \\ 1 - x^2 - \frac{1}{2}x^3 & (-1, 1) \end{cases}$$

f IS DIFF FOR $|x| \neq 1$ THAT $\cup \quad x \neq -1, x \neq 1$

$$f'(x) = \begin{cases} -\frac{3}{2}x^2 + 2x & |x| > 1 \\ -\frac{3}{2}x^2 - 2x & |x| < 1 \end{cases} \quad x \neq -1, 1$$

IS f DIFF IN $1, -1$?

$$\lim_{x \rightarrow 1^+} f'(x) = \frac{1}{2} \quad \lim_{x \rightarrow 1^-} f'(x) = -\frac{7}{2}$$



f IS **NOT** DIFF. IN 1

$$\lim_{x \rightarrow -1^+} f'(x) = \frac{1}{2}$$

$$\lim_{x \rightarrow -1^-} f'(x) = \frac{1}{2}$$



f IS NOT DIFF. IN -1

SIGN OF $f'(x)$

$$|x| > 1 \quad f'(x) > 0 \quad \text{THAT IS} \quad -\frac{3}{2}x^2 + 2x > 0$$

$$x(-\frac{3}{2}x + 2)$$

0 4/3

- + -

1/3

CONSIDERING ONLY $|x| > 1$ $f'(x) < 0$

$$|x| < 1 \quad f'(x) > 0 \quad -\frac{3}{2}x^2 + 2x > 0$$

$$x(-\frac{3}{2}x - 1) > 0$$

$$-4/3$$

0

-

$$(-1, 0) \quad f'(x) < 0$$

$$\lim_{x \rightarrow 1} \frac{e^{(x-1)^2} - 1 - (x-1)^2}{(\sin(2(x-1)))^2}$$

CHANGE OF VARIABLE

$$y = x-1 \quad x \rightarrow 1 \Rightarrow y \rightarrow 0$$

$$\lim_{y \rightarrow 0} \frac{e^{y^2} - 1 - y^2}{(\sin(2y^2))^2}$$

CHANGE OF VARIABLE

$$z = y^2 \quad y \rightarrow 0 \quad z \rightarrow 0^+$$

$$\lim_{z \rightarrow 0^+} \frac{e^z - 1 - z}{(\sin(2z))^2}$$

$$\lim_{z \rightarrow 0^+} \frac{(e^z - 1 - z)(2z)^2}{(\sin(2z))^2 (4z^2)}$$

$$z \rightarrow 0^+ \rightarrow 1$$

$$\lim_{z \rightarrow 0^+} \frac{e^z - 1 - z}{2z^2} \left[\frac{0}{0} \right] \text{ HOPITAL}$$

$$\lim_{z \rightarrow 0} \frac{e^z - 1}{2z} \left[\frac{0}{0} \right]$$

HOPITAL

$$\lim_{z \rightarrow 0} \frac{e^z}{2} = \frac{1}{2}$$