PERMANENCE OF SIGN THEOREM FOR CONTINUOUS FUNCTIONS

LET $f: A \subseteq \mathbb{R} \to \mathbb{R}$ AND LET $x_0 \in A$. SUPPOSE THAT f IS CONTINUOUS IN x_0. SO:

a) IF $f(x_0) > 0$, SO EXISTS $\delta > 0$ SUCH THAT FORALL $x \in A$ WITH $|x - x_0| < \delta$ WE HAVE $f(x) > 0$

b) IF $f(x_0) < 0$, SO EXISTS $\delta > 0$ SUCH THAT FORALL $x \in A$ WITH $|x - x_0| < \delta$ WE HAVE $f(x) < 0$

PROOF:

CHOOSEN $\varepsilon > 0$ SUCH THAT $f(x_0) - \varepsilon > 0$. SO EXISTS $\delta > 0$ SUCH THAT FORALL $x \in A$ WITH $|x - x_0| < \delta$ WE HAVE

$$0 < f(x_0) - \varepsilon < f(x) < f(x_0) + \varepsilon$$