$+ \quad \times$

$- \quad \div$

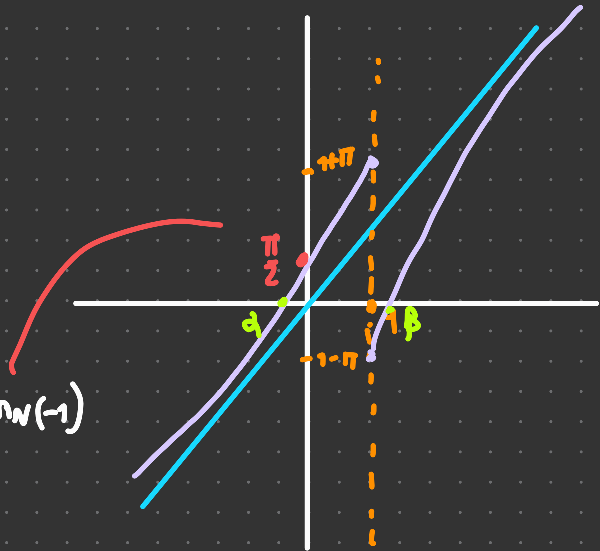
$$f(x) = -2\arctan\left(\frac{1}{x-1}\right) + x$$

DOMAIN

ARCTAN $\mathbb{R}$

DENOMINATOR 1

$\mathbb{R} - \{1\}$



NO SYMMETRIES

SIGN $\quad f(0) = -2\arctan(-1)$

$-2\left(\frac{\pi}{2}\right)$

LIMIT

$x \to +\infty \quad f(x) = +\infty$

$\arctan(+\infty) \to \frac{\pi}{2}$

$(-\infty) \to -\frac{\pi}{2}$

$x \to -\infty \quad f(x) = -\infty$

$\exists\ \alpha < 0$ AND $\beta > 1$ SUCH THAT

$f < 0 \quad x < \alpha$

$f = 0 \quad x = \alpha$

$f > 0 \quad \alpha < x < 1$

$f < 0 \quad 1 < x < \beta$

$f = 0 \quad x = \beta$

$f > 0 \quad x > \beta$

$x \to 1^+ \quad f(x) \quad 1 - \pi$

$x \to 1^- \quad f(x) \quad 1 + \pi$

ASYMPTOTES $\quad \frac{f(x)}{x} = 1$

$\lim_{x \to \pm\infty} (f(x) - x) = 0$

$y = x$ ASYMPTOTE

f IS DIFFERENTIABLE? COMPUTE DERIVATE

$$f(x) = -2\arctan\left(\frac{1}{x-1}\right) + x$$

f IS DIFFERENTIABLE IN $\mathbb{R} - \{1\}$

$$f' - 2\left(-\frac{1}{(x-1)^2}\right)\frac{1}{1+\left(\frac{1}{x-1}\right)^2} + 1$$

$$\left(\frac{2}{\cancel{(x-1)^2}}\right)\frac{1}{\frac{(x-1)^2+1}{\cancel{(x-1)^2}}} + 1$$

$$\frac{2}{(x-1)^2+1} + 1 \quad x \neq 1$$

$f'(x)$ ALWAYS POSITIVE

$f'(x) > 0 \quad \forall x \neq 1$

NO CRITIC POINT

NO LOCAL EXTREMUM

f STRICTLY INCREASING ON $(-\infty, 1)$ $(1, +\infty)$

f IS NOT INCREASIN ON $\mathbb{R}\{-1\}$

$$\lim_{x \to 0} \frac{\log(x^2 - sen^2(x) + 1)}{e^{x^2} - 1 - x^2}$$

$$\lim_{x \to 0} \underbrace{\frac{x^2 - sen^2(x)}{x^2 - sen^2(x)} \cdot \frac{\log(1 + x^2 - sin^2(x))}{e^{x^2} - 1 - x^2}}_{1 \quad x \to 0}$$

WE NEED TO CHECK THAT FOR $x \in (-\delta, \delta) \setminus \{0\}$

$$x^2 - sen^2(x) \neq 0$$

$$\lim_{x \to 0} \frac{x^2 - sen^2(x)}{e^{x^2} - 1 - x^2} \quad \left[\frac{0}{0}\right]$$

HOPITAL

$$\lim_{x \to 0} \frac{\cancel{2}x - \cancel{2}\, sin(x)\cos(x)}{\cancel{2}x(e^{x^2} - 1)}$$

$$\lim_{x \to 0} \underbrace{\frac{x^2}{e^{x^2} - 1}}_{\substack{x \to 0 \\ 1}} \cdot \frac{x - sin(x)}{x^3} \quad \left(\frac{x^2}{x^2}\right)$$

HOPITAL

$$\lim_{x \to 0} \frac{1 - \cos^2(x) + \sin^2(x)}{3x^2} = \lim_{x \to 0} \frac{(1 - \cos(x))(1 + \cos(x) + \sin^2(x))}{3x^2}$$

$$\frac{1}{3} + \frac{1}{3} = \frac{2}{3}$$

$$\frac{d}{dx} \cos(1 + x^2 - \sin(x)^2) = \frac{\frac{d}{dt}(x^2 - \sin(x)^2)}{1 + x^2 - \sin(x)^2}$$

1

$x \to 0$

$$f(x) = \begin{cases} \dfrac{\cos(1-2x)}{\arctan(3x)} & x < 0 \\ a & x = 0 \\ -\dfrac{9}{4}a^3 + 2 + a\,\dfrac{2\cos(\sqrt{x}) - 2 + x}{x^2} & x > 0 \end{cases}$$

CONTINITY OF f AS a VARIES

f IS CONTINOUS ON $\mathbb{R} - \{0\}$

$$f(0) = \lim_{x \to 0} f(x)$$

$$f(0) = \lim_{x \to 0^+} f(x) = \lim_{x \to 0^-} f(x)$$

$$f(0) = a$$

$$\lim_{x \to 0^-} \frac{\cos(1-2x)}{\arctan(3x)} \frac{(-2x)}{(-2x)} \frac{(3x)}{(3x)} = -\frac{2}{3}$$

$$\lim_{x \to 0^+} \frac{2\cos(\sqrt{x}) - 2 + x}{x^2} \quad \left[\frac{0}{0}\right]$$

HOPITAL

$$\frac{-\dfrac{\cancel{2}}{\cancel{2}\sqrt{x}}\sin(\sqrt{x}) + 1}{2x} = \frac{1 - \dfrac{\sin\sqrt{x}}{\sqrt{x}}}{2x} \quad \left[\frac{0}{0}\right]$$

$$\lim_{x \to 0^+} \frac{\sqrt{x} - \sin\sqrt{x}}{2x^{3/2}}$$

HOPITAL

$$\frac{\frac{1}{2\sqrt{x}}\left[1-\cos(\sqrt{x})\right]}{3\sqrt{x}} = \lim_{x\to 0^+} \frac{1}{6}\left[\frac{1-\cos\sqrt{x}}{x}\right]$$

$$\frac{1}{12}$$

$$-9a\, a^3 + 2a)$$

$a; -\frac{2}{3}; \quad -\frac{9}{4}a^3 + 2a$

$\exists\, a \in \mathbb{R}$ SUCH THAT THESE 3 NUMBERS ARE EQUAL?

$$a = -\frac{2}{3}$$

$$-\frac{9}{4}\left(-\frac{2}{3}\right)^3 + 2\left(-\frac{2}{3}\right) = -\frac{2}{3}$$

$$\cancel{\frac{9}{4}} \cdot \cancel{\frac{8}{27}} - \frac{4}{3} = -\frac{2}{3}$$

$$\frac{2}{3} - \frac{4}{3} = -\frac{2}{3} \quad \checkmark$$

USE TYLOR INSTEAD

$$\cos(y) = 1 - \frac{y^2}{2} + \frac{y^4}{4!} + R(y)$$

$$\lim_{y \to 0} \frac{R(y)}{y^4} = 0$$

$\cos(\sqrt{x})$

$$1 - \frac{x}{2} + \frac{x^2}{4!} + R(\sqrt{x})$$

$$\lim_{x \to 0} \frac{R(\sqrt{x})}{x^2} = 0$$

$$2\cos(\sqrt{x}) - 2 + x = \frac{2x^2}{4!} + 2R(\sqrt{x}) = \frac{1}{12}x^2 + 2R(\sqrt{x})$$

$$\lim_{x \to 0} \frac{\operatorname{sen}(x) + \log(1-x)}{x^2} \quad \left[\frac{0}{0}\right]$$

HOPITAL

$$\frac{\cos(x) + \frac{-1}{1-x}}{2x}$$

$$\frac{\cos(x) - \frac{1}{1-x}}{2x}$$

$$\frac{(1-x)(\cos(x)) - 1}{(1-x)\,2x} \quad \left[\frac{0}{0}\right]$$

$x \to 0$

1

$$\frac{(1-x)(\cos(x)) - 1}{2x} \quad \left[\frac{0}{0}\right]$$

Hopital

$$\frac{-\cos(x) - (1-x)\operatorname{sen}(x)}{2} = -\frac{1}{2}$$

SVILUPPI TAYLOR

$$\sin(x) = x - \frac{x^3}{3!} + R(x)$$

$$\lim_{x \to 0} \frac{R(x)}{x^3} = 0$$

$$\cos(1+y) = y - \frac{y^2}{2} + \frac{y^3}{3} + \tilde{R}(y) \qquad \lim_{y \to 0} \frac{R(y)}{y^3} = 0$$

$$\sin(x) + \log(1-x) = \cancel{x} - \frac{x^3}{3!} + R(x) - \cancel{x} - \frac{x^2}{2} - \frac{x^3}{3} + R(-x)$$

$$\underbrace{-\frac{x^2}{2} - \left[\frac{1}{6} + \frac{1}{3}\right]x^3}_{P_3(x)} + R(x) + R(-x)$$

$$\lim_{x \to +\infty} \frac{\cos^3(x)}{\sqrt{x}}$$

$$\frac{(\log(x))^a}{x^b} \xrightarrow{x \to +\infty} 0 \qquad a > 0 \quad b > 0$$