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INTEGRABLE FUNCTIONS AND DEFINITE INTEGRAL (SHORT)

- $a, b \in \mathbb{R}$ $a<b$
- $f : [a,b] \longrightarrow \mathbb{R}$
- f BOUNDED

* f IS INTEGRABLE IN $[a,b] \iff \forall \varepsilon > 0$ $\exists P_\varepsilon$ PARTITION SUCH THAT $0 \le S(P_\varepsilon) - s(P_\varepsilon) < \varepsilon$
* WE SAY THAT f IS RIEMANN INTEGRABLE ON $[a,b]$ IF $\underline{I}(f,[a,b]) = \overline{I}(f,[a,b])$
  * IN THIS CASE, THE NUMBER $I = \overline{I} = \underline{I}$ IS CALLED THE DEFINITE INTEGRAL OF f ON $[a,b]$ WHICH IS DENOTED BY $\int_a^b f(x)\,dx$ OR $\int_a^b f$

INTEGRABLE FUNCTIONS

$f : [a,b] \longrightarrow \mathbb{R}$ BOUNDED

f IS INTEGRABLE IN $[a,b] \iff \forall \varepsilon > 0$ $\exists P_\varepsilon$ PARTITION SUCH THAT $0 \le S(P_\varepsilon) - s(P_\varepsilon) < \varepsilon$

DEFINITE INTEGRAL

- LET $f : [a,b] \rightarrow \mathbb{R}$, $a, b \in \mathbb{R}$, $a<b$
- f BOUNDED

WE SAY THAT f IS RIEMANN INTEGRABLE ON $[a,b]$ IF $\underbrace{\underline{I}(f,[a,b])}_{\text{INFERIOR (RIEMANN) INTEGRAL OF } f \text{ ON } [a,b]} = \overbrace{\overline{I}(f,[a,b])}^{\text{SUPERIOR (RIEMANN) INTEGRAL OF } f \text{ ON } [a,b]}$

IN THIS CASE, THE NUMBER $I = \underline{I} = \overline{I}$ IS CALLED THE DEFINITE INTEGRAL OF f ON $[a,b]$ WHICH IS DENOTED BY $\int_a^b f(x)\,dx$ OR $\int_a^b f$