$+$ $\times$

$-$ $\div$

- **N EVEN**

$f_N : [0,+\infty) \to [0,+\infty)$ IS BIJECTIVE

$\Rightarrow \exists\ f_N^{-1} : [0,+\infty) \to [0,+\infty)$ INVERSE $f_N^{-1} = \sqrt[N]{\ }$

THAT IS $\forall a \geq 0$ $\exists$ A UNIQUE $b \in \mathbb{R}$, $b \geq 0$, SUCH THAT $b^N = a$ THAT IS

$b = f_N^{-1}(a) = \sqrt[N]{a} = a^{1/N}$ N-TH ROOT OF a

$f_N^{-1} : [0,+\infty) \to [0,+\infty)$ STRICTLY INCREASING AND BIJECTIVE

- **N ODD**

$f_N : \mathbb{R} \to \mathbb{R}$ BIJECTIVE (STRICTLY INCREASING)

$\Rightarrow \exists\ f_N^{-1} : \mathbb{R} \to \mathbb{R}$ THAT IS

$\forall a \in \mathbb{R}$ $\exists$ A UNIQUE $b \in \mathbb{R}$ SUCH THAT $b^N = a$

$b = f_N^{-1}(a) = \sqrt[N]{a} = a^{1/N}$ N-TH ROOT OF a

$f_N^{-1}: \mathbb{R} \to \mathbb{R}$ STRICTLY INCREASING AND BIJECTIVE

NOTATION:

$\sqrt{\ } = \sqrt[2]{\ }$ SQUARE ROOT

$\sqrt[3]{\ }$ CUBIC ROOT

REMARKS: $b^N = 0 \iff b = 0$ SO $\sqrt[N]{0} = 0 \ \forall N \in \mathbb{R}$

$b^N = 1,\ b > 0 \iff b = 1$, SO $\sqrt[N]{1} = 1 \ \forall N \in \mathbb{R}$

- $a > 0$

IF N EVEN $\nexists\, b \in \mathbb{R}$ SUCH THAT $b^N = -a$,

THAT IS, $\nexists \sqrt[N]{-a}$

IF N ODD $\sqrt[N]{-a} = -\sqrt[N]{a}$, THAT IS $\sqrt[N]{\ }$ IS ODD

EXAMPLE:

$\sqrt{4} = 2$ (NOT EQUIVALENT TO SOLVING $x^2 = 4$)

$\sqrt[4]{16} = 2 \quad \sqrt[3]{8} = 2 \quad \sqrt[3]{-8} = -2 \quad \nexists \sqrt[4]{-16}$

- **N EVEN**

$$\sqrt[N]{x^N} = x \quad \forall x \geq 0 \qquad \left(\sqrt[N]{x^N} = |x| \quad \forall x \in \mathbb{R}\right)$$

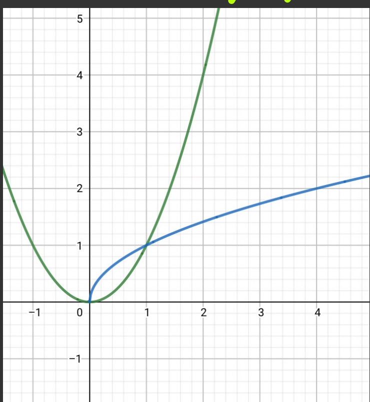
$$\left(\sqrt[N]{y}\right)^N = y \quad \forall y \geq 0$$

- **N ODD**

$$\sqrt[N]{x^N} = x \quad \forall x \in \mathbb{R}$$

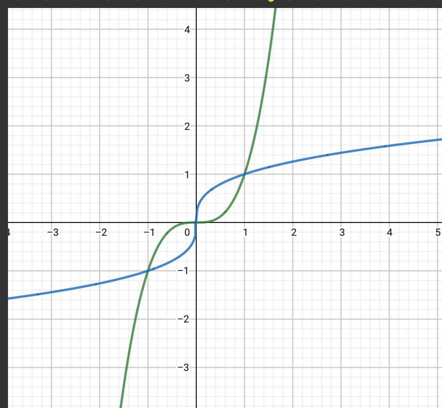
$$\left(\sqrt[N]{y}\right)^N = y \quad \forall y \in \mathbb{R}$$

$y = x^2$



$y = \sqrt{x} = x^{1/2}$

$y = x^3$



$y = \sqrt[3]{x} = x^{1/3}$

DEFINITION

LET m $N \in \mathbb{R}$ WITH NO COMMON FACTOR

WE DEFINE

N EVEN

$x \geq 0 \quad x^{m/n} := \left(\sqrt[n]{x}\right)^m = \sqrt[n]{x^m}$

$x > 0 \quad x^{-m/n} := \frac{1}{x^{m/n}}$

N ODD

$x \in \mathbb{R} \quad x^{m/n} := \left(\sqrt[n]{x}\right)^m = \sqrt[n]{x^m}$

$x \neq 0 \quad x^{-m/n} := \frac{1}{x^{m/n}}$

$x > 0 \quad x^0 = 1$

REMARK

a^q IS WELL DEFINED IN THE FOLLOWING CASE

- $a \in \mathbb{R}, \ a > 0 \quad \forall q \in \mathbb{Q}$
- $a \in \mathbb{R}, \ a \geq 0 \quad \forall q \in \mathbb{Q}, q > 0$
- $a \in \mathbb{R}, \ a < 0$ ONLY IF $q = \pm m/n$

  $m, n \in \mathbb{N}$ WITH NO COMMON FACTORS N ODD

$0^q = 0 \quad \forall q \in \mathbb{Q},\ q > 0$

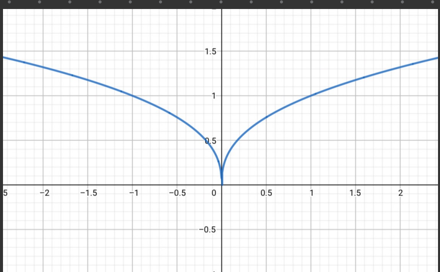
$1^q = 1 \quad \forall q \in \mathbb{Q}$

- $m, n \in \mathbb{N}$ WITH NO COMMON FACTOR N ODD

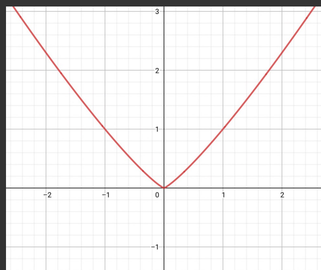
$\mathbb{R} \ni a \to a^{m/n}$ EVEN IF m EVEN

$0 \neq a \to a^{-m/n}$ ODD IF m ODD

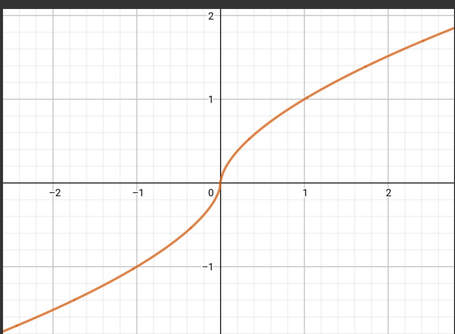
$y = x^{2/5}$



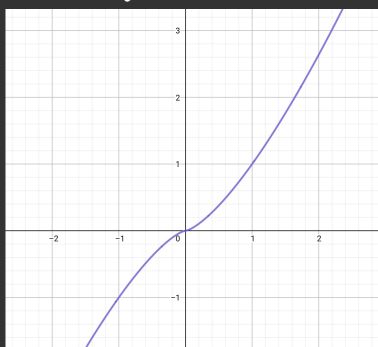
$y = x^{6/5}$



$y = x^{3/5}$



$y = x^{7/5}$



PROPERTIES OF THE POWER FUNCTION $a, b \in \mathbb{R}, a, b > 0$

$p, q \in \mathbb{Q}$ THEN

$a^p > 0$ AND $a^0 = 1$

$a^p \cdot a^q = a^{p+q}$ $\quad (a^p)^q = a^{pq}$ $\quad (a \cdot b)^p = a^p \cdot b^p$

$a^p = \left(\frac{1}{a}\right)^{-p}$

- $0 \leq a < b$ AND $p > 0 \Rightarrow 0 \leq a^p < b^p$
- $0 < a < b$ AND $p < 0 \Rightarrow a^p > b^p$
- $a > 1$ AND $p < q \Rightarrow a^p < a^q$

PROOF

$a^p < a^q \Leftrightarrow 1 < \frac{a^q}{a^p} = a^{q-p}$

BUT $q - p > 0$ AND $1 = 1^{q-p} < a^{q-p}$

$0 < a < 1$ AND $p < q \Rightarrow a^p > a^q$

QUESTION

LET $a \in \mathbb{R}$, $a > 0$ LET $b \in \mathbb{R}$

HOW TO DEFINE a^b?

DEFINITION

$a \in \mathbb{R}$, $a > 1$ $\forall b \in \mathbb{R}$ DEFINE

$$a^b := \sup \{ a^q : q \in \mathbb{Q}, q \leq b \}$$

ONE CAN SHOW THAT

$$0 < a^b < +\infty$$

- $a = 1$ $\forall b \in \mathbb{R}$ $1^b := 1$
- $0 < a < 1$ $\forall b \in \mathbb{R}$ DEFINE

$$a^b := \frac{1}{(1/a)^b}$$

- $0^b := 0$ $\forall b \in \mathbb{R}$, $b > 0$

REMARK

$a < 0$ $b \in \mathbb{R}$ WE CAN NOT DEFINE a^b **EXCEPT** IN THE CASE ALREADY DISCUSSED

($a^{\pm m/n}$ $m, n \in \mathbb{N}$ WITH NO COMMON FACTOR n ODD)

$(-2)^{\pi}$ **NOT** DEFINED $(-2)^0$ **NOT** DEFINED

- $a=0$ $\quad 0^b$ IS NOT DEFINE $\forall b\leq 0$

IN PARTICULAR 0^0 NOT DEFINED

RECALL THAT $a^0=1 \ \forall a>0 \quad 0^b=0 \ \forall b>0$

IMPORTANT REMARK

BY CONVENTION, USUALLY WE WRITE

$$a^0 := 1 \quad \forall a\in\mathbb{R}$$

$$\begin{array}{cccc} 1 & x & x^2 & x^3 \\ \| & \| & \| & \| \\ x^0 & x^1 & x^2 & x^3 \end{array}$$

IMPORTANT REMARK

PROPERTIES OF POWER FUNCTION HOLD

$\forall p,q\in\mathbb{R}$

DEFINITION

$b\in\mathbb{R} \quad f_b:(0,+\infty)\to\mathbb{R}$

$f_b(x)=x^b \quad \forall x\in(0,+\infty)$ POWER FUNCTION WITH EXPONENT b

- $b \in \mathbb{R} \quad b > 0 \quad f_b : [0, +\infty) \to \mathbb{R}$

  $f_b(x) = x^b \quad \forall x \in [0, +\infty)$ POWER FUNCTION WITH EXPONENT b

  $0^b = 0$

- $a \in \mathbb{R}, a > 0 \quad \exp_a : \mathbb{R} \to \mathbb{R}$

  $\exp_a(x) = a^x \quad \forall x \in \mathbb{R}$ EXPONENTIAL FUNCTION WITH BASE $a \in \mathbb{R}, a > 0$

PROPERTIES

- $b > 0 \quad [0, +\infty) \ni x \to x^b$ STRICTLY INCREASING

  $f_b([0, +\infty)) = [0, +\infty)$ ITS INVERSE $f_{1/b}$

- $b = 0 \quad (0, +\infty) \ni x \longrightarrow x^0 = 1$ CONSTANT
- $b < 0 \quad (0, +\infty) \ni x \longrightarrow x^b$ STRICTLY INCREASING

  $f_b((0, +\infty)) = (0, +\infty)$ ITS INVERSE $f_{1/b}$

- $a > 1$ $\quad \mathbb{R} \ni x \to a^x$ POSITIVE AND STRICTLY INCREASING

$$\exp_a(\mathbb{R}) = (0, +\infty)$$

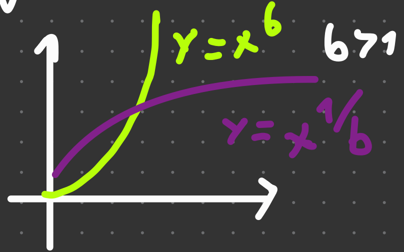
- $a = 1$ $\quad \mathbb{R} \ni x \to 1^x = 1$ CONSTANT

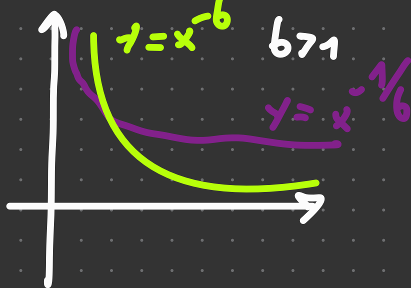
- $0 < a < 1$ $\quad \mathbb{R} \ni x \to a^x$ POSITIVE AND STRICTLY DECREASING

$$\exp_a(\mathbb{R}) = (0, +\infty)$$

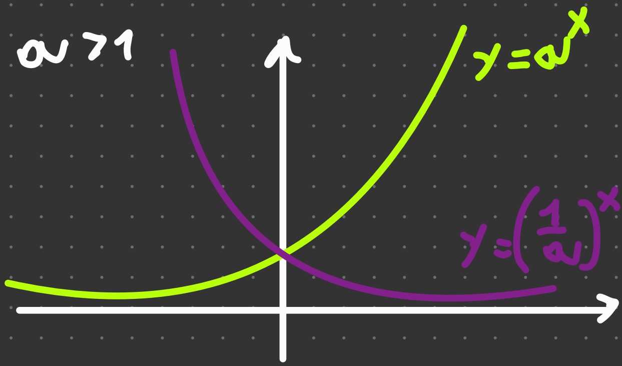
IN PARTICULAR, $\exp_a : \mathbb{R} \to (0, +\infty)$ IS BIJECTIVE

$\forall a > 0$ $\quad a \neq 1$





$a > 1$



THE LOGARITMIC FUNCTION

DEFINITION

LET $a \in \mathbb{R}$ $a > 0$ $a \neq 1$

LET $c \in \mathbb{R}$ $c > 0$

$\exists$ A UNIQUE $b \in \mathbb{R}$ SUCH THAT $a^b = c$ WE DENOTE

$b = \log_a c$ LOGARITHM IN BASE a OF c

IN FACT $\exp_a : \mathbb{R} \to (0, +\infty)$ IS BIJECTIVE AND

$\log_a = \exp_a^{-1} : (0, +\infty) \longrightarrow \mathbb{R}$ LOGARITMIC FUNCTION WITH BASE a

IN PARTICULAR

$\forall c > 0$ $\exp_a(\log_a c) = c$ $a^{\log_a c} = c$

$\forall x \in \mathbb{R}$ $\log_a(\exp_a(x)) = x$ $\log_a(a^x) = x$

EXAMPLE

$\log_2 8 = 3$ $(2^3 = 8)$ $\log_2 \frac{1}{8} = -3$ $\left(2^{-3} = \frac{1}{8}\right)$

$\nexists \log_2(-8)$ $(2^b > 0 \ \forall b \in \mathbb{R})$

$\nexists \log_2(0)$

REMARK

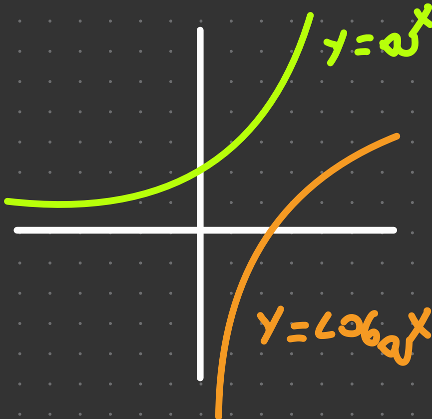
- $a > 1$ $\quad (0,+\infty) \ni x \longrightarrow \log_a x$ STRICTLY INCREASING

- $0 < a < 1$

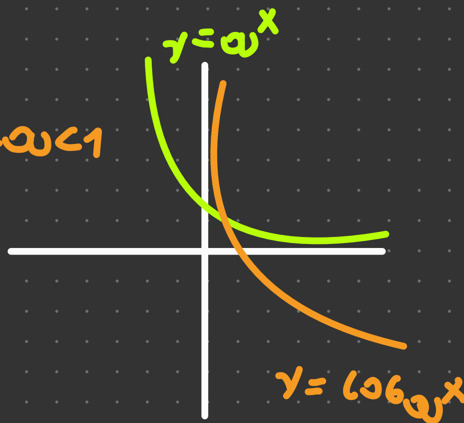
  $(0,+\infty) \ni x \to \log_a x$ STRICTLY DECREASING

IN BOTH CASES:

$$\log_a : (0,+\infty) \to \mathbb{R} \text{ IS BIJECTIVE}$$



$0<a<1$



FILE B

PROPERTIES OF LOGARITHMS

$a \in \mathbb{R}$, $a > 0$ $a \neq 1$

$x, y \in \mathbb{R}$

- $\log_a 1 = 0$

PROOF

$b = \log_a 1$ THEN $a^b = 1$ BUT $a^0 = 1$ SO $b = 0$

- $\log_a\left(\frac{1}{x}\right) = -\log_a(x)$ $\forall x > 0$

PROOF

$b = \log(x)$, THAT IS, $a^b = x$. SO $\frac{1}{a^b} = \frac{1}{x}$

BUT $\frac{1}{a^b} = a^{-b} = \frac{1}{x}$ SO $-b = \log_a\left(\frac{1}{x}\right)$

- $\log_a(xy) = \log_a x + \log_a y$ $\forall x, y > 0$

PROOF

$b = \log_a(xy)$ $x = a^{\log_a x}$ $y = a^{\log_a y}$

$xy = a^{\log_a x} \cdot a^{\log_a y} = a^{\log_a x + \log_a y}$

- $\log_a\left(\frac{x}{y}\right) = \log_a(x) - \log_a(y) \quad \forall x, y > 0$
- $\log_a(x^y) = y \log_a(x) \qquad x > 0, y \in \mathbb{R}$

PROOF

$$x = a^{\log_a x} \qquad x^y = \left(a^{\log_a x}\right)^y = a^{y \log_a x}$$

- $\log_{\frac{1}{a}}(x) = \log_a\left(\frac{1}{x}\right) = -\log_a(x)$

PROOF

$$b = \log_a x \qquad a^b = x$$

$$\frac{1}{a^b} = \left(\frac{1}{a}\right)^b = \frac{1}{x} \Rightarrow b = \log_{\frac{1}{a}}\left(\frac{1}{x}\right)$$

$$\Rightarrow \log_{\frac{1}{a}}(x) = -b = -\log_a x$$

EXAMPLE

$$\log_2(2^3) = 3$$

$$\log_2(4^3) = 3 \cdot \log_2 4 = 3 \cdot 2 = 6$$

$$\log_2(2^3 \cdot 2^5) = \log_2(2^3) + \log_2(2^5) = 3 + 5 = 8$$

$$\log_2\left(\frac{1}{2^3}\right) = -\log_2(2^3) = -3$$

$$\log_{\frac{1}{2}}(2^5) = \log_{\frac{1}{2}}\left(\frac{1}{\left(\frac{1}{2}\right)^5}\right) = \log_{\frac{1}{2}}\left(\left(\frac{1}{2}\right)^{-5}\right) = -5$$

- $a > 1$ $\quad 0 < x < y \Rightarrow \log_a x < \log_a y$
- $0 < a < 1$ $\quad 0 < x < y \Rightarrow \log_a x > \log_a y$

IN PARTICULAR

$$\log_a x = 0 \Leftrightarrow x = 1$$

$a > 1$

$\log_a x < 0 \Leftrightarrow 0 < x < 1 \qquad \log_a x > 0 \Leftrightarrow x > 1$

$0 < a < 1$

$\log_a x < 0 \Leftrightarrow x > 1 \qquad \log_a x > 0 \Leftrightarrow 0 < x < 1$

FORMULA FOR THE CHANGE OF BASE

$a, b > 0$

$a, b \neq 1$

$b = a^{\log_a b}$ $(b \neq 1 \Rightarrow \log_a b \neq 0)$

$b^x = (a^{\log_a b})^x = a^{x \log_a b}$ $\forall x \in \mathbb{R}$

LET $c > 0$

$c = a^{\log_a c} = b^{\log_b c} = a^{(\log_b c)(\log_a b)}$

$\Rightarrow \log_a c = (\log_b c)(\log_a b)$

THAT IS

$$\log_b c = \frac{\log_a c}{\log_a b}$$

EXAMPLE:

$$\log_4(2^9) = \frac{\log_2(2^9)}{\log_2(4)} = \frac{9}{2}$$

$a = 2$

$b = 4$

$c = 2^9$

IMPORTANT

WE ONLY USE THE EXPONENTIAL FUNCTION WITH BASE e (NAPIER NUMBER e IS IRRATIONAL NUMBER $e = 2.78\ldots$) OR EXPONENTIAL FUNCTION

$$\exp : \mathbb{R} \to (0, +\infty)$$

$$\exp(x) = e^x \quad \forall x \in \mathbb{R}$$

EXPONENTIAL FUNCTION

ITS INVERSE, LOGARITHM WITH BASE e OR NATURAL LOGARITHM

$$\log : (0, +\infty) \to \mathbb{R}$$

LOGARITHMIC FUNCTION

$$\log(x) = \log_e(x) \quad \forall x \in (0, +\infty)$$

SOMETIMES THE NOTATION

$\ln(x) = \log(x)$ IS USED

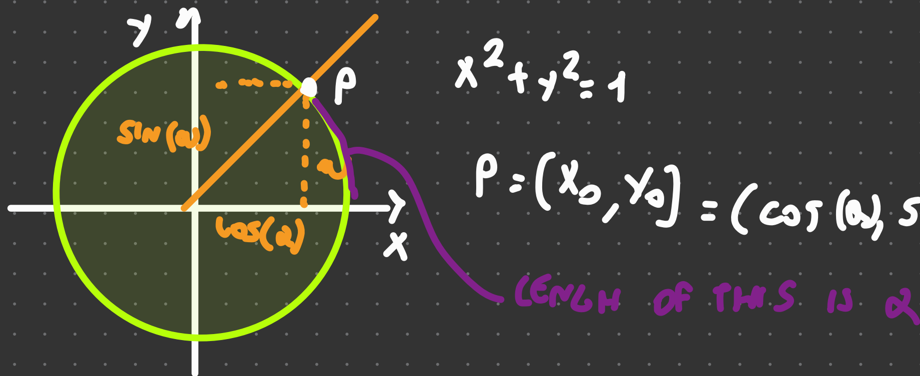
SO $\log(10^3) \neq 3$! $\log(10^3) = 3\log_e 10$

$n \in \mathbb{N} \quad a_n^{b_n} \quad a_n > 0 \quad b_n \in \mathbb{R}$

$a_n = e^{\cos a_n}$

$a_n^{b_n} = \left(e^{\cos a_n}\right)^{b_n} = e^{b_n \cdot \cos a_n}$

TRIGONOMETRIC FUNCTIONS



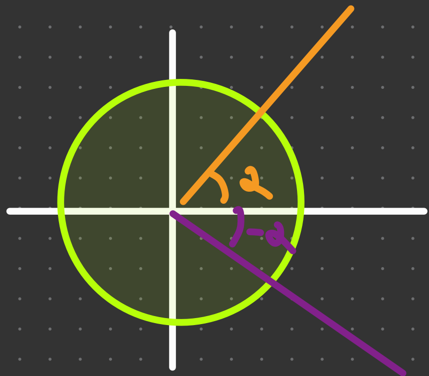
$x^2 + y^2 = 1$

$P = (x_0, y_0) = (\cos(\alpha), \sin(\alpha))$

α ANGLE IN RADIANTS WITH RESPECT TO THE POSITIVE X-AXIS ROTATING IN THE COUNTER CLOCKWISE DIRECTORY

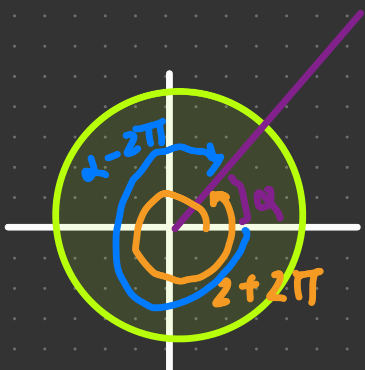
FULL CIRCLE 360° IS 2π RADIANTS

2π LENGTH OF THE CIRCUNFERENCE



$\alpha > 0$ THE ANGLE $-\alpha$ GIVEN BY ROTATING OF AN ANGLE α IN THE CLOCKWISE DIRECTION

$$(\cos(-\alpha), \sin(-\alpha)) = (\cos(\alpha), -\sin(\alpha))$$



$\alpha > 0$

$\alpha + 2k\pi$ $k \in \mathbb{Z}$ GIVE THE SAME SINE

$\forall \alpha \in \mathbb{R}$ LET $P = (x_0, y_0)$ THE POINT ON THE UNIT CIRCLE DETERMINATED BY ANGLE α THEN

$\cos(\alpha) = x_0$ COSINE FUNCTION

$\sin(\alpha) = x_0$ SINE FUNCTION

PROPERTY OF TRIGONOMETRIC FUNCTIONS

$$\cos^2(\alpha) + \sin^2(\alpha) = 1 \quad \forall \alpha \in \mathbb{R}$$

HENCE

$$-1 \leq \cos\alpha \leq 1 \quad \text{AND} \quad -1 \leq \sin\alpha \leq 1$$

$\cos(-\alpha) = \cos(\alpha)$ COS IS EVEN

$\sin(-\alpha) = -\sin(\alpha)$ SIN IS ODD

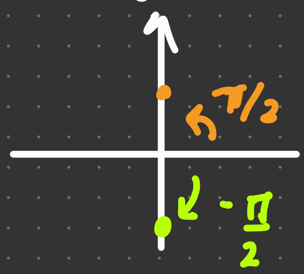
SIN AND COS ARE PERIODIC WITH PERIOD 2π

$$\cos(\alpha + 2\pi) = \cos(\alpha), \quad \sin(\alpha+2\pi) = \sin(\alpha)$$

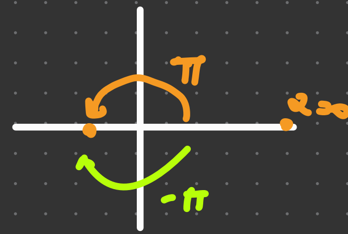
SO

$$\cos(\alpha + 2k\pi) = \cos(\alpha), \quad \sin(\alpha + 2k\pi) = \sin(\alpha) \quad \forall k \in \mathbb{Z}$$

$$\cos\alpha = 0 \iff \alpha = \frac{\pi}{2} + k\pi, \quad k \in \mathbb{Z}$$



$\sin \alpha = 0 \iff \alpha = k\pi \quad k \in \mathbb{Z}$



THE TANGENT FUNCTION

$A = \mathbb{R} \setminus \left\{ \frac{\pi}{2} + k\pi,\ k \in \mathbb{Z} \right\}$ WE DEFINE

$$\tan(\alpha) = \frac{\sin(\alpha)}{\cos(\alpha)} \quad \forall \alpha \in A$$ TANGENT FUNCTION

PROPERTIES OF TANGENT FUNCTION

$\tan(-\alpha) = -\tan(\alpha) \quad \forall \alpha \in A$ tan is ODD

TAN IS PERIODIC WITH PERIOD π

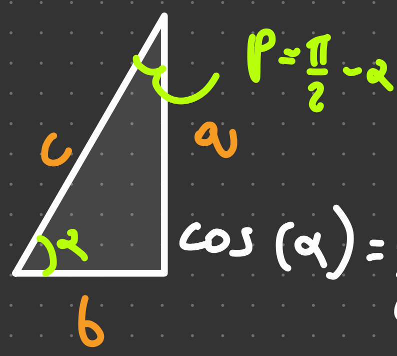
HENCE

$$\tan(\alpha + k\pi) = \tan(\alpha) \quad \forall \alpha \in A \quad \forall k \in \mathbb{Z}$$

$$\tan(\alpha) = 0 \iff \alpha = k\pi \quad k \in \mathbb{Z}$$

ANOTHER DEFINITION OF TRIGONOMETRIC FUNCTIONS

$0 < \alpha < \frac{\pi}{2}$



$\cos(\alpha) = \frac{b}{c}$, $\sin(\alpha) = \frac{a}{c}$, $\tan(\alpha) = \frac{a}{b}$

$\beta = \frac{\pi}{2} - \alpha$

$\cos\left(\frac{\pi}{2} - \alpha\right) = \frac{a}{c} = \sin(\alpha)$

$\sin\left(\frac{\pi}{2} - \alpha\right) = \frac{b}{c} = \cos(\alpha)$

ACTUALLY

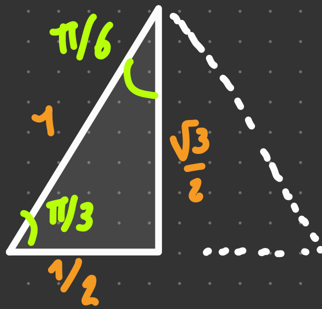
$\cos\left(\frac{\pi}{2} - \alpha\right) = \sin(\alpha)$ AND $\sin\left(\frac{\pi}{2} - \alpha\right) = \cos(\alpha)$ $\forall \alpha \in \mathbb{R}$

YOU NEED TO KNOW THE VALUE OF SIN, COS, TAN

FOR THE FOLLOWING VALUES OF α

$0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$

$\frac{\pi}{3}$, $\frac{\pi}{6}$



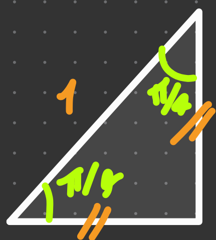
EQUILATERAL TRIANGLE

$$\cos\left(\frac{\pi}{3}\right) = \frac{1}{2} \qquad \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

$$\parallel \qquad\qquad \parallel$$

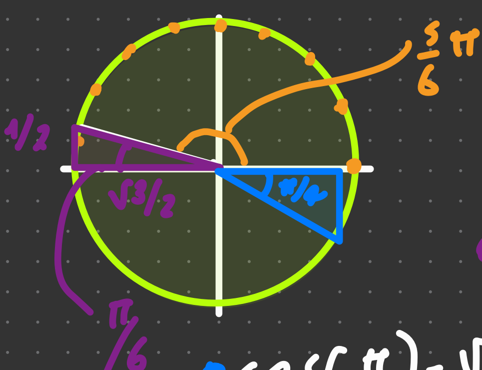
$$\sin\left(\frac{\pi}{6}\right) = \frac{1}{2} \qquad \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$$

$\pi/4$



ISOSCELES TRIANGLE

$$\cos\left(\frac{\pi}{4}\right) = \sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$$



- $\cos\left(\frac{5}{6}\pi\right) = -\frac{\sqrt{3}}{2}$
- $\sin\left(\frac{5}{6}\pi\right) = \frac{1}{2}$
- $\cos\left(-\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$ $\quad \sin\left(-\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

OTHER USEFUL FORMULAS: SUM AND COSEQUENCES

- $\sin(\alpha \pm \beta) = \sin(\alpha)\cos(\beta) \pm \sin(\beta)\cos(\alpha)$
- $\cos(\alpha \pm \beta) = \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta)$

TAKING $\alpha = \beta$

- $\sin(2\alpha) = 2\sin(\alpha)\cos(\alpha)$
- $\cos(2\alpha) = \cos^2\alpha - \sin^2\alpha = 1 - 2\sin^2\alpha = 2\cos^2\alpha - 1$

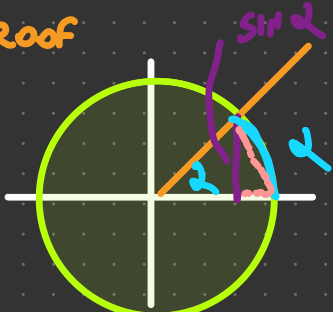
$(\cos^2\alpha + \sin^2\alpha = 1)$

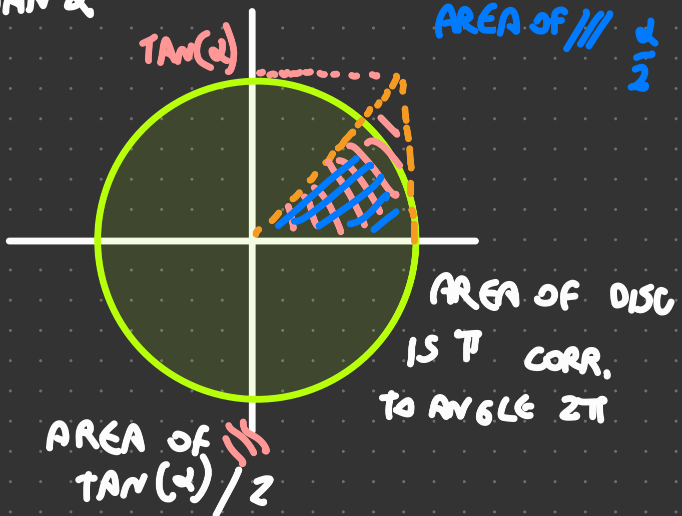
PROPOSITION:

$0 < \alpha < \frac{\pi}{2}$ THEN

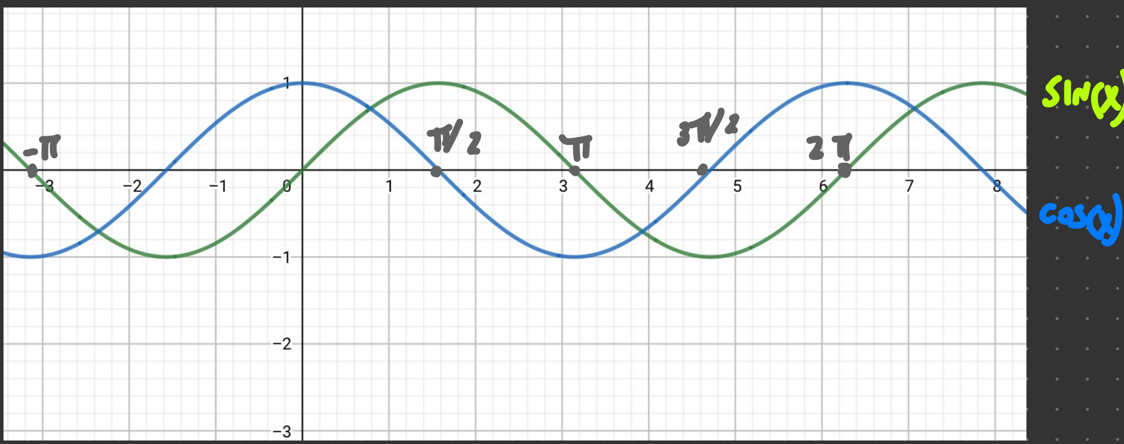
$0 < \sin\alpha < \alpha < \tan\alpha$

PROOF





GRAPHS

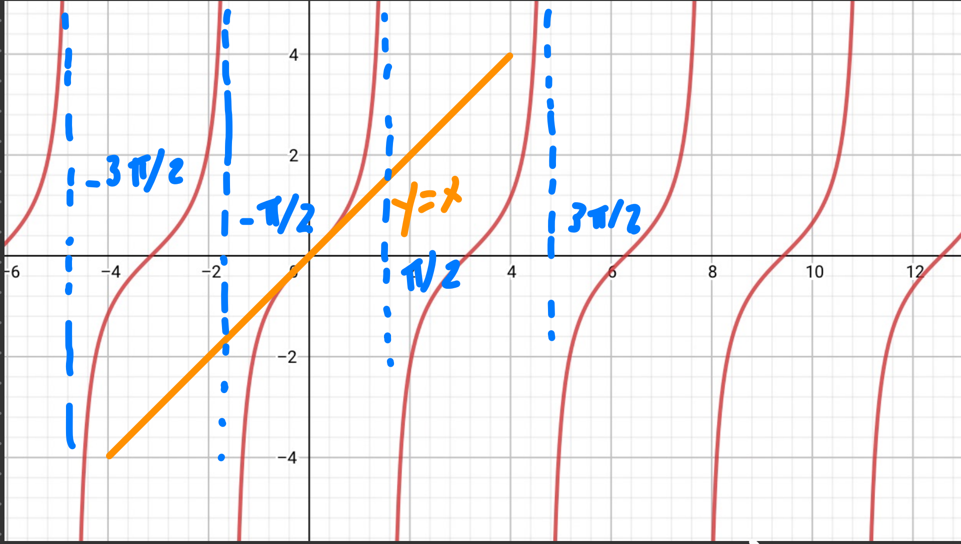


$$\sin\left(\frac{\pi}{2}-\alpha\right)=\cos\alpha$$

$$\parallel$$

$$-\sin\left(\alpha-\frac{\pi}{2}\right)$$

$$\sin(\alpha)=\cos\left(\frac{\pi}{2}-\alpha\right)=\cos\left(\alpha-\frac{\pi}{2}\right)$$



TAN(x)

SUM, PRODUCT, QUOTIENT OF FUNCTIONS

A ARBITRARY SET, $A \neq \emptyset$ $f, g : A \to \mathbb{R}$, $\alpha \in \mathbb{R}$

$f = g \iff f(x) = g(x) \quad \forall x \in A$

$f + g : A \to \mathbb{R} \quad (f+g)(x) := f(x) + g(x) \quad \forall x \in A$

$\alpha f : A \to \mathbb{R} \quad (\alpha f)(x) = \alpha f(x) \quad \forall x \in A$

IF $\alpha = -1$

$-1 f = -f : A \to \mathbb{R} \quad (-f)(x) := -f(x) \quad \forall x \in A$

$fg : A \to \mathbb{R} \quad (fg)(x) := f(x)g(x) \quad \forall x \in A$

$A_1 = \{x \in A : g(x) \neq 0\}$

$\frac{f}{g} : A_1 \to \mathbb{R} \quad \left(\frac{f}{g}\right)(x) := \frac{f(x)}{g(x)} \quad \forall x \in A$

$\text{MAX}\{f, g\}(x) := \text{MAX}\{f(x), g(x)\} \quad \text{MIN}\{f, g\}$

$|f|(x) = |f(x)| \; : \; f^2(x) = (f(x))^2$

$(\cos(f))(x) = \cos(f(x)) = (\cos f)(x)$