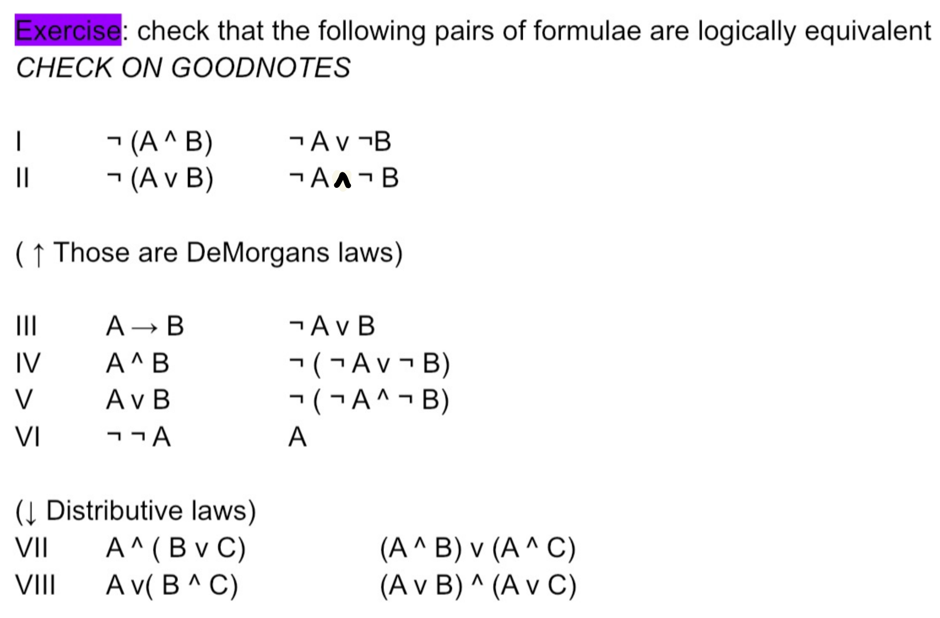
FROM COMPUTATIONAL LOGIC - LESSON 3 1/OTT/2021



Exercise: check that the following pairs of formulae are logically equivalent

CHECK ON GOODNOTES

I ¬ (A ^ B) ¬ A v ¬B

II ¬ (A v B) ¬ A ∧ ¬ B

(↑ Those are DeMorgans laws)

III A → B ¬ A v B

IV A ^ B ¬ (¬ A v ¬ B)

V A v B ¬ (¬ A ^ ¬ B)

VI ¬ ¬ A A

(↓ Distributive laws)

VII A ^ (B v C) (A ^ B) v (A ^ C)

VIII A v(B ^ C) (A v B) ^ (A v C)

✓ I. $\neg(A \land B)$ $\neg A \lor \neg B$

A	B	$A \land B$	$\neg(A \land B)$	$\neg A$	$\neg B$	$\neg A \lor \neg B$
1	1	1	0	0	0	0
1	0	0	1	0	1	1
0	1	0	1	1	0	1
0	0	0	1	1	1	1

✓ II. $\neg(A \vee B)$ $\neg A \wedge \neg B$

A	B	$A \vee B$	$\neg(A \vee B)$	$\neg A$	$\neg B$	$\neg A \wedge \neg B$
1	1	1	0	0	0	0
1	0	1	0	0	1	0
0	1	1	0	1	0	0
0	0	0	1	1	1	1

✓ III. $A \to B$ $\neg A \vee B$

A	B	$A \to B$	$\neg A$	$\neg A \vee B$
1	1	1	0	1
1	0	0	0	0
0	1	1	1	1
0	0	1	1	1

✓ IV. $A \cap B$ $\neg(\neg A \vee \neg B)$

A	B	$A \wedge B$	$\neg A$	$\neg B$	$\neg A \vee \neg B$	$\neg(\neg A \vee \neg B)$
1	1	1	0	0	0	1
1	0	0	0	1	1	0
0	1	0	1	0	1	0
0	0	0	1	1	1	0

√ V. $A \vee B$ $\neg(\neg A \wedge \neg B)$

A	B	$A \vee B$	$\neg A$	$\neg B$	$\neg A \cap \neg B$	$\neg(\neg A \cap \neg B)$
1	1	1	0	0	0	1
1	0	1	0	1	0	1
0	1	1	1	0	0	1
0	0	0	1	1	1	0

√ VI. $\neg\neg A$ A

A	$\neg A$	$\neg\neg A$
1	0	1
0	1	0

√ VII. $A \wedge (B \vee C)$ $(A \wedge B) \vee (A \wedge C)$

A	B	C	$B \vee C$	$A \wedge (B \vee C)$	$A \wedge B$	$A \wedge C$	$(A \wedge B) \vee (A \wedge C)$
1	1	1	1	1	1	1	1
1	1	0	1	1	1	0	1
1	0	1	1	1	0	1	1
1	0	0	0	0	0	0	0
0	1	1	1	0	0	0	0
0	0	1	1	0	0	0	0
0	1	0	1	0	0	0	0
0	0	0	0	0	0	0	0

✓ VIII. $A \vee (B \wedge C)$ $(A \vee B) \wedge (A \vee C)$

A	B	C	$B \wedge C$	$A \vee (B \wedge C)$	$(A \vee B)$	$(A \vee C)$	$(A \vee B) \wedge (A \vee C)$
1	1	1	1	1	1	1	1
1	1	0	0	1	1	1	1
1	0	1	0	1	1	1	1
0	1	1	1	1	1	1	1
1	0	0	0	1	1	1	1
0	0	1	0	0	0	1	0
0	1	0	0	0	1	0	0
0	0	0	0	0	0	0	0

$(p \lor q) \to r) \to ((p \to r) \land (q \to r))$ TAUTOLOG

$\neg[(p \lor q) \to r) \to ((p \to r) \land (q \to r))]$

$(p \lor q) \to r) \land \neg((p \to r) \land (q \to r))$

$(\neg(p \lor q) \lor r) \land \neg((p \to r) \land (q \to r))$

$(\neg(p \lor q) \lor r) \land (\neg(p \to r) \lor \neg(q \to r))$

$((\neg p \land \neg q) \lor r) \land (\neg(p \to r) \lor \neg(q \to r))$

$((\neg p \land \neg q) \lor r) \land ((p \land \neg r) \lor \neg(q \to r))$

$((\neg p \land \neg q) \lor r) \land ((p \land \neg r) \lor (q \land \neg r))$

DISTRIBUTIVE LAW

$\underset{\checkmark}{(\neg p \lor r)} \land \underset{\checkmark}{(\neg q \lor r)} \land ((\underbrace{p}_{A} \land \underbrace{\neg r}_{B}) \lor \underbrace{(q \land \neg r)}_{C}$ $(A \land B) \lor C \rightsquigarrow (A \lor C) \land (B \lor C)$

$(\neg p \lor r) \land (\neg q \lor r) \land (p \lor (q \land \neg r)) \land (\neg r \lor (q \land \neg r)))$

CAN BE REMOVED BECAUSE $\land$ IS INSIDE

$(\neg p \lor r) \land (\neg p \lor r) \land (p \lor (q \land \neg r)) \land (\neg r \lor (q \land \neg r)))$

$A \lor (B \land C) \rightsquigarrow (A \lor B) \land (A \lor C)$

$\underset{\checkmark}{(\neg p \lor r)} \land \underset{\checkmark}{(\neg q \lor r)} \land \underset{\checkmark}{(p \lor q)} \land \underset{\checkmark}{(p \lor \neg r)} \land (\neg r \lor (q \land \neg r))$

$$A \lor (B \land C) \rightsquigarrow (A \lor B) \land (A \lor C)$$

$$\underset{\checkmark}{(\neg p \lor r)} \land \underset{\checkmark}{(\neg q \lor r)} \land \underset{\checkmark}{(p \lor q)} \land \underset{\checkmark}{(p \lor \neg r)} \land \underset{\checkmark}{(\neg r \lor q)} \land \underset{\checkmark}{(\neg r \lor \neg r)}$$

1) $\cancel{p \Rightarrow r}$ BECAUSE 7
2) $\cancel{q \Rightarrow r}$ BECAUSE 8
3) $\Rightarrow p, q$
4) $\cancel{r \Rightarrow p}$ BECAUSE 6
5) $\cancel{r \Rightarrow q}$
6) $r \Rightarrow$
7) $p \Rightarrow$
8) $q \Rightarrow$
9) $\Rightarrow p$

$$\frac{p \Rightarrow r \qquad r \Rightarrow}{p \Rightarrow}$$

2 AND 6

$$\frac{q \Rightarrow \qquad r \Rightarrow}{q \Rightarrow}$$

3 AND 8

$$\frac{\Rightarrow p, q \qquad q \Rightarrow}{\Rightarrow p}$$

9 AND 7

$$\frac{p \Rightarrow \qquad \Rightarrow p}{\Rightarrow}$$

UNSAT

$\Downarrow$

A IS A TAUTOLOGY

CHECK TAUTOLOGY

$(p \to (q \land r)) \to ((p \to q) \land (p \to r))$

$(p \to q) \land (p \to r)) \to (p \to (q \land r))$

$(p \to q) \to q) \to p \lor q$

$p \lor q \to ((p \to q) \to q)$

$\Gamma \Rightarrow$ $\Gamma \Rightarrow$ HORN CLAUSES

RESTRICT RESOLUTION TO UNIT RESOLUTION

$\downarrow$

WHEN APPLYING RESOLUTION ONE CLAUSE SHOULD HAVE ONLY ONE LITERAL

$p, q \Rightarrow r$

~~$p \Rightarrow q$~~

$\Rightarrow p$

$r \Rightarrow$

$\Rightarrow q$

CAN REMOVE

$$\frac{\Rightarrow p \qquad p \Rightarrow q}{\Rightarrow q}$$

THE RESULT SUBSUME THE PREMISESES

UNIT RESOLUTION CAN BE FOLLOWED BY SUBSUMPTION

ONLY LINEARLY MANY UNIT RESOLUTION CAN BE DONE!

REFUTATIONAL COMPLETENESS HOLDS FOR UNIT RESOLUTION FOR HORN CLAUSES

HENCE SATISFIABILITY OF HORN CLAUSES CAN BE DECIDED QUICKLY (POLYNOMIAL TIME)

$$(p \to (q \land r)) \to ((p \to q) \land (p \to r))$$

$$\neg\left[(p \to (q \land r)) \to ((p \to q) \land (p \to r))\right]$$

$A \to B \longmapsto \neg A \lor B$

$\neg(A \to B) \longmapsto A \land \neg B$

$$(p \to (q \land r)) \land \neg((p \to q) \land (p \to r))$$

$$(p \to (q \land r)) \land (\neg(p \to q) \lor \neg(p \to r))$$

$$(p \to (q \land r)) \land ((p \land \neg q) \lor (p \land \neg r))$$

$$(\neg p \lor (q \land r)) \land ((p \land \neg q) \lor (p \land \neg r))$$

$$(\neg p \lor q) \land (\neg p \lor r) \land ((p \land \neg q) \lor (p \land \neg r))$$

$$(\neg p \lor q) \land (\neg p \lor r) \land (((p \land \neg q) \lor p) \land ((p \land \neg q) \lor \neg r))$$

$$(\neg p \lor q) \land (\neg p \lor r) \land (((p \lor p) \land (\neg q \lor p) \land (p \lor \neg r) \land (\neg r \lor \neg q)))$$

$$(\neg p \lor q) \land (\neg p \lor r) \land (p \lor p) \land (\neg q \lor p) \land (p \lor \neg r) \land (\neg r \lor \neg q)$$

$p \Rightarrow q$

$p \Rightarrow r$

$\Rightarrow p$

$q \Rightarrow p$

$r \Rightarrow p$

$r, q \Rightarrow$

$\Rightarrow q \quad \Rightarrow p$

$$\frac{p \Rightarrow q \qquad \Rightarrow p}{\Rightarrow q}$$

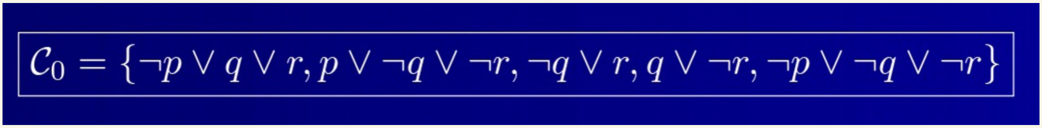
$$\frac{q \Rightarrow p \qquad \Rightarrow q}{\Rightarrow p}$$

$$\frac{r, q \Rightarrow \qquad \Rightarrow q}{r \Rightarrow}$$

$$\frac{p \Rightarrow r \qquad r \Rightarrow}{p \Rightarrow}$$

$$\frac{p \Rightarrow \qquad \Rightarrow p}{\Rightarrow}$$

EXAMPLE 2



$$\mathcal{C}_0 = \{\neg p \vee q \vee r, p \vee \neg q \vee \neg r, \neg q \vee r, q \vee \neg r, \neg p \vee \neg q \vee \neg r\}$$

$\Rightarrow a, r$

$\Rightarrow b, c$

$a, p \Rightarrow$

$a, q \Rightarrow$

$b \Rightarrow p$

$b, z \Rightarrow$

$c \Rightarrow q$

$c, x \Rightarrow$

$\Rightarrow c P$

$p \Rightarrow q$

$q \Rightarrow x$

$\Rightarrow q, b$

$\Rightarrow p, q$

$\Rightarrow c, q$

$x \Rightarrow q$

$$\frac{\Rightarrow b, c \qquad b \Rightarrow p}{p \Rightarrow c \ P}$$

$$\frac{c \Rightarrow q \qquad \Rightarrow c p}{p \Rightarrow q}$$

$$\frac{a, q \Rightarrow \qquad \Rightarrow a, r}{q \Rightarrow r}$$

$$\frac{c \Rightarrow q \qquad \Rightarrow b, c}{\Rightarrow q, b}$$

$$\frac{b \Rightarrow p \qquad \Rightarrow q, b}{\Rightarrow p, q}$$

$$\frac{p \Rightarrow c \qquad \Rightarrow p, q}{\Rightarrow c, q}$$

$$\frac{c, x \Rightarrow \qquad \Rightarrow c, q}{x \Rightarrow q}$$

$\Rightarrow$

22/10/21

$$C_0 \equiv \{ \overset{\textcircled{1}}{P_1 \vee P_2},\ \overset{\textcircled{2}}{\neg P_1 \vee P_2},\ \overset{\textcircled{3}}{P_1 \vee \neg P_2},\ \overset{\textcircled{4}}{\neg P_1 \vee \neg P_2} \}$$

$$(\emptyset | C_0 | *) \mapsto (P_1^d | C_0 | *)$$

START PROPAGATING SINCE P1 IS TRUE

THE ② P2 NEED TO BE

DECIDED

$$\to (P_1^d, P_2^{\textcircled{2}} | C_0 | *) \mapsto (P_1^d, P_2^{\textcircled{2}} | \neg P_1 \vee P_2)$$

DECIDED IN THE WRONG WAY

$$\frac{\neg P_1 \vee \neg \cancel{P_2} \qquad \neg P_1 \vee \cancel{P_2}}{\neg P_1}$$

← REPLACE CONFLICT CLAUSE W/ RESOLVE

$$\to (P_1^d, P_2^{\textcircled{2}} | C_0 | \neg P_1) \longmapsto (\neg P_1^{\textcircled{5}} | C_0, \overset{\textcircled{5}}{\neg P_1} | *)$$

USE BACKJUMPING

$$\to (\neg P_1^{\textcircled{5}}, \neg P_2^{\textcircled{3}} | C_0, \neg P_1 | *) \xrightarrow{\text{CONFLICT STATE}} (\underbrace{\neg P_1^{\textcircled{5}}, \neg P_2^{\textcircled{3}}}_{} | C_0\ \neg P_1 | P_1 \vee P_2)$$

BOTH PROPAGATE

ALMOST SURE TO GET EMPTY CLAUSE

$$\frac{P_1 \vee \cancel{P_2} \qquad P_1 \vee \neg \cancel{P_2}}{P_1}$$

$$\longmapsto \left(\neg \underline{\overset{\textcircled{5}}{P_1}}, \neg \overset{\textcircled{3}}{P_2} \;\middle|\; C_0, \neg \overset{\textcircled{3}}{P_1} \;\middle|\; \underline{P_1} \right)$$

RESOLUTION STEP

$$\frac{\neg \underline{P_1} \qquad \underline{P_1}}{\square}$$

$\rightarrow (\ldots\ldots \mid \square)$ ND UNSAT

$$C_0 = \{ \overset{\textcircled{1}}{\neg p_1 \vee p_2}, \overset{\textcircled{2}}{\neg p_3 \vee p_4}, \overset{\textcircled{3}}{\neg p_5 \vee \neg p_6}, \overset{\textcircled{4}}{p_6 \vee \neg p_5 \vee \neg p_2}, \overset{\textcircled{5}}{\neg p_4 \vee p_1 \vee p_5 \vee p_3} \}$$

$$(\emptyset, C_0 \mid *) \rightsquigarrow (p_1^d \mid C_0 \mid *) \xrightarrow{\text{PROPAG}} (p_1^d, \overset{\textcircled{1}}{p_2} \mid C_0 \mid *)$$

p_1 FALSE p_2 MUST BE TRUE

$$\mapsto (p_1^d, \overset{\textcircled{1}}{p_2}, p_3^d \mid C_0 \mid *) \to (p_1^d, \overset{\textcircled{1}}{p_2}, p_3^d, \overset{\textcircled{2}}{p_4} \mid C_0 \mid *)$$

$$\mapsto (p_1^d, \overset{\textcircled{1}}{p_2}, p_3^d, \overset{\textcircled{2}}{p_4}, p_5^d \mid C_0 \mid *)$$

$$\mapsto (p_1^d, \overset{\textcircled{1}}{p_2}, p_3^d, \overset{\textcircled{2}}{p_4}, p_5^d, \overset{\textcircled{3}}{\neg p_6} \mid C_0 \mid *)$$

TRUE

CONFLICT $\textcircled{4}$

$$\mapsto (p_1^d, \overset{\textcircled{1}}{p_2}, p_3^d, \overset{\textcircled{2}}{p_4}, p_5^d, \overset{\textcircled{3}}{\neg p_6} \mid C_0 \mid \overset{\textcircled{4}}{p_6 \vee \neg p_5 \vee \neg p_2})$$

MAKE A RESOLUTION STEP

$$\frac{\underset{\textcircled{3}}{\neg p_5 \vee p_6} \qquad \underset{\textcircled{4}}{p_6 \vee \neg p_5 \vee \neg p_2}}{\neg p_5 \vee \neg p_2}$$

$$\to (p_1^d, \overset{1}{p_2}, p_3^d, \overset{\textcircled{2}}{p_4}, p_5^d, \overset{\textcircled{3}}{\neg p_6} \mid C_0 \mid \neg p_5 \vee \neg p_2)$$

$$\to (p_1^d, \overset{\textcircled{1}}{p_2}, \overset{\textcircled{6}}{\neg p_5} \mid C_0, \overset{\textcircled{6}}{\neg p_5 \vee \neg p_2} \mid *)$$

$$\to (p_1^d, \overset{\textcircled{1}}{p_2}, \overset{\textcircled{6}}{\neg p_3}, p_3^d \mid C_0, \neg p_5 \vee \neg p_2 \mid *)$$

$$\to (p_1^d, \overset{\textcircled{1}}{p_2}, \overset{\textcircled{6}}{\neg p_5}, p_3^d, \overset{\textcircled{5}}{p_4} \mid C_0, \neg p_5 \vee \neg p_2 \mid *$$

$\rightarrow (p_1^{d}, p_2^{⑦}, \neg p_8^{⑤}, p_3^{d}, p_4^{⑤}, p_6^{d} \mid C_0\ \neg p_5 \vee \neg p_2 \mid *)$

SAT

EXERCISE

$\{a \vee r, b \vee c, \neg a \vee \neg p, \neg a \vee \neg q, \neg b \vee p, \neg b \vee \neg r, \neg c \vee q, \neg c \vee \neg r\}$

$\boxed{\text{UNSAT}}$

+ PAG $\boxed{126}$