$+$ $\times$

$-$ $\div$

EMANUELE DOLERA   LAST LECTURE $\boxed{2 \text{ DEC}}$

AIM: CHAPTERS 5 → 10

1 SESSION IN JAN
2 SESSON IN FEB
6 SESSION

MAIN DIFFERENCE:

URN CONTAINIG BALLS OF 2 COLORS (BLACK AND WHITE)

EXPERIMENT: PICK N BALLS AT RANDOM (W OR W/OUT REPLACEMENT)

	PROBABILITY	VS STATISTIC
ASSUMPTION	THE COMPOSITION OF THE URN IS KNOWN (KNOW HOW MANY BLACK AND HOW MANY WHITE)	THE EXPERIMENT ARE REALLY OBSERVED I KNOW A SPECIFIC OUTCOME
GOALS	GUESS THE OUTCOME OF THE EXPERIMENT (ATTACH TO EVERY OUTCOME THEIR PROBABILITY	NOW THE COMPOSITION OF THE URN IS UNKNOWN, THE GOAL IS TO GUESS THE REAL COMPOSITION OF THE URN

TWO KEY WORDS: $\boxed{\text{EXPERIMENT}}$, $\boxed{\text{GUESS}}$

EXPERIMENT ↓ RANDOM VARIABLE
GUESS ↓ PROBABILITY

BRIEF RECAP ON PROBABILITY AND RANDOM VARIABLE
PROBABILITY: CH. 1, CH. 2
RANDOM VARIABLE: CH. 3

CHAPTER 1 - RECAP

$(\Omega, \mathcal{F}, P)$

↑ PROBABILITY

- Ω = SET (NON EMPTY)
- $\mathcal{F}$ = COLLECTION OF ALL THE EVENTS THAT ARE WORTH OF CONSIDERATION ⟶ IS A COLLECTION OF SUBSET OF Ω "THE INTERESTING SET"

PROPERTY OF THE ELEMENTS OF $\mathcal{F}$

- Ω BELONGS TO $\mathcal{F}$
- IF A BELONGS "$\in$" TO $\mathcal{F}$ THEN, A^c BELONGS "$\in$" TO $\mathcal{F}$
- IF $A_1, A_2, A_3 \in \mathcal{F}$, THEN $\bigcup_{n=1}^{+\infty} A_n \in \mathcal{F}$
- IF $A \subseteq \Omega$; $P(A) \in [0,1]$ P IS A FUNCTION

INPUT $\xrightarrow{P}$ OUTPUT

EVENT ⟶ NUMBER IN $[0,1]$

AXIOMS

1) $P(\Omega) = 1$
2) $A_1, A_2, \dots \in \mathcal{F}$ WITH $A_i \cap A_j = \emptyset$ IF $i \neq j$

THEN $P\left(\bigcup_{n=1}^{+\infty} A_n\right) = \sum_{n=1}^{+\infty} P(A_n)$

$P(A_1) + P(A_2) + P(A_3) = P(\bigcup A_i)$

ANY EXPERIMENT CAN BE OF TWO TYPES

QUALITATIVE OR QUANTITATIVE

QUALITATIVE	QUANTITATIVE
FEATURES THAT CAN BE OBSERVED ABOUT SOME CHARACTERS	CAN BE MEASURED BY AN INSTRUMENT OR COUNTING SOMETHING
[COLOR OF EYES]	[TEMPERATURE]

- A DESCRETE RANDOM VARIABLE MODELIZE EITHER A QUALITATIVE CHARACTER OR SOME COUNTING
- A CONTINOUS RANDOM VARIABLE MODELIZE THE MEASUREMENT OF A PHYSICAL QUANTITY (LIKE TIME, HEIGHT, MONEY)

WHEN I PERFORM AN EXPERIMENT I HAVE TO TAKE INTO ACCOUNT A RANDOM VARIABLE $x_1, x_2, \dots x_N$ EACH ONE DESCRIBING THE OUTCOMES OF THE RELATIVE EXPERIMENT

x_1 = OUTCOME OF THE FIRST EXPERIMENT

x_2 = " " SECOND " "

⋮

x_N = " " N " "

- FOR A DISCRETE RANDOM VARIABLE, FIRST WE HAVE TO FIX THE CODOMAIN OF THIS VARIABLE
- IF I OBSERVE A QUALITATIVE CHARACTER WITH K-DIFFERENT MODALITIES, I CAN FIRST ATTACH TO THESE MODALITIES THE (FICTISIUS) NAME 1, 2, ... K

COLOR OF EYES $\in \{\underset{1}{\text{BROWN}}, \underset{2}{\text{BLACK}}, \underset{3}{\text{GREEN}}, \underset{4}{\text{BLUE}}\}$

- FOR COUNTING THE OUTCOME IS ALREADY AN INTEGER

- FOR QUALITATIVE CHARACTERS WITH K MODALITIES — CODOMAIN $= \{1, 2, \dots, K\}$
- FOR COUNTING — CODOMAIN $\to \mathbb{N}_0 = \{0, 1, 2, \dots\}$
  $\to \mathbb{N} = \{1, 2, 3, \dots\}$

A DISCRETE RANDOM VARIABLE IS A FUNCTION DEFINED ON Ω AND TAKING VALUES IN $\{1, 2, 3, \dots K\}$ OR IN $\mathbb{N}_0$ (OR $\mathbb{N}$)

$X : \Omega \to \{1, 2, 3, \dots K\}$ [FOR QUALITATIVE CHARACTERS]

$: \Omega \to \mathbb{N}_0$ (OR $\mathbb{N}$)

WE ALSO DEFINE THE SO-CALLED PROBABILITY MASS FUNCTION GIVEN BY

QUAL. CHAR

$p : \{1, \dots K\} \to [0, 1)$

$p(i) = P[X = i]$ FOR $i \in \{1, \dots K\}$

COUNTING

$p : \mathbb{N}_0 \to [0, 1]$

$p(i) = P[X = i)$

THE SIMPLEST DISCRETE RANDOM VARIABLE IS BERNOULLI RANDOM VARIABLE. IN THIS CASE, I AM OBSERVING SOME QUALITATIVE CHARACTERS WITH TWO MODALITIES. THEY CAN BE CALLED ON/OFF OR SUCCESS/FAILURE OR 1/0

$X : \Omega \to \{0, 1\}$

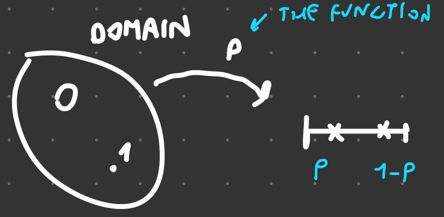
IF I CONSIDER N EXPERIMENTS WITH MODALITY 0/1 THEN I CAN DESCRIBE THIS MULTIPLE OBSERVATION BY CONSIDERING

$X_1, X_2, \dots X_N$ $\quad X_i$ = OUTCOME OF i-TH EXPERIMENT

FOR A SINGLE BERNULLI VARIABLE THE PROBABILITY MASS FUNCTION IS JUST (P∈(0,1) FIXED A PRIORI)

$P(0) = 1-p \quad P(1) = p$

THE PROBABILITY



FOR EXAMPLE $p = 0,4$ THEN $P(0) = 0,6$, $P(1) = 0,4$

PROBABILITY

FUNCTION

FOR BERNULLI RANDOM VARIABLE

$$E[X] = \underset{\text{VALUE}}{0} \cdot \underset{\text{PROB.}}{P(0)} + \underset{\text{VALUE}}{1} \cdot \underset{\text{PROB.}}{P(1)} = p$$

$$VAR(X) = E[X^2] - (E[X])^2 = E[(X - E[X])^2]$$

USEFUL FOR EXERCISE

$$E[X^2] = 0^2 P(0) + 1^2 P(1) = 0(1-p) + 1 \cdot p = p$$

$$VAR(X) = p - p^2 = p(1-p)$$

- $E[X] = 0,4$
- $VAR(X) = 0,4 \cdot 0,6 = 0,24$

NEXT TIME
- BINOMIAL
- HYPERGEOMETRIC
- POISSON
- GEOMETRIC