$+$ $\times$

$-$ $\div$

CONTINOUS DYNAMICAL SYSTEM

x, t CONTINOUS

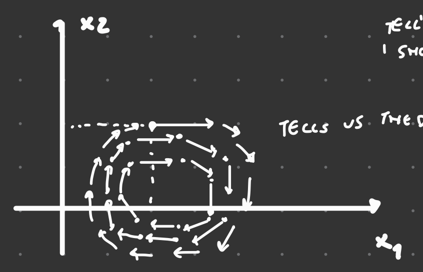
$t \in [0, \infty) \subset \mathbb{R}$

$x \in \mathbb{R}^D$ ← DIMENSION = DEGREE OF FREEDOM

$x_{n+1} = g(x_n)$

$[x(t) = g(x(t-?))]$ DON'T HAVE A WAY TO DEFINE THE INCREMENT FOR CONTINOUS TIME

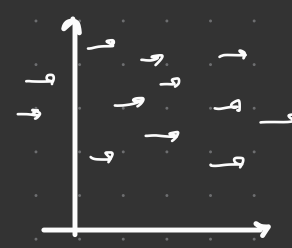
AVOID THIS



SAME DIRECTION AND SAME AMPLITUDE

⇓

TRAJECTORY IS STRAIGHT LINE



$\| f \| = \sqrt{f_x^2 + f_y^2}$

THE • IS DERIVATIVE

$$\vec{f}(\vec{x}(t)) = \underbrace{\dot{\vec{x}}(t)}_{\text{TELLS THE DIRECTION}} = \frac{d}{dt}\vec{x}(t)$$

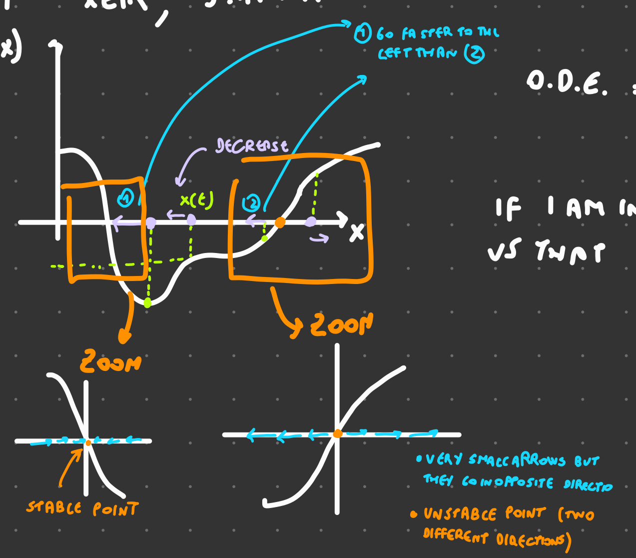
ALL VECTORS

AUTONOMOUS SYSTEM

$\vec{f}(\vec{x}) =$ "VECTOR FIELD"

$D = 1 \quad x \in \mathbb{R}^1, \quad f: \mathbb{R} \to \mathbb{R}$

$\frac{d}{dt} x(t) = f(x)$



O.D.E. = ORDINARY DIFFERENTIAL EQUATION

IF I AM IN $\bar{x}$ WHERE $f(\bar{x}) = 0$ [ON X AXIS] TELLS US THAT $\left.\frac{d}{dt} x(t)\right|_{t=0}$

AT TIME ZERO (CAN BE TODAY OR 30 YEARS AGO, DEPENDS ON HOW WE DEFINE IT)

IF $x(0) = \bar{x} \implies x(t) = \bar{x}$ **FIXED POINT**

THAT MEANS

IF I START ON $\bar{x}$ I WILL STAY IN $\bar{x}$

SECOND DERIVATIVE $\frac{d^2}{dt^2} x$

$a = \frac{F}{m} \Rightarrow \ddot{x} = \frac{F}{m}$

$$\frac{d}{dt}\underbrace{\frac{d}{dt} x}_{} = \frac{d}{dt} y$$

$$\begin{cases} \dot{x} = y \\ \dot{y} = a = \frac{F(x)}{m} \end{cases}$$

F IS FUNCTION OF x

$\dot{\vec{X}} = \vec{f}(x, y)$

$\vec{X} = (x, y)$

STABILITY

$\forall \epsilon > 0 \ \exists \delta > 0 \ \| x(t) - \bar{x} \| < \delta$

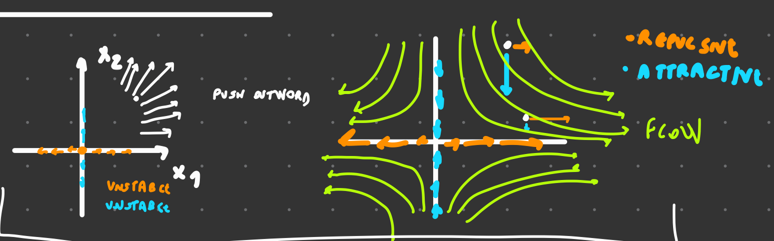
WILL FIND A DELTA S.T.

HOW FAR YOU ARE FROM THE FIXED POINT

$\forall t > 0 \quad \forall x(0) \quad \forall x(0) \quad \| x(0) - \bar{x} \| < \delta$

D = 1
- STABLE
- UNSTABLE

D = 2



BOTH UNSTABLE (SUFFICIENT HAVING ONE AXIS UNSTABLE)