# Knowledge Representation and Reasoning

Exercise Session 2

**Exercise 1. Knowledge Representation** (∗)

Assume that all facts in a knowledge base will have the forms

$$parent(a, b) \leftarrow$$
$$female(a) \leftarrow$$
$$male(a) \leftarrow$$

meaning that "$a$ is a parent of $b$," "$a$ is female," and "$a$ is male" respectively.

1. Using predicate rules, create a knowledge base which describes family relations including **at least**: `aunt`, `uncle`, `grandmother`, `sibling`, and `ancestor`.
2. If, in addition, facts of the form $married(a, b)$ are allowed, extend the knowledge base to allow legal family within the notions of `aunt` and `uncle`.

**Exercise 2. Canonical Model** (∗)

1. Add the following facts to the KB from Exercise 1 and build the **canonical model**:

$$\begin{array}{rr} parent(efraim, ana) \leftarrow & parent(ana, ingrid) \leftarrow \\ parent(ingrid, denis) \leftarrow & parent(ana, claudia) \leftarrow \\ parent(denis, hans) \leftarrow & parent(claudia, bob) \leftarrow \\ parent(francis, greta) \leftarrow & married(claudia, francis) \leftarrow \end{array}$$

$$\begin{array}{rr} female(ana) \leftarrow & male(bob) \leftarrow \\ female(claudia) \leftarrow & male(denis) \leftarrow \\ male(efraim) \leftarrow & male(francis) \leftarrow \\ female(greta) \leftarrow & male(hans) \leftarrow \\ female(ingrid) \leftarrow & \end{array}$$

2. Answer the following queries using this canonical model:
   - $ancestor(efraim, denis)$
   - $ancestor(efraim, great)$
   - $uncle(francis, bob)$
   - $uncle(francis, denis)$
   - $grandmother(X)$
   - $sibling(X, Y)$

**Exercise 3. Models** (∗)

1. Build a model of the KB from the previous exercises, whose domain has only **7** elements.
2. Build a model of the KB from the previous exercises, whose domain has only **4** elements.

## Exercise 4. Consistency (**)

A knowledge base is *consistent* if it has a model. Tell whether the following statements are true or false, justifying your answer.

1. Every set of predicate rules is consistent
2. Every set of predicate rules has a model with one element

## Exercise 5. Canonical model size (***)

1. Construct a KB with 4 facts and 1 rule such that its canonical model construction must add $4^2 = 16$ facts.
2. Generalise the construction to work for any number $n$ of facts in the KB (and $n^2$ facts in the canonical model)

## Exercise 6. Query Expressivity (***)

Suppose that we are interested in deducing whether a rule $p(x) \leftarrow q(x)$ is entailed by a KB $K$; that is, whether every model of $K$ also satisfies this rule.

Devise a reasoning method that can derive this consequence using the tools that we have seen in the lecture.

## Exercise 7. Disjoint unions (**)

Consider two interpretations $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$ and $\mathcal{J} = (\Delta^{\mathcal{J}}, \cdot^{\mathcal{J}})$ such that $\Delta^{\mathcal{I}} \cap \Delta^{\mathcal{J}}$. The *disjoint union* of $\mathcal{I}$ and $\mathcal{J}$ is the interpretation $\mathcal{I} \oplus \mathcal{J} = (\Delta^{\mathcal{I}} \cup \Delta^{\mathcal{J}}, \cdot^{\mathcal{I}\mathcal{J}})$ where for every predicate $P$, $P^{\mathcal{I}\mathcal{J}} = P^{\mathcal{I}} \cup P^{\mathcal{J}}$.

In other words, the disjoint union of $\mathcal{I}$ and $\mathcal{J}$ is the graph obtained by putting together the two graphs defined by $\mathcal{I}$ and $\mathcal{J}$.

Is it true that if $\mathcal{I}$ and $\mathcal{J}$ are both models of a knowledge base $K$, then $\mathcal{I} \oplus \mathcal{J}$ is also a model of $K$? **Justify.**

## Exercise 8. Representing Constraints (*)

1. Add constraints to your knowledge base from Exercise 1 to remove any unexpected consequences you have observed.
2. Do your answers to Exercise 3 change?

**Exercise 9. Model sizes with constraints** (∗∗∗)

1. Using constraints, build a knowledge base $K$ such that all models have at least 3 elements
2. Generalise the construction to models with $n$ elements, for any arbitrary $n$
3. How many constraints are needed?

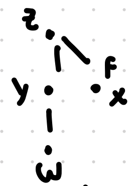
**Exercise 1. Knowledge Representation** (*)

Assume that all facts in a knowledge base will have the forms

$$parent(a, b) \leftarrow$$
$$female(a) \leftarrow$$
$$male(a) \leftarrow$$

meaning that "$a$ is a parent of $b$," "$a$ is female," and "$a$ is male" respectively.

1. Using predicate rules, create a knowledge base which describes family relations including **at least**: `aunt`, `uncle`, `grandmother`, `sibling`, and `ancestor`.
2. If, in addition, facts of the form $married(a, b)$ are allowed, extend the knowledge base to allow legal family within the notions of `aunt` and `uncle`.

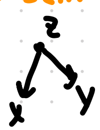



AUNT: AUNT(X,W) ← FEMALE(X), PARENT(Y,W), PARENT(Z,Y), PARENT(Z,F)

UNCLE: UNCLE(X,W) ← MALE(X), PARENT(Y,W), PARENT(Z,Y), PARENT(Z,X)

GRANDMOTHER: GRANDMOTHER(X,Z) ← FEMALE(X), PARENT(X,Y), PARENT(Y,Z)

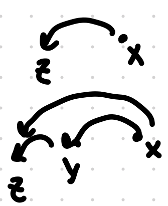

SIBLIN




SIBLIN(X,Y) ← PARENT(Z,X), PARENT(Z,Y)

OR TECNICALLY: SIBLIN(X,Y) ← SIBLIN(Y,X)

ANCESTOR




ANCESTOR(X,Z) ← PARENT(X,Z)

ANCESTOR(X,Z) ← PARENT(X,Y), ANCESTOR(Y,Z)

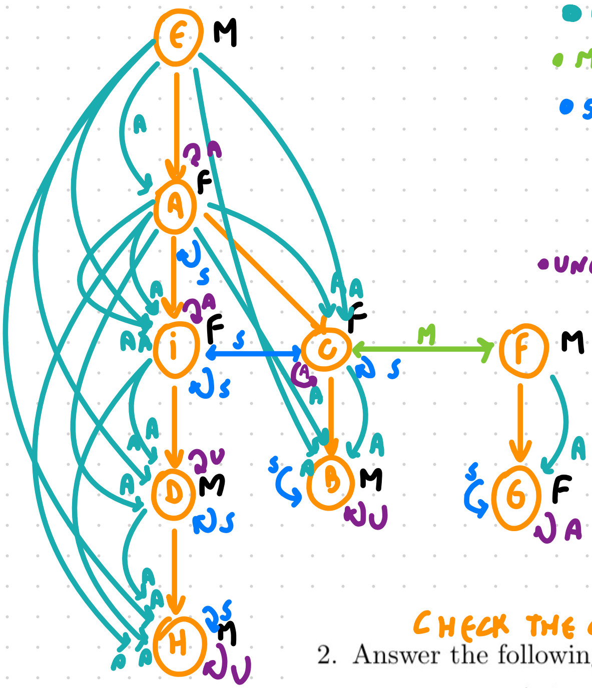
**Exercise 2. Canonical Model** (*)

1. Add the following facts to the KB from Exercise 1 and build the **canonical model**:

$$
\begin{array}{rr}
parent(efraim, ana) \leftarrow & parent(ana, ingrid) \leftarrow \\
parent(ingrid, denis) \leftarrow & parent(ana, claudia) \leftarrow \\
parent(denis, hans) \leftarrow & parent(claudia, bob) \leftarrow \\
parent(francis, greta) \leftarrow & married(claudia, francis) \leftarrow \\
\\
female(ana) \leftarrow & male(bob) \leftarrow \\
female(claudia) \leftarrow & male(denis) \leftarrow \\
male(efraim) \leftarrow & male(francis) \leftarrow \\
female(greta) \leftarrow & male(hans) \leftarrow \\
female(ingrid) \leftarrow &
\end{array}
$$

BUILD THE CANONICAL MODEL

- ANCESTOR
- MARRIED (IS SYMMETRIC)
- SIBLIN (EVERY PERSON THAT HAS A PARENT IS A SIBLIN OF THEMSELF). IT IS ALSO SYMMETRIC
- UNCLE/AUNT (EVERY PERSON THAT HAVE A PARENT IS AN UNCLE/AUNT WITH HIM/HERSELF BECAUSE HE/SHE IS SIBLIN WITH HIM/HERSELF)




CHECK THE CANONICAL MODEL AND ANSWER YES OR NO

2. Answer the following queries using this canonical model:
   - $ancestor(efraim, denis)$ YES
   - $ancestor(efraim, great)$ NO
   - $uncle(francis, bob)$ YES! BECAUSE IS MARRIED WITH AN AUNT OF BOB SO IS AN UNCLE
   - $uncle(francis, denis)$ YES
   - $grandmother(X)$
   - $sibling(X, Y)$

THOSE ARE GENERAL QUERIES GIVE ALL THE SUBSTITUTION THAT MAKES IT TRUE

GRANDMOTHER(X)
- INGRID
- ANA

SIBLIN(X, Y)
- (INGRIND, CLAUDIA)
- (CLAUDIA, INGRID)

THE ONLY ONE IN THE NATURAL SENSE

- (INGRID, INGRID)
- (BOB, BOB)
- ...

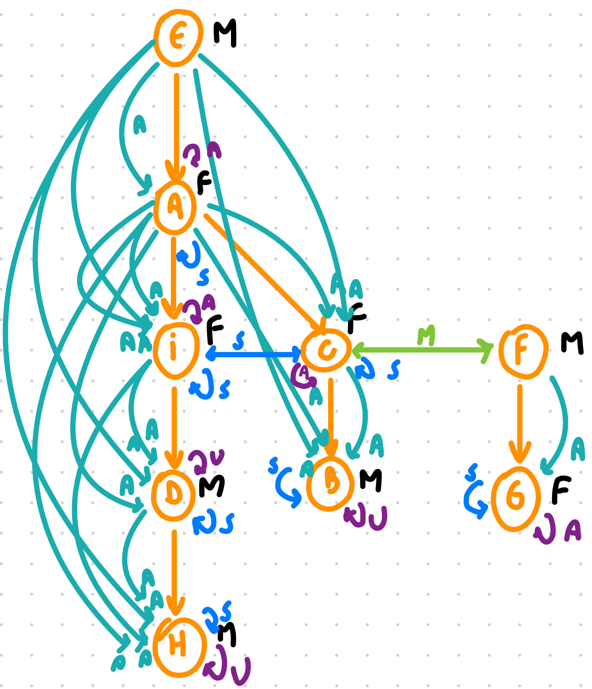
EVERYONE THAT HAVE A PARENT IS A SIBLIN WITH HIM/HER SELF

THE DO NOT HAVE A PARENT →
- NO (E, E) AND NO (F, F)

**Exercise 3. Models** (*)

1. Build a model of the KB from the previous exercises, whose domain has only **7** elements.

2. Build a model of the KB from the previous exercises, whose domain has only **4** elements.




THE DOMAIN OF THIS MODEL HAS 9 ELEMENTS, IT IS POSSIBLE TO BUILD A MODEL OF THE SAME KB THAT ONLY HAVE 7 ELEMENTS

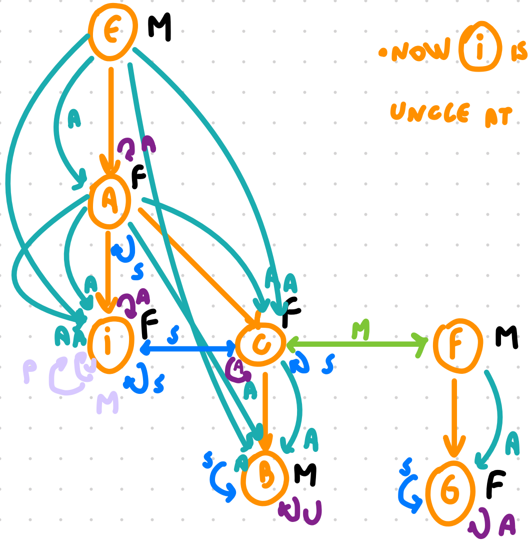
WE NEED TO REMOVE TWO ELEMENTS FROM THE CANONICAL MODEL

REMEMBER!! A MODEL WITH LESS ELEMENTS IT IS A WEAKER MODEL, PUT TOGHETER TWO NODES HAVE MORE PROPRIETIES

I CAN REMOVE D AND H AND SQUEZE THEM TOGHETER IN I

THE NEW DOMAIN WILL BE $\Delta^I = \{a, b, c, e, f, g\}$

- THE INTERPRETATION OF H AND D IS NOW $DENIS^I = i$ $HANS^I = i$




• NOW (i) IS A PARENT OF ITSELF, MALE, FEMALE, UNCLE AT AUNT <u>AT THE SAME TIME</u>

↳ NOBODY SAID THAT YOU CAN'T BE MALE AND FEMALE AT THE SAME TIME. IN THIS KIND OF LANGUAGE WE CAN NOT HAVE NEGATIONS

**Exercise 4. Consistency** (**)

A knowledge base is *consistent* if it has a model. Tell whether the following statements are true or false, justifying your answer.

1. Every set of predicate rules is consistent
2. Every set of predicate rules has a model with one element

MODEL: AN INTERPRETATION THAT SATISFIES ALL THE RULE

1. T EVERY KB IS CONSISTEN BECAUSE IT HAS THE CANONICAL MODEL

2. T WE CAN BUILD A SMALLER MODEL, THERE IS NO LIMITATION WE CAN ALWAYS PUT EVERYTHING TOGHETER IN 1 NODE. THERE IS ALWAYS A MODEL WITH ONE ELEMENT

$\mathcal{I} = (\{\varepsilon\}, \cdot^{\mathcal{I}})$

THE INTERPRETATION FUNCTION MUST MAP EACH CONSTANT TO AN OBJECT

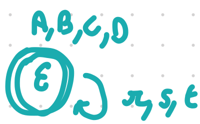
ONE ELEMENT IN THE DOMAIN

$a^{\mathcal{I}} = \varepsilon$

$b^{\mathcal{I}} = \varepsilon$

$\dots$

FOR ALL CONSTANTS




FOR ALL UNARY PREDICATES: $P^{\mathcal{I}} = \{\varepsilon\}$

FOR ALL BINARY PREDICATES: $Q^{\mathcal{I}} = \{(\varepsilon, \varepsilon)\}$

THIS KIND OF MODEL IS USELESS FOR REASONING, IT IS AN EXTREME CASE

**Exercise 5. Canonical model size** (***)

1. Construct a KB with 4 facts and 1 rule such that its canonical model construction must add $4^2 = 16$ facts.

2. Generalise the construction to work for any number $n$ of facts in the KB (and $n^2$ facts in the canonical model)

WHEN WE BUILD THE CANONICAL MODEL WE NEED TO ADD 16 FACTS

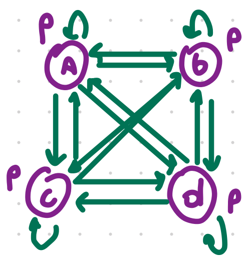
- 4 RULE
- 1 FACT

IF I HAVE 4 ELEMENTS THERE ARE 16 WAYS IN WHICH I CAN CHOOSE A PAIR.

$|\Sigma| = 4$ ↳ SET SIGMA $\Sigma$ WITH CARDINALITY 4

$|\underbrace{\Sigma \cdot \Sigma}| = 16$ ↳ THIS IS TRUE FOR ANY N. IF I HAVE A SET WITH N ELEMENTS I HAVE $N^2$ POSSIBLE PAIRS

THOSE ARE THE 4 FACTS
$\left\{ \begin{array}{l} P(a) \leftarrow \\ P(b) \leftarrow \\ P(c) \leftarrow \\ P(d) \leftarrow \end{array} \right.$




$q(x, y) \leftarrow P(x), P(y)$

- $q(a, a)$
- $q(a, b)$
- $q(a, c)$
- ...

**Exercise 6. Query Expressivity** (***)

Suppose that we are interested in deducing whether a rule $p(x) \leftarrow q(x)$ is entailed by a KB $K$; that is, whether every model of $K$ also satisfies this rule.

Devise a reasoning method that can derive this consequence using the tools that we have seen in the lecture.

TWO POSSIBLE QUERIES

- $P(a)$ - WANT TO KNOW IF A GROUND FACT FOLLOWS FROM THE KB
- $P(x)$ - GIVEN ALL THE CONSTANT THAT SATISFY THIS PROPRIETY

WE CAN CREATE AN ABSTRACT OBJECT (CONSTANT) THAT IS NOT IN K

ADD $q(\epsilon)$ (e IS A NEW CONSTANT) IN K. E IS GUARANTEE TO HAVE THE PROPRIETY q BUT NOTHING ELSE. THEN WE CAN ASK ?$P(\epsilon)$

- IF THE ANSWER IS YES THE $P(\epsilon)$
- IF IT DOES NOT WE HAVE A COUNTER-EXAMPLE WE JUST FIND AN OBJECT THAT IS $q(\epsilon)$ BUT NOT $P(\epsilon)$ AND SO THE IMPLICATION DOES NOT HOLDS

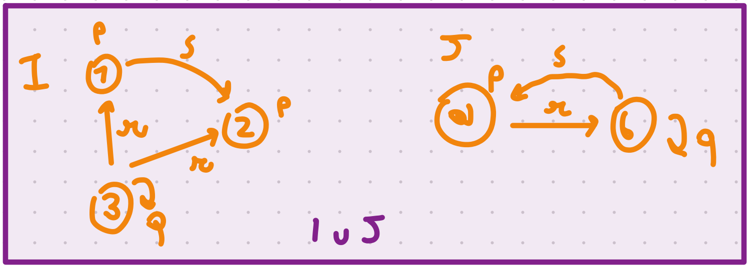
**Exercise 7. Disjoint unions** (**)

Consider two interpretations $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$ and $\mathcal{J} = (\Delta^{\mathcal{J}}, \cdot^{\mathcal{J}})$ such that $\Delta^{\mathcal{I}} \cap \Delta^{\mathcal{J}}$. The *disjoint union* of $\mathcal{I}$ and $\mathcal{J}$ is the interpretation $\mathcal{I} \oplus \mathcal{J} = (\Delta^{\mathcal{I}} \cup \Delta^{\mathcal{J}}, \cdot^{\mathcal{I}\mathcal{J}})$ where for every predicate $P$, $P^{\mathcal{I}\mathcal{J}} = P^{\mathcal{I}} \cup P^{\mathcal{J}}$.

In other words, the disjoint union of $\mathcal{I}$ and $\mathcal{J}$ is the graph obtained by putting together the two graphs defined by $\mathcal{I}$ and $\mathcal{J}$.

Is it true that if $\mathcal{I}$ and $\mathcal{J}$ are both models of a knowledge base $K$, then $\mathcal{I} \oplus \mathcal{J}$ is also a model of $K$? **Justify.**

WE HAVE TWO DISJOINT GRAPH, THE NOTES DOES NOT CORRESPOND TO EACHOTHER. THE IDEA OF THE DISJOINT UNION IS TO LOOK AT THOSE TWO SEPARATED GRAPH AS 1.



**Exercise 8. Representing Constraints** (*)

1. Add constraints to your knowledge base from Exercise 1 to remove any unexpected consequences you have observed.

2. Do your answers to Exercise 3 change?

NEED SOME CONSTRAINTS TO AVOID PEOPLE THAT ARE SIBLIN WITH THEMSELF

1. MAKE A RULE WITH NO HEAD

$\leftarrow sibling(x, x)$

NO OBJECT CAN BE A SIBLIN OF THEMSELF

OF COURSE THE ANSWER WILL CHANGE

- FRANCIS IS NOT ANYMORE AN UNCLE OF BOB
- ONLY TWO SIBLIN RELATION (INGRID, CLAUDIA) AND (CLAUDIA, INGRID)

NOT TRUE THAT IS FEMALE AND MALE AT THE SAME TIME

$\leftarrow male(x), female(x)$

NOBODY IS A PARENT OF THEMSELF

$\leftarrow parent(x, x)$

OF COURSE NOW FROM EX. 3 (WHEN WE SQUEZED D AND H W i) WE HAVE A PROBLEM BECAUSE D RESULTS TO BE MALE AND FEMALE AND PARENT OF HIM/HERSELF

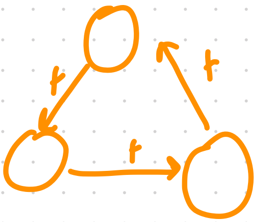
CAN TRY TO SQUEZE b AND c BUT c WILL BECOME PARENT OF HIMSELF

NOW (MAYBE) WE CAN NOT BUILD ANYMORE A MODEL WITH 7 ELEMENTS (DEFINETLY NOT ONE WITH 4 ELEMENTS)

**Exercise 9. Model sizes with constraints** (***)

1. Using constraints, build a knowledge base $K$ such that all models have at least 3 elements
2. Generalise the construction to models with $n$ elements, for any arbitrary $n$
3. How many constraints are needed?

• CAN NOT CREATE A MODEL WITH LESS THAN 3 ELEMENTS




FACTS $\begin{cases} r(a,b) \leftarrow \\ r(b,c) \leftarrow \\ r(c,a) \leftarrow \end{cases}$

AVOID MAPPING TO THE SAME OBJECT $\} \leftarrow r(x,x)$

THERE ARE MANY POSSIBLE SOLUTIONS