Exercises - Calculus
Academic Year 2021-2022

Sheet 13

1. Let I be a bounded closed interval and f integrable on I. Let a, b, $c \in I$. Prove that
$$\int_a^b f = \int_a^c f + \int_c^b f$$
no matter what the order of a, b and c is.

2. Compute the primitive of $\sqrt{1-y^2}$, $y \in [1,1]$, using the substitution $y = \sin(x)$ and the substitution $y = \cos(x)$. Then compute
$$\int_{-1/2}^{\sqrt{3}/2} \sqrt{1-y^2}\, dy$$

3. Prove that
$$\int_0^{\pi/2} \cos^2(x)\, dx = \int_0^{\pi/2} \sin^2(x)\, dx$$
Show that the same formula holds for the intervals $[0,\pi]$, $[-\pi/2, 0]$, $[0, 2\pi]$.
Hint: show that $\int_0^{\pi/2} \sin^2(x)\, dx = \int_{-\pi/2}^0 \sin^2(x)\, dx$ and use the formula $\cos(x) = -\sin(x - \pi/2)$.

4. Compute $\int \sqrt{2-x-x^2}\, dx$
Hint: by changing variables, try to transform it into $\int \sqrt{1-y^2}\, dy$

5. Compute the following definite or indefinite integrals
   (a) $\displaystyle\int \frac{\log(x)}{x}\, dx$; $\displaystyle\int \arcsin(x)\, dx$; $\displaystyle\int \frac{1}{\sqrt{7-5x^2}}\, dx$
   (b) $\displaystyle\int \sqrt{x-2}\, dx$; $\displaystyle\int \sqrt[3]{x-2}\, dx$; $\displaystyle\int \frac{1}{\sqrt{x-2}}\, dx$; $\displaystyle\int \frac{1}{\sqrt[3]{x-2}}\, dx$
   (c) $\displaystyle\int_0^{\pi/4} \frac{3\cos(x)}{\sin(x)-1}\, dx$; $\displaystyle\int_{\pi/4}^{\pi/2} \frac{2\sin(x)}{\cos(x)-1}\, dx$; $\displaystyle\int \frac{\cos(x)}{1+\sin^2(x)}\, dx$; $\displaystyle\int \frac{\sin(2x)}{1+\sin^2(x)}\, dx$
   (d) $\displaystyle\int \frac{e^x}{\sqrt{e^x-1}}, dx$; $\displaystyle\int x^2\cos(x^3+1)\, dx$; $\displaystyle\int \frac{x}{\cos^2(x)}\, dx$; $\displaystyle\int_{\pi^2/4}^{\pi^2} \frac{\cos(\sqrt{x})}{\sqrt{x}}\, dx$
   (e) $\displaystyle\int \frac{2x}{1+x^4}\, dx$; $\displaystyle\int \frac{2}{5x+3}\, dx$; $\displaystyle\int \frac{5x+2}{2x+1}\, dx$; $\displaystyle\int \frac{2x}{x^2+x+1}\, dx$
   (f) $\displaystyle\int \frac{1}{x^2-2x+3}\, dx$; $\displaystyle\int \frac{1}{x^2-4x+4}\, dx$; $\displaystyle\int \frac{3}{x^2+2x+5}\, dx$
   (g) $\displaystyle\int \frac{2x^3+3x^2+4x+3}{x^2+1}\, dx$
   (h) $\displaystyle\int \frac{4x}{(x^2+4)^4}\, dx$; $\displaystyle\int_0^1 \frac{2x-1}{x^2-2x+10}\, dx$; $\displaystyle\int_0^1 \frac{2x-1}{x^2-2x+5}\, dx$

(i) $\displaystyle\int \frac{e^x}{e^{2x}+3e^x+2}\,dx$; $\displaystyle\int_0^1 \frac{e^{2x}}{e^x+1}\,dx$; $\displaystyle\int \frac{3e^x}{e^{2x}-5e^x+6}\,dx$

(j) $\displaystyle\int_0^1 (x^2-2x)\sin(x)\,dx$; $\displaystyle\int_0^1 (x^2+2x)\cos(x)\,dx$; $\displaystyle\int \sin^2(x)\cos^3(x)\,dx$

(k) $\displaystyle\int_e^{e^2} \frac{1}{x((\log(x))^2-2\log(x)-3)}\,dx$; $\displaystyle\int_e^{e^2} \frac{1}{x((\log(x))^2-3\log(x)-4)}\,dx$

(l) $\displaystyle\int_0^{\pi/4} \frac{(\arctan(x)-1)^2}{1+x^2}\,dx$; $\displaystyle\int_0^{\pi/2} e^{-\sin(x)}(\cos(x))^3\,dx$;

(m) $\displaystyle\int_0^{\pi} \cos(x)(x^2-x+1)\,dx$; $\displaystyle\int_0^{\pi/2} e^{\cos(x)}\big(\sin(x)\big)^3\,dx$

(n) $\displaystyle\int_0^{\pi/2} \frac{5\sin(x)}{1+4\cos^2(x)}\,dx$; $\displaystyle\int_0^{\pi} \frac{\sin(x)\cos^2(x)}{1+\cos^2(x)}\,dx$;

(o) $\displaystyle\int \sin(x)e^{2x}\,dx$; $\displaystyle\int_0^1 \frac{\sin(\log(x+1))}{x+1}\,dx$; $\displaystyle\int_1^e \frac{(\log(x))^2-1}{x\,(1+(\log(x))^2)}\,dx$

(p) $\displaystyle\int x\big(\log(x)\big)^2\,dx$; $\displaystyle\int_1^e \log(x)(x^2+x-1)\,dx$; $\displaystyle\int_0^1 2\log(x^2+1)x^3\,dx$

2. Compute the primitive of $\sqrt{1-y^2}$, $y \in [1,1]$, using the substitution $y = \sin(x)$ and the substitution $y = \cos(x)$. Then compute

$$\int_{-1/2}^{\sqrt{3}/2} \sqrt{1-y^2}\, dy$$

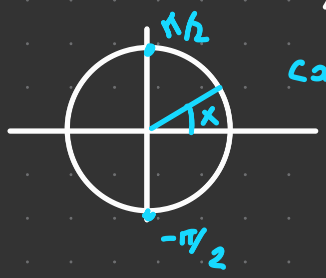
$$\int_{-1/2}^{\sqrt{3}/2} \sqrt{1-y^2}\, dy = \int_{-1/2}^{\sqrt{3}/2} \sqrt{\underbrace{1-\sin^2 x}_{\cos^2 x}} \cdot \cos\, dx = \int_{-1/2}^{\sqrt{3}/2} \cos^2(x)\, dx$$

$y = \sin(x)$

$dy = \cos(x)\,dx$

$\hookrightarrow |\cos x|$

$x = \arcsin y$ $\quad y \in [-\pi/2, \pi/2]$



$\cos(x) > 0$

$$\int \cos^2(x)\,dx = \int \underbrace{\cos(x)}_{f} \underbrace{\cos(x)}_{g}\, dx = -\sin(x)\cos(x) - \int -\sin^2(x)\,dx = \sin(x)\cos(x) + \underbrace{\int 1\,dx}_{x} - \int \cos^2(x)\,dx$$

$-\sin^2(x) \downarrow 1-\cos^2(x)$

$f = \cos(x)$ $\quad F = \sin(x)$

$g = \cos(x)$ $\quad g' = -\sin(x)$

$$2\int \cos^2(x)\,dx = \sin(x)\cos(x) + x$$

$$\int \cos^2(x)\,dx = \frac{1}{2}\sin(x)\cos(x) + \frac{x}{2} + c$$

$y = \sin x \Rightarrow x = \arcsin$

$$\int \sqrt{1-y^2}\, dy = \frac{1}{2}\sin(\arcsin y)\cos(\arcsin y) + \frac{\arcsin y}{2} + c$$

4. Compute $\int \sqrt{2 - x - x^2}\, dx$

Hint: by changing variables, try to transform it into $\int \sqrt{1 - y^2}\, dy$

$$y = x + \frac{1}{2} \qquad dy = dx$$

$$\int \sqrt{2 - x - x^2}\, dx = \int \sqrt{2 + \frac{1}{4} - \frac{1}{4} - x - x^2}\, dx = \int \sqrt{\frac{9}{4} - \underbrace{\left(x^2 + x + \frac{1}{4}\right)}_{\left(x + \frac{1}{2}\right)^2}}\, dx =$$

$$= \int \sqrt{\frac{9}{4} - y^2} = \frac{3}{2} \int \sqrt{1 - \left(\frac{2}{3} y\right)^2}\, dy$$

$$\frac{2}{3} y = z$$

$$\frac{2}{3} dy = dz$$

$$= \frac{3}{2} \int \sqrt{1 - z^2}\ \frac{3}{2} dz = \frac{9}{4} \int \sqrt{1 - z^2}\, dz = \frac{9}{4} \left[\frac{1}{2} z \sqrt{1 - z^2} + \frac{\arcsin z}{2}\right] + c$$

$$\frac{9}{8} z \sqrt{1 - z^2} + \frac{9}{8} \arcsin z + c$$

$$z = \frac{2}{3} y = \frac{2}{3}\left(x + \frac{1}{2}\right)$$

$$= \frac{9}{8} \cdot \frac{2}{3}(x + 1/2) \sqrt{1 - \frac{4}{9}\left(x + \frac{1}{2}\right)^2} + \frac{9}{8} \arcsin\left(\frac{2}{3}\left(x + \frac{1}{2}\right)\right) + c$$

S a1)

$u = \log(x) \qquad du = \frac{1}{x}dx$

$$\int \frac{\log(x)}{x}dx = \int u\,du = \frac{u^2}{2} + c \quad c \in \mathbb{R} = \boxed{\frac{\log(x)^2}{2} + c \quad c \in \mathbb{R}}$$

S a2)

$$\int \arcsin(x)dx = \int \underbrace{1}_{g'} \cdot \underbrace{\arcsin(x)}_{f} dx = x\arcsin(x) - \int \frac{x}{\sqrt{1-x^2}}dx = x\arcsin(x) - \int -1\,dt = x\arcsin(x) + t + c \in \mathbb{R}$$

$$= \boxed{x\arcsin(x) + \sqrt{1-x^2} + c \quad c \in \mathbb{R}}$$

$f \ \arcsin(x) \quad f' \ 1/\sqrt{1-x^2}$

$g' \ 1 \quad g \ x$

$t = \sqrt{1-x^2} \ ; \ \sqrt{1-t^2} = x$

$dt = \frac{-x}{2\sqrt{1-x^2}}dx$

S a3)

$$\int \frac{1}{\sqrt{7-5x^2}}dx = \int \frac{1}{\sqrt{5(\frac{7}{5}-x^2)}}dx = \int \frac{1}{\sqrt{5}\sqrt{\frac{7}{5}-x^2}}dx = \frac{1}{\sqrt{5}}\int \frac{1}{\sqrt{\frac{7}{5}-x^2}}dx = \boxed{\frac{1}{\sqrt{5}}\arcsin\left(\sqrt{\frac{5}{7}}\,x\right) + c \quad c \in \mathbb{R}}$$

$$\int \frac{1}{\sqrt{a^2-x^2}}dx = \arcsin\left(\frac{x}{a}\right)$$

S b1)

$$\int \sqrt{x-2}\,dx = \int \sqrt{t}\,dt = \frac{2t^{3/2}}{3} + c = \boxed{\frac{2}{3}\sqrt{(x-2)^3} + c \quad c \in \mathbb{R}}$$

$t = x-2$

$dt = dx$

S b2)

$$\int \sqrt[3]{x-2}\,dx = \int \sqrt[3]{t}\,dt = \frac{3t^{4/3}}{4} + c = \boxed{\frac{3}{4}\sqrt[3]{(x-2)^4} + c \quad c \in \mathbb{R}}$$

$t = x-2$

$dt = dx$

S b3)

$$\int \frac{1}{\sqrt{x-2}}dx = \int (x-2)^{-1/2}dx = \int t^{-1/2}dt = 2t^{1/2} + c = \boxed{2\sqrt{x-2} + c \quad c \in \mathbb{R}}$$

$t = x-2$

$dt = dx$

S b4)

$$\int \frac{1}{\sqrt[3]{x-2}}dx = \int (x-2)^{-1/3}dx = \int t^{-1/3}dt = \frac{3t^{2/3}}{2} + c \quad c \in \mathbb{R} = \boxed{\frac{3}{2}\sqrt[3]{(x-2)^2} + c \quad c \in \mathbb{R}}$$

$t = x-2$

$dt = dx$

S c1)

$$3\int_0^{\pi/4} \frac{\cos(x)}{\sin(x)-1}dx = 3\int_{-1}^{\frac{\sqrt{2}}{2}-1} \frac{1}{w}dw = 3\ln|w|\Big|_{-1}^{\frac{\sqrt{2}}{2}-1} = 3\ln|\sin(x)-1|\Big|_0^{\pi/4} = 3\ln\left|\frac{\sqrt{2}}{2}-1\right| - 3\ln|-1| = \boxed{3\ln\left(\left|\frac{\sqrt{2}}{2}-1\right|\right)}$$

$w = \sin(x) - 1$

$dw = \cos(x)\,dx$

$x_1 = \pi/4 \quad w_1 = \sin(\pi/4) - 1 = \frac{\sqrt{2}}{2} - 1$

$x_2 = 0 \quad w_2 = \sin(0) - 1 = -1$

WHY

S c2)

$$2\int_{\pi/4}^{\pi/2} \frac{\sin(x)}{\cos(x)-1}dx = -2\int_{\pi/4}^{\pi/2} \frac{-\sin(x)}{\cos(x)-1}dx = -2\int_{\sqrt{2}/2}^{1} \frac{1}{y-1}dy = -2\ln|y-1|\Big]_{\sqrt{2}/2}^{1} = -2\ln|\cos(x)-1|\Big]_{\pi/4}^{\pi/2} = \boxed{0 + 2\ln\left|\frac{\sqrt{2}}{2}-1\right|}$$

$y = \cos(x)$

$dy = -\sin(x)dx$

$y_1 = \cos(\pi/2) = 1$

$y_2 = \cos(\pi/4) = \sqrt{2}/2$

sc3) $\displaystyle\int \frac{\cos(x)}{1+\sin^2(x)}\,dx = \int \frac{1}{1+y^2}\,dy = \arctan(y) + c = \arctan(\sin(x)) + c$

$y = \sin(x)$

$dy = \cos(x)\,dx$

sc4) $\displaystyle\int \frac{\sin(2x)}{1+\sin^2(x)}\,dx = \int \frac{2\sin(x)\cos(x)}{1+\sin^2(x)}\,dx = \int \frac{2y}{1+y^2}\,dy = \ln|1+y^2| + c = \ln|1+\sin^2(x)| + c$

$\sin(2x) = 2\sin(x)\cos(x)$

$y = \sin(x)$

$dy = \cos(x)\,dx$

sd1) $\displaystyle\int \frac{e^x}{\sqrt{e^x-1}}\,dx = \int \frac{1}{\sqrt{y-1}}\,dy = \int (y-1)^{-1/2}\,dy = \frac{(y-1)^{1/2}}{1/2} + c = 2(y-1)^{1/2} = 2\sqrt{e^x-1} + c$

$y = e^x$

$dy = e^x\,dx$

sd2) $\displaystyle\int \underbrace{x^2}_{f'} \underbrace{\cos(x^3+1)}_{g}\,dx = -3x^4\sin(x^3+1) + \int \frac{x^3}{\cancel{3}}\,\cancel{3}x^2\sin(x^3+1)\,dx = -3x^4\sin(x^3+1) + \int \underbrace{x^5}_{f'} \underbrace{\sin(x^3+1)}_{g}\,dx$

BRUTTO

$f'\ x^2 \qquad f\ x^3/3$

$g\ \cos(x^3+1) \qquad g'\ -3x^2\sin(x^3+1)$

$f'\ x^5 \qquad f\ x^6/6$

$g\ \sin(x^3+1) \qquad g'\ 3x^2\cos(x^3+1)$

$\displaystyle\int x^2\cos(x^3+1)\,dx = \frac{1}{3}\int 3x^2\cos(x^3+1)\,dx = \frac{1}{3}\int \cos(y)\,dy = \frac{1}{3}\sin(y) + c = \frac{1}{3}\sin(x^3+1) + c$

$y = x^3+1$

$dy = 3x^2\,dx$

sd3) $\displaystyle\int \frac{x}{\cos^2(x)}\,dx = \int \underbrace{x}_{f}\,\underbrace{\frac{1}{\cos^2(x)}}_{g'}\,dx = x\tan(x) - \int \tan(x)\,dx = x\tan(x) - \int \frac{\sin(x)}{\cos(x)}\,dx = x\tan(x) + \int \frac{1}{y}\,dy = x\tan(x) + \ln|\cos(x)| + c$

$y = \cos(x)$

$dy = -\sin(x)\,dx$

$f\ x \qquad f'\ 1$

$g'\ 1/\cos^2(x) \qquad g\ \tan(x)$

sd4) $\displaystyle\int_{\pi^2/4}^{\pi^2} \frac{\cos(\sqrt{x})}{\sqrt{x}}\,dx = 2\int_{\pi/2}^{\pi} \cos(y)\,dy = 2\sin(y)\Big]_{\pi/2}^{\pi} = 2\sin(\sqrt{x})\Big]_{\pi^2/4}^{\pi^2} = 0 - 2\sin(\pi/2) = -2$

$y = \sqrt{x}$

$dy = \frac{1}{2\sqrt{x}}\,dx$

⊕

sc1) $\displaystyle\int \frac{2x}{1+x^4}\,dx = \int \frac{1}{1+y^2}\,dy = \arctan(y) + c = \arctan(x^2) + c$

$y = x^2$

$dy = 2x\,dx$

NO

* se2) $\displaystyle\int \frac{2}{5x+3}\,dx = \int \frac{2}{3\left(\frac{5}{3}x+1\right)}\,dx = \frac{2}{3}\int \frac{1}{\frac{5}{3}x+1}\,dx$ $\boxed{\displaystyle \cancel{\frac{2}{3}}\,\frac{3}{5}\int \frac{5/3}{\frac{5}{3}x+1}\,dx = \frac{2}{5}\ln\left|\frac{5}{3}x+1\right| + c}$

$\displaystyle\frac{2}{3}\,\frac{3}{5}\int \frac{5/3}{5/3\,x+1}\,dx = \frac{2}{5}\int \frac{1}{y+1}\,dy = \frac{2}{5}\ln|y+1| + c = \frac{2}{5}\ln\left|\frac{5}{3}x+1\right| + c$

$y = \frac{5}{3}x$

$dy = 5/3\,dx$

* 5e3) $\int \frac{5x+2}{2x+1}\,dx = \int \frac{5}{2} - \frac{1/2}{2x+1}\,dx = \frac{5}{2}x - \frac{1}{2}\int \frac{1}{2x+1}\,dx = \frac{5}{2}\boxed{-\frac{1}{4}\int \frac{2}{2x+1}\,dx = \frac{5}{2} - \frac{1}{4}\ln|2x+1| + c}$

NO

$$\begin{array}{l|l} 5x+2 & 2x+1 \\ \hline 5x+5/2 & 5/2 \\ \hline / \ -1/2 & \end{array}$$

$y = 2x$
$dy = 2\,dx$

$$\frac{5}{2}x - \frac{1}{4}\int \frac{1}{y+1}\,dy = \frac{5}{2}x - \frac{1}{4}\ln|y+1| + c = \frac{5}{2}x - \frac{1}{4}\ln|2x+1| + c$$

* 5e4) $\int \frac{2x\,dx}{x^2+x+1} =$??

CHECK

5f1) $\int \frac{1}{x^2-2x+3}\,dx = \int \frac{1}{x^2-2x+3+1-1}\,dx = \int \frac{1\,dx}{x^2-2x+4-1} = \int \frac{1\,dx}{(x-2)^2-1} = \int \frac{1}{y^2-1}\,dy = \arctan(y) + c = \arctan(x-2) + c$

$\Delta < 0$

$y = x-2$
$dy = dx$

5f2) $\int \frac{1\,dx}{x^2-4x+4} = \int \frac{1\,dx}{(x-2)^2} = \int \frac{1\,dy}{y^2} = \int y^{-2}\,dy = \frac{y^{-1}}{-1} + c = -(x-2)^{-1} + c = -\frac{1}{x-2} + c$

$y = x-2$
$dy = dx$

CHECK

5f3) $\int \frac{3}{x^2+2x+5}\,dx = \int \frac{3\,dx}{x^2+2x+4+1} = 3\int \frac{1}{(x+2)^2+1}\,dx = 3\int \frac{dy}{y^2+1} = 3\arctan(y) + c = 3\arctan(x+2) + c$

$y = x+2$
$dy = dx$

5g1) $\int \frac{2x^3+3x^2+4x+3}{x^2+1}\,dx = \int 2x+3+\frac{2x}{x^2+1}\,dx = x^2+3x+\ln(x^2+1) + c$

$$\begin{array}{l|l} 2x^3+3x^2+4x+3 & x^2+1 \\ \hline 2x^3 \qquad 2x & 2x+3 \\ \hline / \ \ 3x^2+2x+3 & \\ 3x^2 \qquad +3 & \\ \hline / \ \ 2x & \end{array}$$

(e) $\int \frac{2x}{1+x^4}\,dx$; $\int \frac{2}{5x+3}\,dx$; $\int \frac{5x+2}{2x+1}\,dx$; $\int \frac{2x}{x^2+x+1}\,dx$
(f) $\int \frac{1}{x^2-2x+3}\,dx$; $\int \frac{1}{x^2-4x+4}\,dx$; $\int \frac{3}{x^2+2x+5}\,dx$
(g) $\int \frac{2x^3+3x^2+4x+3}{x^2+1}\,dx$
(h) $\int \frac{4x}{(x^2+4)^4}\,dx$; $\int_0^1 \frac{2x-1}{x^2-2x+10}\,dx$; $\int_0^1 \frac{2x-1}{x^2-2x+5}\,dx$

5h1) $\int \frac{4x\,dx}{(x^2+4)^4}$ $\int \frac{\quad}{\quad}$

$y = x^2+4$

$dy = 2x$

$$\text{(g)} \int \frac{2x^3+3x^2+4x+3}{x^2+1}\,dx$$

$$\begin{array}{l|l} 2x^3+3x^2+4x+3 & x^2+1 \\ \hline 2x^3 \quad\quad +2x & 2x+3 \\ \hline / \; 3x^2+2x+3 & \\ 3x^2 \quad\quad 3 & \\ \hline / \; 2x & \end{array}$$

$$\int 2x+3\,dx + \int \frac{2x}{x^2+1}\,dx = x^2+3x+\ln(x^2+1)+c$$

$c \in \mathbb{R}$

$$\text{(f)} \int \frac{1}{x^2-2x+3}\,dx;$$

$$\int \frac{1}{x^2-2x+3}\,dx = \int \frac{1}{\underbrace{x^2-2x+1}_{(x-1)^2}\underbrace{-1+3}_{2}}\,dx = \int \frac{1}{\underbrace{(x-1)^2}+2}\,dx = \int \frac{1}{y^2+2}\,dy = \frac{1}{2}\int \frac{1}{\frac{y^2}{2}+1}\,dy =$$

$\hookrightarrow x_{1,2} \quad 2 \pm \sqrt{\underbrace{4-4\cdot 3}_{<0}}$

$Y = x-1$

$dy = dx$

NEED 1 TO HAVE THE DERIVATIVE OF ARCTAN

$z = \frac{y}{\sqrt{2}}$

$dz = \frac{1}{\sqrt{2}}dy \; ; \; dy = \sqrt{2}\,dz$

$$= \frac{\sqrt{2}}{2}\int \frac{1}{z^2+1}\,dz = \underbrace{\frac{\sqrt{2}}{2}\arctan(z)}+c$$

$z = \frac{x-1}{\sqrt{2}}$

$$\frac{\sqrt{2}}{2}\arctan\left(\frac{1}{\sqrt{2}}(x-1)\right)+c$$

d3

$$\int \frac{x}{\cos^2 x}\,dx = x\tan x - \int 1\, tg(x)\,dx$$

↳ $tg\,x = \frac{\sin(x)}{\cos(x)}$

$$x\tan(x) + \int \frac{-\sin(x)}{\cos(x)}\,dx$$

$y = \cos(x)$

$dy = -\sin(x)\,dx$

$$x\tan(x) + \int \frac{1}{y}\,dy$$

$$x\tan(x) + \ln|y| + c$$

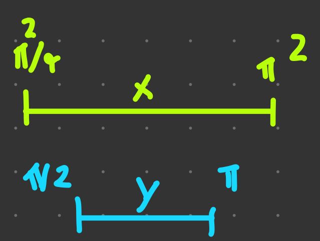
$$x\tan(x) + \ln|\cos(x)| + c$$

d4)

$$\int_{\frac{\pi^2}{4}}^{\pi^2} \frac{\cos(\sqrt{x})}{\sqrt{x}}\,dx$$

$y = \sqrt{x} = x^{1/2}$

$dy = \frac{1}{2}x^{-1/2} = \frac{1}{2}\frac{1}{\sqrt{x}}\,dx$



$y = \sqrt{x}$

$$2\int \frac{1}{2}\cos(\sqrt{x})\,\frac{dx}{\sqrt{x}}$$

$$2\int_{\pi/2}^{\pi} \cos y\,dy = 2\sin y\Big|_{\pi/2}^{\pi} = 0 - 2 = -2$$

$$\int \frac{\sin(2x)}{1+\sin^2(x)}\,dx$$

$\cos(2x) = \cos^2(x) - \sin^2(x)$

$\sin(2x) = 2\sin(x)\cos(x)$

5x4)

$$\int \frac{\sin(2x)}{1+\sin^2(x)}\,dx = \int \frac{2\sin(x)\cos(x)}{1+\sin^2(x)}\,dx$$

$y = \sin(x)$
$dy = \cos(x)\,dx$

$$= 2\int \frac{y}{1+y^2}\,dy$$

$z = y^2$
$dz = 2y\,dy$

$$= \int \frac{1}{1+z}\,dz = \log|1+z| + C = \log|\sin^2 x + 1| + C$$

$$\int \frac{4x}{(x^2+4)^4}\,dx$$

$y = x^2$
$dy = 2x\,dx$

$$2\int \frac{1}{(y+4)^4}\,dy = 2\int (y+4)^{-4}\,dy = \frac{2(y+4)^{-3}}{-3} + C = -\frac{2}{3}(y+4)^{-3} + C = -\frac{2}{3}(x^2+4)^{-3} + C$$

$$\text{(o)} \quad \int \sin(x)e^{2x}\,dx;$$

BY PARTS

$f = e^{2x}$ $\quad f' = 2e^{2x}$
$g' = \sin(x)$ $\quad g = -\cos(x)$

BY PARTS 2

$$\underbrace{\int \sin(x)e^{2x}\,dx}_{} = -\cos(x)e^{2x} - \int -\cos(x)2e^{2x}\,dx =$$

$$= -e^{2x}\cos(x) - 2\left(e^{2x}(-\sin(x)) - \int 2e^{2x}(-\sin(x))\,dx\right.$$

$$-e^{2x}\cos(x) + 2e^{2x}\sin(x) - 4\int e^{2x}\sin(x)\,dx$$

$$5\int \sin(x)\,e^{2x}\,dx = -e^{2x}\cos(x) + 2e^{2x}\sin(x)$$

$$\int \sin(x)\,e^{2x}\,dx = -\frac{e^{2x}}{5}\cos(x) + \frac{2}{5}e^{2x}\sin(x) + \mathcal{C}$$