Knowledge Representation and Reasoning

Exercise Session 6

Exercise 1. Probabilistic Reasoning

(*)

Consider the KB $K = K_P \cup K_C$ where

$$K_P := \{0.5 :: x, \quad 0.5 :: y, \quad 0.5 :: y \leftarrow z\}$$

$$K_C := \{z \leftarrow x, \quad w \leftarrow x, y\}$$

Compute the probability of the following consequences assuming **probabilistic independence** between clauses:

- 1. w
- 2. *y*
- 3. $w \leftarrow x$

Exercise 2. Probability Distribution

(**)

Consider again the KB from Exercise 1. Suppose that 0.5 :: y is **not** a probabilistic clause, but rather a constraint on the joint distribution of the clauses. That is, we want P(y) = 0.5.

- 1. Find an adequate probability distribution and compute P(w)
- 2. How would the result change if we required P(y)=p for some p>0.5? And for p<0.5?

Exercise 3. Extreme Probabilities

(*)

Following the **open world approach** compute the extreme probabilities for w from the KB of Exercise 1.

Exercise 4. DLs (**)

Consider the probabilistic \mathcal{EL}_{\perp} TBox

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\mathcal{T} := \{ 0.5 :: \top \sqsubseteq Male, \quad 0.5 :: \top \sqsubseteq Female, \\ Male \sqcap Female \sqsubseteq \bot, \quad \top \sqsubseteq \exists.hasParent. \top \}
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- 1. Explain in words what the GCIs in this TBox mean
- 2. Compute the probabilities of:
 - $Male \sqsubseteq \exists hasParent.Female$ and
 - $\bullet \ \mathit{Male} \sqsubseteq \exists \mathit{hasParent}. \mathit{Male}$

Exercise 5. Semantics

(***)

Suppose that you want to represent the (uncertain) knowledge about the spread, consequences, and cost of a recently discovered disease.

- 1. Which probabilistic semantics do you think is more adequate? why?
- 2. Identify constructors necessary to express all relevant notions

Exercise 6. Probabilities

(***)

A uniform probability distribution is one that assigns the same probability to events of the same size (e.g., assigning 1/6 to each face in a die). Suppose that we define a uniform distribution over all the natural numbers.

- 1. what is the probability of finding an even number? And a multiple of 5?
- 2. how do you define the probability of a set N?
- 3. what is the probability of observing the number 42?
- 4. is it impossible to observe 42?

Consider the KB $K = K_P \cup K_C$ where

$$\begin{split} K_P := \{0.5 :: x, \quad 0.5 :: y, \quad 0.5 :: y \leftarrow z\} \\ K_C := \{z \leftarrow x, \quad w \leftarrow x, y\} \end{split}$$

(*)

Compute the probability of the following consequences assuming **probabilistic independence** between clauses:

- 1. w
- 2. y
- 3. $w \leftarrow x$

$$P(w) = \frac{3}{8} P(y) = \frac{5}{8} P(w + 2) = \frac{1}{8} P(w) = \frac{3}{8} P(w) = \frac{3}{8}$$

Exercise 2. Probability Distribution

Consider again the KB from Exercise 1. Suppose that 0.5 :: y is **not** a probabilistic clause, but rather a constraint on the joint distribution of the clauses. That is, we want P(y) = 0.5.

- 1. Find an adequate probability distribution and compute P(w)
- 2. How would the result change if we required P(y) = p for some p > 0.5? And for p < 0.5?

Exercise 3. Extreme Probabilities

(*)

Following the open world approach compute the extreme probabilities for w from the KB of Exercise 1.

$$K_P := \{0.5 :: x, \quad 0.5 :: y, \quad 0.5 :: y \leftarrow z\}$$

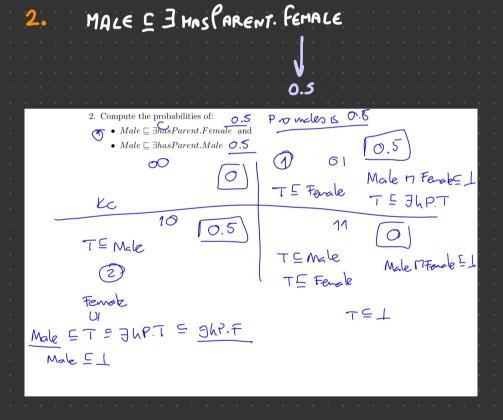
Exercise 4. DLs

(**

Consider the probabilistic \mathcal{EL}_{\perp} TBox

$$\mathcal{T} := \{ \begin{array}{ll} 0.5 :: \top \sqsubseteq Male, & 0.5 :: \top \sqsubseteq Female, \\ Male \sqcap Female \sqsubseteq \bot, & \top \sqsubseteq \exists.hasParent. \top \ \} \end{array}$$

- 1. Explain in words what the GCIs in this TBox mean
- 2. Compute the probabilities of:
 - $Male \sqsubseteq \exists hasParent.Female$ and
 - $Male \sqsubseteq \exists hasParent.Male$



Exercise 6. Probabilities

A uniform probability distribution is one that assigns the same probability to events of the same size (e.g., assigning 1/6 to each face in a die). Suppose that we define a uniform distribution over all the natural numbers.

- 1. what is the probability of finding an even number? And a multiple of 5?
- 2. how do you define the probability of a set N?
- P({42 }) = 0 3. what is the probability of observing the number 42?
- 4. is it impossible to observe 42?