

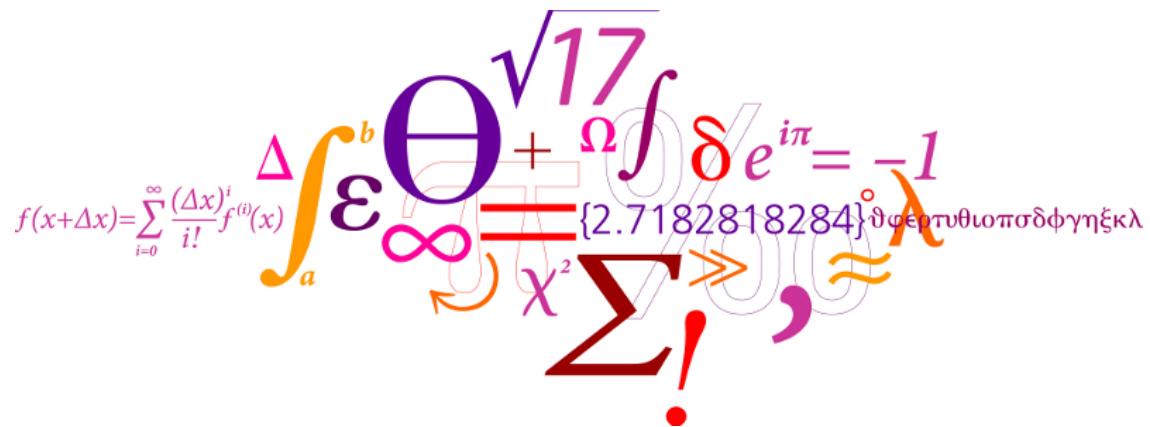
# 31310 LINEAR CONTROL DESIGN 2

## COMPULSORY ASSIGNMENT 2015 : LOUDSPEAKER CONTROL

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# Exercice 1

## Distortion Attenuation for Loudspeakers

### 1.1 Moving-coil Loudspeakers

#### 1.1.1 Loudspeakers electrical equivalent circuit

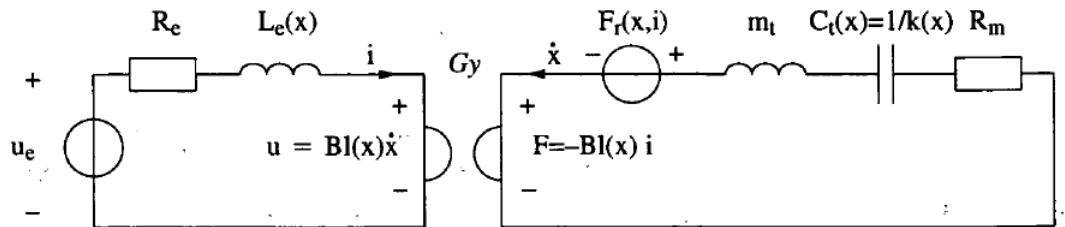


Figure 1.1: Electrical equivalent lumped element model of the voltage driven electrodynamic loudspeaker for low frequencies. The coupling between the electrical and mechanical domain is performed through the gyrator with gyration constant  $Bl(x)$ .

$$u_e = R_e i + \frac{dL_e(x)}{dx} \frac{dx}{dt} i + L_e(x) \frac{di}{dt} + Bl(x) \frac{dx}{dt} \quad (1.1)$$

$$Bl(x)i = m_t \frac{d^2x}{dt^2} + R_m \frac{dx}{dt} + k(x)x - \frac{1}{2} \frac{dL_e(x)}{dx} i^2 \quad (1.2)$$

where

$$Bl(x) = Bl_0 + b_1 x + b_2 x^2 \quad (1.3)$$

$$L_e(x) = L_{e0} + l_1 x + l_2 x^2 \quad (1.4)$$

$$k(x) = k_0 + k_1 x + k_2 x^2 \quad (1.5)$$

## Problem 1

By means of Eqs 1.1, 1.2, 1.3, 1.4 and 1.5, we can identify 3 state variables  $x$ ,  $\dot{x}$  and  $i$ . We can also identify the input  $u_e$ .

$$\mathbf{x} = \begin{pmatrix} x \\ \dot{x} \\ i \end{pmatrix} \text{ and } \mathbf{u} = (u_e)$$

Then, we can derive the nonlinear dynamical state space model to obtain

$$\dot{x} = \dot{x} \quad (1.6)$$

$$\ddot{x} = \frac{(Bl_0 + b_1x + b_2x^2)i - R_m\dot{x} - (k_0 + k_1x + k_2x^2)x + \frac{1}{2}(l_1 + 2l_2x)\dot{x}^2}{m_t} \quad (1.7)$$

$$\dot{i} = \frac{u_e - (R_e + (l_1 + 2l_2x)\dot{x}^2)i - (Bl_0 + b_1x + b_2x^2)\dot{x}}{L_{e0} + l_1x + l_2x^2} \quad (1.8)$$

In matrix format, we have

$$\dot{\mathbf{x}} = f(\mathbf{x}) + g(\mathbf{x})\mathbf{u} \quad (1.9)$$

with

$$f(\mathbf{x}) = \begin{pmatrix} \dot{x} \\ \ddot{x} \\ \dot{i} \end{pmatrix} = \begin{pmatrix} \dot{x} \\ \frac{(Bl_0 + b_1x(1) + b_2x(1)^2)x(3) - R_m x(2) - (k_0 + k_1x(1) + k_2x(1)^2)x(1) + \frac{1}{2}(l_1 + 2l_2x(1))x(2)x(3)^2}{m_t} \\ \frac{-(R_e + (l_1 + 2l_2x(1))x(2)^2)x(3) - (Bl_0 + b_1x(1) + b_2x(1)^2)x(2)}{L_{e0} + l_1x(1) + l_2x(1)^2} \end{pmatrix} \quad (1.10)$$

$$g(\mathbf{x}) = \begin{pmatrix} 0 \\ 0 \\ \frac{1}{L_{e0} + l_1x(1) + l_2x(1)^2} \end{pmatrix} \quad (1.11)$$

## Problem 2

### Problem 3

The input voltage is set to  $u_e = A_u \sin(2\pi f_c t)$ , where  $A_u = 5 \text{ V}$  and  $f_c = 20 \text{ Hz}$ . Then the nonlinear system implemented in PB2 is simulated to get the states responses. To visualise the signals in the frequency domain, we used the *power\_spectral\_density* function given in the assignment [1] with  $F_s = \frac{1}{\text{TIME\_SIM}} = 10 \text{ kHz}$ . The function output gives the power spectral density  $P_{xx}$  in  $[\text{Amplitude}^2/\text{Hz}]$ . In order to convert

it in [dB/Hz], we use:

$$P_{dB} = 10 \log_{10}(P_{xx})$$

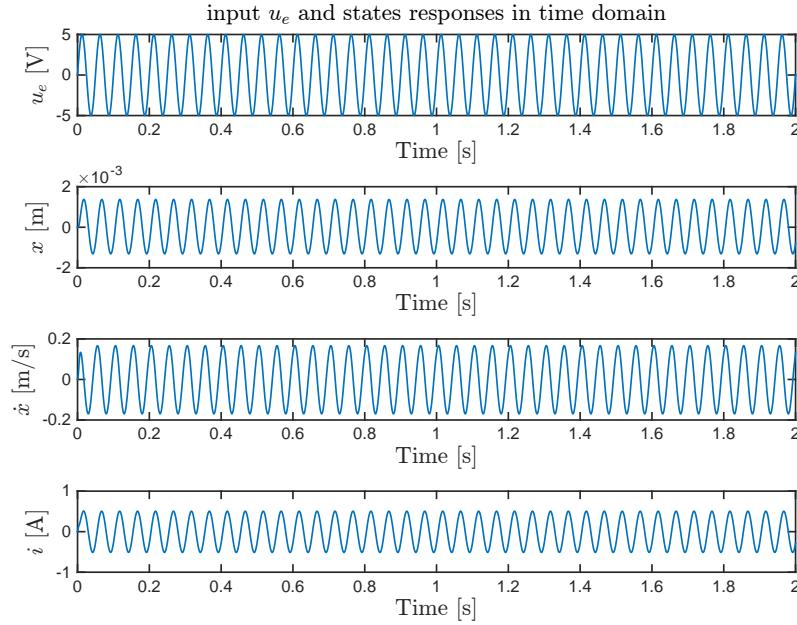


Figure 1.2: Nonlinear Model: input  $u_e$  and the states responses in the time domain

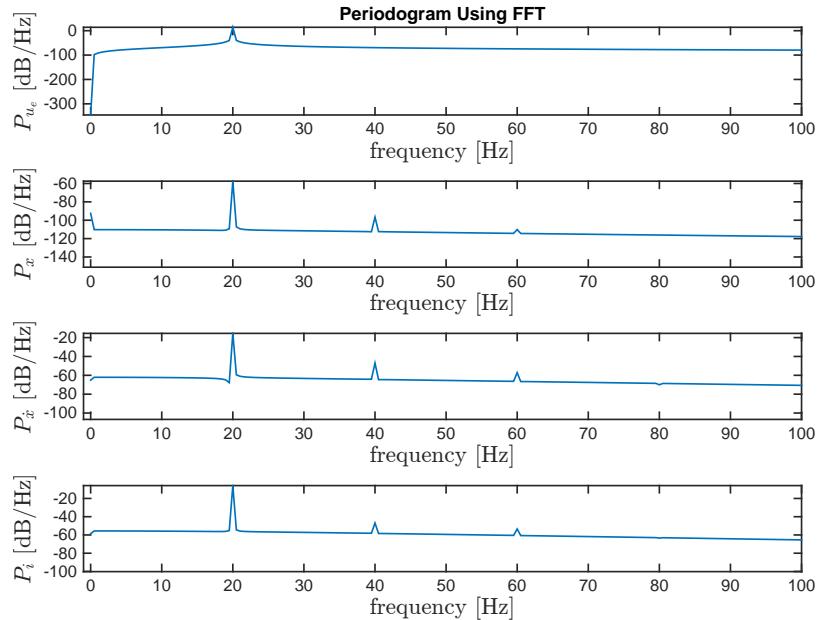


Figure 1.3: Nonlinear Model: input  $u_e$  and the states responses in the frequency domain

In the time domain (see figure 1.2), we notice that the states responses are sinusoidal, with the same amplitude. The system looks stable. Moreover, the frequency

of the states seems similar to the frequency of the input signal, with slight variations. The explanation for this changes is given in the frequency domain, figure 1.3, where we can see several more spikes at the frequencies  $f_n = nf_c$ , smaller than the fundamental one at  $f_c = 20 \text{ Hz}$ . The presence of other harmonics in the states responses, which are not in the input, shows the effects of the nonlinearity of the system. It may due to the fact that the voice coil displacement is limited: when it goes too far, the spider prevents it from leaving the magnetic field, which can cause the harmonic distortion.

### 1.1.2 Harmonic Distortion

We are now going to study the harmonic distortion observed in the states responses in the frequency domain.

#### Problem 4

The analysis will be restricted to the voice coil velocity  $\dot{x}$ . The Total Harmonic Distortion (THD) is described by

$$THD = \frac{\sqrt{\sum_{n=2}^N A_n^2}}{\sqrt{\sum_{n=1}^N A_n^2}} \cdot 100\% \quad (1.12)$$

where  $A_1$  the amplitude of the fundamental frequency and  $A_n$  the amplitudes of the harmonics.

To compute THD for the 5 harmonics after the fundamental frequency  $f_c$  we need to find the 6 amplitudes corresponding to the fundamental frequency pikes and the 5 following harmonics. To that purpose, a function *amplitude* was implemented (see below) with  $x$  the signal and  $fr$  the frequency at which we desire to know the amplitude of the spike. The Fast Fourier Transform is computed but not the spectral power density as previously.

```
function [ res ] = amplitude(x, TIME_SIM, fr)
    NFFT = length(x);
    X = fft(x,NFFT) / (NFFT);
    X = 2*abs(X(1:NFFT/2+1));
    res = X(fr*TIME_SIM+1);
end
```

Finally for a  $TIME\_SIM = 5\text{s}$ , for the voice coil velocity  $THD = 2.57\%$ . We then use the equations (1.13) and (1.14) to compute the second and third order

harmonic distortion. We obtain  $d_2 = 2.48\%$  and  $d_3 = 0.65\%$ .

$$d_2 = \frac{A_2}{\sqrt{A_1^2 + A_2^2}} \cdot 100\% \quad (1.13)$$

$$d_3 = \frac{A_3}{\sqrt{A_1^2 + A_3^2}} \cdot 100\% \quad (1.14)$$

We can notice that  $d_2$  corresponds to the THD calculated for  $N = 2$ . Because the amplitudes of the harmonics decrease when the order gets bigger, it is logical that the values of THD and  $d_2$  are quite similar. On the contrary,  $A_3$  is negligible in front of  $A_1$ , that's why  $d_3$  is small and far from the THD.

## Problem 5

In order to study the effect of the variations of the frequency  $f_c$  and the amplitude  $A_u$  of the input  $u_e$  on the second and third order harmonic distortion, we have computed  $d_2$  and  $d_3$  for a range of  $A_u$  and  $f_c$ . The results can be seen on the figure 1.4. According to the graphic, the variations of the amplitude of the input do not affect the distortion levels. On the opposite, when the fundamental frequency increases, the distortion level

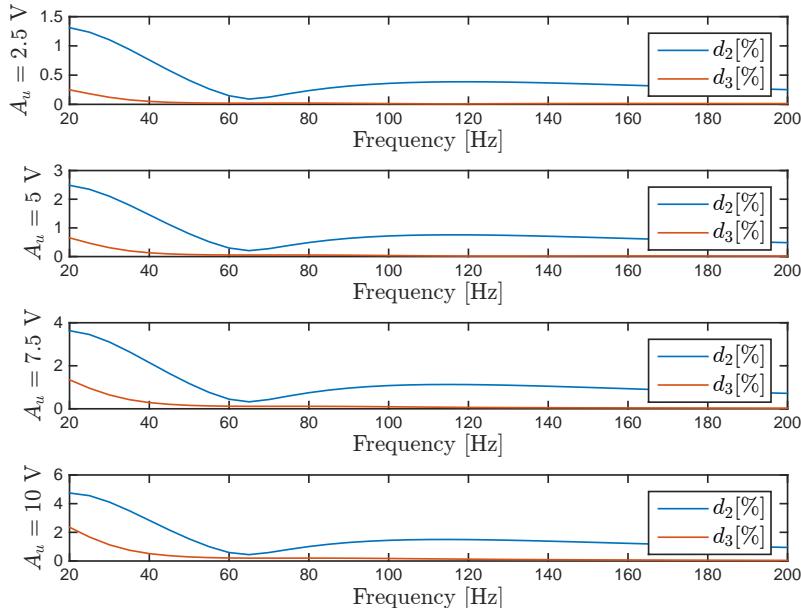


Figure 1.4: Effect of the variations of the frequency  $f_c$  and the amplitude  $A_u$  of the input  $u_e$  on the second and third order harmonic distortion

### 1.1.3 Linearised Model

The measured output is set to the voice coil current. Therefore  $y(t) = i(t)$ . We also take  $u_e = 0$  for the analysis of the linear and nonlinear model around the resting position of the voice coil.

### Problem 6

All time derivatives are set to zero in order to determine the stationary states. Therefore, we have  $\frac{dx}{dt} = 0$  and equations (1.1) and (1.2) are rewritten below:

$$u_e = R_e i \quad (1.15)$$

$$Bl(x)i = k(x)x \quad (1.16)$$

As  $u_e = 0$ , from (1.15) we obtain  $i = 0$  and we deduce by substituting in (1.16) that  $k(x)x = 0$ . Then  $k(x) = 0$  or  $x = 0$ . The discriminant of the polynomial  $k(x)$  of degree 2 is  $\Delta = k_1^2 - 4k_2k_0 < 0$ . The voice coil displacement  $x$  being real, we discard this value and get:

$$\mathbf{x}_0 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

We linearise the model  $\dot{\mathbf{x}} = h(\mathbf{x}, u)$  with  $h(\mathbf{x}) = f(\mathbf{x}) + g(\mathbf{x})u$  around the stationary states:

$$x(t) = x_0 + \Delta x(t) = \Delta x(t)$$

$$\dot{x}(t) = \dot{x}_0 + \Delta \dot{x}(t) = \Delta \dot{x}(t)$$

$$i(t) = i_0 + \Delta i(t) = \Delta i(t)$$

The linear model can then be written in the form:

$$\dot{\mathbf{x}} = A\mathbf{x} + B\mathbf{u}$$

$$y = C\mathbf{x}$$

where

$$A = \begin{pmatrix} \frac{\partial h_1}{\partial x_1} & \frac{\partial h_1}{\partial x_2} & \frac{\partial h_1}{\partial x_3} \\ \frac{\partial h_2}{\partial x_1} & \frac{\partial h_2}{\partial x_2} & \frac{\partial h_2}{\partial x_3} \\ \frac{\partial h_3}{\partial x_1} & \frac{\partial h_3}{\partial x_2} & \frac{\partial h_3}{\partial x_3} \end{pmatrix}_{x_0} \quad B = \begin{pmatrix} \frac{\partial h_1}{\partial u} \\ \frac{\partial h_2}{\partial u} \\ \frac{\partial h_3}{\partial u} \end{pmatrix}_{x_0} \quad C = \begin{pmatrix} \frac{\partial r}{\partial x_1} & \frac{\partial r}{\partial x_2} & \frac{\partial r}{\partial x_3} \end{pmatrix}_{x_0}$$

We obtain:

$$\frac{\partial h_1}{\partial x_1} = 0 \quad \frac{\partial h_1}{\partial x_2} = 1 \quad \frac{\partial h_1}{\partial x_3} = 0$$

$$\frac{\partial h_2}{\partial x_1} = \frac{b_1 x_{30} + 2b_2 x_{10} x_{30} - (k_0 + 2k_1 x_{10} + 3k_2 x_{10}^2) + l_2 x_{20} x_{30}^2}{m_t}$$

$$\frac{\partial h_2}{\partial x_2} = \frac{-R_m + \frac{1}{2} \times (l_1 + 2l_2 x_{10}) x_{30}^2}{m_t}$$

$$\frac{\partial h_2}{\partial x_3} = \frac{Bl_0 + b_1 x_{10} + b_2 x_{10}^2 + (l_1 + 2l_2 x_{10}) x_{20} x_{30}}{m_t}$$

$$\begin{aligned} \frac{\partial h_3}{\partial x_1} &= \frac{(-2l_2 x_{20}^2 x_{30} - (b_1 + 2b_2 x_{10}) x_{20}) \times (L_{e0} + l_1 x_{10} + l_2 x_{10}^2)}{(L_{e0} + l_1 x_{10} + l_2 x_{10}^2)^2} - (l_1 + 2l_2 x_{10}) \times \\ &\quad \frac{(-(R_e + (l_1 + 2l_2 x_{10}) x_{20}^2) x_{30} - (Bl_0 + b_1 x_{10} + b_2 x_{10}^2) x_{20})}{(L_{e0} + l_1 x_{10} + l_2 x_{10}^2)^2} \end{aligned}$$

$$\frac{\partial h_3}{\partial x_2} = -\frac{2(l_1 + 2l_2 x_{10}) x_{20} x_{30} + Bl_0 + b_1 x_{10} + b_2 x_{10}^2}{L_{e0} + l_1 x_{10} + l_2 x_{10}^2}$$

$$\frac{\partial h_3}{\partial x_3} = -\frac{(l_1 + 2l_2 x_{10}) x_{20}^2 + R_e}{L_{e0} + l_1 x_{10} + l_2 x_{10}^2}$$

$$\frac{\partial h_1}{\partial u} = 0 \quad \frac{\partial h_2}{\partial u} = 0 \quad \frac{\partial h_3}{\partial u} = \frac{1}{L_{e0} + l_1 x_{10} + l_2 x_{10}^2}$$

$$\frac{\partial r}{\partial x_1} = 0 \quad \frac{\partial r}{\partial x_2} = 0 \quad \frac{\partial r}{\partial x_3} = 1$$

We then substitute  $x_{10} = x_{20} = x_{30} = 0$ . Finally, we get:

$$A = \begin{pmatrix} 0 & 1 & 0 \\ -\frac{k_0}{m_t} & -\frac{R_m}{m_t} & \frac{Bl_0}{m_t} \\ 0 & -\frac{Bl_0}{L_{e0}} & -\frac{R_e}{L_{e0}} \end{pmatrix} \quad B = \begin{pmatrix} 0 \\ 0 \\ \frac{1}{L_{e0}} \end{pmatrix} \quad C = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}$$

Numerically

$$A = \begin{pmatrix} 0 & 1 & 0 \\ -1.20 \cdot 10^5 & -50.46 & 279.19 \\ 0 & -2.57 \cdot 10^3 & -1.40 \cdot 10^3 \end{pmatrix} \quad B = \begin{pmatrix} 0 \\ 0 \\ 177 \end{pmatrix} \quad C = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix} \quad (1.17)$$

We can check these results with the matlab function *linmod(model,x0,u<sub>e</sub>)*:

```
[A, B, C, D] = linmod('nonLinearModel', [0; 0; 0], 0);
```

## Problem 7

In this problem, we draw the block diagram of the linearised loudspeaker (see figure 1.5) showing the couplings between the different states.

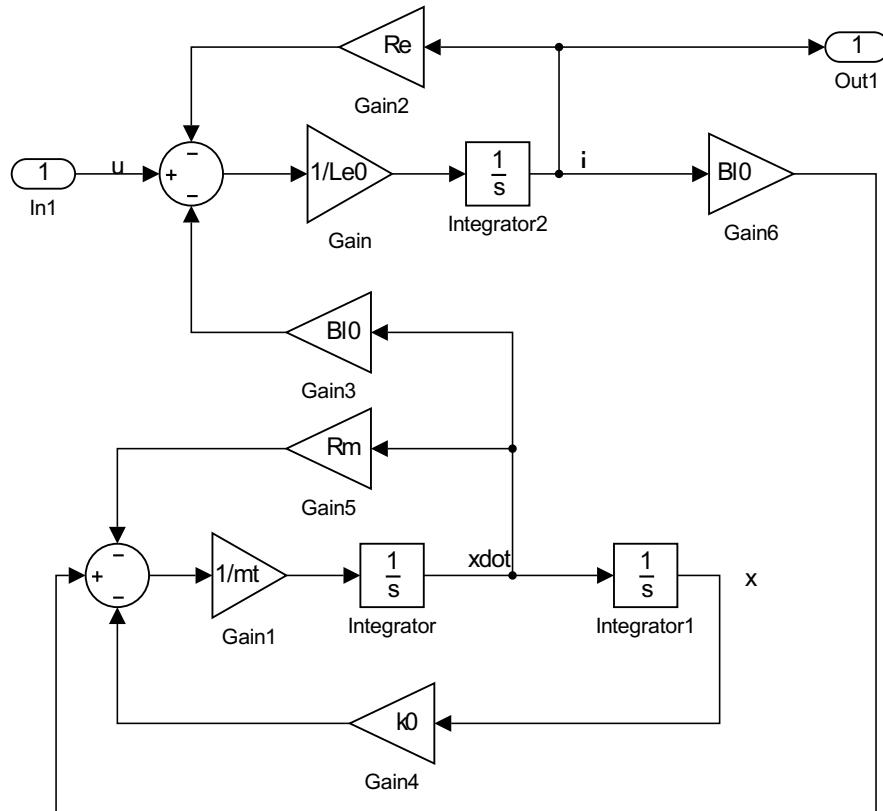


Figure 1.5: block diagram of the linearised loudspeaker

By means of this block diagram, we can make preliminary assessments of the system. Indeed, we can see that the state  $i$  seems to be controllable and observable as it is connected to the input  $u_e$  and to the output. However, the states  $x$  and  $\dot{x}$  seems to be controllable as they are also connected to the input  $u_e$  but not to be observable because they are not connected to any output. Moreover the 3 states seems to be stable because of the 2 backloops.

We can verify our assumptions regarding the controllability and the observability by calculating the *rank* of  $M_c$  and  $M_o$ .

```
Mc = [B A*B A^2*B];
```

```

rank (Mc) % = 3 controllable
Mo=[C
     C*A
     C*A^2];
rank (Mo) % = 3 observable
    
```

## Problem 8

We can derive analytically the eigenvalues of the system dynamical matrix  $A$ .

$$\det(\lambda I - A) = \det \begin{pmatrix} \lambda & -1 & 0 \\ \frac{k_0}{m_t} & \lambda + \frac{R_m}{m_t} & -\frac{Bl_0}{m_t} \\ 0 & \frac{Bl_0}{Le_0} & \lambda + \frac{Re}{Le_0} \end{pmatrix} \quad (1.18)$$

$$= \lambda^3 + \left( \frac{Re}{Le_0} + \frac{Rm}{m_t} \right) \lambda^2 + \left( \frac{RmRe + Bl_0}{m_t Le_0} + \frac{k_0}{m_t} \right) \lambda + \frac{k_0 Re}{m_t Le_0} \quad (1.19)$$

We therefore have a third-order polynomial which can be solved with MATLAB but the eigenvalues can also be calculated using

```
lambda = eig (A);
```

Thus, we obtain

$$\lambda = 1, 0.10^2 \begin{pmatrix} -2.9573 \\ -5.7648 + 4.8571i \\ -5.7648 - 4.8571i \end{pmatrix} \quad (1.20)$$

We can notice that

$$Re(\lambda_i) < 0, \quad i = 1, 2, 3 \quad (1.21)$$

which means that the system is asymptotically stable. Moreover, using Euler decomposition, the eigenmodes of the system are  $e^{\lambda_1 t}$ ,  $e^{\Re(\lambda_2)t} (\cos(\Im(\lambda_2)t) + \sin(\Im(\lambda_2)t))$  and  $e^{\Re(\lambda_3)t} (\cos(\Im(\lambda_3)t) - \sin \Im(\lambda_3)t))$ . We can notice that

$$\frac{1}{\lambda_1} = \tau_1 > \tau_2, \tau_3, \quad \text{with } \tau_i = \frac{1}{\sqrt{\Re(\lambda_i)^2 + \Im(\lambda_i)^2}}, \quad i = 2, 3$$

which means that the response of the state  $x$  to an input  $u_e$  will be slower than the one of  $i$  and  $\dot{x}$ . We can also see that as  $\lambda_2$  and  $\lambda_3$  have an imaginary part which means that the eigenmodes associated to  $\lambda_2$  and  $\lambda_3$  will have an oscillatory

behavior.

### Problem 9

Using the results of Problem 6 (1.17), we can implement a SIMULINK model (see figure A.1) of the linear system.

Then, we can simulate the linearised model with an input  $u_e = A_u \sin(2\pi f_c t)$ , where  $A_u = 5V$  and  $f_c = 20Hz$ . The states response in the time domain is plotted figure 1.6 and the PSD is plotted figure 1.7.

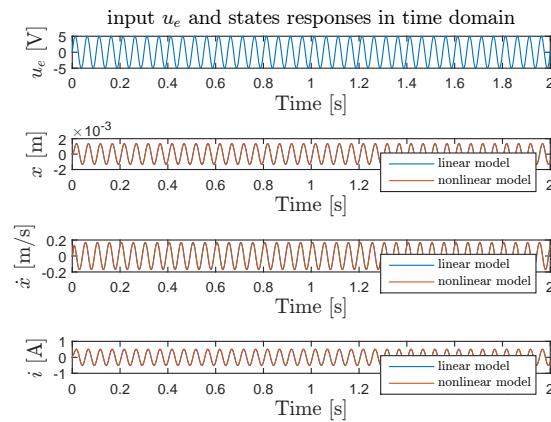
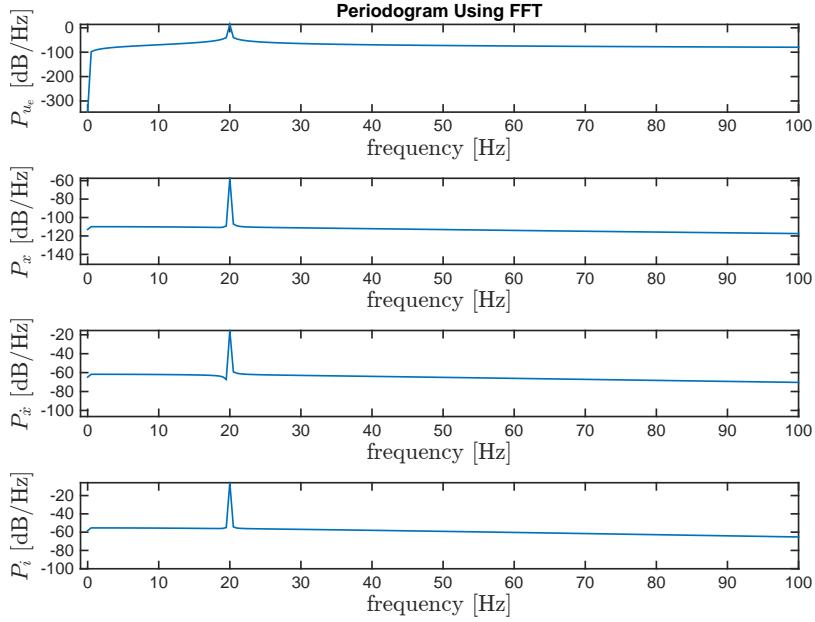


Figure 1.6:  $u_e$  and states reponse to  $u_e$  in time domain


 Figure 1.7: PSD of  $u_e$  and states

First, comparing the response from the nonlinear model with the linear model to the input  $u_e$  in the time domain (figure 1.2), the states response is almost exactly the same, but comparing the PSD of the states response (figure 1.3 and figure 1.7), we can see that the states response is not the same. Indeed, the linearised system is not affected by the harmonic distortion, there is only one frequency present in the state response, the one of  $20\text{Hz}$  (which has the same magnitude). This result was expected because the nonlinear distortion only affects non-linear systems. Moreover, we have linearised the system with an input  $u_e = 0$  which means that we have a linearised system without harmonic, and then, we used a new input  $u_e$  with a frequency  $f_c = 20\text{Hz}$ , which means we only obtain a state response with an harmonic of a frequency of  $f_c$ .

#### 1.1.4 Harmonic distortion and fictitious disturbances

In this section, we basically want to add a disturbance in order to obtain the same output with the linearised system than with the nonlinear system (the analysis is restrained to the second and third order harmonics). Thus, the output  $i(t)$  should be

$$y_{nl}(t) = A_1 \sin(2\pi f_c t + \psi_1) + A_2 \sin(4\pi f_c t + \psi_2) + A_3 \sin(6\pi f_c t + \psi_3) \quad (1.22)$$

## Problem 10

First, we extend our model with two input disturbances such that the linear output will also show the second and third order harmonics. The new linearised model is

$$\dot{\mathbf{x}} = A\mathbf{x} + Bu + B_d\mathbf{d}, \quad \text{with } \mathbf{d} = [d_{i1}, d_{i2}].$$

$$y = C\mathbf{x}$$

We know that  $d_{i1} = A_{i1} \sin(4\pi f_c t)$  and  $d_{i2} = A_{i2} \sin(4\pi f_c t)$  but we have to determine the magnitudes  $A_{i1}$  and  $A_{i2}$ . As this disturbance can be considered as an input, we choose to take  $B_d = [B \ B]$  and to find the right magnitudes to use.

In order to find  $A_{i1}$  and  $A_{i2}$ , we evaluate the gain of the transfer function for the three different frequencies. Then, knowing the desired amplitude for each harmonic, we can find the magnitudes  $A_{i1}$  and  $A_{i2}$ .

```
sys=ss(A,B,C,[0]);
w = [2:2:6]*pi*fc;
[MAG,PHASE] = bode(sys,w)
MAG=[MAG(1) MAG(2) MAG(3)];
Amplitude(3,1:3)./MAG % = 5.0212    0.0723    0.0792
```

## Problem 11

The disturbance is now added to our SIMULINK model (see figure B.1). The response to the input can be seen figure 1.8 and the PSD figure 1.9.

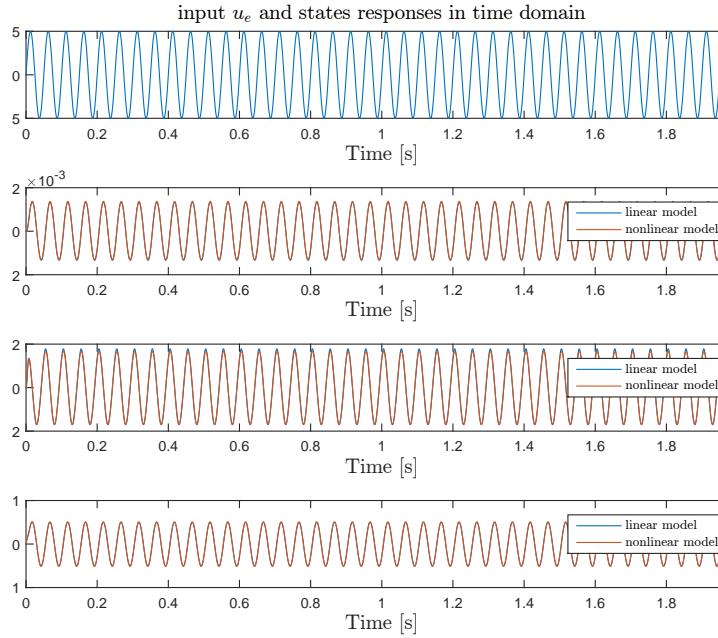


Figure 1.8: response of the states to the input  $u_e$  using the linearised model extended with disturbances and the nonlinear model

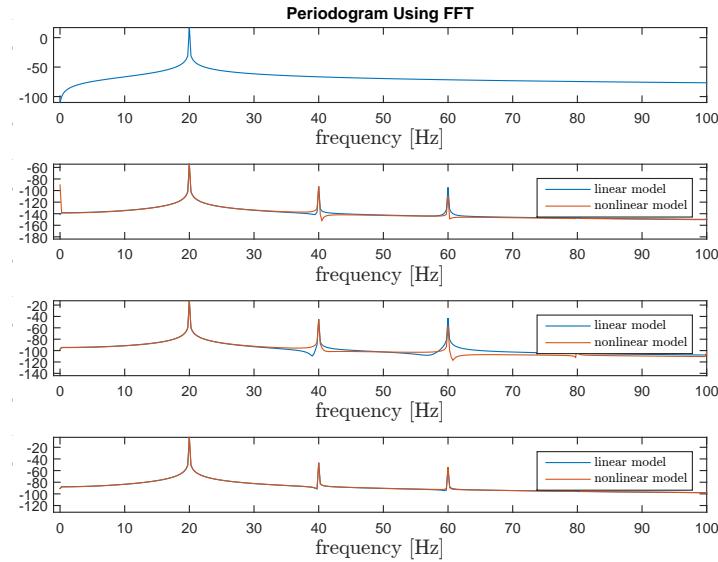


Figure 1.9: PSD of the states using the linearised model extended with disturbances and the nonlinear model

We can see on both figures (1.8 and 1.9) that the responses are almost exactly the same (the plot of the response of the nonlinear model is over the one of the linear model). As expected, the two harmonic distortions of 40 and 60 Hz are present with the right magnitude. The next distortions are of course missing. Also, the magnitude of the harmonics of state  $i$  are almost perfect, but not the ones of the

other states which is normal because the disturbance is designed for  $i$ .

### 1.1.5 System discretization

#### Problem 12

We now want to discretize the continuous time linear model. As the smallest constant time is  $\tau_2 = 0.0013$  s, we will choose a sampling time  $T_s = 0.0001$  s, that is to say 10 times faster. However, this sampling time is the *STEP\_SIZE* of the simulation, so the sampling time should be different. Therefore, we will choose  $T_s = 0.0002$  s.

Then, we can discretize the linear system using *c2d* MATLAB function.

```
Ts=0.0002; %[s]
[F, G]=c2d(A, B, Ts);
[F, Gd] = c2d(A, Bd, Ts);
lambdad=eig(F);
```

The eigenvalues,  $\lambda_1 = 0.9426$ ,  $\lambda_2 = 0.8869 + 0.0864i$  and  $\lambda_3 = 0.8869 - 0.0864i$ , of the discrete time system are inside the unit circle, which means that the system is still asymptotically stable. This result was expected because a discrete time system with a well chosen sampling time is supposed to have the same behavior and be the same system.

#### Problem 12

#### Problem 16

In order to test the performance of the discrete controller, it has been implemented on the continuous time linear system (SIMULINK model figure C.1).

As we can see figure 1.10 and figure 1.11, the controller works well, that is to say, the reference position is well followed. Indeed, with or without the noise  $\mathbf{d}$ , the tracking is the same, with a dilay  $\tau = 0.0026$  s

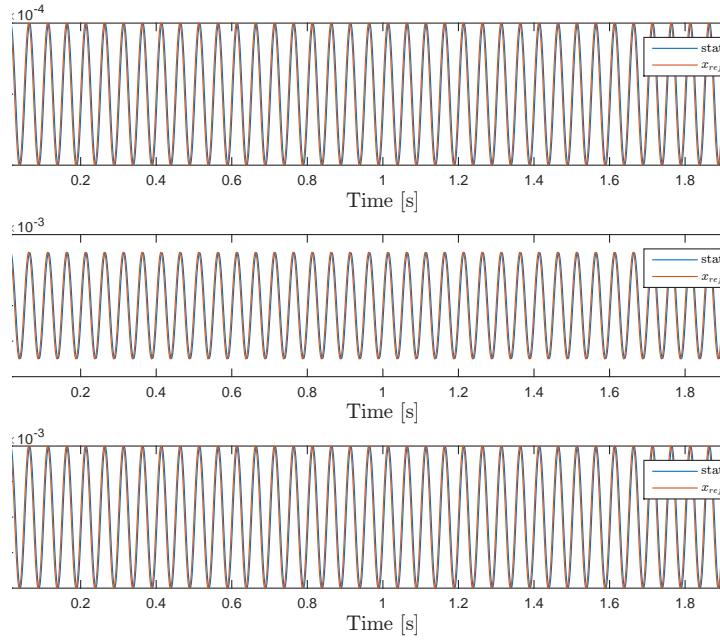


Figure 1.10: response of the state  $x$  to the input  $x_{ref}$  with  $Ax = [0.0005, 0.00075, 0.001]$  (from the top to the bottom) with the controller without noise

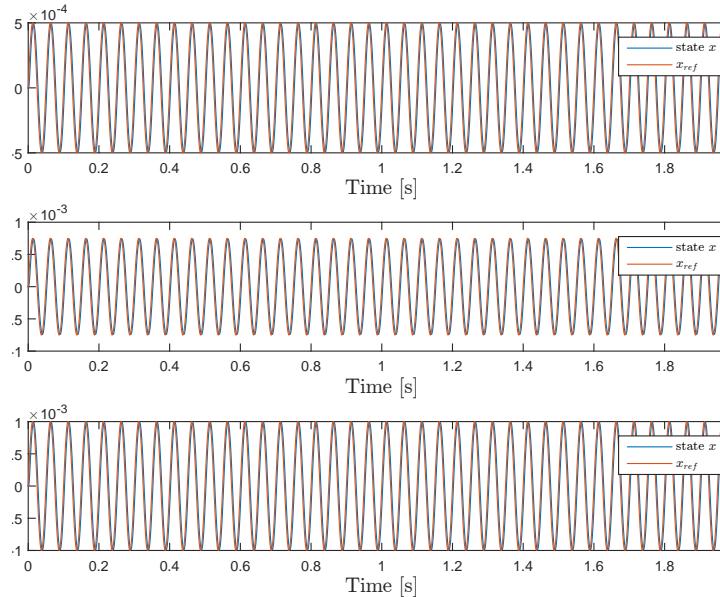


Figure 1.11: response of the state  $x$  to the input  $x_{ref}$  with  $Ax = [0.0005, 0.00075, 0.001]$  (from the top to the bottom) with the controller with the noise  $\mathbf{d}$

We can notice that it is odd that there is no difference between the simulation with or without the noise  $\mathbf{d}$ .

## Problem 17

In this problem, the discrete time feedback is tested on the nonlinear model. The controller has been implemented on the SIMULINK model of the nonlinear system (see figure D.1).

The state responses to  $x_{ref}$  has been plotted figure 1.12 in the time domain and figure 1.13 in the frequency domain.

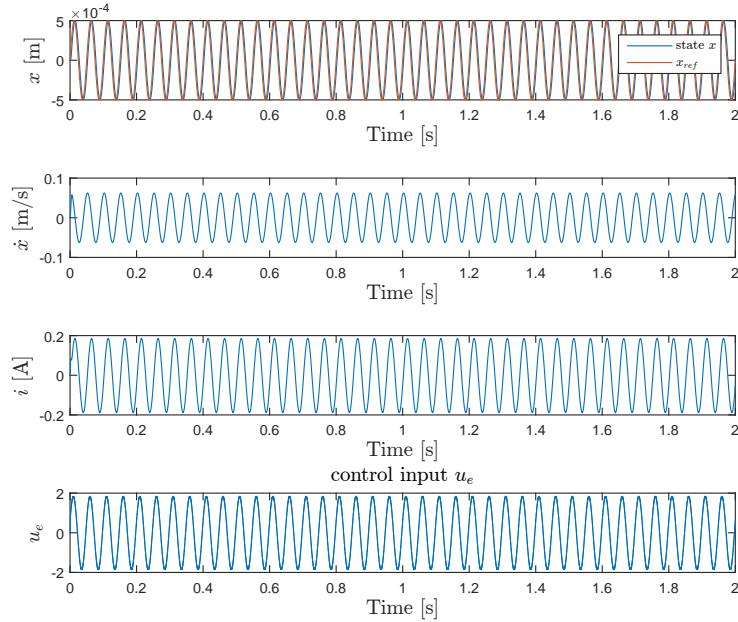


Figure 1.12:  $u_e$  and state responses to  $x_{ref}$  with  $Ax = 0.0005$  m using the controller on the nonlinear model.

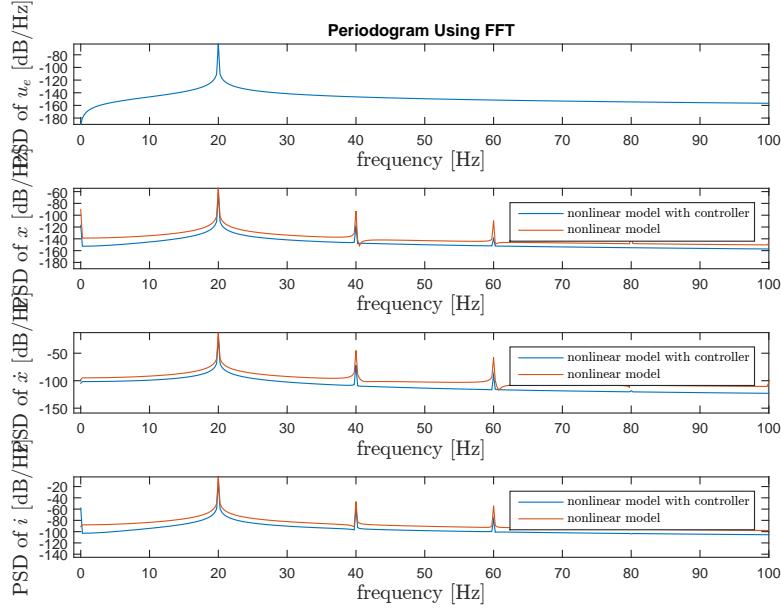


Figure 1.13: PSD of  $u_e$  and the states response to  $x_{ref}$  with  $Ax = 0.0005$  m using the controller on the nonlinear model.

As we can see figure 1.12, the reference is well tracked with a delay  $\tau = 0.0034$  s. Moreover,  $u_e$  has also been plotted figure 1.12 and we can notice that  $\max(u_e) \simeq 2 < 5$  V, which respects the condition of the maximum voltage. The simulation has also been run with  $Ax = 0.001$  m and  $\max(u_e) \simeq 3.7 < 5$  V too.

Also, we can see figure 1.13 that the harmonic distortion is attenuated. Indeed, we can see tab 1.14 that the percentage of the *Total Harmonic Distortion (THD)* has decreased a lot, there is only 12% of the initial THD left. Moreover, the distortion level of the second and third harmonic ( $d_2$ ,  $d_3$ ) have decreased a lot too (13 and 7% left). Therefore, we can say that the controller succeeded in attenuating the harmonic distortion.

	with controller	without controller
THD	0.3029 %	2.4350 %
$d_2$	0.3005 %	2.3735 %
$d_3$	0.0383 %	0.5432 %

Figure 1.14: THD,  $d_2$  and  $d_3$  of the coil velocity with the nonlinear model with and without controller (include the first 5 harmonics after the fundamental frequency  $f_c = 20$  Hz)

## Appendix A

# SIMULINK Model of the Linear System

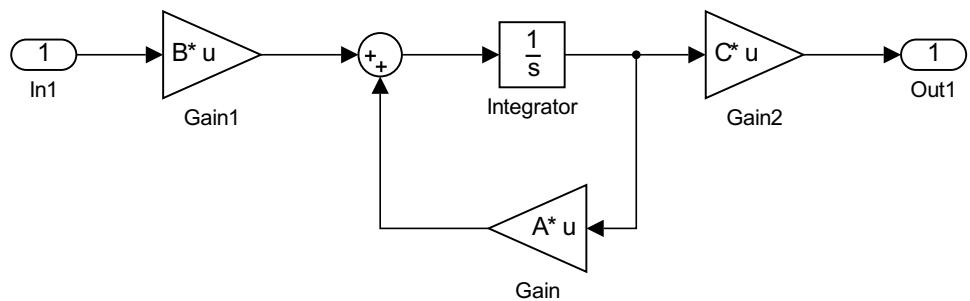


Figure A.1: SIMULINK model of the linearised loudspeaker

## Appendix B

# SIMULINK Model of the Linear System with Disturbance

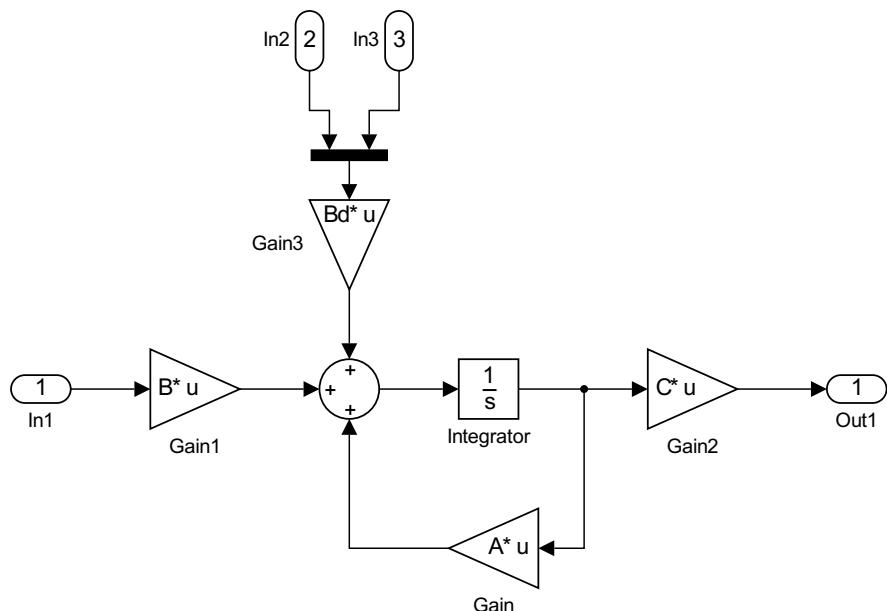


Figure B.1: SIMULINK model of the linearised loudspeaker with noise

## Appendix C

# SIMULINK Model of the Continuous Time Linear System with the Discrete Time Feedback

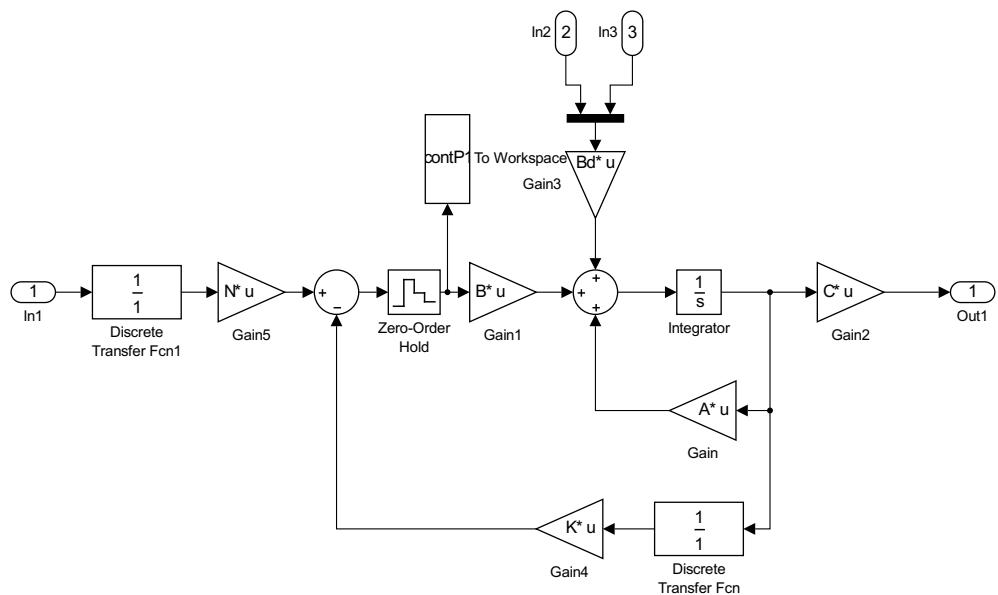


Figure C.1: SIMULINK Model of the Continuous Time Linear System with the Discrete Time Feedback

## Appendix D

# SIMULINK Model of the Nonlinear Model with the Discrete Time Feedback

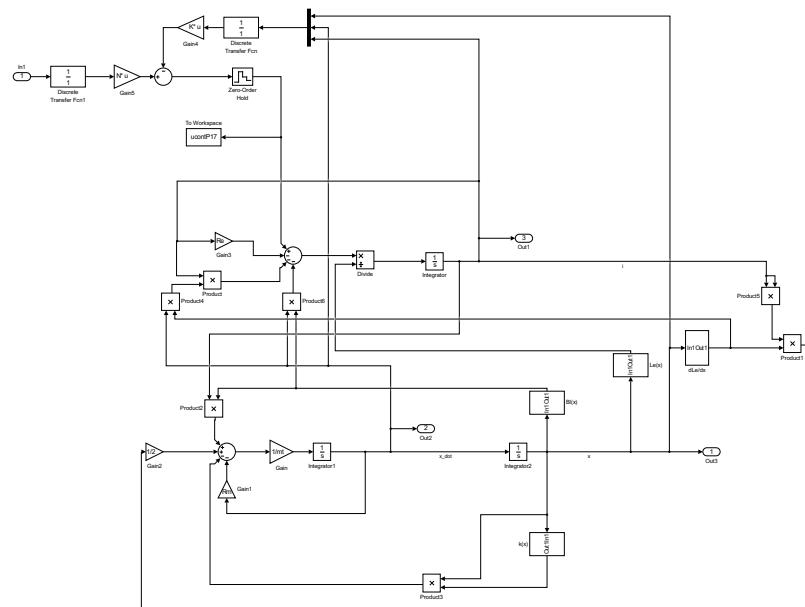


Figure D.1: SIMULINK Model of the Nonlinear Model with the Discrete Time Feedback

# Bibliography

- [1] Roberto Galeazzi, *Manual For Compulsory Exercise*, 2015.