Лабораторная работа №2
Ряды Фурье.
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Вариант 7

Процедура-функция построения тригонометрического ряда Фурье.

> $BuildFourierSeries := \mathbf{proc}(func, leftBorder, rightBorder, count)$
local $l, a0, fourierSeries$:
description "Построение ряда Фурье для функции 'func' на заданном промежутке" :

$$l := \frac{rightBorder - leftBorder}{2} :$$

$$a0 := \frac{1}{l} \cdot \int_{leftBorder}^{rightBorder} func(x)\,\mathrm{d}x :$$

$assume(n \in \mathbb{Z})$:

$$a(n) := \frac{1}{l} \cdot \int_{leftBorder}^{rightBorder} func(x) \cdot \cos\left(\frac{\pi \cdot n \cdot x}{l}\right) \mathrm{d}x :$$

$$b(n) := \frac{1}{l} \cdot \int_{leftBorder}^{rightBorder} func(x) \cdot \sin\left(\frac{\pi \cdot n \cdot x}{l}\right) \mathrm{d}x :$$

$$fourierSeries := \frac{a0}{2} + \sum_{n=1}^{count} \left(a(n) \cdot \cos\left(\frac{\pi \cdot n \cdot x}{l}\right) + b(n) \cdot \sin\left(\frac{\pi \cdot n \cdot x}{l}\right) \right) :$$

return $fourierSeries$:
end proc:

Построение графика.

> $PlotFourierSeries := \mathbf{proc}(func, leftBorder, rightBorder, count, selected\_color)$
local $l, fourierSeries$:
description "Построение графика ряда Фурье для функции 'func' на заданном промежутке" :

$fourierSeries(x) := BuildFourierSeries(func, leftBorder, rightBorder, count)$:

$$l := \frac{rightBorder - leftBorder}{2} :$$

```
return plot(fourierSeries(x), x = leftBorder − 2·l ..rightBorder + 2·l, discont
    = [showremovable], color = selected_color, legend ='Тригонометрический ряд Фурье')
    :
end proc:
```

Процедура для анимации построения графика.

```
> AnimatePlottingProcess := proc(func, leftBorder, rightBorder, count)
local l, a0, fourierSeries :
description "Построение графика ряда Фурье для функции 'func' на заданном
    промежутке" :
```

$$l := \frac{rightBorder - leftBorder}{2} :$$

$$a0 := \frac{1}{l} \cdot \int_{leftBorder}^{rightBorder} func(x)\,dx :$$

$$assume(n \in \mathbb{Z}) :$$

$$a(n) := \frac{1}{l} \cdot \int_{leftBorder}^{rightBorder} func(x) \cdot \cos\left(\frac{\pi \cdot n \cdot x}{l}\right) dx :$$

$$b(n) := \frac{1}{l} \cdot \int_{leftBorder}^{rightBorder} func(x) \cdot \sin\left(\frac{\pi \cdot n \cdot x}{l}\right) dx :$$

$$fourierSeries(x, number) := \frac{a0}{2} + \sum_{n=1}^{number} \left(a(n) \cdot \cos\left(\frac{\pi \cdot n \cdot x}{l}\right) + b(n) \cdot \sin\left(\frac{\pi \cdot n \cdot x}{l}\right) \right) :$$

```
return plots[animate](plot, [[func(x), fourierSeries(x, number)], x = leftBorder − 2·l
    ..rightBorder + 2·l], number = 1 ..count, digits = 1, frames = count) :
end proc:
```

Задание1 (получить разложение в тригонометрический ряд Фурье для 2Pi-периодической функции, построить на промежутке [-3*Pi, 3*Pi] графики $S_1(x)$, $S_3(x)$, $S_7(x)$ и $S(x)$)

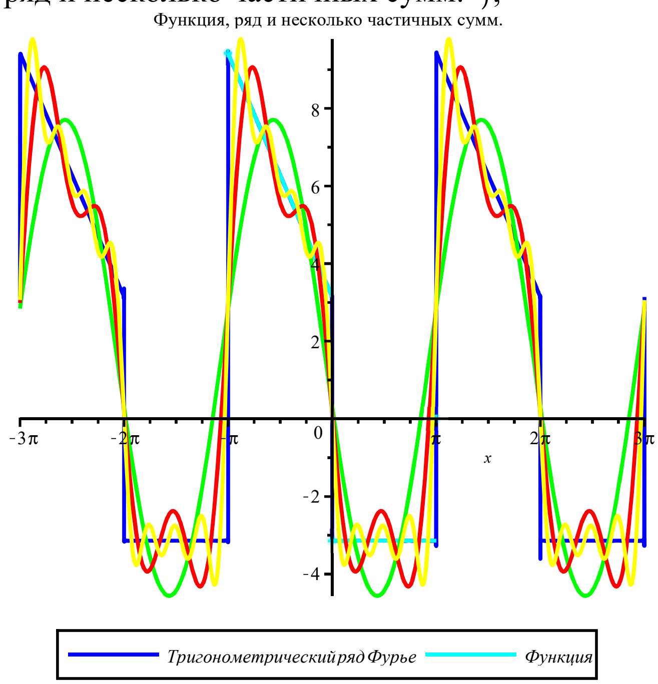
$$> f1 := x \to \begin{cases} -2 \cdot x + \pi & -\pi \le x < 0 \\ -\pi & 0 \le x < \pi \end{cases}$$

$$f1 := x \mapsto \begin{cases} -2\cdot x + \pi & -\pi \le x < 0 \\ -\pi & 0 \le x < \pi \end{cases} \tag{4.1}$$

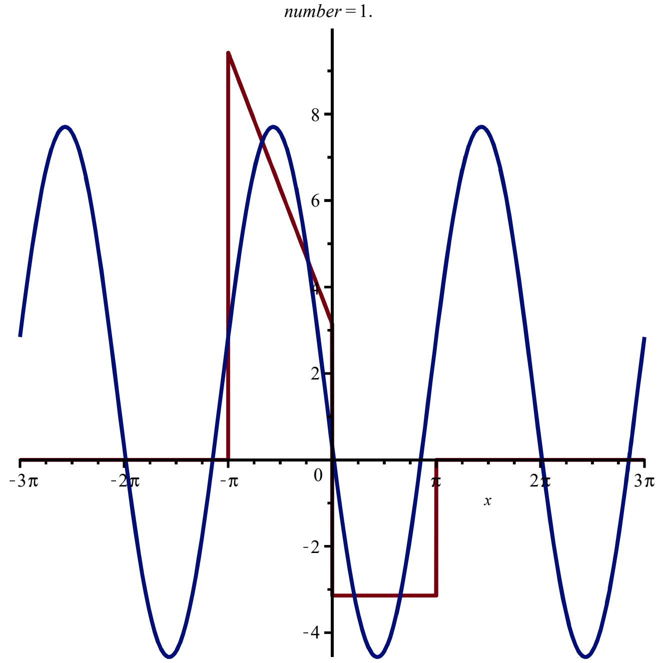
> $BuildFourierSeries(f1, -\pi, \pi, \infty);$

$$\frac{\pi}{2} + \sum_{n\sim=1}^{\infty} \left(\frac{2\left(\left(-1\right)^{n\sim} - 1\right)\cos(n\sim x)}{\pi\, n\sim^{2}} + \frac{\left(\frac{\pi\left(3\left(-1\right)^{n\sim} - 1\right)}{n\sim} + \frac{\pi\left(\left(-1\right)^{n\sim} - 1\right)}{n\sim}\right)\sin(n\sim x)}{\pi} \right) \tag{4.2}$$

> $fourierPlot1 := PlotFourierSeries(f1, -\pi, \pi, 10000, blue):$
$partSum1 := PlotFourierSeries(f1, -\pi, \pi, 1, green):$
$partSum3 := PlotFourierSeries(f1, -\pi, \pi, 3, red):$
$partSum7 := PlotFourierSeries(f1, -\pi, \pi, 7, yellow):$
$funcPlot1 := plot(f1(x), x = -\pi..\pi, discont = [showremovable], color = cyan, legend =$ 'Функция'$):$
$plots[display]([fourierPlot1, funcPlot1, partSum1, partSum3, partSum7], title$ = "Функция, ряд и несколько частичных сумм."$);$



> $AnimatePlottingProcess(f1, -\pi, \pi, 10);$



Задание2 (получить разложение тригонометрический в ряд Фурье для x2-периодической функции, заданной на промежутке (0, x1) формулой y=ax+b, а на промежутке [x1, x2] — y = c. Построить на промежутке [-2*x2, 2*x2] графики $S_1(x)$, $S_3(x)$, $S_7(x)$ и $S(x)$)

> $f2 := x \to \begin{cases} -2\cdot x - 3 & 0 < x < 4 \\ 2 & 4 \le x \le 6 \end{cases}$

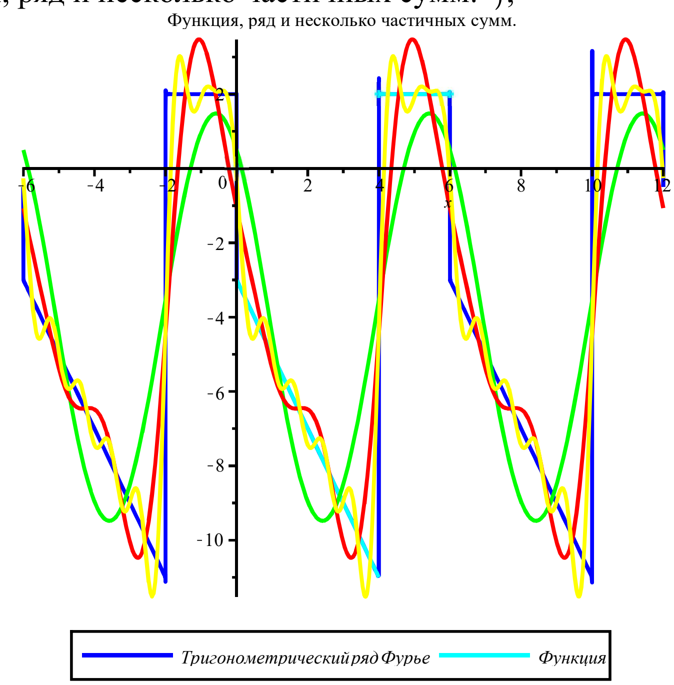
$$f2 := x \mapsto \begin{cases} -2\cdot x - 3 & 0 < x < 4 \\ 2 & 4 \le x \le 6 \end{cases} \tag{5.1}$$

> $BuildFourierSeries(f2, 0, 6, \infty);$

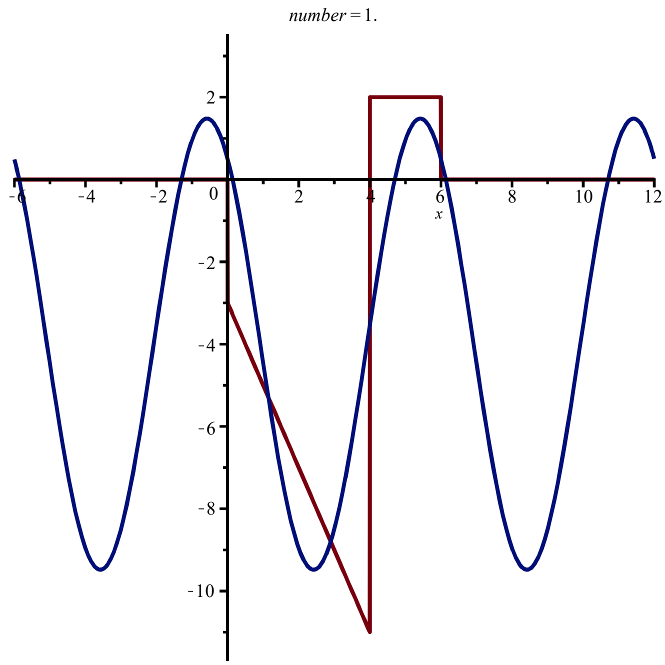
$$-4 + \sum_{n\sim=1}^{\infty} \left(\left(-\frac{11\,\pi\, n\sim \sin\left(\frac{4\,\pi\, n\sim}{3}\right) + 6\cos\left(\frac{4\,\pi\, n\sim}{3}\right) - 6}{\pi^2\, n\sim^2} - \frac{2\sin\left(\frac{4\,\pi\, n\sim}{3}\right)}{\pi\, n\sim} \right) \cos\left(\frac{\pi\, n\sim\, x}{3}\right) + \left(\frac{11\,\pi\, n\sim \cos\left(\frac{4\,\pi\, n\sim}{3}\right) - 3\,\pi\, n\sim - 6\sin\left(\frac{4\,\pi\, n\sim}{3}\right)}{\pi^2\, n\sim^2} \right. \right. \tag{5.2}$$

$$+\frac{2\left(\cos\left(\frac{4\,\pi\,n\sim}{3}\right)-1\right)}{\pi\,n\sim}\Bigg)\sin\left(\frac{\pi\,n\sim x}{3}\right)\Bigg)$$

```
> fourierPlot2 := PlotFourierSeries(f2, 0, 6, 10000, blue) :
  partSum1_2 := PlotFourierSeries(f2, 0, 6, 1, green) :
  partSum3_2 := PlotFourierSeries(f2, 0, 6, 3, red) :
  partSum7_2 := PlotFourierSeries(f2, 0, 6, 7, yellow) :
  funcPlot2 := plot(f2(x), x = 0 .. 6, discont = [showremovable], color = cyan, legend =
      'Функция') :
  plots[display]([fourierPlot2, funcPlot2, partSum1_2, partSum3_2, partSum7_2], title
      = "Функция, ряд и несколько частичных сумм.");
```




```
> AnimatePlottingProcess(f2, 0, 6, 10);
```



Задание3 (для графически заданной функции построить три разложения в тригонометрический ряд Фурье, считая, что функция определена:
— на полном периоде,
— на полупериоде и является четной,
— на полупериоде и является нечетной.
Построить графики сумм полученных рядов на промежутке, превышающем длину заданного в 3 раза)

$$> f3\_default := x \to \begin{cases} -(x-1)^2 & 0 \le x < 2 \\ \frac{1}{3}\cdot x - \frac{5}{3} & 2 \le x \le 5 \end{cases}$$

$$f3\_default := x \mapsto \begin{cases} -(x-1)^2 & 0 \le x < 2 \\ \frac{x}{3} - \frac{5}{3} & 2 \le x \le 5 \end{cases} \tag{6.1}$$

$> BuildFourierSeries(f3\_default, 0, 5, \infty);$

$$-\frac{13}{30} + \sum_{n\sim=1}^{\infty}\Bigg(\Bigg(\tag{6.2}$$

$$-\frac{2\pi^2 n\sim^2 \sin\left(\frac{4\pi n\sim}{5}\right) + 10\pi n\sim \cos\left(\frac{4\pi n\sim}{5}\right) + 10\pi n\sim - 25\sin\left(\frac{4\pi n\sim}{5}\right)}{2\pi^3 n\sim^3}$$

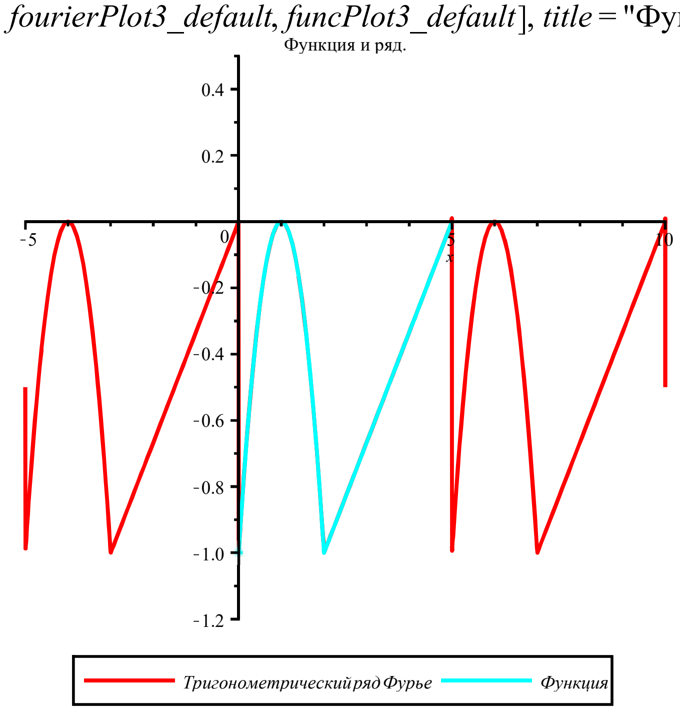
$$+\frac{6\pi n{\sim}\sin\left(\frac{4\pi n{\sim}}{5}\right)-5\cos\left(\frac{4\pi n{\sim}}{5}\right)+5}{6\pi^{2}n{\sim}^{2}}\Bigg)\cos\left(\frac{2\pi n{\sim}x}{5}\right)+\Bigg($$

$$-\frac{1}{2\pi^{3}n{\sim}^{3}}\left(-2\pi^{2}n{\sim}^{2}\cos\left(\frac{4\pi n{\sim}}{5}\right)+2\pi^{2}n{\sim}^{2}+10\pi n{\sim}\sin\left(\frac{4\pi n{\sim}}{5}\right)\right.$$

$$\left.+25\cos\left(\frac{4\pi n{\sim}}{5}\right)-25\right)-\frac{6\pi n{\sim}\cos\left(\frac{4\pi n{\sim}}{5}\right)+5\sin\left(\frac{4\pi n{\sim}}{5}\right)}{6\pi^{2}n{\sim}^{2}}\Bigg)$$

$$\sin\left(\frac{2\pi n{\sim}x}{5}\right)\Bigg)$$

> *fourierPlot3_default* := *PlotFourierSeries*(*f3_default*, 0, 5, 10000, *red*) :
funcPlot3_default := *plot*(*f3_default*(*x*), *x* = 0 .. 5, *y* =-1.2 ..0.5, *discont* = [*showremovable*], *color* = *cyan*, *legend* ='*Функция*') :
plots[*display*]([*fourierPlot3_default*, *funcPlot3_default*], *title* = "Функция и ряд.")



> $f3\_even := x \to \begin{cases} f3\_default(-x) & -5 \le x < 0 \\ f3\_default(x) & 0 \le x \le 5 \end{cases}$

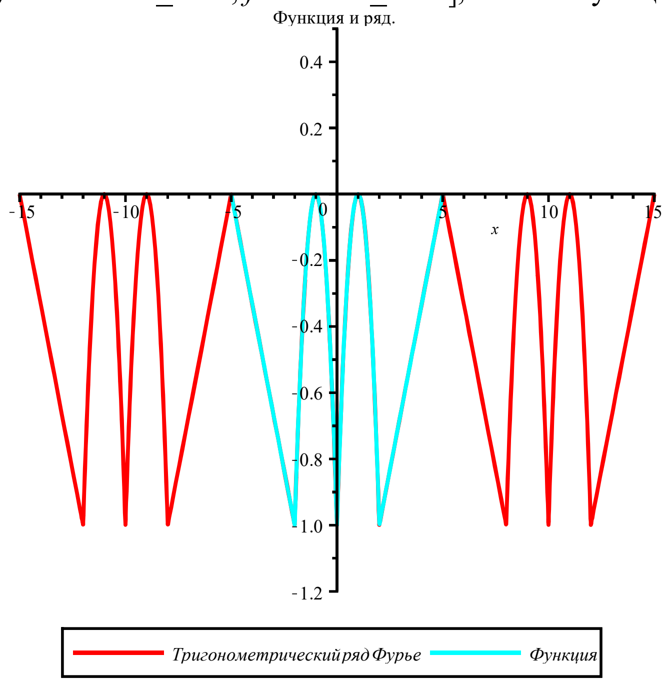
$$f3\_even := x \mapsto \begin{cases} f3\_default(-x) & -5 \le x < 0 \\ f3\_default(x) & 0 \le x \le 5 \end{cases} \tag{6.3}$$

> *BuildFourierSeries*(*f3_even*, -5, 5, ∞);

(6.4)

$$-\frac{13}{30}+\left(\sum_{n\sim=1}^{\infty}\left(\frac{2\left(3\,n\sim\pi\sin\left(\frac{2\,n\sim\pi}{5}\right)-5\cos\left(\frac{2\,n\sim\pi}{5}\right)+5\,(-1)^{n\sim}\right)}{3\,n\sim^{2}\pi^{2}}-\frac{1}{n\sim^{3}\pi^{3}}\left(2\left(n\sim^{2}\pi^{2}\sin\left(\frac{2\,n\sim\pi}{5}\right)+10\,n\sim\pi\cos\left(\frac{2\,n\sim\pi}{5}\right)+10\,n\sim\pi-50\sin\left(\frac{2\,n\sim\pi}{5}\right)\right)\right)\right)\cos\left(\frac{\pi\,n\sim x}{5}\right)\right) \tag{6.4}$$

> *fourierPlot3_even* := *PlotFourierSeries*(*f3_even*, -5, 5, 10000, *red*) :
funcPlot3_even := *plot*(*f3_even*(*x*), *x* = -5 .. 5, *y* =-1.2 ..0.5, *discont* = [*showremovable*], *color* = *cyan*, *legend* ='*Функция*') :
plots[*display*]([*fourierPlot3_even*, *funcPlot3_even*], *title* = "Функция и ряд.")



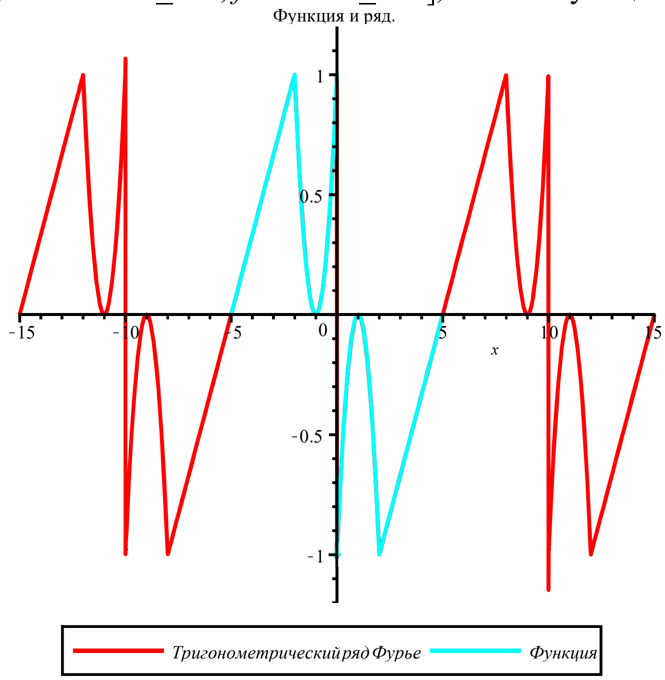
> $f3\_odd := x \rightarrow \begin{cases} -f3\_default(-x) & -5 \le x < 0 \\ f3\_default(x) & 0 \le x \le 5 \end{cases}$

$$f3\_odd := x \mapsto \begin{cases} -f3\_default(-x) & -5 \le x < 0 \\ f3\_default(x) & 0 \le x \le 5 \end{cases} \tag{6.5}$$

> *BuildFourierSeries*(*f3_odd*, -5, 5, ∞);

$$\sum_{n\sim=1}^{\infty}\left(-\frac{2\left(3\,\pi\,n\sim\cos\left(\frac{2\,\pi\,n\sim}{5}\right)+5\sin\left(\frac{2\,\pi\,n\sim}{5}\right)\right)}{3\,\pi^{2}\,n\sim^{2}}+\frac{1}{\pi^{3}\,n\sim^{3}}\left(2\left(\pi^{2}\,n\sim^{2}\cos\left(\frac{2\,\pi\,n\sim}{5}\right)-\pi^{2}\,n\sim^{2}-10\,\pi\,n\sim\sin\left(\frac{2\,\pi\,n\sim}{5}\right)-50\cos\left(\frac{2\,\pi\,n\sim}{5}\right)+50\right)\right)\right)\sin\left(\frac{\pi\,n\sim x}{5}\right) \tag{6.6}$$

```
> fourierPlot3_odd := PlotFourierSeries(f3_odd, -5, 5, 10000, red) :
  funcPlot3_odd := plot(f3_odd(x), x = -5 .. 5, y = -1.2 .. 1.2, discont = [showremovable],
      color = cyan, legend = 'Функция') :
  plots[display]([fourierPlot3_odd, funcPlot3_odd], title = "Функция и ряд.")
```



Процедура разложения в ряд Фурье по многочленам Лежандра

> *LegendreFourierSeries* := **proc**(*func*, *count*)
> **local** *fourierSeries* :
> **description** "Разложение в ряд Фурье по многочленам Лежандра" :
>
> *assume*$(n \in \mathbb{Z})$:
>
> $$P(n) := \frac{1}{n!\cdot 2^n}\cdot\frac{d^n}{dx^n}\left(\left(x^2-1\right)^n\right):$$
>
> $$C(n) := \frac{2\cdot n+1}{2}\cdot\int_{-1}^{1} func(x)\cdot P(n)\,dx:$$
>
> $$fourierSeries := \sum_{n=0}^{count}\left(C(n)\cdot P(n)\right):$$
>
> **return** *simplify*(*fourierSeries*) :
> **end proc**:

Процудура разложения в ряд Фурье по многочленам Чебышёва

> *ChebyshevFourierSeries* := **proc**(*func*, *count*)

local $C0$, $chebyshev$:

$C(k) := \frac{2}{\pi} \cdot \int_{-1}^{1} \frac{1}{\sqrt{1-x^2}} \cdot func(x) \cdot \cos(k \cdot \arccos(x))\,dx$:

$C0 := \frac{1}{\pi} \cdot \int_{-1}^{1} \frac{1}{\sqrt{1-x^2}} \cdot func(x)\,dx$:

$chebyshev := C0 + \sum_{k=1}^{count} (C(k) \cdot \cos(k \cdot \arccos(x)))$:

return $chebyshev$:

end proc:

Процедура разложения в ряд Тейлора

\> $TaylorSeries :=$ **proc**($func$, $x0$, $count$)

local $tSeries$:

$tSeries := \sum_{k=0}^{count} \frac{1}{k!} \cdot func^{(k)}(x0) \cdot (x - x0)^k$:

return $tSeries$:

end proc:

Задание4 (разложить функцию в ряд Фурье по многочленам Лежандра и Чебышёва на промежутке [-1, 1].)

\> $f4(x) := 3 \cdot (\sin(2 \cdot x))^3$;

$$f4 := x \mapsto 3 \cdot \sin(2 \cdot x)^3 \qquad \textbf{(10.1)}$$

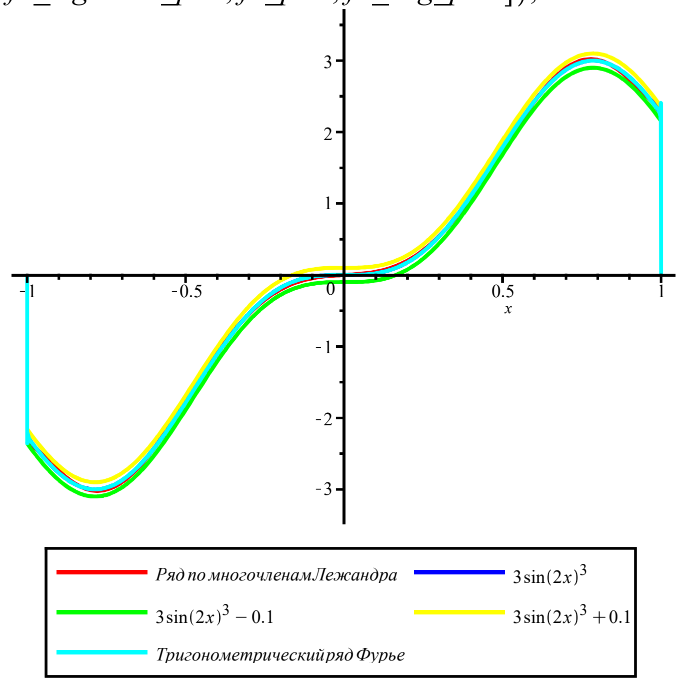
\> $f4\_legendre := LegendreFourierSeries(f4, 8)$;

$$f4\_legendre := \frac{1}{6144}\left(1200485\, x \left(\left(x^6 - \frac{15771}{8395} x^4 + \frac{22981}{21827} x^2 - \frac{109669}{720291}\right) \cos(2)^3 \right.\right. \qquad \textbf{(10.2)}$$
$$+ \frac{24661 \sin(2) \left(x^6 - \frac{5907}{3523} x^4 + \frac{35969}{45799} x^2 - \frac{130721}{1511367}\right) \cos(2)^2}{10074}$$
$$+ \left(\frac{4361745}{1679} x^6 - \frac{35369397}{8395} x^4 + \frac{1828029}{949} x^2 - \frac{51882425}{240097}\right) \cos(2)$$
$$\left.\left. + \frac{11974403 \sin(2) \left(x^6 - \frac{2774373}{1710629} x^4 + \frac{16490647}{22238177} x^2 - \frac{61040935}{733859841}\right)}{10074}\right)\right)$$

```
> ser := BuildFourierSeries(f4, -1, 1, 5000) :
```

```
> f4_trig_plot := plot(ser, x = -1..1, color = cyan, legend = 'Тригонометрический ряд Фурье') :
```

```
> f4_legendre_plot := plot(f4_legendre, x = -1..1, color = red, legend = 'Ряд по многочленам Лежандра') :
  f4_plot := plot([f4(x), f4(x) - 0.1, f4(x) + 0.1], x = -1..1, color = [blue, green, yellow],
      legend = ['3·(sin(2·x))^3', '3·(sin(2·x))^3 - 0.1', '3·(sin(2·x))^3 + 0.1']) :
  plots[display]([f4_legendre_plot, f4_plot, f4_trig_plot]);
```




```
> f4_2(x) := 3·arccos(x) + 1;
```

$$f4\_2 := x \mapsto 3 \cdot \arccos(x) + 1 \tag{10.3}$$

```
> f4_2_legendre := LegendreFourierSeries(f4_2, 20);
```

$$\begin{aligned} f4\_2\_legendre := 1 + \frac{1}{9007199254740992} \big((& -3054900150021795075\, x^{19} \\ & + 13116474399787483725\, x^{17} - 23640302034122433660\, x^{15} \\ & + 23208790338781254900\, x^{13} - 13476772533718780650\, x^{11} \\ & + 4702538480186096550\, x^{9} - 957919230820737900\, x^{7} + 104121527259449700\, x^{5} \\ & - 6594638734318635\, x^{3} - 8527509307392075\, x + 13510798882111488) \pi \big) \end{aligned} \tag{10.4}$$

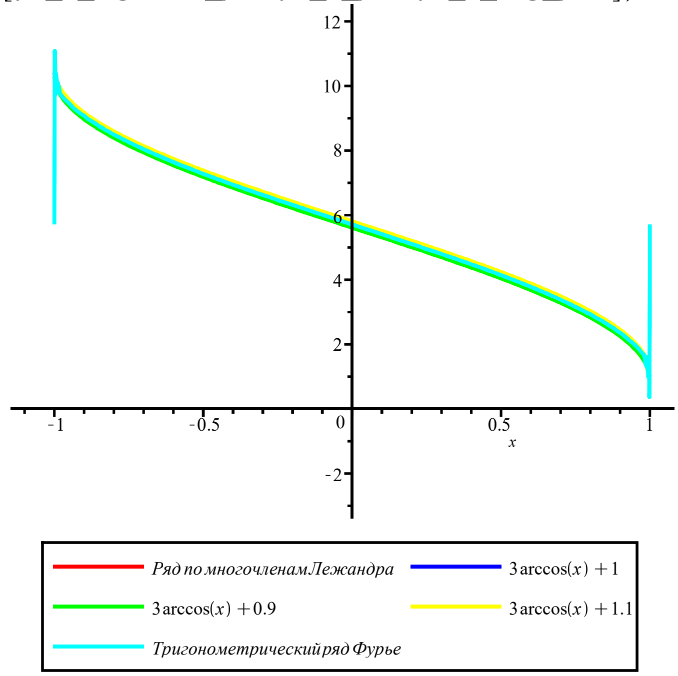
```
> ser2 := BuildFourierSeries(f4_2, -1, 1, 600) :
```

```
> f4_2_trig_plot := plot(ser2, x = -1..1, color = cyan, legend = 'Тригонометрический ряд Фурье') :
```

```
> f4_2_legendre_plot := plot(f4_2_legendre, x = -1..1, color = red, legend = 'Ряд по многочленам Лежандра') :
```

> $f4\_2\_plot := plot([f4\_2(x), f4\_2(x) - 0.1, f4\_2(x) + 0.1], x = -1..1, color = [blue, green, yellow], legend = ['3\cdot \arccos(x) + 1', '3\cdot \arccos(x) + 1 - 0.1', '3\cdot \arccos(x) + 1 + 0.1']) :$

> $plots[display]([f4\_2\_legendre\_plot, f4\_2\_plot, f4\_2\_trig\_plot]);$



> $f4\_fourierSeries := BuildFourierSeries(f4, -1, 1, \infty);$
$f4\_2\_fourierSeries := BuildFourierSeries(f4\_2, -1, 1, \infty);$

$$f4\_fourierSeries := \sum_{n\sim=1}^{\infty} \left(-\frac{1}{2\left(\pi^4 n\sim^4 - 40\pi^2 n\sim^2 + 144\right)} \left(3\pi(-1)^{n\sim} n\sim \left(3\pi^2 \sin(2) n\sim^2 - \sin(6)\pi^2 n\sim^2 - 108\sin(2) + 4\sin(6)\right) \sin(\pi n\sim x)\right) \right)$$

$$f4\_2\_fourierSeries := 1 + \frac{3\pi}{2} + \left(\sum_{n\sim=1}^{\infty} \left(\int_{-1}^{1} (3\arccos(x) + 1)\sin(\pi n\sim x)\, dx \right) \sin(\pi n\sim x) \right) \quad \textbf{(10.5)}$$

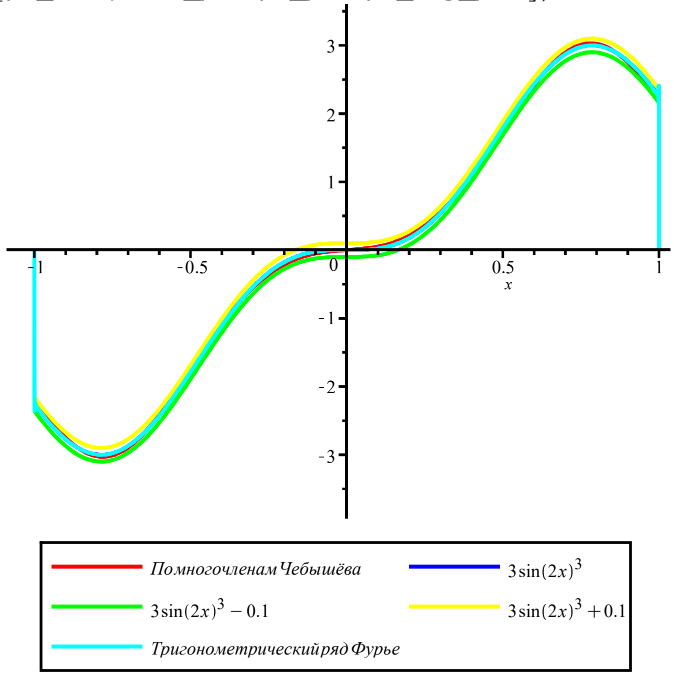
Ниже разложения по многочленам Чебышёва

> $f4\_chebyshev := ChebyshevFourierSeries(f4, 7) :$
$f4\_chebyshev\_infinity := ChebyshevFourierSeries(f4, \infty);$

$$f4\_chebyshev\_infinity := \sum_{k=1}^{\infty} \frac{2\left(\int_{-1}^{1} \frac{3\sin(2x)^3 \cos(k\arccos(x))}{\sqrt{-x^2+1}}\, dx\right)\cos(k\arccos(x))}{\pi} \quad \textbf{(10.6)}$$

> $f4\_chebyshev\_plot := plot(f4\_chebyshev, x = -1..1, color = red, legend =$

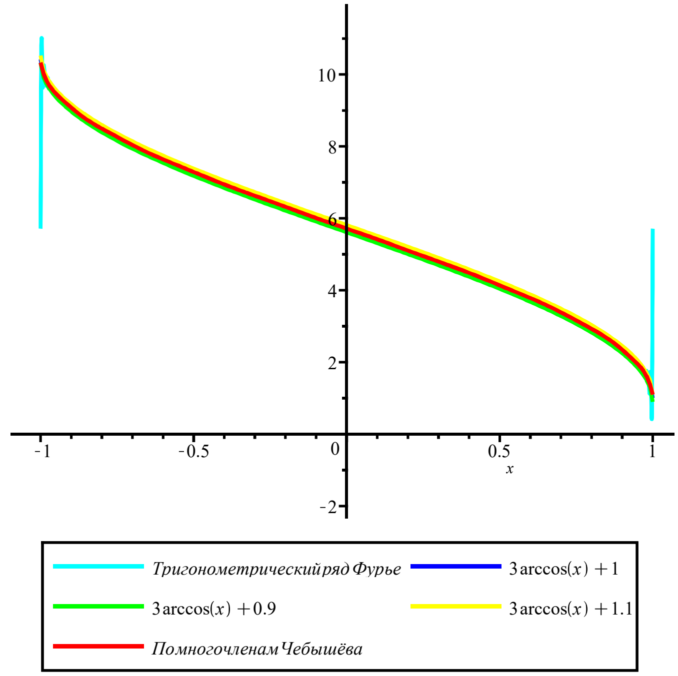
'По многочленам Чебышёва') :
plots[display]([f4_chebyshev_plot, f4_plot, f4_trig_plot]);



> f4_2_chebyshev := ChebyshevFourierSeries(f4_2, 19) :
f4_2_chebyshev_infinity := ChebyshevFourierSeries(f4_2, ∞);

$$f4\_2\_chebyshev\_infinity := \frac{\frac{3}{2}\pi^2 + \pi}{\pi} + \left(\sum_{k=1}^{\infty} \frac{2\left(3\pi k \sin(\pi k) + \sin(\pi k)\, k + 3\cos(\pi k) - 3\right) \cos(k \arccos(x))}{\pi k^2} \right) \tag{10.7}$$

> f4_2_chebyshev_plot := plot(f4_2_chebyshev, x = -1 ..1, color = red, legend = 'По многочленам Чебышёва') :
plots[display]([f4_2_trig_plot, f4_2_plot, f4_2_chebyshev_plot]);



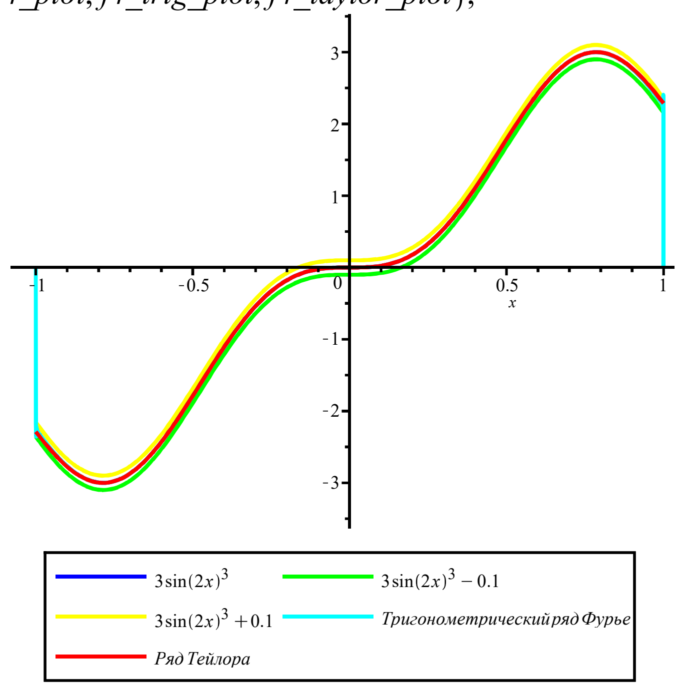
Ниже разложение в ряд Тейлора

> $f4\_taylor := TaylorSeries(f4, 0, 15);$

$$f4\_taylor := 24\,x^3 - 48\,x^5 + \frac{208}{5}\,x^7 - \frac{1312}{63}\,x^9 + \frac{10736}{1575}\,x^{11} - \frac{2336}{1485}\,x^{13} + \frac{19131872}{70945875}\,x^{15} \tag{10.8}$$

> $f4\_taylor\_plot := plot(f4\_taylor, x = -1..1, color = red, legend = 'Ряд\ Тейлора') :$

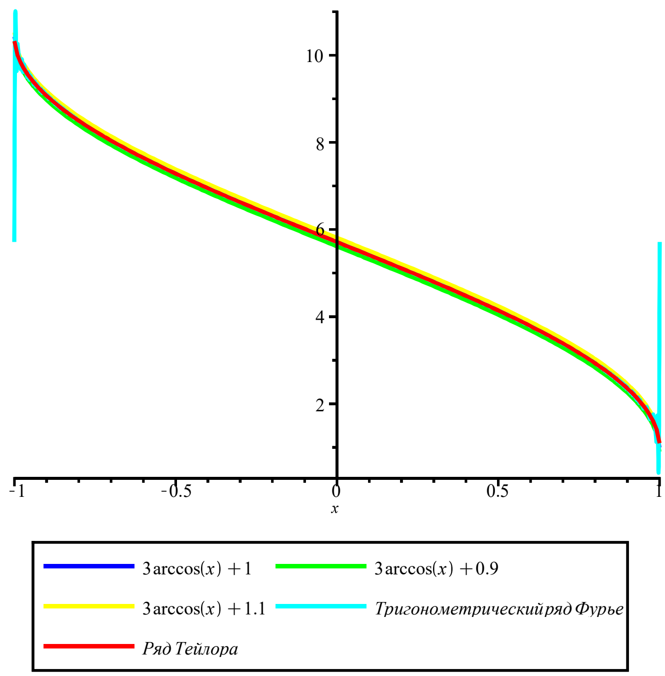
> $plots[display](f4\_plot, f4\_trig\_plot, f4\_taylor\_plot);$



> $f4\_2\_taylor := TaylorSeries(f4\_2, 0, 600) :$

> $f4\_2\_taylor\_plot := plot(f4\_2\_taylor, x = -1..1, color = red, legend = 'Ряд\ Тейлора') :$

> $plots[display](f4\_2\_plot, f4\_2\_trig\_plot, f4\_2\_taylor\_plot);$



Задание4 (используя orthopoly)

> $LegendreBuiltIn := \mathbf{proc}(func, count)$
local $fourierSeries$:
description "Разложение в ряд Фурье по многочленам Лежандра" :

$assume(n \in \mathbb{Z})$:
$with(orthopoly)$:

$$C(n) := \frac{2 \cdot n + 1}{2} \cdot \int_{-1}^{1} func(x) \cdot P(n, x)\, \mathrm{d}x :$$

$$fourierSeries := \sum_{n=0}^{count} (C(n) \cdot P(n, x)) :$$

return $fourierSeries$:
end proc:

> $ChebyshevBuiltIn := \mathbf{proc}(func, count)$
local $C0$, $chebyshev$:
$assume(n \in \mathbb{Z})$:
$with(orthopoly)$:

$$C(k) := \frac{2}{\pi} \cdot \int_{-1}^{1} \frac{1}{\sqrt{1 - x^2}} \cdot func(x) \cdot T(k, x)\mathrm{d}x :$$

$$C0 := \frac{1}{\pi} \cdot \int_{-1}^{1} \frac{1}{\sqrt{1 - x^2}} \cdot func(x)\mathrm{d}x :$$

$$chebyshev := C0 + \sum_{k=1}^{count} (C(k) \cdot T(k, x)) :$$

return *chebyshev* :

end proc:

> $F1(x) := 3 \cdot (\sin(2 \cdot x))^3;$
> $F2(x) := 3 \cdot \arccos(x) + 1;$

$$F1 := x \mapsto 3 \cdot \sin(2 \cdot x)^3$$

$$F2 := x \mapsto 3 \cdot \arccos(x) + 1 \qquad \textbf{(11.1)}$$

> $L1 := LegendreBuiltIn(F1, 8) :$

> $L2 := LegendreBuiltIn(F2, 29) :$

> $C1 := ChebyshevBuiltIn(F1, 7) :$

> $C2 := ChebyshevBuiltIn(F2, 19) :$

> $Four1 := BuildFourierSeries(F1, -1, 1, 4000) :$

> $Four2 := BuildFourierSeries(F2, -1, 1, 300) :$

> $F1\_Taylor := TaylorSeries(F1, 0, 15) :$

> $F2\_Taylor := TaylorSeries(F2, 0, 590) :$

> *F1_L1plot* := *plot*(*L1*, *x* = -1 ..1, *color* = *red*, *legend* = '*Лежандра*') :

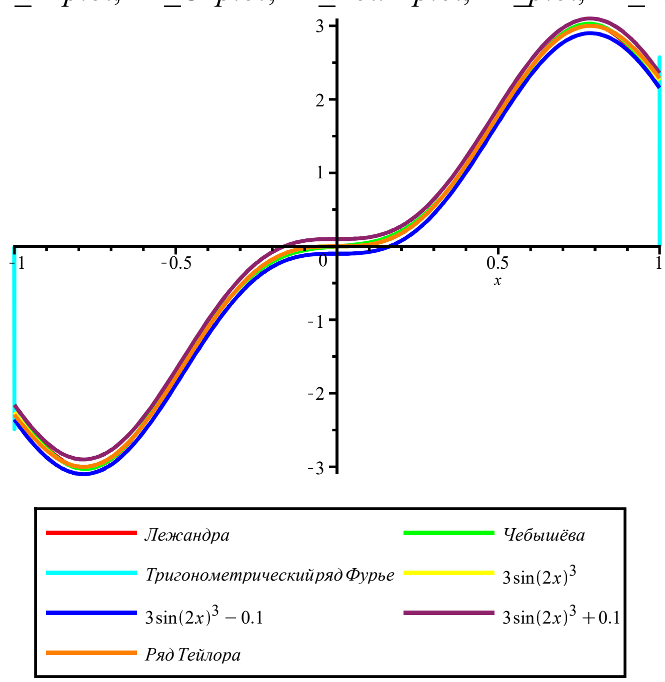
> *F1_C1plot* := *plot*(*C1*, *x* = -1 ..1, *color* = *green*, *legend* = '*Чебышёва*') :

> *F1_Four1plot* := *plot*(*Four1*, *x* = -1 ..1, *color* = *cyan*, *legend* = '*Тригонометрический ряд Фурье*') :

> *F1_Taylorplot* := *plot*(*F1_Taylor*, *x* = -1 ..1, *color* = *coral*, *legend* = '*Ряд Тейлора*') :

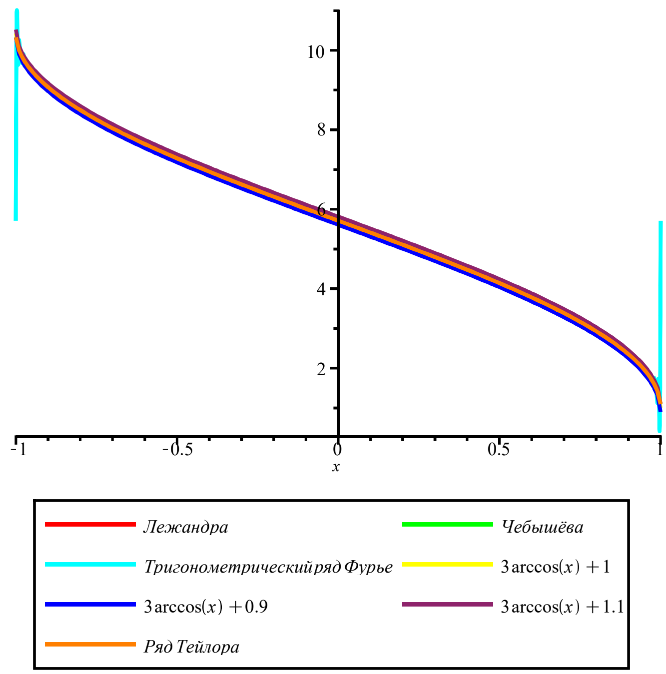
> $F1\_plot := plot\left([F1(x), F1(x) - 0.1, F1(x) + 0.1], x = -1 ..1, color = [yellow, blue, maroon], legend = \left['3 \cdot (\sin(2 \cdot x))^3', '3 \cdot (\sin(2 \cdot x))^3 - 0.1', '3 \cdot (\sin(2 \cdot x))^3 + 0.1' \right] \right) :$

> $plots[display](F1\_L1plot, F1\_C1plot, F1\_Four1plot, F1\_plot, F1\_Taylorplot);$



> *F2_L2plot* := *plot*(*L2*, *x* = -1 ..1, *color* = *red*, *legend* = '*Лежандра*') :

```
> F2_C2plot := plot(C2, x = -1 ..1, color = green, legend = 'Чебышёва') :
> F2_Four2plot := plot(Four2, x = -1 ..1, color = cyan, legend =
    'Тригонометрический ряд Фурье') :
> F2_Taylorplot := plot(F2_Taylor, x = -1 ..1, color = coral, legend = 'Ряд Тейлора') :
> F2_plot := plot([F2(x), F2(x) - 0.1, F2(x) + 0.1], x = -1 ..1, color = [yellow, blue,
    maroon], legend = ['3·arccos(x) + 1','3·arccos(x) + 1 - 0.1','3·arccos(x) + 1 + 0.1'])
    :
> plots[display](F2_L2plot, F2_C2plot, F2_Four2plot, F2_plot, F2_Taylorplot);
```

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>
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