Лабораторная работа №3.2
Обыкновенные ДУ высших порядков
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Вариант 7

Задание1 (решить уравнения, построить в одной системе координат несколько интегральных кривых)

1.1 x = ch(y'') + (y'')^2, y'' = t

> $x\_ := \cosh(t) + t^2;$

$$x\_ := \cosh(t) + t^2 \quad \textbf{(1.1)}$$

> $dx := diff(x\_, t)$

$$dx := \sinh(t) + 2\,t \quad \textbf{(1.2)}$$

dy'=tdx

> $dy\_1 := t \cdot dx$

$$dy\_1 := t\,(\sinh(t) + 2\,t) \quad \textbf{(1.3)}$$

> $y\_1 := rhs(dsolve(diff(\_y(t), t) = dy\_1))$

$$y\_1 := t\cosh(t) - \sinh(t) + \frac{2\,t^3}{3} + \_C1 \quad \textbf{(1.4)}$$

> $y\_func := rhs(dsolve(diff(\_y(t), t) = y\_1))$

$$y\_func := t\sinh(t) - 2\cosh(t) + \frac{t^4}{6} + \_C1\,t + \_C2 \quad \textbf{(1.5)}$$

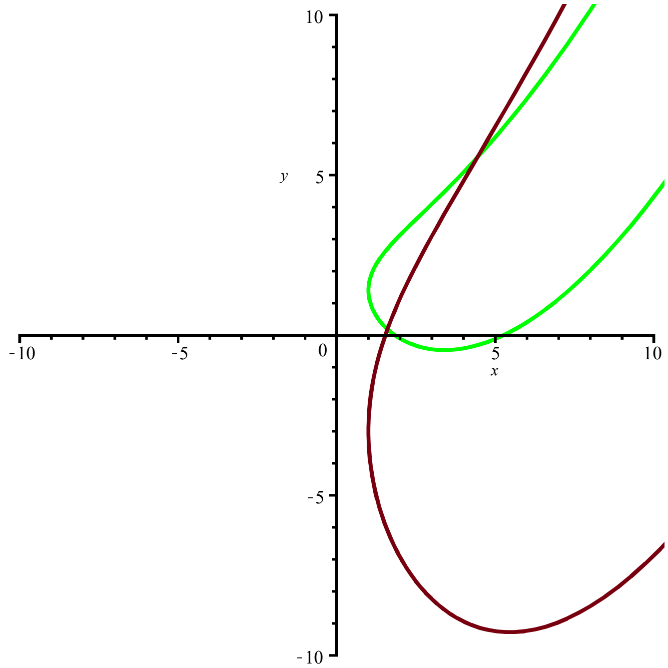
> $y\_ := subs(\_C1 = -2, \_C2 = 3.4, y\_func) :$

> $curve1 := plot([x\_, y\_, t = -5..5], x = -10..10, y = -2..10, color = green) :$

> $y\_ := subs(\_C1 = 5, \_C2 = -1, y\_func) :$

> $curve2 := plot([x\_, y\_, t = -5..5], x = -10..10, y = -10..10) :$

> $plots[display](curve1, curve2);$



1.2 sinx*cosx*(yy'' - y'^2)=2yy'

> $de12 := \sin(x)\cdot\cos(x)\cdot\left(y(x)\cdot y''(x) - (y'(x))^2\right) = 2\cdot y(x)\cdot y'(x)$

$$de12 := \sin(x)\cos(x)\left(y(x)\left(\frac{d^2}{dx^2}y(x)\right) - \left(\frac{d}{dx}y(x)\right)^2\right) = 2\,y(x)\left(\frac{d}{dx}y(x)\right) \quad \textbf{(1.6)}$$

> $dsolve(de12, y(x))$

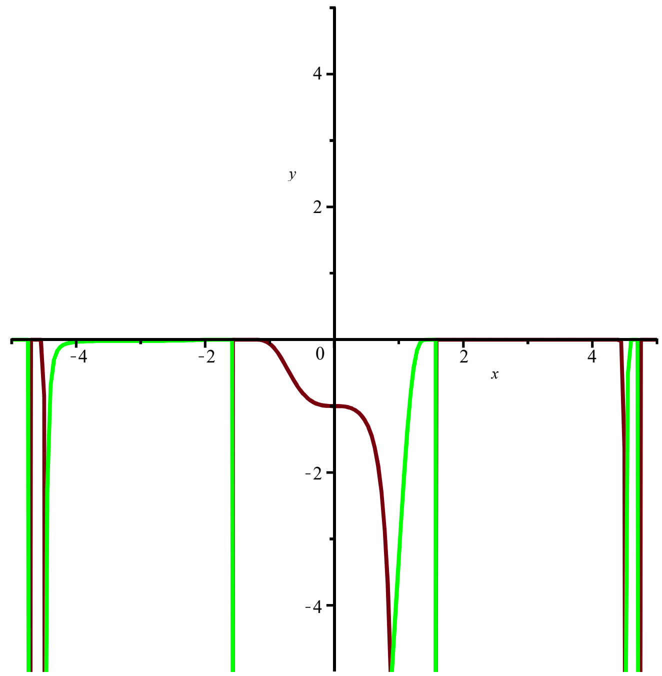
$$y(x) = \frac{\_C2\, e^{\frac{\_C1}{\sin(2x)}}}{e^{-\_C1\,x}\, e^{\frac{\_C1\cos(2x)}{\sin(2x)}}} \quad \textbf{(1.7)}$$

> $y\_func := rhs(\textbf{(1.7)})$:

> $curve1 := plot(subs(\_C1 = 5, \_C2 = -1, y\_func), x = -5..5, y = -5..5)$:

> $curve2 := plot(subs(\_C1 = -2, \_C2 = -10, y\_func), x = -5..5, y = -5..5, color = green)$:

> $plots[display](curve1, curve2);$



1.3

> $de13 := y''(x) \cdot x \cdot \ln(x) - y'(x) = 0$

$$de13 := \left(\frac{d^2}{dx^2}\, y(x)\right) x \ln(x) - \frac{d}{dx}\, y(x) = 0 \qquad (1.8)$$

> $dsolve(de13, y(x))$

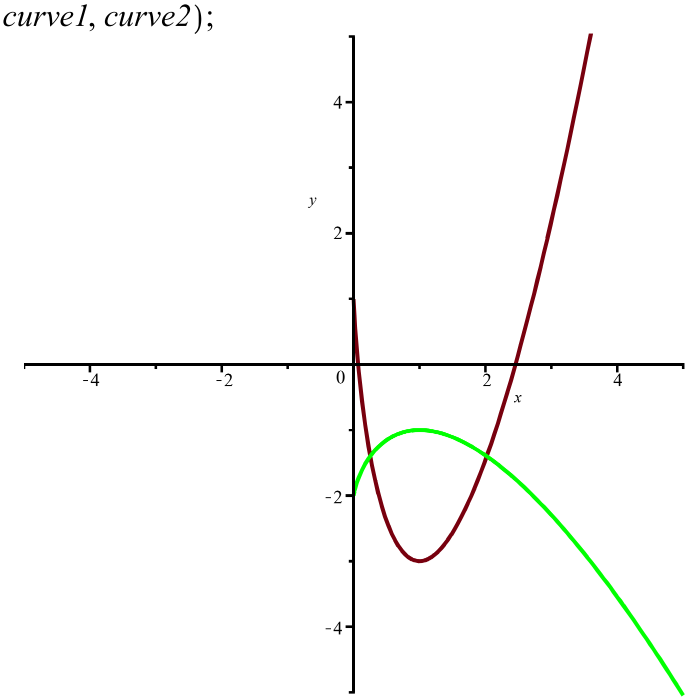
$$y(x) = \_C1 + (x \ln(x) - x)\ \_C2 \qquad (1.9)$$

> $y\_func := rhs(\mathbf{(1.9)})$:

> $curve1 := plot(subs(\_C1 = 1, \_C2 = 4, y\_func), x = -5..5, y = -5..5)$:

> $curve2 := plot(subs(\_C1 = -2, \_C2 = -1, y\_func), x = -5..5, y = -5..5, color = green)$:

> $plots[display](curve1, curve2);$



1.4

> $de14 := y''(x) + \frac{3}{x}\cdot y'(x) - \frac{3}{x^2}\cdot y(x) = 16\cdot x^3\cdot e^{x^4}$

$$de14 := \frac{d^2}{dx^2}\, y(x) + \frac{3\left(\frac{d}{dx}\, y(x)\right)}{x} - \frac{3\, y(x)}{x^2} = 16\, x^3\, e^{x^4} \tag{1.10}$$

> $dsolve(de14, y(x))$

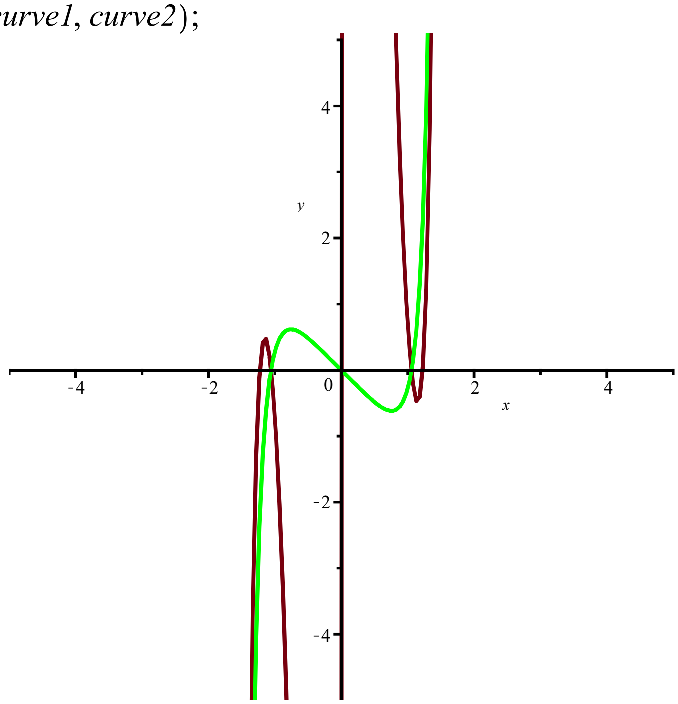
$$y(x) = \frac{\_C2}{x^3} + \_C1\, x + \frac{e^{x^4}}{x^3} \tag{1.11}$$

> $y\_func := rhs(\textbf{(1.11)}):$

> $curve1 := plot(subs(\_C1 = -6,\ \_C2 = 4,\ y\_func),\ x = -5..5,\ y = -5..5):$

> $curve2 := plot(subs(\_C1 = -2,\ \_C2 = -1,\ y\_func),\ x = -5..5,\ y = -5..5,\ color = green):$

> $plots[display](curve1, curve2);$



Задание2 (найти общее решение уравнения)

> $de := y'''(x)\cdot \cot(2\cdot x) + 2\cdot y''(x) = 0$

$$de := \left(\frac{d^3}{dx^3}\, y(x)\right) \cot(2\, x) + 2\, \frac{d^2}{dx^2}\, y(x) = 0 \tag{2.1}$$

> $dsolve(de, y(x))$

$$y(x) = \frac{\_C1\, \sin(4\, x)}{4\sqrt{-\frac{2}{\cos(4\, x) - 1}}\, (\cos(4\, x) - 1)} + \_C2\, x + \_C3 \tag{2.2}$$

> $y\_func := rhs(\textbf{(2.2)}):$

> $curve1 := plot(subs(\_C1 = 7,\ \_C2 = 4,\ \_C3 = 2,\ y\_func),\ x = -5..5,\ y = -5..5):$

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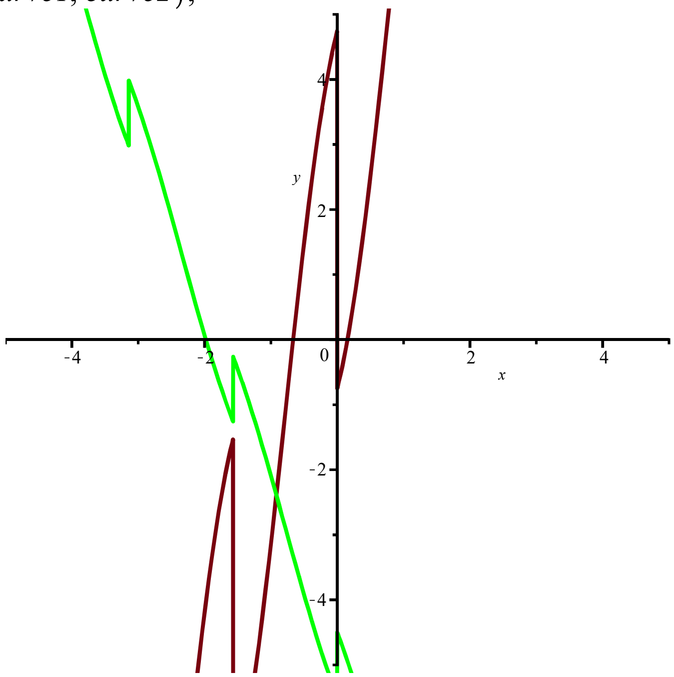
> curve2 := plot(subs(_C1 =-2, _C2 =-2.7, _C3 =-5, y_func), x =-5 ..5, y =-5 ..5, color
     = green) :
> plots[display](curve1, curve2);
```



Задание3 (найти общее решение ДУ)

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> de := y''(x) + 2·y'(x) = e^x·(sin(x) + cos(x));
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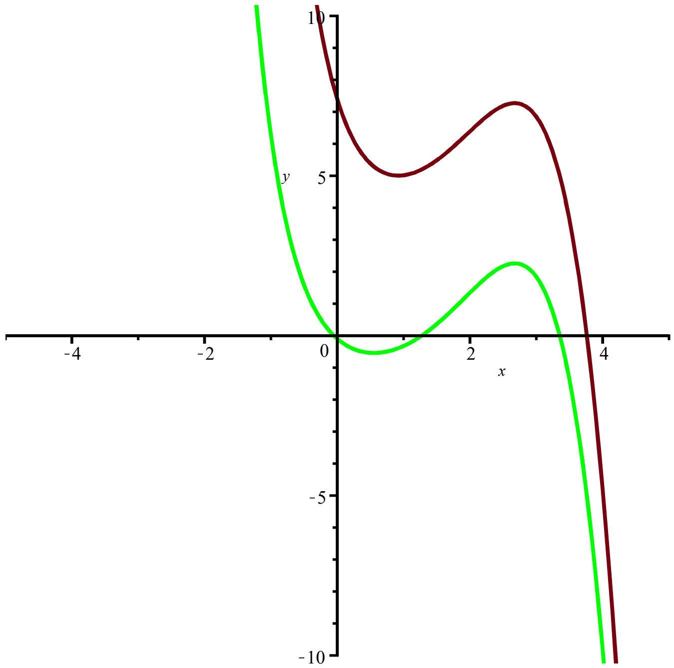
$$de := \frac{d^2}{dx^2}\, y(x) + 2\, \frac{d}{dx}\, y(x) = e^x\, (\sin(x) + \cos(x)) \quad \mathbf{(3.1)}$$

```
> dsolve(de, y(x));
```

$$y(x) = -\frac{e^x \cos(x)}{10} + \frac{3\, e^x \sin(x)}{10} - \frac{\_C1}{2\, \left(e^x\right)^2} + \_C2 \quad \mathbf{(3.2)}$$

```
> y_func := rhs((3.2)) :
> curve1 := plot(subs(_C1 =-7, _C2 = 4, y_func), x =-5 ..5, y =-10 ..10) :
> curve2 := plot(subs(_C1 =-2, _C2 =-1, y_func), x =-5 ..5, y =-5 ..5, color = green) :
> plots[display](curve1, curve2);
```



10
5
y
-4
-2
0
2
4
x
-5
-10