1.1 912 or 841

1.2 $\{\{1,1,1,1,1\}\ \{1,1,1,1,2\}\ \cdots\ \{N,N,N,N,N\}\}$: N^5

1.3 $\frac{1}{N^5}$ because $\frac{1}{N}\cdot\frac{1}{N}\cdot\frac{1}{N}\cdot\frac{1}{N}\cdot\frac{1}{N} = \frac{1}{N^5}$

1.4 912 or 841

1.5 unreasonable

1.6 range of $(n_1 \cdots n_k)$ should be the total number

1.7 $P(N=500) = 0$, false

1.8 ① $P(N=n) : C \cdot F(n)$

$= C \cdot \frac{1}{n^5}$

$= C \sum_{n=912}^{\infty} \frac{1}{n^5} = 1$

$C \int_{912}^{\infty} \frac{1}{n^5}\, dn$

$C \frac{1}{4 \cdot 912^4} = 1$

$C = 4 \cdot 912^4 = 2.767 \cdot 10^{12}$

② As $n \to \infty$,

$C \cdot F(n) = \frac{C}{n^5}$

$= \frac{4 \cdot 912^4}{n^5}$

As $n \to \infty$

$\frac{C}{n^5} = \frac{4 \cdot 912^4}{\infty^5}$

then, it will goes to 0

1.9. $P(N < n_{max}) =$

$0.95 = 2.767 \cdot 10^{12} \cdot F(n_{max})$

$= 2.767 \cdot 10^{12} \cdot \frac{1}{4 (n_{max})^4}$

$= 6.918 \cdot 10^{11} \cdot \frac{1}{(n_{max})^4}$

$(n_{max}) = \sqrt[4]{\frac{6.918 \cdot 10^{11}}{0.95}} = 923.7708$

2.1 530

2.2 Assume, we have total of N drones in wild, then total ~~[illegible]~~ outcome of 400 in N is $\binom{N}{400} = \frac{N!}{400!(N-400)!}$

2.3 catch 270 tagged drone in 400 will be : $\binom{400}{270}$

catch 130 untagged drone ~~in 400 will be~~ will be : $\binom{N-400}{130}$

where you catch 400 drone in N : $\binom{N}{400}$

$\therefore$ catch 270 tagged & 130 untagged : $\dfrac{\binom{400}{270}\cdot\binom{N-400}{130}}{\binom{N}{400}}$

$$= \frac{\frac{400!}{270!(400-270)!}\cdot\frac{(N-400)!}{130!((N-400)-130)!}}{\frac{N!}{400!(N-400)!}}$$

2-4 when $F(N) \geq F(N+1)$:

$$\frac{\frac{400!}{270!\,130!}\cdot\frac{(N-400)!}{130!((N-400)-130)!}}{\frac{N!}{400!(N-400)!}} \geq \frac{\frac{400!}{270!\,130!}\cdot\frac{((N+1)-400)!}{130!(((N+1)-400)-130)!}}{\frac{(N+1)!}{400!((N+1)-400)!}}$$

$$= \frac{\frac{400!}{270!\,130!}\cdot\frac{(N-400)!}{130!((N-400)-130)!}}{\frac{N!}{400!(N-400)!}} \cdot \frac{\frac{(N+1)!}{400!((N+1)-400)!}}{\frac{400!}{270!\,130!}\cdot\frac{((N+1)-400)!}{130!(((N+1)-400)-130)!}} \geq 1$$

$$= \frac{(N+1)(N-529)}{(N-399)^2} \geq 1$$

$= 270N \geq 159730$ $\quad N \geq 591.59259$

$N \geq 592$

$\therefore$ the answer is $591.59 \leq N \leq 592.59$

when $F(N) \geq F(N-1)$

$$\frac{\frac{400!}{270!\,130!}\cdot\frac{(N-400)!}{130!((N-400)-130)!}}{\frac{N!}{400!(N-400)!}} \cdot \frac{\frac{(N-1)!}{400!((N-1)-400)!}}{\frac{400!}{270!\,130!}\cdot\frac{((N-1)-400)!}{130!(((N-1)-400)-130)!}}$$

$$= \frac{(N-400)^2}{N(N-530)} \geq 1$$

$N \geq 592.59$

$N \geq 593$ ~~is the answer, because~~

$\therefore \frac{F(N)}{F(N+1)} = \frac{F(1000)}{F(1000+1)} = 1.0002$

$\boxed{\frac{F(N)}{F(N-1)}} = \frac{F(1000)}{F(1000-1)} = 0.999\ldots$

→ more close to 1

we need to put 1000 in two functions, and compare with 1

2.5 For $P(N \le n_{max}) = 0.95$

$$\therefore C \cdot F(N) = 0.95$$

$$C \cdot F(N) = F(N) \cdot \frac{1}{F(N-1)} =$$

$$\frac{\dfrac{400!}{270!\,130!}\;\dfrac{(N-400)!}{130!\,((N-400)-130)!}}{\dfrac{N!}{400!\,(N-400)!}} \cdot \frac{\dfrac{(N-1)!}{400!\,((N-1)-400)!}}{\dfrac{400!}{270!\,130!}\;\dfrac{((N-1)-400)!}{130!\,(((N-1)-400)-130)!}}$$

$$= \frac{(N-400)^2}{N(N-530)} \qquad \therefore \frac{(n_{max}-400)^2}{n_{max}(n_{max}-530)} = 0.95$$

$$(n_{max}-400)^2 = 0.95(n_{max}(n_{max}-530))$$

$$n_{max,1} = \frac{29650 + \sqrt{559122500}}{10}$$

$$n_{max,2} = \frac{29650 - \sqrt{559122500}}{10}$$

$$n_{max,1} = 5329.577$$

$$n_{max,2} = 600.423 \approx 601$$

$\therefore$ ~~smallest will be 601~~

2-5 For $P(N \leq \Lambda_{max}) = 0.95$

$\therefore \quad C \cdot F(N) = 0.95$

From 2.4, we know that $F(N) \cdot \frac{1}{F(N+1)} = \frac{(N+1)(N-529)}{(N-399)^2}$

$\because C \cdot F(N) = F(N) \cdot \frac{1}{F(N+1)}$

$$\therefore \frac{(N+1)(N-529)}{(N-399)^2} = \frac{(\Lambda_{max}+1)(\Lambda_{max}-529)}{(\Lambda_{max}-399)^2} = 0.95$$

$$= (\Lambda_{max}+1)(\Lambda_{max}-529) = 0.95(\Lambda_{max}-399)^2$$

$$\Lambda_{max_1} = \frac{-23010 + \sqrt{1833000000}}{10}$$

$$\Lambda_{max_2} = \frac{-23010 - \sqrt{1833000000}}{10}$$

$\Lambda_{max_1} = 585.174$

$\Lambda_{max_2} = -5187.174$

$\therefore$ based on the last page, the range of the Λ_{max} is between 585.174 to 600.423

so, smallest Λ_{max} is between 585.174 — 600.423

3.1 ①

p of fair coin is $\frac{1}{2}$

p of bais coin is head is $\frac{2}{3}$

If 10 flips, threshold will be $\frac{10}{2} = 5$

$$p(\text{win}) = p(\text{fair}) \cdot p\left(\frac{\text{win}}{\text{fair}}\right) + p(\text{bias}) \cdot p\left(\frac{\text{win}}{\text{bias}}\right)$$

$$= \frac{1}{2} \cdot \left(p\left(\frac{X<5}{0.5}\right) + p\left(\frac{X \geq 5}{\frac{2}{3}}\right)\right)$$

$$= \frac{1}{2}\left(\left(\sum_{k=0}^{4} \underbrace{\binom{10}{k}}_{\frac{10!}{k!(10-k)!}} \left(\frac{1}{2}\right)^{k} \cdot \left(1-\frac{1}{2}\right)^{10-k}\right) + \left(\sum_{k=5}^{10} \underbrace{\binom{10}{k}}_{\frac{10!}{k!(10-k)!}} \left(\frac{2}{3}\right)^{k} \cdot \left(1-\frac{2}{3}\right)^{10-k}\right)\right)$$

$$= \frac{1}{2}\left(\frac{193}{512} + \frac{18176}{19683}\right) = 0.65019$$

② If 20 filps, threshold will be $\frac{20}{2} = 10$

$$p(\text{win}) = \frac{1}{2}\left(p\left(\frac{X<10}{0.5}\right) + p\left(\frac{X \geq 10}{\frac{2}{3}}\right)\right)$$

$$= \frac{1}{2}\left(\left(\sum_{k=0}^{9} \underbrace{\binom{20}{k}}_{\frac{20!}{k!(20-k)!}} \left(\frac{1}{2}\right)^{k} \cdot \left(1-\frac{1}{2}\right)^{20-k}\right) + \left(\sum_{k=10}^{20} \underbrace{\binom{20}{k}}_{\frac{20!}{k!(20-k)!}} \left(\frac{2}{3}\right)^{k} \left(1-\frac{2}{3}\right)^{20-k}\right)\right)$$

$$= \frac{1}{2}\left(\frac{215955}{524288} + 0.96236\right)$$

$$= 0.68713$$

①

3.2 For 10 filps,

$$\frac{1}{2}\left(P\left(\frac{k-1}{0.5}\right) + P\left(\frac{k}{\frac{2}{3}}\right)\right)$$

$$= \frac{1}{2}\left(\left(\sum_{k=0}^{n-1} \binom{10}{k}\left(\frac{1}{2}\right)^k \cdot \left(1-\frac{1}{2}\right)^{10-k}\right) + \left(\sum_{n}^{10} \binom{10}{k}\left(\frac{2}{3}\right)^k \left(1-\frac{2}{3}\right)^{10-k}\right)\right)$$

$$\binom{10}{k} = \frac{10!}{k!(10-k)!}$$

We already knows that if we set our threshold to 5 then probability of winning is 0.65019

If we set our threshold to 6, then the function will looks like

$$\frac{1}{2}\left(\left(\sum_{k=0}^{5} \left(\frac{10!}{k!(10-k)!}\right)\left(\frac{1}{2}\right)^k \left(\frac{1}{2}\right)^{10-k}\right) + \left(\sum_{k=6}^{10} \left(\frac{10!}{k!(10-k)!}\right)\left(\frac{2}{3}\right)^k \left(1-\frac{2}{3}\right)^{10-k}\right)\right)$$

$$= \frac{1}{2}\left(\frac{319}{512} + \frac{15488}{19683}\right) = 0.705$$

However, if we set our threshold to 7, then:

$$\frac{1}{2}\left(\left(\sum_{k=0}^{6} \left(\frac{10!}{k!(10-k)!}\right)\left(\frac{1}{2}\right)^k \left(\frac{1}{2}\right)^{10-k}\right) + \left(\sum_{k=7}^{10} \frac{10!}{k!(10-k)!}\right)\left(\frac{2}{3}\right)^k \left(\frac{1}{3}\right)^{10-k}\right)$$

$$= \frac{1}{2}\left(\frac{53}{64} + \frac{11008}{19683}\right) = 0.6937$$

As we can see, when threshold is 6, $P(win) = 0.705$ which is bigger than threshold of 5 and 7

∴ For 10 filps, threshold should set to 6

3.2 ① For 20 filps,

$$\frac{1}{2}\left(\left(\sum_{k=0}^{n-1}\binom{20}{k}\left(\frac{1}{2}\right)^k\cdot\left(1-\frac{1}{2}\right)^{20-k}\right)+\left(\sum_{n}^{20}\binom{20}{k}\left(\frac{2}{3}\right)^k\left(1-\frac{2}{3}\right)^{20-k}\right)\right)$$

Same logic as 3.2①, we know that threshold 10, P(win) = 0.68713.

If we set threshold to 11, function will be.

$$\frac{1}{2}\left(\left(\sum_{k=0}^{10}\binom{20}{k}\left(\frac{1}{2}\right)^k\cdot\left(\frac{1}{2}\right)^{20-k}\right)+\left(\sum_{11}^{20}\binom{20}{k}\left(\frac{2}{3}\right)^k\left(\frac{1}{3}\right)^{20-k}\right)\right)$$

$$=\frac{1}{2}\left(\frac{308333}{524288}+0.90810\right)=0.7481$$

If we set threshold to 12,

$$\frac{1}{2}\left(\left(\sum_{k=0}^{11}\binom{20}{k}\left(\frac{1}{2}\right)^k\cdot\left(\frac{1}{2}\right)^{20-k}\right)+\left(\sum_{12}^{20}\binom{20}{k}\left(\frac{2}{3}\right)^k\left(\frac{1}{3}\right)^{20-k}\right)\right)$$

$$=\frac{1}{2}\left(\frac{392313}{524288}+0.80945\right)=0.77886$$

If we set to 13

$$\frac{1}{2}\left(\left(\sum_{k=0}^{12}\binom{20}{k}\left(\frac{1}{2}\right)^k\cdot\left(\frac{1}{2}\right)^{20-k}\right)+\left(\sum_{13}^{20}\binom{20}{k}\left(\frac{2}{3}\right)^k\left(\frac{1}{3}\right)^{20-k}\right)\right)$$

$$=\frac{1}{2}\left(\frac{227649}{262144}+0.66147\right)=0.7649$$

we can see that when threshold is 12, P(win) = 0.77886 which is bigger than threshold at 10, 11 and 13

∴ For 20 filps, threshold should be 12.

3.3 $F(N,T,p) \geq F(N,T+1,p)$

N is number of flips, T is threshold, p is probability

$F(N,T,p)$

$= \frac{1}{2}\left(\sum_{T=0}^{T-1} \binom{N}{T} (\frac{1}{2})^T (\frac{1}{2})^{N-T} + \sum_{T=T}^{N} \binom{N}{T} (p)^T (1-p)^{N-T}\right)$

$F(N,T+1,p)$

$= \frac{1}{2}\left(\sum_{T=0}^{T} \binom{N}{T} (\frac{1}{2})^T (\frac{1}{2})^{N-T} + \sum_{T=T+1}^{N} \binom{N}{T} (p)^T (1-p)^{N-T}\right)$

$F(N,T-1,p)$

$= \frac{1}{2}\left(\sum_{T=0}^{T-2} \binom{N}{T} (\frac{1}{2})^T (\frac{1}{2})^{N-T} + \sum_{T=T-1}^{N} \binom{N}{T} (p)^T (1-p)^{N-T}\right)$

$F(N,T,p) - F(N,T+1,p) \leq 0$

$\frac{1}{2}\left(\sum_{T=0}^{T-1} \binom{N}{T} (\frac{1}{2})^T (\frac{1}{2})^{N-T} + \sum_{T=T}^{N} \binom{N}{T} (p)^T (1-p)^{N-T}\right) - \frac{1}{2}\left(\sum_{T=0}^{T-2} \binom{N}{T} (\frac{1}{2})^T (\frac{1}{2})^{N-T} + \sum_{T=T+1}^{N} \binom{N}{T} (p)^T (1-p)^{N-T}\right)$

$= \underbrace{\frac{1}{2}\binom{N}{T+1}\left(\frac{1}{2^N}\right)}_{\text{we know this part is negative so we don't care this part}} \cdot \left(-1 + \underbrace{2^N \cdot (p)^{T+1} \cdot (1-p)^{N-T+1}}\right) \Rightarrow \therefore 2^N (p)^{T+1} \cdot (1-p)^{N-T+1} \leq 1$

$F(N,T,p) - F(N,T+1,p) \geq 0$

$\frac{1}{2}\left(\sum_{T=0}^{T-1} \binom{N}{T} (\frac{1}{2})^T (\frac{1}{2})^{N-T} + \sum_{T=T}^{N} \binom{N}{T} (p)^T (1-p)^{N-T}\right) - \frac{1}{2}\left(\sum_{T=0}^{T} \binom{N}{T} (\frac{1}{2})^T (\frac{1}{2})^{N-T} + \sum_{T=T+1}^{N} \binom{N}{T} (p)^T (1-p)^{N-T}\right)$

$\cancel{\frac{1}{2}\binom{N}{T+1} \cdot (\frac{1}{2^N}) \cdot (-1 + 2^N (p)^{T+1} \cdots)}$

I don't know !!!