Data Mining

Self-Organizing Maps

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NOVA-IMS

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AGENDA

- Cluster analysis
  - Clustering techniques
    - Self-organizing maps

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Clustering

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Clustering

- Grouping
  - Clustering
    - Hierarchical Methods
    - Partition Methods
    - DBScan
    - Shift mean
    - Self-Organizing Maps
  - Cohort Analysis
  - Cell-based segments
    - Variables
    - Through time
    - RFM Analysis

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Self-Organizing Maps

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Self-Organizing Maps

- Unsupervised neural networks;
- Closely related to clustering;
- The inputs are connected to a two-dimensional (it may have several dimensions) matrix of units (neurons);
- Each unit is connected to its neighbors.
- What is its use?
  - Multidimensional data visualization;
  - Cluster detection;
  - Market segmentation;
  - Outlier detection;
  - Solve TSP, robot control, alarm detection, etc., etc., etc.

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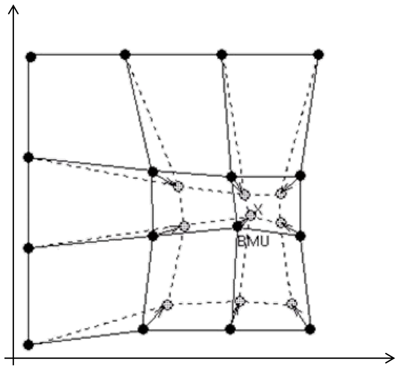
Self-Organizing Maps

- Each neuron is a **vector in the input space**, just as the data patterns;
- During training, neurons are **pulled** to the positions of the input data, **dragging** with them their neighbors in the output space;
- The map can be seen as a **rubber sheet**, stretched and twisted, so that it passes in (or at least near) the data patterns.



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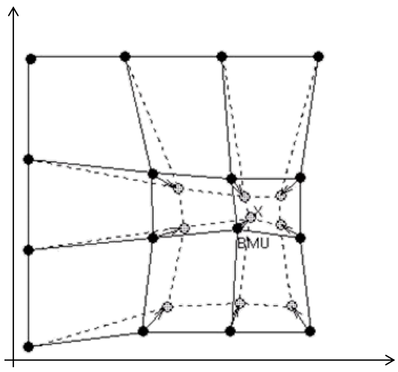
Self-Organizing Maps

- Input patterns are compared with all neurons and the **closest is considered to be the winning neuron.**
- We consider that the input pattern is **mapped to the winning neuron**.
- The **winner is updated** (so that it resembles even more the data pattern that it represents), and its neighbors are also updated a little.
- There is always a slight difference between the data and the neurons that represent them. That difference is the **quantization error**.



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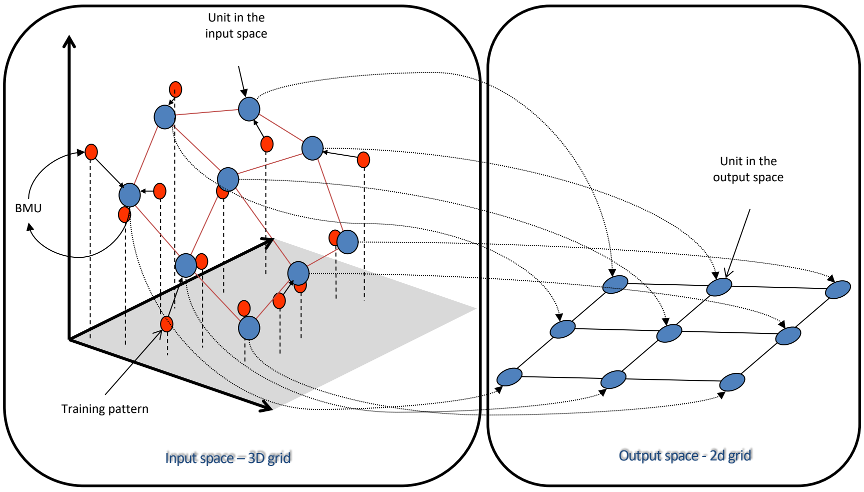
Self-Organizing Maps



Self-Organizing Maps

- **Comparisons with biology**
  - Biological systems have to use some kind of self-organization and adaptation.
  - There is evidence of:
    - Layered structure in the brain;
    - Those layers seem to spatially organize the information;
    - Similar “Concepts” are mapped to adjacent areas;
    - Experimental work with animals points to an organization similar to SOM in the visual cortex.

The SOM Algorithm

- **SOM Algorithm:**

Step 0: Randomly initialize the weights w_{ij}
Set the neighborhood topological parameters
Set the learning rate

Step 1: While stop condition false, do steps 2-7

Step 2: For each input vector x, do steps 3-5

Step 3: For each j, execute:

$$D(j) = \sum (w_{ij} - x_i)^2$$

Step 4: Find the unit that minimizes $D(j)$

Step 5: For every j unit within the predefined and for all the i:

$$w_{iJ}\text{ (new)} = w_{iJ}\text{ (old)} + \alpha\, [x_i - w_{iJ}\text{ (old)}]$$

Step 6: Update the learning rate

Step 7: Update (reduce) the radius of the topological neighborhood

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Self-Organizing Maps

- **SOM Algorithm:**
  - What happens in the Input space?

https://claude.ai/public/artifacts/69173914-bac8-4564-aca5-5f9b58466a81

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Self-Organizing Maps

- **SOM Algorithm:**
  - Key outputs of the SOM:
    - U-Matrices;
    - Component planes;
    - Hit Plots.

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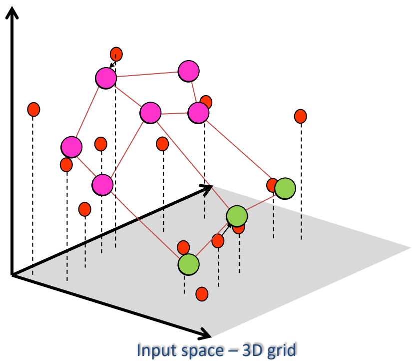
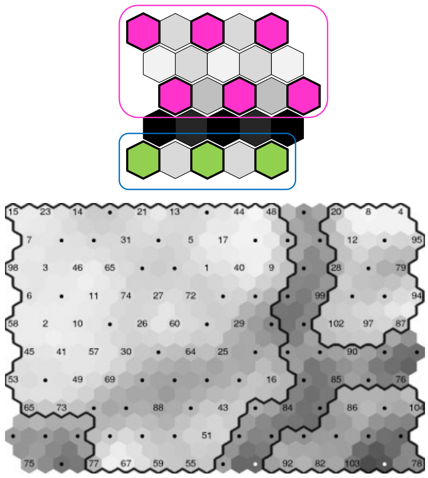
Self-Organizing Maps – U-Matrix





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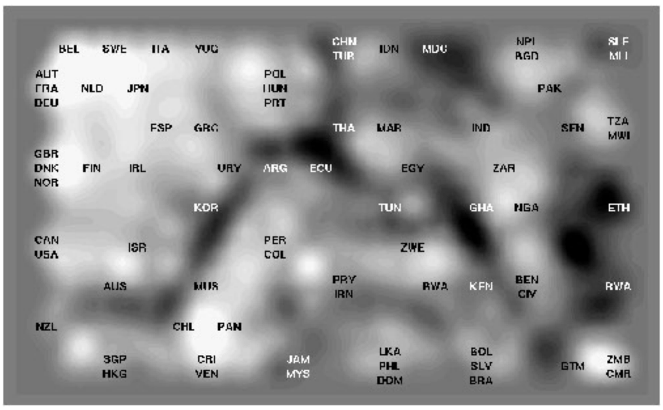
Self-Organizing Maps – U-Matrix



Kaski, S. and Kohonen, T. 1996. Exploratory Data Analysis by the Self-Organizing Map: Structures of Welfare and Poverty in the World. In: Apostolos, P. N., Refenes, Y. A., Moody, J. and Weigend, A. (eds.) Neural Networks in Financial Engineering. Singapore: World Scientific, 498-507.

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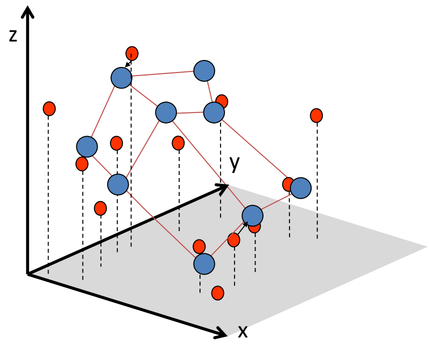
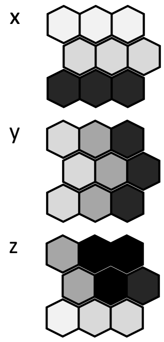
Self-Organizing Maps – Component Planes



Input space – 3D grid



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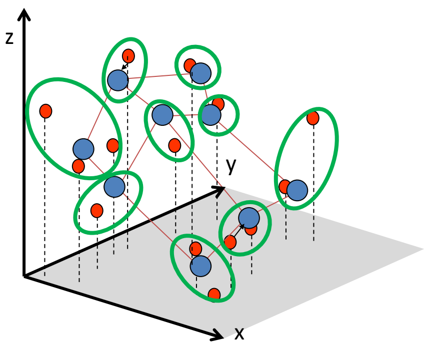
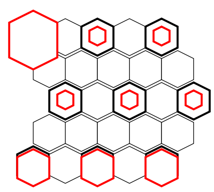
Self-Organizing Maps – Hit Maps



Input space – 3D grid



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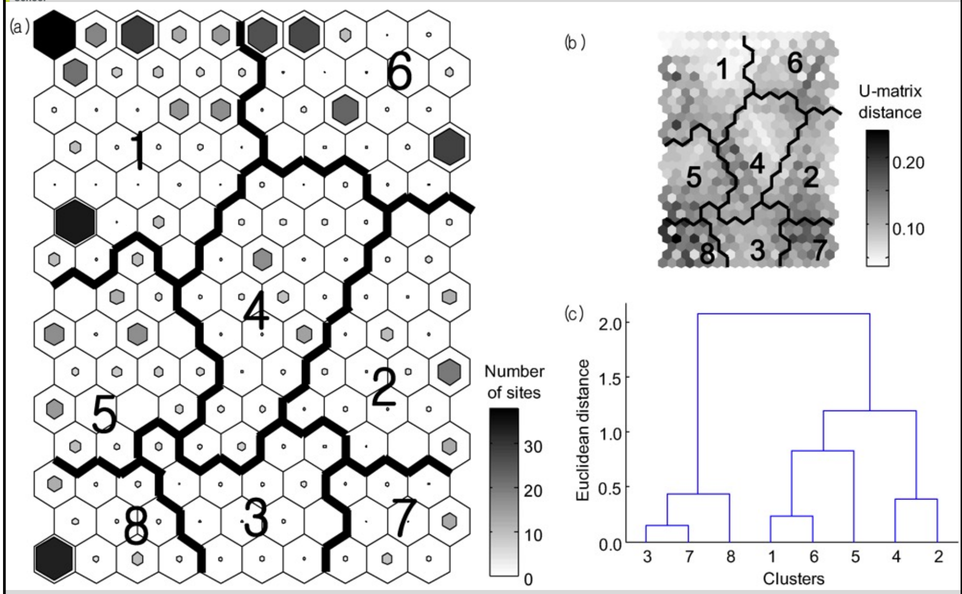
Self-Organizing Maps – Interpreting the U-Matrix



Self-Organizing Maps

- **SOM Algorithm:**

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Step 5: For every j unit within the predefined and for all the i:

$$w_{iJ}\,(\text{new}) = w_{iJ}\,(\text{old}) + \alpha\,[x_i - w_{iJ}\,(\text{old})]$$

Step 6: Update the learning rate

Step 7: Update (reduce) the radius of the topological neighborhood

- **SOM Algorithm:**

Vectors to classify:

(1, 1, 0, 0); (0, 0, 0, 1); (1, 0, 0, 0); (0, 0, 1, 1)

Maximum number of clusters to form:

m = 2

Learning rate:

$\alpha(0) = .6$, $\alpha(t+1) = .5\ \alpha(t)$

- **SOM Algorithm:**

Step 0: Initial matrix of weights

(0.2, 0.6, 0.5, 0.9);

(0.8, 0.4, 0.7, 0.3);

Initial radius: R = 0

Initial learning rate:

$\alpha(0) = .6$

Self-Organizing Maps

- **SOM Algorithm:**

Step 1: Initialize training

Step 2: first vector (1, 1, 0, 0);

Step 3:

$D(1) = (.2-1)^2 + (.6-1)^2 + (.5-0)^2 + (.9-0)^2$

$= 1.86;$

$D(2) = (.8-1)^2 + (.4-1)^2 + (.7-0)^2 + (.3-0)^2$

$= 0.98$

Vectors of weights:

(0.2, 0.6, 0.5, 0.9)

(0.8, 0.4, 0.7, 0.3)

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Self-Organizing Maps

- **SOM Algorithm:**

Step 4: The input vector is closest to unit 2, therefore

$j = 2 = (0.8, 0.4, 0.7, 0.3)$

Step 5: The weights of the winning unit are adjusted

$w_{i2}\text{ (new)} = w_{i2}\text{ (old)} + .6\,[x_i - w_{i2}\text{ (old)}]$

$= .4\, w_{i2}\text{ (old)} + .6x_i$

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- **SOM Algorithm:**

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$$= .4\,w_{i2}\text{ (old)} + .6x_i$$

- **SOM Algorithm:**

Thus, the weight matrix is adjusted:

.2	.92
.6	.76
.5	.28
.9	.12

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Self-Organizing Maps

- **SOM Algorithm:**

Thus, the weight matrix is adjusted:

.08	.92
.24	.76
.20	.28
.96	.12

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