

Java 2 Red-Black Trees

- **Comparing R-B to AVL trees**

- Max height:
 - AVL
 - $\sim 1.44 \log_2 N$
 - R-B
 - $2 \log_2(N + 1)$
- Insertion
 - AVL
 - Insert according to value, then check balance with a reverse walk.
 - $bf = +/- 2?$
 - R-B
 - Insert according to value, then check balance with a reverse walk.
 - red-red?
- If unbalanced
 - AVL
 - Identify a 3-node neighborhood
 - R-B
 - Identify a 4-node neighborhood
- Repair
 - AVL
 - Rotations
 - At most one repair per insertion
 - Many repairs possible for deletion
 - R-B
 - Rotations and recoloring
 - Many repairs possible for insertion
 - Many repairs possible for deletion

- **Red-Black Trees**

- A red-black is BST with the following node color rules
 - Each node is either red or black.
 - The root and all empty trees are black.
 - All paths from the root to an empty tree contain the same number of black nodes.
 - A red node can't have a red child.
- Rule 1 tells us what types of nodes are legal: red ones and black ones.
- Rule 2 specifies the root must be black and, since empty trees are valid trees, it gives them a color (black). We know what the "boundaries" of a R-B tree look like.
- Rule 3 + 4 = Balance
 - Rule 3 is half of the balance requirement. It makes a statement about the height of tree in terms of black node. This is often called the tree's black height.
 - Without red nodes, R-B tree could only be full.
 - A red node is used like "filler". It allows a R-B tree to obey rules 1, 2, and 3 without being a perfect triangle
 - Rule 4 prevents a red node from having a red child.
- Use the standard BST algorithm to insert the new node. Make the addition red.
 - Walk up the tree and look for a red node with a red parent
 - Stop at the first (lowest) red node that has a red parent. Go the grandparent, then its other child (This is the 4 node neighborhood)

- **5 Cases for Repair**

- A is red
 - Repaired by only recoloring nodes.
 - Re-color the top three nodes in the neighborhood (toggle their state)
- A is black
 - Repaired by rotations and recoloring Using the same scheme as an AVL tree

- Case 1 from AVL (zig-zag left - right)
 - rotate left
- Case 2 from AVL (left)
 - rotate right
- Case 3 from AVL (zig-zag right - left)
 - rotate right
- Case 4 from AVL (right)
 - rotate left