1) $T(n) = 3T(n/2) + n^2$

$T(n) = aT(n/b) + f(n^2)$

$a \geq 1, b \geq 1$

on comparing

$a = 3, b = 2, f(n) = n^2$

Now, $C = \log_b a = \log_2 3$

$= 1.584$

$n^C = n^{1.584} < n^2$

$f(n) > n^C$

$T(n) = \Theta(n^2)$

2) $T(n) = 4T(n/2) + n^2$

→ $a \geq 1, b > 1$

$a = 4, b = 2, f(n) = n^2$

$C = \log_2 4 = 2$

$n^C = n^2 = f(n) = n^2$

∴ $T(n) = \Theta(n^2 \log_2 n)$

3) $T(n) = T(n/2) + 2^n$

$a = 1, b = 2$

$f(n) = 2^n$

$C = \log_b a = \log_2 1 = 0$

$n^C = n^b = 1$

4) $T(n) = 2^n T(n/2) + n^2$

$a = 2^n$

$b = 2, f(n) = n^2$

$C = \log_b a = \log_2 2^n$

$= n$

$n^C + n^2$

$f(n) = n^C$

$T(n) = \Theta(n^2 \log_2 n)$

5) $T(n) = 16T(n/4) + n$

$a = 16, b = 4$

$f(n) = n$

$C = \log_4 16 = \log_4 (4)^2$

$2 \log_4 k = 2^4$

$n^C = n^2$

$f(n) < n^C$

∴ $T(n) = \Theta(n^2)$

6) $T(n) = 2T(n/2) + n \log n$

$\quad a = 2, \ b = 2$

$\quad f(n) = n \log n$

$\quad c = \log_2 2 = 1$

$\quad n^c = n^1 = n$

$\quad n \log n > n$

$\quad f(n) > n^c$

$\quad T(n) = \theta(n \log n)$

7) $T(n) = 2T(n/2) + n \log n$

$\quad a = 2, \ b = 2 \ \text{if} \ f(n) = n \log n.$

$\quad c = \log_2 2 = 1$

$\quad n^c = n^1 = n$

$\quad \dfrac{n}{n \log n} < n$

$\quad \therefore f(n) < n^c$

$\quad \therefore T(n) = \theta(n)$

8) $T(n) = 2T(n/4) + n^{0.51}$

$\quad a = 2, \ b = 4, \ f(n) = n^{0.51}$

$\quad c = \log_4 2 = \log 4 \, 2 = 0.5$

$\quad n^c = n^{0.5}$

$\quad n^{0.5} < n^{0.51}$

$\quad \therefore T(n) = \theta(n^{0.51})$

9) $T(n) = 0.5 \, T(n/2) + 1/n$

$\quad a = 0.5, \ b = 2$

$\quad a \geqslant 1 \ \text{but here}$
$\quad \quad a < 0.5$

$\quad$ so we can't apply
$\quad$ Master method too.

10) $T(n) = 16 \, T(n/4) + n!$

$\quad a = 16, \ b = 4, \ f(n) = n!$

$\quad \therefore c = \log_b a = \log_4 16 = 2$

$\quad n^c = n^2$

$\quad \text{As} \quad n! > n^2$

$\quad \therefore T(n) = \theta(n!)$

11) $4 \, T(n/2) + \log n$

$\quad a = 4, \ b = 2, \ f(n) = \log n.$

$\quad c = \log_b a = \log_2 4 = 2$

$\quad n^c = n^2$

$\quad f(n) = \log n.$

$\quad \therefore \log n < n^2$

$\quad \quad f(n) < n^c$

$\quad \quad T(n) = \theta(n^c)$

$\quad \quad \quad = \theta(n^2)$

$$\text{for } \sqrt{q} + (n)T(n/2) + \log n$$

$a = \sqrt{n}, \ b = 2$

$c = \log_2 \sqrt{n} = \frac{1}{2} \log_2 n$

$\therefore \ \frac{1}{2} \log_2 n < \log n$

$\therefore \ f(n) > n^c$

$T(n) = \Theta(\log(n))$

(3) $T(n) = 3T(n/2) + n$

$a = 3, \ b = 2$

$c = \log_2 3 = 1.584$

$n^c = n^{1.58}$

$n < n^{1.5} = f(n) < n^c$

$T(n) = \Theta(n^{1.58})$

(4) $T(n) = 3T(n/3) + \sqrt{q} t(n)$

$a = 3, \ b = 3$

$c = \log_3 3 = 1$

$n^c = n$

$\sqrt{q} t(n) < n$

$n > f(n)$

$T(n) = \Theta(n)$

(5) $T(n) = 4(T(n/2) + n$

$a = 4, \ b = 2$

$c = \log_n a = \log_2 4 = 2$

$n^c = n^2$

$n^c = f(n)$

$\boxed{T(n) = \Theta(n^2)}$

(6) $T(n) = 3T(n/4) + n \log n$

$a = 3, \ b = 4$

$c = \log_4 3 = 0.792$

$n^c = n^{0.792}$

$n^c < f(n)$

$\boxed{T(n) = \Theta(n \log n)}$

(7) $T(n) = 3T(n/3) + n/2$

$a = 3, \ b = 3$

$c = \log_3 3 = 1$

$n^c = n$

$n > f(n)$

$\boxed{T(n) = \Theta(n)}$

18) $T(n) = 6T(n/3) + n^2 \log n$

$a = 6, \ b = 3$

$C = \log_3 6 = 1.6309$

$n^C < n^2 \log n$

$\boxed{T(n) = \Theta(n^2 \log n)}$

19) $T(n) = 4T(n/2) + \dfrac{n}{\log n}$

$a = 4, \ b = 2, \ f(n) = n/\log n$

$C = \log_b a = \log_2 4 = 2$

$n^C = n^2$

$\dfrac{n}{\log n} < n^2$

$\boxed{T(n) = \Theta(n^2)}$

20) $T(n) = 64 T(n/8)$

$\qquad - n^2 (\log n)$

$a = 64, \ b = 8$

$C = \log_8 64 = 2$

$n^C = n^2$

$n^2 \log n > n^2$

$T(n) = \Theta(n^2 \log n)$