Tutorial-6

Q:1- What do you mean by Minimum Spanning tree? What are application MST?

Ans: Minimum Spanning tree is a
a weighted, connected, undirected graph is a spanning tree having a weight less or equal to the weight of every other possible spanning tree. The weight of a spanning tree is sum of weights given to each edge of the spanning tree.

Application:

i) Consider stations are to be linked using a communication network and laying of communication link between any two stations involves a cost. The ideal solution would be the extract a subgraph termed as minimum cost spanning tree.

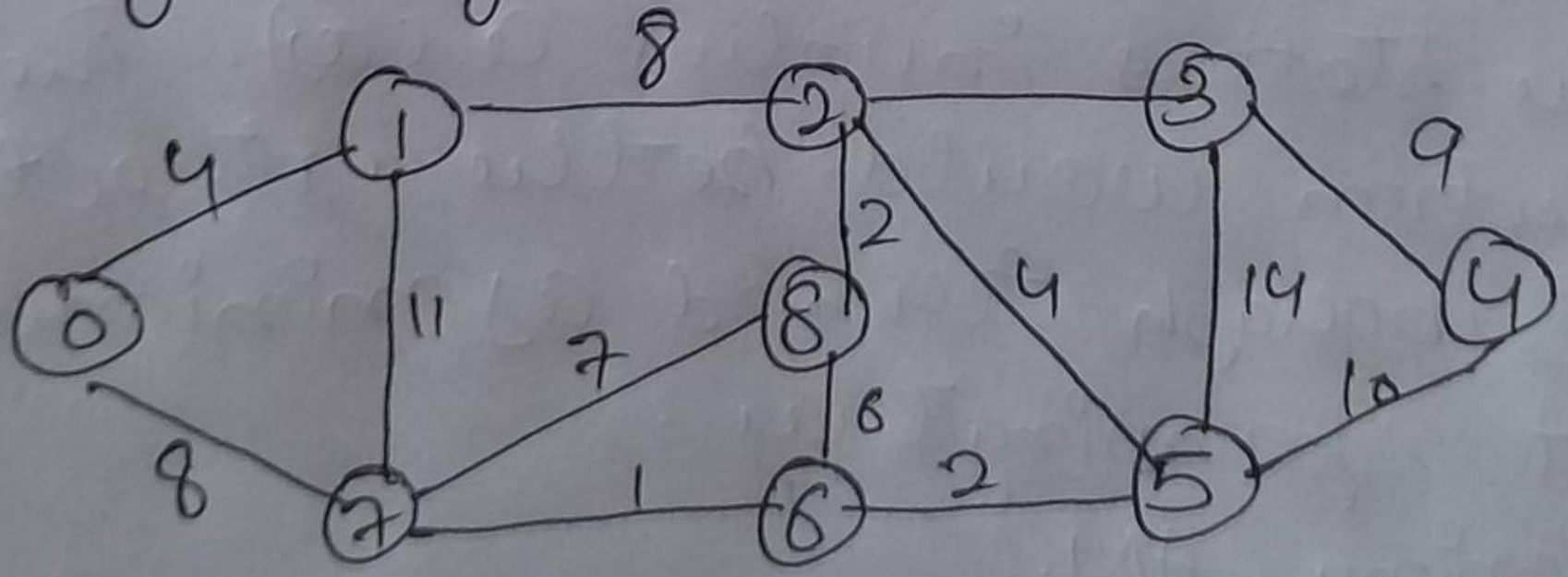
ii) Designing LAN.

iii) Suppose you want to construct highway or railroads spanning several cities, then we can use concept of MST.

Q2: Analyze time and space complexity of Prim, Kruskal, Dijkstra and Bellman Ford Algorithm.

Ans: → Time Complexity of Prim's Algo = O(|E| log |V|)
→ Space complexity of Prim's Algo : O|V|
→ Time complexity of Kruskal's Algo: O|E| log |E|)
→ Space complexity of Kruskal's Algo: O|V|
→ Time complexity of Dijkstra's Algo: O(V²)
→ Space Complexity of Dijkstra's Algo: O(V²)
→ Time Complexity of bellman Ford's Algo: O(VE)
→ Space Complexity of bellmanFord's Algo: O(E)

Q3: Apply Kruskal and Prim's Algorithm on given graph to comp MST and wt.



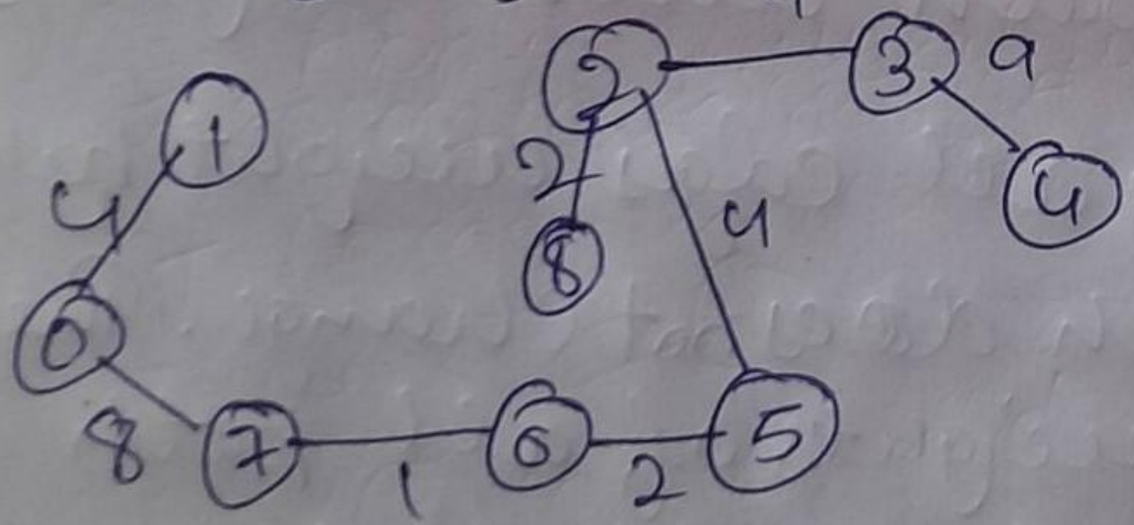
Ans: Kruskal's Algo

0	1	4	✓
6	7	1	✓
5	6	2	✓
2	8	2	✓
0	1	4	✓
2	5	4	✓
6	8	6	X
2	3	7	✓
7	8	7	X
0	7	8	✓
1	2	8	X
4	3	9	✓
4	5	10	X
1	7	11	X
3	5	14	X

Prim's Algo

weight = 4+8+2+4+2+7+9

= 37



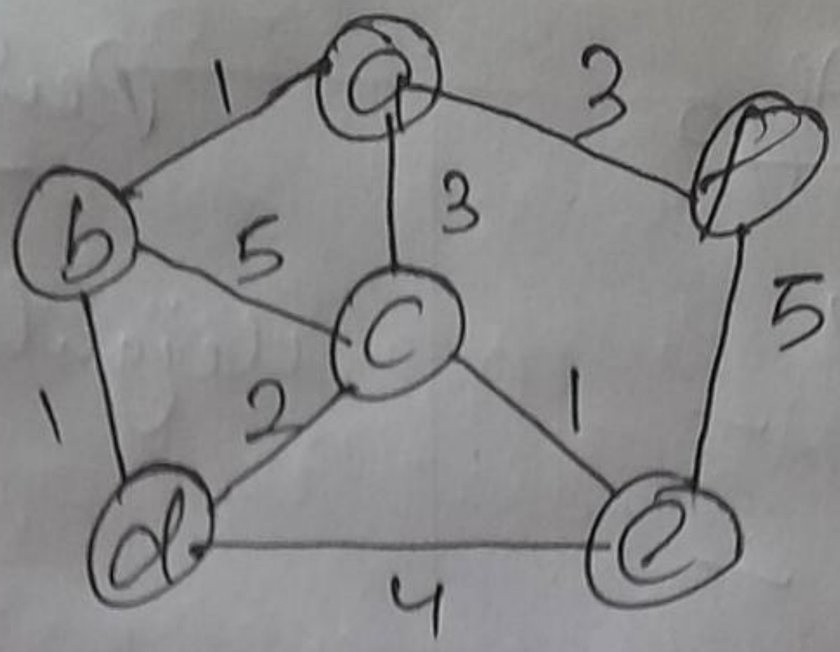
weight = 1+2+2+4+4+7+8+9

= 37

Q4: Given a directed..... in following cases:

(i) if weight of every edge is increased by 10 units

(ii) if weight of every edge is multiplied by 10 units.



Ans 8 (i) The shortest path may change.

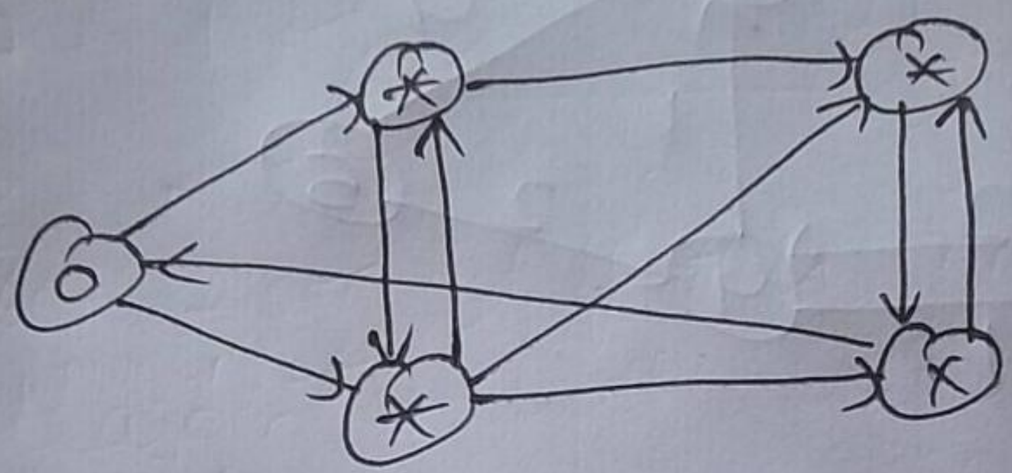
The reason is that there may be different no. of edges in different paths from '5' to 't'.

For eg: let the shortest path weight 15 and has edges 5.

The weight of shortest path is increased by 5*10 and becomes 15+50. Weight of other path is increased by 2'10 and becomes 26+26. So, the shortest path changes to other path with weight as 45.

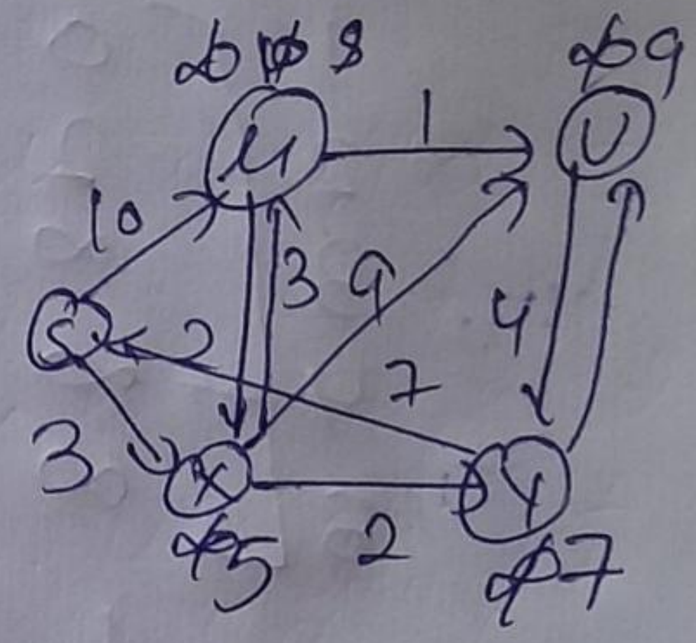
(ii) If we multiply all edges weight by 10, the shortest path doesnot change. The reason is that weights of all path from '5' to 't' gets multiplied by same unit. The number of edges on path doesn't matter.

Q5: Apply Dijkstra and Bellman Ford algo on graph given right side to compute shortest path to all nodes from node 5.



Ans: Dijkstra's Algorithm:

NODE	SHORTEST DIST FROM SOURCE NODE
u	8
x	5
v	9
y	7



Bellman Ford Algorithm:

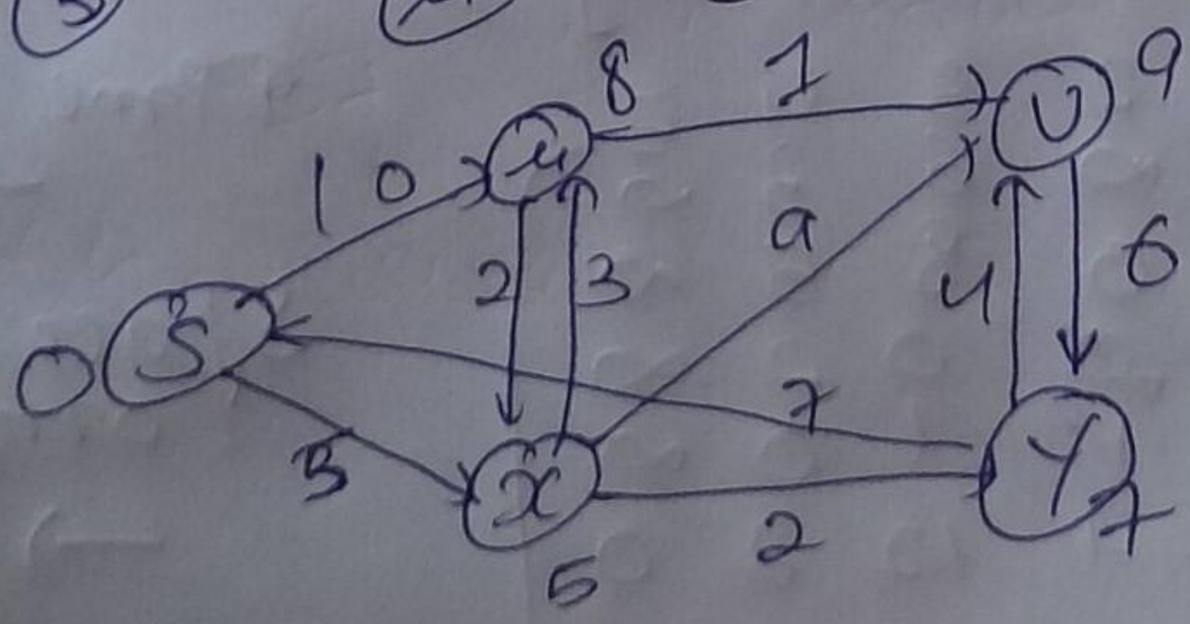
1st → (s) (u) ∞ (v) ∞ (x) 5 (y) ∞

2nd → (s) (u) 10 (v) 14 (x) 5 (y) ∞

3rd → (s) (u) 10 (v) 9 (x) 5 (y) 7

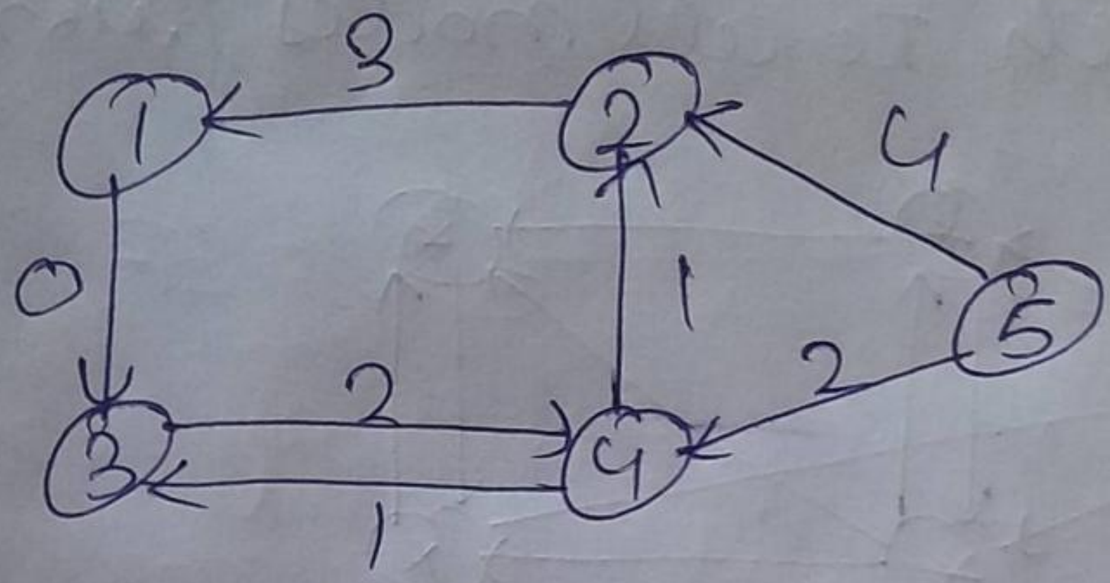
4th → (s) (u) 8 (v) 9 (x) 5 (y) 7

graph doesn't have negative cycle.



→ Final graph

Q6: Apply all pair ---- time complexity of any



	1	2	3	4	5
1	0	∞	6	3	∞
2	2	0	∞	∞	∞
3	∞	∞	0	2	∞
4	∞	1	1	0	∞
5	∞	4	∞	2	0

	1	2	3	4	5
1	0	∞	6	3	∞
2	2	0	8	5	∞
3	∞	∞	0	2	∞
4	3	1	1	0	∞
5	6	4	12	2	0

	1	2	3	4	5
1	0	∞	6	3	∞
2	2	0	8	5	∞
3	∞	∞	0	2	∞
4	3	1	1	0	∞
5	6	4	12	2	0

Time Complexity
→ $O(|V|^3)$

Space Complexity
→ $O(|V|^3)$