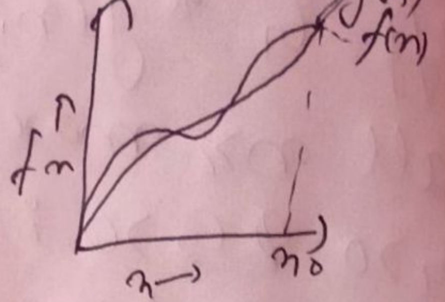
Tutorial-1

Q1: What do you understand by asymptotic notation ... with example.

i) Big $O(n)$



$f(n) = O(g(n))$

if $f(n) \le g(n) * c \quad \forall n \geq n_0$

for const, $c > 0$

g(n) is tight upper bound of f(n)

eg. $f(n) = n^2 + n$

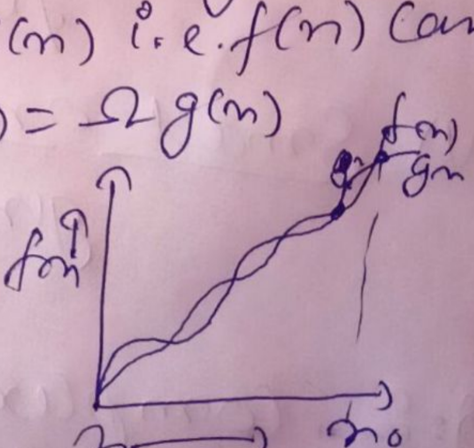
$g(n) = n^3$

$n^2 + n \le c * n^3$

$n^2 + n = O(n^3)$

ii) Big Omega(Ω)

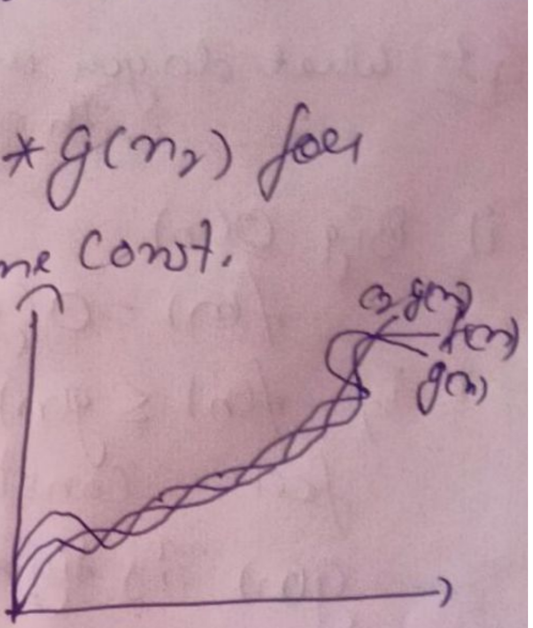
when $f(n) = \Omega(g(n))$ means $(g(n))$ is "tight" lowerbound of f(n) i.e. f(n) can go beyond g(n) i.e. $f(n) = \Omega\, g(n)$



if $f(n) \geq c \cdot g(n)$

$\forall n \geq n_0$ & $c = \text{const} > 0$

iii) **Big Theta (Θ)** - When $f(n) = \Theta(g(n))$ gives the tight upperbound & lowerbound both. i.e. $f(n) = \Theta(g(n))$ iff. $C_1 * g(n_1) \leq f(n) \leq C_2 * g(n_2)$ for all $n \geq \max(n_1, n_2)$, some const. $C_1 > 0$ & $C_2 > 0$. i.e. $f(n)$ can never go beyond $C_2 g(n)$ & will never come down of $C_2 C_1 g(n)$.



eg. $3n+2 = \Theta(n)$ as $3n+2 \geq 3n$
$3n+2 \leq 4n$ for n, $C_1 = 3$, $C_2 = 4$ & $n_0 = 2$

iv) **Small oh (o)**:- $f(n) = o(g(n))$
$g(n)$ is upper bound of $f(n)$
$f(n) < g(n)$ $\forall$ $n > n_0$ & $c > 0$

v) **Small-omega (ω)**- $f(n) = \omega(g(n))$
$g(n)$ is lower bound of $f(n)$
$f(n) > g(n)$ $\forall$ $n > n_0$ & $c > 0$

Q=2 Complexity of:

```
for( i=1 to n)
{   i = i×2.
}
```

$i = 1, 2, 4 ---- n$

$a = 1$, $r = 2$.

k^{th} term of G.P,

$$t_k = a r^{k-1}$$

$$n = \frac{2^k}{2}$$

$$2n = 2^k$$

$$\log_2 2(n) = k \log_2 2.$$

$$1 + \log_2(n) = k$$

[Complexity = O(log n)]

Q3: $T(n) = 3T(n-1)$

$T(1) = 1$

$T(n) = 3T(n-2)$ — ①

put $n = n-1$ in eq^n(1)

$T(n-1) = 3T(n-2)$

put value of $T(n-1)$ is eq ①

$T(n) = 3[3T(n-2)]$

$T(n) = 3^2 T(n-2)$ — ②

put $n = n-2$ is eq — ①

$T(n-2) = 3T(n-2)$

put $T(n-2)$ in eq^n — ②

$T(n) = 3^2(3T(n-3)) = 3^3 T(n-3)$ — (iii)

from (i), (ii), & (iii)

$T(n) = 3^k[T(n-k)]$ —— (a)

$T(1) = 1$

$n-k = 1$ $[k = n-1]$

put value of k is eq — (a)

$T(n) = 3^{n-1}[T(1)]$

$T(n) = 3^{n-1}$

$[\text{Complexity} = O(3^n)]$

Q5:- what should be time complexity of

```
int i=1, s=1;
while (s<=n)
{  i++;
   s=s+i;
   print("#");
}
```

$i = 1, 2, 3, 4 \ldots$

$s = 1+3+6+10 \ldots + n$ (1)

also $s = \cancel{1}+3+6 \ldots T_{n-1} + T_n$

$s = 0 + 2 + ?\cdot 6 \ldots 10$

$0 = 1+2+3+4 \ldots n - T_n$

$T_k = 1+2+3+\ldots+k$

$T_k = \frac{1}{2}k(k+2)$

for k iterations

$1+2+3+\ldots k <= n$

$\frac{k(k+1)}{2} <= n$ $\qquad ak^2 <= n$

$\frac{k+k^2}{2} <= n$ $\qquad k = a(\sqrt{n})$

$[T(k) = a\sqrt{n}]$

Q:6 Time Complexity:

```
void f(int n)
{ int i, count=0;
  for( i=1; i*i<=n; ++i)
}
```

as $i^2 = n$

$i = \sqrt{n}$

$i = 1, 2, 3, 4, \ldots \sqrt{n}$

$\sum_{i=1} 1+2+3+4 \ldots \sqrt{n}$

$$T(n) = \frac{\sqrt{n} * (\sqrt{n}+1)}{2}$$

$$T(n) = \frac{n * \sqrt{n}}{2}$$

$$\boxed{T(n) = O(n)}$$

Q7: Time Complexity

```
void f(int n)
{ int i, j, k, count=0;
  for( int i=n/2; i<=n; ++i)
    for( j=1 ; j<=n; j=j*2 )
    for( k=1; k<=n; k=k*2)
            count++;
}
```

→ Since for $k=k2$.

$k = 1, 2, \ldots \ldots n$

∴ series in G.P.

So, $a=1, r=2$ $\quad \frac{a(r^n-1)}{r-1}$

$$= \frac{1(2^k-1)}{1}$$

$$n = 2^k - 1$$

$$n+1 = 2^k$$

$$\log_2(n) = k$$

i	j	k
1	$\log(n)$	$\log(n) \times \log(n)$
2	⋮	⋮
⋮	⋮	⋮
n	$\log(n)$	$\log(n) \times \log(n)$

$$T.C = O(n \times \log n \times \log n)$$

$$T.C = O(n \log^2(n)) \text{ ans}$$

Q8: Time Complexity of

```
void function(int n)
{   if(n==1) return;
    for(i=1 to n) {
    for (j=1 to n {
       printf("*");
    }
  }
  function(n-3);
}
```

(5) for (i = 1 to n)

we get j = n times everytime

$\therefore$ $i \times j = n^2$

Now, $T(n) = n^2 + T(n-3)$;

k^{th} $T(n-3) = (n-3)^2 + T(n-6)$;

$T(n-6) = (n-6)^2 + T(n-9)$;

and $T(1) = 1$

Now, subs.. each value in $T(n)$

$T(n) = n^2 + (n-3)^2 + (n-6)^2 + \cdots + 1$

let $k^{n} - 3k = 1$

$k = (n-1)/3$ total terms $= k+1$

$T(n) = n^2 + (n-3)^2 + (n-6)^2 + \cdots + 1$

$T(n) \simeq kn^2$

$T(n) \simeq (k-1)/3 \times n^2$

So, $T(n) = O(n^3)$ ans

Q: For the function n^{k} & c^{n}, what is the asymptotic relationship b/w these f^n? assume that $k >= 1$ & $c > 1$ are const. Find out value c & no. of which relationship holds.

→ As given n^k and c^n
relationship b/w n^k & c^n is

$n^k = O(c^n)$

$n^k \leq a(c^n)$

$\forall\ n \geq n_0$ & const^n^ $a > 0$

for $n_0 = 1$; $c = 2$

$\Rightarrow 1^k \leq a2$

$\Rightarrow n_0 = 1$ & $c = 2$ ans