



Project Owner	Rob de Ligt
Science Maintainers	Rob de Ligt Juha Metsaranta Cris Brack Marcela Olguin
Science Editors	Rob de Ligt Sarah Buchan
Science Review Panel	Stephen Roxburgh
Science Approval	Werner Kurz
Code maintainers	Max Fellows James Leitch Malcolm Francis
Code Coach	
Documentation Lead	
Release Approval	Werner Kurz

PLEASE READ THIS BEFORE YOU CONTINUE:

This is an open source document. Everybody is welcome to contribute. To allow us to be efficient we have only a few rules:

- All contributors use comments to ask questions or make substantive and structural suggestions. Always suggest solutions in your comments not problems.
- Only Maintainers can resolve comments.
- All contributors make edits in “[suggesting mode](#)” (even if you are a maintainer or editor, so the other maintainers or editors can cross-check and accept your contribution.)
- Write suggested edits directly into the text in ‘[suggesting mode](#)’ do NOT use comments to suggest edits
- Only Editors assigned to the document can approve edits
- Spelling will be UK English
- Referencing Style is Harvard (author-date) type. Tools such as [Zotero](#) may be useful.

More info about how the moja global community works on projects can be found [here](#).

Design for Monte Carlo and Propagation of Error Uncertainty in the Full Lands Integration Tool

Abstract

Uncertainty assessment is important for land based greenhouse gas estimation, being a component and is incorporated in all aspects of reporting, from national greenhouse gas inventories through to results based payments for REDD+. The purpose of this design document is to describe the requirements for implementing two commonly used IPCC methods for combining uncertainties within the FLINT framework. These are the Propagation of Error and Monte Carlo methods for combining uncertainties. A key design requirement is that the methods are consistent with the 2006 IPCC Guidelines for national greenhouse gas inventories, and are also capable of supporting other common land sector MRV programs and reporting requirements.

Abstract	2
Introduction	5
High level needs for countries and organisations	5
IPCC Guidelines and Uncertainty in UNFCCC mechanisms	5
Uncertainty assessment for mitigation programs and protocols	6
Initiative for Sustainable Forest Landscape (ISFL). BioCarbon Fund.	6
Overview of the program	6
Requirements for uncertainty estimation	6
Frequency of use	7
Spatial coverage	8
Types of outputs	8
Forest Carbon Partnership Facility. Carbon Fund.	8
Overview of the program	8
Requirements for uncertainty estimation	8
Frequency of use	9
Spatial coverage	9
Verra (Verified Carbon Standard - VCS)	10
Overview of the program	10
Requirements for uncertainty estimation	10
Frequency of use	11
Spatial coverage	11
Types of outputs	11
Gold Standard	11
Overview of the program	11
Requirements for uncertainty estimation	12
Frequency of use	13
Spatial coverage	13
Types of outputs	13
Overview of key Uncertainty requirements	13
Outline of the FLINT framework	15
What is FLINT?	15
FLINT Modules	15
FLINT Uncertainty Assessment	16
Propagation of Error	16
Propagation of Error in FLINT	17
Monte Carlo	19
FLINT framework	19

Define Model Configuration	21
Determining the inputs to be included in the Uncertainty Analysis	21
Specifying PDFs for Inputs	21
PDFs for coefficients vs equations	22
Correlation of Inputs	23
Number of iterations for Monte Carlo simulations	23
Determining which pixels to simulate	23
Simulate land units and aggregate outputs	25
Calculate Uncertainty Statistics	25
Monte Carlo Infrastructure for FLINT	25
Monte Carlo Local Domain Controller	26
Monte Carlo Transform	26
Monte Carlo Output Module	28
Specific design elements for Spatially Referenced modes	28
Key Criteria for assessing different software options for developing the modules	28
References	30
Annex 1 - Probability Density Functions (PDFs) that are available in the Boost C++ Library	31

Introduction

The purpose of this document is to describe the design elements of two commonly used methods of uncertainty assessment for implementation in the Full Lands Integration Tool (FLINT). Uncertainty analysis is an important component of National Greenhouse Gas Inventories under the United Nations Framework Convention on Climate Change (UNFCCC) including activities such as REDD+¹. It has also become an important component of emissions mitigation mechanisms such as the Forest Carbon Partnership Facility and the Initiative for Sustainable Forest Landscapes.

High level needs for countries and organisations

Uncertainty is used for different purposes in land sector greenhouse gas reporting and accounting. This introductory section details the key requirements from the most common land sector greenhouse gas (GHG) reporting and accounting frameworks for national and jurisdictional levels. The key requirements for the Uncertainty design in FLINT are then summarised.

IPCC Guidelines and Uncertainty in UNFCCC mechanisms

Volume 1, Chapter 3 of the 2006 IPCC Guidelines for National Greenhouse Gas Inventories (NGGI) provides guidance on uncertainties. The guidance focuses on uncertainties related to annual emissions and removals of GHG from all sectors, as well as their trends over time.

Uncertainty estimates are viewed as a fundamental component of complete national greenhouse gas inventories, with the main intention of uncertainty assessment being to support inventory compilers to continuously improve their NGGI, by enabling them to prioritise their efforts to reduce uncertainties and to help select inventory methods. This is also true for the use of uncertainty assessment in greenhouse gas emissions estimates provided through Biennial Update Reports and National Communications including the UNFCCC REDD+ technical annex.

There are no specified uncertainty levels that must be adhered to, and there are no punitive measures in place in relation to uncertainty in greenhouse gas inventories. Therefore, uncertainty assessment in this context is seen as a tool to help inventory compilers to best use limited resources to make the greatest improvement to reduce uncertainty.

The IPCC Guidelines are used by a wider range of mechanisms than just national inventories, and these different mechanisms may have a different purpose for estimating uncertainty. For

¹ REDD+, Reduce emissions from deforestation, forest degradation, and increasing removals from conservation of forest carbon stocks, sustainable management of forests and enhancement of forest carbon stocks.

example, uncertainty may be used to determine if the potential credits a party may receive for reducing emissions should be discounted due to having relatively high uncertainty.

The design of the Propagation of Error method in this document is based off the guidance in Volume 1 Chapter 3 of the 2006 IPCC Guidelines for National Greenhouse Gas Inventories. The design of the Monte Carlo analysis is also consistent with the guidance, noting that the guidance provided for Monte Carlo analysis is limited in the guidelines.

The IPCC guidance (Volume 1 Chapter 3) states that “Inventories should be accurate in the sense that they are neither over- nor underestimated as far as can be judged, and precise in the sense that uncertainties are reduced as far as practicable”. The purpose of the Uncertainty assessment in FLINT is to provide methods for assessing the precision element that is due to random error/variability and assumes that ‘accuracy’ or systematic error (or bias) has been addressed prior to the uncertainty analysis being performed.

Uncertainty assessment for mitigation programs and protocols

Recently, a number of programs and protocols have or are being established for the purpose of results based payments for emissions mitigation in the land sector. In these programs uncertainty is treated as an abatement risk and is typically used for applying discounts or equivalent mechanisms to reduce the number of potential credits that can be used for payments (eg results based payments). This differs from the purpose of uncertainty in NGGIs and perhaps provides a stronger driver for reducing uncertainty.

The main programs and protocols are described in this section together with a summary of their use of uncertainty.

Initiative for Sustainable Forest Landscape (ISFL). BioCarbon Fund.

Overview of the program

A multilateral fund for results based payments managed by the World Bank with existing capital of \$342 million and covering all AFOLU². To date, ISFL supports programs in Colombia, Ethiopia, Mexico and Zambia and is expected to expand into additional jurisdictions as the fund develops.

Requirements for uncertainty estimation

To be eligible to receive results-based finance, a country must comply with the ISFL Emission Reduction Program. This program is informed by an analysis of significant changes in GHG fluxes (quantitative or qualitative) over the Program’s base period and what is likely to change in the future. The Program GHG inventory will include all AFOLU categories, subcategories,

² AFOLU, Agriculture, Forestry and Other Land Use.

gases, and pools in the Program Area, using the most recent IPCC guidance and guidelines and based on best available methods and data.

The Program also specifies that a country “*shall systematically identify and assess sources of uncertainty in the determination of the Emissions Baseline and the monitoring of emissions and removals following the most recent IPCC guidance and guidelines*” and indicates that, to the extent feasible, the uncertainty on the estimates of the Emissions Baseline and the monitoring of GHG fluxes is managed and reduced (e.g., the emissions reduction used to request payment should be free of biases as far as possible).

The uncertainty of the emissions reduction shall be quantified using Monte Carlo and “*shall be combined into a single combined uncertainty estimate and reported at the two-tailed 90% confidence interval*”, in accordance with the IPCC guidelines.

The emission reduction during an Emission Reductions Payment Agreements (ERPA) Phase, a country has to consider a buffer reserve called “uncertainty set-aside factor” to reflect the level of uncertainty estimate of the total net emissions reductions (*actual GHG net emissions minus net emission baseline for the Program Area*), across the eligible subcategories (Table 1).

If the ISFL ER Program account uses Tier 1 methods and data for some pools and gases, or if the share of net emissions reductions is based on Activity Data proxies and methods, and their uncertainty is not included in the aggregate uncertainty of emissions reduction, an additional portion of emissions reduction needs to be set aside (e.g., 15% discount in the case of Activity Data).

Table 1. Percent uncertainty and set-aside factors for ISFL.

Aggregate uncertainty of emission reductions using a Monte Carlo simulation and reported at the two-tailed 90% confidence interval	Uncertainty set-aside factor
≤ 15%	0%
> 15% and ≤ 30%	4%
> 30 and ≤ 60%	8%
> 60 and ≤100%	12%
> 100%	15%

Frequency of use

Inventory during program design and every second year during the term of ERPA.

Spatial coverage

Jurisdictional (state, province, or region within a country).

Types of outputs

Uncertainty presented as a percent uncertainty relative to a two-tailed 90% confidence interval estimate.

Forest Carbon Partnership Facility. Carbon Fund.

Overview of the program

This program aims to support developing countries in their efforts on the implementation of REDD+ activities, *“by building their capacity and developing a methodological and policy framework that provides incentives for the implementation of REDD+ programs”* (Charter of the FCPF, page 1). In this sense, the Facility has a Carbon Fund which is designed to pilot the implementation of Emission Reduction Programs (ER Programs) using positive incentives such as the Emission Reduction Payment Agreement (ERPA). To date, 47 developing countries and 17 financial contributors participate under this Partnership. Of the USD\$ 1.3 billion capital for this Program, about USD \$900 million is committed to the Carbon Fund and 19 countries are currently in the Carbon Fund pipeline.

Requirements for uncertainty estimation

The ER Program uses the most recent IPCC guidance to account for emissions and removals for setting a Reference Level (RL) and Measurement, Monitoring and Reporting (MMR) systems. Thus, a REDD+ Country Participant is required to report on accuracy, precision and/or confidence level (as applicable) and discuss key uncertainties.

To guide the selection and implementation of the REDD+ programs, the Facility provides a Methodological Framework (MF), which rather than consisting of detailed calculation methods and protocols, it acts as *“a standard that is designed to achieve a consistent approach to carbon accounting and programmatic characteristics”* as reflected in a set of criteria and indicators selected by the Carbon Fund participants.

Although a REDD+ Country Participant can select which sources and sinks will be accounted for in their ER Program, at minimum they must include emissions from deforestation (i.e., using IPCC Approach 3 for activity data and Tier 2 or higher methods for emissions factors) and may consider forest degradation if it contributes to more than 10% of total forest-related emissions in

the accounting area of the Reference Level and ERPA periods. In addition, countries can exclude carbon pools and GHG if collectively they amount to *“less than 10% of total forest-related emissions in the Accounting Area during the Reference Period”* or *“the ER Program can demonstrate that excluding such Carbon Pools and greenhouse gases would underestimate total emissions reductions”*.

To account for uncertainty in the estimates of emissions and removals of GHG, participant countries are required to follow a 3-step process. First, countries need to identify and assess sources of uncertainty. Second, they need to minimize uncertainty as far as feasible and cost-effective (e.g., include a set of quality assessment and quality control processes to minimize systematic errors). Third, countries need to quantify the remaining uncertainty.

REDD+ Country Participant are required to use Monte Carlo methods to estimate the uncertainty in the Emission Reductions (ERs). If ERs resulting from avoided deforestation and forest degradation and from carbon stock enhancements are measured through separate (i.e., non-integrated) approaches and when degradation is estimated using proxy data, the uncertainty estimates need to be reported separately. However, if the ER is measured through integrated approaches (e.g., a national forest inventory), then the uncertainty is quantified as a single combined uncertainty estimate and reported at a two-tailed 90% confidence interval.

To reflect the level of uncertainty associated with the ER estimates over the Term of the ERPA, the ER Program accounts for a portion of ERs that need to be set aside in a buffer reserve. This amount is called a “conservativeness factor” and is assigned in the same manner to the five classes of aggregate uncertainty of emissions reductions as in the ISFL ER Program for deforestation and forest degradation (see Table 1, section on ISFL).

Finally, the ER Program is designed to address the risk of Reversals and the long-term sustainability of Emission Reductions. Based on a Reversal risk assessment, an additional amount of ERs generated and verified are transferred to an ER Carbon Fund Buffer so that they can be used to compensate should a reversal event occur during the Term of the ERPA. The ER Program Carbon Fund Buffer will set aside between 10% and 40% of the ERs generated and verified at each time of ER transfer.

Frequency of use

Before each periodic verification, *“activity data are determined periodically at least twice during the Term of the ERPA and allow for ERs to be estimated from the beginning of the Term of the ERPA.”*

Spatial coverage

Jurisdictional scale, programmatic scale or national-government-designated area (i.e., multiple land areas, landowners or managers within one or several jurisdictions, or a particular

ecoregion).

Verra (Verified Carbon Standard - VCS)

Overview of the program

The VCS Program is an international voluntary GHG program, with almost 1500 certified GHG projects. The credits from these projects amount to ~200 million tonnes of GHG removed from the atmosphere and are available on the open market to individuals and companies that wish to offset their own emissions.

To ensure credibility of the Program, certified projects must comply with the VCS Standard to generate GHG credits (Verified Carbon Units -VCUs). This global standard for GHG emission reductions and removal is consistent with the International Organization for Standardization requirements (i.e. ISO 14064-2:2006, ISO 14064-3:2006 and ISO 14065:2007), as well as with other requirements specific to the AFOLU sector (e.g., VCS document AFOLU Requirements, IPCC 2006 Guidelines for National GHG Inventories).

Requirements for uncertainty estimation

AFOLU projects are required to follow the methodologies approved under the VCS Program. Among the key methodological elements are that projects must include a clear statement of the assumptions, parameters and procedures that contribute the most to the overall uncertainty, as well as a description on how such uncertainty will be addressed.

The VCS Program requires that the uncertainty estimates follow recognized statistical approaches (e.g., IPCC Good Practice Guidance and Uncertainty Management in National Greenhouse Gas Inventories) and, where applicable, provide a means to estimate confidence intervals at the 90% or 95% confidence level. In addition, projects should apply “conservative factors” (Table 2) to deduct tradable GHG credits when a confidence interval (CI) is applied and the width of the confidence interval exceeds a percent of the estimated value (e.g., if 90% CI and the width is > 20% of the estimate or if 95% and the width is > 30% of the estimate).

Finally, when the selected methodology requires simulation processes that generate GHG emissions using specific models, the project shall assess model uncertainty following recognized statistical approaches, such as those described in 2006 IPCC Guidelines for National Greenhouse Gas Inventories, Volume 1, Chapter 3. The project must apply conservative assumptions and parameters on the GHG emission reductions or removals, as well as conservative factors to discount for model uncertainty³. For example, if the random uncertainty of overall emissions reduction is greater than 15% at a 95%CI, the project is

³ CDM Meth Panel guidance on addressing uncertainty in its Thirty Second Meeting Report, Annex 14. https://cdm.unfccc.int/Panels/meth/meeting/08/032/mp_032_an14.pdf

required to apply the conservative factors shown in the table below. However, if the random uncertainty value is less than 15% at a 95%CI, no further action is required to deal with random uncertainty.

Table 2. Acceptable uncertainty limits for random uncertainty. From: “CDM Meth Panel guidance on addressing uncertainty in its 32nd Meeting Report, Annex 14.”

Estimated uncertainty range at 95% confidence level of overall emissions reductions	Conservativeness factor
> +/- 15%, ≤ +/- 30%	0.943
> +/- 30%, ≤ +/- 50%	0.893
> +/- 50%, ≤ +/- 100%	0.836
> +/- 100%	To be addressed in the methodology

Frequency of use

Within 5 years of project registry and then as often as the project proponent wishes to apply for VCU issuance.

Spatial coverage

Subnational / project level. Depending on the size of their estimated average annual GHG emissions reductions or removals, projects are categorized in “projects” ($\leq 300 \text{ Kt CO}_2 \text{ yr}^{-1}$) and “large projects” ($> 300 \text{ Kt CO}_2 \text{ yr}^{-1}$).

Types of outputs

A two-tailed 90% or 95% confidence interval estimate.

Gold Standard

Overview of the program

Created by a group of international NGOs in 2003, the aim of this standard is to provide best practice guidance for climate change mitigation and sustainable development interventions, such as projects under the UN’s Clean Development Mechanism and the Sustainable Development Goals. To date, the standard is supported by 80 NGOs, and used in over 80 countries and more than a thousand projects.

In order to obtain a Gold Standard certification for projects focusing on forestry and agriculture the activities must comply with the “[Global Goals Principles & Requirements](#)”, as well as with the

“Land Use Forest Activity Requirements”. Projects aimed to generate GHG credits from Afforestation/Reforestation (A/R) activities, can include planting trees, single-species plantations and silvicultural systems (e.g., conservation of forests, selective harvesting), and agroforestry or silvopastoral activities. To enhance environmentally sound, social and economically viable management practices, projects can seek the certification of Forest Stewardship Council and the Gold Standard (dual certification). In the case of Agricultural (AGR) activities, projects need to fulfill the approved activities under the Gold Standard for the global goals of the “Agriculture Methodology”. In addition, A/R and AGR projects need to contribute to key principles such as, safeguards (a minimum of 10% of the total Project Area should protect or enhance biodiversity), climate security & sustainable development (AGR projects should promote adaptive capacity to climate change), stakeholder consultation & engagement and demonstrate real outcomes complying with additionality assessments.

Requirements for uncertainty estimation

The Gold Standard requires that the estimates of GHG emissions and removals from Land Use and Forestry activities of a project comply with a 20% target precision of the mean at a 90% confidence level. The Standard assumes that the uncertainties associated with measurements/estimates of several input parameters used to calculate GHG fluxes are known through statistical measurements, published values, or default IPCC values (i.e., 2003 GPG-LULUCF, 2006 Guidelines). However, the Standard offers three approaches to accommodate that the estimated uncertainty is not always known and that the baseline and project activity quantification required may not be met, including through the use of default IPCC values. In general terms, these approaches are:

- *Approach 1.* On-site measurements for pre-project and project activity data (e.g., land management change), following accepted sampling and analysis protocols. If the estimated uncertainty is >20% of the mean value, then the project owner can either apply an “Uncertainty Deduction” (Table 3) or increase the sampling effort to meet the 20% target. There is no deduction for uncertainty $\leq 20\%$.
- *Approach 2.* Peer-reviewed literature for baseline and project activity data quantification. The project owner shall prove that the selected information is conservative and suitable for the project area (e.g., climate factors, soil and vegetation types) and management activity (e.g., fertilization, crops, etc.), and shall use a 20% target precision of the mean at a 90% confidence level.
- *Approach 3.* Applicable IPCC default factors to account for pre-project and project management changes are used. Uses of an uncertainty deduction if the estimated uncertainty is >20% of the mean value at a 90% confidence interval.

Finally, regardless of the selected approach, the application of the uncertainty deduction shall be used in the most conservative manner to achieve “limiting the activities’ GHG benefits to the lower end of the confidence interval”.

Table 3. Uncertainty discount approach to deduct tradable GHG credits.

Uncertainty (U)	Uncertainty Deduction (% of U)
$20 < U \leq 30\%$	50%
$30 < U \leq 40\%$	75%
$40 < U \leq 50\%$	100%

Frequency of use

At least once within 5 years of certification cycle for agricultural projects or afforestation/reforestation projects.

Spatial coverage

Sub-national and or project level.

Types of outputs

A two-tailed 90% confidence interval estimate.

Overview of key Uncertainty requirements

The purpose of conducting uncertainty analysis varies, with the UNFCCC programs (NGGI and REDD+) utilising uncertainty as a way to support countries to efficiently improve their estimates, while the results based payment and voluntary offsets use uncertainty as a discounting mechanism, where larger uncertainties result in larger discounts. This difference however, does not create any fundamental differences in the way that the uncertainty is calculated.

The Uncertainty requirements for the different programs and protocols are overall generally consistent (Table 4). In particular they all require Uncertainty analysis methods to be consistent with the 2006 IPCC Guidelines on Uncertainties. They also allow for both Propagation of Error and Monte Carlo approaches, although some programs such as the World Bank FCPF encourage the use of Monte Carlo where appropriate.

The frequency of reporting includes annual, biennial and up to 5 yearly reporting. Conducting Uncertainty analysis can therefore either be a regular or occasional exercise and providing an intuitive method will be important for supporting users. All of the mechanisms require uncertainty to be reported as a percentage relative to a two-tailed confidence interval. The confidence interval varies and is typically in the range of a 90% or 95% confidence interval. Some flexibility in the FLINT uncertainty will need to be incorporated to support users to specify their confidence level.

Finally, the scale of spatial coverage ranges from project level, through to jurisdictional and national. The nesting of project, jurisdiction and national is therefore of interest, and calculating uncertainty at multiple levels of aggregation will be important.

The design presented in this document has focused on the base functionality needed for Uncertainty analysis consistent with the IPCC guidelines. There is certainly scope to provide expanded functionality, however as this will also introduce additional complexity. As such this document focuses on the core functionality requirements and identifies some potential functionality options that may be explored in future design processes.

All of the programs and protocols require Uncertainty to be presented as a percent uncertainty. This presents one potential issue, given that it is possible for the net emissions and removals for the land sector to be zero, it may not be mathematically possible to calculate a percent Uncertainty. Some programs address this issue by calculating Uncertainty separately for emissions and removals. This may work for activities such as reforestation and deforestation, however, for forest management type activities, zero or very small net emissions or removals are plausible. This is an issue with the requirement to provide percent uncertainties and is not an issue that can necessarily be solved by the FLINT Uncertainty module. However, having the ability in FLINT to output confidence intervals as well as percent uncertainties, may further assist in the assessment of uncertainty where emissions and removals are small.

Table 3. Summary of the key requirements for Uncertainty (U) analysis from different programs

Program	Use	Method	Frequency	Outputs	Spatial coverage
UNFCCC NGGI	U in annual emissions and trends for inventory improvement	IPCC Propagation of error and Monte Carlo	Annual/ biennial	Uncertainty expressed as a % (relative to a two-tailed confidence interval)	National, spatially explicit or spatially referenced
UNFCCC REDD+	U in annual emissions and trends to guide improvements	IPCC Propagation of error and Monte Carlo	Biennial	Uncertainty expressed as a % (relative to a two-tailed confidence interval)	National or subnational, spatially explicit or spatially referenced
World Bank FCPF	U in baseline emissions and uncertainty of mitigation for discounting	IPCC Propagation of Error and Monte Carlo	At project establishment then at least twice during the term of the ERPA ~2.5 years	U expressed as a % (relative to a two-tailed 90% confidence interval estimate).	Jurisdictional, programmatic scale or national-government-designated area, spatially explicit
World Bank ISFL Bio Carbon Fund	U in baseline emissions and uncertainty of mitigation for discounting	IPCC Propagation of Error and Monte Carlo	Biennial (during the term of the ERPA)	U expressed as a % (relative to a two-tailed 90% confidence interval estimate).	Jurisdictional (state, province, or region within a country), spatially explicit.
Verra (VCS)	U in emissions for mitigation discounting	IPCC Propagation of error and Monte Carlo	At least once every five-years of project registry	U expressed as a % (relative two-tailed 90% or 95% confidence interval estimate).	Subnational / project level, spatially explicit.
Gold Standard	U in emissions for mitigation discounting	IPCC Propagation of error and Monte Carlo	At least once every five-years until the end of the crediting period	U expressed as a % (relative to a two-tailed 90% confidence interval estimate).	Regional to microscale, spatially explicit.

Outline of the FLINT framework

What is FLINT?

The FLINT is an open-source modelling platform that provides tools to integrate multiple data types (including remote sensing, other spatial data and empirical data) with FLINT-compatible modules to produce spatially-explicit or spatially referenced calculations of emissions and other variables.

The FLINT is managed by [moja global](#), and open-sourced under the Mozilla Public License (MPL) 2.0. This means that the FLINT software is free to be used and modified, but any improvements to the FLINT software must be submitted back to moja global for consideration for inclusion in the FLINT open-source.

The FLINT provides a framework for coordinating the interaction of data (spatial and aspatial data, pools, variables, fluxes) and modules, and managing the outputs of any computations.

FLINT Modules

A FLINT-compatible module is a discrete software package attached to the FLINT that makes specific calculations (e.g. GHG emissions) or enhances functionality (e.g. data aggregation and reporting). FLINT modules may be open source or proprietary.

Modules have access to FLINT resources (eg., pools, variables, timing). While there isn't always a clear distinction between the two, there are generally two types of Modules: Calculation Modules and Functionality Modules.

Calculation Modules change the state of variables (e.g., land cover status, "has tree", etc.) and pools (e.g., above-ground biomass, below-ground biomass etc.) for a simulation unit (pixel), over a sequence of steps. The module is separate from location and sequence, simply using available information (variables, pools, stored state information) to perform specific calculations (e.g. required carbon movements). Once the calculations have been completed, the module sends the new state of variables and pools to the FLINT.

Functionality Modules affect the utility of the FLINT, but generally don't make any changes to variables or pools. This can be a utility for managing outputs or performance of the system. Several open-source Functionality Modules have been developed for tasks such as managing output databases, error logging, developing spatial outputs, and distributed processing.

FLINT Uncertainty Assessment

This uncertainty design deals with two methods of estimating uncertainty, Propagation of Error and Monte Carlo Uncertainty assessment. The IPCC guidance for these two methods is almost entirely focused on the context where emissions are calculated from activity data (AD) and emission factor (EF) information. The key difference in the use of the two methods is that the IPCC propagation of error approach assumes that the usual assumptions about the independence of variables and normally distributed errors apply, whereas the Monte Carlo approach provides methods for incorporating covariance and allows analysis of errors that may be asymmetrical. Uncertainty assessment in FLINT also needs to deal with a third situation, where assessments are based on models that may contain numerous uncertainties attributable to model input data, parameters, or algorithms. In this situation, the IPCC provides little specific guidance. This is because there are not yet general methods that universally apply, and therefore model users and developers must typically create customized solutions appropriate to their needs. FLINT provides the ability to do this and the Monte Carlo design in this document is one method for doing this. As noted in the introduction, the FLINT uncertainty module has been developed for the purpose of assessing the 'precision' component of uncertainty related to random error/variability. It is assumed that any systematic errors, or issues of 'accuracy' in IPCC terms have been identified and corrected prior to analysis in FLINT.

The Uncertainty Assessment is described here as a module, however, in practice, the code development required for implementation will require changes and functionality enhancements in existing FLINT libraries as well as new modules. The enhancements represent both functionality and calculation aspects of FLINT modules, as the Monte Carlo needs to control simulations and aggregate outputs consistent with the functionality module definition; as well as, specifying and changing the state of input variables.

Propagation of Error

IPCC Approach 1 Propagation of Error is relatively straightforward to apply and suits applications where the simplified AD X EF approach is used. For a full explanation of the key assumptions and requirements of Approach 1 Propagation of Error, refer to section 3.2.3.1, Volume 1 Chapter 3 of the 2006 IPCC Guidelines. This approach can be used to quantify uncertainty in estimates for any year as well as uncertainty in trends.

In the Propagation of Error approach, uncertainty in GHG emissions or removals is calculated by propagating the uncertainty of the inputs. This requires estimates of the mean and the standard deviation of all inputs.

There are two main steps to the method for calculating the uncertainty for any particular year. The first is combining the uncertainties of the emission factor and the activity data by

multiplication (Equation 1) and the second is combining these uncertainties through addition to arrive at an aggregate uncertainty (Equation 2).

Equation 1

$$U_{total} = \sqrt{U_1^2 + U_2^2 + \dots + U_n^2}$$

Where:

U_{total} = the percentage uncertainty in the product of the quantities (half the 95 percent confidence interval divided by the total and expressed as a percentage);

U_i = the percentage uncertainties associated with each of the quantities

Equation 2

$$U_{total} = \frac{\sqrt{(U_1 \times x_1)^2 + (U_2 \times x_2)^2 + \dots + (U_n \times x_n)^2}}{|x_1 + x_2 + \dots + x_n|}$$

Where:

U_{total} = the percentage uncertainty in the sum of the quantities (half the 95 percent confidence interval divided by the total (i.e., mean) and expressed as a percentage). This term 'uncertainty' is thus based upon the 95 percent confidence interval;

X_i and U_i = the uncertain quantities and the percentage uncertainties associated with them respectively (According to the IPCC guidance the quantities $X_1 \dots X_n$ should be independent i.e., the covariances are assumed to be zero).

Propagation of Error in FLINT

In implementations of FLINT that involve simple activity data and emission factor calculations, the propagation of error approach may be appropriate (depending upon meeting the assumptions identified in the IPCC 2006 Guidelines). For more advanced process based or empirical/process based FLINT implementations, Monte Carlo is likely to be more appropriate. As such the Propagation of Error approach in FLINT has only been designed for simple AD * EF implementations. This means that any particular flux (emission source) can be directly related to an AD * EF combination, making the application of the propagation of error equations possible. For Uncertainty in the level for any particular year in the inventory, the steps involved with the method are shown in Figure 3. These 5 main steps are:

1. Define model configuration. Model configuration is described in more detail under Monte Carlo methods in FLINT; for Propagation of Error, model configuration will involve a combination of emission factor or carbon stock factor and activity data (e.g., land use change).

2. Determine inputs to be included in the uncertainty assessment. The user should be able to choose which inputs they wish to include in the assessment, as they may not need to conduct an uncertainty assessment for all emissions or removals estimates.
3. Specify the input values and % Uncertainties for the selected inputs that are required for Equation 1. These will be stored in a database that FLINT will use for both emissions calculations and uncertainty calculations. Guidance on the input values and uncertainties can be found in the IPCC guidelines.
4. Calculate the emissions/removals separately for each simulation unit and each emission/removal source.
5. Aggregate the fluxes (emissions/removals) for each source (e.g., above ground biomass, below ground biomass, dead organic matter, soil organic carbon) in each year up to a level where the uncertainties can be considered uncorrelated (e.g., where there is a single emission factor and uncertainty value) and convert to CO₂-e. Then using Equation 1 calculate the uncertainty for the aggregated emissions using the specified percent Uncertainty value for the relevant emission factor and activity data combinations.
6. The Uncertainty for each aggregated source can then be combined at the desired level using Equation 2. A user may wish to aggregate Uncertainty for different pools or sources, as well as categories and sub-categories.

FLINT Propagation of Error

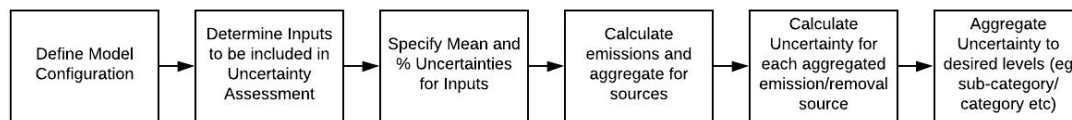


Figure 3. Workflow for FLINT propagation of error method

The IPCC 2006 Guidelines provide guidance on calculating the Uncertainty in the trend for any particular year in relation to a particular base year. The method provided in the guidelines analyses trend uncertainty of each emission category relative to the uncertainty introduced into the trend of the entire NGGI. It is therefore not feasible to implement the trend uncertainty method directly into FLINT because FLINT does not calculate emissions for all sectors. The method provided for calculating Uncertainty in the trend relates the uncertainty due to aggregated emissions/removals categories to total national emissions. This is done partly to avoid issues of correlation between emissions/removals estimates between sources. The methodology for doing this has therefore not been designed for FLINT at this stage, as FLINT does not calculate total national emissions across the entire inventory. Conceptually a modified method could be designed based on the IPCC approach, to calculate the uncertainty in the trend at the sectoral level, eg LULUCF, once all land uses have been implemented within the FLINT framework.

Monte Carlo

Monte Carlo analysis involves repeated model simulations where the values of model inputs are determined on a stochastic basis by randomly selecting their value from within their individual probability density functions. The outputs of these repeated model simulations are then used to calculate uncertainty statistics for the particular variables of interest (IPCC, 2006; McMurray et al., 2017).

The conceptual process for Monte Carlo is relatively straight forward:

1. Specify probability density functions for model inputs and any correlation between inputs
2. Run repeated simulations
 - a. Select input values randomly from within their PDF
 - b. Run model using the selected input values
 - c. Repeat specified number of times
3. Combine inputs from repeated simulations and calculate Uncertainty Statistics

In the context of FLINT for greenhouse gas estimation, we use the term model here in its broad sense and therefore model inputs can range from inputs for basic 'models' such as emission factors through to inputs for more advanced empirical and process based 'models' such as decomposition rates, growth curve parameters, and turnover fractions etc.

The execution of this process within the FLINT framework, disaggregated into component steps is described below.

FLINT framework

FLINT has two main modes of operation, spatially referenced and spatially explicit. The process for conducting Monte Carlo analysis in the two modes is very similar. The spatially referenced mode performs Monte Carlo simulations for all simulation units (spatially referenced units) while the spatially explicit mode requires additional steps to deal with the need to sample spatial units to include in the Monte Carlo simulation (Figure 4). Each of these steps is detailed below.

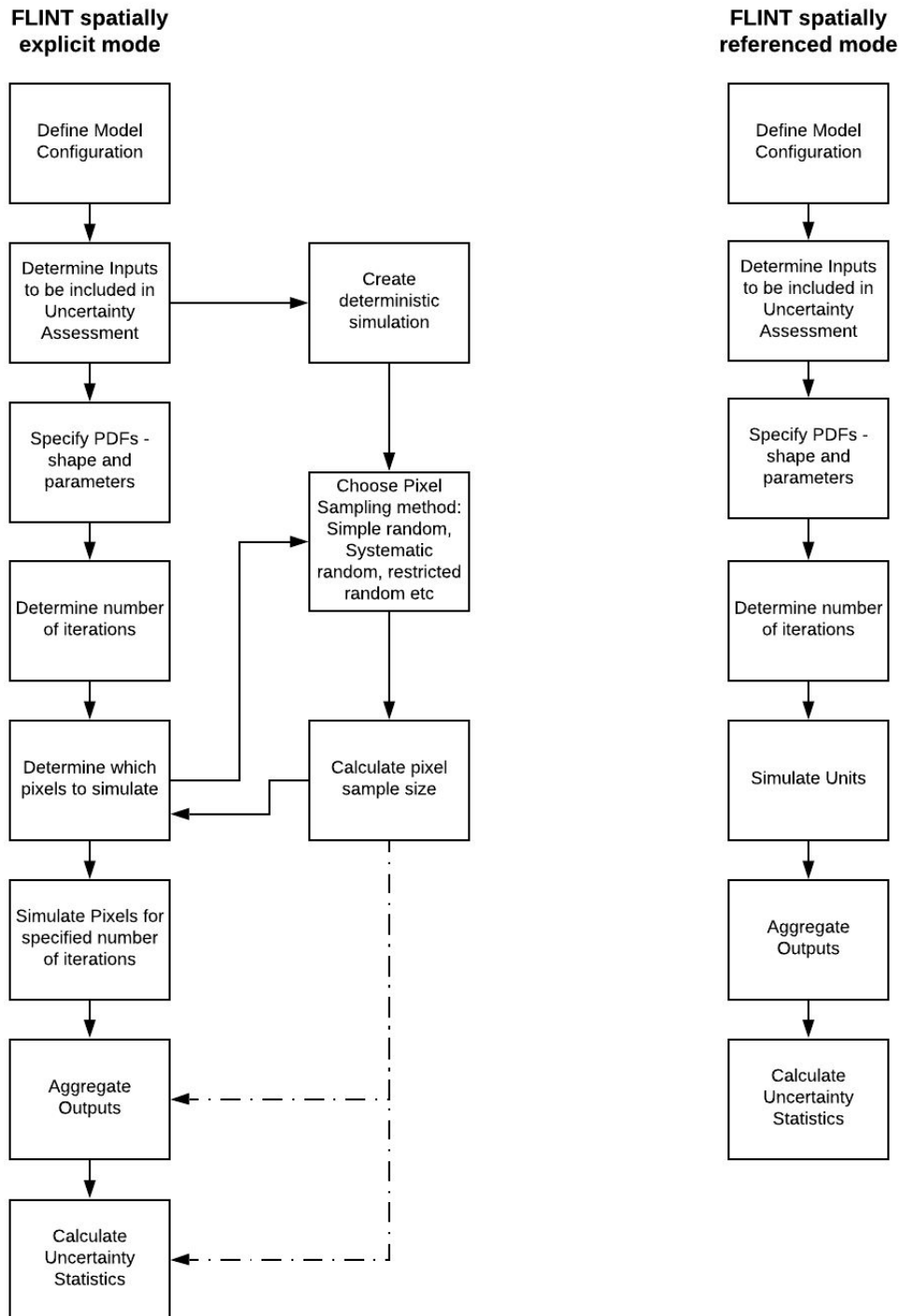


Figure 4. Process steps for performing Monte Carlo Analysis in the Full Lands Integration Tool for spatially explicit and spatially referenced modes.

Define Model Configuration

As FLINT is a modular framework, each individual implementation of FLINT is likely to include different modules, together with different data sources and inputs. As such, the first step in the FLINT uncertainty process is to determine the configuration of the FLINT implementation, and in so doing determine the input parameters of the implementation that can potentially be analysed through the Monte Carlo simulation.

The three general types of inputs to FLINT include:

- Spatial datasets
 - Static layers that do not change through time (e.g., soil classification)
 - Time-series layers that represent variables through time (e.g., land-use over time, or weather variables like rainfall)
- Model inputs
 - Managed through a database these are variables required by the models implemented through modules attached to the FLINT (e.g., decomposition rate constants or emission factors).
- Equations
 - Equations and their coefficients can be specified, for example above-ground forest biomass may be modelled using an empirical growth curve (e.g., a Chapman-Richards growth function with appropriate coefficients for particular forest types and/or locations).

Defining the model configuration is therefore the initial step that needs to be taken to determine the model inputs that can potentially be analysed through the Monte Carlo simulation.

The outcome of defining the model configuration step should be a list of all of the inputs used in the FLINT implementation.

Determining the inputs to be included in the Uncertainty Analysis

Once a full list of inputs has been compiled, the inputs that the user wishes to include in the Uncertainty analysis need to be able to be selected. It may not be necessary or desirable to include all inputs in a Monte Carlo simulation; for example, the user may only be interested in the variables that affect a particular model output, or they may know a-priori that certain inputs have no significant influence on the model uncertainty.

Specifying PDFs for Inputs

The development of Probability Density Functions (PDFs) for the inputs for which uncertainty is being analysed is a key requirement of Monte Carlo simulation. Monte Carlo simulation allows the use of any physically possible shape and width of these distributions, and also needs to be able to deal with correlation between the distributions of the inputs.

The standard C++ library contains approximately 20 functions in the random float generator library. In addition, the Boost C++ library supports more than 30 different PDFs (Annex 1). The most common of these that will likely be used in FLINT include the following:

- Normal
- Triangular
- Uniform
- Log Normal
- Weibull
- Beta

The PDF is specified by the user for each input of interest together with the parameters of the PDF. The choice of PDF will depend on a combination of the state of scientific or technical knowledge about a particular model input and the characteristics of the system being modelled. If little is known and uncertainty is based mainly on expert judgement, then simple bounded distributions like the triangular or uniform are likely to be used. If more is known, then more complex distributions may be used to describe the uncertainty. An additional consideration is whether or not it is appropriate to allow for the possibility of both positive and negative unbound values. If this is physically appropriate for the system being modelled, then normally distributed errors may be appropriate. If it is not, then PDFs with a lower bound of zero (e.g., Log Normal or Weibull) may be more appropriate.

Additional functionality in future versions of the module could include the ability for a user to enter an empirical distribution function directly. For example, one might know the distribution of errors for a series of model parameters obtained through some optimization process, and one might wish to use those directly without fitting a distribution.

PDFs for coefficients vs equations

As noted above, the input variables for FLINT can take a number of different forms. For example in the simplest methods, emissions for a given unit of activity may be specified by an emission factor. Therefore PDFs can be specified explicitly for the variable. However, some methods are more advanced and for example may be represented by an equation that has multiple coefficients. For these there are two potential approaches, specify PDFs for each of the equation coefficients, or specify a function that describes the variance of the input in relation to the predictor variable, that inherently incorporates the PDF for each discrete value of the input. For the initial implementation of the Monte Carlo module, it is proposed to provide support for the PDFs for equation coefficients, and if at a later point there is a need, development of a method to allow for a variance equation method could be undertaken.

Correlation of Inputs

The Monte Carlo method implemented for FLINT should be capable of handling correlation between input variables. The proposed method is that of Iman-Conover (Iman and Conover, 1982) which is a commonly used method that can be used to develop a rank correlation matrix for input variables. There is a selection of R packages (eg.

<https://rdrr.io/cran/mc2d/man/cornode.html>) and gitHub repositories for the Iman-Conover method that may be used or adapted for the FLINT Monte Carlo module. This method allows the user to specify the correlation between inputs.

Number of iterations for Monte Carlo simulations

As Monte Carlo analysis requires iterative simulation using randomly sampled input values from the PDFs, FLINT users need to be able to specify the number of iterations to perform for each sample point. This choice may be a function of the desire to ensure a degree of stability or convergence in the derived uncertainty statistics. As a result some users may also wish to specify criteria for this. It may also be a function of computing restrictions like the time available for conducting simulations or the available space for storing results.

The simple specification by the user of a number of iterations is the baseline functionality required for the initial implementation and is consistent with the IPCC guidelines. This places the responsibility on the user to determine how many iterations are needed to reach the degree of stability they would like. By storing all simulation results for each spatial unit, it will be possible for the user to assess the simulation results to determine if individual spatial units have reached an acceptably stable estimate. In this method all spatial units would need to be subjected to the same number of iterations so that the uncertainty statistics can be calculated.

In a subsequent implementation a second proposed method could be based on convergence, where the user specifies convergence criteria that must be met before simulation iterations are stopped. The proposed criteria for convergence would be to use the Standard Deviation as a measure of providing a stable estimate of the mean of a particular output of interest for the land units. The user will therefore need to be able to specify the variable of interest (for which the mean and Standard Deviation will need to be calculated) as well as the Standard Deviation that must be achieved to determine convergence. To ensure that the Uncertainty statistics can be calculated, all pixels will need to have the same number of iterations performed and the outputs for each pixel and iteration stored.

Determining which pixels to simulate

This step is the key difference between the spatially explicit method and spatially referenced method in FLINT. In a spatially referenced simulation all simulation units can be included in the Monte Carlo simulation, however, it may not be possible or efficient to include every pixel within a spatially explicit Monte Carlo simulation. Because the areas of interest to the key reporting mechanisms (e.g., UNFCCC NGGI) can be whole countries, the potential number of pixels can

be in the order of billions. As such it would be inefficient and potentially prohibitive to simulate all of these pixels potentially many hundreds or thousands of times as required for a Monte Carlo Simulation. For the initial implementation of the FLINT uncertainty module, a simple random sampling approach to selecting the pixels to include in the Monte Carlo simulation is proposed. This is a relatively simple method to implement and can utilise the existing C++ libraries for the random selection of pixels. Where all pixels cannot be simulated, then any process whereby only a subsample is selected will add an additional uncertainty terms that needs to be addressed.

The user will need to be able to specify the number of pixels to include in the Monte Carlo simulation. To support the user to do this it is proposed that a pre-run simulation or deterministic simulation will be run, from which the variance of all pixels can be calculated. A user may be interested in the overall variance associated with an individual emission source (eg carbon stock change in aboveground biomass) or they may be interested in the combined emissions/removals for all emission sources. It should therefore be possible to calculate the variance for all emission/removal sources as well as combined sources. The user could then choose the emission/removal source with the highest variance to determine the sample size. This variance can then be used together with the common sample size equations to determine the number of pixels that will need to be sampled in the Monte Carlo simulations. This assumes that there is a strong correlation between the variation in the population and the uncertainty associated with the input parameters. By storing all simulation outputs, it will be possible for the user to test this assumption by plotting the incremental effect on the mean of adding additional pixels.

More advanced probability based sampling methods are also possible for future implementation. Two methods which may offer the greatest advantages are Restricted Random Sampling and Unequal Probability. In restricted Random Sampling, the study area is divided into equal areas, and samples are taken randomly within each subdivision. This has the advantage that the sample is still random, but the samples are more evenly dispersed than in simple random sampling. In unequal probability sampling, sample units have different probabilities of being sampled. One example is the probability-proportional-to-size method that is commonly used in forestry. In this method a covariate is chosen related to the size of the sample unit. One generalised option to do this in FLINT could be to run a complete spatially explicit simulation for the study area, and use an output from this run as the covariate for determining the probability. The main advantage of this approach is to improve the efficiency of sampling, however, the calculations are more complicated and the efficiency benefit may not be as important where computer resources are not a severe limiting factor. Following the implementation of the simple random sampling method, it will be possible to determine if efficiency is an issue and if more advanced probability based sampling methods are warranted in future uncertainty module development.

Simulate land units and aggregate outputs

This process is conceptually simple, as the emissions calculations are performed in the 'normal' way, it is just performed multiple times using the values for each input variable selected from the PDF for the variable. As identified in the Number of iterations section, there are two methods for determining the number of iterations for each land unit Monte Carlo process. The first simply involves simulating the land unit the fixed number of times specified by the user. The second convergence method would require FLINT to monitor the output results for each land unit to determine when convergence criteria for the Standard Deviation is met.

Calculate Uncertainty Statistics

In terms of implementation, the calculation of Uncertainty statistics will be occur as part of the process of aggregating outputs, as the uncertainty calculations need to occur during the process of aggregation of outputs. The specific requirements for the Uncertainty outputs are listed below in relation to the aggregated outputs. Users need to be able to report their Uncertainty statistics with some level of flexibility to meet the requirements of the different programs and protocols.

The main characteristics are:

- All of the frameworks require uncertainties to be reported as a percent uncertainty relative to a particular two-tailed confidence interval.
 - Therefore users should be able to specify the confidence level (eg., 90%, 95% that they would like to use
 - The percent uncertainty should be calculated relative to the specified confidence level
- Because percent uncertainty is not always a perfect measure in the land sector, where net zero, or very small emissions are plausible, it should be possible to report the confidence intervals and not just the percent uncertainty.
- In Spatially Explicit mode, where Uncertainty has been calculated on the basis of sampling the pixels included in the Monte Carlo simulation, the additional uncertainty term introduced due to this sampling must be included in the overall uncertainty estimate.
- Trend uncertainty can be calculated by assessing the trend uncertainty for a base year and the year of interest (e.g., the inventory year). This is facilitated by storing the results for each Monte Carlo Simulation for each pixel through time.

Monte Carlo Infrastructure for FLINT

To implement Monte Carlo analysis in FLINT, new infrastructure will need to be developed. The Monte Carlo infrastructure in the FLINT consists of three major components, a Monte Carlo Local Domain Controller, Monte Carlo Transform and Monte Carlo Transform. These are described below.

Monte Carlo Local Domain Controller

The Monte Carlo local domain controller is a specialized version of the standard local domain controller which is responsible for managing the simulation lifecycle: iterating through the land units in the landscape and running the simulation for each land unit.

This specialized version of the controller includes the standard functionality and additionally manages the Monte Carlo run:

- Methods to select pixels to sample (for spatially explicit mode)
- Configurable with number of Monte Carlo iterations.
- Makes Monte Carlo information available to the system in the same way that land unit or timing information is available: current Monte Carlo iteration #,
- For each land unit, repeats the simulation for each Monte Carlo iteration.

Monte Carlo Transform

A variable in the FLINT can read its value from a Transform, which is a type of module that produces a value in a way that the programmer defines - for example, by reading a spatial layer, or by querying a database.

The Monte Carlo Transform produces a value based on the current land unit and Monte Carlo iteration # made available by the Monte Carlo Local Domain Controller.

Possible implementations could include:

- 1) The transform is configured with an upper and lower bound, and the step is calculated based on the number of Monte Carlo iterations - meaning that each Monte Carlo iteration for a land unit is a step through its distribution of values.
- 2) Similar to #1, the transform is configured with a distribution supported by a statistical library, essentially internalizing the work of producing distributions of values that used to be generated by external scripts. Each Monte Carlo iteration for a land unit is a step through the distribution of values.
- 3) The transform is configured with a fixed set of values produced externally to the FLINT, organized by Monte Carlo iteration #.

Monte Carlo configuration should be separate from the main configuration of a simulation to preserve as much of the original configuration as possible, including variable names. A new top-level configuration section (MonteCarlo) should be added to the current list (LocalDomain, Libraries, Variables, etc.), and the presence of this causes the FLINT to replace the standard LocalDomainController with the Monte Carlo version - allowing Monte Carlo to easily be toggled on or off by using the FLINT's multi-config file support.

Preserving the original variable names and configuration details means that the system needs to intercept and modify values being returned by the original variables in some way - either through hooks built into the system, or by wrapping the original variable under the same name in an additional transform, as defined above, when the system builds up the simulation objects from configuration files.

Variables return values in a variety of formats, each of which should be configurable for Monte Carlo simulation:

1) Single value

The simplest type of variable simply returns a single value that possibly varies by spatial location. Variables of this type are straightforward to intercept or wrap for Monte Carlo simulation.

2) Dictionary of key/value pairs

Some variables return a set of values equivalent to a database record, accessed by attribute name, often coming from a query.

Variables of this type do not have their attribute names exposed explicitly; instead, they are created at run-time as the query output is retrieved - meaning that the user must have detailed knowledge of the structure of the data in order to name and configure one or more attributes for Monte Carlo simulation.

To expose the attribute names returned by this type of variable in order to make Monte Carlo configuration easier, for example in a graphical interface, a preliminary non-Monte Carlo (deterministic run, i.e. one assuming certainty of knowledge) run could be performed, where the FLINT captures the details of each variable and records them in a JSON file or database at the end. The outputs of the deterministic run could also be stored to provide a baseline to compare to the Monte Carlo simulation results (e.g., to compare the mean of the Monte Carlo simulations to).

3) List of dictionaries of key/value pairs

Some variables return a complex dataset consisting of multiple records, for example the age/volume pairs making up a yield curve. Two alternative options should be supported here. First, the internal consistency of the database should be maintained in a Monte Carlo configuration, so the distribution should be an offset from the original values, where a single value is drawn for each Monte Carlo iteration and applied uniformly to the selected attribute. The second, is to allow the Monte Carlo values for complex records such as age/volume pairs to be drawn independently to allow users to test the situation where specific years may be over or under predictions.

4) Timeseries of values

Timeseries variables contain a list of values organized by simulation timestep. It is possible to retrieve either a single value for the current timestep the simulation is on, or the entire list of values at once.

Configuring this type of variable for Monte Carlo simulation should preserve the temporal autocorrelation pattern of the timeseries: the distribution should represent an offset from the original values, applied uniformly to the whole timeseries.

Monte Carlo Output Module

Collecting the results for Monte Carlo iterations works virtually the same way as for standard simulations and may in fact be able to re-use the same modules with some enhancements - for module configurations that use “classifiers” to stratify their output (GCBM, ...), it could be as simple as adding the Monte Carlo iteration # and optionally the Monte Carlo variable values for the iteration to the classifier set.

Another possibility is that most module configurations (GCBM, ...) have a concept of “location” (in both space and time) in their results, which every pool and flux record is attached to - so the Monte Carlo iteration could become part of the “location” of a results record.

Specific design elements for Spatially Referenced modes

Monte Carlo in spatially referenced mode uses the same design as the spatially explicit mode, except that it simulates all inventory records instead of a sample.

The only extra requirement is to implement a spatially referenced Monte Carlo local domain controller that extends the functionality of the standard spatially referenced LDC.

Key Criteria for assessing different software options for developing the modules

In the process of developing this design document, it became apparent that the most appropriate way to implement the Monte Carlo functionality in FLINT is through C++. The reasoning behind using existing C++ libraries as opposed to using R libraries or the required functionality already exists within the C++ language. Of the key criteria, existing libraries in C++ are able to satisfy key requirements:

- There are existing C++ libraries to generate the PDFs that users are interested in, as well as many more.
- In terms of being user-friendly, it is considered better to have an integrated toolset, rather than an add-on that needs to be managed separately.
- The existing libraries in C++ are industry standard, and are used across a wide range of computer science applications.
- Many elements for implementing the Monte Carlo analysis in FLINT will require FLINT specific code, and therefore the reusability of code/tools from external tools will be limited.

While existing Monte Carlo libraries exist in R, it is unlikely to result in a more efficient process to try to integrate C++ and R languages. In addition the R libraries typically only deal with part of the process required, for example, the Vignette Monte Carlo library in R automates the process of looping over simulations. As this part of the process will be specialised in FLINT anyway, there is likely to be no advantage in trying to adapt the R library to work with the C++ code. Core FLINT functionality (Monte Carlo Local Domain Controller, Transforms, output modules) will be developed in C++.

References

Iman, R.L., Conover, W.J., 1982. A distribution-free approach to inducing rank correlation among input variables. *Commun. Stat. - Simul. Comput.* 11, 311–334.
<https://doi.org/10.1080/03610918208812265>

IPCC, 2006. 2006 IPCC guidelines for national greenhouse gas inventories.

McMurray, A., Pearson, T., Casarim, F., 2017. Guidance on applying the Monte Carlo approach to uncertainty analyses in forestry and greenhouse gas accounting. World Resources Institute.

Annex 1 - Probability Density Functions (PDFs) that are available in the Boost C++ Library

Probability Density Function Distribution	
Arcsine	Laplace
Bernoulli	Logistic
Beta	Log Normal
Binomial	Negative Binomial
Cauchy-Lorentz	Noncentral Beta
Chi Squared	Noncentral Chi-Squared
Exponential	Noncentral F
Extreme Value	Noncentral T
F	Normal (Gaussian)
Gamma (and Erlang)	Pareto
Geometric	Poisson
Hyperexponential	Rayleigh
Hypergeometric	Skew Normal
Inverse Chi Squared	Students t
Inverse Gamma	Triangular
Inverse Gaussian (or Inverse Normal)	Uniform
	Weibull