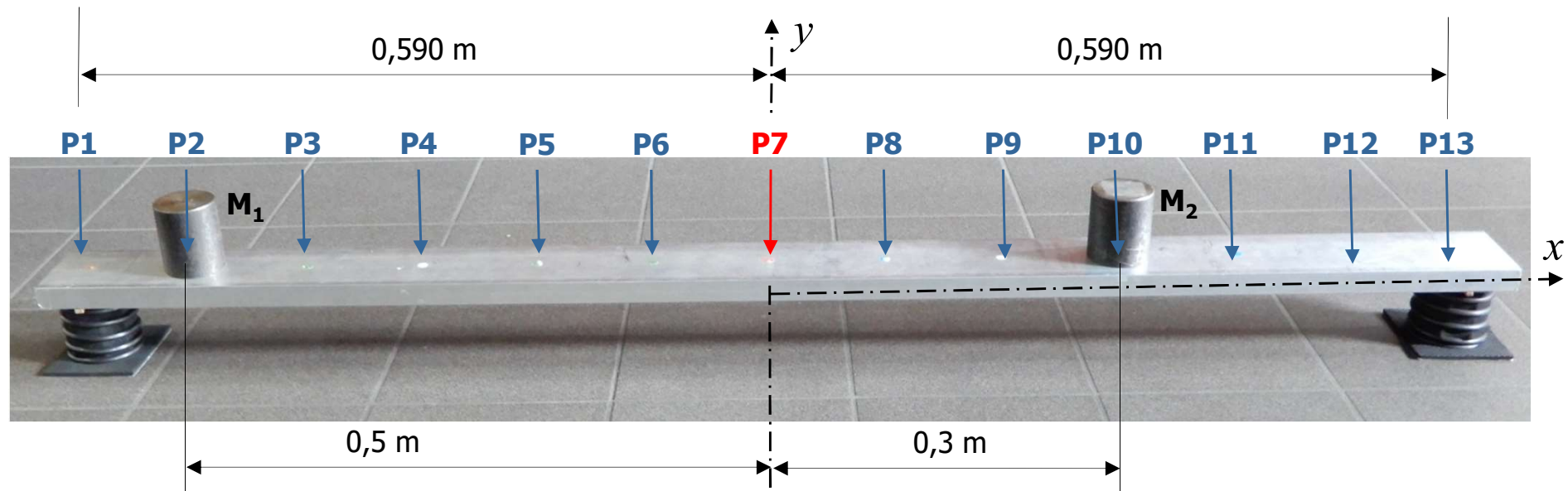


# **Dynamics of Mechanical Systems**

**J.C. GOLINVAL**

**Assignment of Project n°2**

The system consists of a beam on two supports located at both ends at coordinates P1 and P13 respectively. Two steel cylinders of mass  $M_1$  and  $M_2$  are mounted at locations P2 and P10 as illustrated in Figure 1.



**Figure 1.- Beam supporting two cylinders of mass  $M_1$  and  $M_2$**

Point P7 is located at the centre of the beam, P1 and P13 at a distance of 0,590 m from P7. The distance between two consecutive points (from P2 to P12) is equal to 0,1 m.

The beam has a rectangular cross-section and is made of aluminium.

## Geometry of the beam

Length = 1,25 *m*

Width = 80 *mm*

Height = 20 *mm*

## Material (aluminium)

Density = 2 690 *kg/m<sup>3</sup>*

Elasticity modulus  $\sim 6,45 \cdot 10^{10}$  *N/m<sup>2</sup>*

Poisson ratio = 0,39

## Steel cylinders

Mass ( $M_1$ ) = 1 *kg*; height = 61 *mm*; diameter = 51 *mm*

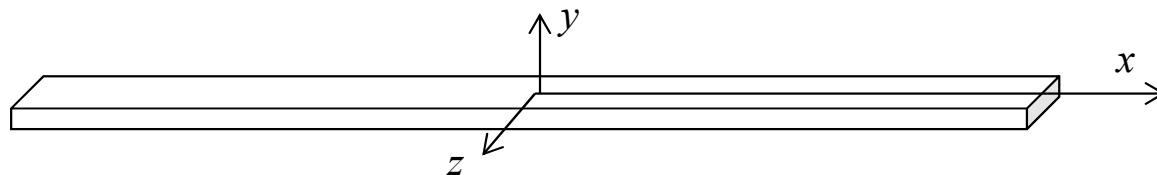
Mass ( $M_2$ ) = 1 *kg*; height = 61 *mm*; diameter = 51 *mm*

## Supports

Each support will be modelled by a single stiffness element with the following characteristic:

$$k_{support} = 10\,800 \text{ N/m}$$

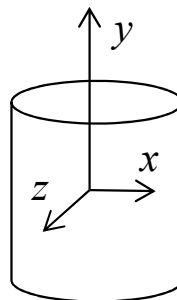
Moment of inertia of a beam about the  $z$ -axis passing through its centre of gravity



$$J_{beam} = \frac{M L^2}{12}$$

where  $M$  and  $L$  are the mass and the length of the beam respectively.

Moment of inertia of a cylinder about the  $z$ -axis passing through its centre of gravity



$$J_{cylinder} = \frac{M (3 R^2 + h^2)}{12}$$

where  $M$  is the mass,  $R$  the radius and  $h$  the height of the cylinder.

## Project 2 (Part 1)

1. Based on the assumption that the beam is rigid, write the analytical expressions of the kinetic and potential energies of the system in terms of displacements and rotations located on the neutral axis of the beam.
2. Choose two generalized coordinates and compute the associated stiffness and mass matrices.
3. Calculate the natural frequencies and the corresponding mode-shapes.
4. Calculate the coordinate of the node of vibration for the second mode.
5. The experimental results in terms of natural frequencies and mode-shapes are available on MyULiège website in the files 'Project\_2019\_freq.txt' and 'Project\_2019\_modes.txt' as explained hereafter.
6. Compare the theoretical predictions for the first two modes with the corresponding experimental results in terms of frequencies and mode-shapes.

## Experimental data available on MyULiège

- The file 'Project\_2019\_freq.txt' contains 5 identified frequencies (1<sup>st</sup> column) and modal damping factors (2<sup>nd</sup> column).
- The file 'Project\_2019\_modes.txt' contains the amplitudes of the corresponding identified mode-shapes (columns 1 to 5) under the format:

Mode 1	Mode 2	Mode 3	Mode 4	Mode 5
...	...	...	...	...
...	...	...	...	...
...	...	...	...	...

13 lines  
corresponding to  
locations P1 to P13

- 1) The report must be concise (max 3 pages according to the template given in the file Project 2\_part1\_form.pdf).
- 2) The figures must be clear (legend, readability).
- 3) The text must be well structured and should be free of spelling and grammatical mistakes.
- 4) Penalty for late report: -1 point/24h

A PDF version of the report will be sent by e-mail at the following address: [JC.Golinval@uliege.be](mailto:JC.Golinval@uliege.be) and will be named as follows:

DSM2\_part 1\_LAST NAME\_first name.pdf

The deadline for the submission of the form is fixed to

**November 21, 2019 at 13:45 pm.**



## Project 2 (Part 2)

1. Relaxing the assumption of a rigid beam, compute the first 5 natural frequencies and mode-shapes of the system using the Rayleigh-Ritz method; to this purpose, at least 10 polynomial functions will be chosen as approximation functions.
2. The FRF are measured by accelerometers at coordinates P4 and P10. The excitation consists of an impact produced successively at each point from P1 to P13 using the so-called «roving hammer technique». The responses are measured in the frequency range from 0 to 400 Hz with a frequency resolution of 0,1 Hz. The results of the experimental modal identification in terms of natural frequencies and mode-shapes are available on MyULiège website in the files 'Project\_2019\_freq.txt' and 'Project\_2019\_modes.txt'.
3. Compare the theoretical predictions with the experimental results in terms of frequencies and mode-shapes. Enumerate at least three reasons why discrepancies occur between the numerical and the experimental results, except the measurement accuracy.

- 1) The report must be concise (max 3 pages according to the template given in the file Project 2\_part2\_form.pdf).
- 2) Figures must be clear (legend, readability).
- 3) The text must be well structured and should be free of spelling and grammatical mistakes.
- 4) Penalty for late report: -1 point/24h

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[JC.Golinval@uliege.be](mailto:JC.Golinval@uliege.be) and will be named as follows:

DSM2\_Part 2\_LAST NAME\_first name.pdf

The deadline for the submission of the report is fixed to

**December 12, 2019 at 13:45 pm.**