MECH3750 Engineering Analysis II
Assignment 1: Buckling analysis

[This is essentially problem 2.29 from the text by Carnahan, Luther and Wilkes. The answer, however, is not in that book.]

A very light spring of length L has Young's modulus E and cross-sectional moment of inertia I; it is rigidly clamped at its lower end B and is initially vertical (Figure 1). A downward force P at the free end A causes the spring to bend over.

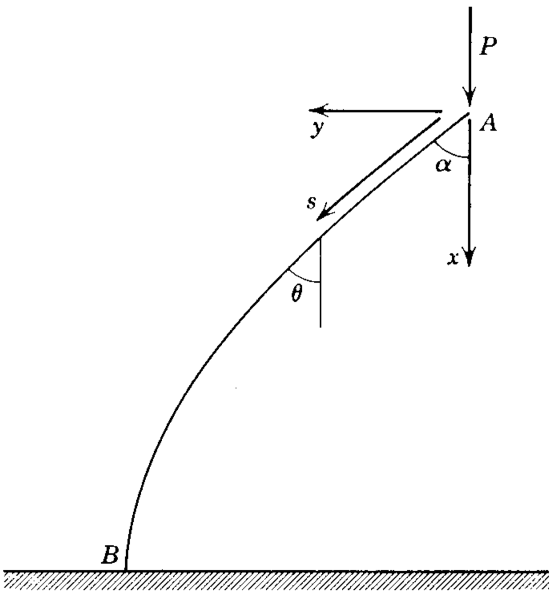

Figure 1: Buckling spring.

If θ is the angle of slope at any point and s is the distance along the spring measured from A, then the integration of the exact governing equation

$$EI\left(\frac{d\theta}{ds}\right) = -Py \quad ,$$

noting that $\frac{dy}{ds} = \sin\theta$, leads to

$$ds = -\frac{d\theta}{\left[\left(\frac{2P}{EI}\right)(\cos\theta - \cos\alpha)\right]^{\frac{1}{2}}} \quad ,$$

and

$$L = \sqrt{\frac{EI}{P}} \int_0^{\pi/2} \left[1 - \sin^2\left(\frac{\alpha}{2}\right)\sin^2\phi\right]^{-1/2} d\phi \quad ,$$

where α is the value of θ at A.

For the following tasks, you may work individually or in pairs, submitting one brief report by the due date shown on BlackBoard. The marking criteria will be on how completely and neatly you do the following tasks.

Task 1: Show from the above that the Euler load P_e for which the spring just begins to bend is given by

$$P_e = \frac{\pi^2 EI}{4L^2} \quad .$$

Task 2: Write an adaptive quadrature function in Python. Model it on the simple adaptive scheme that was shown as a recipe in MECH2700 but use a pair of Gaussian rules as the basic integration formulas. Remember to include documentation, along with some test and demonstration code.

Let x_g and y_b denote the vertical distance of A above the datum plane and the horizontal distance of B from A, respectively.

Task 3: Compute the values of $\frac{P}{P_e}$ for which $\frac{x_g}{L} = 0.99$, 0.95, 0.9, 0.5 and 0. What are the corresponding values of $\frac{y_b}{L}$ and α ?

[Note that the above expression for L and a related expression for $\frac{x_g}{L}$ involve elliptic integrals.]