Assignment-Part A

Apply one of the following numerical methods

(a) domain-free discretization (DFD) method,

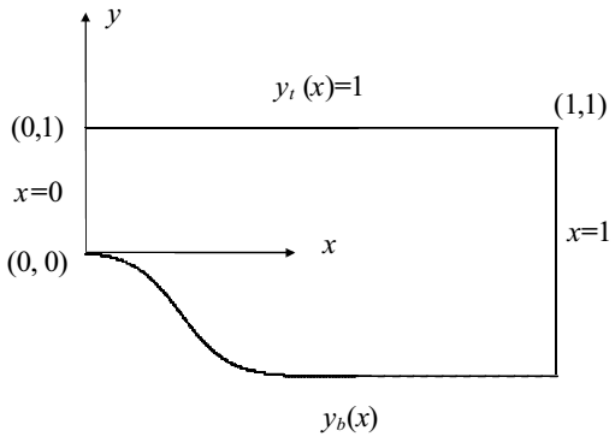
(b) least square-based finite difference (LSFD) method,

(c) local radial basis function-based differential quadrature (RBFDQ) method

to solve the two-dimensional nonlinear equation

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+u\left(\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}\right)-2(x+y)u=4$$

The computational domain is given below



$$\begin{cases} y_t(x)=1 \\ y_b(x)=\dfrac{1}{2}\left[\tanh(2-10x)-\tanh(2)\right] \end{cases}$$

The boundary conditions for the problem are

$$\begin{aligned} &u(x,y)=y^2, && \text{at } x=0 \\ &u(x,y)=1+y^2, && \text{at } x=1 \\ &u(x,y)=\sin(\pi x/2)+[y_b(x)]^2, && \text{at } y=y_b(x) \\ &u(x,y)=1+x^2, && \text{at } y=y_t(x) \end{aligned}$$

The assignment has to be completed with following requirements,

(1) You can use either direct method or iterative method to solve resultant algebraic equations;

(2) Consider the solution at the point $x = 0.5$, $y = 0.5$, and study the effect of increasing the number of points on the numerical solution. For the DFD method, this study can be made by increasing the number of lines and distributing more points along each line, while for the LSFD and local RBF-DQ methods, the study can be conducted by increasing the total number of points in the whole domain;

(3) For the LSFD and local RBF-DQ methods, study the effect of increasing the number of supporting points on the numerical solution. Again, you can use the solution at $x = 0.5$, $y = 0.5$ to illustrate your study;

(4) Plot the convergence history of the residuals if iterative methods are used;

(5) Show the numerical results by the figures or by the tables. It is preferable to plot contours of solution in the whole domain and the curves of solution along the vertical line of $x = 0.5$ and the horizontal line of $y = 0.5$;

Submit a report which shows clearly the methodology, solution procedure and the discussion of numerical results.