Chapter 3 Notes

3.1:
A simple pendulum consists of:

- A particle of mass mmm
- A massless string connected to a rigid support

Angle θ\thetaθ:

- Defined as the angle between the string and the vertical
- Assumed that the string remains taut at all times

Forces acting on the pendulum:

1. Gravity
2. Tension in the string

Force components:

- Parallel components sum to zero (since the string is inextensible)
- Perpendicular force to the string is: $F\theta = -mg\sin\theta$
- The negative sign indicates that the force is always directed opposite to the displacement from the vertical ($\theta = 0$).

3.2:
Realistic modification:

- The previous equation (3.2) describes a frictionless pendulum.
- Damping is added to make the system more realistic.

Sources of friction:

- Bearing friction at the pivot point.
- Air resistance.

Frictional force assumption:

- Proportional to velocity → modeled as −q(dθ/dt)-q (d\theta / dt)−q(dθ/dt).
- Here, qqq is a damping parameter that measures the strength of the damping.
- The negative sign ensures that friction opposes the motion.

Equation of motion for damped pendulum:

$$\frac{d^2\theta}{dt^2} = -\frac{g}{\ell}\theta - q\frac{d\theta}{dt}$$

- Second term (-q dθ/dt) models friction.

Solution and regimes:

- The equation is still linear and can be solved analytically.
- Three distinct physical regimes exist.

3.3:
Advancing the model:

- Moving beyond previous simplifications.
- Incorporating damping, nonlinearity, and external driving force.

Key modifications:

1. Nonlinearity:
    - No small-angle approximation → keeps sinθ instead of using θ.
2. Damping:
    - Friction term remains -q(dθ/dt).
3. Driving Force:
    - Added sinusoidal force: FDsin(ΩDt).

Equation of Motion:

$$\frac{d^2\theta}{dt^2} = -\frac{g}{\ell}\sin\theta - q\frac{d\theta}{dt} + F_D\sin(\Omega_D t)$$

- This defines the nonlinear, damped, driven pendulum.

Rewriting into first-order differential equations.

- Introducing ω=dθ/dt:

$$\frac{d\omega}{dt} = -\frac{g}{\ell}\sin\theta - q\omega + F_D\sin(\Omega_D t)$$
$$\frac{d\theta}{dt} = \omega$$

- These equations allow for numerical computation of θ(t).