# BE3 - partie 1

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On reprend le jeu de données concernant la valeur des logements des villes aux alentours de Boston. On cherche à identifier un bon modèle part régression pénalisée, CART, Boosting et Random Forest.

Les variables utilisées sont les suivantes:

CRIM taux de criminalité par habitant

ZN proportion de terrains résidentiels

INDUS proportion de terrains industriels

CHAS 1 si ville en bordure de la rivière Charles 0 sinon

NOX concentration en oxydes d'azote

RM nombre moyen de pièces par logement

AGE proportion de logements construit avant 1940

**DIS** distance du centre de Boston

RAD accessibilité aux autoroutes de contournement

TAX taux de l'impôt foncier

PTRATIO rapport élèves-enseignant par ville

LSTAT % de la population 'a faibles revenus

class valeur du logement en 1000 $\!\$$ 

L'objectif de ce BE est de comparer différents modèles de machines learning : le modèle linéaire, le modèle linéaire pénalisé (Ridge et Lasso), les modèles linéaires de réduction de dimension (PCR et PLS), les modèles à base d'arbres (CART, Boosting, Bagging et Random Forest).

Lasso = L1 Ridge = L2

#### Modèles linéaires

On commence par ajuster un modèle linéaire sans et avec interactions à partir de l'ensemble d'apprentissage constitué de 300 observations. Evaluer la qualité prédictive de ces deux modèles sur l'ensemble test constitué de 206 observations. On pourra utiliser la commande RMSE du package DiceEval.

library(DiceEval)

## Loading required package: DiceKriging

```
library(car)
## Loading required package: carData
library(MASS)
housing = read.table("housing_new.txt", header = T)
p = ncol(housing)
summary(housing)
                                                           CHAS
##
        CRIM
                           ZN
                                          INDUS
   Min.
          : 0.0060
                           : 0.00
                                      Min.
                                            : 0.46
                                                      Min.
                                                             :0.00000
                     Min.
                                                      1st Qu.:0.00000
   1st Qu.: 0.0820
                     1st Qu.: 0.00
                                      1st Qu.: 5.19
## Median : 0.2565
                     Median: 0.00
                                      Median: 9.69
                                                      Median :0.00000
## Mean : 3.6135
                     Mean : 11.36
                                      Mean :11.14
                                                      Mean :0.06917
   3rd Qu.: 3.6770
                     3rd Qu.: 12.50
                                      3rd Qu.:18.10
                                                      3rd Qu.:0.00000
                     Max. :100.00
##
   Max.
          :88.9760
                                      Max.
                                            :27.74
                                                      Max. :1.00000
##
        NOX
                          RM
                                         AGE
                                                          DIS
##
  \mathtt{Min}.
          :0.3850
                    Min.
                           :3.561
                                    Min.
                                           : 2.90
                                                     Min.
                                                            : 1.130
##
   1st Qu.:0.4490
                    1st Qu.:5.886
                                    1st Qu.: 45.02
                                                     1st Qu.: 2.100
   Median :0.5380
                    Median :6.208
                                    Median : 77.50
                                                     Median : 3.208
          :0.5547
##
  Mean
                                          : 68.57
                                                           : 3.795
                    Mean
                           :6.285
                                    Mean
                                                     Mean
   3rd Qu.:0.6240
                    3rd Qu.:6.623
                                    3rd Qu.: 94.08
                                                     3rd Qu.: 5.189
##
  Max.
          :0.8710
                          :8.780
                                    Max.
                                          :100.00
                                                     Max.
                                                           :12.126
                    Max.
##
        RAD
                         TAX
                                       PTRATIO
                                                        LSTAT
##
                                          :12.60
  Min. : 1.000
                    Min.
                           :187.0
                                    Min.
                                                    Min. : 1.73
   1st Qu.: 4.000
                    1st Qu.:279.0
                                    1st Qu.:17.40
                                                    1st Qu.: 6.95
## Median : 5.000
                    Median :330.0
                                    Median :19.05
                                                    Median :11.36
## Mean
         : 9.549
                    Mean
                          :408.2
                                    Mean
                                         :18.46
                                                    Mean :12.65
  3rd Qu.:24.000
                    3rd Qu.:666.0
                                    3rd Qu.:20.20
                                                    3rd Qu.:16.95
## Max.
          :24.000
                    Max. :711.0
                                    Max. :22.00
                                                    Max. :37.97
##
       class
## Min.
          : 5.00
  1st Qu.:17.02
## Median :21.20
## Mean
         :22.53
## 3rd Qu.:25.00
## Max.
          :50.00
dim(housing)
## [1] 506 13
set.seed(23)
u = sample(1:nrow(housing), 300)
housing.train = housing[u,]
housing.test = housing[-u,]
library(caret)
```

## Loading required package: ggplot2

```
## Loading required package: lattice
## Attaching package: 'caret'
## The following objects are masked from 'package:DiceEval':
##
##
       MAE, R2, RMSE
# Ajuster un modèle linéaire simple sans interactions
mod_lm <- lm(class ~., data = housing.train)</pre>
Y_pred_lm = predict(mod_lm, housing.test[, -p]) -p = excluir a coluna de previsão
RMSE_lm = RMSE(housing.test[, p], Y_pred_lm) # root mean square error
RMSE_lm # 5.9
                       p = pegar os dados de previsão
## [1] 5.900007
# Ajuster un modèle linéaire avec interactions
#a completer :
                             todos os parametros e suas iterações duplas
mod_lm_inter <- lm(class ~.^2, data = housing.train)</pre>
length(na.omit(mod_lm_inter$coefficients))
## [1] 79
#a completer :
Y_pred_lm_inter = predict(mod_lm_inter, housing.test[, -p])
#a completer :
RMSE_lm_inter = RMSE(housing.test[, p], Y_pred_lm_inter)
RMSE_lm_inter # 4.41
```

## [1] 4.416989

On remarque qu'en ajoutant les termes d'interaction l'erreur a diminué. Cependant le modèle avec interaction est très complexe (79 termes) donc certainement trop variable. L'idée est de chercher un modèle plus prédictif, i.e. avec une RMSE plus faible que 4.836.

# Modèles linéaires pénalisés

Ajuster une régression lasso (commande lars du package lars ). Pour ce faire, il est nécessaire de créer une matrice contenant tous les termes (variables principales et les interactions).

## Loaded lars 1.3

```
y = as.matrix(housing.train[, p])
x = as.matrix(housing.train[, -p])
x_extend = x

# makes the binary combinations of all columns
for (i in 1:(p-2)) {
   for (j in 2:(p-1)) {
        x_extend = cbind(x_extend, x[, i] * x[, j])
    }
}

mod_lasso = lars(x_extend, y, type = "lasso")

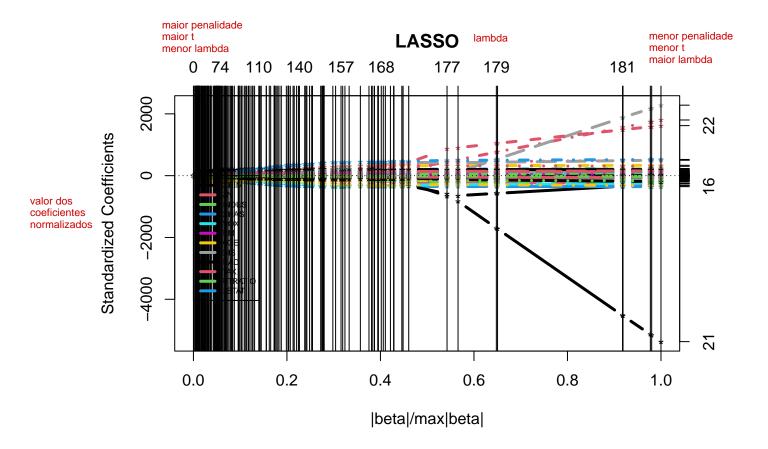
lambda ~ 1/(soma dos coefficientes)
   t >= soma do modulo dos coefficientes = penalidade
```

— Etudier la matrice des coefficients (commande coef.lars) et tracer le modèle (commande plot).

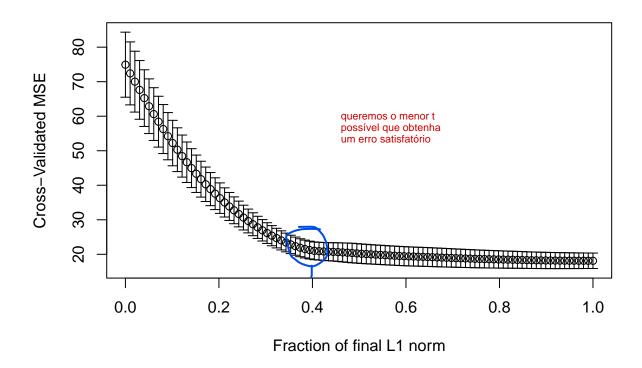
```
## [1] 127
```

With l = 10, sum equals 127, whereas with l = 100, sum equals 82. This shows that with a lesser l, more coefficients are getting 0 as a value.

```
par(mfrow = c(1, 1))
plot(mod_lasso, cex = 1, lwd = 3, col = 1:(p-1))
legend(
    0,
    0,
    names(housing.train[, -p]),
    col = 1:(p-1),
    lty = rep(1, p-1),
    lwd = rep(3, p-1),
    cex = 0.5
)
```



— Faire un choix du meilleur modèle en étudiant la décroissance de l'erreur par validation croisée (commande cv.lars). Faire un choix de la meilleure fraction (directement liée à la valeur du lambda. Fraction = 1 (resp. Fraction = 0) correspond à  $\lambda = 0$  (resp.  $\lambda = +\infty$ )).



```
#a completer :
value = 0.4
```

• Donner le nombre de coefficients non nuls du modèle final retenu. La pénalité a-t-elle jouer son rôle ?

```
#a completer :
coef_lasso_beter <- predict.lars(mod_lasso, s = value, mode = "frac", type = "coef")
#a completer :
nb_coef_nul = sum(coef_lasso_beter$coefficients == 0)
nb_coef_nul</pre>
```

#### ## [1] 51

Obtemos 51 coeficientes nulos. Um valor menor que os valores que obtivemos antes. L portanto seria maior que 100.

— Evaluer la qualité prédictive du modèle sur l'ensemble test (commande predict.lars ).

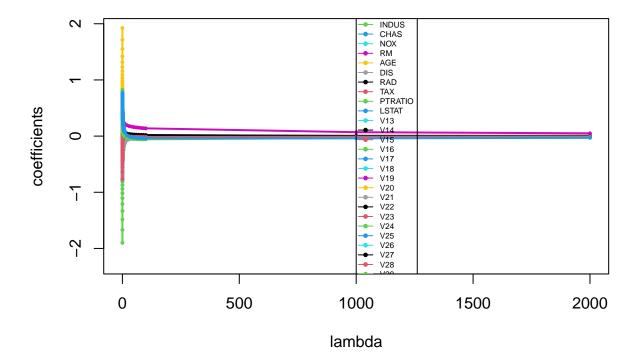
```
newx = as.matrix(housing.test[,-p])
for (i in 1:(p-2))
{
    for (j in 2:(p-1))
    {
        newx = cbind(newx,newx[,i]*newx[,j])
    }
```

```
#a completer :
fits <- predict.lars(mod_lasso, newx, s = value, mode = "frac")
#a completer :
RMSE_lasso = RMSE(housing.test[, p], fits$fit)
c(RMSE_lm_inter, RMSE_lasso)</pre>
```

#### ## [1] 4.416989 4.581404

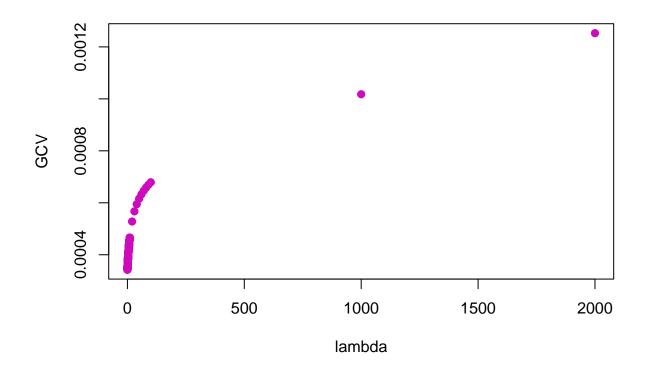
Ajuster maintenant une régression Ridge. La pénalité Ridge ne s'applique pas sur <u>l'intercept</u>. Il est donc nécessaire d'enlever le terme constant avant d'appliquer la procédure lm.ridge de la library MASS.

• Tracer les différents beta en fonction du  $\lambda$ . Que remarquez-vous ?

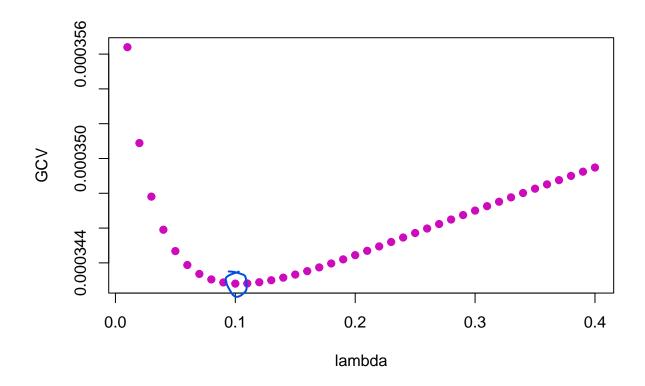


Nesse caso os termos não vão diretamente à zero, por mais que se aproximem dele. Le choix du meilleur  $\lambda$  se fait par validation croisée.

```
# tracer du GCV en fonction du lambda
plot(lambda, mod_ridge$GCV, pch = 19, col = 6, xlab = "lambda", ylab = "GCV")
```



plot(lambda[1:40], mod\_ridge\$GCV[1:40], pch = 19, col = 6, xlab = 'lambda', ylab = "GCV")



La remise à l'échelle est nécessaire pour la prédiction.

```
#prédiction du modèle et remise à l'échelle pour comparer à diab.test
ypred = as.matrix((x_extend-t(matrix(rep(mean.train[-p2],300),p2-1,300)))/t(matrix(rep(sd.train[-p2],300),p2-1,200)))
X_test = as.matrix(newx - t(matrix(rep(mean.train[-p2], 206), p2-1, 206)))
X_test = X_test / t(matrix(rep(sd.train[-p2], 206), p2-1, 206))
Y_pred_ridge = (as.matrix(X_test))**matrix(beta,p2-1,1))*sd.train[p2]+mean.train[p2]

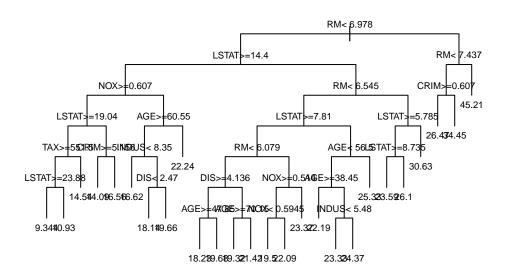
RMSE_ridge = RMSE(housing.test[, p], Y_pred_ridge)
c(RMSE_lm_inter, RMSE_lasso, RMSE_ridge)
```

## [1] 4.416989 4.581404 4.653380

## Modèles à base d'arbres

On commence par ajuster un modèle CART de la librairie Rpart.

Classification and regression tree

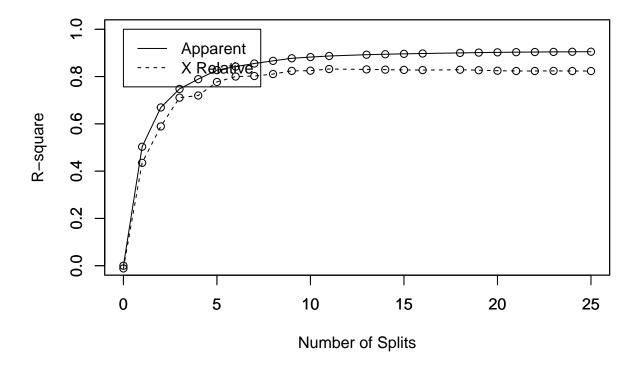


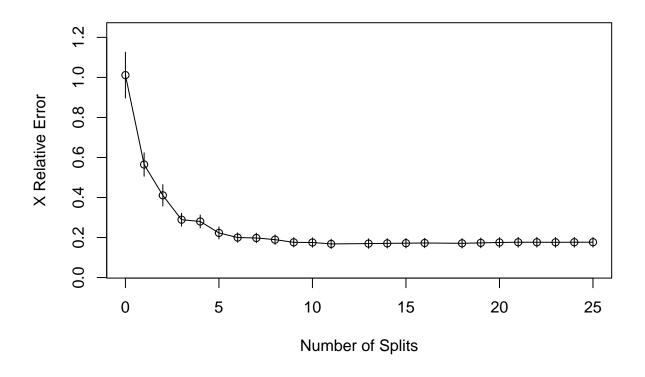
- A quoi correspond le paramètre cp? Il contrôle la complexité de l'arbre (complexity parameter).
- Le choix de la valeur 0.0001 conduit-il à un arbre grossier ou détaillé ? Un *cp* petit entraine un arbre profond avec beaucoup de noeuds.
- Quelles sont les variables les plus influentes ? RM et LSTAT (look both the height and frequency)
- Pourquoi n'est-il pas nécessaire ici de mettre en place un modèle avec interaction ? Trees already consider the interactions effects
- Faire un choix du meilleur *cp* par validation croisée (quatrième colonne de l'attribut *cptable* d'un objet de type *rpart*).

```
\#Apparent: apprentissage, X relative: par validation croisée \\ mod_tree$cptable # ele testa cps maiores (árvores mais simples), antes de chegar em cp=0.0001
```

```
CP nsplit rel error
##
                                        xerror
## 1 0.5030656136
                        0 1.00000000 1.0114478 0.11451713
     0.1662686916
                        1 0.49693439 0.5647737 0.05827073
## 3
     0.0782727816
                        2 0.33066569 0.4107676 0.05333136
## 4
     0.0414868184
                        3 0.25239291 0.2892056 0.03197709
                        4 0.21090609 0.2802356 0.03224474
## 5
    0.0382249961
                        5 0.17268110 0.2229881 0.03005734
## 6
    0.0148019951
                        6 0.15787910 0.1998693 0.02349025
## 7
     0.0130550489
## 8
     0.0111708195
                        7 0.14482405 0.1978706 0.02406715
## 9 0.0108478201
                        8 0.13365324 0.1894953 0.02378675
## 10 0.0051137140
                        9 0.12280542 0.1760483 0.02352917
## 11 0.0047095058
                       10 0.11769170 0.1752187 0.02347684
## 12 0.0027083741
                       11 0.11298220 0.1681780 0.02349925
## 13 0.0019607608
                       13 0.10756545 0.1697190 0.02364828
## 14 0.0015930841
                       14 0.10560469 0.1708474 0.02350104
## 15 0.0014471054
                       15 0.10401160 0.1718676 0.02353299
                       16 0.10256450 0.1727270 0.02363719
## 16 0.0013119307
## 17 0.0012952061
                      18 0.09994064 0.1710708 0.02362531
## 18 0.0009682042
                       19 0.09864543 0.1732953 0.02440385
## 19 0.0008134445
                       20 0.09767723 0.1753319 0.02442750
## 20 0.0005885364
                       21 0.09686378 0.1764818 0.02444670
## 21 0.0005644762
                       22 0.09627524 0.1764812 0.02444766
                       23 0.09571077 0.1761164 0.02445440
## 22 0.0004751045
## 23 0.0002569774
                       24 0.09523566 0.1766070 0.02447482
## 24 0.0001000000
                       25 0.09497869 0.1766223 0.02447450
#affichage uniquement cp>0.01
#summary(mod_tree, cp=0.01)
par(mfrow = c(1, 1)) # one plot on one page
rsq.rpart(mod_tree) # visualize cross-validation results
##
## Regression tree:
## rpart(formula = class ~ ., data = housing.train, control = cont)
## Variables actually used in tree construction:
           CRIM DIS
                        INDUS LSTAT NOX
  [1] AGE
##
## Root node error: 22208/300 = 74.025
##
## n= 300
##
##
              CP nsplit rel error xerror
                                              xstd
## 1
     0.50306561
                        1.000000 1.01145 0.114517
     0.16626869
                         0.496934 0.56477 0.058271
## 2
                      1
## 3
     0.07827278
                         0.330666 0.41077 0.053331
## 4
                      3
                         0.252393 0.28921 0.031977
     0.04148682
## 5
     0.03822500
                         0.210906 0.28024 0.032245
                         0.172681 0.22299 0.030057
## 6
     0.01480200
                     5
## 7
     0.01305505
                      6
                         0.157879 0.19987 0.023490
## 8 0.01117082
                      7
                         0.144824 0.19787 0.024067
## 9 0.01084782
                     8 0.133653 0.18950 0.023787
## 10 0.00511371
                     9 0.122805 0.17605 0.023529
```

```
## 11 0.00470951
                         0.117692 0.17522 0.023477
                         0.112982 0.16818 0.023499
## 12 0.00270837
                     11
                         0.107565 0.16972 0.023648
## 13 0.00196076
## 14 0.00159308
                         0.105605 0.17085 0.023501
## 15 0.00144711
                         0.104012 0.17187 0.023533
## 16 0.00131193
                         0.102564 0.17273 0.023637
## 17 0.00129521
                         0.099941 0.17107 0.023625
## 18 0.00096820
                     19
                         0.098645 0.17330 0.024404
## 19 0.00081344
                     20
                         0.097677 0.17533 0.024427
## 20 0.00058854
                     21
                         0.096864 0.17648 0.024447
## 21 0.00056448
                         0.096275 0.17648 0.024448
                     23
                         0.095711 0.17612 0.024454
## 22 0.00047510
## 23 0.00025698
                     24
                         0.095236 0.17661 0.024475
                     25
## 24 0.00010000
                         0.094979 0.17662 0.024474
```



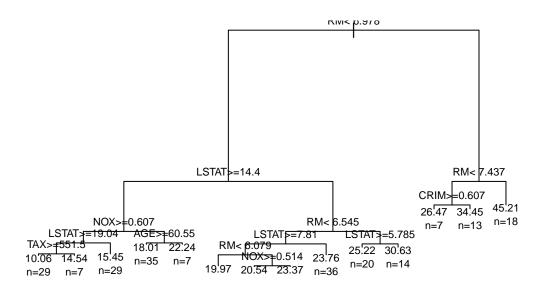


```
# the best result is given by nsplit = 11
mod_tree$cptable[12, 1]
```

#### ## [1] 0.002708374

- Elaguer l'arbre avec ce choix de  $\it cp$  (fonction  $\it prune$ )

```
#a completer :
mod_tree_pruned = prune(mod_tree, cp = 0.002708374)
par(mfrow = c(1, 1))
plot(mod_tree_pruned)
text(mod_tree_pruned, use.n = TRUE, cex = 0.7)
```



```
#a completer :
y_pred_tree <- predict(mod_tree_pruned, housing.test[, -p])
#a completer :
RMSE_cart = RMSE(housing.test[, p], y_pred_tree)
RMSE_cart</pre>
```

#### ## [1] 5.479321

- Mettre en place un modèle de boosting en utilisant la routine gbm de la librairie gbm.
- Combien d'arbres doit-on considérer ? 726
- Quelles sont les variables les plus influentes avec ce modèle ? RM et LSTAT

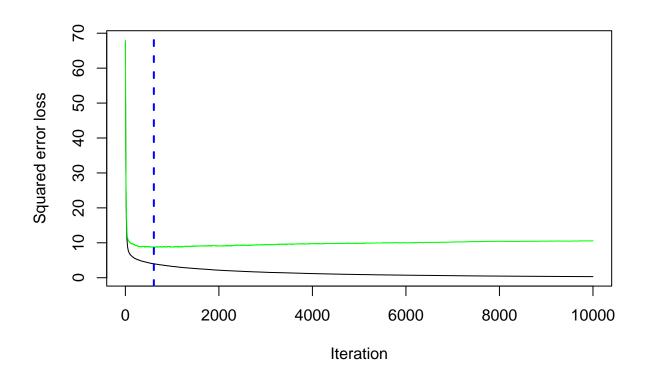
```
#modèle boosted trees
library(gbm)

## Loaded gbm 2.1.8.1

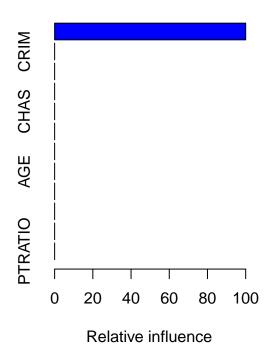
mod_boosted <- gbm(class ~., data = housing.train, n.trees = 10000, cv.folds = 10)

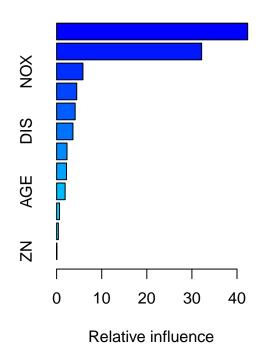
## Distribution not specified, assuming gaussian ...

par(mfrow = c(1, 1))
best.iter <- gbm.perf(mod_boosted, method = "cv")</pre>
```



```
par(mfrow = c(1, 2))
summary(mod_boosted, n.trees = 1)
                                             # based on the first tree
##
               var rel.inf
                        100
## LSTAT
             LSTAT
## CRIM
              CRIM
                          0
                          0
## ZN
                ZN
## INDUS
             INDUS
                          0
## CHAS
              CHAS
                          0
## NOX
               NOX
                          0
                RM
                          0
## RM
## AGE
               AGE
                          0
                         0
## DIS
               DIS
## RAD
               RAD
                          0
## TAX
               TAX
                          0
## PTRATIO PTRATIO
summary(mod_boosted, n.trees = best.iter) # based on the estimated best number of trees
```





```
##
                        rel.inf
                var
                 RM 42.36820423
## RM
## LSTAT
             LSTAT 32.15865779
                    5.83833014
## NOX
               NOX
## CRIM
               {\tt CRIM}
                     4.46132510
## PTRATIO PTRATIO
                     4.10908279
## DIS
               DIS
                     3.61888325
## TAX
                TAX
                     2.30171792
## INDUS
             INDUS
                     2.19312476
  AGE
                AGE
                     1.88764093
                     0.64305519
## RAD
                RAD
## CHAS
                     0.38004279
               CHAS
## ZN
                 ZN
                     0.03993511
```

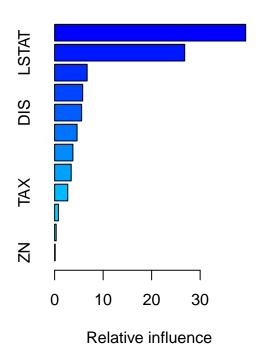
```
summary(mod_boosted, n.trees = 3000) # based on something
```

```
##
                       rel.inf
                var
## RM
                RM 39.3864807
## LSTAT
             LSTAT 26.8133470
## CRIM
              CRIM
                     6.7066555
## NOX
               NOX
                     5.7859931
## DIS
               DIS
                     5.5650171
## PTRATIO PTRATIO
                     4.6317920
## AGE
                AGE
                     3.7671004
             INDUS
## INDUS
                    3.4174186
```

```
## TAX TAX 2.7119535
## RAD RAD 0.7809762
## CHAS CHAS 0.3185178
## ZN ZN 0.1147481
```

```
#print(pretty.gbm.tree(mod_boosted,1))
#print(pretty.gbm.tree(mod_boosted,mod_boosted$n.trees))
f.predict1 <- predict(mod_boosted, housing.test[, -p], 1)
f.predict2 <- predict(mod_boosted, housing.test[, -p], 3000)
f.predict3 <- predict(mod_boosted, housing.test[, -p], best.iter)
RMSE_b1 = RMSE(housing.test[, p], f.predict1)
RMSE_b2 = RMSE(housing.test[, p], f.predict2)
RMSE_b3 = RMSE(housing.test[, p], f.predict3)
c(RMSE_b1, RMSE_b2, RMSE_b3)</pre>
```

## [1] 9.618313 5.024867 4.853227



bagging = selecionar aleatoriamente conjunto de linhas dos dados de treino, para treinar modelos em sequencia boosting = selecionar priorizando dados que foram pior preditos, para treinar modelos em sequencia

On construit maintenant une procédure de bagging. L'idée est de moyenner des arbres CART construits sur des échantillons bootstrap des données de départ. Un échantillon Bootstrap est un tirage avec remise de N points parmi les N points de l'échantillon initial.

```
#modèle arbre bagging
cont = rpart.control(minsplit = 2, cp = 0.0001)
B = 2000
Y <- matrix(0, nrow(housing.test), B)</pre>
```

```
for (i in 1:B) {
  u = sample(1:nrow(housing.train), nrow(housing.train), replace = TRUE)
  appren <- housing.train[u, ]</pre>
  mod_tree <- rpart(class ~., data = appren, control = cont)</pre>
  Y[, i] <- predict(mod_tree, newdata = housing.test)
#Y est de taille 206*B, 206 etant le nombre de points tests
#a completer :
Y_pred_bag = apply(Y, 1, mean) # mean of each tree`s result
#a completer :
RMSE_bag = RMSE(housing.test[, p], Y_pred_bag)
RMSE_bag
                        Bagging is an ensemble algorithm that fits multiple models on different subsets
                        of a training dataset, then combines the predictions from all models.
## [1] 4.864175
                        Random forest is an extension of bagging that also randomly selects subsets of
                        features used in each data sample.
Construire un modèle de Type Random Forest. - quelle est la différence entre un modèle type bagging et un
modèle de forets aléatoires?
#modèle arbres random forest
library(randomForest)
## randomForest 4.7-1.1
## Type rfNews() to see new features/changes/bug fixes.
##
## Attaching package: 'randomForest'
## The following object is masked from 'package:ggplot2':
##
##
       margin
mod_RF <- randomForest(class~., data = housing.train, ntree = 300, sampsize = nrow(housing.train))</pre>
summary(mod RF)
##
                     Length Class Mode
## call
                       5
                            -none- call
## type
                       1
                            -none- character
## predicted
                     300
                            -none- numeric
                     300
## mse
                            -none- numeric
## rsq
                     300
                            -none- numeric
                     300
## oob.times
                            -none- numeric
## importance
                     12
                            -none- numeric
## importanceSD
                            -none- NULL
                      0
## localImportance
                       0
                            -none- NULL
                       0
## proximity
                            -none- NULL
## ntree
                       1
                            -none- numeric
                            -none- numeric
## mtry
                       1
```

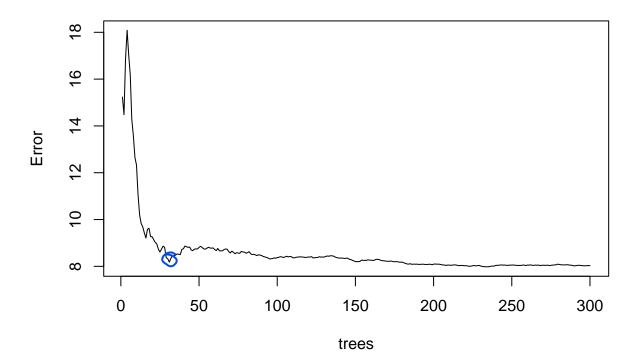
```
## forest
                    11
                          -none- list
## coefs
                    0
                          -none- NULL
## y
                   300
                          -none- numeric
## test
                     0
                          -none- NULL
## inbag
                     0
                          -none- NULL
## terms
                     3
                          terms call
```

## importance(mod\_RF)

##		IncNodePurity
##	CRIM	1289.00260
##	ZN	194.06135
##	INDUS	1409.38286
##	CHAS	41.82852
##	NOX	1309.59676
##	RM	7471.05953
##	AGE	656.74447
##	DIS	1036.39422
##	RAD	144.51651
##	TAX	1059.09902
##	PTRATIO	1306.87398
##	LSTAT	5852.45326

plot(mod\_RF)

# $mod\_RF$



```
which(mod_RF$trees == min(mod_RF$trees), arr.ind = TRUE)

## Warning in min(mod_RF$trees): no non-missing arguments to min; returning Inf

## integer(0)

#a completer:
Y_pred_RF <- predict(mod_RF,housing.test)
#a completer:
RMSE_RF = RMSE(housing.test[, p], Y_pred_RF)
RMSE_RF</pre>
## [1] 4.931825
```

## Méthode de réduction de dimension par orthogonalisation

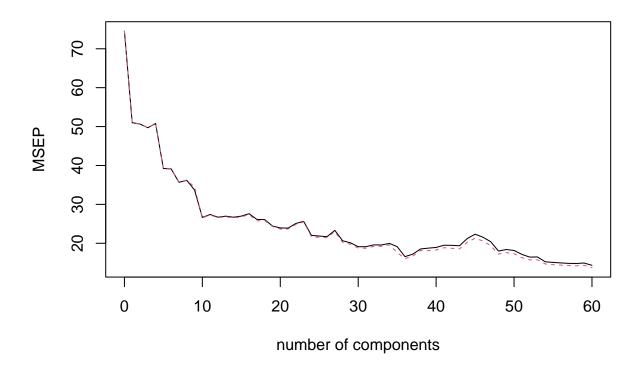
On s'intéresse maintenant aux modèles PCR et PLS construits à partir de la matrice augmentée des interactions.

```
library(pls)
##
## Attaching package: 'pls'
## The following object is masked from 'package:caret':
##
##
       R2
## The following object is masked from 'package:DiceEval':
##
##
       R2
## The following object is masked from 'package:stats':
##
##
       loadings
housing.pcr <- pcr(class ~ .^2, 60, data = housing.train)</pre>
```

Faire le choix du nombre de composantes par validation croisée en utilisant la routine crossval.

```
#a completer :
housing.cv <- crossval(housing.pcr,segments = 10)
plot(MSEP(housing.cv))</pre>
```

## class



Faire la prédiction du modèle pour le nombre de composantes sélectionnées.

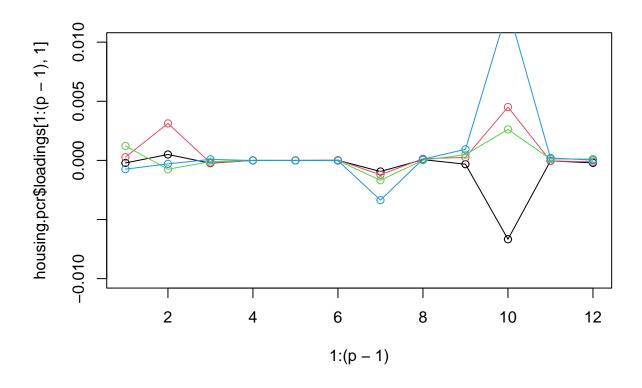
```
#a completer :
nbcomp = 36
Y_PCR = predict(housing.pcr, newdata = housing.test, ncomp = nbcomp, type = "response")
#a completer :
RMSE_PCR = RMSE(housing.test[, p], Y_PCR)
RMSE_PCR
```

#### ## [1] 5.487672

Tracer les premières fonctions propres et interpréter.

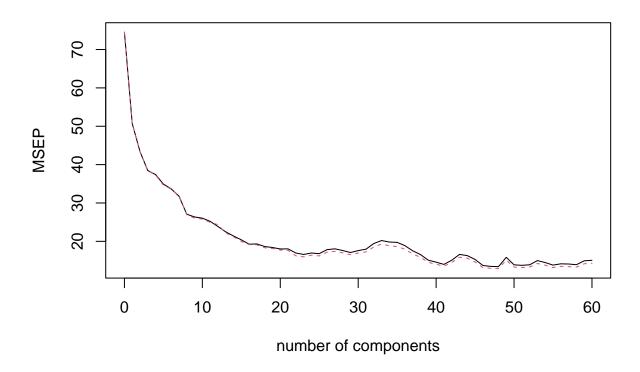
```
housing.pcr <- pcr(class ~ .^2, nbcomp, data = housing.train)
#coef(housing.pcr)

#tracer des premières fonctions propres
par(mfrow =c(1,1))
plot(1:(p-1),housing.pcr$loadings[1:(p-1),1],type ="l",col = 1,ylim =c(-0.01,0.01))
points(1:(p-1),housing.pcr$loadings[1:(p-1),1],col = 1)
for (i in 2:4)
{
    points(1:(p-1),housing.pcr$loadings[1:(p-1),i],col = i)
    lines(1:(p-1),housing.pcr$loadings[1:(p-1),i],col = i)
}</pre>
```



```
#PLS
housing.pls <- plsr(class ~ .^2, 60, data = housing.train)
housing.pls.cv <- crossval(housing.pls,segments =10)
plot(MSEP(housing.pls.cv))</pre>
```

# class



```
#a completer :
Y_PLS = predict(housing.pls, newdata = housing.test, ncomp = nbcomp, type = "response")
#a completer :
RMSE_PLS = RMSE(housing.test[, p], Y_PLS)
RMSE_PLS
```

## [1] 4.993442