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Homework 1

1.
   1. False
   2. False
   3. False
   4. True
   5. True
   6. True

2.
   1. {a,c,e}
   2. {a,b,c,d,e}
   3. {}
   4. {a,c}
   5. {{},{a},{c},{e},{a,c},{a,e},{c,e},{a,c,e}} – 8
   6. {(a, b),(a,d),(a,e),(a,f)(b,b),(b,d),(b,e),(b,f),(c,b),(c,d),(c,e),(c,f),(d,b),(d,d),(d,e),(d,f)}
   7. {a,b,c,d,f}
   8. 6

3. Proof:

Basis Step: If n = 0, then LHS = 1, and RHS =$\frac{3^{0+1}-1}{2}$ = 1.

Hence LHS = RHS.

Induction: Assume that for an arbitrary integer number k, $\sum_{i=0}^{k} 3^i = \frac{3^{k+1}-1}{2}$

Prove k+1

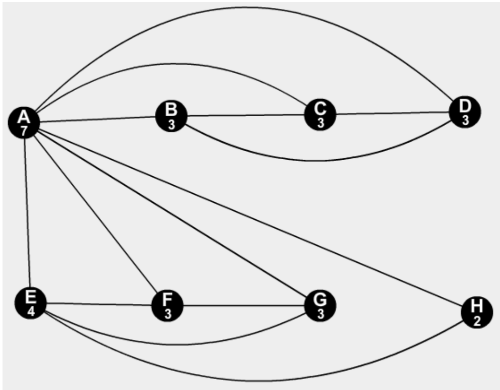
$\sum_{i=0}^{k+1} 3^i = \frac{3^{(k+1)+1}-1}{2}$

$\frac{3^{k+1}-1}{2} + 3^{k+1} = \frac{3^{(k+1)+1}-1}{2} => \frac{3^{k+1}-1+2(3^{k+1})}{2} = \frac{3^{(k+1)+1}-1}{2} => \frac{3^{k+2}-1}{2} = \frac{3^{k+2}-1}{2}$

Therefore by induction $\sum_{i=0}^{n} 3^i = \frac{3^{n+1}-1}{2}$ is true.

4.
   1. Reflexive: Yes, it is reflexive because it has (a,a),(b,b),(c,c),(d,d) inside the set.
   2. Symmetric: Yes, it is symmetric, it has (a,a)-(a,a); (b,b)-(b,b); (c,c)-(c,c); (d,d)-(d,d); (a,d)-(d,a); (d,b)-(b,d); (c,d)-(d,c) inside the set.
   3. Anti-symmetric: No, It is not symmetric because a < b and b < a can never both be true.
   4. Transitive: d can reach any letter so by calling "d" you can get a transitive relationship.
   5. Equivalence relation: Yes, it is an equivalence relation because it is reflexive, symmetric and transitive.

5.
   1. 7, 7, 3, 3, 3, 3, 3, 1 – **NO** - There are 2 7's nodes that have a degree of 7 and must have an edge to each node. (Since you can't do a loop) Therefore since we have 1 node with degree 1 that must accept 2 this problem is impossible
   2. 7, 4, 3, 3, 3, 3, 3, 2 – **YES** – graph below



3. 6, 5, 4, 3, 3, 3, 3, 2 – **NO** – This has an odd number of odd degree, and by the degree sum formula this graph is impossible
4. 6, 6, 6, 6, 6, 5, 4, 1 – **NO** – AN undirected 8 node graph can contain $\frac{V*(V-1)}{2} => \frac{8*(8-1)}{2} = 28$ and in this case we have a total of (6+6+6+6+6+5+4+1) = 40 therefore this is graph is impossible.