COMP 310 Homework 1

Solve the following problems, type them up, convert into a pdf file, and submit via moodle by 2355 on Monday 6/6. Clearly submissions made by the deadline are counted as on time, but additionally submissions made **before** I begin grading the item are also counted as on time. If moodle allows you to submit then I have not begun grading the item. My suggestion would be to submit something by the deadline and if you improve your answers then resubmit. Homework is not accepted after I begin to grade it. A subset of the questions will be graded. While your score will primarily be determined by whether your answers are correct, a portion of the score will be determined by the appearance of your submission and whether you followed directions. For example, submitting a word document instead of a pdf will cause you to lose some points.

Problem 1. (10 points) For each of the following statements determine whether it is true or false.

1. $\{\} = \{\emptyset\}$.
2. It is possible for a finite set and its power set (S and 2^S) to have the same number of elements.
3. If $|A \triangle B| = 5$ then A must contain at least 6 elements.
4. The union of two finite sets must be finite.
5. It is possible for a relation to be both symmetric and anti-symmetric.
6. The relation $\{(a, a)\}$ over the base set $S = \{a\}$ is reflexive, symmetric, antisymmetric.

Problem 2. (10 points) Let $X = \{a, b, c, d\}$, $Y = \{b, d, e, f\}$, and $Z = \{a, c, e\}$. Find each of the following.

1. $(X \cup Y) \cap Z$
2. $(X \cap Y) \cup Z$
3. $(X \setminus Y) \setminus Z$
4. $X \setminus (Y \setminus Z)$
5. 2^Z
6. $X \times Y$
7. $Y \triangle Z$
8. $|X \cup Y \cup Z|$

Problem 3. (10 points) Use induction to prove $\sum_{i=0}^{n} 3^i = (3^{n+1} - 1)/2$.

Problem 4. (10 points) Let $R = \{(a, a), (a, d), (b, b), (b, d), (c, c), (c, d), (d, a), (d, b), (d, c), (d, d)\}$ be a relation over $S \times S = \{a, b, c, d\} \times \{a, b, c, d\}$.

Is R reflexive? Is R symmetric? Is R anti-symmetric? Is R transitive? Is R an equivalence relation? Briefly explain each answer.

Problem 5. (10 points) For each of the lists below either draw an connected, undirected graph with eight nodes having one node of each degree listed or give a convincing argument why it is impossible.

- 7, 7, 3, 3, 3, 3, 3, 1
- 7, 4, 3, 3, 3, 3, 3, 2
- 6, 5, 4, 3, 3, 3, 3, 2
- 6, 6, 6, 6, 6, 5, 4, 1