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(CMMSE paper) A finite-difference model for indoctrination dynamics

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In this work, a system of non-linear difference equations is employed to model the opinion dynamics between a small group of agents (the target group) and a very persuasive agent (the indoctrinator). Two scenarios are investigated: the indoctrination of a homogeneous target group, in which each agent grants the same weight to his (or her) partner's opinion and the indoctrination of a heterogenous target group, in which each agent may grant a different weight to his or her partner's opinion. Simulations are performed to study the required times by the indoctrinator to convince a group. Initially, these groups are in a consensus about a doctrine different to that of the ideologist. The interactions between the agents are pairwise.

KEYWORDS

agent-based model, indoctrination, non linear difference equation, opinion dynamics

1 | INTRODUCTION

Indoctrination is a recurrent theme of discussion by some philosophers, although there is not an agreement about what indoctrination means; see the study of Thiessen.[1] For example, Peters argues, "whatever else indoctrination may mean, it obviously has something to do with doctrines."[2] In America, indoctrination means a method of teaching; see the study of Flew.[3] Other authors argue that the presentation of contentious beliefs constitutes indoctrination.[4]

Recently, a great interest has arisen in the study of social phenomena using the models and tools from complex systems science, statistical physics, and quantum field theory.[5-8] Among these social phenomena, the opinion dynamics has become an important field by itself.[9,10] Opinions can be interpreted as assessments made by the agents about prices, levels of preference (or agreement) with political parties, or controversial topics. Polarization into opinion clusters or the arising of a consensus in a social group has become of great interest in political science, economics, sociology, and sociophysics; see the study of Girejko, Machado, Malinowska, and Martins.[11] Of particular interest are the conditions that lead to the emergence of consensus, and the time required to its arising if it can be finally reached; see the study of Lu, Sun, and Liu.[12] For example, Medina, Macías, Gallegos, and Vargas[13] have studied the emergence of a consensus about three political options. Other authors have studied the control of the opinion and the social influence using opinion dynamics models; see Albi, Pareschi, Toscani, and Zanella.[14]

The extremism and indoctrination are also recurrent themes in opinion dynamics; for example, Deffuant, Amblard, Weisbuch, and Faure[15] have studied the emergence of extremism using an agent based model, while Short, McCalla, and D'Orsogna[16] using game theoretical methods have studied how small sects develop into large indoctrinated societies.

In this work, we study the indoctrination of small groups. To this purpose, we adapt an agent-based model, proposed by Medina et al,[17] to model the perturbation of consensus in the social group (target group) by a very persuasive agent who introduces a new opinion contrary to the common one shared for the group; for the sake of simplicity, this agent will be called the indoctrinator. This model uses the logistic map to introduce different fixed points on the opinion space, which are interpreted as different doctrines. Depending on the logistic map's parameter, agents are attracted or repelled from these doctrines. The logistic map's parameter is called the personal parameter. Each agent has two attributes an opinion and a personal parameter; both can evolve in time. Target groups are of two kinds: homogeneous ones, in which all agents give the same trust to each other agent's opinion and heterogeneous ones, in which, agents give, in general, a different trust to each other agent's opinion. These groups are considered being open to new opinions as long as good arguments are provided. In this model, the interactions among agents are pairwise; in any temporal step, any agent is free to interact with any other agent, then the opinions belonging to agents are updated through a system of first order non-linear difference equations; if an agent is convinced by the indoctrinator, her (his) personal parameter changes to that of the indoctrinator, while an agent, convinced by the group, changes her (his) personal parameter to that of the group. Initially, all agents in the target group have the same personal parameter and share the same opinion that corresponds to one of the equilibrium points. The opinion promoted by the indoctrinator corresponds to an unstable equilibrium point, a repeller, from the target group point of view, in this sense the indoctrinator promotes a contentious belief to the target group.

This work is sectioned as follows. In preliminaries section, the opinion dynamics model from the study of Medina et al[17] is presented. In section 3, we modify the previous model to investigate the dynamics of indoctrination, the social groups under consideration are characterized and a measure of the indoctrinator's influence: the charisma is defined. In results section, the required average times to convince groups of different sizes are given. Finally, a section of concluding remarks.

2 | PRELIMINARIES

The logistic model described in,[17] consist of a set of N-agents, in the temporal step $n \in 0 \cup \mathbb{Z}^+$, the ith-agent is characterized by a pair of attributes: an opinion $x_n^i \in [0, 1]$, and a personal parameter $\kappa^i \in [0, 4]$ which imitates the presence of strong attitudes by introducing attractors and repellers in the opinion space. Agents also use a logistic term to update their very own opinion, imitating an internal reflection processes, which allow them to manifest a concrete posture (acceptance or rejection) regarding certain opinions; see below and Figure 1.

In each temporal step, opinions are updated, when randomly chosen pairs of agents interact considering each other's opinion with some relative weights. The latter being subjective trusts that agents give to both opinions.

Thus, each pair of agents update their opinions through the following rules:

$$\begin{cases} x_{n+1}^i = a_{ii}\kappa^i x_n^i(1 - x_n^i) + a_{ij}x_n^j, \\ x_{n+1}^j = a_{ji}x_n^i + a_{jj}\kappa^j x_n^j(1 - x_n^j), \end{cases} \tag{2.1}$$

here, x_n^i and x_n^j are respectively opinions of i-th agent and j-th agent, while κ^i and κ^j are their personal parameters.

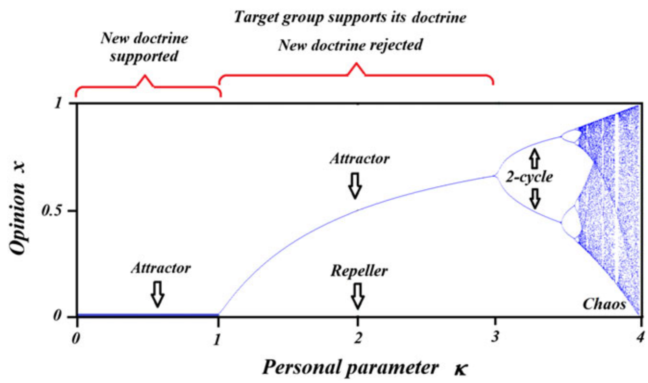

FIGURE 1 Bifurcation diagram of logistic map (3.1). Logistic map, in the dynamic equations, models an internal reflexion process in each agent. Without interaction with other agents, one who has a personal parameter $\kappa \in (0, 1)$ supports opinion $x = 0$, while other with $\kappa \in (1, 3)$ supports opinion $x = 1 - 1/\kappa$ rejecting $x = 0$. An agent with personal parameter $\kappa \in (3, 1 + \sqrt{6})$ is insecure, and her opinion is attracted to a 2-cycle, jumping between two different opinions, this could be interpreted as a dilemma. For $\kappa > 1 + \sqrt{6}$, a cascade of double period-doubling bifurcation appears, leading to chaos. In these regimes, agent's opinion jumps among different postures, becoming more and more insecure and finally falling in a completely erratic behavior [Colour figure can be viewed at wileyonlinelibrary.com]

The relative weights a_{ij} are those relative trusts given to the opinions of the pair of agents, they are defined as follows:

$$a_{ii} = \frac{c_{ii}}{c_{ii} + c_{ij}}, \qquad a_{ij} = \frac{c_{ij}}{c_{ii} + c_{ij}}, \tag{2.2}$$

$$a_{ji} = \frac{c_{ji}}{c_{jj} + c_{ji}}, \qquad a_{jj} = \frac{c_{jj}}{c_{jj} + c_{ji}}. \tag{2.3}$$

For example, a_{ij} represents the relative importance (or trust) given by ith agent to the opinion of jth-agent. Notice that $a_{ii} + a_{ij} = 1$. Here, c_{ij} are the absolute weights, which are positive and fulfill

$$\sum_{j=1}^{N} c_{ij} = 1. \tag{2.4}$$

The absolute weights, c_{ij}, represent the subjective importance given by each $ith-$agent to the opinion of each $jth-$agent in the social network; these weights were firstly introduced by DeGroot[18] to study the emergence of a consensus.

The following theorem[17] gives necessary and sufficient conditions to emergence of consensus in the social network described by the logistic model:

Theorem. *Consensus will form in (2.1)*

1. *Around* $x = 0$ *for* $0 < \kappa^i < 1$ *for* $i = 1, .., N$.
2. *Around* $x = 1 - 1/\kappa$ *for* $\kappa^i = \kappa, 1 < \kappa < 3,$ *for* $i = 1, \ldots, N$.

3 | INDOCTRINATION DYNAMICS

The logistic map,[19]

$$x_{n+1} = \kappa x_n (1 - x_n), \tag{3.1}$$

has a stable fixed point $x = 0$ for $0 < \kappa < 1$. If $\kappa = 1$, it bifurcates in a repeller and a new stable fixed point $x = 1 - 1/\kappa$ for $1 < \kappa < 3$. Logistic map converges to the stable fixed point without oscillation if $1 < \kappa < 2$ and oscillating if $2 < \kappa < 3$. If $\kappa = 3$, the stable fixed point bifurcates in a stable 2-cycle for $3 < \kappa < 1 + \sqrt{6}$; the logistic map converges jumping between two values. For $\kappa > 1 + \sqrt{6}$, a period-doubling bifurcation occurs, leading finally to chaos; see Figure 1.

The logistic map is used in the stuyd of Medina et al[17] to imitate an internal reflection process in the agents; it allows them to fix a posture regarding certain theme. For example, if there were no interactions among the agents, those agents with personal parameter $\kappa \in [0, 1]$ would be attracted to the stable fixed point $x = 0$, consequently supporting the opinion $x = 0$. Agents with $\kappa \in [1, 3]$ would support the opinion $x = 1 - 1/\kappa$, while they would reject the opinion $x = 0$. Agents with $\kappa > 3$ would be attracted to these n-cycles; they would be insecure about their opinions, jumping among different n-postures. Agents with a personal parameter in the chaotic regime would have an erratic opinion.

The stable fixed points, $x = 0$ for $0 < \kappa < 1$ and $x = 1 - 1/\kappa$ for $1 < \kappa < 3$, are interpreted here as two different and opposed doctrines. In this work, it is considered a social group integrated by one indoctrinator and a target group with $N - 1$ agents. Initially, the indoctrinator has the opinion $x_0^1 = 0$, and the personal parameter $\kappa_0^1 = 0.1$, while all agents in the target group have the same personal parameter $\kappa_0^i = \kappa$ and the same opinion $x_0^i = 1 - 1/\kappa$ for $i = 2, \ldots, N$ and $1 < \kappa < 4$. Hence, according to the Theorem, initially, all agents in the target group have a stable consensus if $1 < \kappa < 3$, and unstable if $3 < \kappa < 4$, in either case, they reject indoctrinator's opinion.

Two kinds of social groups are considered:

1. The homogeneous one which has the following features:
   (a) It is integrated by an even number of agents, N, conformed by one indoctrinator (the agent number one) and a subgroup of $N - 1$ agents called the target group.
   (b) The indoctrinator is considered a strong speaker, whose arguments are well supported. In this sense, all agents in target group give him (her) the same high absolute weight to his (her) opinion.
   (c) The indoctrinator gives himself (herself) the same high absolute weight to his (her) opinion.

(d) The indoctrinator gives the same absolute weight to opinions of target group members.
(e) The absolute weight matrix for this group is

$$C = \frac{1}{a+N-1}\begin{pmatrix} a & 1 & \dots & 1 \\ a & 1 & \dots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ a & 1 & \dots & 1 \end{pmatrix}_{N\times N}. \tag{3.2}$$

2. The heterogeneous one which has the following features:

(a) It is integrated by an even number of agents, N, conformed by one indoctrinator (the agent number one) and a subgroup of $N-1$ agents called the target group.
(b) The indoctrinator is considered a strong speaker, whose arguments are well supported. In this sense, all agents in target group give him (her) a high absolute weights to his (her) opinion.
(c) The indoctrinator gives himself (herself) a high absolute weight to his (her) own opinion.
(d) The indoctrinator gives different and smaller absolute weights to opinions of target group members (in the simulations below, these absolute weights are randomly chosen).
(e) There is at least one very influential agent in the target group, the agent number two; it is introduced to establish indoctrinator's charisma in the same way in all heterogeneous groups.
(f) The absolute weight matrix for this group is as follows:

$$C = \begin{pmatrix} ab/\Gamma_1 & b/\Gamma_1 & \gamma_{13}/\Gamma_1 & \dots & \gamma_{1N}/\Gamma_1 \\ ab/\Gamma_2 & b/\Gamma_2 & \gamma_{23}/\Gamma_1 & \dots & \gamma_{2N}/\Gamma_2 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ ab/\Gamma_N & b/\Gamma_N & \gamma_{N3}/\Gamma_N & \dots & \gamma_{NN}/\Gamma_N \end{pmatrix}_{N\times N}, \tag{3.3}$$

where $a > 1$, $b > 0$ and γ_{ij} for $i = 1, 2, \dots, N$ and $j = 3, 4, \dots, N$ are randomly chosen constants obeying $0 \le \gamma_{ij} \le b$, and Γ_i normalize the matrix's rows to add to one,

$$\Gamma_i = b(1+a) + \sum_{j=3}^{N} \gamma_{ij}. \tag{3.4}$$

Now, to have some way to measure the influence of indoctrinator on both target groups, we define the charisma of the $jth-$agent as perceived by $ith-$agent.

Definition. The *charisma* η_{ji} of the $jth-$agent as perceived by $ith-$agent is the ratio of the absolute weight that the $ith-$agent gives to $jth-$agent, to the weight that $ith-$agents grants to his (her) own opinion.

$$\eta_{ij} = \frac{C_{ij}}{C_{ii}}. \tag{3.5}$$

From this definition, one could see that the indoctrinator always have the same charisma $\eta_{1i} = a$ regarding all agents in homogeneous target group; while in heterogeneous target group, he (she) has a charisma $\eta_{12} = a$ only regarding the most influential member in the target group, the agent number two. In general, indoctrinator has a greater charisma regarding any other agent (different from agent two) in the heterogeneous target group since the weights of the other $N-2$ agents are randomly chosen.

For a very large indoctrinator's charisma, let us say, $a \to \infty$, the absolute weights for both target groups are for $i = 1, 2 \dots, N$; $C_{i1} \to 1$ and for $j = 2, 3, \dots, N$, $C_{ij} \to 0$. However, relative weights between agents in target groups remain constant; *ie*, they do not depend on a; this is interpreted as existence of strong bonds among those agents. This allows the agents in target group to convince back already indoctrinated agents. For example, for homogeneous target group relative weights are as follows:

$$a_{ij} = \frac{1}{2}, \tag{3.6}$$

for $i = 2, \dots, N$ and $j = 2, \dots, N$. While in the heterogeneous target group

$$a_{ij} = \frac{\gamma_{ij}}{\gamma_{ii} + \gamma_{ij}}, \quad a_{ii} = \frac{\gamma_{ii}}{\gamma_{ii} + \gamma_{ij}}, \tag{3.7}$$

with $\gamma_{i2} = b$; for $i = 2, \dots, N$ and $j = 2, \dots, N$.

To allow the agent number one to convince agents in target group, we allow the personal parameters to change in time, then the recursive model consists in the following:

1. A set of an even number of agents.
2. Initially, a weight matrix is generated; the value of charisma a is chosen; all agents in either target group have the same initial opinion $x_0^i = 1 - 1/\kappa$ and the same personal parameter $\kappa_0^i = \kappa$.
3. In each temporal step, randomly chosen pairs of agents interact through:

$$\begin{cases} x_{n+1}^i = a_{ii}\kappa_n^i x_n^i(1 - x_n^i) + a_{ij}x_n^j, \\ x_{n+1}^j = a_{ji}x_n^i + a_{jj}\kappa_n^j x_n^j(1 - x_n^j), \end{cases} \tag{3.8}$$

4. In each temporal step, if $x_n^i < \sigma$, then $\kappa_n^i = 0.1$ otherwise $\kappa^i = \kappa$ for $i = 1, 2, \dots, N$.

Here, $\sigma = (1 - 1/\kappa)/2$.

4 | RESULTS

Several simulations were performed to determine the average times required by a single charismatic indoctrinator to convince either a homogeneous target group or a heterogeneous target group. Homogeneous and heterogeneous target groups with personal parameters $\kappa \in \{1.5, 2.0, 2.9, 3.4, 4.0\}$ are considered. Initially, groups with personal parameters in

TABLE 1 Average time vs homogeneous group size with $\kappa = 1.5$

Agents	Average Time	Standard Deviation	Average Time	Standard Deviation	Average Time	Standard Deviation
10	16.6	3.2	13.4	1.3	12.6	2.0
20	34.3	6.1	21.5	2.0	18.5	1.7
30	419.1	247.0	29.9	2.1	24.8	1.4
40			40.3	3.7	31.5	2.3
50			55.5	6.3	40.0	3.2
60			101.8	28.0	48.2	3.4
70			2204.1	1996.9	69.7	7.6
80					99.7	24.1
Charisma	2		3		10	

TABLE 2 Average time vs homogeneous group size with $\kappa = 2.0$

Agents	Average Time	Standard Deviation	Average Time	Standard Deviation	Average Time	Standard Deviation
2	2	0	2	0	2	0
4	7.9	1.7	7.0	2.6	5.1	1.7
6	14.8	2.9	10.7	2.7	9.6	1.3
8	22.9	8.6	14.7	2.6	11.7	2.0
10	35.4	7.6	18.4	2.5	15.0	1.7
12	54.9	26.9	20.6	2.8	18.0	2.4
14	238.0	163.0	23.2	3.0	18.9	2.1
16	1345.4	722.4	28.3	3.8	21.9	2.2
18			31.2	5.8	27.0	2.7
20			41.5	9.2	31.4	4.0
22			50.0	14.9	33.5	3.5
24			54.7	7.7	39.6	4.8
26			93.9	43.1	44.0	8.2
28			123.2	92.2	49.1	8.4
30			322.8	233.4	66.2	10.1
Charisma	2		3		10	

the subset $\{1.5, 2.0, 2.9\}$ are in a stable consensus, while those with personal parameters in the subset $\{3.4, 4.0\}$ are in an unstable consensus, in fact $\kappa = 3.4$ correspond to the stable 2-cycle region while $\kappa = 4.0$ to the chaotic region in the bifurcation diagram in Figure 1. For each case, an indoctrinator with charismas 2, 3, and 10 is used.

For the homogeneous target group case and the considered charismas, results are presented in Table 1 for $\kappa = 1.5$, in Table 2 for $\kappa = 2.0$, in Table 3 for $\kappa = 2.9$, in Table 4 for $\kappa = 3.4$, and in Table 5 for $\kappa = 4.0$. For the heterogeneous target group case and the considered charismas, results are presented in Table 6 for $\kappa = 1.5$, in Table 7 for $\kappa = 2.0$, in Table 8 for $\kappa = 2.9$, in Table 9 for $\kappa = 3.4$, and in Table 10 for $\kappa = 4.0$.

Figure 2 compares the two cases, when the target groups are initially in a stable consensus, while Figure 3 compares the two cases, when the target groups are in an unstable consensus.

TABLE 3 Average time vs homogeneous group size with $\kappa = 2.9$

Agents	Average Time	Standard Deviation	Average Time	Standard Deviation	Average Time	Standard Deviation
4	15.6	4.7	8.0	2.7	7.1	2.3
6	38.5	16.9	15.8	5.9	12.6	3.2
8	183.2	131.2	24.5	6.1	23.2	7.1
10	10 304.1	10 026.7	40.4	11.8	34.1	6.2
12			62.4	24.5	51.8	12.7
14			134.1	77.8	80.0	24.1
16			602.8	498.9	229.8	133.2
18					2157.8	2051.9
Charisma	2		3		10	

TABLE 4 Average time vs homogeneous group size with $\kappa = 3.4$

Agents	Average Time	Standard Deviation	Average Time	Standard Deviation	Average Time	Standard Deviation
2	2	0	2	0	2	0
4	17.0	4.1	7.7	1.5	6.9	2.7
6	52.6	26.2	18.9	6.1	17.7	6.9
8	204.4	144.6	32.2	10.23	27.2	11.45
10	3593.1	2408.9	39.3	11.7	35.3	8.43
12			77.1	35.6	54.0	18.3
14			280.6	170.0	126.1	44.7
16			2773.6	2410.8	220.8	182.2
18					1960.3	1627.4
Charisma	2		3		10	

TABLE 5 Average time vs homogeneous group size with $\kappa = 4.0$

Agents	Average Time	Standard Deviation	Average Time	Standard Deviation	Average Time	Standard Deviation
2	2	0	2	0	2	0
4	21.7	10.75	9.4	2.7	6.7	2.5
6	40.7	19.8	15.4	3.9	18.6	8.2
8	58.9	26.0	29.4	9.1	26.6	11.7
10	127.0	79.8	43.4	11.4	35.7	13.5
12	394.9	262.8	73.3	26.1	53.3	15.9
14	1917.4	1750.0	104.3	46.1	80.7	31.0
16			180.1	72.6	112.6	63.4
18			462.1	363.1	184.0	107.0
20			2148.1	1827.9	309.3	204.6
Charisma	2		3		10	

TABLE 6 Average time vs heterogeneous group size with $\kappa = 1.5$

Agents	Average Time	Standard Deviation	Average Time	Standard Deviation	Average Time	Standard Deviation
10	15.7	4.7	12.3	2.5	12.2	2.4
20	23.2	7.0	21.5	2.8	22.2	4.6
30	34.2	5.1	27.1	4.6	26.5	5.5
40	43.3	8.8	35.8	4.2	32.9	6.9
50	49.0	10.8	43.6	8.3	39.6	7.4
60	59.2	18.5	49.2	10.2	49.4	11.1
70	70.5	12.3	61.8	9.3	59.2	12.8
80	83.4	33.8	75.8	12.0	70.4	16.6
90	106.7	36.3	83.0	27.6	80.7	33.8
100	126.8	43.8	94.7	25.1	87.9	29.1
110	188.6	61.2	109.0	36.3	107.0	39.3
120	205.2	171.2	155.6	79.2	117.5	33.9
130	257.1	128.4	173.3	90.0	127.7	45.8
Charisma	2		3		10	

TABLE 7 Average time vs heterogeneous group size with $\kappa = 2.0$

Agents	Average Time	Standard Deviation	Average Time	Standard Deviation	Average Time	Standard Deviation
10	33.1	5.4	17.2	10.2	14.2	4.0
20	35.3	10.9	32.3	6.7	28.8	5.8
30	59.0	19.7	49.6	18.0	45.4	11.9
40	67.9	28.2	68.2	16.0	69.8	19.2
50	136.3	58.5	102.9	44.1	78.9	21.5
60	227.2	117.4	126.5	46.3	147.2	130.3
70	455.1	458.3	222.9	199.0	182.1	124.1
Charisma	2		3		10	

TABLE 8 Average time vs heterogeneous group size with $\kappa = 2.9$

Agents	Average Time	Standard Deviation	Average Time	Standard Deviation	Average Time	Standard Deviation
4	8.8	3.79	6.25	2.05	5.6	2.04
6	16.65	6.02	13.0	5.20	10.75	4.08
8	20.75	8.03	16.4	6.00	16.65	4.98
10	26.66	10.34	22.75	11.74	18.95	8.48
12	33.68	14.96	30.72	15.07	24.15	9.04
14	40.86	21.99	34.98	12.23	34.3	11.05
16	57.84	28.27	45.82	18.62	39.50	13.82
18	60.86	21.10	47.84	20.32	39.10	8.49
20	79.25	40.08	61.50	28.83	57.30	20.08
22	89.01	35.80	77.96	37.73	66.85	19.65
24	115.68	80.18	81.2	33.27	76.22	34.28
26	145.12	86.16	84.96	32.70	83.78	34.13
28	188.18	180.16	109.64	55.28	102.69	58.69
30	257.14	231.62	130.54	53.43	123.90	53.91
32	300.77	259.15	177.88	113.81	141.22	81.82
34	441.72	458.30	218.90	155.98	183.29	121.23
Charisma	2		3		10	

TABLE 9 Average time vs heterogeneous group size with $\kappa = 3.4$

Agents	Average Time	Standard Deviation	Average Time	Standard Deviation	Average Time	Standard Deviation
2	2	0	2	0	2	0
4	9.6	3.6	6.3	1.8	6.1	3.3
6	14.7	5.5	14.2	5.2	9.4	3.2
8	23.5	6.7	15.9	10.2	14.8	4.5
10	36.1	14.0	23.2	13.2	23.5	7.6
12	42.1	23.9	28.6	10.1	31.6	14.6
14	57.3	25.6	33.7	8.9	35.6	14.2
16	63.2	24.2	45.9	19.8	40.9	17.1
18	80.9	37.6	52.1	25.0	49.5	23.5
20	88.9	51.6	69.1	29.8	53.6	13.0
22	111.1	36.0	86.4	51.3	70.0	33.0
24	160.1	92.9	95.1	44.2	76.0	27.9
26	211.6	142.9	116.3	32.4	90.3	35.2
28	231.8	220.3	157.2	110.9	115.0	59.5
30	258.5	204.2	171.6	112.1	117.6	76.0
Charisma	2		3		10	

TABLE 10 Average time vs heterogeneous group size with $\kappa = 4.0$

Agents	Average Time	Standard Deviation	Average Time	Standard Deviation	Average Time	Standard Deviation
20	33.0	5.8	38.9	15.3	35.3	16.4
100	70.3	15.2	70.7	13.9	81.8	19.8
200	89.7	21.3	94.9	11.9	87.5	13.6
300	96.9	10.9	97.4	20.4	100.5	14.3
400	111.9	20.3	97.6	9.1	100.4	9.1
500	110.3	10.3	105.8	8.6	110.7	17.0
600	102.8	11.0	115.5	15.7	100.5	9.2
700	120.8	15.5	112.0	14.3	112.0	16.3
800	116.1	12.8	117.9	12.9	115.5	9.6
900	119.0	17.5	111.3	11.1	121.7	14.3
1000	111.3	8.6	119.3	9.7	116.8	13.0
1100	112.2	10.2	122.0	13.7	121.9	14.0
1200	120.2	11.5	119.5	9.8	124.7	13.1
1300	130.1	20.2	128.3	16.6	118.2	9.8
Charisma	2		3		10	

From Figure 2, it can be seen that

1. The required time grows exponentially with the group size.
2. The required time grows also with the personal parameter because with greater personal parameter, the greater the opinion difference among the indoctrinator and the target group, since, initially, the indoctrinator has an opinion $x = 0$ while the target group has an opinion $x = 1 - 1/\kappa$.
3. More time is required to indoctrinate a homogeneous group than a heterogeneous group.
4. The more charismatic the ideologist is, the less time required to convince the group.

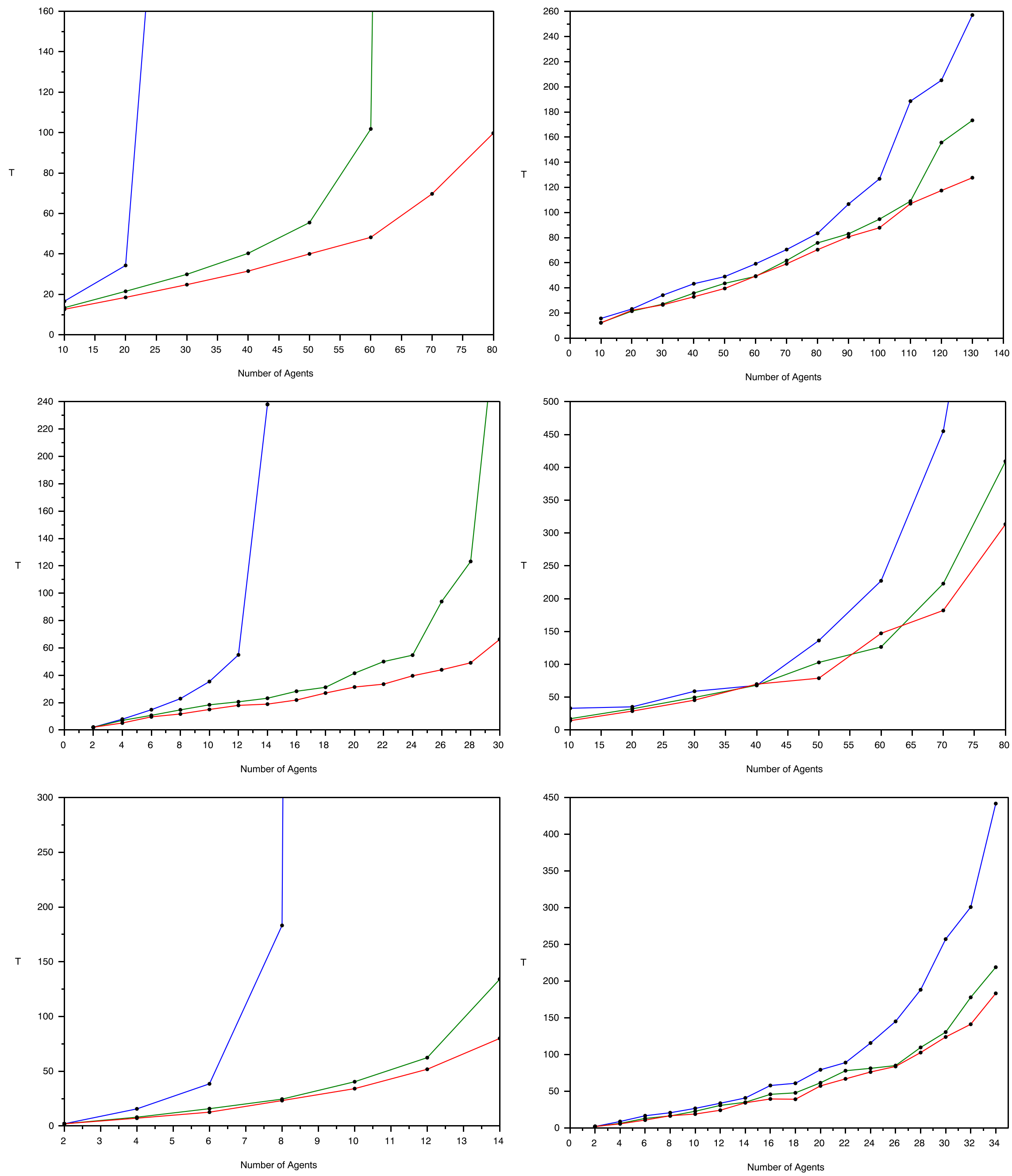

FIGURE 2 Graphs on the left column depicts the average times vs group size for the homogeneous target group case, while those on the right column correspond to heterogeneous target group case. Graphs on the upper row correspond to target groups with personal parameter $\kappa = 1.5$, while those in the middle row to groups with $\kappa = 2.0$, and those on the lower row to $\kappa = 2.9$. Blue lines correspond to indoctrinators with charisma $a = 2$, green lines to indoctrinators with charisma $a = 3$, and red lines to indoctrinators with $a = 10$. In all cases, the average time grows exponentially with the group size. However, the more charismatic the indoctrinator, less average time is required to convince the group. The required average time also grows with the personal parameter because, initially, target groups with greater personal parameter have a greater opinion difference regarding that of the indoctrinator, since they all have a stable consensus about the opinion $x = 1 - 1/\kappa$, while the indoctrinator about $x = 0$ [Colour figure can be viewed at wileyonlinelibrary.com]

In Figure 3, it can be seen practically the same behaviors like those in Figure 2:

1. When the group is in an unstable consensus but in stable 2-cycle, $\kappa = 3.4$, (a) average time also grows exponentially with group size. (b) The more charismatic the ideologist is, the less the time is required to convince the group.

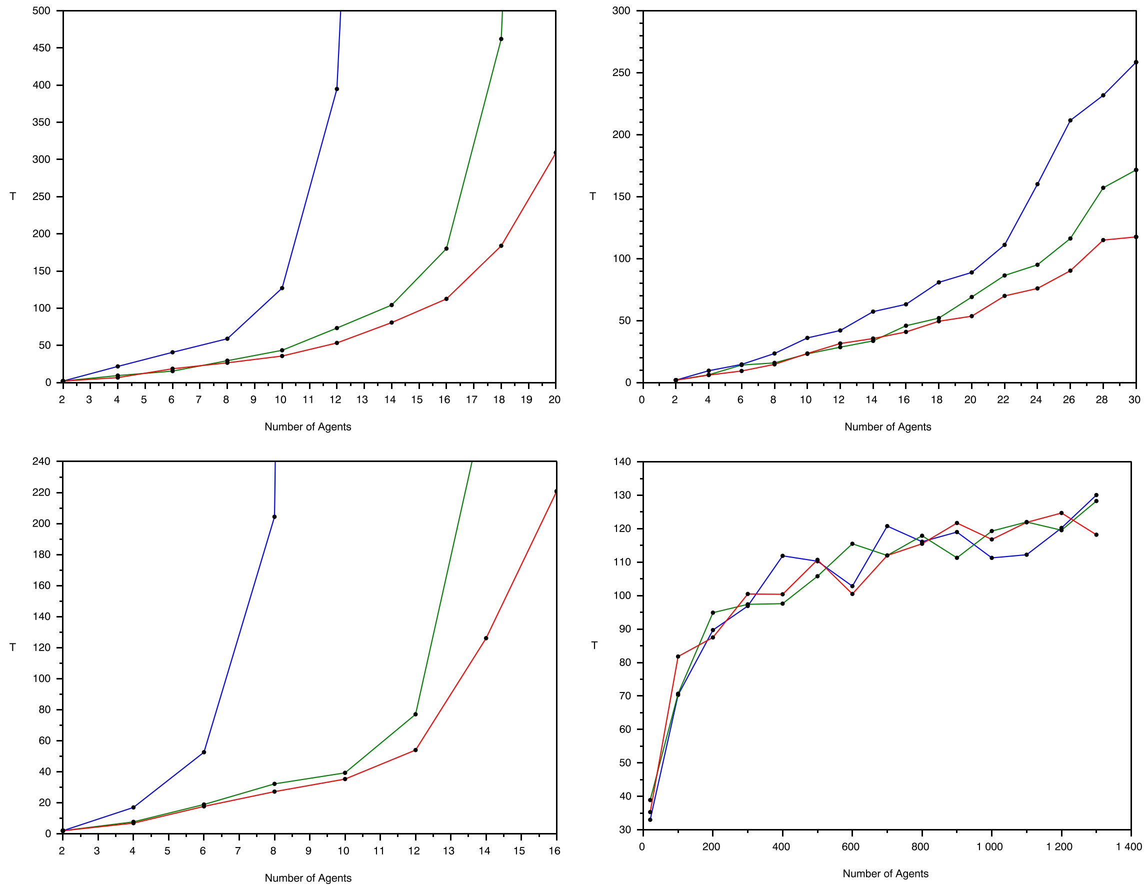

FIGURE 3 Graphs on the left column depicts the average times vs group size for the homogeneous target group case, while those on the right column correspond to heterogeneous target group case. Graphs on the upper row correspond to target groups with personal parameter $\kappa = 3.4$ and those on the lower row to $\kappa = 4.0$. Blue lines correspond to indoctrinators with charisma $a = 2$, green lines to indoctrinators with charisma $a = 3$, and red lines to indoctrinators with $a = 10$. In all cases, initially, all target groups are in an unstable consensus about the opinion $x = 1 - 1/\kappa$. Agents with $\kappa = 3.4$ are in the 2-cycle region of the bifurcation diagram, and their opinions could evolve into a dilemma, jumping between two different opinions; here, the average times are lower than those for the cases with $\kappa = 2.9$. Agents with $\kappa = 4.0$ are in the chaotic region, and their opinions may evolve by themselves to that of the indoctrinator, this happens for heterogeneous target groups but not for the homogeneous target groups which, unexpectedly, behave like the other homogeneous cases [Colour figure can be viewed at wileyonlinelibrary.com]

2. When the groups are in the chaotic regime, $\kappa = 4.0$; agents change their opinion in an erratic manner, consequently, when their opinion is below the threshold σ, its personal parameter must change to that of the ideologist, even without the influence of the latter, since their personal parameter must change according with rule 4 of the recursive model. This happens for the heterogeneous target group case; notice that the average times depend only on the group size, but they do not depend on how charismatic the ideologist is. However, unexpectedly, a homogeneous target group integrated by erratic agents behaves like the other homogeneous groups, despite the chaotic behavior of its agents.

5 | CONCLUSIONS

In this work, we have proposed a model to study how a single and very influential agent (the indoctrinator) convinces a social group (the target group) to replace their accepted doctrine by her (his) own one. In this model, interactions among agents are pairwise, and each agent is free to speak with any other. The group's doctrine is introduced as a fixed point (an attractor or a repeller) in the opinion space, while that of the indoctrinator is chosen as a repeller in the opinion space; hence, from the viewpoint of the target group, the new doctrine is a contentious one. These fixed points are introduced with the help of the logistic map that is also used to model an internal reflection process in the agents. Agents use the logistic map to fix a posture (like acceptance or rejection) regarding the doctrines. The stability of these fixed points depend on the values of the map's parameter; thus, besides the opinion, each agent is provided with a logistic parameter, called here personal parameter.

As initial conditions, all agents in the target group have the same opinion and the same personal parameter; ie, they are in a consensus. So to convince them, the indoctrinator has to change not only their opinions but also their personal parameters. Of course, the target group can convince back its lost members, and also the indoctrinator, if she (he) is not a strong speaker.

To be a strong speaker, the indoctrinator must have the respect of the group, this is achieved, when all agents in the target group give a high weight to the indoctrinator's opinion. As a measure of this strength, we have defined the indoctrinator's charisma, as the ratio of the weight that an agent in the target group gives to the indoctrinator, to the weight that the agent gives to herself (himself). Two target groups are considered here: the homogeneous one whose agents give the indoctrinator's opinion the same high weight and the heterogeneous one, whose agents give the indoctrinator's opinion, in general, different high weights.

To study this indoctrination process, we performed simulations to determine the times required by different charismatic indoctrinators to convince these two types of target groups.

From the simulations, it could be seen that for a stable consensus:

1. The more charismatic the ideologist is, the less time it will take to indoctrinate the group.
2. The average time (the number of iterations), required by the indoctrinator to convince the group, grows exponentially with the group size; in this sense, large groups should become immune to indoctrination, no matter how charismatic the indoctrinator would be.
3. Average time also grows with the personal parameter, since there is a greater difference between the doctrines.
4. Homogeneous target groups require a greater time to become indoctrinated than heterogeneous target groups; hence, a group whose members trust equally each other's opinion is more resistant to be indoctrinated.

while for unstable consensus:

1. Target groups whose members are in the 2-cycle regime, $\kappa = 3.4$ behave in the same manner like those groups in a stable consensus, despite that now their members are insecure about their common shared doctrine.
2. According with the recursive model, members of target groups with personal parameters in the chaotic regime were expected to change by themselves their opinions and personal parameters to those of the indoctrinator, this happened for the heterogeneous target group. However, the homogeneous target group did not behave in this way, actually, it behaved like the other homogeneous target groups considered here. While erratic agents in a heterogeneous target group adopt easily the new doctrine, those in a homogeneous target group are hard to convince.

From the simulations above, it is clear that very large times are required to indoctrinate large groups, when the indoctrinator changes only the opinions and personal parameter's of the group's members. So, in future works, we will consider the indoctrination of large groups, using indoctrinators that are able to change not only the opinions and personal parameters but also the charismas of the new indoctrinated agents.

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