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Lab Report 2

1. **Starting with the outline Matlab programs bisection.m and regfalsi.m (you will need to complete crucial lines in both) use both methods to find the root of the equation**

$$e^x = 2 + x$$

lying in the interval [-2.6,-1.4]. Compare the results of the two.

For the bisection.m the code that I inputted was:

```
X1=-2.6
X2=-1.4
X3=0.5*(x1+x2)
Else
X2=x3
F2=f3
```

Then when I ran the bisection.m program for the function $e^x = 2 + x$
It gave the answer to be -1.8414 after 20 iterations.

Then next program was regfalsi.m the code that I inputted was:

```
F=incline('exp(x)-2.0-x')
X1=-2.6
X2=-1.4
X3=x1-((x2-x1)/(f2-f1))*f1
If (f2*f3 < 0)
X1=x3
F1=f3
Else
X2=x3
F2=f3
```

When this program is run for the same function it gives the same answer of -1.8414 but does it with a smaller number of iterations at 6.

2. **Consider the following method for finding a (real) root of the polynomial $p(x) \equiv x^3 - 2x^2 + x - 2$: solve $p(x) = 0$ by Newton's Method, derive the relation that defines x_{i+1} in terms of x_i for this specific function p.**

The newton.m code can be altered to find the root of this equation by:

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F=incline('x^3-2*x^2+x-2')
Fdash=incline('3*x^2-4*x+1')

When run this gives a solution of 2 after 7 iterations.

a) **Derive the relation that defines x_{i+1} in terms of x_i for this specific function p**

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

$$f(x_i) = x^3 - 2x^2 + x - 2$$

$$f'(x_i) = 3x^2 - 4x + 1$$

$$x_{i+1} = x_i - \frac{x_i^3 - 2x_i^2 + x_i - 2}{3x_i^2 - 4x_i + 1}$$

b) **Show that if $x_{i+1} = 2$ then $x_i = 2$ also**

When $\boldsymbol{x_{i+1} = 2}$,

$$2 = x - \frac{x^3 - 2x^2 + x - 2}{3x^2 - 4x + 1}$$

$$(2 - x)(3x^2 - 4x + 1) = -(x^3 - 2x^2 + x - 2)$$

$$6x^2 - 8x + 2 - 3x^3 + 4x^2 - x = -x^3 + 2x^2 - x + 2$$

$$8x^2 - 8x - 2x^3 = 0$$

$$8x - 8 - 2x^2 = 0$$

$$-2(x - 2)^2 = 0$$

$$x = 2$$

Therefore $\boldsymbol{x_i = 2}$

When $\boldsymbol{x_i = 2}$,

$$x_{i+1} = x_i - \frac{x_i^3 - 2x_i^2 + x_i - 2}{3x_i^2 - 4x_i + 1}$$

$$x_{i+1} = 2 - \frac{8 - 8 + 2 - 2}{12 - 8 + 1} = 2$$

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c) **Show that, if the errors are defined by $\{\epsilon_j\}$ are defined by**

$$\boldsymbol{\epsilon_j \stackrel{\text{def}}{=} x_j - 2 \qquad \forall j}$$

then ϵ_{i+1} is approximately given by

$$\boldsymbol{\epsilon_{i+1} \approx \frac{4}{5}\varepsilon_i^2}$$

$$\epsilon_{i+1} = -\frac{1}{2}\frac{f''(x_i)}{f'(x_i)}\varepsilon_i^2$$

$$f(x_i) = x^3 - 2x^2 + x - 2$$

$$f'(x_i) = 3x^2 - 4x + 1$$

$$f''(x_i) = 6x - 4$$

$$\epsilon_{i+1} = -\frac{1}{2}\frac{6x-4}{3x^2-4x+1}\varepsilon_i^2$$

$$\epsilon_{i+1} = -\frac{1}{2}\frac{2(3x-2)}{x(3x-2)-2x+1}\varepsilon_i^2 = \frac{3(2)-2}{2(4)-4+1}\varepsilon_i^2$$

$$= \frac{4}{5}\varepsilon_i^2$$

d) **Does this match the general result connecting ϵ_{i+1} and ϵ_i that was obtained in the lectures? Justify your answer.**

3. **Let f be the last digit of your University registration number. Let g be the next to last digit of your registration number (if g=0 then take g=2). Use Newton's Method to find the equation of the catenary which passes through the point $(4f, 5g)$.**

As my registration number is 1202807, $f = 7$ **and g=2, which gives the point (28,10)**

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The code that I inputted into the newton.m file was:

```
F=incline('x*cosh(10/x)-x-28')
Fdash=incline('cosh(10/x)+x*sinh(10/x)*(-10/(x^2))-1')
X2=x1-f(x1)/fdash(x1)
```

This gives an answer of 3.4472 after 8 iterations.

4. **Use the Regula Falsi method to find the rate of interest that would be required in the problem considered in slides 7 and 8 of the lecture notes, if the monthly payment is changed to £(250+10g+f) and the target value becomes £[245,000 +1000*(f+g)].**

Just like in question 3 my , $\boldsymbol{f} = \mathbf{7}$ **and g=2**

This gives a monthly payment of £277 and a target value of £254000.

We then have to change some of the code

```
F=incline('((277)/(x/1200))*((1+x/1200)^(240)-1)-254000')
X1=10
X2=15
```

This gives us an answer of 11.4338 after 19 iterations.

5. **Using a Taylor series for f repeat the analysis of Newton's method on slides 35-38 in the lecture notes but this time do it for Regula Falsi**

Let ε be the actual root of the equation f(x)=0, such that f(ε)=0.

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determining the root using

$$x_{n+1} = \frac{x_n f(x_{n-1}) - x_{n-1} f(x_n)}{f(x_{n-1}) - f(x_n)}, \quad n = 1, 2, \ldots,$$

we get

$$\xi + \varepsilon_{n+1} = \frac{(\xi+\varepsilon_n) f(\xi+\varepsilon_{n-1}) - (\xi+\varepsilon_{n-1}) f(\xi+\varepsilon_n)}{f(\xi+\varepsilon_{n-1}) - f(\xi+\varepsilon_n)}$$

and so

$$\varepsilon_{n+1} = \frac{(\xi+\varepsilon_n) f(\xi+\varepsilon_{n-1}) - (\xi+\varepsilon_{n-1}) f(\xi+\varepsilon_n)}{f(\xi+\varepsilon_{n-1}) - f(\xi+\varepsilon_n)} - \xi$$
$$= \frac{\varepsilon_n f(\xi+\varepsilon_{n-1}) - \varepsilon_{n-1} f(\xi+\varepsilon_n)}{f(\xi+\varepsilon_{n-1}) - f(\xi+\varepsilon_n)}.$$

Expanding the right-hand side by Taylor's series, we get

$$\varepsilon_{n+1} = \frac{\varepsilon_n\left[f(\xi) + \varepsilon_{n-1} f'(\xi) + \frac{1}{2}\varepsilon_{n-1}^2 f''(\xi) + \cdots\right] - \varepsilon_{n-1}\left[f(\xi) + \varepsilon_n f'(\xi) + \frac{1}{2}\varepsilon_n^2 f''(\xi) + \ldots\right]}{f(\xi) + \varepsilon_{n-1} f'(\xi) + \frac{1}{2}\varepsilon_{n-1}^2 f''(\xi) + \cdots - f(\xi) - \varepsilon_n f'(\xi) - \frac{1}{2}\varepsilon_n^2 f''(\xi) - \cdots}$$

that is,

$$\varepsilon_{n+1} = k\varepsilon_{n-1}\varepsilon_n + O(\varepsilon_n^2), \qquad (2.7)$$

where

$$k = \frac{1}{2}\frac{f''(\xi)}{f'(\xi)}.$$

6. **Solve the problem $x^2 + 5x + 6 = 0$ using Regula Falsi and Bisection with the interval [-2.5,25.0]. Compare and comment on what happens.**

The code for both the regfalsi.m and bisection.m files have to be changed to accommodate the new problem and interval. For both the code can be changed to:

```
F=incline('x^2+5*x+6')
X1=-2.5
X2=25.0
```

The regfalsi method exceeds the number of itertaions at 50, but when extended gives an answer of -2 after 381 iterations and the bisection method gives an answer of -2 after 21 iterations.