A Hybrid AB-SFC Macroeconomic Model with an explicit distribution of income and wealth

Working Notes

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1 Introduction

1.1 Aims

The long-term goal of this model is to highlight the macroeconomic and distributional effects of some welfare policies. The focus is put in particular on some (recent) policies yet unapplied in the real world as Universal Basic Income, Job Guarantee schemes, or the presence of only cooperative firms.

2 General Hypothesis

2.1 Time

The timescale of the first version of the model should be relevant to calibrate the model on real data.

An adaptive approach for the agents' behaviour may work better using a higher frequency model that covers a shorter timespan (e.g. one month per tick, 15 years length, 180 time steps in total), because of the smaller variations expected at each tick.

Moreover, in a future version of the model, the simulation's timespan has to be long enough to observe the effects of introducing a policy. But, at the same time, it is unreasonable to keep the simulation running over 5-10 years after the policy's introduction because, in any real-world context, a government can tune or revert the policy afterwards.

2.2 Close Economy

The assumption of a close economy strongly reduces the complexity of the model but prevents observing some economic phenomena like export-led growth (such as for Italy or Germany) or the offshoring of labour-intensive productions. Nevertheless, this is a common hypothesis which is used also in this model.

An updated version of this paper and all the source code and the instructions required to replicate the paper are available at https://github.com/TnTo/FE/

Highlighted parts of the text indicate substantial choices to be taken.

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2.3 Sectors

The model includes the core sector of most SFC models (Nikiforos and Zezza 2017). Of those, three (Banks (B), Government (G) and Central Bank (C)) are represented by a single agent because unique in the model or described as an aggregate sector, while the remaining two (Households (H) and Firms (F)) are disaggregated and constitute the Agent-Based part of the model.

Firms are considered as different sectors in the model matrices depending on the goods produced.

2.4 Real Assets

The model comprises two kinds of real assets: Capital Goods (K) and Consumption Goods (G). Only Capital Goods are durable.

2.5 Financial Assets

The model includes five different financial assets. Bank Deposits (D) of Households and Firms, which are not interest-bearing. Loans (L) issued by the Banks to Firms, which interest rate is Firm-specific and fixed by the Bank. Bank Bonds (S, like shares) held by Households, which interest rate is fixed each period by the Bank. Banks Reserves and Government Account at the Central Bank (R), which are not interest-bearing. Government Bonds (T, like treasure bonds) hold by Bank and Central Bank, and which interest rate is fixed by the Central Bank.

3 Matrices

3.1 Balance Sheet Matrix

	H	F_C	F_K	B	G	C	Tot.
D	$+D_H$	$+D_{F_C}$	$+D_{F_K}$	$-D$			0
S	$+S_H$			$-S$			0
L		$-L^{F_C}$	$-L^{F_K}$	$+L$			0
T				$+T_B$	$-T$	$+T_C$	0
R				$+R_B$	$+R_G$	$-R$	0
K		$+pK_{F_C}$	$+pK_{F_K}$				$+pK$
Tot.	$+V_H$	$+V_{F_C}$	$+V_{F_K}$	$+V_B$	$+V_G$	$+V_C$	$+pK$

V is the Net Worth of the sector.

3.2 Transactions Matrix

	H	F_C	F_K	B	G	C	Tot.
Consumption	$-pG_H$	$+pG$			$-pG_G$		0
Investment		$-pK_{F_C}$	$+p(K-K_{F_K})$				0
Wages	$+W$	$-W^{F_C}$	$-W^{F_K}$				0
Taxes	$-T$				$+T$		0
Transfers	$+M$				$-M$		0
F Profits		$-\Pi^{F_C}$	$-\Pi^{F_K}$	$+\Pi$			0
C Profits					$+\Pi$	$-\Pi$	0
S Interests	$+rS$			$-rS$			0
L Interests		$-rL^{F_C}$	$-rL^{F_K}$	$+rL$			0
T Interests				$+rT_B$	$-rT$	$+rT_C$	0

4 Sectors

4.1 Households

In this model, the core agents (consumer, worker, capitalist) represent a household rather than a single individual. This is a very common approximation in economics and I think it is reasonable as long as we are not going into modelling education paths and care work, where the gender asymmetries become very relevant.

Each agent is characterized by an education level assigned when it enters the simulation replacing a retired agent inheriting their wealth, and gains experience when working in the same sector (Capital/Consumption) without an employment gap. The education level is assigned with a probability related to the inherited wealth and provides the starting skill level. Skills s evolve like in Dosi et al. (2018), which means $s_t = (1+\phi)^{\delta} s_{t-1}$ where $\delta = 1$ if the household is employed in the same sector of the previous time step, $\delta = 0$ if the household is still employed but in a different sector, $\delta = -1$ if the household is unemployed.

Households face two choices: if work and which proportion of their income they should consume.

Households flows' balance is $I = W + M + rS = C + T + \Delta S + \Delta D$. I assume, as a heuristic, that Taxes (T) and Transfers (M) can be approximated as constant from the previous period. Additionally, I assume that desired Deposits (D) at the end of the period are a fraction of the desired consumption ($C = \langle p_G \rangle G$) as insurance against unexpected increases in prices or unemployment[1] ($D = \rho\ C,\ \rho > 0$). Subtracting two consecutive periods and ignoring second-order differences, the in-flow income becomes $\Delta I = \Delta W + \Delta rS + r\Delta S$, and calling η the marginal propensity to consume it becomes $\eta \Delta C \approx \Delta W + \Delta rS + r\Delta S$. Fixed the income level I we can write $0 = \Delta C + \Delta D + \Delta S = (1+\rho)\Delta C + \Delta S$ to express the choice the household faces between consumption and saving. Putting these together we find $\eta \Delta C \approx \Delta W + \Delta rS - r(1+\rho)\Delta C$ and so $\Delta C \approx \frac{\Delta W + \Delta rS}{\eta + r(1+\rho)}$, which provides an adaptive rule for monetary consumption.

To translate this in material terms I introduce the material consumption of consumption goods G. We have $\Delta C = \Delta(pG) \approx \Delta(p)G + p\Delta G \approx p(\pi G + \Delta G)$. We can finally rewrite $\Delta G = \frac{\Delta W + \Delta rS}{p(\eta + r(1+\rho))} - \pi G$

ΔW can be approximated by wW_{t-1} where w is the average rate at which wages for the

[1] It is possible to assume that deposits are used only in case of increase in prices, which allows setting ρ smaller than one, assuming that in case of unexpected unemployment a mix of public subsidies and cashing out from Bank Bonds, without getting interests paid, is pursued. Otherwise, if deposits are used also as insurance against unemployment ρ as to be greater than 1.

given skill level are increased in the last year[2]. Δr is communicated by the Bank before the agent has to choose between consumption and saving. η is calibrated from empirical data as an exponential or Pareto distribution as $\eta(\frac{S}{p}, \frac{W}{p})$, where the price is used to get a-dimensional values (Fisher et al. 2020; Carroll et al. 2017).

From this relation, we can model the two choices.

First, a household exits from the labour market if the loss of the wage can anyway grant an increase in consumption, i.e. $\Delta rS - p\pi(\eta + r(1+\rho))G > W$. Similarly, it re-enters the labour market if the expected salary (i.e. the average salary given the skill level) prevents a loss of consumption, i.e. if $\Delta rS - p\pi(\eta + r(1+\rho))G < 0$ and $W + \Delta rS - p\pi(\eta + r(1+\rho))G > 0$.

Second, each household sets the desired total consumption as $\mathbb{E}(G) = G + \Delta G$ (using G from the previous period). From which follows $\mathbb{E}(C) = p(1+\pi)\mathbb{E}(G)$, $\mathbb{E}(D) = \rho p(1+\pi)\mathbb{E}(G)$ and $\Delta S = D + (1+w)W + M - (1+\rho)\mathbb{E}(C)$, where D is the value of the deposit at the end of the previous time step.

4.2 Firms

Firms are characterized by their position in the supply chain (either Capital or Consumption).

In the case of rationed credit, Firms first pay the salaries of workers, then they pay the salaries of researchers, then they acquire new machinery and finally repay the Bank of the Loans contracted.

4.2.1 Consumption Firms

Noted S the number of goods sold in the previous period, each Firm sets the desired production as $\mathbb{E}(G) = k(1+g-\pi)S$, where g is the GDP growth rate.

The maximum output of the available machinery is defined as $B = \sum_{k \in K} \beta_k$. Each Firm set a desired potential output $\mathbb{E}(B) = \frac{1}{u^*}\mathbb{E}(G) + \delta B$, where δ is the depreciation rate of capital and u^* is the target capacity utilization rate. From this follows that $\Delta B = \max(\frac{\mathbb{E}(G)}{u^*} - (1-\delta)B, 0)$ and the expected investment in monetary units is $\mathbb{E}(I) = p_K \frac{\Delta B}{\langle \beta \rangle}$.

The expected expense for salaries is $\mathbb{E}(W) = \frac{\mathbb{E}(G)}{\langle \beta \rangle}\langle w \rangle$, where w are the individual salaries paid.

Loans are required for an amount equal to $\mathbb{E}(L) = \max(k(\mathbb{E}(I) + \mathbb{E}(W)) - D, 0)$.

The markup is set equal to $\mu = \mu_{t-1}(1 + \theta \frac{S_{t-1}}{G})$ after that the production happened.

Called U the balance of flows in the period before distributing profits and D the amount of deposits in the same moment, each firm distributes profits $\Pi = kU\frac{D}{C}$, where $C = I + W$ in the period.

4.2.2 Capital Firms

Capital Firms produce their own machinery, creating a system of two simultaneous equations to determine the desired output. We avoid the problem approximating $\mathbb{E}(B) = \frac{k(1+g-\pi)}{u^*}S + \delta B$, and $\mathbb{E}(K) = k(1+g-\pi)S + \Delta B - K$, where K are the unsold capital goods from the previous period. The machinery produced for own use is kept separate from those to be sold.

Additionally, Capital Firms perform research to improve the machinery they sell. They aim to employ a number of researchers $\mathbb{E}(R) = R(1 + \frac{k}{S}\frac{\Pi}{\langle w_R \rangle})$ where $\langle w_R \rangle$ is the average salaries of the employees with $\sigma \geq \sigma^*$.

Finally, we note that $I = 0$ and so $C = W$.

[2] Or 0 assuming that salaries increase only changing employer which is an unexpected event.

4.3 Bank

Bank agent represents the aggregate banking sector.

Bank is required to maintain both a liquidity ($\Lambda = \frac{R}{D}$) and a capital ratio ($\Gamma = \frac{V}{L}$).

Liquidity is obtained, in case of necessity, by selling Government's Bonds to the Central Bank.

Bank fixes the interest rates on Bank Bonds as $r_S = (1 - \tau_S)(i + \lambda(\Gamma - \Gamma^*))$, where i is the Central Bank interest rate and τ_S is the constant tax rate on financial income. In this model the Bank does not distribute profits and can access all the needed liquidity from the Central Bank, making the liquidity requirement a tautology and needed a way to avoid excessive capitalization. So, it is not possible to have $r_S(\Lambda)$ (because it will be constant) and to make $r_S(\Gamma)$ approximate a profit distribution, without in a bond-like market (rather than a stock-like ownership model).

It also chooses when granting loans to Firms (based on the balance sheet of the applicant) and fixes a different interest rate for each loan. The duration of Loans is fixed. Bank is willing to provide loans to a firm F up to $\hat{l} = \min(\chi_0(D^F + pK^F) - L^F, \max((\chi_1 N_F)^{-1} L(\frac{\Gamma}{\Gamma^*} - 1), 0))$, at a firm-specific interest rate $r_L^F = i + \gamma_1(\Gamma - \Gamma^*) + \gamma_2(\frac{L^F}{V^F}) - \gamma_3(\frac{\Pi^F}{L^F})$. These relations account for the fulfilment of the capital requirement for the Bank and the presence of sufficient collaterals on the Firm side.

4.4 Government

Government fixes the public expenditure. It additionally collects taxes and pays unemployment benefits. It determines the amount to be transferred to Households (both as monetary and non-monetary, as Essential Goods).

When liquidity is needed, Government emits Bonds and sells them at will to the Central Bank.

As an approximation, fiscal policy is kept constant during the simulation and taxes are collected only from the Household sector during the transactions. Particularly, the model includes four taxes: a VAT on the purchase of consumption goods (τ_G); a flat financial income tax on distributed interests on Bank Bonds (τ_S); a progressive income tax computed as $T_W(W) = W \max(\tau_M \tanh(\tau_F(\frac{W}{p} - \tau_T)), 0)$; an inheritance tax on wealth with a constant rate (τ_I).

Fixed unemployment benefit $U_{t+1} = \gamma \max(W, U)$ is paid to those who have not exited the job market and are not employed.

Each period government buys Consumption goods which distribute to the Households. Particularly each Household receives an amount of Consumption goods equal to $G^G = ((1 - q_0) + q_0 e^{-q_1 \frac{V}{p}})\hat{G}$.

We define g as the growth rate of the GDP (measured as Government and Households consumptions, including VAT, plus the variation in Capital stock, noted as Y) in the previous periods, $M = \sum_H U$ and $G = \sum_H G^G$.

Assuming a Maastricht-like scenario in which Government has a target deficit δ to achieve, the expected balance of the next period can be written as $\delta Y(1 + g) = (1 + \pi)(\mathbb{E}(G) + M) + iT_B(1 + \frac{1}{T^G}\delta Y) - (1 + g)(1 + \pi)T$ where it is assumed that expenses for unemployment benefits are approximately constant, T are the taxes raised, T_B the stock of Government bonds held by the Bank and T^G is the total stock of Government Bonds.

Then, $\mathbb{E}(G) = (1 + g)T + \frac{1+g}{1+\pi}(1 - i\frac{T_B}{T^G})\delta Y - \frac{i}{1+\pi}T_B - M$, from which it is possible to write an adaptive rule for Government demand which aims to match the target deficit as $\Delta\hat{G} = q_2 \frac{\mathbb{E}(G) - G}{pN^H}$, where N^H is the number of Households agents.

4.5 Central Bank

In the model the role of the Central Bank is to fix the Government's Bonds interest rate according to a Taylor rule $i = \pi + \alpha_1(\pi - \pi^*) + \alpha_2(c - c^*) - \alpha_3(u - u^*)$, where π is the inflation rate, c is the capacity utilization measured as the fraction of capital goods used in production, u is the unemployment rate computed among those who have not voluntarily exited the job market, and starred variables are the targets.

Additionally, it passively buys and sells Government Bonds on request to the Bank and the Government. Reserves do not grant interests.

In other words, the Central Bank is a lender of last resort for the Government, which then has no accounting limits to spending.

5 Real Assets

5.1 Consumption Goods

Consumption Goods represent all the consumption (goods and services) of Households and are homogeneous and non-durable.

5.2 Capital Goods

Capital goods are characterized by their productivity β and the minimum skill level required to operate them σ. The production happens according to a Leontieff-like function in which one worker with at least σ skill can operate a single capital good getting β goods as output[3]. Each period they have a fixed probability to break and disappear from the model, equal to $\delta = \langle N \rangle^{-1}$, where $\langle N \rangle$ is the expected life of the machinery.

6 Financial Assets

6.1 Deposits

Deposits represent liquidity for Households and Firms and are not interest-bearing. Bank satisfies any transaction as long as the balance of the account remains positive.

6.2 Bank Bonds

Bank Bonds are sold and bought at their nominal value and do not expire. Bank satisfies every transaction, as long as the accounts remain positive. Households can buy or sell Bank Bonds only at the beginning of each period. At the end of each period, interests are paid, according to the rate fixed by the Bank at the beginning of the period.

6.3 Loans

Loans are issued by the Bank to a specific Firm. They have a fixed duration during which an equal share of capital is repaid plus the interest on the remaining debt. Interests are fixed by the Bank at a different value for each Firm at the time of emission.

6.4 Government Bonds

Government Bonds are sold and bought at their nominal value and do not expire. Central Bank satisfies every transaction. Bank can buy Government Bonds only at the beginning of each period. At the end of each period, interests are paid only to the Bank, according to the rate fixed by the Central Bank at the time beginning of the period.

[3] It is possible that to balance the model, the output needs to be β consumption goods or $b\beta$ capital goods.

6.5 Reserves

Reserves represent liquidity for Bank and Government and are not interest-bearing. Central Bank satisfies any transaction as long as the balance of the account remains positive.

7 Dynamics

7.1 Consumption Goods market

Each Household sees χ_C Consumption Firms χ_C times, each time it buys $\frac{\mathbb{E}(G)-G^G}{\chi_C}$ at the lowest possible price until it matches the desired consumption or ends liquidity.

7.2 Capital Goods market

Each Consumption Firm sees χ_K Capital Firms χ_K times (including the last one from which it has bought), each time it buys $\frac{\Delta B}{\chi_C}$ machineries choosing the one with the lowest value of $\beta p_C - \langle w_\sigma \rangle - \frac{p_K}{\langle N \rangle}$ until it matches the desired quantity or ends liquidity.

7.3 Labour market

Households are employed by a Firm until they are fired, they chose to exit the job market or they accept an offer from another Firm.

Firms can fire workers only when profits turn negative. In that case, Firms fire workers until the number of workers reaches u^*K. Additionally, in the case the Firm is not able to pay a worker, they is fired. In both cases, Firms start to fire those with lower skills.

Firms match each employed worker with the most productive machinery they can use, starting with those with higher skills. For this purpose, a researcher is treated like a worker assigned to a machinery with productivity σ^*. Then the current potential output (as the sum of the productivity of the used machinery) is computed and vacancies are filled starting from the most productive unused machinery. All research vacancies are filled if possible. For each vacancy, the Firm sees χ_L workers which have the required skills and earn less than the average salary in the model for the skill level. The Firm employs the one with the higher skills offering the average salary in the model for the skill level.

Each vacancy is filled once and new vacancies are not filled.

7.4 Retirement and inheritance

When a Household reaches a certain age it is considered retired and is replaced by a new agent in the model which inherits their (taxed) wealth and enters the simulation with a random skill level proportional to the inherited wealth, and a random age proportional to the skill level.

7.5 Non Performing Loans

In the case a Firm is unable to repay the principal of a loan or to pay the interest, the unpaid sum is added to the import of the loan and the duration is increased by one period. Additionally, the loan is marked as non-performing.

7.6 Bankrupt

Once the net value of a Firm becomes negative the Firm declares bankruptcy, fires all the employed households and loose all the financial assets (loans and deposits). It is "replaced" by a new firm which inherits the capital stock.

7.7 Innovation

Each Capital Firm can achieve an innovation each period with a probability $p = e^{-\xi R}$ where R is the number of employed workers not assigned to a Capital Good with skills at least $sigma^*$.

If the innovation is achieved the Capital Goods produced by the Firm increase its productivity $\Delta\beta \propto \text{Beta}(1, b_1)$ and modify the required skills as $\Delta\sigma \propto (\Delta\beta - k\text{Beta}(1, b_2))$.

8 Model steps

A Statistics are computed at the relevant frequency
B Agents variables independent from other agents' decisions are updated
C [QUARTERLY] Central Bank sets the Government Bonds interest rate
D.0 Bank sets the interest rate on Bank Bonds
D.1 Bank buys Government Bonds to reach the liquidity requirement
E.0 Households set desired consumption and savings
E.1 Firms set desired output and investment
F.0 Households acquire Bank Bonds
F.1 Firms ask the Bank for loans
G Labour market and wages payment
H Production and price setting
I Innovation
L.0 Government pays unemployment benefits
L.1 Government buys and distributes Consumption Goods
M Households Consumption Goods market
N Capital Goods market
O.0 Firms repay loans and interests
O.1 Government pays interest on Bonds to the Bank
P Firms distribute dividends
Q Bank pays interests on Bonds
R.0 [YEARLY] Income taxes are arbitraged
R.1 Capital depreciation

9 Equations

10 Parameters

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