An AB-SFC Macroeconomic Model with an explicit distribution of income and wealth

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The model

- The model aims to represent personal inequality of income and wealth distribution
- It is Agent-Based Stock-Flow Consistent Macroeconomic Model with two disaggregated sectors (Households, Firms)
- Some choices are made to test different welfare policies in a second stage
- European (Italian) economy is considered as real benchmark for stylized facts

The balance matrix

	$\mathcal{H}$	$\mathcal{F}_{\mathbf{C}}$	$\mathcal{F}_{\mathbf{K}}$	$\mathcal{B}$	$\mathcal{G}$	$\mathcal{C}$	Tot.
D	$+\mathbf{D}_{\mathcal{H}}$	$+\mathbf{D}_{\mathcal{F}_{\mathbf{C}}}$	$+\mathbf{D}_{\mathcal{F}_{\mathbf{K}}}$	$-\mathbf{D}$			0
S	$+\mathbf{S}$			$-\mathbf{S}$			0
L		$-\mathbf{L}^{\mathcal{F}_{\mathbf{C}}}$	$-\mathbf{L}^{\mathcal{F}_{\mathbf{K}}}$	$+\mathbf{L}$			0
B				$+\mathbf{B}_{\mathcal{B}}$	$-\mathbf{B}$	$+\mathbf{B}_{\mathcal{C}}$	0
R				$+\mathbf{R}_{\mathcal{B}}$	$+\mathbf{R}_{\mathcal{G}}$	$-\mathbf{R}$	0
K		$+p_{\mathbf{K}}\mathbf{K}_{\mathcal{F}_{\mathbf{C}}}$	$+p_{\mathbf{K}}\mathbf{K}_{\mathcal{F}_{\mathbf{K}}}$				$+p_{\mathbf{K}}\mathbf{K}$
Tot.	$+V_{\mathcal{H}}$	$+V_{\mathcal{F}_{\mathbf{C}}}$	$+V_{\mathcal{F}_{\mathbf{K}}}$	$+V_{\mathcal{B}}$	$+V_{\mathcal{G}}$	$+V_{\mathcal{C}}$	$+p_{\mathbf{K}}\mathbf{K}$

The transactions matrix

	$\mathcal{H}$	$\mathcal{F}_{\mathbf{C}}$	$\mathcal{F}_{\mathbf{K}}$	$\mathcal{B}$	$\mathcal{G}$	$\mathcal{C}$	Tot.
Consumption	$-p_{\mathbf{C}}\mathbf{C}_{\mathcal{H}}$	$+p_{\mathbf{C}}\mathbf{C}$			$-p_{\mathbf{C}}\mathbf{C}_{\mathcal{G}}$		0
Investment		$-p_{\mathbf{K}}\mathbf{K}_{\mathcal{F}_{\mathbf{C}}}$	$+p_{\mathbf{K}}(\mathbf{K}-\mathbf{K}_{\mathcal{F}_{\mathbf{K}}})$				0
Wages	$+W$	$-W^{\mathcal{F}_{\mathbf{C}}}$	$-W^{\mathcal{F}_{\mathbf{K}}}$				0
Taxes	$-T$				$+T$		0
Transfers	$+M$				$-M$		0
$\mathcal{F}$ Profits		$-\Pi_{\mathcal{B}}^{\mathcal{F}_{\mathbf{C}}}$	$-\Pi_{\mathcal{B}}^{\mathcal{F}_{\mathbf{K}}}$	$+\Pi_{\mathcal{B}}$			0
$\mathcal{C}$ Profits					$+\Pi^{\mathcal{C}}$	$-\Pi^{\mathcal{C}}$	0
S Interests	$+r_{\mathbf{S}}\mathbf{S}$			$-r_{\mathbf{S}}\mathbf{S}$			0
L Interests		$-r_{\mathbf{L}}\mathbf{L}^{\mathcal{F}_{\mathbf{C}}}$	$-r_{\mathbf{L}}\mathbf{L}^{\mathcal{F}_{\mathbf{K}}}$	$+r_{\mathbf{L}}\mathbf{L}$			0
B Interests				$+r_{\mathbf{B}}\mathbf{B}_{\mathcal{B}}$	$-r_{\mathbf{B}}\mathbf{B}$	$+r_{\mathbf{B}}\mathbf{B}_{\mathcal{C}}$	0

Capital goods

A capital good is characterized by:

- a minimum skill level required to operate it σ
- the output produced by the worker operating it β
- constant life-time with constant depreciation

Households

- Heterogeneous in their skill level $\sigma_t = (1 + \Sigma)^{\delta} \sigma_{t-1}$
- Retired at fixed age and get substituted by a single new agent which inherits the wealth
- Enter the simulation with an average initial skill $\mathbb{E}(\sigma_0) = 1 + (\sigma^M - 1) \tanh(e_0 \frac{v}{p})$

Households

Marginal propensity to consume is assumed decreasing in wealth
$\eta_t = (\frac{v_{t-1}}{p_{t-1}} + 1)^{-a}$

$$
\begin{aligned}
\eta \Delta c_t = \Delta y_t &= \Delta z_t + \Delta(r_{\mathbf{S}}\mathbf{s})_t = \Delta z_t + (r_{\mathbf{S}t} + \Delta r_{\mathbf{S}t})\mathbf{s}_t^* - r_{\mathbf{S}t}\mathbf{s}_{t-1} \\
&\approx \Delta z_t + (r_{\mathbf{S}t} + \Delta r_{\mathbf{S}t})(\mathbf{s}_{t-1} + \mathbf{d}_{t-1} + \mathbb{E}(z)_t + \mathbb{E}(m)_t - \mathbf{d}_t^* - c_t^*) \\
&\quad - r_{\mathbf{S}t}\mathbf{s}_{t-1} \\
&= (r_{\mathbf{S}t} + \Delta r_{\mathbf{S}t})(\mathbf{d}_{t-1} + z_{t-1} + \phi m_{t-1} - (1 + \rho_{\mathcal{H}})c_t^*) \\
&\quad + \Delta r_{\mathbf{S}t}\mathbf{s}_{t-1}
\end{aligned}
$$

$$
\Delta c_t \approx \frac{(r_{\mathbf{S}t} + \Delta r_{\mathbf{S}t})(\mathbf{d}_{t-1} + z_{t-1} + \phi m_{t-1} - (1 + \rho_{\mathcal{H}})c_{t-1}) + \Delta r_{\mathbf{S}t}\mathbf{s}_{t-1}}{\eta + (r_{\mathbf{S}t} + \Delta r_{\mathbf{S}t})(1 + \rho_{\mathcal{H}})}
$$

Consumption Firms

$$\mathbf{c}_t^* = \rho_{\mathbf{C}}(1 + g - \psi)s_{t-1} = \rho_{\mathbf{C}}\mathbb{E}(s)_t$$

$$b_t^* = \frac{1}{u^*}\mathbf{c}_t^* + \gamma b_{t-1}$$

$$i_t^* = (1 + \psi)\langle \frac{p_{\mathbf{K}}}{\beta} \rangle_{t-1} \Delta b_t$$

$$w_t^* = \max(w_{t-1}, \frac{\mathbf{c}_t^*}{\langle \beta \rangle_{t-1}} \langle w \rangle_{t-1})$$

$$l_t^* = \max(\rho_{\mathcal{F}} w_t^* - \mathbf{d}_{t-1}, \rho_{\mathcal{F}}(w_t^* + i_t^*) - (\mathbf{d}_{t-1} + (1 + \psi)p_{\mathbf{C}\,t-1}\mathbb{E}(s)_t), 0)$$

$$\mu_t = \mu_{t-1}(1 + \Theta \frac{s_{t-1} - \mathbb{E}(s)_{t-1}}{\mathbb{E}(s)_{t-1}})$$

$$p_{\mathbf{C}\,t} = (1 + \tau_{\mathbf{C}})(1 + \mu_t)\frac{w_t}{\mathbf{c}_t}$$

$$\pi_t = r_{\Pi}(p_{\mathbf{C}\,t}s_t - w_t)$$

Capital Firms

$$\mathbf{k}_t^* = \rho_{\mathbf{K}}(1+g-\psi)s_{t-1} + \frac{\Delta b_t}{\langle\beta\rangle} - \hat{\mathbf{k}}_{t-1}$$

$$b_t^* = \rho_{\mathbf{K}}\frac{(1+g-\psi)}{u^*}s_{t-1} + \gamma b_{t-1}$$

$$i_t^* = 0, \qquad l_t^* = \max(\rho_{\mathcal{F}} w_t^* - \mathbf{d}_{t-1}, 0)$$

$$q_t^* = \lfloor q_{t-1}(1 + \frac{\rho_Q}{s_{t-1}}\frac{\pi_{t-1}}{\langle w_Q\rangle_{t-1}})\rfloor$$

Innovate with probability $\theta = e^{-\zeta q}$ with output $\Delta\beta_{\mathbf{C}} = \text{Beta}(1, b_0)$
$\Delta\sigma = (\Delta\beta_{\mathbf{C}} - b_1\text{Beta}(1, b_2))$

Bank

Represent the aggregate financial sector. No enforceable liquidity ratio. Target capital ratio $\Gamma = \frac{V}{L}$.

$$r_{\mathbf{S}\,t} = (1 - \tau_{\mathbf{S}})(r_{\mathbf{B}\,t} + \lambda(\Gamma - \Gamma^*))$$

$$I^{f^*}_t = \min(\nu_0(p_{\mathbf{K}}\mathbf{k}_f)_{t-1} - \mathbf{I}^f_{t-1}, \max((\nu_1 N_F)^{-1}\mathbf{L}_{t-1}(\frac{\Gamma_{t-1}}{\Gamma^*} - 1), 0))$$

$$r_{\mathbf{L}}{}^f_t = r_{\mathbf{B}} + \nu_2(\Gamma^* - \Gamma_{t-1}) + \nu_3(\frac{\mathbf{I}^f_{t-1}}{v_{ft-1}}) - \nu_4(\frac{\pi_{ft-1}}{\mathbf{I}^f{}_{t-1}})$$

Government

Maastricht-like setting.

$$m_t = \phi \max(w_{t-1}, m_{t-1}), \qquad \mathbf{c}^{\mathcal{G}}_{h\,t} = ((1-\varepsilon_0) + \varepsilon_0 e^{-\varepsilon_1 \frac{v_{h\,t-1}}{p}})\Xi_t$$

$$(1+g)\delta^* Y_{t-1} = \mathbb{E}(G)_t + r_{\mathbf{B}\,t}\mathbf{B}_{\mathcal{B}\,t-1}(1 + \frac{(1+g)\delta^* Y_{t-1}}{\mathbf{B}^{\mathcal{G}}{}_{t-1}}) - (1+g+\psi)T_{t-1}$$

$$\mathbb{E}(G) = (1+\psi+g)\langle T\rangle + (1+g)(1 - r_{\mathbf{B}\,t}\langle\frac{\mathbf{B}_{\mathcal{B}}}{\mathbf{B}^{\mathcal{G}}}\rangle)\delta^* Y_{t-1} - r_{\mathbf{B}\,t}\mathbf{B}_{\mathcal{B}\,t-1}$$

$$\mathbb{E}(G)_t = \mathbb{E}(M)_t + \mathbb{E}(C^{\mathcal{G}})_t \approx M_{t-1} + (1+\psi)p\frac{\mathbf{C}^{\mathcal{G}}{}_{t-1}}{\Xi_{t-1}}\Xi_t$$

$$\Xi_t = \frac{\Xi_{t-1}}{\langle\mathbf{C}^{\mathcal{G}}\rangle}\frac{\mathbb{E}(G) - \langle M\rangle}{(1+\psi)p}$$

Central Bank

$$r_{\mathbf{B}} = \psi + \alpha_1(\psi - \psi^*) + \alpha_2(u - u^*) - \alpha_3(\omega - \omega^*)$$

Time

To smooth the dynamic of the model, two different times are overlapped

- a slow-time (the quarter) which is used to update institutional decision (CB rate, government targets, aggregate statistics update)
- a fast-time (the month) which is used for consumption and production

Good Market

For each transaction, an agent buys χ^{-1} of the desired quantity χ times looking at χ sellers.
The markets in the model are

- Consumption Goods market: Consumption Firms sell Consumptions Goods to Households and the Government
- Capital Goods market: Capital Firms sell Capital Goods to Consumption Firms

Every other transaction is settled at will, given the constraints.

Labour Market

Households are employed by a Firm until they are fired (when $w_t - p_t s_t < 0$), they chose to exit the job market or they accept an offer from another Firm.
For each vacancy (i.e. unmatched capital good required for production target or research position) a Firm sees $\chi_{\mathcal{H}}$ workers which have the required skills and earn less than the average salary in the model for that skill level.
The Firm employs the one with the higher skills offering the average salary in the model for the required skill level.

Implementation

The planned implementation will rely on a stateful description in which it is possible to reconstruct most of the dynamics of the model.
Particularly the state of the model at the each of each time-step will be stored.
The model is written in Julia (`https://github.com/TnTo/FE/`).
This is helping prototyping since the quantities to be measured and recorded are not required to be defined a priori.

Calibration

The calibration will aim not to fit real-world time series, but to match stylized facts.
The target facts will be listed as expected ranges of macroeconomic variables or moments of distributions, taking as reference the Western-European economies.
A Monte-Carlo search on the parameter space will be used to set the parameters that have no clear economic meaning to match the stylized facts.

Policies experiments

The final goal of the work is to compare the macroeconomic effects of different welfare paradigms.

- A Keynesian government with full-employment or poverty-reduction goals
- Minimum wage
- Universal Basic Income
- Universal Basic Dividend
- Universal Basic Service (more significant with two-goods/foundational perspective)
- Job Guarantee scheme (with differentiated firm-property and public ownership)
- Cooperativism (with differentiated firm-property and public ownership)

What is missing

- A gender perspective: the care and domestic work and the education paths
- A spatial perspective: commuting and housing – a mayor factor of financial stress for the many –
- A foundational perspective: differentiated goods and the public infrastructure
- A capital perspective: competitive models of firm ownership and profit distribution
- A development perspective: wage suppression and export-led growth in a multi-country model

Thanks for your attention!
Any question?