A Hybrid AB-SFC Macroeconomic Model with an explicit distribution of income and wealth

Working Notes

Margherita Redigonda*

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Recent developments in Post-Keynesian macroeconomics have found in Stock-Flow Consistent models a general framework to enforce the macroeconomic accounting identities in the models. These models are sometimes extended to include micro-foundations (or a microeconomic description) through the use of Agent-Based Models. But despite the progress and the diffusion of AB-SFC models in the last ten years, very few models can replicate the fat-tail distribution of income and wealth observed empirically. Instead, they usually rely on a Kaleckian distinction between workers and capitalists or a segmented labour market (for example low-, medium- and high-skilled workers). This model aims to reproduce a fat-tail distribution in income and wealth, in addition to other stylized facts, by differentiating workers by their skills and the capital goods used in production by their productivity and the skills required to use them. A realistic distribution of income and wealth allows bringing closer this strain of literature with the literature on income and wealth inequality since it makes measuring indexes like Gini or the top5/bottom50 ratio possible. The model includes endogenous innovation and credit rationing as previous models in the literature. Households' consumption is expressed in material terms (i.e. the amount of goods consumed), rather than in monetary value, to be able to differentiate the goods consumed and better characterize the behaviour of low-income households in a future version of the model.Furthermore, the model is designed to compare different welfare policies, like minimum wage, minimum income, Universal Basic Income, Universal Basic Services or Job Guarantee programs, and different political orientations in public policies, like the austerity principles of the '10s, a Keynesian demand-led public spending or an entrepreneur state. At the time of writing this abstract, the model has been completely outlined on paper and implemented, but a proper calibration and results are missing.

Keywords: Agent-Based Model; Stock-Flow Consistent; Post-Keynesian Macroeconomics; Inequality

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An updated version of this paper and all the source code and the instructions required to replicate the paper will be available at https://github.com/TnTo/FE/

*m@orsorosso.net - https://orcid.org/0000-0003-1485-1204

1. Introduction

In recent years Stock-Flow Consistent (SFC) models have been recognized as a valuable tool to model macroeconomics (Nikiforos & Zezza, 2017). Particularly, they can represent the financial side of the economy, which has been recognized as a necessary feature to model after the financial crisis of the 00s.

Those models are aggregate models which can neither describe microeconomic dynamics and behaviours nor, if it is considered necessary, provide a microfoundation for the model. Agent-Based Models (ABMs) have been used to add a microeconomic description to the macroeconomic model provided by the SFC framework (Caverzasi & Russo, 2018; Dosi & Roventini, 2019).

But in the AB-SFC models literature, very few papers explore the conditions to model a realistic (i.e. fat-tail) distribution of income and wealth among households (e.g. Dafermos & Papatheodorou, 2015; Kinsella et al., 2011), often relying on a Kaleckian framework which studies the distribution between wages and profits (e.g. Dosi et al., 2013) or a segmented labour-market (e.g. Caiani et al., 2019).

Modelling personal income and wealth distribution is a needed step to close the gap between this strain of post-Keynesian literature and the literature on inequality pioneered by Atkinson, Piketty, Zucman and Saez, allowing, for example, to measure the evolution of inequality indexes (like Gini or the Top5/Bottom50) through time or policies changes.

Some choices in the model are dictated by its long-term goals, specifically comparing disruptive welfare policies (Universal Basic Income, Universal Basic Service, Job Guarantee, ...) and focusing on the different consumption patterns between high- and low-income households, including different compositions of the consumption bundle, the relevance of publicly provided goods and services (including infrastructures) in the consumption bundle, following the Foundational Economy framework (Arcidiacono et al., 2018), the incidence of unpaid domestic and care work on the work provided by individuals in the household, especially from a gender perspective.

Finally, the model is inspired more by west-European economies (particularly Italy) rather than by north-American ones.

2. General Hypothesis

To simplify the model a close single-country economy is assumed. This is a common hypothesis for SFC models (and macroeconomics in general) because an open economy does not preserve the balance of flows in the model. So, to preserve the consistency of the model the only alternative is to model a multi-countries system, each with its own dynamics. It is worth keeping in mind that this hypothesis prevents to model to replicate economic dynamics strongly dependent on international trade, like the export-led growth path.

The microeconomic behaviour of the agent is modelled mostly using adaptive expectations, focusing on the variation of behaviours (as desires or expectations) with regard to the previous period. Exploiting this, to smooth the dynamics of the model each period represents a month (a small amount of time compared to other models) to keep the variations small.

The model includes the five sectors: the Households ($\mathcal{H}$), the Firms which produce consumption goods ($\mathcal{F}_\mathbf{C}$), the Firms which produce Capital goods ($\mathcal{F}_\mathbf{K}$), a consolidated (i.e. aggregated) banking and financial sector (the Bank, $\mathcal{B}$), a consolidated public sector which include the Government and the Central Bank ($\mathcal{G}$). Households and Firms constitute the Agent-Based part of the model.

The model comprises two kinds of real assets: Capital Goods ($\mathbf{K}$) and Consumption Goods ($\mathbf{C}$). Capital Goods are durable and Consumption Goods are homogeneous.

The model includes four different financial assets. Bank Deposits ($\mathbf{D}$) of Households and Firms, which are not interest-bearing. Loans ($\mathbf{L}$) issued by the Banks to Firms, which interest rate is Firm-specific and fixed by the Bank. Bank Shares ($\mathbf{S}$) held by Households, which interest rate is fixed each period by the Bank. Government Bonds ($\mathbf{B}$) held by the Bank, and which interest rate is fixed by the Government and can be shorted, acting as Central Bank's advances.

3. Matrices

3.1. Balance Sheet Matrix

	$\mathcal{H}$	$\mathcal{F}_{\mathbf{C}}$	$\mathcal{F}_{\mathbf{K}}$	$\mathcal{B}$	$\mathcal{G}$	Tot.
$\mathbf{D}$	$+\mathbf{D}_{\mathcal{H}}$	$+\mathbf{D}_{\mathcal{F}_{\mathbf{C}}}$	$+\mathbf{D}_{\mathcal{F}_{\mathbf{K}}}$	$-\mathbf{D}$		0
$\mathbf{S}$	$+\mathbf{S}$			$-\mathbf{S}$		0
$\mathbf{L}$		$-\mathbf{L}^{\mathcal{F}_{\mathbf{C}}}$	$-\mathbf{L}^{\mathcal{F}_{\mathbf{K}}}$	$+\mathbf{L}$		0
$\mathbf{B}$				$+\mathbf{B}$	$-\mathbf{B}$	0
$\mathbf{K}$		$+p_{\mathbf{K}}\mathbf{K}_{\mathcal{F}_{\mathbf{C}}}$	$+p_{\mathbf{K}}\mathbf{K}_{\mathcal{F}_{\mathbf{K}}}$			$+p_{\mathbf{K}}\mathbf{K}$
Tot.	$+V_{\mathcal{H}}$	$+V_{\mathcal{F}_{\mathbf{C}}}$	$+V_{\mathcal{F}_{\mathbf{K}}}$	$+V_{\mathcal{B}}$	$+V_{\mathcal{G}}$	$+p_{\mathbf{K}}\mathbf{K}$

V is the net worth of the sector.

3.2. Transactions Matrix

	$\mathcal{H}$	$\mathcal{F}_{\mathbf{C}}$	$\mathcal{F}_{\mathbf{K}}$	$\mathcal{B}$	$\mathcal{G}$	Tot.
Consumption	$-p_{\mathbf{C}}\mathbf{C}_{\mathcal{H}}$	$+p_{\mathbf{C}}\mathbf{C}$			$-p_{\mathbf{C}}\mathbf{C}_{\mathcal{G}}$	0
Investment		$-p_{\mathbf{K}}\Delta^+\mathbf{K}_{\mathcal{F}_{\mathbf{C}}}$	$+p_{\mathbf{K}}(\Delta^+\mathbf{K}-\Delta^+\mathbf{K}_{\mathcal{F}_{\mathbf{C}}})$			0
Wages	$+W$	$-W^{\mathcal{F}_{\mathbf{C}}}$	$-W^{\mathcal{F}_{\mathbf{K}}}$			0
Taxes	$-T$				$+T$	0
Transfers	$+M$				$-M$	0
$\mathcal{F}$ Profits		$-\Pi_{\mathcal{B}}^{\mathcal{F}_{\mathbf{C}}}$	$-\Pi_{\mathcal{B}}^{\mathcal{F}_{\mathbf{K}}}$	$+\Pi_{\mathcal{B}}$		0
$\mathbf{S}$ Interests	$+r_{\mathbf{S}}\mathbf{S}$			$-r_{\mathbf{S}}\mathbf{S}$		0
$\mathbf{L}$ Interests		$-r_{\mathbf{L}}\mathbf{L}^{\mathcal{F}_{\mathbf{C}}}$	$-r_{\mathbf{L}}\mathbf{L}^{\mathcal{F}_{\mathbf{K}}}$	$+r_{\mathbf{L}}\mathbf{L}$		0
$\mathbf{B}$ Interests				$+r_{\mathbf{B}}\mathbf{B}$	$-r_{\mathbf{B}}\mathbf{B}$	0
$\Delta\mathbf{D}$	$-\Delta\mathbf{D}_{\mathcal{H}}$	$-\Delta\mathbf{D}_{\mathcal{F}_{\mathbf{C}}}$	$-\Delta\mathbf{D}_{\mathcal{F}_{\mathbf{K}}}$	$+\Delta\mathbf{D}$		0
$\Delta\mathbf{S}$	$-\Delta\mathbf{S}$			$+\Delta\mathbf{S}$		0
$\Delta\mathbf{L}$		$+\Delta\mathbf{L}^{\mathcal{F}_{\mathbf{C}}}$	$+\Delta\mathbf{L}^{\mathcal{F}_{\mathbf{K}}}$	$-\Delta\mathbf{L}$		0
$\Delta\mathbf{B}$				$-\Delta\mathbf{B}$	$+\Delta\mathbf{B}$	0
$\Delta\mathbf{K}$		$-\Delta(p_{\mathbf{K}}\mathbf{K}_{\mathcal{F}_{\mathbf{C}}})$	$-\Delta(p_{\mathbf{K}}\mathbf{K}_{\mathcal{F}_{\mathbf{K}}})$			$-\Delta(p_{\mathbf{K}}\mathbf{K})$

where $\Delta(p_{\mathbf{K}}\mathbf{K})$ includes capital depreciation.

4. Sectors

4.1. Households

In this model, the core agents (consumers, workers, capitalists) represent a household rather than a single individual. This is a very common approximation in economics and I think it is reasonable as long as we are not going into modelling education paths and care and domestic work, where the gender asymmetries become very relevant.

Each agent is characterized by an education level assigned when it enters the simulation replacing a retired agent inheriting their wealth, and gains experience when working for the same firm for consecutive periods. The education level is assigned with a probability related to the inherited wealth and provides the starting skill level. Particularly, the initial skill level is distributed as a Beta-Binomial distribution characterized by σ^M as the maximum initial skill level, an average value equal to $1+(\sigma^M-2)\tanh(e_0\frac{v}{p})$ with v the net inherited wealth, and a variance described by a parameter e_1 and described in appendix A.

Retirement happens at a fixed A_R age, while the initial age is equal to $A_0 + \sigma_0$, where σ_0 is the initial skill level.

Skills σ evolve $\sigma_t = \max(0, \sigma_{t-1} + \delta\Sigma)$ where $\delta = 1$ if the household is employed by the same firm of the previous period, $\delta = 0$ if the household is still employed but in a different firm, $\delta = -1$ if the household is unemployed.

Each household participates in the labour market and so has to choose a desired consumption given its income and wealth.

This differs from the post-keynesian tradition, in which the upper class of capitalists and rentiers doesn't receive any income from work. On the other hand, more recent studies from Piketty, Saez, Zucman and coauthors highlight how contemporary upper class mix financial and labour income, for example by paying the managers in equities (and so granting them financial income) or appointing owners in apical positions, e.g. board's members, granting them labour income.

Households' flows balance is $w_t^{\mathcal{H}} + m_t + r^{\mathcal{H}}_{\mathbf{S}\,t}\mathbf{s}_t = c_t + \Delta\mathbf{s}_t + \Delta\mathbf{d}_t$, where bold lowercase letters are used to indicate individual value for the correspondent uppercase aggregates and the $\mathcal{H}$ superscript indicate the net value after taxes (i.e. for the net wage $w^{\mathcal{H}} = w^{\mathcal{F}}(1 - \max(\tau_M \tanh(\tau_F(\frac{w^{\mathcal{F}}}{p} - \tau_T)), 0)))$.

I assume, as a heuristic, that wages (w) can be approximated as constant from the previous period (because salaries increase only by changing employers, which is an unexpected event), while transfers (m) are composed only of unemployment benefits. Additionally, I assume that desired deposits ($\mathbf{d}_t^*$) at the end of the period are a fraction of the desired consumption ($c_t^* = \mathbb{E}(p_{\mathbf{C}})\mathbf{c}_t^*$) as insurance against unexpected increases in prices ($\mathbf{d}_t^* = \rho_{\mathcal{H}} c_t^*$, $\rho_{\mathcal{H}} > 0$).

Following for example Fisher et al. (2020) the propensity to consume has to be considered decreasing in income and wealth. This idea is able to improve a classical post-keynesian consumption function by introducing variable coefficients for the propensity to consume out of income and out of wealth. As detailed in the appendix B, the resulting equation for the real desired consumption is

$$\mathbf{c}^* = \frac{1}{(1-a_y)}\left(\left(\frac{\mathbb{E}(y)}{p^{\mathcal{H}}_{\mathbf{C}}(1+\psi)} + 1\right)^{1-a_y} - 1\right) + \frac{1}{(1-a_v)}\left(\left(\frac{v}{p^{\mathcal{H}}_{\mathbf{C}}(1+\psi)} + 1\right)^{1-a_v} - 1\right)$$

where $v = \mathbf{s} + \mathbf{d}$ at the beginning of the period and $\mathbb{E}(y)_t = w^{\mathcal{H}}_{t-1} + \phi m_{t-1}$ is the expected non-financial income.

The financial income does not appear explicitly in the equation because it is determined by the portfolio choice, which relies on the desired consumption, creating a couple of equation without a closed form solution.

From this follows $c_t^* = (1+\psi)p^{\mathcal{H}}_{\mathbf{C}t-1}\mathbf{c}_t^*$ and $\mathbf{d}_t^* = \rho_{\mathcal{H}} c_t^*$. Finally, the household buys shares $\mathbf{s}_t = \mathbf{s}_{t-1} + \mathbf{d}_{t-1} + w^{\mathcal{H}}_{t-1} + \phi m_{t-1} - \mathbf{d}_t^* - c_t^*$.

4.2. Firms

Firms are characterized by their position in the supply chain (either Capital or Consumption).

Loans are asked to the Bank at the beginning of the period to pay for salaries and to make investments and started to be repaid at the end of the period. In the case of rationed credit, Firms first pay the salaries of workers, then they pay the salaries of researchers, then they acquire new machinery then they distribute profits, and finally, they repay the bank the loans contracted.

4.2.1. Consumption Firms

Noted s_{t-1} the number of goods sold in the previous period, each Firm sets the desired production as $\mathbf{c}_t^* = \max(\rho_{\mathbf{C}}(1+g-\psi)s_{t-1}, 1) \approx \rho_{\mathbf{C}}\mathbb{E}(s)_t$, where g is the GDP growth rate and the functional form chosen is the one of a quasi-multiplicative process without a fixed point.

The maximum output of the available machinery is defined as $b = \sum_{\kappa \in \mathbf{k}} \beta^{\mathbf{C}}_{\kappa}$. Each Firm set a desired potential output $b^*_t = \frac{1}{u^*}\mathbf{c}^*_t + \gamma b_{t-1}$, where γ is the depreciation rate of capital and u^* is the target capacity utilization rate. From this follows that $\Delta b^*{}_t = \max(\frac{\mathbf{c}^*_t}{u^*} - (1-\gamma)b_{t-1}, 0)$ and the expected investment in monetary units is $\mathbb{E}(i)_t = (1+\psi)\langle \frac{p^0_\kappa}{\beta^{\mathbf{C}}_\kappa} \rangle_t \Delta b^*{}_t$, where the average is taken on the current capital stock and the price considered is the one not depreciated. Expected expenses for salaries are computed as $\mathbb{E}(w^{\mathcal{F}}_t) = \max(w^{\mathcal{F}}_{t-1}, \frac{\mathbf{c}^*_t}{\langle \beta^{\mathbf{C}} \rangle_t} \langle w^{\mathcal{F}} \rangle_{t-1})$

The markup is set as $\mu_t = \mu_{t-1}(1 + \Theta \frac{s_{t-1} - \mathbb{E}(s)_{t-1}}{\mathbb{E}(s)_{t-1}}) \approx \mu_{t-1}(1 + \Theta(\rho_{\mathbf{C}} \frac{s_{t-1}}{\mathbf{c}^*_{t-1}} - 1))$ and the price $p^{\mathcal{F}}_{\mathbf{C}t} = (1+\mu_t)\frac{w_t}{\mathbf{c}_t}$ after the production (like in Caiani et al., 2016), where w is the sum of the wages paid by the firm, the price paid by households is $p^{\mathcal{H}}_{\mathbf{C}} = (1+\tau_{\mathbf{C}})p^{\mathcal{F}}_{\mathbf{C}}$ and $\tau_{\mathbf{C}}$ is the VAT rate.

The request for loans is set to cover salaries and investments, keeping in account revenues, i.e. $l^*_t = \max(\rho_{\mathcal{F}}\mathbb{E}(w^{\mathcal{F}})_t - \mathbf{d}_{t-1}, \rho_{\mathcal{F}}(\mathbb{E}(w^{\mathcal{F}})_t + \mathbb{E}(i)_t) - (\mathbf{d}_{t-1} + \mathbb{E}(p^{\mathcal{F}}_{\mathbf{C}})_t \mathbb{E}(s)_t), 0)$, where approximating a constant labour cost $\mathbb{E}(p^{\mathcal{F}}_{\mathbf{C}})_t = (1+\mu_t)\frac{w^{\mathcal{F}}_{t-1}}{\mathbf{c}_{t-1}}$.

Each firm distributes profits after they are able to self-finance the expected needed liquidity for the production flows, i.e. $\pi_t = \max(\mathbf{d}_t - \rho_{\mathcal{F}}(p^{\mathcal{F}}_{\mathbf{C}t} s_t - w^{\mathcal{F}}_t), 0)$, where the deposits are those at the time of paying dividends.

4.2.2. Capital Firms

Capital Firms produce their own machinery, creating a system of two simultaneous equations to determine the desired output. The problem is avoided by approximating $b^*_t = \max(\rho_{\mathcal{F}} \frac{(1+g-\psi)}{u^*} s_{t-1} + \gamma b_{t-1}, 1)$, and $\Delta^+ \mathbf{k}^*_t = \rho_{\mathbf{K}}(1+g-\psi)s_{t-1} + \frac{\Delta b_t}{\beta^{\mathbf{K}}_{t-1}} - \hat{\mathbf{k}}_{t-1}$, where $\hat{\mathbf{k}}$ are the inventories unsold from the previous period. The machinery produced for own use is kept separate from those to be sold (the inventories), and valued in the balance sheet with value $p_{\mathbf{K}t}$.

Following the same logic used for Consumption Firms, we can write the mark-up $\mu_t = \mu_{t-1}(1 + \Theta \frac{s_{t-1} - \mathbb{E}(s)_{t-1}}{\mathbb{E}(s)_{t-1}}) = \mu_{t-1}(1 + \Theta(\rho_{\mathbf{K}} \frac{s_{t-1}}{\Delta^+ \mathbf{k}^*_{t-1} + \hat{\mathbf{k}}_{t-2} - \Delta b_{t-1} \beta^{\mathbf{K}}_{t-2}{}^{-1}} - 1))$. This 2-lagged variables are the only one in the model. To simplify the implementation the strong approximation $\mu_t \approx \mu_{t-1}(1 + \Theta(\rho_{\mathbf{K}} \frac{s_{t-1}}{\Delta^+ \mathbf{k}^*_{t-1}} - 1))$ is adopted.

Additionally, Capital Firms perform research to improve the machinery they sell. They aim to employ a number of researchers $q^*_t = q_{t-1} + \lfloor \rho_Q \frac{p_{\mathbf{K}t-1} s_{t-1} - w^{\mathcal{F}}_{t-1}}{\langle w^{\mathcal{F}}_Q \rangle_{t-1}} \rfloor$ where $\langle w^{\mathcal{F}}_Q \rangle$ is the average salaries of the employees with $\sigma \geq \sigma^*$. This makes $\mathbb{E}(w^{\mathcal{F}})_t = \max(w_{t-1}, \frac{\mathbf{k}^*_t}{\langle \beta^{\mathbf{K}} \rangle_{t-1}} \langle w^{\mathcal{F}} \rangle_{t-1} + q^*_t \langle w^{\mathcal{F}}_Q \rangle_{t-1})$.

Finally, we note that $i_t = 0$ and so $l^*_t = \max(\rho_{\mathcal{F}}\mathbb{E}(w^{\mathcal{F}})_t - \mathbf{d}_{t-1}, 0)$.

4.3. Bank

Bank agent represents the aggregate banking and financial sector.

Bank is required to maintain a capital ratio ($\Gamma = \frac{V}{L}$) (see Caiani et al., 2016). Liquidity is obtained, in case of necessity, by selling (and eventually shorting) Government's Bonds.

Bank fixes the interest rates on Bank Shares as $r^{\mathcal{B}}_{\mathbf{S}t} = r_{\mathbf{B}t} + \lambda(\Gamma - \Gamma^*)$, where $r_{\mathbf{B}t}$ is the Bonds interest rate. The net interest rate for the households is $r^{\mathcal{H}}_{\mathbf{S}t} = (1-\tau_{\mathbf{S}})r^{\mathcal{S}}_{\mathbf{S}}$, where $\tau_{\mathbf{S}}$ is the constant tax rate on financial income.

In this model the Bank does not distribute profits and can access all the needed liquidity from the consolidated public sector (the Government). As a consequence, it needs a way to avoid excessive capitalization that does not depends on a liquidity ratio. The proposed behaviour simulates a profit distribution when the Bank is capitalized over the target, without the need to introduce a market for shares.

The Bank does not ration the credit to Firms, accommodating all the demand for Loans. Bank provides new loans ate the interest rate

$$r_{\mathbf{L}t}^{f} = r_{\mathbf{B}t} + \nu_2(\Gamma^* - \Gamma_{t-1})$$

4.4. Government

The consolidated Public sector acts as the Central Bank and as the Government.

As Central Bank, it fixes the Bonds interest rate according to a Taylor rule. The annual interest rate is $r_{\mathbf{B}}^{y} = \psi^y + \alpha_1(\psi^y - \psi^*) + \alpha_2(\langle u \rangle_y - u^*) - \alpha_3(\langle \omega \rangle_y - \omega^*)$, where ψ is the inflation rate, u is the capacity utilization measured as the fraction of capital goods used in production, ω is the unemployment rate computed among those who have not voluntarily exited the job market, and starred variables are the targets. The Public sector do not raise the yearly interest rate by more than 0.005 each quarter. The interest rate used in the model is then $r_{\mathbf{B}t} = (1 + r_{\mathbf{B}t}^{y})^{\frac{1}{12}} - 1$.

Looking at the assets' matrix of the model, any Public expenditure happens directly toward the Bank with the emission of new Bonds, or through the Bank which converts new Bonds in deposits for the other sectors. As consequence, the Government has no accounting limits to spending.

As Government, it fixes the public expenditure. It additionally collects taxes and pays unemployment benefits. It determines the amount to be transferred to households (both as monetary and non-monetary, as consumption goods).

As an approximation, fiscal policy is kept constant during the simulation and taxes are collected only from the Household sector during the transactions. Particularly, the model includes four taxes: a VAT on the purchase of consumption goods ($\tau_{\mathbf{C}}$); a flat financial income tax on distributed interests on Bank shares ($\tau_{\mathbf{S}}$); an inheritance tax on wealth with a constant rate (τ_I); a progressive income tax rate computed as $\tau_W(w) = \max(\tau_M \tanh(\tau_F(\frac{w^{\mathcal{F}}}{p} - \tau_T)), 0)$, where p is the average price of consumption goods since the last update.

Fixed unemployment benefits $m_t = \phi \max(w_{t-1}^{\mathcal{H}}, m_{t-1})$ is paid to those who have not exited the job market and are not employed.

Each period government buys consumption goods which distribute to the Households. Particularly each Household receives an amount of Consumption goods equal to $\mathbf{c}_{h\,t}^{\mathcal{G}} = ((1 - \varepsilon_0) + \varepsilon_0 e^{-\varepsilon_1 \frac{v_{h\,t-1}}{p}})\Xi_t$.

We define g as the growth rate of the GDP (as $Y = p_{\mathbf{C}}\mathbf{C} + p_{\mathbf{K}}\Delta^+\mathbf{K}_{\mathcal{F}_{\mathbf{C}}}$) in the previous periods, $M = \sum_{h\in\mathcal{H}} m_h$, $C^{\mathcal{G}} = \sum_{h\in\mathcal{H}} (p_{\mathbf{C}}\mathbf{c})_h^{\mathcal{G}}$ and $G = M + C^{\mathcal{G}}$.

Assuming a Maastricht-like scenario in which Government has a target deficit δ^* to achieve, the expected balance of the next period can be written as

$$\begin{aligned}(1+g)\delta^* Y_{t-1} &= \mathbb{E}(G)_t + r_{\mathbf{B}t}(\mathbf{B}_{t-1} + (1+g)\delta^* Y_{t-1}) - (1+g)(1+\psi)T_{t-1} \\ \mathbb{E}(G)_t &= (1+\psi)(1+g)T_{t-1} + (1+g)(1 - r_{\mathbf{B}t})\delta^* Y_{t-1} - r_{\mathbf{B}t}\mathbf{B}_{t-1}\end{aligned}$$

But the expected public expenditure can also be written as

$$\begin{aligned}\mathbb{E}(G)_t &= \mathbb{E}(M)_t + \mathbb{E}(C^{\mathcal{G}})_t \approx M_{t-1} + (1+\psi)p\mathbb{E}(\mathbf{C}^{\mathcal{G}})_t \\ &\approx M_{t-1} + (1+\psi)pN_{\mathcal{H}}\langle \mathbf{c}_{h\,t}^{\mathcal{G}} \rangle = M_{t-1} + (1+\psi)pN_{\mathcal{H}}\langle \xi_t \rangle \Xi_t \\ &\approx M_{t-1} + (1+\psi)pN_{\mathcal{H}}\langle \xi_{t-1} \rangle \Xi_t = M_{t-1} + (1+\psi)p\frac{\mathbf{C}^{\mathcal{G}}{}_{t-1}}{\Xi_{t-1}}\Xi_t\end{aligned}$$

where it is assumed that expenses for unemployment benefits are approximately constant.

Finally, the public expenditure is regulated by updating regularly Ξ as

$$\begin{aligned}\Xi_t &= \frac{\Xi_{t-1}}{\langle \mathbf{C}^{\mathcal{G}} \rangle}\frac{\mathbb{E}(G) - \langle M \rangle}{(1+\psi)p} \\ \mathbb{E}(G) &= (1+\psi)(1+g)\langle T \rangle + (1+g)(1 - r_{\mathbf{B}t})\delta^* Y_{t-1} - r_{\mathbf{B}t}\langle \mathbf{B} \rangle\end{aligned}$$

where the averages are taken since the last update.

5. Real Assets

5.1. Consumption Goods

Consumption Goods represent all the consumption (goods and services) of Households and are homogeneous and non-durable.

5.2. Capital Goods

Capital goods are characterized by their productivity β and the minimum skill level required to operate them σ. The production happens according to a Leontieff-like function in which one worker with at least σ skill can operate a single capital good getting $\beta_{\mathbf{C}} = k\beta$ consumption goods as output or $\beta_{\mathbf{K}} = \beta$ capital goods. Each period they are depreciated of a constant fraction $N_{\mathbf{K}}^{-1}$ of their original price and are destroyed after $N_{\mathbf{K}}$ periods.

6. Financial Assets

6.1. Deposits

Deposits represent liquidity for Households and Firms and are not interest-bearing. Bank satisfies any transaction as long as the balance of the account remains positive.

6.2. Bank Shares

Bank Shares are sold and bought at their nominal value. Bank satisfies every transaction both as emission and as buy-back, as long as the accounts remain positive. Households can buy or sell Bank Shares only at the beginning of each period. At the end of each period, interests are paid, according to the rate fixed by the Bank at the beginning of the period.

6.3. Loans

Loans are issued by the Bank to a specific Firm. They have a fixed duration during which an equal share of capital is repaid plus the interest on the remaining debt. Interests are fixed by the Bank at a different value for each Firm at the time of emission.

6.4. Government Bonds

Government Bonds are sold and bought at their nominal value and do not expire. Bank satisfies every transaction and can short. At the end of each period, interests are paid to the Bank in case of net public debt or to the Government in the case the position is short, according to the rate fixed at the beginning of the period.

7. Dynamics

7.1. Consumption Goods market

Each Household sees $\chi_{\mathbf{C}}$ Consumption Firms $\chi_{\mathbf{C}}$ times, each time it buys up to $\frac{\mathbf{c}^*}{\chi_{\mathbf{C}}}$ at the lowest offered price until it matches the desired consumption or ends liquidity.

7.2. Capital Goods market

Each Consumption Firm sees $\chi_{\mathbf{K}}$ Capital Firms $\chi_{\mathbf{K}}$ times, each time it buys machinery with a total production capacity of $\frac{\Delta b}{\chi_C}$, choosing the one with the lowest value of $\beta p_{\mathbf{C}} - \mathbb{E}(w|\sigma) - \frac{p_{\mathbf{K}}}{\langle N_{\mathbf{K}} \rangle}$ until it matches the desired quantity or ends liquidity.

7.3. Labour market

Households are employed by a Firm until they are fired, they chose to exit the job market or they accept an offer from another Firm.

Firms can fire workers only when $w_t - p_t s_t < 0$ or equivalently $\pi_t < 0$. In that case, Firms fire workers until the number of workers reaches $u^* \mathbf{k}$. Additionally, in the case the Firm is not able to pay a worker, they is fired. In both cases, Firms start to fire those with lower skills.

Firms match each employed worker with the most productive machinery they can use, starting with those with higher skills. For this purpose, a researcher is treated like a worker assigned to a machinery with productivity σ^*. Then the current potential output (as the sum of the productivity of the used machinery) is computed and vacancies are open starting from the most productive unused machinery. All research vacancies are filled if possible. For each vacancy, the Firm sees $\chi_{\mathcal{H}}$ workers who have the required skills. The firm offers at first the salary that preserves the unitary cost of the previous period, i.e. $w_h^* = \beta \frac{w_{t-1}}{c_{t-1}}$ or $w_h^* = \langle w_Q \rangle$, and employs the worker with the higher skills earning less than the proposed salary. If none of the workers earns less than the proposed salary, the firm employs the worker with the lower salary offering a salary equal to $\rho_W w_{t-1}^h$.

Each vacancy is filled once and new vacancies are not filled.

7.4. Retirement and inheritance

When a Household reaches a certain age it is considered retired and is replaced by a new agent in the model which inherits their (taxed) wealth and enters the simulation with a random skill level proportional to the inherited wealth, and an age proportional to the skill level.

7.5. Non Performing Loans

In the case a Firm is unable to repay the principal of a loan or to pay the interest, the unpaid sum is added to the import of the loan and the duration is increased by one period. Additionally, the loan is marked as non-performing.

7.6. Bankrupt

Once the net value of a Firm becomes negative the Firm declares bankruptcy, fires all the employed households and loose all the financial assets (loans and deposits). It is “replaced” by a new firm which inherits the capital stock.

7.7. Innovation

Each Capital Firm can achieve an innovation each period with a probability $\theta = 1 - e^{-\zeta Q}$ where Q is the number of researchers employed.

If the innovation is achieved the Capital Goods produced by the Firm increase its productivity $\Delta\beta = \text{Beta}(1, b_0)$ and modify the required skills as $\Delta\sigma = (\Delta\beta - b_1 \text{Beta}(1, b_2))$.

8. Model steps

A [QUARTERLY] Statistics are computed at the relevant frequency
B [QUARTERLY] Government sets the Bonds interest rate
C Bank sets the interest rate on Bank Bonds
D [QUARTERLY] Government update the public spending policy
E.0 Firms bankrupt and get substituted
E.1 Firms set desired output and investment
E.2 Firms ask the Bank for loans

F.0 Households retire and get substituted
F.1 Households set desired consumption and savings
F.2 Households acquire Bank Shares
G.0 Firms open vacancies
G.1 Labour market and wages payment
H Production and price setting
I.0 Government pays unemployment benefits
I.1 Government buys and distributes Consumption Goods
J Households Consumption Goods market
K.0 Capital Goods depreciation
K.1 Capital Goods market
L Innovation
M Skill update
N Firms repay loans and interests
O Firms distribute dividends
P Bank pays interests on Shares
Q Interest on Bonds are paid

9. Parameters

	Description	Value	Source and Notes
$N_{\mathcal{H}}$	Number of Households	2000	~22mln households including those in retirement[1]; scale approximatly 1:10 000
$N_{\mathcal{F}_{\mathbf{C}}}$	Number of Consumption Firms	250	~4.5mln enterprises[2]; 80% of total as in Caiani et al. (2016); number of firms halved to consider 2 workers per households and preserving firms' size
$N_{\mathcal{F}_{\mathbf{K}}}$	Number of Capital Firms	50	See above
$N_{\mathbf{K}}$	Capital Goods lifetime	60	5 years
N_L	Loans duration	60	$N_{\mathbf{K}}$
Σ	Relative skill variation	0.08	To keep the annual increment of skill for a worker constantly employed $\lesssim 1$
σ^M	Maximum initial skill level	10	2 high school years after compulsory education, 5 years in university, 3 years as PhD
e_0	Intergenerational social mobility parameter		Calibration; > 0; characterization of degree holders by previous generation degree
e_1	Shape parameter for initial skill level		Calibration; $1 > \cdot > 0$; fraction of degree holders
A_R	Retirement Age	780	65 years; approximate retirement age in Italy
A_0	Initial age without education	180	15 years; compulsory education in Italy enrols people until 16 y.o., but the minimum skill level (years of education) is 1.
$\rho_{\mathcal{H}}$	Deposits to consumption ratio	2	> 1
a_y	Distribution parameter for propensity to consume out of income		Calibration; > 0; average propensity to consume

	Description	Value	Source and Notes
a_v	Distribution parameter for propensity to consume out of wealth		Calibration; > 0; average propensity to consume
$\rho_{\mathbf{C}}$	Production over expected sales for Consumption Firms	1.1	> 1
$\rho_{\mathbf{K}}$	Production over expected sales for Capital Firms	1.05	> 1
$\rho_{\mathcal{F}}$	Liquidity over expected wages and investment ratio	2	(Calibration); > 1
Θ	Markup growth rate		Calibration; > 0; inflation rate
ρ_{Π}	Profit distribution rate		Calibration; $1 > \cdot > 0$; Bank's capital ratio
ρ_Q	Researchers number adjustment rate		Calibration; > 0
σ^*	Minimum researchers' skill level	10	σ_M
Γ^*	Target capital ratio	0.08	Basel III
λ	Bank share premium		Calibration; > 0; profit share
ν_2	Loan rate premium for Bank capitalization		Calibration; > 0; rate of interest for Firms credit
$\tau_{\mathbf{C}}$	VAT rate	0.2	VAT rate in Italy is between 0.04 and 0.22 depending on the kind of good
$\tau_{\mathbf{S}}$	Financial income tax rate	0.25	Capital income tax rate is between 0.24 and 0.26
τ_I	Inheritance tax rate	0	There is no such thing as taxing again honestly earned wealth
τ_M	Maximum labour income tax rate	0.45	The maximum marginal rate of Italian income tax is 0.43
τ_F	Labour income tax progressiveness		Calibration; > 0; tax revenues on GDP
τ_T	Labour income tax threshold		Calibration; > 0; number of exempted households
ϕ	Unemployment benefits decay rate	0.90	~30% of initial wage after one year
δ^*	Target deficit	0.03	EU
α_1	Inflation coefficient in Taylor rule	0.5	
α_2	Capacity utilization coefficient in Taylor rule	0.25	
α_2	Unemployment coefficient in Taylor rule	0.25	
ε_0	Progressiveness of Government transfers		Calibration; $1 \geq \cdot > 0$; Wealth and income inequality
ε_1	Progressiveness of Government transfers		Calibration; > 0; Wealth and income inequality
ψ^*	Target inflation	0.02	EU
u^*	Target capacity utilization	0.8	
ω^*	Target unemployment rate	0.05	
k	Consumption goods to capital good output ratio		Calibration; ≥ 1; growth and unemployment rates
$\chi_{\mathbf{C}}$	Number of Consumption Firms seen in Consumption Goods market	5	2%
$\chi_{\mathbf{K}}$	Number of Capital Firms seen in Capital Goods market	5	10%

	Description	Value	Source and Notes
$\chi_{\mathcal{H}}$	Number of Households seen in labour market	20	1%
ρ_W	Salary increase if the labour market is short on the offer	1.05	
ζ	Innovation speed		Calibration; > 0; growth and unemployment rate
b_0	Productivity gain distribution parameter		Calibration; > 0; growth and unemployment rate
b_1	Maximum required skill loss		Calibration; > 0; growth and unemployment rate (adjusted for education/skill)
b_2	Required skill loss distribution parameter		Calibration; > 0; growth and unemployment rate (adjusted for education/skill)

[1]Eurostat data on Italy - 2001 https://ec.europa.eu/eurostat/databrowser/view/cens_hndwsize/
[2]Eurostat data on Italy - 2021 https://ec.europa.eu/eurostat/databrowser/view/sbs_sc_ovw/

10. Implementation

This model has to be solved computationally, and, as a consequence, it has to be implemented as software.

It has often been noticed that agents closely resemble the concept of object in Object-Oriented Programming. Both are logically independent pieces of information, which evolve through the execution (the simulation) updating themself in response to external stimuli (the calls of their methods from other agents).

For this reason, some models are implemented using this paradigm, using different patterns for agent communication and different frameworks for generic Agent-Based or Multi-Agent models, leveraging OOP programming languages such as C++ or Java. An example in the stream of literature of interest is Caiani et al. (2016).

A different approach relies on linear algebra routines and represents each variable of the model as a vector whose indexes are each associated with a different agent. Such representation levellers the ability of some libraries and programming languages to apply a function to each element of the vector (i.e. to each agent) in an optimized way. Programming languages suitable to this approach are R, Julia, Fortran or linear algebra libraries like Python's Numpy.

It is interesting to highlight that both these approaches are widely diffused in video game development as Object-Oriented and Entity-Component-System patterns. The industrial investment in improving libraries and frameworks for game development is way bigger than the academic one on simulation models in Economics, and so borrowing the software infrastructure from game development can be an interesting path in case of computationally expensive models (i.e. large-scale or high interactions).

In both cases, the simulation relies only on the current state of the model not storing, in principle, the intermediate states the model has traversed. Practically, it requires one to plan what data from the intermediate states of the model have to be preserved and log them separately. As a consequence, to enhance the analysis of the model and focus on previously ignored questions it is necessary to edit the simulation code and run it again.

An alternative approach is to continuously store the states and transactions of the model in a database and to use an auxiliary programming language to update the database, being able to observe the full evolution of the simulation.

The approach taken is an intermediate one: the final state of the model after each cycle is kept in memory, without tracking every transaction. The resulting object is a vector of states that allows tracking some variables during the evolution of the run without managing a database. Additionally, all the flows and stock for each period are accounted for, allowing to compute the flow of founds and balance sheet matrices for each period.

The last observation is about the need of writing parallelized code, for example, to run simulations with a huge number of agents in High-Performance Clusters. While this idea appears interesting at first look, it is probably an unnecessary overhead, since writing parallel code in the context of interacting objects is a very difficult task. Instead, the same computation capacity can be used to run in parallel multiple runs of the same single-thread model, for example, to explore the sensitivity to a parameter or to obtain statistically robust analysis by comparing different initializations of the random number generator.

10.1. Initialization

To initialize the model a handful of parameters are added to the model to easy the start-up.

Particularly price level is set to some order of magnitude to easy discretization errors, given all monetary values are stored as integers in the implementation.

δ_0 and σ_0 are used to initialize households in order to obtain a different skills levels.

	Description	Value	Source and Notes
T	Simulation length	300	25 years
p_0	Price level	100	
δ_0	Probability of skill increase per age in initialization	Calibration	
β_0	Productivity of Capital Goods at the beginning of the simulation	1.05	(Calibration)
σ_0	Minimum skill level for Capital Goods at the beginning of the simulation	Calibration	

11. Calibration

At the moment of writing the calibration of the model is being attempted.

Different schemes are being tried but generally speaking the calibration is done as following: model is simulated with random parameters, using four different seed for each set of parameters; for each period of each seed a list of stylized facts are checked, and the set of parameter is scored with the fraction of the stylized facts matched over the total possible (number of stylized facts * number of periods per run * number of seeds); the distributions from which the parameters are drawn are updated using the observed runs (for example assuming a truncated normal distribution for each parameter and computing the weighted mean and variance of the observations).

11.1. Stylized facts

	Description	Value	Target	
Intergenerational mobility	$\mathbb{P}(\sigma_0 \geq 5 \vert \sigma_0^{parent} \geq 5)$	$[0.67 : 0.65]$	≥ 0.7	[1]
Degree holders	$\mathbb{P}(\sigma_0 \geq 5)$	$[19,4\% : 20,3\%]$	$[0.2 : 0.35]$	[2]

	Description	Value	Target	
Average propensity to consume out of income - 1st quintile	$\langle c/y^{\mathcal{H}} \rangle$	$[0.974 : 0.976]$	$[0.90 : 1]$	[3]
Average propensity to consume out of income - 5th quintile	$\langle c/y^{\mathcal{H}} \rangle$	$[0.435 : 0.475]$	$[0.4 : 0.5]$	[4]
Growth rate	g^y	$[-0.075 : 0.097]$	$[-0.1 : 0.1]$	[5]
Inflation rate	ψ^y	$[-0.001 : 0.087]$	$[-0.01 : 0.05]$	[6]
Bank's capital ratio	$\Gamma \leq \Gamma^*$		True	
Profit share	$(r^{\mathcal{H}}_{\mathbf{S}}\mathbf{S})/Y^{\mathcal{H}}$	$[0.35 : 0.40]$	$[0.30 : 0.45]$	[7]
Public debt interest rate	$r^y_{\mathbf{B}}$	$[0.006 : 0.049]$	$[0 : 0.05]$	[8]
Debt to GDP ratio	$\mathbf{B}/Y$	$[1.32 : 1.55]$	$[1.0 : 1.5]$	[9]
Average loan interest rate for Firms	$\langle r_{\mathbf{L}} \rangle$	$[0.012 : 0.045]$	$[0.00 : 0.05]$	[10]
Tax revenues over GDP	T/Y	$[0.4165 : 0.438]$	$[0.35 : 0.50]$	[11]
Households exempted from tax payment	$\lvert\{h \vert t^h_W = 0\}\rvert / N^{\mathcal{H}}$	0.24	$[0.15 : 0.25]$	[12]
Unemployment rate	ω	$[0.081 : 0.124]$	$[0.00 : 0.10]$	[13]
Decreasing unemployment rate by skill level	$\rho(\omega, \sigma) < 0$		True	
Increasing salary by skill level	$\rho(w, \sigma) > 0$		True	
Kurtosis of firms' size	$\kappa(\#employed_f)$		> 6	[14]
Kurtosis of income	$\kappa(y^{\mathcal{H}})$		> 6	
Kurtosis of wealth	$\kappa(v_h)$		$> \kappa(y^{\mathcal{H}})$	
Gini index for income	$\cdot$	$[0.295 : 0.307]$	$[0.25 : 0.35]$	[15]
Wealth and income inequality	$\text{Gini}(wealth) > \text{Gini}(income)$		True	
Government spending over GDP	$(M + C^{\mathcal{G}})/Y$	$[0.484 : 0.570]$	$[0.45 : 0.60]$	[16]

[1]Holders of bachelor degree, children of holders of bachelor degree, ITALY 2022 https://www.istat.it/it/files//2023/10/Report-livelli-di-istruzione-e-ritorni-occupazionali.pdf

[2]ITALY 2019-2022 https://www.istat.it/it/files//2023/10/Report-livelli-di-istruzione-e-ritorni-occupazionali.pdf

[3]US 1999-2013 (Fisher et al., 2020)

[4]US 1999-2013 (Fisher et al., 2020)

[5]ITALY 2012-2022 https://ec.europa.eu/eurostat/databrowser/view/tec00001/default/table?lang=en

[6]ITALY 2011-2022 https://ec.europa.eu/eurostat/databrowser/view/tec00118/default/table?lang=en

[7]ITALY 2011-2022 Gross Operating Surplus over Gross Value Added https://ec.europa.eu/eurostat/databrowser/view/nasa_10_nf_tr/default/table?lang=en

[8]ITALY 2014-2022 https://fred.stlouisfed.org/series/INTGSBITM193N

[9]ITALY 2013-2022 Gross Operating Surplus over Gross Value Added https://ec.europa.eu/eurostat/databrowser/view/gov_10dd_edpt1/default/table?lang=en

[10]ITALY 2011-2022 https://data.ecb.europa.eu/data/datasets/MIR/MIR.M.IT.B.A2I.AM.R.A.2240.EUR.N

[11]ITALY 2010-2022 https://data.oecd.org/tax/tax-revenue.htm

[12]ITALY 2021 https://www1.finanze.gov.it/finanze/analisi_stat/public/v_4_0_0/contenuti/analisi_dati_2021_irpef.pdf (§1.8)

[13]ITALY 2011-2022 http://dati.istat.it/Index.aspx?DataSetCode=DCCV_TAXDISOCCU1

[14]Kurtosis of exponential distribution

[15]ITALY 2011-2022 http://dati.istat.it/Index.aspx?DataSetCode=DCCV_GINIREDD

[16]ITALY 2011-2022 https://ec.europa.eu/eurostat/databrowser/view/tec00023/default/table?lang=en
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A. Beta-Binomial Distribution

The Beta-binomial distribution is a probability distribution defined on a discrete and finite support from 0 to N, which describes a Bernoulli process of N trials each with a probability of success drawn from a Beta distribution of parameters α and β. Its probability mass function is $\binom{x}{N}\frac{B(x+\alpha,N-x+\beta)}{B(\alpha,\beta)}$, where B is the Beta function.

It is easy to see that the number of trials is the maximum possible result of a drawing, and so $N = \sigma^M$. The relations to define α and β as functions of the mean μ and the variance s are less easy to derive.

First, it is known that $\mu = \frac{N\alpha}{\alpha+\beta}$ and $s = \frac{N\alpha\beta(\alpha+\beta+N)}{(\alpha+\beta)^2(\alpha+\beta+1)}$, from which it follows $\beta = \frac{\alpha}{\mu}(N-\mu)$ and $\alpha + \beta = \frac{N\alpha}{\mu}$.

$$s = \frac{N\alpha^2\mu^3(N-\mu)(N\alpha - N\mu)}{N^2\alpha^2\mu^2(N\alpha+\mu)} = \frac{\mu(N-\mu)(\alpha-\mu)}{N\alpha+\mu}$$

$$sN\alpha + s\mu = \mu N\alpha + \mu^2 N - \alpha\mu^2 - \mu^3$$

$$\alpha(sN - \mu N + \mu^2) = \mu(\mu N - \mu^2 - s)$$

$$\alpha = \frac{\mu(\mu N - \mu^2 - s)}{sN - \mu N + \mu^2}$$

$$\beta = \frac{(N-\mu)(\mu N - \mu^2 - s)}{sN - \mu N + \mu^2}$$

As additional conditions it must be $\alpha > 0$ and $\beta > 0$, given that $N > \mu > 0$ and $s > 0$, from which it follows

$$\mu(N-\mu) > s > \frac{\mu}{N}(N-\mu)$$

To keep the variance small and so to ensure a bell-shaped mass function, $s = e_1[\mu(N-\mu) - \frac{\mu}{N}(N-\mu)] + \frac{\mu}{N}(N-\mu) = \frac{\mu}{N}(Ne_1 - e_1 + 1)$, for $e_1 \in (0,1)$ and small.

B. Consumption Equation

Empirical literature (Fisher et al., 2020) suggests that the marginal propensity to consume η is decreasing in income and wealth.

By definition $\eta = \frac{\partial c}{\partial x}$ (where x is either income or wealth) and so, as long as c depends on y only directly, it is possible to write $c = \int_0^X \eta(x)dx$. Assuming a persistency of the marginal propensity to consume, and so a fat-tail func rendo tional form like $\eta = (x+1)^{-a}$, it is possible to obtain

$$c(X) = \int_0^X (x+1)^{-a}dx = \int_1^{X+1} y^{-a}dy = \frac{1}{1-a}\, y^{1-a}\Big|_1^{X+1} = \frac{1}{1-a}((X+1)^{1-a} - 1)$$

C. Aggregated quasi-SFC model

C.1. Model 0

$$\mathbf{D}_{\mathcal{H}t} = \mathbf{D}_{\mathcal{H}t-1} + \mathbf{S}_{\mathcal{H}t-1} - \mathbf{S}_{\mathcal{H}t} + W_{\mathcal{H}t} + M_{\mathcal{H}t} - p_{\mathbf{C}t}\mathbf{C}_{\mathcal{H}t} + r_{\mathbf{S}t}\mathbf{S}_{\mathcal{H}t} - T^{\mathcal{H}}{}_t \tag{C.1.1}$$

$$\mathbf{D}_{\mathcal{F}_{\mathbf{C}}t} = \mathbf{D}_{\mathcal{F}_{\mathbf{C}}}{}_t^{II} - \Pi^{\mathcal{F}_{\mathbf{C}}}{}_t \tag{C.1.2}$$

$$\mathbf{D}_{\mathcal{F}_{\mathbf{K}}t} = \mathbf{D}_{\mathcal{F}_{\mathbf{K}}}{}_t^{II} - \Pi^{\mathcal{F}_{\mathbf{K}}}{}_t \tag{C.1.3}$$

$$\mathbf{D}^{\mathcal{B}}{}_t = \mathbf{D}_{\mathcal{H}t} + \mathbf{D}_{\mathcal{F}_{\mathbf{C}}t} + \mathbf{D}_{\mathcal{F}_{\mathbf{K}}t} \tag{C.1.4}$$

$$\mathbf{S}_{\mathcal{H}t} = \mathbf{D}_{\mathcal{H}t-1} + \mathbf{S}_{\mathcal{H}t-1} - (1+\rho_{\mathcal{H}})(1+\psi_{t-1})(1+\tau_{\mathbf{C}})p_{\mathbf{C}t-1}\mathbf{C}_{\mathcal{H}}{}_t^{*} + (1-\tau_W)W_{\mathcal{H}t-1} + \phi M_{\mathcal{H}t-1} \tag{C.1.5}$$

$$\mathbf{S}^{\mathcal{B}}{}_t = \mathbf{S}_{\mathcal{H}t} \tag{C.1.6}$$

$$\mathbf{L}^{\mathcal{F}_{\mathbf{C}}}{}_t = \mathbf{L}^{\mathcal{F}_{\mathbf{C}}}{}_{t-1} + \Delta^+\mathbf{L}^{\mathcal{F}_{\mathbf{C}}}{}_t + \max(0, r_{\mathbf{L}t}\mathbf{L}^{\mathcal{F}_{\mathbf{C}}}{}_{t-1} - \mathbf{D}_{\mathcal{F}_{\mathbf{C}}}{}_t^{I}) - \min(\max(0, \mathbf{D}_{\mathcal{F}_{\mathbf{C}}}{}_t^{I} - r_{\mathbf{L}t}\mathbf{L}^{\mathcal{F}_{\mathbf{C}}}{}_{t-1}), N_{\mathbf{L}}^{-1}\mathbf{L}^{\mathcal{F}_{\mathbf{C}}}{}_{t-1}) \tag{C.1.7}$$

$$\mathbf{L}^{\mathcal{F}_{\mathbf{K}}}{}_t = \mathbf{L}^{\mathcal{F}_{\mathbf{K}}}{}_{t-1} + \Delta^+\mathbf{L}^{\mathcal{F}_{\mathbf{K}}}{}_t + \max(0, r_{\mathbf{L}t}\mathbf{L}^{\mathcal{F}_{\mathbf{K}}}{}_{t-1} - \mathbf{D}_{\mathcal{F}_{\mathbf{K}}}{}_t^{I}) - \min(\max(0, \mathbf{D}_{\mathcal{F}_{\mathbf{K}}}{}_t^{I} - r_{\mathbf{L}t}\mathbf{L}^{\mathcal{F}_{\mathbf{K}}}{}_{t-1}), N_{\mathbf{L}}^{-1}\mathbf{L}^{\mathcal{F}_{\mathbf{K}}}{}_{t-1}) \tag{C.1.8}$$

$$\mathbf{L}_{\mathcal{B}t} = \mathbf{L}^{\mathcal{F}\mathbf{C}}{}_t + \mathbf{L}^{\mathcal{F}\mathbf{K}}{}_t \tag{C.1.9}$$

$$\mathbf{B}_{\mathcal{B}t} = \mathbf{B}^{\mathcal{G}}{}_t \tag{C.1.10}$$

$$\mathbf{B}^{\mathcal{G}}{}_t = (1 + r_{\mathbf{B}t})(\mathbf{B}^{\mathcal{G}}{}_{t-1} - p_{\mathbf{C}t}\mathbf{C}_{\mathcal{G}t} + T_{\mathcal{G}t} - M^{\mathcal{G}}{}_t) \tag{C.1.11}$$

$$\mathbf{K}_{\mathcal{F}_{\mathbf{C}}t} = (1-\gamma)\mathbf{K}_{\mathcal{F}_{\mathbf{C}}t-1} + \Delta^+\mathbf{K}_{\mathcal{F}_{\mathbf{C}}t} \tag{C.1.12}$$

$$\mathbf{K}_{\mathcal{F}_{\mathbf{K}}t} = \mathbf{K}^e_t + \hat{\mathbf{K}}_t \tag{C.1.13}$$

$$\mathbf{C}_{\mathcal{H}t} = \min(\max(0, \mathbf{C}_{\mathcal{H}}{}^*_t - \mathbf{C}_{\mathcal{G}t}), \mathbf{C}^{\mathcal{F}\mathbf{C}}{}_t - \mathbf{C}_{\mathcal{G}t}, \frac{\mathbf{D}_{\mathcal{H}t-1} + \mathbf{S}_{\mathcal{H}t-1} - \mathbf{S}_{\mathcal{H}t} + (1-\tau_W)W_{\mathcal{H}t} + M_{\mathcal{H}t}}{(1+\tau_{\mathbf{C}})p_{\mathbf{C}t}}) \tag{C.1.14}$$

$$\mathbf{C}^{\mathcal{F}\mathbf{C}}{}_t = \mathbf{C}_{\mathcal{H}t} + \mathbf{C}_{\mathcal{G}t} \tag{C.1.15}$$

$$\mathbf{C}_{\mathcal{G}t} = \min\left(\Delta^+\mathbf{C}^{\mathcal{F}\mathbf{C}}{}_t, \frac{(1+\psi_{t-1})(1+g_{t-1})T_{\mathcal{G}t-1} + (1+g_{t-1})(1-r_{\mathbf{B}t-1})\delta^* Y_{t-1} - r_{\mathbf{B}t-1}\mathbf{B}^{\mathcal{G}}{}_t - \phi M^{\mathcal{G}}{}_{t-1}}{(1+\psi_{t-1})p_{\mathbf{C}t-1}}\right) \tag{C.1.16}$$

$$\Delta^+\mathbf{K}_{\mathcal{F}_{\mathbf{C}}t} = \min\left(\Delta^+\mathbf{K}_{\mathcal{F}_{\mathbf{C}}}{}^*_t, \Delta^+\mathbf{K}^{\mathcal{F}\mathbf{K}}{}_t + (1-\gamma)\hat{\mathbf{K}}_{t-1} - \Delta^+\mathbf{K}^e{}_t, \frac{\mathbf{D}_{\mathcal{F}_{\mathbf{C}}t-1} + \Delta^+\mathbf{L}^{\mathcal{F}\mathbf{C}}{}_t - W^{\mathcal{F}\mathbf{C}}{}_t + p_{\mathbf{C}t}\mathbf{C}^{\mathcal{F}\mathbf{C}}{}_t}{p_{\mathbf{K}t}}\right) \tag{C.1.17}$$

$$\Delta^+\mathbf{K}^{\mathcal{F}\mathbf{K}}{}_t = N_{\mathbf{K}t}\langle\beta\rangle_{\mathcal{F}_{\mathbf{K}}t-1} \tag{C.1.18}$$

$$W_{\mathcal{H}t} = W^{\mathcal{F}\mathbf{C}}{}_t + W^{\mathcal{F}\mathbf{K}}{}_t \tag{C.1.19}$$

$$W^{\mathcal{F}\mathbf{C}}{}_t = \langle W\rangle_t N_{\mathbf{C}t} \tag{C.1.20}$$

$$W^{\mathcal{F}\mathbf{K}}{}_t = \langle W\rangle_t (N_{\mathbf{K}t} + N_{Q\,t}) \tag{C.1.21}$$

$$T^{\mathcal{H}}{}_t = \tau_{\mathbf{C}} p_{\mathbf{C}t}\mathbf{C}_{\mathcal{H}t} + \tau_{\mathbf{S}} r_{\mathbf{S}t}\mathbf{S}_{\mathcal{H}t} + \tau_W W_{\mathcal{H}t} \tag{C.1.22}$$

$$T_{\mathcal{G}t} = T^{\mathcal{H}}{}_t \tag{C.1.23}$$

$$M_{\mathcal{H}t} = M^{\mathcal{G}}{}_t \tag{C.1.24}$$

$$M^{\mathcal{G}}{}_t = \omega_t N_{\mathcal{H}} \sum_{n=1}^{t} \langle W\rangle_{t-n}\phi^n \tag{C.1.25}$$

$$\Pi^{\mathcal{F}\mathbf{C}}{}_t = \max(0, \mathbf{D}_{\mathcal{F}_{\mathbf{C}}}{}^{II}_t - \rho_{\mathcal{F}}(W_{\mathcal{F}_{\mathbf{C}}t} + p_{\mathbf{K}t}\Delta^+\mathbf{K}_{\mathcal{F}_{\mathbf{C}}t})) \tag{C.1.26}$$

$$\Pi^{\mathcal{F}\mathbf{K}}{}_t = \max(0, \mathbf{D}_{\mathcal{F}_{\mathbf{K}}}{}^{II}_t - \rho_{\mathcal{F}} W_{\mathcal{F}_{\mathbf{K}}t}) \tag{C.1.27}$$

$$\Pi_{\mathcal{B}t} = \Pi^{\mathcal{F}\mathbf{C}}{}_t + \Pi^{\mathcal{F}\mathbf{K}}{}_t \tag{C.1.28}$$

$$p_{\mathbf{C}t} = (1 + \mu_{\mathcal{F}_{\mathbf{C}}t})\frac{W^{\mathcal{F}\mathbf{C}}{}_t}{\Delta^+\mathbf{C}^{\mathcal{F}\mathbf{C}}{}_t} \tag{C.1.29}$$

$$p_{\mathbf{K}t} = (1 + \mu_{\mathcal{F}_{\mathbf{K}}t})\frac{W^{\mathcal{F}\mathbf{K}}{}_t}{\Delta^+\mathbf{K}^{\mathcal{F}\mathbf{K}}{}_t} \tag{C.1.30}$$

$$r_{\mathbf{S}t} = r_{\mathbf{B}t} + \lambda(\Gamma_{t-1} - \Gamma^*) \tag{C.1.31}$$

$$r_{\mathbf{L}t} = r_{\mathbf{B}t} + \nu_2(\Gamma^* - \Gamma_{t-1}) \tag{C.1.32}$$

$$r_{\mathbf{B}t} = \psi_{t-1} + \alpha_1(\psi_{t-1} - \psi^*) + \alpha_2(u_{t-1} - u^*) - \alpha_3(\omega_{t-1} - \omega^*) \tag{C.1.33}$$

$$\psi_t = \frac{p_{\mathbf{C}t}}{p_{\mathbf{C}t-1}} - 1 \tag{C.1.34}$$

$$u_t = \frac{N_{\mathbf{C}t} + N_{\mathbf{K}t}}{\mathbf{K}_{\mathcal{F}_{\mathbf{C}}t-1} + \mathbf{K}^e_{\mathcal{F}_{\mathbf{K}}t-1}} \tag{C.1.35}$$

$$\omega_t = \frac{N_{\mathbf{C}t} + N_{\mathbf{K}t} + N_{Q\,t}}{N_{\mathcal{H}}} \tag{C.1.36}$$

$$g_t = \frac{Y_t}{Y_{t-1}} - 1 \tag{C.1.37}$$

$$Y_t = p_{\mathbf{C}t}\mathbf{C}^{\mathcal{F}\mathbf{C}}{}_t + p_{\mathbf{K}t}\Delta^+\mathbf{K}_{\mathcal{F}_{\mathbf{C}}t} \tag{C.1.38}$$

$$\Gamma_t = \frac{\mathbf{L}_{\mathcal{B}t} + \mathbf{B}_{\mathcal{B}t} - \mathbf{D}^{\mathcal{B}}{}_t - \mathbf{S}^{\mathcal{B}}{}_t}{\mathbf{L}_{\mathcal{B}t}} \tag{C.1.39}$$

$$\Delta^+\mathbf{C}^{\mathcal{F}\mathbf{C}}{}^*_t = \rho_{\mathbf{C}}(1 + g_{t-1} - \psi_{t-1})\mathbf{C}^{\mathcal{F}\mathbf{C}}{}_{t-1} \tag{C.1.40}$$

$$\Delta^+\mathbf{C}^{\mathcal{F}\mathbf{C}}{}_t = N_{\mathbf{C}t}\langle\beta\rangle_{\mathcal{F}_{\mathbf{C}}t-1} k_{\mathcal{F}_{\mathbf{C}}} \tag{C.1.41}$$

$$\hat{\mathbf{K}}_t = (1-\gamma)\hat{\mathbf{K}}_{t-1} + \Delta^+\mathbf{K}^{\mathcal{F}\mathbf{K}}{}_t - \Delta^+\mathbf{K}^e{}_t - \Delta^+\mathbf{K}_{\mathcal{F}_{\mathbf{C}}t} \tag{C.1.42}$$

$$\mathbf{K}^e_t = (1-\gamma)\mathbf{K}^e_{t-1} + \Delta^+\mathbf{K}^e{}_t \tag{C.1.43}$$

$$\Delta^+\mathbf{K}^{\mathcal{F}\mathbf{K}}{}^*_t = \max(0, \rho_{\mathbf{K}}(1 + g_{t-1} - \psi_{t-1})\Delta^+\mathbf{K}_{\mathcal{F}_{\mathbf{C}}t-1} + \Delta^+\mathbf{K}^e{}^*_t - (1-\gamma)\hat{\mathbf{K}}_{t-1}) \tag{C.1.44}$$

$$\Delta^+\mathbf{K}_{\mathcal{F}_{\mathbf{C}}}{}^*_t = (\beta_{t-1}k_{\mathcal{F}_{\mathbf{C}}})^{-1}\max(0, \frac{\Delta^+\mathbf{C}^{\mathcal{F}\mathbf{C}}{}^*_t}{u^*} + (\gamma-1)\langle\beta\rangle_{\mathcal{F}_{\mathbf{C}}t-1}k_{\mathcal{F}_{\mathbf{C}}}\mathbf{K}_{\mathcal{F}_{\mathbf{C}}t-1}) \tag{C.1.45}$$

$$\Delta^+\mathbf{K}^e{}^*_t = \max\left(0, \frac{\rho_{\mathbf{K}}\Delta^+\mathbf{K}_{\mathcal{F}_{\mathbf{C}}t-1}}{u^*\langle\beta\rangle_{\mathcal{F}_{\mathbf{K}}t-1}} - (1-\gamma)\mathbf{K}^e_{t-1}\right) \tag{C.1.46}$$

$$\Delta^+\mathbf{K}^e{}_t = \min(\Delta^+\mathbf{K}^{\mathcal{F}\mathbf{K}}{}_t + (1-\gamma)\hat{\mathbf{K}}_{t-1}, \Delta^+\mathbf{K}^e{}^*_t) \tag{C.1.47}$$

$$N^*_{Q\,t} = \max\left(0, N_{Q\,t-1} + \rho_Q\frac{p_{\mathbf{K}t-1}\Delta^+\mathbf{K}_{\mathcal{F}_{\mathbf{C}}t-1} - W^{\mathcal{F}\mathbf{K}}{}_{t-1}}{\langle W\rangle_{t-1}}\right) \tag{C.1.48}$$

$$\mu_{\mathcal{F}_{\mathbf{C}}t} = \mu_{\mathcal{F}_{\mathbf{C}}t-1}\left(1 + \Theta\left(\rho_{\mathbf{C}}\frac{\mathbf{C}^{\mathcal{F}\mathbf{C}}{}_{t-1}}{\Delta^+\mathbf{C}^{\mathcal{F}\mathbf{C}}{}^*_{t-1}} - 1\right)\right) \tag{C.1.49}$$

$$\mu_{\mathcal{F}_{\mathbf{K}}t} = \mu_{\mathcal{F}_{\mathbf{K}}t-1}\left(1 + \Theta\left(\rho_{\mathbf{K}}\frac{\Delta^+\mathbf{K}_{\mathcal{F}_{\mathbf{C}}t-1}}{\Delta^+\mathbf{K}^{\mathcal{F}\mathbf{K}}{}^*_{t-1}} - 1\right)\right) \tag{C.1.50}$$

$$\Delta^{+}\mathbf{L}^{\mathcal{F}\mathbf{C}}{}_t = \max\left(0, \rho_{\mathcal{F}}\mathbb{E}(W^{\mathcal{F}\mathbf{C}})_t - \mathbf{D}_{\mathcal{F}\mathbf{C}\,t-1}, \rho_{\mathcal{F}}\left(\mathbb{E}(W^{\mathcal{F}\mathbf{C}})_t + \mathbb{E}(I^{\mathcal{F}\mathbf{C}})_t\right) - \left(\mathbf{D}_{\mathcal{F}\mathbf{C}\,t-1} + (1+\mu_{\mathcal{F}\mathbf{C}\,t})\frac{W^{\mathcal{F}\mathbf{C}}{}_{t-1}}{\mathbf{C}^{\mathcal{F}\mathbf{C}}_{t-1}}\frac{\Delta^{+}\mathbf{C}^{\mathcal{F}\mathbf{C}}{}^{*}_t}{\rho_{\mathbf{C}}}\right)\right) \tag{C.1.51}$$

$$\Delta^{+}\mathbf{L}^{\mathcal{F}\mathbf{K}}{}_t = \max\left(0, \rho_{\mathcal{F}}\mathbb{E}(W^{\mathcal{F}\mathbf{K}})_t - \mathbf{D}_{\mathcal{F}\mathbf{K}\,t-1}\right) \tag{C.1.52}$$

$$\mathbb{E}(W^{\mathcal{F}\mathbf{C}})_t = \Delta^{+}\mathbf{C}^{\mathcal{F}\mathbf{C}}{}^{*}_t \frac{\langle W\rangle_t}{\langle\beta\rangle_{\mathcal{F}\mathbf{C}\,t-1}k_{\mathcal{F}\mathbf{C}}} \tag{C.1.53}$$

$$\mathbb{E}(I^{\mathcal{F}\mathbf{C}})_t = p_{\mathbf{K}\,t-1}\Delta^{+}\mathbf{K}_{\mathcal{F}\mathbf{C}}{}^{*}_t \tag{C.1.54}$$

$$\mathbb{E}(W^{\mathcal{F}\mathbf{K}})_t = \Delta^{+}\mathbf{K}^{\mathcal{F}\mathbf{K}}{}^{*}_t \frac{\langle W\rangle_t}{\langle\beta\rangle_{\mathcal{F}\mathbf{K}\,t-1}} \tag{C.1.55}$$

$$\mathbf{C}_{\mathcal{H}}{}^{*}_t = \frac{a_y((1-\tau_W)W_{\mathcal{H}\,t-1} + \phi M_{\mathcal{H}\,t-1}) + a_v(\mathbf{S}_{\mathcal{H}\,t-1} + \mathbf{D}_{\mathcal{H}\,t-1})}{(1-\tau_{\mathbf{C}})p_{\mathbf{C}\,t-1}(1+\psi_{t-1})} \tag{C.1.56}$$

$$\mathbf{D}_{\mathcal{F}\mathbf{C}}{}^{I}_t = \mathbf{D}_{\mathcal{F}\mathbf{C}\,t-1} + \Delta^{+}\mathbf{L}^{\mathcal{F}\mathbf{C}}{}_t - W^{\mathcal{F}\mathbf{C}}{}_t + p_{\mathbf{C}\,t}\mathbf{C}^{\mathcal{F}\mathbf{C}}{}_t - p_{\mathbf{K}\,t}\Delta^{+}\mathbf{K}_{\mathcal{F}\mathbf{C}\,t} \tag{C.1.57}$$

$$\mathbf{D}_{\mathcal{F}\mathbf{C}}{}^{II}_t = \max(0, \mathbf{D}_{\mathcal{F}\mathbf{C}}{}^{I}_t - (r_{\mathbf{L}\,t} + N_{\mathbf{L}}^{-1})\mathbf{L}^{\mathcal{F}\mathbf{C}}{}_{t-1}) \tag{C.1.58}$$

$$\mathbf{D}_{\mathcal{F}\mathbf{K}}{}^{I}_t = \mathbf{D}_{\mathcal{F}\mathbf{K}\,t-1} + \Delta^{+}\mathbf{L}^{\mathcal{F}\mathbf{K}}{}_t - W^{\mathcal{F}\mathbf{K}}{}_t + p_{\mathbf{K}\,t}\Delta^{+}\mathbf{K}_{\mathcal{F}\mathbf{C}\,t} \tag{C.1.59}$$

$$\mathbf{D}_{\mathcal{F}\mathbf{K}}{}^{II}_t = \max(0, \mathbf{D}_{\mathcal{F}\mathbf{K}}{}^{I}_t - (r_{\mathbf{L}\,t} + N_{\mathbf{L}}^{-1})\mathbf{L}^{\mathcal{F}\mathbf{K}}{}_{t-1}) \tag{C.1.60}$$

$$\langle W\rangle_t = \langle W\rangle_{t-1}\rho_W(\omega^{*} - \omega_t) \tag{C.1.61}$$

$$N_{\mathbf{C}\,t} = \min\left(N_{\mathcal{H}}, \frac{\mathbf{D}_{\mathcal{F}\mathbf{C}\,t-1} + \Delta^{+}\mathbf{L}^{\mathcal{F}\mathbf{C}}{}_t}{\langle W\rangle_t}, \frac{\Delta^{+}\mathbf{C}^{\mathcal{F}\mathbf{C}}{}^{*}_t}{\mathbf{K}_{\mathcal{F}\mathbf{C}\,t-1}\langle\beta\rangle_{\mathcal{F}\mathbf{C}\,t-1}k_{\mathcal{F}\mathbf{C}}}\right) \tag{C.1.62}$$

$$N_{\mathbf{K}\,t} + N_{Q\,t} = \min\left(N_{\mathcal{H}} - N_{\mathbf{C}\,t}, \frac{\mathbf{D}_{\mathcal{F}\mathbf{K}\,t-1} + \Delta^{+}\mathbf{L}^{\mathcal{F}\mathbf{K}}{}_t}{\langle W\rangle_t}, \frac{\Delta^{+}\mathbf{K}^{\mathcal{F}\mathbf{K}}{}^{*}_t}{\mathbf{K}^{e}_{\mathcal{F}\mathbf{K}\,t-1}\langle\beta\rangle_{\mathcal{F}\mathbf{K}\,t-1}} + N^{*}_{Q\,t}\right) \tag{C.1.63}$$

$$N_{\mathbf{K}\,t} = (N_{\mathbf{K}\,t} + N_{Q\,t}) - N_{Q\,t} \tag{C.1.64}$$

$$N_{Q\,t} = \max\left(0, N_{\mathbf{K}\,t} + N_{Q\,t} - \frac{\Delta^{+}\mathbf{K}^{\mathcal{F}\mathbf{K}}{}^{*}_t}{\mathbf{K}^{e}_{\mathcal{F}\mathbf{K}\,t-1}\langle\beta\rangle_{\mathcal{F}\mathbf{K}\,t-1}}\right) \tag{C.1.65}$$

$$\beta_t = \beta_{t-1}\rho_{\beta}(1 - e^{-\zeta N_{Q\,t}}) \tag{C.1.66}$$

$$\langle\beta\rangle_{\mathcal{F}\mathbf{C}\,t} = \frac{\langle\beta\rangle_{\mathcal{F}\mathbf{C}\,t-1}(1-\gamma)\mathbf{K}_{\mathcal{F}\mathbf{C}\,t-1} + \beta_{t-1}\Delta^{+}\mathbf{K}_{\mathcal{F}\mathbf{C}\,t-1}}{\mathbf{K}_{\mathcal{F}\mathbf{C}\,t}} \tag{C.1.67}$$

$$\langle\beta\rangle_{\mathcal{F}\mathbf{K}\,t} = \frac{\langle\beta\rangle_{\mathcal{F}\mathbf{K}\,t-1}(1-\gamma)\mathbf{K}^{e}{}_{t-1} + \beta_{t-1}(\mathbf{K}^{e}{}_t - (1-\gamma)\mathbf{K}^{e}{}_{t-1})}{\mathbf{K}^{e}{}_t} \tag{C.1.68}$$

With additional constrains

$$\omega_t = \omega \tag{C.1.69}$$

$$g_t = g \tag{C.1.70}$$

$$\psi_t = \psi \tag{C.1.71}$$

$$r_{\mathbf{B}\,t} = r_{\mathbf{B}} \tag{C.1.72}$$

$$r_{\mathbf{L}\,t} = r_{\mathbf{L}} \tag{C.1.73}$$

$$\frac{\mathbf{B}^{\mathcal{G}}{}_t}{Y_t} = k_0 \tag{C.1.74}$$

$$\frac{r_{\mathbf{S}\,t}(1-\tau_{\mathbf{S}})\mathbf{S}_{\mathcal{H}\,t}}{r_{\mathbf{S}\,t}(1-\tau_{\mathbf{S}})\mathbf{S}_{\mathcal{H}\,t} + (1-\tau_W)W_{\mathcal{H}\,t} + M_{\mathcal{H}\,t}} = k_1 \tag{C.1.75}$$

$$\frac{T^{\mathcal{H}}{}_t}{Y_t} = k_2 \tag{C.1.76}$$

$$\frac{M^{\mathcal{G}}{}_t + p_{\mathbf{C}\,t}\mathbf{C}^{\mathcal{G}}{}_t}{Y_t} = k_3 \tag{C.1.77}$$

$$\rho_W(\omega^{*} - \omega_t) = k_4 \tag{C.1.78}$$

Leaving as additional variables (parameters not *a priori* fixed): τ_W, a_y, a_v, Θ, ζ, ρ_β, ρ_Q, ρ_W, ν_2, λ, $k_{\mathcal{F}\mathbf{C}}$.

C.2. First iteration and inequality checks

First iteration assuming satisfied demand.

$$\mathbf{D}_{\mathcal{H}\,t} = \mathbf{D}_{\mathcal{H}\,t-1} + \mathbf{S}_{t-1} - \mathbf{S}_t + W_{\mathcal{H}\,t} + M_t - p_{\mathbf{C}\,t}\mathbf{C}_{\mathcal{H}\,t} + r_{\mathbf{S}\,t}\mathbf{S}_t - T_t \tag{C.2.1}$$

$$\mathbf{D}_{\mathcal{F}\mathbf{C}\,t} = \mathbf{D}_{\mathcal{F}\mathbf{C}\,t-1} + \Delta^{+}\mathbf{L}^{\mathcal{F}\mathbf{C}}{}_t - W^{\mathcal{F}\mathbf{C}}{}_t + p_{\mathbf{C}\,t}\mathbf{C}^{\mathcal{F}\mathbf{C}}{}_t - p_{\mathbf{K}\,t}\Delta^{+}\mathbf{K}_{\mathcal{F}\mathbf{C}\,t} - (r_{\mathbf{L}\,t} + N_{\mathbf{L}}^{-1})\mathbf{L}^{\mathcal{F}\mathbf{C}}{}_{t-1} - \Pi^{\mathcal{F}\mathbf{C}}{}_t \tag{C.2.2}$$

$$\mathbf{D}_{\mathcal{F}\mathbf{K}\,t} = \mathbf{D}_{\mathcal{F}\mathbf{K}\,t-1} + \Delta^{+}\mathbf{L}^{\mathcal{F}\mathbf{K}}{}_t - W^{\mathcal{F}\mathbf{K}}{}_t + p_{\mathbf{K}\,t}\Delta^{+}\mathbf{K}_{\mathcal{F}\mathbf{C}\,t} - (r_{\mathbf{L}\,t} + N_{\mathbf{L}}^{-1})\mathbf{L}^{\mathcal{F}\mathbf{K}}{}_{t-1} - \Pi^{\mathcal{F}\mathbf{K}}{}_t \tag{C.2.3}$$

$$\mathbf{D}^{\mathcal{B}}{}_t = \mathbf{D}_{\mathcal{H}\,t} + \mathbf{D}_{\mathcal{F}\mathbf{C}\,t} + \mathbf{D}_{\mathcal{F}\mathbf{K}\,t} \tag{C.2.4}$$

$$\mathbf{S}_t = \mathbf{D}_{\mathcal{H}\,t-1} + \mathbf{S}_{t-1} - (1+\rho_{\mathcal{H}})(1+\psi)(1+\tau_{\mathbf{C}})p_{\mathbf{C}\,t-1}\mathbf{C}_{\mathcal{H}\,t} + (1-\tau_W)W_{\mathcal{H}\,t-1} + \phi M_{t-1} \tag{C.2.5}$$

$$\mathbf{L}^{\mathcal{F}\mathbf{C}}{}_t = \mathbf{L}^{\mathcal{F}\mathbf{C}}{}_{t-1} + \Delta^{+}\mathbf{L}^{\mathcal{F}\mathbf{C}}{}_t - N_{\mathbf{L}}^{-1}\mathbf{L}^{\mathcal{F}\mathbf{C}}{}_{t-1} \tag{C.2.6}$$

$$\mathbf{L}^{\mathcal{F}\mathbf{K}}{}_t = \mathbf{L}^{\mathcal{F}\mathbf{K}}{}_{t-1} + \Delta^+\mathbf{L}^{\mathcal{F}\mathbf{K}}{}_t - N_{\mathbf{L}}^{-1}\mathbf{L}^{\mathcal{F}\mathbf{K}}{}_{t-1} \tag{C.2.7}$$

$$\mathbf{L}_{\mathcal{B}t} = \mathbf{L}^{\mathcal{F}\mathbf{C}}{}_t + \mathbf{L}^{\mathcal{F}\mathbf{K}}{}_t \tag{C.2.8}$$

$$\mathbf{B}_t = (1 + r_{\mathbf{B}})(\mathbf{B}_{t-1} - p_{\mathbf{C}t}\mathbf{C}_{\mathcal{G}t} + T_t - M_t) \tag{C.2.9}$$

$$\mathbf{K}_{\mathcal{F}_{\mathbf{C}}t} = (1-\gamma)\mathbf{K}_{\mathcal{F}_{\mathbf{C}}t-1} + \Delta^+\mathbf{K}_{\mathcal{F}_{\mathbf{C}}t} \tag{C.2.10}$$

$$\mathbf{K}_{\mathcal{F}_{\mathbf{K}}t} = \mathbf{K}^e_t + \hat{\mathbf{K}}_t \tag{C.2.11}$$

$$\mathbf{C}_{\mathcal{H}t} = \frac{a_y((1-\tau_W)W_{\mathcal{H}t-1} + \phi M_{t-1}) + a_v(\mathbf{S}_{t-1} + \mathbf{D}_{\mathcal{H}t-1})}{(1-\tau_{\mathbf{C}})p_{\mathbf{C}t-1}(1+\psi)} - \mathbf{C}_{\mathcal{G}t} \tag{C.2.12}$$

$$\mathbf{C}^{\mathcal{F}\mathbf{C}}{}_t = \mathbf{C}_{\mathcal{H}t} + \mathbf{C}_{\mathcal{G}t} \tag{C.2.13}$$

$$\mathbf{C}_{\mathcal{G}t} = \frac{(1+\psi)(1+g)T_{t-1} + (1+g)(1-r_{\mathbf{B}})\delta^* Y_{t-1} - r_{\mathbf{B}}\mathbf{B}_t - \phi M_{t-1}}{(1+\psi)p_{\mathbf{C}t-1}} \tag{C.2.14}$$

$$\Delta^+\mathbf{K}_{\mathcal{F}_{\mathbf{C}}t} = \frac{\rho_{\mathbf{C}}(1+g-\psi)\mathbf{C}^{\mathcal{F}\mathbf{C}}{}_{t-1}}{u^*\beta_{t-1}k_{\mathcal{F}_{\mathbf{C}}}} - (1-\gamma)\frac{\langle\beta\rangle_{\mathcal{F}_{\mathbf{C}}t-1}}{\beta_{t-1}}\mathbf{K}_{\mathcal{F}_{\mathbf{C}}t-1} \tag{C.2.15}$$

$$\Delta^+\mathbf{K}^{\mathcal{F}\mathbf{K}}{}_t = N_{\mathbf{K}t}\langle\beta\rangle_{\mathcal{F}_{\mathbf{K}}t-1} \tag{C.2.16}$$

$$W_{\mathcal{H}t} = W^{\mathcal{F}\mathbf{C}}{}_t + W^{\mathcal{F}\mathbf{K}}{}_t \tag{C.2.17}$$

$$W^{\mathcal{F}\mathbf{C}}{}_t = \langle W\rangle_t N_{\mathbf{C}t} \tag{C.2.18}$$

$$W^{\mathcal{F}\mathbf{K}}{}_t = \langle W\rangle_t(N_{\mathbf{K}t} + N_{Qt}) \tag{C.2.19}$$

$$T_t = \tau_{\mathbf{C}}p_{\mathbf{C}t}\mathbf{C}_{\mathcal{H}t} + \tau_{\mathbf{S}}r_{\mathbf{S}t}\mathbf{S}_t + \tau_W W_t \tag{C.2.20}$$

$$M_t = \omega N_{\mathcal{H}}\sum_{n=1}^{t}\langle W\rangle_{t-n}\phi^n \tag{C.2.21}$$

$$\Pi^{\mathcal{F}\mathbf{C}}{}_t = \mathbf{D}_{\mathcal{F}_{\mathbf{C}}t-1} + \Delta^+\mathbf{L}^{\mathcal{F}\mathbf{C}}{}_t - W^{\mathcal{F}\mathbf{C}}{}_t + p_{\mathbf{C}t}\mathbf{C}^{\mathcal{F}\mathbf{C}}{}_t - p_{\mathbf{K}t}\Delta^+\mathbf{K}_{\mathcal{F}_{\mathbf{C}}t} - (r_{\mathbf{L}t} + N_{\mathbf{L}}^{-1})\mathbf{L}^{\mathcal{F}\mathbf{C}}{}_{t-1} - \rho_{\mathcal{F}}(W^{\mathcal{F}\mathbf{C}}{}_t + p_{\mathbf{K}t}\Delta^+\mathbf{K}_{\mathcal{F}_{\mathbf{C}}t}) \tag{C.2.22}$$

$$\Pi^{\mathcal{F}\mathbf{K}}{}_t = \mathbf{D}_{\mathcal{F}_{\mathbf{K}}t-1} + \Delta^+\mathbf{L}^{\mathcal{F}\mathbf{K}}{}_t - W^{\mathcal{F}\mathbf{K}}{}_t + p_{\mathbf{K}t}\Delta^+\mathbf{K}_{\mathcal{F}_{\mathbf{C}}t} - (r_{\mathbf{L}t} + N_{\mathbf{L}}^{-1})\mathbf{L}^{\mathcal{F}\mathbf{K}}{}_{t-1} - \rho_{\mathcal{F}}W^{\mathcal{F}\mathbf{K}}{}_t \tag{C.2.23}$$

$$\Pi_{\mathcal{B}t} = \Pi^{\mathcal{F}\mathbf{C}}{}_t + \Pi^{\mathcal{F}\mathbf{K}}{}_t \tag{C.2.24}$$

$$p_{\mathbf{C}t} = (1 + \mu_{\mathcal{F}_{\mathbf{C}}t})\frac{W^{\mathcal{F}\mathbf{C}}{}_t}{N_{\mathbf{C}t}\langle\beta\rangle_{\mathcal{F}_{\mathbf{C}}t-1}k_{\mathcal{F}_{\mathbf{C}}}} \tag{C.2.25}$$

$$p_{\mathbf{K}t} = (1 + \mu_{\mathcal{F}_{\mathbf{K}}t})\frac{W^{\mathcal{F}\mathbf{K}}{}_t}{\Delta^+\mathbf{K}^{\mathcal{F}\mathbf{K}}{}_t} \tag{C.2.26}$$

$$r_{\mathbf{S}t} = r_{\mathbf{B}} + \lambda(\Gamma - \Gamma^*) \tag{C.2.27}$$

$$r_{\mathbf{L}} = r_{\mathbf{B}} + \nu_2(\Gamma^* - \Gamma) \tag{C.2.28}$$

$$r_{\mathbf{B}} = \psi + \alpha_1(\psi - \psi^*) + \alpha_2(u_{t-1} - u^*) - \alpha_3(\omega - \omega^*) \tag{C.2.29}$$

$$\psi = \frac{p_{\mathbf{C}t}}{p_{\mathbf{C}t-1}} - 1 \tag{C.2.30}$$

$$u_t = \frac{N_{\mathbf{C}t} + N_{\mathbf{K}t}}{\mathbf{K}_{\mathcal{F}_{\mathbf{C}}t-1} + \mathbf{K}^e_{\mathcal{F}_{\mathbf{K}}t-1}} \tag{C.2.31}$$

$$\omega = \frac{N_{\mathbf{C}t} + N_{\mathbf{K}t} + N_{Qt}}{N_{\mathcal{H}}} \tag{C.2.32}$$

$$g = \frac{Y_t}{Y_{t-1}} - 1 \tag{C.2.33}$$

$$Y_t = p_{\mathbf{C}t}\mathbf{C}^{\mathcal{F}\mathbf{C}}{}_t + p_{\mathbf{K}t}\Delta^+\mathbf{K}_{\mathcal{F}_{\mathbf{C}}t} \tag{C.2.34}$$

$$\Gamma = \frac{\mathbf{L}_{\mathcal{B}t} + \mathbf{B}_t - \mathbf{D}^{\mathcal{B}}{}_t - \mathbf{S}_t}{\mathbf{L}_{\mathcal{B}t}} \tag{C.2.35}$$

$$\Delta^+\mathbf{C}^{\mathcal{F}\mathbf{C}}{}^*_t = \rho_{\mathbf{C}}(1+g-\psi)\mathbf{C}^{\mathcal{F}\mathbf{C}}{}_{t-1} \tag{C.2.36}$$

$$\Delta^+\mathbf{C}^{\mathcal{F}\mathbf{C}}{}_t = N_{\mathbf{C}t}\langle\beta\rangle_{\mathcal{F}_{\mathbf{C}}t-1}k_{\mathcal{F}_{\mathbf{C}}} \tag{C.2.37}$$

$$\hat{\mathbf{K}}_t = (1-\gamma)\hat{\mathbf{K}}_{t-1} + \Delta^+\mathbf{K}^{\mathcal{F}\mathbf{K}}{}_t - \frac{\rho_{\mathbf{K}}\Delta^+\mathbf{K}_{\mathcal{F}_{\mathbf{C}}t-1}}{u^*\langle\beta\rangle_{\mathcal{F}_{\mathbf{K}}t-1}} - (1-\gamma)\mathbf{K}^e_{t-1} - \Delta^+\mathbf{K}_{\mathcal{F}_{\mathbf{C}}t} \tag{C.2.38}$$

$$\mathbf{K}^e_t = (1-\gamma)\mathbf{K}^e_{t-1} + \frac{\rho_{\mathbf{K}}\Delta^+\mathbf{K}_{\mathcal{F}_{\mathbf{C}}t-1}}{u^*\langle\beta\rangle_{\mathcal{F}_{\mathbf{K}}t-1}} - (1-\gamma)\mathbf{K}^e_{t-1} \tag{C.2.39}$$

$$\Delta^+\mathbf{K}^{\mathcal{F}\mathbf{K}}{}^*_t = \rho_{\mathbf{K}}(1+g-\psi)\Delta^+\mathbf{K}_{\mathcal{F}_{\mathbf{C}}t-1} + \frac{\rho_{\mathbf{K}}\Delta^+\mathbf{K}_{\mathcal{F}_{\mathbf{C}}t-1}}{u^*\langle\beta\rangle_{\mathcal{F}_{\mathbf{K}}t-1}} - (1-\gamma)\mathbf{K}^e_{t-1} - (1-\gamma)\hat{\mathbf{K}}_{t-1} \tag{C.2.40}$$

$$\Delta^+\mathbf{K}^e{}_t = \frac{\rho_{\mathbf{K}}\Delta^+\mathbf{K}_{\mathcal{F}_{\mathbf{C}}t-1}}{u^*\langle\beta\rangle_{\mathcal{F}_{\mathbf{K}}t-1}} - (1-\gamma)\mathbf{K}^e_{t-1} \tag{C.2.41}$$

$$\mu_{\mathcal{F}_{\mathbf{C}}t} = \mu_{\mathcal{F}_{\mathbf{C}}t-1}\left(1 + \Theta\left(\rho_{\mathbf{C}}\frac{\mathbf{C}^{\mathcal{F}\mathbf{C}}{}_{t-1}}{\rho_{\mathbf{C}}(1+g-\psi)\mathbf{C}^{\mathcal{F}\mathbf{C}}{}_{t-2}} - 1\right)\right) \tag{C.2.42}$$

$$\mu_{\mathcal{F}_{\mathbf{K}}t} = \mu_{\mathcal{F}_{\mathbf{K}}t-1}\left(1 + \Theta\left(\rho_{\mathbf{K}}\frac{\Delta^+\mathbf{K}_{\mathcal{F}_{\mathbf{C}}t-1}}{\rho_{\mathbf{K}}(1+g-\psi)\Delta^+\mathbf{K}_{\mathcal{F}_{\mathbf{C}}t-2} + \frac{\rho_{\mathbf{K}}\Delta^+\mathbf{K}_{\mathcal{F}_{\mathbf{C}}t-2}}{u^*\langle\beta\rangle_{\mathcal{F}_{\mathbf{K}}t-2}} - (1-\gamma)\mathbf{K}^e_{t-2} - (1-\gamma)\hat{\mathbf{K}}_{t-2}} - 1\right)\right) \tag{C.2.43}$$

$$\Delta^+\mathbf{L}^{\mathcal{F}\mathbf{C}}{}_t = \max\left(\rho_{\mathcal{F}}\frac{\rho_{\mathbf{C}}(1+g-\psi)\mathbf{C}^{\mathcal{F}\mathbf{C}}{}_{t-1}\langle W\rangle_t}{\langle\beta\rangle_{\mathcal{F}_{\mathbf{C}}t-1}k_{\mathcal{F}_{\mathbf{C}}}} - \mathbf{D}_{\mathcal{F}_{\mathbf{C}}t-1},\right. \tag{C.2.44}$$

$$\rho_{\mathcal{F}}\Bigg(\frac{\rho_{\mathbf{C}}(1+g-\psi)\mathbf{C}^{\mathcal{F}\mathbf{C}}{}_{t-1}\langle W\rangle_t}{\langle\beta\rangle_{\mathcal{F}_{\mathbf{C}}\,t-1}k_{\mathcal{F}_{\mathbf{C}}}} + p_{\mathbf{K}\,t-1}\left(\frac{\rho_{\mathbf{C}}(1+g-\psi)\mathbf{C}^{\mathcal{F}\mathbf{C}}{}_{t-1}}{u^*\beta_{t-1}k_{\mathcal{F}_{\mathbf{C}}}} - (1-\gamma)\frac{\langle\beta\rangle_{\mathcal{F}_{\mathbf{C}}\,t-1}}{\beta_{t-1}}\mathbf{K}_{\mathcal{F}_{\mathbf{C}}\,t-1}\right)\Bigg) -$$

$$\Bigg(\mathbf{D}_{\mathcal{F}_{\mathbf{C}}\,t-1} + (1+\mu_{\mathcal{F}_{\mathbf{C}}\,t})\frac{W^{\mathcal{F}\mathbf{C}}{}_{t-1}}{\mathbf{C}^{\mathcal{F}\mathbf{C}}_{t-1}}\frac{\rho_{\mathbf{C}}(1+g-\psi)\mathbf{C}^{\mathcal{F}\mathbf{C}}{}_{t-1}}{\rho_{\mathbf{C}}}\Bigg)\Bigg)$$

$$\Delta^+\mathbf{L}^{\mathcal{F}\mathbf{K}}{}_t = \rho_{\mathcal{F}}\left(\rho_{\mathbf{K}}(1+g-\psi)\Delta^+\mathbf{K}_{\mathcal{F}_{\mathbf{C}}\,t-1} + \frac{\rho_{\mathbf{K}}\Delta^+\mathbf{K}_{\mathcal{F}_{\mathbf{C}}\,t-1}}{u^*\langle\beta\rangle_{\mathcal{F}_{\mathbf{K}}\,t-1}} - (1-\gamma)\mathbf{K}^e_{t-1} - (1-\gamma)\hat{\mathbf{K}}_{t-1}\right)\frac{\langle W\rangle_t}{\langle\beta\rangle_{\mathcal{F}_{\mathbf{K}}\,t-1}} - \mathbf{D}_{\mathcal{F}_{\mathbf{K}}\,t-1} \tag{C.2.45}$$

$$\mathbb{E}(W^{\mathcal{F}\mathbf{C}})_t = \rho_{\mathbf{C}}(1+g-\psi)\mathbf{C}^{\mathcal{F}\mathbf{C}}{}_{t-1}\frac{\langle W\rangle_t}{\langle\beta\rangle_{\mathcal{F}_{\mathbf{C}}\,t-1}k_{\mathcal{F}_{\mathbf{C}}}} \tag{C.2.46}$$

$$\mathbb{E}(I^{\mathcal{F}\mathbf{C}})_t = p_{\mathbf{K}\,t-1}\left(\frac{\rho_{\mathbf{C}}(1+g-\psi)\mathbf{C}^{\mathcal{F}\mathbf{C}}{}_{t-1}}{u^*\beta_{t-1}k_{\mathcal{F}_{\mathbf{C}}}} - (1-\gamma)\frac{\langle\beta\rangle_{\mathcal{F}_{\mathbf{C}}\,t-1}}{\beta_{t-1}}\mathbf{K}_{\mathcal{F}_{\mathbf{C}}\,t-1}\right) \tag{C.2.47}$$

$$\mathbb{E}(W^{\mathcal{F}\mathbf{K}})_t = (\rho_{\mathbf{K}}(1+g-\psi)\Delta^+\mathbf{K}_{\mathcal{F}_{\mathbf{C}}\,t-1} + \frac{\rho_{\mathbf{K}}\Delta^+\mathbf{K}_{\mathcal{F}_{\mathbf{C}}\,t-1}}{u^*\langle\beta\rangle_{\mathcal{F}_{\mathbf{K}}\,t-1}} - (1-\gamma)\mathbf{K}^e_{t-1} - (1-\gamma)\hat{\mathbf{K}}_{t-1})\frac{\langle W\rangle_t}{\langle\beta\rangle_{\mathcal{F}_{\mathbf{K}}\,t-1}} \tag{C.2.48}$$

$$\mathbf{D}_{\mathcal{F}_{\mathbf{C}}\,t}{}^{I} = \mathbf{D}_{\mathcal{F}_{\mathbf{C}}\,t-1} + \Delta^+\mathbf{L}^{\mathcal{F}\mathbf{C}}{}_t - W^{\mathcal{F}\mathbf{C}}{}_t + p_{\mathbf{C}\,t}\mathbf{C}^{\mathcal{F}\mathbf{C}}{}_t - p_{\mathbf{K}\,t}\Delta^+\mathbf{K}_{\mathcal{F}_{\mathbf{C}}\,t} \tag{C.2.49}$$

$$\mathbf{D}_{\mathcal{F}_{\mathbf{C}}\,t}{}^{II} = \mathbf{D}_{\mathcal{F}_{\mathbf{C}}\,t-1} + \Delta^+\mathbf{L}^{\mathcal{F}\mathbf{C}}{}_t - W^{\mathcal{F}\mathbf{C}}{}_t + p_{\mathbf{C}\,t}\mathbf{C}^{\mathcal{F}\mathbf{C}}{}_t - p_{\mathbf{K}\,t}\Delta^+\mathbf{K}_{\mathcal{F}_{\mathbf{C}}\,t} - (r_{\mathbf{L}} + N_{\mathbf{L}}^{-1})\mathbf{L}^{\mathcal{F}\mathbf{C}}{}_{t-1} \tag{C.2.50}$$

$$\mathbf{D}_{\mathcal{F}_{\mathbf{K}}\,t}{}^{I} = \mathbf{D}_{\mathcal{F}_{\mathbf{K}}\,t-1} + \Delta^+\mathbf{L}^{\mathcal{F}\mathbf{K}}{}_t - W^{\mathcal{F}\mathbf{K}}{}_t + p_{\mathbf{K}\,t}\Delta^+\mathbf{K}_{\mathcal{F}_{\mathbf{C}}\,t} \tag{C.2.51}$$

$$\mathbf{D}_{\mathcal{F}_{\mathbf{K}}\,t}{}^{II} = \mathbf{D}_{\mathcal{F}_{\mathbf{K}}\,t-1} + \Delta^+\mathbf{L}^{\mathcal{F}\mathbf{K}}{}_t - W^{\mathcal{F}\mathbf{K}}{}_t + p_{\mathbf{K}\,t}\Delta^+\mathbf{K}_{\mathcal{F}_{\mathbf{C}}\,t} - (r_{\mathbf{L}} + N_{\mathbf{L}}^{-1})\mathbf{L}^{\mathcal{F}\mathbf{K}}{}_{t-1} \tag{C.2.52}$$

$$\langle W\rangle_t = \langle W\rangle_{t-1}\rho_W(\omega^* - \omega) \tag{C.2.53}$$

$$N_{\mathbf{C}\,t} = \frac{\rho_{\mathbf{C}}(1+g-\psi)\mathbf{C}^{\mathcal{F}\mathbf{C}}{}_{t-1}}{\mathbf{K}_{\mathcal{F}_{\mathbf{C}}\,t-1}\langle\beta\rangle_{\mathcal{F}_{\mathbf{C}}\,t-1}k_{\mathcal{F}_{\mathbf{C}}}} \tag{C.2.54}$$

$$N_{\mathbf{K}\,t} = \frac{\rho_{\mathbf{K}}(1+g-\psi)\Delta^+\mathbf{K}_{\mathcal{F}_{\mathbf{C}}\,t-1} + \frac{\rho_{\mathbf{K}}\Delta^+\mathbf{K}_{\mathcal{F}_{\mathbf{C}}\,t-1}}{u^*\langle\beta\rangle_{\mathcal{F}_{\mathbf{K}}\,t-1}} - (1-\gamma)\mathbf{K}^e_{t-1} - (1-\gamma)\hat{\mathbf{K}}_{t-1}}{\mathbf{K}^e_{\mathcal{F}_{\mathbf{K}}\,t-1}\langle\beta\rangle_{\mathcal{F}_{\mathbf{K}}\,t-1}} \tag{C.2.55}$$

$$N_{Q\,t} = N_{Q\,t-1} + \rho_Q\frac{p_{\mathbf{K}\,t-1}\Delta^+\mathbf{K}_{\mathcal{F}_{\mathbf{C}}\,t-1} - W^{\mathcal{F}\mathbf{K}}{}_{t-1}}{\langle W\rangle_{t-1}} \tag{C.2.56}$$

$$\beta_t = \beta_{t-1}\rho_\beta(1 - e^{-\zeta N_{Q\,t}}) \tag{C.2.57}$$

$$\langle\beta\rangle_{\mathcal{F}_{\mathbf{C}}\,t} = \frac{\langle\beta\rangle_{\mathcal{F}_{\mathbf{C}}\,t-1}(1-\gamma)\mathbf{K}_{\mathcal{F}_{\mathbf{C}}\,t-1} + \beta_{t-1}\Delta^+\mathbf{K}_{\mathcal{F}_{\mathbf{C}}\,t-1}}{\mathbf{K}_{\mathcal{F}_{\mathbf{C}}\,t}} \tag{C.2.58}$$

$$\langle\beta\rangle_{\mathcal{F}_{\mathbf{K}}\,t} = \frac{\langle\beta\rangle_{\mathcal{F}_{\mathbf{K}}\,t-1}(1-\gamma)\mathbf{K}^e{}_{t-1} + \beta_{t-1}(\mathbf{K}^e{}_t - (1-\gamma)\mathbf{K}^e{}_{t-1})}{\mathbf{K}^e{}_t} \tag{C.2.59}$$

$$\mathbf{B}_t = k_0 Y_t \tag{C.2.60}$$

$$r_{\mathbf{S}\,t}(1-\tau_{\mathbf{S}})\mathbf{S}_t = k_1(r_{\mathbf{S}\,t}(1-\tau_{\mathbf{S}})\mathbf{S}_t + (1-\tau_W)W_{\mathcal{H}\,t} + M_t) \tag{C.2.61}$$

$$T_t = k_2 Y_t \tag{C.2.62}$$

$$M_t + p_{\mathbf{C}\,t}\mathbf{C}^{\mathcal{G}}{}_t = k_3 Y_t \tag{C.2.63}$$

$$\rho_W(\omega^* - \omega) = k_4 \tag{C.2.64}$$

Where the hypothesis of equilibrium made determine the following inequality constraints

$$\mathbf{D}_{\mathcal{F}_{\mathbf{C}}\,t-1} + \Delta^+\mathbf{L}^{\mathcal{F}\mathbf{C}}{}_t - W^{\mathcal{F}\mathbf{C}}{}_t + p_{\mathbf{C}\,t}\mathbf{C}^{\mathcal{F}\mathbf{C}}{}_t - p_{\mathbf{K}\,t}\Delta^+\mathbf{K}_{\mathcal{F}_{\mathbf{C}}\,t} \geq r_{\mathbf{L}}\mathbf{L}^{\mathcal{F}\mathbf{C}}{}_{t-1} \tag{C.2.65}$$

$$\mathbf{D}_{\mathcal{F}_{\mathbf{C}}\,t-1} + \Delta^+\mathbf{L}^{\mathcal{F}\mathbf{C}}{}_t - W^{\mathcal{F}\mathbf{C}}{}_t + p_{\mathbf{C}\,t}\mathbf{C}^{\mathcal{F}\mathbf{C}}{}_t - p_{\mathbf{K}\,t}\Delta^+\mathbf{K}_{\mathcal{F}_{\mathbf{C}}\,t} - r_{\mathbf{L}}\mathbf{L}^{\mathcal{F}\mathbf{C}}{}_{t-1} \geq N_{\mathbf{L}}^{-1}\mathbf{L}^{\mathcal{F}\mathbf{C}}{}_{t-1} \tag{C.2.66}$$

$$\mathbf{D}_{\mathcal{F}_{\mathbf{K}}\,t-1} + \Delta^+\mathbf{L}^{\mathcal{F}\mathbf{K}}{}_t - W^{\mathcal{F}\mathbf{K}}{}_t + p_{\mathbf{K}\,t}\Delta^+\mathbf{K}_{\mathcal{F}_{\mathbf{C}}\,t} \geq r_{\mathbf{L}}\mathbf{L}^{\mathcal{F}\mathbf{K}}{}_{t-1} \tag{C.2.67}$$

$$\mathbf{D}_{\mathcal{F}_{\mathbf{K}}\,t-1} + \Delta^+\mathbf{L}^{\mathcal{F}\mathbf{K}}{}_t - W^{\mathcal{F}\mathbf{K}}{}_t + p_{\mathbf{K}\,t}\Delta^+\mathbf{K}_{\mathcal{F}_{\mathbf{C}}\,t} - r_{\mathbf{L}}\mathbf{L}^{\mathcal{F}\mathbf{K}}{}_{t-1} \geq N_{\mathbf{L}}^{-1}\mathbf{L}^{\mathcal{F}\mathbf{K}}{}_{t-1} \tag{C.2.68}$$

$$\frac{a_y((1-\tau_W)W_{\mathcal{H}\,t-1} + \phi M_{t-1}) + a_v(\mathbf{S}_{t-1} + \mathbf{D}_{\mathcal{H}\,t-1})}{(1-\tau_{\mathbf{C}})p_{\mathbf{C}\,t-1}(1+\psi)} \geq \mathbf{C}_{\mathcal{G}\,t} \tag{C.2.69}$$

$$\mathbf{C}^{\mathcal{F}\mathbf{C}}{}_t - \mathbf{C}_{\mathcal{G}\,t} \geq \frac{a_y((1-\tau_W)W_{\mathcal{H}\,t-1} + \phi M_{t-1}) + a_v(\mathbf{S}_{t-1} + \mathbf{D}_{\mathcal{H}\,t-1})}{(1-\tau_{\mathbf{C}})p_{\mathbf{C}\,t-1}(1+\psi)} - \mathbf{C}_{\mathcal{G}\,t} \tag{C.2.70}$$

$$\frac{\mathbf{D}_{\mathcal{H}\,t-1} + \mathbf{S}_{t-1} - \mathbf{S}_t + (1-\tau_W)W_{\mathcal{H}\,t} + M_{\mathcal{H}\,t}}{(1+\tau_{\mathbf{C}})p_{\mathbf{C}\,t}} \geq \frac{a_y((1-\tau_W)W_{\mathcal{H}\,t-1} + \phi M_{t-1}) + a_v(\mathbf{S}_{t-1} + \mathbf{D}_{\mathcal{H}\,t-1})}{(1-\tau_{\mathbf{C}})p_{\mathbf{C}\,t-1}(1+\psi)} - \mathbf{C}_{\mathcal{G}\,t} \tag{C.2.71}$$

$$\frac{(1+\psi)(1+g)T_{t-1} + (1+g)(1-r_{\mathbf{B}})\delta^* Y_{t-1} - r_{\mathbf{B}}\mathbf{B}_t - \phi M_{t-1}}{(1+\psi)p_{\mathbf{C}\,t-1}} \geq N_{\mathbf{C}\,t}\langle\beta\rangle_{\mathcal{F}_{\mathbf{C}}\,t-1}k_{\mathcal{F}_{\mathbf{C}}} \tag{C.2.72}$$

$$\Delta^+\mathbf{K}^{\mathcal{F}\mathbf{K}}{}_t + (1-\gamma)\hat{\mathbf{K}}_{t-1} - \Delta^+\mathbf{K}^e{}_t \geq \frac{\rho_{\mathbf{C}}(1+g-\psi)\mathbf{C}^{\mathcal{F}\mathbf{C}}{}_{t-1}}{u^*\beta_{t-1}k_{\mathcal{F}_{\mathbf{C}}}} - (1-\gamma)\frac{\langle\beta\rangle_{\mathcal{F}_{\mathbf{C}}\,t-1}}{\beta_{t-1}}\mathbf{K}_{\mathcal{F}_{\mathbf{C}}\,t-1} \tag{C.2.73}$$

$$\frac{\mathbf{D}_{\mathcal{F}_{\mathbf{C}}\,t-1} + \Delta^+\mathbf{L}^{\mathcal{F}\mathbf{C}}{}_t - W^{\mathcal{F}\mathbf{C}}{}_t + p_{\mathbf{C}\,t}\mathbf{C}^{\mathcal{F}\mathbf{C}}{}_t}{p_{\mathbf{K}\,t}} \geq \frac{\rho_{\mathbf{C}}(1+g-\psi)\mathbf{C}^{\mathcal{F}\mathbf{C}}{}_{t-1}}{u^*\beta_{t-1}k_{\mathcal{F}_{\mathbf{C}}}} - (1-\gamma)\frac{\langle\beta\rangle_{\mathcal{F}_{\mathbf{C}}\,t-1}}{\beta_{t-1}}\mathbf{K}_{\mathcal{F}_{\mathbf{C}}\,t-1} \tag{C.2.74}$$

$$\frac{\rho_{\mathbf{C}}(1+g-\psi)\mathbf{C}^{\mathcal{F}\mathbf{C}}{}_{t-1}}{u^*} + (\gamma-1)\langle\beta\rangle_{\mathcal{F}_{\mathbf{C}}\,t-1}k_{\mathcal{F}_{\mathbf{C}}}\mathbf{K}_{\mathcal{F}_{\mathbf{C}}\,t-1} \geq 0 \tag{C.2.75}$$

$$\mathbf{D}_{\mathcal{F}_{\mathbf{C}}\,t-1} + \Delta^+\mathbf{L}^{\mathcal{F}\mathbf{C}}{}_t - W^{\mathcal{F}\mathbf{C}}{}_t + p_{\mathbf{C}\,t}\mathbf{C}^{\mathcal{F}\mathbf{C}}{}_t - p_{\mathbf{K}\,t}\Delta^+\mathbf{K}_{\mathcal{F}_{\mathbf{C}}\,t} - (r_{\mathbf{L}} + N_{\mathbf{L}}^{-1})\mathbf{L}^{\mathcal{F}\mathbf{C}}{}_{t-1} \geq \rho_{\mathcal{F}}(W_{\mathcal{F}_{\mathbf{C}}\,t} + p_{\mathbf{K}\,t}\Delta^+\mathbf{K}_{\mathcal{F}_{\mathbf{C}}\,t}) \tag{C.2.76}$$

$$\mathbf{D}_{\mathcal{F}_{\mathbf{K}}\,t-1} + \Delta^+\mathbf{L}^{\mathcal{F}\mathbf{K}}{}_t - W^{\mathcal{F}\mathbf{K}}{}_t + p_{\mathbf{K}\,t}\Delta^+\mathbf{K}_{\mathcal{F}_{\mathbf{C}}\,t} - (r_{\mathbf{L}} + N_{\mathbf{L}}^{-1})\mathbf{L}^{\mathcal{F}\mathbf{K}}{}_{t-1} \geq \rho_{\mathcal{F}}W_{\mathcal{F}_{\mathbf{K}}\,t} \tag{C.2.77}$$

$$\Delta^+\mathbf{K}^{\mathcal{F}_\mathbf{K}}{}_t + (1-\gamma)\hat{\mathbf{K}}_{t-1} \geq \frac{\rho_\mathbf{K}\Delta^+\mathbf{K}_{\mathcal{F}_\mathbf{C}}{}_{t-1}}{u^*\langle\beta\rangle_{\mathcal{F}_\mathbf{K}}{}_{t-1}} - (1-\gamma)\mathbf{K}^e_{t-1} \tag{C.2.78}$$

$$\frac{\rho_\mathbf{K}\Delta^+\mathbf{K}_{\mathcal{F}_\mathbf{C}}{}_{t-1}}{u^*\langle\beta\rangle_{\mathcal{F}_\mathbf{K}}{}_{t-1}} - (1-\gamma)\mathbf{K}^e_{t-1} \geq 0 \tag{C.2.79}$$

$$\rho_\mathbf{K}(1+g-\psi)\Delta^+\mathbf{K}_{\mathcal{F}_\mathbf{C}}{}_{t-1} + \frac{\rho_\mathbf{K}\Delta^+\mathbf{K}_{\mathcal{F}_\mathbf{C}}{}_{t-1}}{u^*\langle\beta\rangle_{\mathcal{F}_\mathbf{K}}{}_{t-1}} - (1-\gamma)\mathbf{K}^e_{t-1} - (1-\gamma)\hat{\mathbf{K}}_{t-1} \geq 0 \tag{C.2.80}$$

$$\Delta^+\mathbf{L}^{\mathcal{F}_\mathbf{C}}{}_t \geq 0 \tag{C.2.81}$$

$$\Delta^+\mathbf{L}^{\mathcal{F}_\mathbf{K}}{}_t \geq 0 \tag{C.2.82}$$

$$\mathbf{D}_{\mathcal{F}_\mathbf{C}}{}_{t-1} + \Delta^+\mathbf{L}^{\mathcal{F}_\mathbf{C}}{}_t - W^{\mathcal{F}_\mathbf{C}}{}_t + p_{\mathbf{C}t}\mathbf{C}^{\mathcal{F}_\mathbf{C}}{}_t - p_{\mathbf{K}t}\Delta^+\mathbf{K}_{\mathcal{F}_\mathbf{C}}{}_t - (r_\mathbf{L} + N_\mathbf{L}^{-1})\mathbf{L}^{\mathcal{F}_\mathbf{C}}{}_{t-1} \geq 0 \tag{C.2.83}$$

$$\mathbf{D}_{\mathcal{F}_\mathbf{K}}{}_{t-1} + \Delta^+\mathbf{L}^{\mathcal{F}_\mathbf{K}}{}_t - W^{\mathcal{F}_\mathbf{K}}{}_t + p_{\mathbf{K}t}\Delta^+\mathbf{K}_{\mathcal{F}_\mathbf{C}}{}_t - (r_\mathbf{L} + N_\mathbf{L}^{-1})\mathbf{L}^{\mathcal{F}_\mathbf{K}}{}_{t-1} \geq 0 \tag{C.2.84}$$

$$N_\mathcal{H} \geq N_{\mathbf{C}t} + N_{\mathbf{K}t} + N_{Qt} \tag{C.2.85}$$

$$N_{\mathbf{C}t} \geq 0 \tag{C.2.86}$$

$$N_{\mathbf{K}t} \geq 0 \tag{C.2.87}$$

$$N_{Qt} \geq 0 \tag{C.2.88}$$

$$\frac{\mathbf{D}_{\mathcal{F}_\mathbf{C}}{}_{t-1} + \Delta^+\mathbf{L}^{\mathcal{F}_\mathbf{C}}{}_t}{\langle W\rangle_t} \geq \frac{\rho_\mathbf{C}(1+g-\psi)\mathbf{C}^{\mathcal{F}_\mathbf{C}}{}_{t-1}}{\mathbf{K}_{\mathcal{F}_\mathbf{C}}{}_{t-1}\langle\beta\rangle_{\mathcal{F}_\mathbf{C}}{}_{t-1}k_{\mathcal{F}_\mathbf{C}}} \tag{C.2.89}$$

$$\frac{\mathbf{D}_{\mathcal{F}_\mathbf{K}}{}_{t-1} + \Delta^+\mathbf{L}^{\mathcal{F}_\mathbf{K}}{}_t}{\langle W\rangle_t} \geq \frac{\rho_\mathbf{K}(1+g-\psi)\Delta^+\mathbf{K}_{\mathcal{F}_\mathbf{C}}{}_{t-1} + \frac{\rho_\mathbf{K}\Delta^+\mathbf{K}_{\mathcal{F}_\mathbf{C}}{}_{t-1}}{u^*\langle\beta\rangle_{\mathcal{F}_\mathbf{K}}{}_{t-1}} - (1-\gamma)\mathbf{K}^e_{t-1} - (1-\gamma)\hat{\mathbf{K}}_{t-1}}{\mathbf{K}^e_{\mathcal{F}_\mathbf{K}}{}_{t-1}\langle\beta\rangle_{\mathcal{F}_\mathbf{K}}{}_{t-1}} + N_{Q}{}_{t-1} + \rho_Q\frac{p_{\mathbf{K}t-1}\Delta^+\mathbf{K}_{\mathcal{F}_\mathbf{C}}{}_{t-1} - W^{\mathcal{F}_\mathbf{K}}{}_{t-1}}{\langle W\rangle_{t-1}} \tag{C.2.90}$$

C.3. Second iteration

Variables are reduced to the required ones

$$\mathbf{D}_{\mathcal{H}t} = \mathbf{D}_{\mathcal{H}t-1} + \mathbf{S}_{t-1} - \mathbf{S}_t + W_{\mathcal{H}t} + M_t - p_{\mathbf{C}t}\mathbf{C}_{\mathcal{H}t} + r_{\mathbf{S}t}\mathbf{S}_t - T_t \tag{C.3.1}$$

$$\mathbf{D}_{\mathcal{F}_\mathbf{C}}{}_t = \rho_\mathcal{F}(W^{\mathcal{F}_\mathbf{C}}{}_t + p_{\mathbf{K}t}\Delta^+\mathbf{K}_{\mathcal{F}_\mathbf{C}}{}_t) \tag{C.3.2}$$

$$\mathbf{D}_{\mathcal{F}_\mathbf{K}}{}_t = \rho_\mathcal{F}W^{\mathcal{F}_\mathbf{K}}{}_t \tag{C.3.3}$$

$$\mathbf{D}^\mathcal{B}{}_t = \mathbf{D}_{\mathcal{H}t} + \mathbf{D}_{\mathcal{F}_\mathbf{C}}{}_t + \mathbf{D}_{\mathcal{F}_\mathbf{K}}{}_t \tag{C.3.4}$$

$$\mathbf{S}_t = \mathbf{D}_{\mathcal{H}t-1} + \mathbf{S}_{t-1} - (1+\rho_\mathcal{H})(1+\psi)(1+\tau_\mathbf{C})p_{\mathbf{C}t-1}\mathbf{C}_{\mathcal{H}t} + (1-\tau_W)W_{\mathcal{H}t-1} + \phi M_{t-1} \tag{C.3.5}$$

$$\mathbf{L}^{\mathcal{F}_\mathbf{C}}{}_t = \mathbf{L}^{\mathcal{F}_\mathbf{C}}{}_{t-1} + \Delta^+\mathbf{L}^{\mathcal{F}_\mathbf{C}}{}_t - N_\mathbf{L}^{-1}\mathbf{L}^{\mathcal{F}_\mathbf{C}}{}_{t-1} \tag{C.3.6}$$

$$\mathbf{L}^{\mathcal{F}_\mathbf{K}}{}_t = \mathbf{L}^{\mathcal{F}_\mathbf{K}}{}_{t-1} + \Delta^+\mathbf{L}^{\mathcal{F}_\mathbf{K}}{}_t - N_\mathbf{L}^{-1}\mathbf{L}^{\mathcal{F}_\mathbf{K}}{}_{t-1} \tag{C.3.7}$$

$$\mathbf{L}_{\mathcal{B}t} = \mathbf{L}^{\mathcal{F}_\mathbf{C}}{}_t + \mathbf{L}^{\mathcal{F}_\mathbf{K}}{}_t \tag{C.3.8}$$

$$\mathbf{B}_t = (1+r_\mathbf{B})(\mathbf{B}_{t-1} - p_{\mathbf{C}t}\mathbf{C}_{\mathcal{G}t} + T_t - M_t) \tag{C.3.9}$$

$$\mathbf{K}_{\mathcal{F}_\mathbf{C}}{}_t = (1-\gamma)\mathbf{K}_{\mathcal{F}_\mathbf{C}}{}_{t-1} + \Delta^+\mathbf{K}_{\mathcal{F}_\mathbf{C}}{}_t \tag{C.3.10}$$

$$\mathbf{K}_{\mathcal{F}_\mathbf{K}}{}_t = \mathbf{K}^e_t + \hat{\mathbf{K}}_t \tag{C.3.11}$$

$$\mathbf{C}_{\mathcal{H}t} = \frac{a_y((1-\tau_W)W_{\mathcal{H}t-1} + \phi M_{t-1}) + a_v(\mathbf{S}_{t-1} + \mathbf{D}_{\mathcal{H}t-1})}{(1-\tau_\mathbf{C})p_{\mathbf{C}t-1}(1+\psi)} - \mathbf{C}_{\mathcal{G}t} \tag{C.3.12}$$

$$\mathbf{C}^{\mathcal{F}_\mathbf{C}}{}_t = \mathbf{C}_{\mathcal{H}t} + \mathbf{C}_{\mathcal{G}t} \tag{C.3.13}$$

$$\mathbf{C}_{\mathcal{G}t} = \frac{(1+\psi)(1+g)T_{t-1} + (1+g)(1-r_\mathbf{B})\delta^*Y_{t-1} - r_\mathbf{B}\mathbf{B}_t - \phi M_{t-1}}{(1+\psi)p_{\mathbf{C}t-1}} \tag{C.3.14}$$

$$\Delta^+\mathbf{K}_{\mathcal{F}_\mathbf{C}}{}_t = \frac{\rho_\mathbf{C}(1+g-\psi)\mathbf{C}^{\mathcal{F}_\mathbf{C}}{}_{t-1}}{u^*\beta_{t-1}k_{\mathcal{F}_\mathbf{C}}} - (1-\gamma)\frac{\langle\beta\rangle_{\mathcal{F}_\mathbf{C}}{}_{t-1}}{\beta_{t-1}}\mathbf{K}_{\mathcal{F}_\mathbf{C}}{}_{t-1} \tag{C.3.15}$$

$$\Delta^+\mathbf{K}^{\mathcal{F}_\mathbf{K}}{}_t = N_{\mathbf{K}t}\langle\beta\rangle_{\mathcal{F}_\mathbf{K}}{}_{t-1} \tag{C.3.16}$$

$$W_{\mathcal{H}t} = W^{\mathcal{F}_\mathbf{C}}{}_t + W^{\mathcal{F}_\mathbf{K}}{}_t \tag{C.3.17}$$

$$W^{\mathcal{F}_\mathbf{C}}{}_t = \langle W\rangle_t N_{\mathbf{C}t} \tag{C.3.18}$$

$$W^{\mathcal{F}_\mathbf{K}}{}_t = \langle W\rangle_t(N_{\mathbf{K}t} + N_{Qt}) \tag{C.3.19}$$

$$T_t = \tau_\mathbf{C}p_{\mathbf{C}t}\mathbf{C}_{\mathcal{H}t} + \tau_\mathbf{S}r_{\mathbf{S}t}\mathbf{S}_t + \tau_W W_t \tag{C.3.20}$$

$$M_t = \omega N_\mathcal{H}\sum_{n=1}^{t}\langle W\rangle_{t-n}\phi^n \tag{C.3.21}$$

$$\Pi^{\mathcal{F}_\mathbf{C}}{}_t = \mathbf{D}_{\mathcal{F}_\mathbf{C}}{}_{t-1} + \Delta^+\mathbf{L}^{\mathcal{F}_\mathbf{C}}{}_t - W^{\mathcal{F}_\mathbf{C}}{}_t + p_{\mathbf{C}t}\mathbf{C}^{\mathcal{F}_\mathbf{C}}{}_t - p_{\mathbf{K}t}\Delta^+\mathbf{K}_{\mathcal{F}_\mathbf{C}}{}_t - (r_{\mathbf{L}t} + N_\mathbf{L}^{-1})\mathbf{L}^{\mathcal{F}_\mathbf{C}}{}_{t-1} - \rho_\mathcal{F}(W^{\mathcal{F}_\mathbf{C}}{}_t + p_{\mathbf{K}t}\Delta^+\mathbf{K}_{\mathcal{F}_\mathbf{C}}{}_t) \tag{C.3.22}$$

$$\Pi^{\mathcal{F}_\mathbf{K}}{}_t = \mathbf{D}_{\mathcal{F}_\mathbf{K}}{}_{t-1} + \Delta^+\mathbf{L}^{\mathcal{F}_\mathbf{K}}{}_t - W^{\mathcal{F}_\mathbf{K}}{}_t + p_{\mathbf{K}t}\Delta^+\mathbf{K}_{\mathcal{F}_\mathbf{C}}{}_t - (r_{\mathbf{L}t} + N_\mathbf{L}^{-1})\mathbf{L}^{\mathcal{F}_\mathbf{K}}{}_{t-1} - \rho_\mathcal{F}W^{\mathcal{F}_\mathbf{K}}{}_t \tag{C.3.23}$$

$$\Pi_{\mathcal{B}t} = \Pi^{\mathcal{F}_\mathbf{C}}{}_t + \Pi^{\mathcal{F}_\mathbf{K}}{}_t \tag{C.3.24}$$

$$p_{\mathbf{C}t} = (1+\mu_{\mathcal{F}_\mathbf{C}}{}_t)\frac{W^{\mathcal{F}_\mathbf{C}}{}_t}{N_{\mathbf{C}t}\langle\beta\rangle_{\mathcal{F}_\mathbf{C}}{}_{t-1}k_{\mathcal{F}_\mathbf{C}}} \tag{C.3.25}$$

$$p_{\mathbf{K}t} = (1+\mu_{\mathcal{F}_\mathbf{K}}{}_t)\frac{W^{\mathcal{F}_\mathbf{K}}{}_t}{\Delta^+\mathbf{K}^{\mathcal{F}_\mathbf{K}}{}_t} \tag{C.3.26}$$

$$r_{\mathbf{S}} = \left(1 + \frac{\lambda}{\nu_2}\right) r_{\mathbf{B}} - \frac{\lambda}{\nu_2} r_{\mathbf{L}} \tag{C.3.27}$$

$$\Gamma = \Gamma^* - \frac{r_{\mathbf{L}} - r_{\mathbf{B}}}{\nu_2} \tag{C.3.28}$$

$$u = \frac{r_{\mathbf{B}} - (1+\alpha_1)\psi + \alpha_1 \psi^* + \alpha_2 u^* + \alpha_3(\omega - \omega^*)}{\alpha_2} r_{\mathbf{B}} \tag{C.3.29}$$

$$p_{\mathbf{C}\,t} = (1+\psi)^t p_{\mathbf{C}\,0} \tag{C.3.30}$$

$$u = \frac{N_{\mathbf{C}\,t} + N_{\mathbf{K}\,t}}{\mathbf{K}_{\mathcal{F}_{\mathbf{C}}\,t-1} + \mathbf{K}^e_{\mathcal{F}_{\mathbf{K}}\,t-1}} \tag{C.3.31}$$

$$\omega = \frac{N_{\mathbf{C}\,t} + N_{\mathbf{K}\,t} + N_{Q\,t}}{N_{\mathcal{H}}} \tag{C.3.32}$$

$$Y_t = (1+g)^t Y_0 \tag{C.3.33}$$

$$Y_t = p_{\mathbf{C}\,t} \mathbf{C}^{\mathcal{F}_{\mathbf{C}}}{}_t + p_{\mathbf{K}\,t} \Delta^+ \mathbf{K}_{\mathcal{F}_{\mathbf{C}}\,t} \tag{C.3.34}$$

$$\Gamma = \frac{\mathbf{L}_{\mathcal{B}\,t} + \mathbf{B}_t - \mathbf{D}^{\mathcal{B}}{}_t - \mathbf{S}_t}{\mathbf{L}_{\mathcal{B}\,t}} \tag{C.3.35}$$

$$\hat{\mathbf{K}}_t = (1-\gamma)\hat{\mathbf{K}}_{t-1} + \Delta^+ \mathbf{K}^{\mathcal{F}_{\mathbf{K}}}{}_t - \frac{\rho_{\mathbf{K}} \Delta^+ \mathbf{K}_{\mathcal{F}_{\mathbf{C}}\,t-1}}{u^* \langle\beta\rangle_{\mathcal{F}_{\mathbf{K}}\,t-1}} - (1-\gamma)\mathbf{K}^e_{t-1} - \Delta^+ \mathbf{K}_{\mathcal{F}_{\mathbf{C}}\,t} \tag{C.3.36}$$

$$\mathbf{K}^e_t = \frac{\rho_{\mathbf{K}} \Delta^+ \mathbf{K}_{\mathcal{F}_{\mathbf{C}}\,t-1}}{u^* \langle\beta\rangle_{\mathcal{F}_{\mathbf{K}}\,t-1}} \tag{C.3.37}$$

$$\mu_{\mathcal{F}_{\mathbf{C}}\,t} = \mu_{\mathcal{F}_{\mathbf{C}}\,t-1} \left(1 + \Theta\left(\rho_{\mathbf{C}} \frac{\mathbf{C}^{\mathcal{F}_{\mathbf{C}}}{}_{t-1}}{\rho_{\mathbf{C}}(1+g-\psi)\mathbf{C}^{\mathcal{F}_{\mathbf{C}}}{}_{t-2}} - 1\right)\right) \tag{C.3.38}$$

$$\mu_{\mathcal{F}_{\mathbf{K}}\,t} = \mu_{\mathcal{F}_{\mathbf{K}}\,t-1} \left(1 + \Theta\left(\rho_{\mathbf{K}} \frac{\Delta^+ \mathbf{K}_{\mathcal{F}_{\mathbf{C}}\,t-1}}{\rho_{\mathbf{K}}(1+g-\psi)\Delta^+ \mathbf{K}_{\mathcal{F}_{\mathbf{C}}\,t-2} + \frac{\rho_{\mathbf{K}} \Delta^+ \mathbf{K}_{\mathcal{F}_{\mathbf{C}}\,t-2}}{u^* \langle\beta\rangle_{\mathcal{F}_{\mathbf{K}}\,t-2}} - (1-\gamma)\mathbf{K}^e_{t-2} - (1-\gamma)\hat{\mathbf{K}}_{t-2}} - 1\right)\right) \tag{C.3.39}$$

$$\begin{aligned}
\Delta^+ \mathbf{L}^{\mathcal{F}_{\mathbf{C}}}{}_t = \max &\left(\rho_{\mathcal{F}} \frac{\rho_{\mathbf{C}}(1+g-\psi)\mathbf{C}^{\mathcal{F}_{\mathbf{C}}}{}_{t-1}\langle W\rangle_t}{\langle\beta\rangle_{\mathcal{F}_{\mathbf{C}}\,t-1} k_{\mathcal{F}_{\mathbf{C}}}} - \mathbf{D}_{\mathcal{F}_{\mathbf{C}}\,t-1},\right. \\
&\rho_{\mathcal{F}} \left(\frac{\rho_{\mathbf{C}}(1+g-\psi)\mathbf{C}^{\mathcal{F}_{\mathbf{C}}}{}_{t-1}\langle W\rangle_t}{\langle\beta\rangle_{\mathcal{F}_{\mathbf{C}}\,t-1} k_{\mathcal{F}_{\mathbf{C}}}} + p_{\mathbf{K}\,t-1}\left(\frac{\rho_{\mathbf{C}}(1+g-\psi)\mathbf{C}^{\mathcal{F}_{\mathbf{C}}}{}_{t-1}}{u^* \beta_{t-1} k_{\mathcal{F}_{\mathbf{C}}}} - (1-\gamma)\frac{\langle\beta\rangle_{\mathcal{F}_{\mathbf{C}}\,t-1}}{\beta_{t-1}} \mathbf{K}_{\mathcal{F}_{\mathbf{C}}\,t-1}\right)\right) - \\
&\left.\left(\mathbf{D}_{\mathcal{F}_{\mathbf{C}}\,t-1} + (1+\mu_{\mathcal{F}_{\mathbf{C}}\,t})\frac{W^{\mathcal{F}_{\mathbf{C}}}{}_{t-1}}{\mathbf{C}^{\mathcal{F}_{\mathbf{C}}}_{t-1}} \frac{\rho_{\mathbf{C}}(1+g-\psi)\mathbf{C}^{\mathcal{F}_{\mathbf{C}}}{}_{t-1}}{\rho_{\mathbf{C}}}\right)\right)
\end{aligned} \tag{C.3.40}$$

$$\Delta^+ \mathbf{L}^{\mathcal{F}_{\mathbf{K}}}{}_t = \rho_{\mathcal{F}} \left(\rho_{\mathbf{K}}(1+g-\psi)\Delta^+ \mathbf{K}_{\mathcal{F}_{\mathbf{C}}\,t-1} + \frac{\rho_{\mathbf{K}} \Delta^+ \mathbf{K}_{\mathcal{F}_{\mathbf{C}}\,t-1}}{u^* \langle\beta\rangle_{\mathcal{F}_{\mathbf{K}}\,t-1}} - (1-\gamma)\mathbf{K}^e_{t-1} - (1-\gamma)\hat{\mathbf{K}}_{t-1}\right) \frac{\langle W\rangle_t}{\langle\beta\rangle_{\mathcal{F}_{\mathbf{K}}\,t-1}} - \mathbf{D}_{\mathcal{F}_{\mathbf{K}}\,t-1} \tag{C.3.41}$$

$$\langle W\rangle_t = k_4^t \langle W\rangle_0 \tag{C.3.42}$$

$$N_{\mathbf{C}\,t} = \frac{\rho_{\mathbf{C}}(1+g-\psi)\mathbf{C}^{\mathcal{F}_{\mathbf{C}}}{}_{t-1}}{\mathbf{K}_{\mathcal{F}_{\mathbf{C}}\,t-1} \langle\beta\rangle_{\mathcal{F}_{\mathbf{C}}\,t-1} k_{\mathcal{F}_{\mathbf{C}}}} \tag{C.3.43}$$

$$N_{\mathbf{K}\,t} = \frac{\rho_{\mathbf{K}}(1+g-\psi)\Delta^+ \mathbf{K}_{\mathcal{F}_{\mathbf{C}}\,t-1} + \frac{\rho_{\mathbf{K}} \Delta^+ \mathbf{K}_{\mathcal{F}_{\mathbf{C}}\,t-1}}{u^* \langle\beta\rangle_{\mathcal{F}_{\mathbf{K}}\,t-1}} - (1-\gamma)\mathbf{K}^e_{t-1} - (1-\gamma)\hat{\mathbf{K}}_{t-1}}{\mathbf{K}^e_{\mathcal{F}_{\mathbf{K}}\,t-1} \langle\beta\rangle_{\mathcal{F}_{\mathbf{K}}\,t-1}} \tag{C.3.44}$$

$$N_{Q\,t} = N_{Q\,t-1} + \rho_Q \frac{p_{\mathbf{K}\,t-1} \Delta^+ \mathbf{K}_{\mathcal{F}_{\mathbf{C}}\,t-1} - W^{\mathcal{F}_{\mathbf{K}}}{}_{t-1}}{\langle W\rangle_{t-1}} \tag{C.3.45}$$

$$\beta_t = \beta_{t-1}\rho_\beta(1 - e^{-\zeta N_{Q\,t}}) \tag{C.3.46}$$

$$\langle\beta\rangle_{\mathcal{F}_{\mathbf{C}}\,t} = \frac{\langle\beta\rangle_{\mathcal{F}_{\mathbf{C}}\,t-1}(1-\gamma)\mathbf{K}_{\mathcal{F}_{\mathbf{C}}\,t-1} + \beta_{t-1}\Delta^+ \mathbf{K}_{\mathcal{F}_{\mathbf{C}}\,t-1}}{\mathbf{K}_{\mathcal{F}_{\mathbf{C}}\,t}} \tag{C.3.47}$$

$$\langle\beta\rangle_{\mathcal{F}_{\mathbf{K}}\,t} = \frac{\langle\beta\rangle_{\mathcal{F}_{\mathbf{K}}\,t-1}(1-\gamma)\mathbf{K}^e{}_{t-1} + \beta_{t-1}(\mathbf{K}^e{}_t - (1-\gamma)\mathbf{K}^e{}_{t-1})}{\mathbf{K}^e{}_t} \tag{C.3.48}$$

$$\mathbf{B}_t = k_0 Y_t \tag{C.3.49}$$

$$\mathbf{S}_t = \frac{k_1}{1-k_1} \frac{(1-\tau_W)W_{\mathcal{H}\,t} + M_t}{r_{\mathbf{S}\,t}(1-\tau_{\mathbf{S}})} \tag{C.3.50}$$

$$T_t = k_2 Y_t \tag{C.3.51}$$

$$M_t = k_3 Y_t - p_{\mathbf{C}\,t} \mathbf{C}^{\mathcal{G}}{}_t \tag{C.3.52}$$

$$\rho_W = \frac{k_4}{(\omega^* - \omega)} \tag{C.3.53}$$

Where the hypothesis of equilibrium made determine the following inequality constraints

$$\mathbf{D}_{\mathcal{F}_{\mathbf{C}}\,t-1} + \Delta^+ \mathbf{L}^{\mathcal{F}_{\mathbf{C}}}{}_t - W^{\mathcal{F}_{\mathbf{C}}}{}_t + p_{\mathbf{C}\,t} \mathbf{C}^{\mathcal{F}_{\mathbf{C}}}{}_t - p_{\mathbf{K}\,t} \Delta^+ \mathbf{K}_{\mathcal{F}_{\mathbf{C}}\,t} \geq r_{\mathbf{L}} \mathbf{L}^{\mathcal{F}_{\mathbf{C}}}{}_{t-1} \tag{C.3.54}$$

$$\mathbf{D}_{\mathcal{F}_{\mathbf{C}}\,t-1} + \Delta^+ \mathbf{L}^{\mathcal{F}_{\mathbf{C}}}{}_t - W^{\mathcal{F}_{\mathbf{C}}}{}_t + p_{\mathbf{C}\,t} \mathbf{C}^{\mathcal{F}_{\mathbf{C}}}{}_t - p_{\mathbf{K}\,t} \Delta^+ \mathbf{K}_{\mathcal{F}_{\mathbf{C}}\,t} - r_{\mathbf{L}} \mathbf{L}^{\mathcal{F}_{\mathbf{C}}}{}_{t-1} \geq N_{\mathbf{L}}^{-1} \mathbf{L}^{\mathcal{F}_{\mathbf{C}}}{}_{t-1} \tag{C.3.55}$$

$$\mathbf{D}_{\mathcal{F}_{\mathbf{K}}\,t-1} + \Delta^+ \mathbf{L}^{\mathcal{F}_{\mathbf{K}}}{}_t - W^{\mathcal{F}_{\mathbf{K}}}{}_t + p_{\mathbf{K}\,t} \Delta^+ \mathbf{K}_{\mathcal{F}_{\mathbf{C}}\,t} \geq r_{\mathbf{L}} \mathbf{L}^{\mathcal{F}_{\mathbf{K}}}{}_{t-1} \tag{C.3.56}$$

$$\mathbf{D}_{\mathcal{F}_{\mathbf{K}}\,t-1} + \Delta^+ \mathbf{L}^{\mathcal{F}_{\mathbf{K}}}{}_t - W^{\mathcal{F}_{\mathbf{K}}}{}_t + p_{\mathbf{K}\,t} \Delta^+ \mathbf{K}_{\mathcal{F}_{\mathbf{C}}\,t} - r_{\mathbf{L}} \mathbf{L}^{\mathcal{F}_{\mathbf{K}}}{}_{t-1} \geq N_{\mathbf{L}}^{-1} \mathbf{L}^{\mathcal{F}_{\mathbf{K}}}{}_{t-1} \tag{C.3.57}$$

$$\frac{a_y((1-\tau_W)W_{\mathcal{H}\,t-1}+\phi M_{t-1})+a_v(\mathbf{S}_{t-1}+\mathbf{D}_{\mathcal{H}\,t-1})}{(1-\tau_{\mathbf{C}})p_{\mathbf{C}\,t-1}(1+\psi)} \geq \mathbf{C}_{\mathcal{G}\,t} \tag{C.3.58}$$

$$\mathbf{C}^{\mathcal{F}_{\mathbf{C}}}{}_t - \mathbf{C}_{\mathcal{G}\,t} \geq \frac{a_y((1-\tau_W)W_{\mathcal{H}\,t-1}+\phi M_{t-1})+a_v(\mathbf{S}_{t-1}+\mathbf{D}_{\mathcal{H}\,t-1})}{(1-\tau_{\mathbf{C}})p_{\mathbf{C}\,t-1}(1+\psi)} - \mathbf{C}_{\mathcal{G}\,t} \tag{C.3.59}$$

$$\frac{\mathbf{D}_{\mathcal{H}\,t-1}+\mathbf{S}_{t-1}-\mathbf{S}_t+(1-\tau_W)W_{\mathcal{H}\,t}+M_{\mathcal{H}\,t}}{(1+\tau_{\mathbf{C}})p_{\mathbf{C}\,t}} \geq \frac{a_y((1-\tau_W)W_{\mathcal{H}\,t-1}+\phi M_{t-1})+a_v(\mathbf{S}_{t-1}+\mathbf{D}_{\mathcal{H}\,t-1})}{(1-\tau_{\mathbf{C}})p_{\mathbf{C}\,t-1}(1+\psi)} - \mathbf{C}_{\mathcal{G}\,t} \tag{C.3.60}$$

$$\frac{(1+\psi)(1+g)T_{t-1}+(1+g)(1-r_{\mathbf{B}})\delta^* Y_{t-1}-r_{\mathbf{B}}\mathbf{B}_t-\phi M_{t-1}}{(1+\psi)p_{\mathbf{C}\,t-1}} \geq N_{\mathbf{C}\,t}\langle\beta\rangle_{\mathcal{F}_{\mathbf{C}}\,t-1}k_{\mathcal{F}_{\mathbf{C}}} \tag{C.3.61}$$

$$\Delta^+\mathbf{K}^{\mathcal{F}_{\mathbf{K}}}{}_t+(1-\gamma)\hat{\mathbf{K}}_{t-1}-\Delta^+\mathbf{K}^e{}_t \geq \frac{\rho_{\mathbf{C}}(1+g-\psi)\mathbf{C}^{\mathcal{F}_{\mathbf{C}}}{}_{t-1}}{u^*\beta_{t-1}k_{\mathcal{F}_{\mathbf{C}}}} - (1-\gamma)\frac{\langle\beta\rangle_{\mathcal{F}_{\mathbf{C}}\,t-1}}{\beta_{t-1}}\mathbf{K}_{\mathcal{F}_{\mathbf{C}}\,t-1} \tag{C.3.62}$$

$$\frac{\mathbf{D}_{\mathcal{F}_{\mathbf{C}}\,t-1}+\Delta^+\mathbf{L}^{\mathcal{F}_{\mathbf{C}}}{}_t-W^{\mathcal{F}_{\mathbf{C}}}{}_t+p_{\mathbf{C}\,t}\mathbf{C}^{\mathcal{F}_{\mathbf{C}}}{}_t}{p_{\mathbf{K}\,t}} \geq \frac{\rho_{\mathbf{C}}(1+g-\psi)\mathbf{C}^{\mathcal{F}_{\mathbf{C}}}{}_{t-1}}{u^*\beta_{t-1}k_{\mathcal{F}_{\mathbf{C}}}} - (1-\gamma)\frac{\langle\beta\rangle_{\mathcal{F}_{\mathbf{C}}\,t-1}}{\beta_{t-1}}\mathbf{K}_{\mathcal{F}_{\mathbf{C}}\,t-1} \tag{C.3.63}$$

$$\frac{\rho_{\mathbf{C}}(1+g-\psi)\mathbf{C}^{\mathcal{F}_{\mathbf{C}}}{}_{t-1}}{u^*} + (\gamma-1)\langle\beta\rangle_{\mathcal{F}_{\mathbf{C}}\,t-1}k_{\mathcal{F}_{\mathbf{C}}}\mathbf{K}_{\mathcal{F}_{\mathbf{C}}\,t-1} \geq 0 \tag{C.3.64}$$

$$\mathbf{D}_{\mathcal{F}_{\mathbf{C}}\,t-1}+\Delta^+\mathbf{L}^{\mathcal{F}_{\mathbf{C}}}{}_t-W^{\mathcal{F}_{\mathbf{C}}}{}_t+p_{\mathbf{C}\,t}\mathbf{C}^{\mathcal{F}_{\mathbf{C}}}{}_t-p_{\mathbf{K}\,t}\Delta^+\mathbf{K}_{\mathcal{F}_{\mathbf{C}}\,t}-(r_{\mathbf{L}}+N_{\mathbf{L}}^{-1})\mathbf{L}^{\mathcal{F}_{\mathbf{C}}}{}_{t-1} \geq \rho_{\mathcal{F}}(W_{\mathcal{F}_{\mathbf{C}}\,t}+p_{\mathbf{K}\,t}\Delta^+\mathbf{K}_{\mathcal{F}_{\mathbf{C}}\,t}) \tag{C.3.65}$$

$$\mathbf{D}_{\mathcal{F}_{\mathbf{K}}\,t-1}+\Delta^+\mathbf{L}^{\mathcal{F}_{\mathbf{K}}}{}_t-W^{\mathcal{F}_{\mathbf{K}}}{}_t+p_{\mathbf{K}\,t}\Delta^+\mathbf{K}_{\mathcal{F}_{\mathbf{C}}\,t}-(r_{\mathbf{L}}+N_{\mathbf{L}}^{-1})\mathbf{L}^{\mathcal{F}_{\mathbf{K}}}{}_{t-1} \geq \rho_{\mathcal{F}}W_{\mathcal{F}_{\mathbf{K}}\,t} \tag{C.3.66}$$

$$\Delta^+\mathbf{K}^{\mathcal{F}_{\mathbf{K}}}{}_t+(1-\gamma)\hat{\mathbf{K}}_{t-1} \geq \frac{\rho_{\mathbf{K}}\Delta^+\mathbf{K}_{\mathcal{F}_{\mathbf{C}}\,t-1}}{u^*\langle\beta\rangle_{\mathcal{F}_{\mathbf{K}}\,t-1}} - (1-\gamma)\mathbf{K}^e_{t-1} \tag{C.3.67}$$

$$\frac{\rho_{\mathbf{K}}\Delta^+\mathbf{K}_{\mathcal{F}_{\mathbf{C}}\,t-1}}{u^*\langle\beta\rangle_{\mathcal{F}_{\mathbf{K}}\,t-1}} - (1-\gamma)\mathbf{K}^e_{t-1} \geq 0 \tag{C.3.68}$$

$$\rho_{\mathbf{K}}(1+g-\psi)\Delta^+\mathbf{K}_{\mathcal{F}_{\mathbf{C}}\,t-1} + \frac{\rho_{\mathbf{K}}\Delta^+\mathbf{K}_{\mathcal{F}_{\mathbf{C}}\,t-1}}{u^*\langle\beta\rangle_{\mathcal{F}_{\mathbf{K}}\,t-1}} - (1-\gamma)\mathbf{K}^e_{t-1} - (1-\gamma)\hat{\mathbf{K}}_{t-1} \geq 0 \tag{C.3.69}$$

$$\Delta^+\mathbf{L}^{\mathcal{F}_{\mathbf{C}}}{}_t \geq 0 \tag{C.3.70}$$

$$\Delta^+\mathbf{L}^{\mathcal{F}_{\mathbf{K}}}{}_t \geq 0 \tag{C.3.71}$$

$$\mathbf{D}_{\mathcal{F}_{\mathbf{C}}\,t-1}+\Delta^+\mathbf{L}^{\mathcal{F}_{\mathbf{C}}}{}_t-W^{\mathcal{F}_{\mathbf{C}}}{}_t+p_{\mathbf{C}\,t}\mathbf{C}^{\mathcal{F}_{\mathbf{C}}}{}_t-p_{\mathbf{K}\,t}\Delta^+\mathbf{K}_{\mathcal{F}_{\mathbf{C}}\,t}-(r_{\mathbf{L}}+N_{\mathbf{L}}^{-1})\mathbf{L}^{\mathcal{F}_{\mathbf{C}}}{}_{t-1} \geq 0 \tag{C.3.72}$$

$$\mathbf{D}_{\mathcal{F}_{\mathbf{K}}\,t-1}+\Delta^+\mathbf{L}^{\mathcal{F}_{\mathbf{K}}}{}_t-W^{\mathcal{F}_{\mathbf{K}}}{}_t+p_{\mathbf{K}\,t}\Delta^+\mathbf{K}_{\mathcal{F}_{\mathbf{C}}\,t}-(r_{\mathbf{L}}+N_{\mathbf{L}}^{-1})\mathbf{L}^{\mathcal{F}_{\mathbf{K}}}{}_{t-1} \geq 0 \tag{C.3.73}$$

$$N_{\mathcal{H}} \geq N_{\mathbf{C}}+N_{\mathbf{K}}+N_Q \tag{C.3.74}$$

$$N_{\mathbf{C}} \geq 0 \tag{C.3.75}$$

$$N_{\mathbf{K}} \geq 0 \tag{C.3.76}$$

$$N_Q \geq 0 \tag{C.3.77}$$

$$\frac{\mathbf{D}_{\mathcal{F}_{\mathbf{C}}\,t-1}+\Delta^+\mathbf{L}^{\mathcal{F}_{\mathbf{C}}}{}_t}{\langle W\rangle_t} \geq \frac{\rho_{\mathbf{C}}(1+g-\psi)\mathbf{C}^{\mathcal{F}_{\mathbf{C}}}{}_{t-1}}{\mathbf{K}_{\mathcal{F}_{\mathbf{C}}\,t-1}\langle\beta\rangle_{\mathcal{F}_{\mathbf{C}}\,t-1}k_{\mathcal{F}_{\mathbf{C}}}} \tag{C.3.78}$$

$$\frac{\mathbf{D}_{\mathcal{F}_{\mathbf{K}}\,t-1}+\Delta^+\mathbf{L}^{\mathcal{F}_{\mathbf{K}}}{}_t}{\langle W\rangle_t} \geq \frac{\rho_{\mathbf{K}}(1+g-\psi)\Delta^+\mathbf{K}_{\mathcal{F}_{\mathbf{C}}\,t-1}+\frac{\rho_{\mathbf{K}}\Delta^+\mathbf{K}_{\mathcal{F}_{\mathbf{C}}\,t-1}}{u^*\langle\beta\rangle_{\mathcal{F}_{\mathbf{K}}\,t-1}}-(1-\gamma)\mathbf{K}^e_{t-1}-(1-\gamma)\hat{\mathbf{K}}_{t-1}}{\mathbf{K}^e_{\mathcal{F}_{\mathbf{K}}\,t-1}\langle\beta\rangle_{\mathcal{F}_{\mathbf{K}}\,t-1}} + N_{Q\,t-1} + \rho_Q\frac{p_{\mathbf{K}\,t-1}\Delta^+\mathbf{K}_{\mathcal{F}_{\mathbf{C}}\,t-1}-W^{\mathcal{F}_{\mathbf{K}}}{}_{t-1}}{\langle W\rangle_{t-1}} \tag{C.3.79}$$