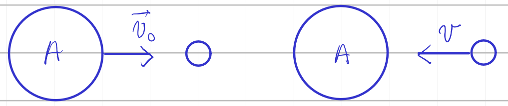
Sadursmes

1



L-sistēmā   A-sistēmā

$M \gg m \Rightarrow$ maza bumbiņa atlēcienā maina ātrumu uz pretējo.

$\vec u = -\vec v$; $\quad \Delta\vec p = m(\vec u - \vec v)$
$\quad = 2m\vec v_0$

2 Sistēma pēc sadursmes apstāsies, ja tās masas centrs pirms sadursmes nekustējās, t.i.

$$m_1\vec v_1 = -m_2\vec v_2 .$$

3 (a)

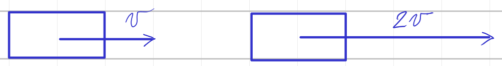
$mv = 2mu$; $\quad u = \frac{v}{2}$

$$Q = \frac{mv^2}{2} - \frac{2mu^2}{2} = \frac{mv^2}{2}\left(1 - \frac{1}{2}\right) = \frac{mv^2}{4}$$

$$\vec v_{com} = \frac{1}{2}\vec v \ ; \qquad \vec v_1' = \vec v - \vec v_{com} = \frac{1}{2}\vec v \ ; \qquad \vec v_2' = -\vec v_{com} = -\frac{1}{2}\vec v$$

$$\vec u' = \vec 0 \ ; \qquad Q' = 2\,\frac{m}{2}\left(\frac{v}{2}\right)^2 = \frac{mv^2}{4} = Q$$

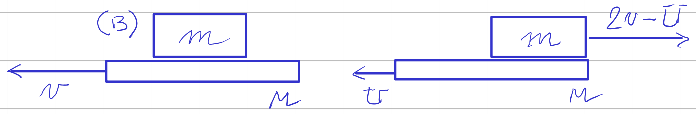
(b)



$$\Delta W^{(L)} = \frac{m(2v)^2}{2} - \frac{mv^2}{2} = \frac{3mv^2}{2}$$

$$\Delta W^{(B)} = \frac{mv^2}{2} - 0 = \frac{mv^2}{2} \neq \Delta W^{(L)}$$

Daļa enerģijas aiziet uz Zemes ātruma izmaiņu.



$$\begin{cases} Mv = MU + mU - 2mv \\ Mv^2 = MU^2 + 4mv^2 + mU^2 - 4mvU - 2Q \end{cases}$$

$$U = v\,\frac{M+2m}{M+m}$$

$$(4m - M)v^2 + (M+m)v^2\left(\frac{M+2m}{M+m}\right)^2 - 4mv^2\,\frac{M+2m}{M+m} = 2Q$$

$$Q = \frac{3}{2}\,\frac{mM}{m+M}v^2 \xrightarrow[M\to\infty]{} \frac{3\,mv^2}{2} \overset{!}{=} \Delta W^{(L)}$$

4 Vienai daļiņai būs lielākais impulss, ja tā lidos $\vec v_0$ virzienā, bet daļiņas 2 un 3 — pretēji $\vec v_0$ virzienam. Pēc simetrijas $u_2 = u_3$.

$$\begin{cases} 3\cancel{m}v_0 = \cancel{m}u_1 - 2\cancel{m}u_2 \\ 3\eta\cancel{m}v_0^2 = \cancel{m}u_1^2 + 2\cancel{m}u_2^2 \end{cases} \rightsquigarrow \quad u_2 = (u_1 - 3v_0)\frac{1}{2}, \quad 3\eta v_0^2 = u_1^2 + \frac{1}{2}(u_1 - 3v_0)^2$$

$$u_1^2 - 2v_0u_1 + (3-2\eta)v_0^2 = 0$$

$$\underline{\underline{u_1 = v_0\left[1 + \sqrt{2(\eta-1)}\right]}}$$

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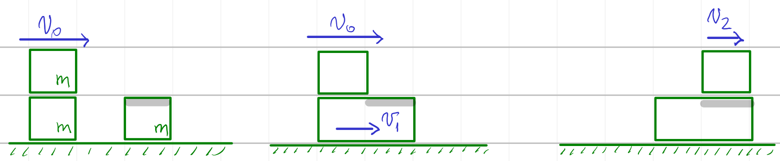


$0 \to 1$: $\quad 2mv_0 = 2mv_1 + mv_0 \rightsquigarrow v_1 = \frac{1}{2}v_0$

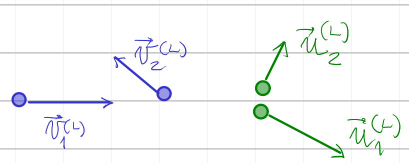
$1 \to 2$: $\quad 2mv_0 = 3mv_2 \rightsquigarrow v_2 = \frac{2}{3}v_0$

$$3mv_2^2 - mv_0^2 - 2mv_1^2 = 2A_\mu$$

$$\frac{4\cancel{m}v_0^2}{3} - \cancel{m}v_0^2 - \frac{\cancel{m}v_0^2}{2} = -2\int_0^L \frac{\mu m g x}{L}\,dx = -\mu\cancel{m}gL$$

$$\underline{\underline{L = \frac{v_0^2}{6\mu g}}}$$

6 Sākumā apskatīsim absolūti elastīgu sadursmi vispārīgā formā. To ir ērti darīt ar masu centru saistītajā sistēmā (C-sistēmā). Sākotnējie dati parasti ir doti laboratorijas L-sistēmā.



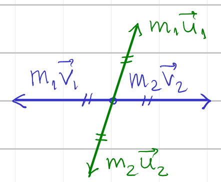
$$\vec v^{(C)} = \vec v^{(L)} - \vec v_{com}$$

$$\vec v_{com} = \frac{m_1\vec v_1 + m_2\vec v_2}{m_1 + m_2} = \vec u_{com}$$

$$(C): \begin{cases} m_1\vec v_1 = -m_2\vec v_2 \\ m_1\vec u_1 = -m_2\vec u_2 \\ m_1v_1^2 + m_2v_2^2 = m_1u_1^2 + m_2u_2^2 \end{cases}$$

$$\Rightarrow v_1^2 = u_1^2 \quad \wedge \quad v_2^2 = u_2^2$$

T.i. C-sistēmā sadursme izskatās šādi:



Centrālās (1D) absolūti elastīgas sadursmes gadījumā C-sistēmā

$$\begin{cases} v_{1x} = -u_{1x} \\ v_{2x} = -u_{2x} \end{cases}$$

Pārejot uz L-sistēmu,

$$\begin{cases} v_{1x}^{(L)} - v_{com\,x} = -\left(u_{1x}^{(L)} - v_{com\,x}\right) \\ v_{2x}^{(L)} - v_{com\,x} = -\left(u_{2x}^{(L)} - v_{com\,x}\right) \end{cases}$$

$$\begin{cases} u_{1x}^{(L)} = 2v_{com\,x} - v_{1x}^{(L)} = \dfrac{2m_1 v_{1x}^{(L)} + 2m_2 v_{2x}^{(L)}}{m_1+m_2} - v_{1x}^{(L)} \\ \qquad = \dfrac{m_1-m_2}{m_1+m_2} v_{1x}^{(L)} + \dfrac{2m_2}{m_1+m_2} v_{2x}^{(L)} \\ u_{2x}^{(L)} = \dfrac{m_2-m_1}{m_1+m_2} v_{2x}^{(L)} + \dfrac{2m_1}{m_1+m_2} v_{1x}^{(L)} \end{cases}$$

Atgriežoties pie uzdevuma, pēc pirmās sadursmes lodītes apmainīsies ar impulsiem C-sistēmā. Pēc otrās apmaiņas impulsi būs vienādi ar sākotnējiem. Tātad,

$$\begin{cases} u_1^{(2N+1)} = \left|\dfrac{m_1-m_2}{m_1+m_2} v_{1x} + \dfrac{2m_2}{m_1+m_2} v_{2x}\right| \\ u_2^{(2N+1)} = \left|\dfrac{m_2-m_1}{m_1+m_2} v_{2x} + \dfrac{2m_1}{m_1+m_2} v_{1x}\right| \\ u_1^{(2N)} = v_1 \\ u_2^{(2N)} = v_2 \end{cases}$$

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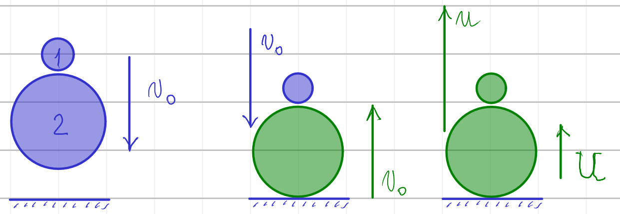


(a) $W_1' = W \Rightarrow U = 0$

$$\begin{cases} (m_2 - m_1) v_0 = m_1 u \\ (m_1 + m_2) v_0^2 = m_1 u^2 \end{cases}$$

$$\frac{m_2^2 + m_1^2 - 2m_1 m_2}{m_1+m_2} = m_1$$

$$1 + \cancel{\mu^2} - 2\mu = \cancel{\mu^2} + \mu \rightsquigarrow \underline{\underline{\mu = \frac{1}{3}}}$$

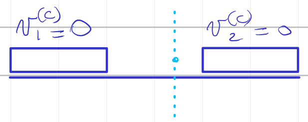
(b) $$u_1 = \left(\frac{1-\mu}{1+\mu} + \frac{2}{1+\mu}\right) v_0 = \frac{3-\mu}{1+\mu} = \left(-1 + \frac{4}{1+\mu}\right) v_0 \longrightarrow \max$$

$\underline{\underline{\mu \longrightarrow 0}}$

8 Apskatīsim uzdevumu c-sistēmā.

$$\vec a_{com} = \frac{\vec F}{m_1+m_2}; \qquad \vec a_1^{(c)} = \vec a_1^{(L)} - \vec a_{com} = \frac{\vec F m_2}{m_1(m_1+m_2)}$$

$$\vec a_2^{(c)} = \vec a_2^{(L)} - \vec a_{com} = \frac{-\vec F}{m_1+m_2}$$



Atsitiena momentā ātrums mainās uz pretējo. Paātrinājums ir konstants. Salīdzinot ar brīvu krišanu ar atsitienu, var secināt, ka $t_\rightarrow = t_\leftarrow = \tau$, un tātad $\underline{\underline{\Delta t = 2\tau}}$.

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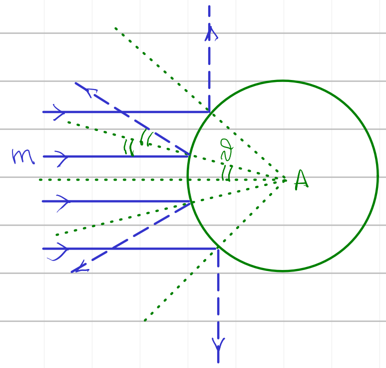

A-sistēmā $\Delta p_x = mv(\cos 2\vartheta + 1)$

$$\Delta P_x = nv\Delta t S_\perp \cdot \Delta p_x$$
$$= nmv^2 \Delta t \pi r^2 \langle \cos 2\vartheta + 1 \rangle_{-\pi/2}^{+\pi/2}$$
$$= nmv^2 \Delta t \pi r^2$$

$$F_{Dx} = \frac{\Delta P_x}{\Delta t} = \underline{\underline{\rho v^2 \cdot \pi r^2}}$$

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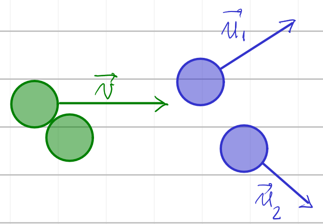

$$\begin{cases} \vec v = \vec u_1 + \vec u_2 \\ v^2 = u_1^2 + u_2^2 \end{cases}$$

$$\cancel{u_1^2} + \cancel{u_2^2} + 2\vec u_1 \vec u_2 = \cancel{u_1^2} + \cancel{u_2^2}$$
$$2\vec u_1 \vec u_2 = 0 \implies \angle(\vec u_1, \vec u_2) = \underline{\underline{\pi/2}}$$

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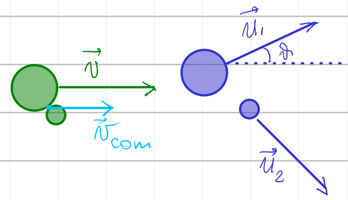
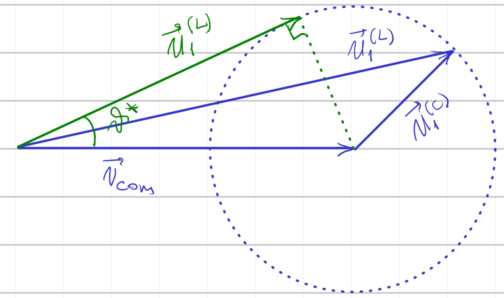


C-sistēmā $u_1^{(c)} = v_1^{(c)}$ un $\vec u_1^{(c)}$ var būt vērsts patvaļīgi.



$$\sin\vartheta^* = \frac{u_1^{(c)}}{v_{com}} = \frac{v_1^{(c)}}{v_{com}}$$
$$= \frac{v - v_{com}}{v_{com}} = \frac{v}{v_{com}} - 1$$
$$= \frac{\cancel{v}(m_1+m_2)}{m_1 \cancel{v}} - 1 = \frac{m_2}{m_1}$$

$$\underline{\underline{\vartheta^* = \arcsin \frac{m_2}{m_1}}}$$

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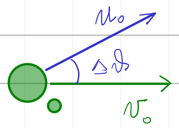

No iepriekšējā uzdevuma maksimālajam izkliedes leņķim $\sin\vartheta^* = m/m_0$. Tātad,

$$\frac{\pi}{N} \le \frac{m}{m_0} = \frac{M}{N m_0} \rightsquigarrow \underline{\underline{\frac{M}{m_0} \ge \pi}}$$

$$v_0^{(c)\,2} + u_0^2 = v_{com}^2$$

$$v_0^{(c)} = v_0 - v_0 \frac{m_0}{m_0+m} = \frac{v_0 m}{m_0+m} \quad ; \quad v_{com} = \frac{v_0 m_0}{m_0+m}$$

$$u_0^2 = \frac{v_0^2 m_0^2}{(m_0+m)^2} - \frac{v_0^2 m^2}{(m_0+m)^2} = v_0^2 \frac{m_0^2 - m^2}{(m_0+m)^2} = v_0^2 \frac{m_0 - m}{m_0+m}$$

$$u_0 \xrightarrow[m\to 0]{} v_0\left(1 - \frac{m}{m_0}\right) \; ; \qquad \Delta v_0 = -v_0\, m/m_0 = -v_0\, \Delta\vartheta$$

$$\frac{dv_0}{v_0} = -d\vartheta \rightsquigarrow \underline{\underline{v = v_0 e^{-\pi}}}$$