Svārstības

2023-12-30

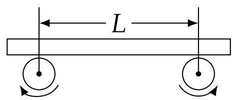
Iesildīšanās

1° Nostieptas stīgas vidū ir iestiprināta maza lodīte ar masu m. Stīgas garums ir L, sastiepuma spēks F. Lodīti novirza par attālumu $x_0 \ll L$ perpendikulāri stīgai. Aprakstiet lodītes kustību pēc tam, kad to atlaiž. Cik liels būs lodītes maksimālais kustības ātrums?

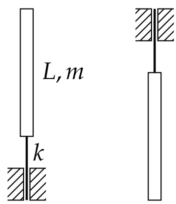
Harmoniskās svārstības

2° Divas vienādas lodītes ir savienotas ar nedeformējamu stieni, kura garums ir 2ℓ, un uzlādētas ar pretējiem pēc zīmes un vienādiem pēc moduļa lādiņiem $\pm q$. Šī sistēma ir ievietota elektriskajā laukā, kas iedarbojas uz lodītēm ar spēku $\pm q\boldsymbol{E}$. Nosakiet (a) kā ir orientēts stienis līdzsvara stāvoklī; (b) vienas lodītes masu, ja mazu svārstību ap līdzsvara stāvokli laikā lodītes maksimālā novirze no līdzsvara ir x_0 un maksimālais ātrums ir v_0.

3° Dēlis, kura masa ir m, ir nolikts uz diviem vienādiem rullīšiem, kas griežas ar lieliem leņķiskajiem ātrumiem pretējos virzienos. Attālums starp rullīšu asīm ir L, slīdes berzes koeficients starp rullīšiem un dēli ir μ. Nosakiet dēļa garenisko svārstību frekvenci.



4° Homogēna tieva stieņa, kura garums ir L un masa m, galā ir piestiprināta maza elastīga plāksnīte. Plāksnīti iestiprina spīlēs tā, ka stienis atrodas vertikāli: pirmajā gadījumā uz augšu, otrajā — uz leju. Cik liela ir mazu svārstību periodu attiecība abos gadījumos? Lieces moments plāksnītē $M = k\varphi$, kur k ir zināms koeficients un φ ir nolieces leņķis.

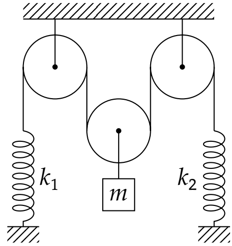


5° One end of a uniform spring of spring constant k and total mass m is attached to the wall, and the other end is attached to a mass M. Show that when $m \ll M$, the oscillation frequency is approximately

$$\omega = \sqrt{\frac{k}{M + m/3}}.$$

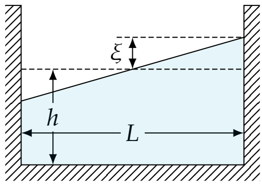
Vispārinātās koordinātas

6° The system shown in the figure consists of a mass m, two ideal springs with spring constants k_1 and k_2 and three massless frictionless pulleys. An unstretchable massless cable goes around the pulleys and its ends are attached to the springs. Segments of the cable that are not touching pulleys, as well as axes of springs, are vertical. Determine the maximum possible amplitude A so that the vertical oscillations of the mass are harmonic.



7° Suppose a particle is constrained to move on a curve $y(x)$ with a minimum at $x = 0$. We know that if $y(x)$ is a circular arc, then the motion is not exactly simple harmonic, for the same reason that pendulum motion is not. Find a differential equation relating y' and y, so that the motion is exactly simple harmonic for arbitrary amplitudes; you don't have to solve it. (Hint: work in terms of the coordinate s, the arc length along the curve.)

8° (*IPhO 1984, Q2*) In certain lakes there is a strange phenomenon called 'seiching' which is an oscillation of the water. Lakes in which you can see this phenomenon are normally long compared with the depth and also narrow. It is natural to see waves in a lake but not something like the seiching, where the entire water volume oscillates, like the coffee in a cup that you carry to a waiting guest.

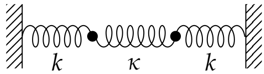


h/mm	T/s
30	1,78
50	1,40
69	1,18
88	1,08
107	1,00
124	0,91
142	0,82

In order to create a model of the seiching we look at water in a rectangular container. The length of the container is L and the depth of the water is h. Assume that the surface of the water to begin with makes a small angle with the horizontal. The seiching will then start, and we assume that the water surface continues to be plane but oscillates around an axis in the horizontal plane and located in the middle of the container. Create a model of the movement of the water and derive a formula for the oscillation period T. Initial conditions are given in figure. Assume that $\xi \ll h$. The table below the figure show experimental oscillation periods for different water depths in a container of length $L = 479\,\mathrm{mm}$. Check – in some reasonable way – how well the formula that you have derived agrees with the experimental data.

Normālās modas

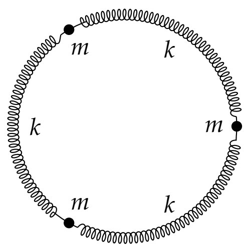
9° Three springs and two equal masses lie between two walls, as shown. The spring constant k of the two outside springs is much larger than the spring constant $\kappa \ll k$ of the middle spring. Let x_1 and x_2 be the positions of the left and right masses, respectively, relative to their equilibrium positions. If the initial positions are given by $x_1(0) = A$ and $x_2(0) = 0$, and if both masses are released from rest, show that



$$x_1(t) \approx A\cos[(\omega+\varepsilon)t]\cos\varepsilon t, \quad x_2(t) \approx A\sin[(\omega+\varepsilon)t]\sin\varepsilon t; \quad \omega = \sqrt{\frac{k}{m}}, \quad \varepsilon = \frac{\omega\kappa}{2k}.$$

Explain qualitatively what the motion looks like. This is an example of beats, which result from superposition two oscillations of nearly equal frequencies.

10° (*IPhO 1986, Q3*) N identical masses m are constrained to move on a horizontal circular hoop connected by N identical springs with spring constant k. The setup for $N = 3$ is shown in the figure. Displacement of n-th mass from equilibrium position along the hoop is s_n.



(a) Find normal modes and their frequencies for $N = 2$.

(b) Do the same for $N = 3$.

(c) For arbitrary N, write down the equations of motion and show that the ℓ-th normal mode is given by

$$\xi_\ell(t) = A_\ell \sum_{n=1}^{N} s_n(t) = A_\ell \sum_{n=1}^{N} \exp \mathrm{i}\left(\omega_\ell t + \frac{2\pi\ell n}{N}\right).$$

Determine the frequencies ω_ℓ.

(d) If one of the masses is replaced with a mass $m' \ll m$, estimate any major change one would expect to occur to the frequency distribution (spectrum).

(e) Describe the form of the spectrum one would predict for a diatomic chain with alternate masses m and m' on the basis of the previous result.

Adiabatiskās izmaiņas

11° A mass m oscillates with amplitude A_0 on a spring with spring constant k_0. Over a very long period of time, the spring smoothly and continuously weakens until its spring constant is $k_0/2$. Find the new amplitude of oscillations.