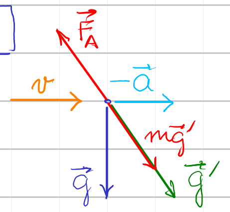
Šķidrumi un gāzes

1



Neinerciālajā AS darbojas efektīvais brīvās krišanas paātrinājums $\vec g' = \vec g - \vec a$.
Efektīvais smaguma spēks ir $\vec F_g' = m\vec g'$
Arhimēda spēks $\vec F_A = -\varrho_0 \vec g' V$
Sastiepuma spēks kompensē $\vec F_A + m\vec g'$. Tātad,
$\vec T = \varrho_0 \vec g' V - m\vec g' = [\varrho_0 V > m] \uparrow\uparrow \vec g$, kas nozīmē, ka diegs noliecsies pretēji kustības virzienam.

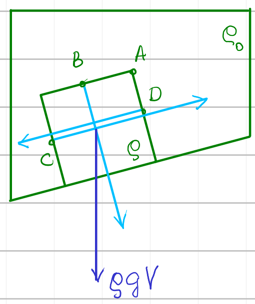
2

$$dp = -\varrho g dz = -\frac{apg\,dz}{T_0 - bz} \rightsquigarrow \int_{p_0}^{p} \frac{d\tilde p}{\tilde p} = -ag\int_0^z \frac{d\tilde z}{T_0 - b\tilde z}$$

$$\ln\frac{p}{p_0} = \frac{ag}{b}\ln\frac{T_0 - bz}{T_0} \rightsquigarrow p = p_0\left(1 - \frac{bz}{T_0}\right)^{\frac{ag}{b}}$$

$$z = H \Rightarrow p = 0 \Rightarrow z = T_0/b$$

3



Spiediens ar dziļumu pieaug lineāri.

$$p_A = \varrho_0 g h + p_0$$

$$p_B = \varrho_0 g\left(h + \tfrac{1}{2}a\sin\varphi\right) + p_0$$

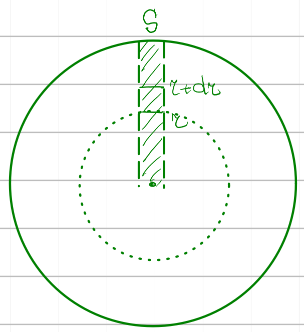
$$p_C = \varrho_0 g\left(h + a\sin\varphi + \tfrac{1}{2}a\cos\varphi\right) + p_0$$

$$p_D = \varrho_0 g\left(h + \tfrac{1}{2}a\cos\varphi\right) + p_0$$

$$F_\perp = \varrho g a^3\cos\varphi + a^2 p_B = \varrho_0 g a^3\left(\frac{\varrho}{\varrho_0}\cos\varphi + \frac{h}{a} + \frac{1}{2}\sin\varphi\right) + p_0 a^2$$

$$F_\parallel = \varrho g a^3 - a^2(p_C - p_D) = (\varrho - \varrho_0) g a^3 \sin\varphi$$

4



$$g = G\frac{4\pi r^3}{3r^2}\varrho = G\varrho\frac{4\pi r}{3}$$

$$dp = -\varrho g\,dr = -\frac{4\pi G\varrho^2}{3} r\,dr$$

$$p(r) = \frac{2\pi G\varrho^2 r^2}{3}$$

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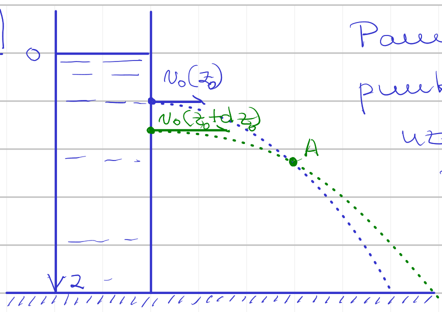

Pamainot sākuma augstumu pa dz_0, punkts, kas atrodas uz apliecējas, paliek uz vietas (ekstrēma nosacījums).

Pēc Bernulli vienādojuma $\frac{1}{2}\rho v_0^2 = \rho g z_0$ un $v_0(z) = \sqrt{2g z_0}$.

Trajektorijas v-jums:

$$\begin{cases} x(t) = v_0 t \\ z(t) = z_0 + \frac{1}{2} g t^2 \end{cases} \rightsquigarrow z(x) = z_0 + \frac{g x^2}{2 v_0^2}.$$

P-tā A $$\cancel{z_0} + \frac{gx^2}{4g z_0} = (\cancel{z_0} + dz_0) + \frac{gx^2}{4g(z_0+dz_0)}$$

$$\frac{x^2}{z_0} = \frac{4z_0 dz_0 + x^2}{z_0 + dz_0} \rightsquigarrow \cancel{x^2 z_0} + x^2 \cancel{dz_0} = 4z_0^2 \cancel{dz_0} + \cancel{x^2 z_0} \qquad [x \geq 0;\ z \geq 0]$$

$$x = 2z_0.$$

Ņemot vērā cilindrisku simetriju, prasītais telpas apgabals ir nošķeltais konuss ar asi, kas sakrīt ar trauka asi, augstumu H un pamatu rādiusiem $R_1 = R$ un $R_2 = R + 2H$.

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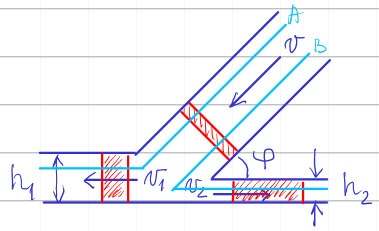

Plūsmas nepārtrauktība:

$$vh = v_1 h_1 + v_2 h_2$$

Apskatīsim iesvītrota plūsmas elementu.

Impulsa horizontālās komp. saglabāšanās (laika vienība, saīsinot ar blīvumu un strūklas platumu):

$$vh \cdot v\cos\varphi = v_1 h_1 \cdot v_1 - v_2 h_2 \cdot v_2 \rightsquigarrow v^2 h \cos\varphi = v_1^2 h_1 - v_2^2 h_2$$

Enerģijas saglabāšanās:

$$vh \cdot v^2 = v_1 h_1 \cdot v_1^2 + v_2 h_2 v_2^2 \rightsquigarrow v^3 h = v_1^3 h_1 + v_2^3 h_2$$

Bernulli v-jums līnijai A: $\cancel{p_0} + \rho v^2/2 + \cancel{\rho g z_A}^{\,0} = \cancel{p_0} + \rho v_1^2/2$

līnijai B: $\cancel{p_0} + \rho v^2/2 + \cancel{\rho g z_B}^{\,0} = \cancel{p_0} + \rho v_2^2/2$

$$\rightsquigarrow v = v_1 = v_2.$$

Kopā sanāk

$$\begin{cases} h = h_1 + h_2 \\ h\cos\varphi = h_1 - h_2 \end{cases} \rightsquigarrow \begin{aligned} h_1 &= \tfrac{1}{2} h (1+\cos\varphi) \\ h_2 &= \tfrac{1}{2} h (1-\cos\varphi) \end{aligned}$$

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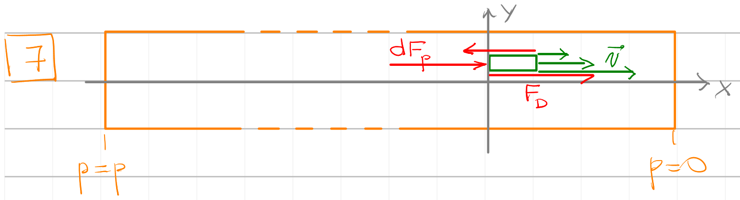

$$dF_p = a\,dy\,dp$$

$$F_D(y) = \eta \left.\frac{dv}{dy}\right|_y \cdot a\,dx \; ; \quad F_D(y+dy) = \eta \left.\frac{dv}{dy}\right|_{y+dy} \cdot a\,dx$$

$$a\,\cancel{dy}\,dp = \eta\, a\,dx \left(\left.\frac{dv}{dy}\right|_{y+dy} - \left.\frac{dv}{dy}\right|_y \right) = \eta\,\cancel{a}\,dx\,\frac{d^2v}{dy^2}\,\cancel{dy}$$

$$\frac{dp}{dx} = \eta \frac{d^2v}{dy^2} \underset{\int dy}{\rightsquigarrow} \frac{dp}{dx}y + C_1 = \eta\frac{dv}{dy} \underset{\int dy}{\rightsquigarrow} \frac{dp}{dx}\frac{y^2}{2} + C_1 y + C_2 = \eta v$$

$$v(b/2) = v(-b/2) = 0 \Rightarrow C_1 = 0 \; ; \; C_2 = -\frac{1}{8}\frac{dp}{dx}b^2.$$

Tātad ātruma profils ir parabolisks:

$$v(y) = -\frac{1}{8}\frac{dp}{dx}\left(b^2 - 4y^2\right).$$

Plūsma bez sieniņas:

$$Q = \int_{-b/2}^{+b/2} \frac{4y^2 - b^2}{8}\frac{dp}{dx}\,a\,dy = \frac{a}{4}\frac{dp}{dx}\left(\frac{b^3}{6} - b^3\right) = -\frac{5ab^3}{24}\frac{dp}{dx}$$

Ar sieniņu

$$Q' = 2Q\left(b \to \frac{b}{2}\right) = \left[Q \sim b^3\right] = \frac{2Q}{2^3} = \underline{\underline{\frac{Q}{4}}}$$

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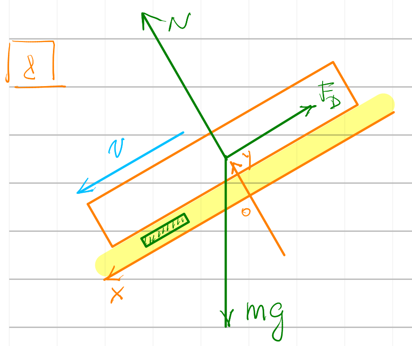

Ripas apakšējā virsma kustas ar ātrumu v, ar tādu pašu ātrumu kustas arī smēres augšējais slānis. Smēres apakšējais slānis ir kontaktā ar virsmu, tātad nekustas.

Smēres elements kustas vienmērīgi:

$$\left.\frac{dv}{dy}\right|_y = \left.\frac{dv}{dy}\right|_{y+dy} \rightsquigarrow \frac{dv}{dy} = C \rightsquigarrow v(y) = Cy.$$

Ripa kustas vienmērīgi, tātad

$$mg\sin\varphi = \eta\frac{v}{b}\pi R^2 \rightsquigarrow \underline{\underline{v = \frac{mgb\sin\varphi}{\eta\pi R^2}}}$$