Datu apstrāde

Atrisinājumi

1° According to the second Newton's law, $M = J\alpha$, where $M = mg(x + L)$ and $J = m(x^2 + L^2)$. When quantity α reaches its extremum, $\mathrm{d}\alpha/\mathrm{d}x = 0$. Thus

$$\frac{\mathrm{d}\alpha}{\mathrm{d}x} = g\frac{\mathrm{d}}{\mathrm{d}x}\frac{x+L}{x^2+L^2} = g\frac{(x^2+L^2) - 2x(x+L)}{(x^2+L^2)^2} = 0,$$
$$x^{*2} + 2Lx^* - L^2 = 0,$$
$$x^* = -L \pm \sqrt{L^2 + L^2} = L\left(\sqrt{2} - 1\right).$$

2° Partial derivatives due to symmetry will be identical.

$$\begin{aligned}\frac{\partial V}{\partial x} &= -GMm \cdot \left(-\frac{1}{2}\right)\left(x^2+y^2+z^2\right)^{-3/2} \cdot 2x \\ &= GMm\frac{x}{\left(x^2+y^2+z^2\right)^{3/2}}, \\ \frac{\partial V}{\partial y} &= GMm\frac{y}{\left(x^2+y^2+z^2\right)^{3/2}}, \\ \frac{\partial V}{\partial x} &= GMm\frac{z}{\left(x^2+y^2+z^2\right)^{3/2}}.\end{aligned}$$

3° As changes are small relative to the actual values, we can use a first-order approximation $(1+x)^\alpha \doteq 1 + \alpha x$. Then

$$\begin{aligned}(R_1 + \Delta R)^{-1} &= R_1^{-1}\left(1 + \frac{\Delta R}{R_1}\right)^{-1} \doteq R_1^{-1}\left(1 - \frac{\Delta R}{R_1}\right), \\ (R_2 + \Delta R)^{-1} &\doteq R_2^{-1}\left(1 - \frac{\Delta R}{R_2}\right), \\ R^{-1} &\doteq R_1^{-1}\left(1 - \frac{\Delta R}{R_1}\right) + R_2^{-1}\left(1 - \frac{\Delta R}{R_2}\right) \\ &= 1\,\Omega^{-1} \cdot 0{,}99 + 0{,}50\,\Omega^{-1} \cdot 0{,}995 \\ &= 1{,}4875\,\Omega^{-1} \\ R &= 0{,}672\,\Omega.\end{aligned}$$

4° Uncertainties can be estimated as a Pythagorean sum of partial uncertainties as long as uncertainties of parameters are relatively small.

$$\Delta f = \frac{k}{x} \cdot \Delta x$$

$$\Delta f = \sqrt{\left(\ln y \cdot \Delta x\right)^2 + \left(\frac{x}{y} \cdot \Delta y\right)^2}$$

$$\Delta f = C\gamma \mathrm{e}^{\gamma x} \cdot \Delta x$$

$$\begin{aligned}\Delta f &= \sqrt{\left(\frac{-2xy}{(x^2+y^2)^2} \cdot \Delta x\right)^2 + \left(\frac{x^2+y^2-2y^2}{(x^2+y^2)^2} \cdot \Delta y\right)^2} \\ &= \frac{\sqrt{\left(2xy \cdot \Delta x\right)^2 + \left[(x^2-y^2) \cdot \Delta y\right]^2}}{(x^2+y^2)^2}\end{aligned}$$

5° The volume is given by the integral

$$\begin{aligned}\iint f(x,y)\,\mathrm{d}x\mathrm{d}y &= \begin{bmatrix} x = \rho\cos\varphi \\ y = \rho\sin\varphi \\ \dfrac{\partial(x,y)}{\partial(\rho,\varphi)} = \rho \end{bmatrix} = \iint \mathrm{e}^{-\rho^2} \rho\,\mathrm{d}\rho\mathrm{d}\varphi \\ &= 2\pi \cdot \frac{1}{2}\int_0^\infty \mathrm{e}^{-\rho^2}\,\mathrm{d}\rho^2 = -\pi\mathrm{e}^{x}\Big|_0^{-\infty} = \pi.\end{aligned}$$

6° Parameters for both datasets:

$$\begin{aligned}\mu &= 20{,}9\,\mathrm{m} \\ s &= 4{,}4\,\mathrm{m} \\ \Delta L &= 1{,}1\,\mathrm{m}\end{aligned} \qquad \begin{aligned}\mu &= 5{,}39\,\mathrm{V} \\ s &= 0{,}60\,\mathrm{V} \\ \Delta U &= 0{,}15\,\mathrm{V}\end{aligned}$$

7° Linearisation parameters are A and B so that there is a linear relation between new variables: $y = Ax + B$.

(a) $H = \frac{1}{2}gt^2 \rightsquigarrow H = \left(\frac{1}{2}g\right)t^2$, i. e. $A = \frac{1}{2}g$ and $B = 0$. Then $g = 2A$.

(b) $a = g\sin\phi \rightsquigarrow a = g\sin\phi$, i. e. $A = g$ and $B = 0$. Then $g = A$.

(c) $T = 2\pi\sqrt{\frac{L}{g}} \rightsquigarrow T^2 = \frac{4\pi^2}{g}L$, i. e. $A = \frac{4\pi^2}{g}$ and $B = 0$. Then $g = \frac{4\pi^2}{A}$.

(d) $U = \frac{\mathscr{E}r}{R+r} \rightsquigarrow \frac{1}{U} = \frac{1}{\mathscr{E}r}R + \frac{1}{\mathscr{E}}$, i. e. $A = \frac{1}{\mathscr{E}r}$ and $B = \frac{1}{\mathscr{E}}$.
Then $\mathscr{E} = \frac{1}{B}$ and $r = \frac{B}{A}$.

(e) $m = \mu_\mathrm{i} L_\mathrm{i} + \mu_\mathrm{v} L_\mathrm{v} \rightsquigarrow \frac{L_\mathrm{i}}{m} = -\frac{\mu_\mathrm{v}}{\mu_\mathrm{i}} \frac{L_\mathrm{v}}{m} + \frac{1}{\mu_\mathrm{i}}$, i. e. $A = -\frac{\mu_\mathrm{v}}{\mu_\mathrm{i}}$ and $B = \frac{1}{\mu_\mathrm{i}}$.
Then $\mu_\mathrm{i} = \frac{1}{B}$ and $\mu_\mathrm{v} = -\frac{A}{B}$.

(f) $X = X_0 \cdot 10^{\lambda t} \rightsquigarrow \lg X = \lg X_0 + \lambda t$, i. e. $A = \lambda$ and $B = \lg X_0$.
Then $\lambda = A$ and $X_0 = 10^B$.

(g) $E = E_0 r^\gamma \rightsquigarrow \ln E = \ln E_0 + \gamma \ln r$, i. e. $A = \gamma$ and $B = \ln E_0$.
Then $E_0 = \exp B$ and $\gamma = A$.

8° First of all, we linearise the $Y(X)$ function by introducing new variables $y = \ln Y$ and $x = \ln X$ to get $y = kx$. The new data are then

X	Y	x	y	Δy
16,0	104,4	2,773	4,648	0,192
18,0	146,1	2,890	4,984	0,137
20,0	172,8	2,996	5,152	0,116
21,0	174,9	3,045	5,164	0,114
25,0	248,0	3,219	5,513	0,081
26,0	256,4	3,258	5,547	0,078
28,0	287,5	3,332	5,661	0,070
29,0	314,3	3,367	5,750	0,064
32,0	357,0	3,466	5,878	0,056
35,0	431,6	3,555	6,067	0,046

where uncertainty of y is $\Delta y = \Delta Y / Y$. After statistical analysis, $k = 1{,}68 \pm 0{,}12$. As $\chi^2/\mathrm{dof} = 0{,}13 \ll 1$, we can conclude that uncertainties are overestimated.

9° As all measurements have different precision, the mean value will be given by the weighted average with the weights being reciprocals of corresponding variances. Thus,

$$L = \frac{\sum_i w_i L_i}{\sum_i w_i} = \frac{20^2 \cdot 2{,}0 + 100^2 \cdot 2{,}1 + 100^2 \cdot 1{,}9}{20^2 + 100^2 + 100^2} = 2{,}00\,\mathrm{m}.$$

Variance is a linear function, so the variance of L is

$$s_L^2 = \frac{\sum_i w_i s_i^2}{\sum_i w_i} = \frac{N}{\sum_i w_i}$$

and the standard error

$$s_{\bar{L}} = \sqrt{\frac{s_L^2}{N}} = \sqrt{\frac{1}{\sum_i w_i}} = \sqrt{\frac{1}{20^2 + 100^2 + 100^2}} = 0{,}007\,\mathrm{m}.$$

Thus, the result of the set of measurements is $L = (2{,}000 \pm 0{,}007)\,\mathrm{m}$.

10° Along the lines of the previous problem,

$$s_{\bar{L}} = \lim_{N\to\infty} \sqrt{\frac{1}{\sum_i w_i}} = \lim_{N\to\infty} \sqrt{\frac{1}{s_1 \sum_{i=0}^{N-1} (1/5)^i}} = \sqrt{\frac{1 - 1/5}{0{,}1}} = 2{,}8\,\mathrm{m}.$$

11° $L = (5897 \pm 87)\,\mathrm{cm}$, $U = (589 \pm 23) \times 10^{12}\,\mathrm{V}$.