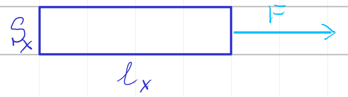
Līdzsvars un elastība

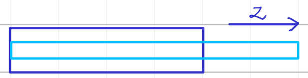
1



$$\frac{F_x}{S_x} = E\varepsilon_x$$

$$W_p = \frac{F_x x}{2} = \frac{ES_x\varepsilon_x \cdot l_x\varepsilon_x}{2} = \frac{EV\varepsilon_x^2}{2} \Rightarrow w_p = \frac{W_p}{V} = \frac{E\varepsilon^2}{2}$$

2



$$V = l_x l_y l_z = (1+\varepsilon_x)l_x(1+\varepsilon_y)l_y(1+\varepsilon_z)l_z$$

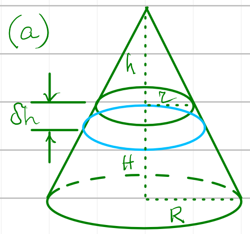
$$1 \doteq 1+\varepsilon_x+\varepsilon_y+\varepsilon_z$$

Simetrijas dēļ $\varepsilon_x = \varepsilon_y$ un tad $\varepsilon_z = -2\varepsilon_x$.

$$\nu = -\varepsilon_x/\varepsilon_z = \frac{1}{2}$$

3 Virtuālo pārvietojumu metode: veic virtuālo mazu pārvietojumu δx un pielieto enerģijas sagl. likumu.

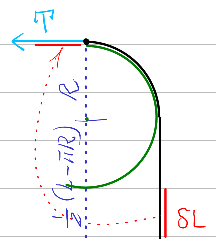
(a)



$$mg\delta h = T\cdot[2\pi(r+\delta r) - 2\pi r] = 2\pi T\delta r$$

$$T = \frac{mg}{2\pi}\cdot\frac{\delta h}{\delta r} = \frac{mg}{2\pi}\frac{H}{R}$$

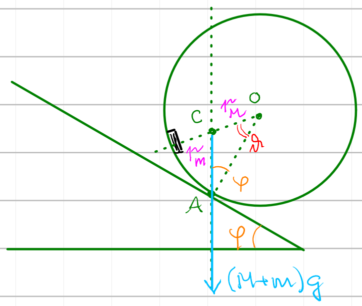
(b)



$$\frac{m}{L}\delta L g\left(\frac{L-\pi R}{2}+R\right) = T\delta L$$

$$T = mg\frac{L-(\pi-2)R}{2L}$$

4



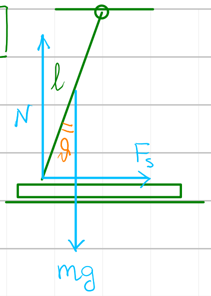
Apskatīsim sistēmu caurule–ripa kā veselu. Ja apskata rotāciju ap A, tad vienīgais spēks, kuram potenciāli var būt nenulles moments ir $(m+M)\vec{g}$, kas ir pielikts masu centrā C. Sistēma nerotē, tātad arī šī spēka moments ir 0, tātad tā darbības līnija iet caur A. Apzīmēsim $\angle AOC = \vartheta$. Pēc sinusu teorēmas

$$\frac{r_M}{\sin\varphi} = \frac{R}{\sin(\varphi+\vartheta)}; \quad r_M = R\frac{M}{m+M}; \quad \frac{\sin\varphi}{\sin(\varphi+\vartheta)} = \frac{M}{m+M}.$$

Ripa izslīd, ja $\tan(\varphi+\vartheta) > \mu$. Tātad,

$$\left(1+\frac{m}{M}\right)\sin\varphi^* = \frac{\mu}{\sqrt{1+\mu^2}} \rightsquigarrow \varphi^* = \arcsin\left(\frac{M}{m+M}\cdot\frac{\mu}{\sqrt{1+\mu^2}}\right).$$

5



Apskatīsim spēku momentus uz stieni ap šarnīru, kad dēlis mēģina kustēties pa labi:

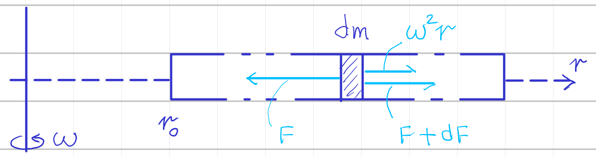
$$mg\frac{l}{2}\sin\vartheta + F_s l\cos\vartheta = Nl\sin\vartheta.$$

Ja dēlis ir uz izslīdēšanas robežas, $F_s=\mu_1 N$ un

$$mg\sin\vartheta = 2N(\sin\vartheta-\mu_1\cos\vartheta) \rightsquigarrow N=\frac{mg\sin\vartheta}{2(\sin\vartheta-\mu_1\cos\vartheta)}.$$

Ja $\underline{\underline{\mu_1\ge\tan\vartheta}}$, tad uz izslīdēšanas robežas $N<0$, t.i. fizikāli nav iespējams gadījums, kad dēlis izslīd.

6



$$\left.\begin{aligned}-dF &= \omega^2 r\cdot dm\\ F &= k' dr = k\frac{M}{dm}dr\end{aligned}\right\}$$

$$\frac{dF}{dm}+\omega^2 r=0$$

$$\frac{d}{dm}\left(kM\frac{dr}{dm}\right)+\omega^2 r=0 \rightsquigarrow r''+\frac{\omega^2}{kM}r=0.$$

$$r(m)=A\cos\left(\frac{\omega}{\sqrt{kM}}m+B\right)$$

$$r(0)=r_0 \Rightarrow r_0=A\cos B$$

$$F(M)=0 \Rightarrow Mkr'(M)=0 \Rightarrow -A\sin\left(\omega\sqrt{\tfrac{M}{k}}+B\right)=0$$

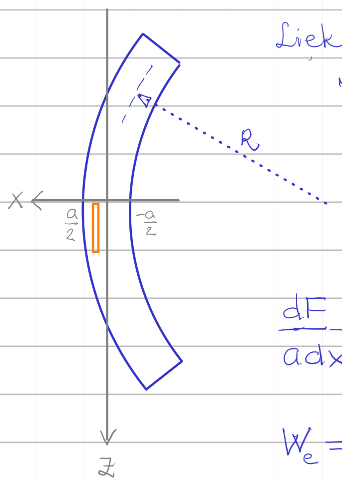
$$\Rightarrow B=-\omega\sqrt{M/k}\ ;\quad A=r_0/\cos\omega\sqrt{M/k}$$

$$r(m)=\frac{r_0}{\cos\omega\sqrt{M/k}}\cos\left[\omega\sqrt{\frac{M}{k}}\left(\frac{m}{M}-1\right)\right]$$

$$l=r(M)-r(0)=\underline{\underline{r_0\left(\frac{1}{\cos\omega\sqrt{M/k}}-1\right)}}\geq 0$$

$$\frac{1}{\cos\omega\sqrt{M/k}}\geq 1 \rightsquigarrow \omega\sqrt{\frac{M}{k}}<\frac{\pi}{2} \rightsquigarrow \underline{\underline{\omega<\frac{\pi}{2}\sqrt{\frac{k}{M}}=\omega^*}}$$

7

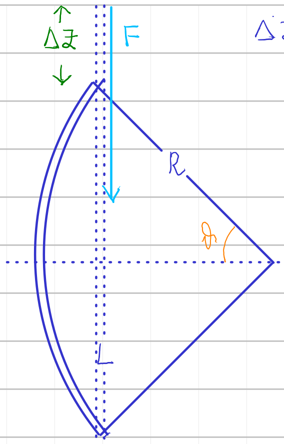


Liekuma rādiuss attālumā x no viduslīnijas $r(x)=R+x$. Elementa garums ir proporc. rādiusam $\Rightarrow dz'=dz\frac{R+x}{R}=dz\left(1+\frac{x}{R}\right)$

$$\varepsilon_z=\frac{dz'-dz}{dz}=\frac{dz'}{dz}-1=\underline{\underline{\frac{x}{R}}}$$

$$\frac{dF}{a\,dx}=E\frac{x}{R}\ ;\quad dA=dF\cdot\varepsilon_z dz=\underline{\underline{\frac{Eax\,dx}{R}\cdot\frac{x}{R}dz}}$$

$$W_e=A=\int_{-l/2}^{l/2}dz\int_{-a/2}^{a/2}dx\,\frac{Ea}{R^2}x^2=\frac{2Eal}{R^2}\frac{a^3}{3\cdot 2^3}=\underline{\underline{\frac{1}{12}\frac{Ea^4l}{R^2}}}$$



$$\Delta z = L - 2R\sin\vartheta = [\vartheta \ll 1] = L - 2R\left(\vartheta - \tfrac{1}{6}\vartheta^3\right)$$
$$= [\vartheta = L/2R] = L - L + \tfrac{1}{3}\cdot L^3/8R^2 = L^3/24R^2$$

Saspiežot stieni bez lieces par Δz ar F,

$$W_e = F\Delta z = \frac{FL^3}{24R^2}.$$

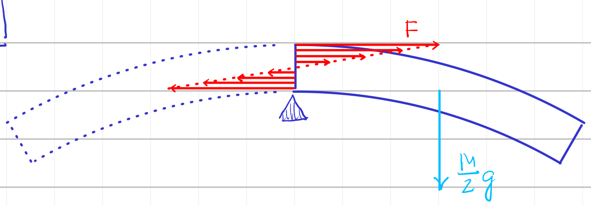
Saspiežot stieni ar lieci,

$$W_e' = \frac{Ea^4L}{12R^2}$$

Sistēma patvaļīgi nonāk stāvoklī ar zemāku potenciālo enerģiju, tātad stienis izliecsies, kad $W_e' < W_e$.

$$\frac{Ea^4\cancel{L}}{\cancel{12}\cancel{R^2}} < \frac{FL^{\cancel{3}\,2}}{\cancel{24}\cancel{R^2}_{\,2}} \rightsquigarrow \underline{\underline{F > \frac{2Ea^4}{L^2}}}.$$

$\boxed{8}$



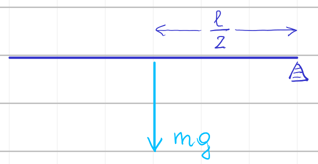
The straw will break in the middle. The mass $m \sim \ell d^2$. The moment of weight $M_g \sim m\ell \sim \ell^2 d^2$.

The moment of weight should be compensated by the bending moment in the middle cross-section $M_b \sim Fd \sim \ell^2 d^2$

The force $F = \sigma S_z \sim d^2$. Combining,

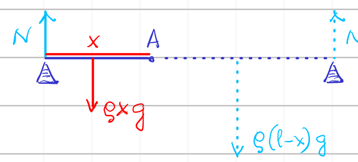
$$d^3 \sim \ell^2 d^2 \rightsquigarrow \ell \sim \sqrt{d} \implies \underline{\underline{\ell' = \ell\sqrt{d'/d} = 160\ \text{cm}}}$$

$\boxed{9}$ (a)



$$M_0 = \frac{mg\ell}{2} = \underline{\underline{\frac{\varrho g \ell^2}{2}}}$$

(b)



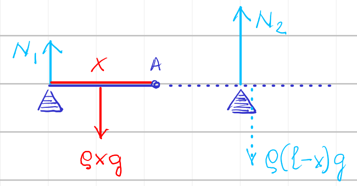
$N = \frac{1}{2}mg$. Apskatām stieņa daļu no kreisā atbalsta līdz p-tam A. Tā ir līdzsvarā, tātad uz to no pārējās stieņa daļas darbojas spēks $F = N - \varrho x g$ un lieces moments

$$M_0(x) = Nx - \frac{\varrho x^2 g}{2} = \frac{\varrho g x}{2}(\ell - x),$$

kas ir maksimāls, kad $x = \ell/2$. Tātad,

$$\underline{\underline{\varrho_{max} = \frac{8M_0}{g\ell^2}}}.$$

(c)



$$\begin{cases} N_1 + N_2 = \varrho l g \\ N_1 d = \varrho l g (d - l/2) \end{cases}$$

$$N_1 = \varrho l g \left(1 - \frac{\ell}{2d}\right) \; ; \; N_2 = \frac{\varrho \ell^2 g}{2d}$$

$$F(x) = N_1 - \varrho x g = \varrho l g \left(1 - \frac{\ell}{2d} - \frac{x}{\ell}\right)$$

$$M_0(x) = N_1 x - \frac{\varrho x^2 g}{2} = \underline{\underline{\varrho l g x \left(1 - \frac{\ell}{2d} - \frac{x}{2\ell}\right)}}.$$

(d) Gadījumā, kad $x \leq d$,

$$x^* = \ell\left(1 - \frac{\ell}{2d}\right) \quad \text{un} \quad M_0^* = \frac{\varrho \ell^2 g}{2}\left(1 - \frac{\ell}{2d}\right).$$

Gadījumā, kad $x \geq d$,

$$M_0'(x) = M_0(x) + N_2(x - d) = \varrho l g x \left(1 - \cancel{\frac{\ell}{2d}} - \frac{x}{2\ell} + \cancel{\frac{\ell}{2d}} - \frac{\ell}{2x}\right)$$

$$= \frac{1}{2}\varrho g\left(2\ell x - x^2 - \ell^2\right) = -\frac{1}{2}\varrho g(\ell - x)^2$$

$$x'^* = d \quad \text{un} \quad M_0'^* = \frac{\varrho g(\ell - d)^2}{2}$$

$\max(M_0^*, M_0'^*)$ ir vismazākais, kad $M_0^* = M_0'^*$. Tātad,

$$1 - \frac{\ell}{2d^*} = 1 - \frac{d^*}{\ell} \rightsquigarrow \underline{\underline{d^* = \frac{\ell}{\sqrt{2}}}}.$$

Piezīme. $M_0(x)$ grafiks kvalitatīvi izskatās šādi.