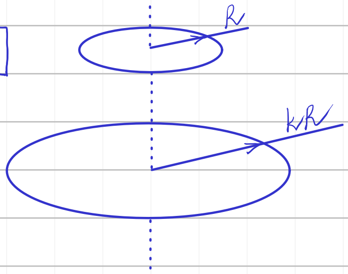
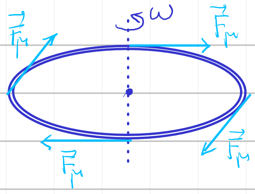
Absolūti ciets ķermenis

1

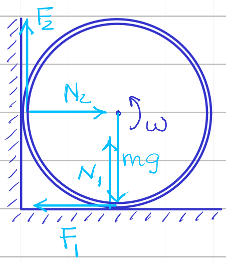


$$W_k = J\frac{\omega^2}{2} \sim J \qquad \frac{R'}{R} = \frac{h'}{h} = k$$
$$J \sim mR^2$$
$$m \sim V \sim R^2 h$$
$$J \sim R^4 h$$
$$W_k \sim R^4 h \Rightarrow \underline{\underline{\frac{W_k'}{W_k} = k^5}}$$

2



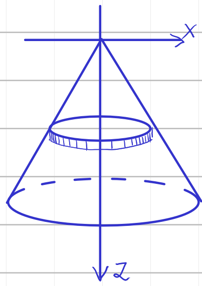
$$\vec{F}_\mu \uparrow\downarrow d\vec{s} \Rightarrow \vec{F}_\mu \perp \vec{r} \Rightarrow M_\mu = R\mu mg$$
$$\text{II NL}: \quad R\mu mg = (mR^2)\alpha \rightsquigarrow \alpha = \frac{\mu g}{R}$$
$$N = \frac{\varphi}{2\pi} = \frac{1}{2\pi}\cdot\frac{\omega^2}{2\alpha} = \underline{\underline{\frac{\omega^2 R}{4\pi\mu g}}}$$



Masu centrs nekustas $\Rightarrow F_{net} = 0$

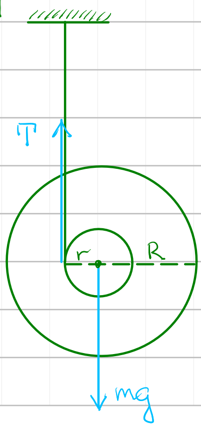
$$\begin{cases} \mu N_2 + N_1 = mg \\ \mu N_1 = N_2 \\ (\mu N_1 + \mu N_2)R = (mR^2)\alpha \end{cases}$$
$$N_1 = \frac{mg}{1+\mu^2}$$
$$\alpha = \frac{\mu N_1 + \mu^2 N_1}{mR} = \frac{\mu(1+\mu)}{1+\mu^2}\cdot\frac{g}{R}$$
$$N = \frac{\omega^2}{4\pi\alpha} = \underline{\underline{\frac{\omega^2 R}{4\pi g}\cdot\frac{1+\mu^2}{\mu(1+\mu)}}}$$

3



$$J = \int_{z=0}^{z=H} \frac{1}{2}\,dm\cdot x^2 = \frac{\pi\rho}{2}\int_0^H x^2 dz\cdot x^2 = \left[x = kz\right]$$
$$= \frac{\pi\rho k^4}{2}\int_0^H z^4 dz = \frac{\pi\rho k^4 H^5}{10} = \frac{\pi\rho R^4 H}{10} = \left[m = \frac{\pi\rho R^2 H}{3}\right]$$
$$= \underline{\underline{\frac{3}{10}mR^2}}$$

4



$$\text{II NL (tr.)}: \qquad mg - T = ma$$
$$\text{II NL (rot. ap A)}: \quad \cancel{m}gr = \frac{1}{2}\cancel{m}R^2\cdot\frac{a}{r} \rightsquigarrow \underline{\underline{T = mg\left(1 - \frac{2r^2}{R^2}\right)}}$$

Kustībā pirms un pēc „atlēciena" $\vec{\omega}_\downarrow = \vec{\omega}_\uparrow$. Tātad varam uzskatīt, ka „atlēcienā" laikā notiek vienmērīga rotācija ar ātrumu

$$\omega = \frac{v}{r} = \frac{\sqrt{2aL}}{r} = \frac{1}{r}\cdot\frac{r}{R}\sqrt{2\cdot 2g\cdot L} = \frac{2\sqrt{gL}}{R}.$$

„Atlēciena" laikā ass pagriežas par π, līdz ar to „atlēciena" ilgums ir

$$\tau = \frac{\varphi}{\omega} = \frac{\pi R}{2\sqrt{gL}}$$

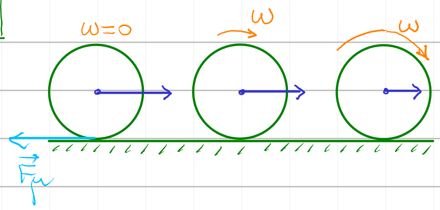
un kopspēka impulss

$$(\tilde{T} - mg)\tau = m|\Delta\vec{v}| = 2mv = 2m\omega r = 4m\frac{r}{R}\sqrt{gL}.$$

Tātad,

$$\tilde{T} = \frac{8mr\cdot gL}{\pi R^2} + mg = \underline{\underline{mg\left(1 + \frac{8rL}{\pi R^2}\right)}}.$$

$\boxed{5}$

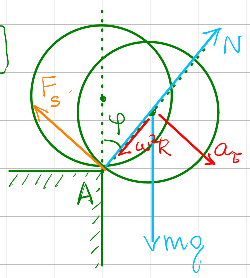


Sākotnēji cilindrs slīd bez rotācijas, tad berzes spēka ietekmē tā masu centra ātrums sāk samazināties, bet berzes spēka momenta dēļ leņķiskais ātrums sāk palielināties. Tas notiek līdz momentam, kad izpildās neizslīdēšanas nosacījums. Tālāk cilindrs vienmērīgi ripo.

$$\left\{\tilde{v} = v_0 - \mu g\tau;\quad \tilde{\omega} = \frac{\mu \cancel{m} g \cancel{R}}{\frac{1}{2}\cancel{m}R^{\cancel{2}}}\tau;\quad \tilde{\omega} = \frac{\tilde{v}}{R}\right\} \Rightarrow v_0 - \mu g\tau = 2\mu g\tau$$

$$\underline{\underline{\tau = \frac{v_0}{3\mu g}}} : \begin{cases} t < \tau & \text{ripo ar izslīdēšanu} \\ t \geq \tau & \text{ripo bez izslīdēšanas} \end{cases}$$

$\boxed{6}$



II NL (tr., rad.): $mg\cos\varphi - N = m\omega^2 R$

II NL (tr., tang.): $mg\sin\varphi - \mu N = ma_\tau$

II NL (rot. ap A): $mgR\sin\varphi = J_A\alpha$

Šteinera t-ma: $J_A = J_C + mR^2 = \frac{7}{5}mR^2$

Neizslīdēšana: $a_\tau = \alpha R$

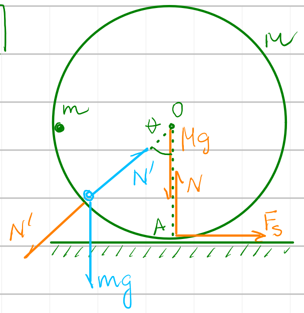
Enerģijas NL: $\frac{1}{2}J_A\omega^2 = mgR(1-\cos\varphi)$

$$g\sin\varphi - \mu(g\cos\varphi - \omega^2 R) = \alpha R = \frac{5}{7}g\sin\varphi$$

$$\omega^2 R = 2\cancel{m}gR^{\cancel{2}}(1-\cos\varphi)/(\tfrac{7}{5}\cancel{mR^2}) = \frac{10}{7}g(1-\cos\varphi)$$

$$\mu = \frac{\frac{2}{7}\sin\varphi}{\cos\varphi - \frac{\omega^2 R}{g}} = \frac{2\sin\varphi}{7\cos\varphi - 10(1-\cos\varphi)} = \underline{\underline{\frac{2\sin\varphi}{17\cos\varphi - 10}}}$$

7



Energy: $mgR = \frac{1}{2}mv^2 + \frac{1}{2}J_A\left(\frac{V}{R}\right)^2$

II NL (m): $ma_x = N'\sin\theta \rightsquigarrow ma_x = 2MA$

II NL (M): $MA = N'\sin\theta - F_s \rightsquigarrow 2MA = N'\sin\theta$

II NL (r.): $J_0 A/R = F_s R = MAR \rightsquigarrow F_s = MA$

$$\int_0^t ma_x\,dt = \int_0^t 2MA\,dt \rightsquigarrow mv = 2MV$$

$$2mgR = mv^2 + 2MV^2 = mv^2\left(1+\frac{m}{2M}\right)$$

$$v = 2\sqrt{\frac{gRM}{2M+m}}\ ;\quad v' = v + V = v\left(1+\frac{m}{2M}\right)$$

$$F = mg\left[1+\frac{4M}{2M+m}\cdot\left(\frac{2M+m}{2M}\right)^2\right] = \underline{\underline{3mg\left(1+\frac{m}{3M}\right)}}$$

8 See 200 More Puzzling Physics Problems, P48

9

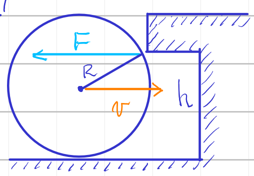
Leņķ. mom. SL: $mvL = \left(\frac{1}{3}ML^2 + mL^2\right)\omega'$

Enerģijas SL: $\frac{1}{2}\omega'^2\left(\frac{1}{3}ML^2+mL^2\right) = \left(M\cdot\frac{1}{2}L + mL\right)(1-\cos\varphi)\,g$

$$\frac{m^2v^2}{\left(\frac{1}{3}M+m\right)} = (M+2m)(1-\cos\varphi)\,gL$$

$$\underline{\underline{\cos\varphi = 1 - \frac{3m^2}{(M+3m)(M+2m)}\cdot\frac{v^2}{gL}}}$$

10



Elastīga sadursme.

Enerģijas SL: $mv^2 + J\omega^2 = mv'^2 + J\omega'^2$

Neizslīdēšana: $v = \omega R \ \wedge\ v' = \omega' R$

$\frac{7}{5}mv^2 = \frac{7}{5}mv'^2 \rightsquigarrow v' = v$

Impulsa SL: $2mv = F\tau$

Leņķ. mom. SL: $2\cdot\frac{2}{5}mR^2\cdot\frac{v}{R} = F(h-R)\tau$ $\Big\}\ \div$

$$5(h-R) = 2R$$

$$\underline{\underline{h = \frac{7}{5}R}}$$