
IDEĀLĀ GĀZE
TERMISKĀ IZPLĒŠANĀS

1° The situation when there are no intermolecular bonds is well described by the ideal gas model. The mass and the volume of water do not change, so the density also stays the same and the pressure in the container becomes

$$p = \frac{mRT}{\mu V} = \frac{\rho_w RT}{\mu} = \frac{10^3 \cdot 8,3 \cdot 300}{18 \cdot 10^{-3}} = 1,4 \cdot 10^8 \text{ Pa.}$$

2° Note that relative changes in thermodynamic parameters are small, so we can neglect higher powers of these changes. From the state equation,

$$d(pV) = d(\nu RT), \quad p dV + V dp = \nu R dT, \quad \frac{dV}{V} + \frac{dp}{p} = \frac{\nu R dT}{pV} = \frac{dT}{T}.$$

Thus the relative change of temperature is $+3\% - 2\% = +1\%$.

3° From the state equation, it follows that

$$\frac{T_0}{T_0 + \Delta T} = \frac{p_0}{p} \frac{V_0}{V} = \left(\frac{V}{V_0} \right)^2 \frac{V_0}{V} = \frac{V}{V_0} = \alpha, \quad T_0 = \Delta T \frac{\alpha}{1 - \alpha}.$$

4° The equation of the process can be rewritten as $p(pV) = \text{const}$. As temperature increases, pV also increases, so p decreases. But $p^2 V = \text{const}$, so V increases. Thus the maximal volume is the final volume, i. e. $V_2 = V_{\text{max}}$. From equation of state,

$$\alpha = \frac{T_2}{T_1} = \frac{V_2 p_2}{V_1 p_1} = \frac{V_2}{V_1} \sqrt{\frac{V_1}{V_2}} = \sqrt{\frac{V_2}{V_1}} \quad \text{so} \quad V_1 = \frac{V_{\text{max}}}{\alpha^2}.$$

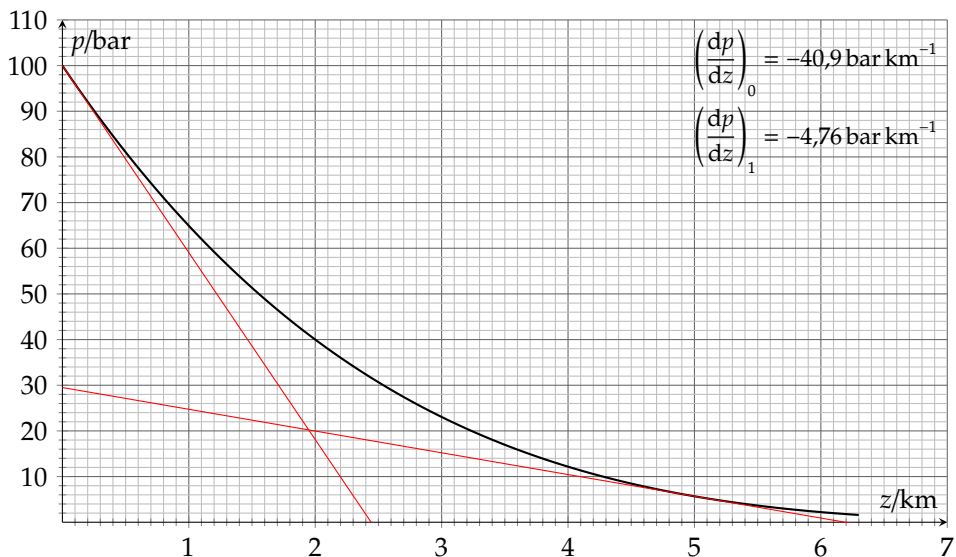
Internal energy of an ideal gas $U = (3/2)pV = (3/2)\nu RT$, thus

$$\frac{U_1 + \Delta U}{U_1} = \frac{T_2}{T_1} = \alpha, \quad U_1 = \frac{\Delta U}{\alpha - 1} = \frac{3}{2} p_1 V_1, \quad p_1 = \frac{2}{3} \frac{\Delta U}{\alpha - 1} \frac{\alpha^2}{V_{\text{max}}}.$$

5° Suppose we are at some altitude z where pressure is p and temperature is T . Moving up by dz , the (hydrostatic) pressure will change by $dp = -\rho g dz$. From the state equation, $\rho = p\mu/(RT)$ thus pressure gradient

$$\frac{dp}{dz} = -\frac{p\mu}{RT}g \quad \text{and} \quad \frac{T_0}{T_1} = \frac{(dp/dz)_1 p_0}{(dp/dz)_0 p_1},$$

where we assumed that g is constant as $R \gg z_1$. Pressure gradients at $z_0 = 0$ and z_1 can be obtained from the graph by constructing tangent lines at given points and calculating their slopes (see fig.).



Thus, finally,

$$T_0 = 250 \cdot \frac{4,76}{40,9} \cdot \frac{100}{5,7} = 510 \text{ K.}$$

6° Suppose the air in the tube has already been heated up and some of the mercury has flowed out. Let x be the current height of air column. Then air pressure in the tube $p = p_{\text{atm}} + \rho g(2L - x) = \rho g(3L - x) = \nu RT/(xS)$. Thus the temperature at which the system will be at equilibrium when the height of air column is x is

$$T(x) = \frac{\rho g S}{\nu R} (3Lx - x^2).$$

This function reaches its maximum at $x^* = 3L/2$ and

$$T_{\text{max}} = T(x^*) = \frac{\rho g S}{\nu R} \frac{9L^2}{4} = \left[(2\rho g L)(SL) = \nu RT_0 \right] = \frac{9}{8} T_0.$$

After T_{max} is reached, the rest of mercury will continue flowing out on its own accord, so no additional heating is necessary.

7° Consider the case when the piston is displaced from its equilibrium position. The volume and the pressure on one side become, respectively, $V + dV$ and $p + dp$. The system is oscillating at constant temperature, so $d(pV) = p dV + V dp = 0$ and $dp = -(p/V)dV$. The change in volume on the other side $dV' = -dV$, so $dp' = (p/V)dV$. The force acting on the piston $dF = S(dp - dp') = -(2p/L)(Sdx)$, where

dx is the linear displacement of the piston. Dividing both sides of the equation by the mass of the piston we get

$$\ddot{x} + \frac{2pS}{mL}x = 0, \quad \omega = \sqrt{\frac{2pS}{mL}}, \quad \tau = 2\pi\sqrt{\frac{mL}{2pS}}.$$

8°

- (a) Using ideal gas state equation, isobaric cubic expansion coefficient

$$\alpha = \frac{1}{V} \left(\frac{\partial V}{\partial T} \right)_p = \left[V = \frac{\nu RT}{p} \right] = \frac{\nu RT}{pV} = \frac{1}{T}.$$

- (b) The state equation can be used to express temperature, pressure or volume as a function of the other two parameters, i. e. $p = p(V, T)$, $V = V(p, T)$ and $T = T(p, V)$. Consider the first two functions and their differentials

$$\begin{aligned} dp &= \left(\frac{\partial p}{\partial V} \right)_T dV + \left(\frac{\partial p}{\partial T} \right)_V dT, \\ dV &= \left(\frac{\partial V}{\partial p} \right)_T dp + \left(\frac{\partial V}{\partial T} \right)_p dT \\ &= \left(\frac{\partial V}{\partial p} \right)_T \left[\left(\frac{\partial p}{\partial V} \right)_T dV + \left(\frac{\partial p}{\partial T} \right)_V dT \right] + \left(\frac{\partial V}{\partial T} \right)_p dT \\ &= dV + \left[\left(\frac{\partial V}{\partial p} \right)_T \left(\frac{\partial p}{\partial T} \right)_V + \left(\frac{\partial V}{\partial T} \right)_p \right] dT. \end{aligned}$$

Assuming dT is non-zero, the square bracket should be zero, which leads to the *cyclic chain rule*

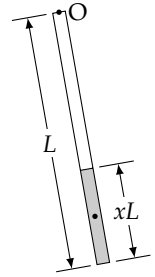
$$\left(\frac{\partial V}{\partial p} \right)_T \left(\frac{\partial p}{\partial T} \right)_V \left(\frac{\partial T}{\partial V} \right)_p = -1.$$

From that and the definitions of parameters, it follows that

$$-V\beta_T \cdot p\lambda_T \cdot \frac{1}{\alpha_p V} = -1 \quad \rightsquigarrow \quad \lambda_T = \frac{\alpha}{p\beta_T} = 46 \text{ K}^{-1}.$$

9° Suppose initially the length of the pipe is L , its inner volume is V , the volume of mercury is V_{Hg} and $x = V_{\text{Hg}}/V$. The centre of mass of mercury is located at the distance

$$l = L \left(1 - \frac{x}{2}\right)$$



from the pivot point. By definitions of thermal expansion coefficients,

$$\frac{dL}{d\theta} = \alpha_{\text{Ni}}L, \quad \frac{dV}{d\theta} = 3\alpha_{\text{Ni}}V, \quad \frac{dV_{\text{Hg}}}{d\theta} = \alpha_{\text{Hg}}V_{\text{Hg}}.$$

- (a) In the framework of the simple pendulum model, the period remains the same, if l does not change with temperature, so

$$\begin{aligned} \frac{dl}{d\theta} &= \frac{dL}{d\theta} - \frac{1}{2} \left(x \frac{dL}{d\theta} + L \frac{dx}{d\theta} \right) = 0, & \alpha_{\text{Ni}}L(2-x) &= L \frac{dx}{d\theta}, \\ \alpha_{\text{Ni}}(2-x) &= \frac{d}{d\theta} \frac{V_{\text{Hg}}}{V} = \frac{\alpha_{\text{Hg}}V_{\text{Hg}}V - 3\alpha_{\text{Ni}}VV_{\text{Hg}}}{V^2} = (\alpha_{\text{Hg}} - 3\alpha_{\text{Ni}})x, \\ x &= \frac{2}{\alpha_{\text{Hg}}/\alpha_{\text{Ni}} - 2} = 0,125. \end{aligned}$$

- (b) In the framework of the physical pendulum model, the period remains the same, if the effective length λ of the pendulum does not change with the temperature. The effective length is the length of a simple pendulum that would have the same period as the given physical pendulum, so

$$\begin{aligned} \frac{mgl}{J} &= \frac{g}{\lambda}, & \lambda &= \frac{J}{ml} = \frac{(xL)^2}{12l} + l = l \left[\frac{x^2}{12} \left(\frac{1}{1-x/2} \right)^2 + 1 \right] \\ & & &= \frac{4}{3} \frac{x^2 - 3x + 3}{(2-x)^2} l. \end{aligned}$$

Along the lines of part (a),

$$\begin{aligned} \frac{d\lambda}{d\theta} &= - \frac{2L \left[x^3(\alpha_{\text{Hg}} - 2\alpha_{\text{Ni}}) + x^2(7\alpha_{\text{Ni}} - 4\alpha_{\text{Hg}}) + 3\alpha_{\text{Hg}}x - 6\alpha_{\text{Ni}} \right]}{3(x-2)^2} = 0, \\ 16x^3 - 65x^2 + 54x - 6 &= 0. \end{aligned} \tag{9.1}$$

In order to solve this equation, assume that the actual value of x that would make the period independent of temperature is only slightly different from

the one obtained in the simpler model of part (a), i. e. $x = x_{(a)} + \delta$ and $\delta \ll x_{(a)}$. Substituting in (9.1) and neglecting higher orders of δ , we get

$$(48x_{(a)}^2 - 130x_{(a)} + 54)\delta + (16x_{(a)}^3 - 65x_{(a)}^2 + 54x_{(a)} - 6) = 0.$$

Numerically, substituting $x_{(a)} = 0,125$, we finally get

$$\delta = 0,006 \quad \text{and} \quad x = 0,131.$$