Absolūti ciets ķermenis

2023-11-25

Iesildīšanās

1° Divi līdzīgi spararati ir izgatavoti no viena un tā paša metāla un iegriezti līdz vienādiem leņķiskiem ātrumiem. Spararatu lineāro izmēru attiecība ir k. Cik liela ir to kinētisko enerģiju attiecība?

2° Tievu gredzenu, kura rādiuss ir R iegrieza līdz leņķiskajam ātrumam ω. Cik daudz apgriezienu gredzens veiks līdz apstāšanas brīdim, kad to noliks

(a) horizontāli uz galda ar berzes koeficientu μ?

(b) vertikāli stūrī pie sienas, ja berzes koeficients gan ar sienu, gan ar grīdu ir μ?

3° Nosakiet konusa, kura rādiuss ir R, inerces momentu ap tā simetrijas asi.

Rotācijas kustības dinamika

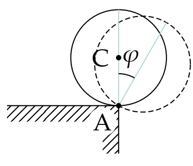
4° *Yo-yo* ir salikts no diviem koaksiāliem diskiem ar rādiusu R un masu $m/2$. Diski ir savienoti ar vieglu asi, kuras rādiuss ir r. Uz ass ir uztīta aukla ar garumu $L \gg R$, kuras viens gals ir piesiets pie ass. *Yo-yo* „palaiž" no miera stāvokļa.

(a) Cik liels ir auklas sastiepuma spēks T, kad *yo-yo* kustas uz leju?

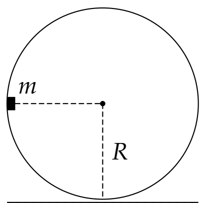
(b) Novērtējiet T raviena laikā apakšējā punktā.

5° Homogēnu cilindru, kura rādiuss ir R un masa ir m, iestuma tā, ka tas sāka slīdēt bez rotācijas ar ātrumu v pa horizontālu virsmu. Kvalitatīvi un kvantitatīvi aprakstiet cilindra kustību, ja berzes koeficients starp cilindru un virsmu ir μ, bet rites berzi var neievērot.

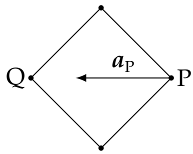
6° Homogēna lodīte ir novietota uz galda tā, ka tās centrs C atrodas tieši virs galda malas A. Lodīti viegli pastum, un tā sāk krist, griežoties ap punktu A. Cik liels ir berzes koeficients starp lodīti un galdu, ja lodīte sāk izslīdēt, kad tā ir pagriezusies pa leņķi $\varphi = 30°$?



7° (*IPhO 2014*) A small puck of mass m is carefully placed onto the inner frictionless surface of a thin hollow cylinder of mass M and radius R. Initially, the cylinder rests on a horizontal plane and the puck is located at height R above the plane as shown in the figure. Find the interaction force F between the puck and the cylinder at the moment when the puck passes the lowest point of its trajectory. Assume that the cylinder rolls on the plane without slipping. The free fall acceleration is g.



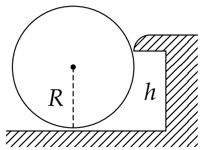
8° Four identical homogeneous rods are connected by four light frictionless knuckle joints to form a square, which is then placed on a horizontal frictionless table-top. Vertex P is pushed in the direction of the diagonal PQ of the square and, as a result, acquires an initial acceleration of a_{P}. In which direction and with how much acceleration does the opposite vertex Q of the square start to move?



Saglabāšanās likumi

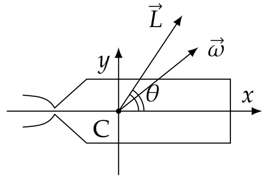
9° Lineāls, kura masa ir M un garums ir L, ir iekarināts augšējā galā. Tā apakšējā galā ar horizontāli vērstu ātrumu v trāpa maza plastilīna lodīte, kuras masa ir m. Cik liels ir lineāla maksimālais noliekšanas leņķis no vertikāles? Lodītes sadursme ar lineālu ir absolūti neelastīga.

10° Cik lielam ir jābūt biljarda galda malas augstumam h, lai biljarda bumba, kuras rādiuss ir R, ripojot bez izslīdēšanas un elastīgi atsitoties pret galda malu, turpinātu ripot bez izslīdēšanas? Pieņemiet, ka spēks, kas darbojas no galda malas uz bumbu sadursmes laikā, ir horizontāls.



Extra

11° (*APhO 2017, Q3.A*) Consider the upper stage of a rocket that has served its purpose and has become space debris (schematically shown in the figure). We introduce a body-fixed reference frame Cxy with the origin in the centre of mass C and x being the symmetry axis of the stage. Moments of inertia with respect to x and y axes are J_x and $J_y < J_x$.



Consider an arbitrary initial rotation of the stage with angular momentum L, where θ is the angle between the symmetry axis and the angular momentum. No forces or torques act upon the stage.

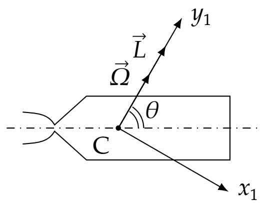
(a) Find the projections of angular velocity $\vec{\omega}$ on x and y, given that $\vec{L} = J_x\omega_x\vec{e}_x + J_y\omega_y\vec{e}_y$. Express the answer in terms of $L = |\vec{L}|$, θ, J_x and J_y.

(b) Find the rotational energy E_x associated with rotation ω_x and E_y associated with rotation ω_y. Find the total rotational kinetic energy $E = E_x + E_y$ of the stage as a function of L and $\cos\theta$.

In the following questions consider the free rotation of the stage with the initial angular momentum L and $\theta(0) = \theta_0$.

(c) Let x_0 be the initial orientation of the symmetry axis Cx of the stage with respect to inertial reference frame. Using conservation laws find the maximum angle ψ that the Cx axis makes with x_0 while the stage freely rotates.

Let us now introduce the reference frame C$x_1y_1z_1$ with y_1 along the constant angular momentum vector $\vec{L}$ (see figure). This reference frame rotates about y_1 in such a way that the symmetry axis of the stage always stays in the Cx_1y_1 plane.



(d) Given L, $\theta(0) = \theta_0$, J_x and J_y, find the angular velocity $\Omega(t)$ of the reference frame Cx_1y_1 about y_1 and direction and magnitude of angular velocity $\vec{\omega}_s(t)$ of the stage relative to the reference frame Cx_1y_1 as functions of time. Express the answer for $\vec{\omega}_s$ direction in terms of the angle $\gamma_s(t)$ it makes with the symmetry axis Cx. *Note. Angular velocity vectors are additive, thus* $\vec{\omega} = \vec{\omega}_x + \vec{\omega}_y = \vec{\Omega} + \vec{\omega}_s$.

12° (*IPhO 2002, Q3*)