Tshwane University of Technology

We empower people

Student Number

Surname & Initials

YEAR: 2021

SEMESTER: 2

ASSESSMENT: SUMMATIVE ASSESSMENT 1

SUBJECT NAME:

SUBJECT CODE:

QUALIFICATION(S):

PAPER DESCRIPTION: Closed Book **DURATION:** 2 HOURS **PAPER:** Only

SPECIAL REQUIREMENTS

- ☐ **NONE**
- ☐ **NON-PROGRAMMABLE POCKET CALCULATOR**
- ☒ **SCIENTIFIC CALCULATOR**
- ☐ **COMPUTER ANSWER SHEET**
- ☐ **GRAPH PAPER**
- ☐ **DRAWING INSTRUMENTS**

OTHER:

INSTRUCTIONS TO CANDIDATES: ANSWER ALL QUESTIONS

ANSWER ALL THE QUESTIONS ON EXAM SCRIPT OR CLEAN PAPER WITH A PEN. PENCIL WRITTEN WORK WILL NOT BE MARKED. See Appendix for formulas and tables for logical equivalences and rules of inference

TOTAL NUMBER OF PAGES INCLUDING COVER PAGE: 11

EXAMINER: L CRONJE **FULL MARKS:** 100

MODERATOR: TP MSIMANGA **TOTAL MARKS:** 102

STUDENT TOTAL: ___

STUDENT %: ___

Special instructions if writing ONLINE (ignore if writing on campus):

1. Once you have the QR code (on the next page), start your invigilator app (BEFORE 9h30)
2. If you have more than 1 device, view the question paper on myTUTor. If you have only 1 device, from inside the invigilator app, click 'Access LMS' (top right corner), navigate to Assignments and view the question paper through the invigilator app
3. Make sure you have enough paper to write your answers on
4. Start the 1st page with your student number, Surname and initials, and sign that you are adhering to the rules and regulations of TUT
5. Start each page with your student number AND number each page clearly (use only one side of the paper)
6. When you are done writing, click 'Finish Assessment' in the invigilator app. It will ask you to scan your pages – please do that in the correct order of your page numbers
7. Then you can exit the invigilator app.
8. Now you must scan and upload your pages to myTUTor. Using Adobe Scan (or similar tool), scan your pages to your phone (keep track of where you save it) in a single PDF. Go to myTUTor, to your assignments tab, and upload the PDF file and submit.
9. You MUST scan your pages TWICE – once on the invigilator app AND for upload to myTUTor
10. NB! You cannot upload to myTUTor while the invigilator app is still running
11. The invigilator app will give you 8min to scan your question paper
12. On myTUTor you have until 11h30 to upload your PDF file (take note of this if you start later than 9h00!!!)

	DCT115D/DCTF15D/HSP115 Summative Assessment
Date:	**11 Dec 2021**
Duration:	**120 Minutes**
QR Window:	**You MUST scan the QR code 9h00-9h30 on the day, otherwise you are considered late for your test**

Please take note that The Invigilator App will request you take a picture of every page of your answer sheet at the end of the assessment.

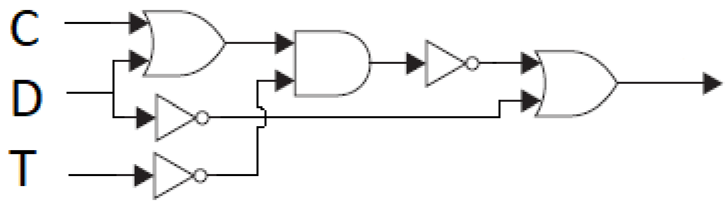
Please take note that this does not replace your formal scan and upload to myTUTor (Learning Management System - LMS).

YOUR EXAM QR CODE & QR ACCESS CODE →

QR Access Code: 03597ff3

1. Logic and proofs [32]

1.1. Write down the logical expression of the following circuit: (6)



1.2. Using propositional equivalences, prove that: $\neg[(\neg q \wedge p) \vee \neg(p \vee q)] \equiv q$

(Show one step per line with a reason for the step) (8)

1.3. A set of premises and a conclusion is given. Use the rules of inference to deduce the conclusion from the premises, giving a reason for each step. (8)

i) $\neg M \vee \neg N \to O$

ii) $(M \wedge N) \to P$

iii) $\neg S$

iv) $\neg P \vee S$

v) $\therefore O$

1.4. Prove, using Mathematical induction, for any $n \geq 0$, that

$$1 + \left(\frac{2}{3}\right) + \left(\frac{2}{3}\right)^2 + \cdots + \left(\frac{2}{3}\right)^n = -3\left[\left(\frac{2}{3}\right)^{n+1} - 1\right]$$

(10)

2. Sets, functions and matrices [30]

2.1. Calculate the following

2.1.1. $\binom{9}{6}$ (2)

2.1.2. $\frac{(k-1)!}{(k-3)!}$ (3)

2.2. How many 3-digit numbers can be made up from the even digits from 0 to 9 (you can include numbers starting with a 0) (4)

2.3. If $f(x) = \sqrt{4x} + 3$, find the inverse $f^{-1}(y)$ (4)

2.4. If $f(x) = -x^2$ and $g(x) = x + 5$ find $f \circ g$ and $g \circ f$ (4)

2.5. Write the following in expanded form and calculate: $\sum_{k=1}^{3}(2^{2k} + 1)$ (4)

2.6. Write out the binomial expansion of: $(2 - h)^3$ (4)

NB! No marks given if the binomial theorem wasn't used!

2.7. Given the following matrices: $A = \begin{bmatrix} 5 & 10 & -5 \\ -3 & 1 & 0 \end{bmatrix}, B = \begin{bmatrix} 2 & 6 & 4 \\ -1 & 0 & 0 \end{bmatrix}$ $and\ C = \begin{bmatrix} -1 & 1 \\ -2 & 0 \end{bmatrix}$, find:

2.7.1. $A - 3B$ (2)

2.7.2. CA (3)

3. Algorithms and trace tables [20]

3.1. Use the table provided to trace the values of the variables in the following pseudo code:

```
a:=25
b:=10
for k:=12 to 15
        if b>a
        then b:=b-5
        else do b:=b*3
                a:=a-4 end do
next k
```

(5)

a											
b											
k											

3.2. Rewrite and fill in the missing parts of the given linear search algorithm

procedure *linear search(x: integer,* $a_1, a_2, \ldots, a_n$*: distinct integers)*

i := _____

while (i ___ n and x ___ a_i)

    i := ______

if i ___ n **then** *location* := i

else *location* := 0

return *location*{location is the subscript of the term that equals x, or is 0 if x is not found}

(5)

3.3. The binary search algorithm is given:

```
procedure binary search (x: integer, a_1, a_2, ..., a_n: increasing integers)
i := 1{i is left endpoint of search interval}
j := n {j is right endpoint of search interval}
while i < j
    m := ⌊(i + j)/2⌋
    if x > a_m then i := m + 1
    else j := m
if x = a_i then location := i
else location := 0
return location{location is the subscript i of the term a_i equal to x, or 0 if x is not found}
```

3.3.1. What is the condition for the algorithm to work? (1)

3.3.2. Refer to the following data set: 3 6 9 12 15 18 21 24 27 30. Show the midpoint calculations and resulting i and j values as the algorithm searches through the list for the value 12. Start by writing down i, j and x. (5)

3.4. Given the bubble sort algorithm and list of values: 500, 710, 402, 313, 254; what will be the state of the list after each pass (i.e., for i=1 to 4)?

```
procedure bubblesort(a_1, . . . , a_n : real numbers with n ≥ 2)
for i := 1 to n − 1
    for j := 1 to n − i
        if a_j > a_j+1 then interchange a_j and a_j+1
{a_1, . . . , a_n is in increasing order}
```

(4)

i=1	
i=2	
i=3	
i=4	

4. Divisibility, modular arithmetic, and cryptography [20]

4.1. Find $124^6 \bmod 416$ (4)

4.2. Find the prime factors of 1260 (show steps) (4)

4.3. Make use of prime factors to find the greatest common divisor (GCD) of 270 and 198 (4)

4.4. Say whether the following is true or false

4.4.1. $2 \equiv 8 (\bmod 3)$ (1)

4.4.2. $12 \equiv 18 (\bmod 4)$ (1)

4.4.3. $8 \equiv -12 (\bmod 5)$ (1)

4.4.4. $22 \equiv 33 (\bmod 11)$ (1)

4.5. Assume that a modified Caesar cipher is given by $C = (M + 7) \bmod 26$

4.5.1. What will the decrypting formula be? (1)

4.5.2. Decrypt the following ciphertext: NVVK SBJR AV HSS (3)

Appendix

Logical Equivalences Given any statement variables p, q, and r, a tautology **t** and a contradiction c, the following logical equivalences hold.		
1. Commutative laws:	$p \wedge q \equiv q \wedge p$	$p \vee q \equiv q \vee p$
2. Associative laws:	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	$(p \vee q) \vee r \equiv p \vee (q \vee r)$
3. Distributive laws:	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
4. Identity laws:	$p \wedge \boldsymbol{t} \equiv p$	$p \vee \boldsymbol{c} \equiv p$
5. Negation laws:	$p \vee \neg p \equiv \boldsymbol{t}$	$p \wedge \neg p \equiv \boldsymbol{c}$
6. Double negative law:	$\neg(\neg p) \equiv p$	
7. Idempotent laws:	$p \wedge p \equiv p$	$p \vee p \equiv p$
8. Universal bound laws:	$p \vee \boldsymbol{t} \equiv \boldsymbol{t}$	$p \wedge \boldsymbol{c} \equiv \boldsymbol{c}$
9. De Morgan's laws:	$\neg(p \wedge q) \equiv \neg p \vee \neg q$	$\neg(p \vee q) \equiv \neg p \wedge \neg q$
10. Absorption laws:	$p \vee (p \wedge q) \equiv p$	$p \wedge (p \vee q) \equiv p$
11. Negations of **t** and **c**:	$\neg \boldsymbol{t} \equiv \boldsymbol{c}$	$\neg \boldsymbol{c} \equiv \boldsymbol{t}$

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$$

<table>
<tr><th colspan="6">Rules of Inference</th></tr>
<tr><td>Modus Ponens</td><td colspan="2">$p \rightarrow q$
p
$\therefore q$</td><td>Elimination</td><td>a. $p \vee q$
$\neg q$
$\therefore p$</td><td>b. $p \vee q$
$\neg p$
$\therefore q$</td></tr>
<tr><td>Modus Tollens</td><td colspan="2">$p \rightarrow q$
$\neg q$
$\therefore \neg p$</td><td>Transitivity</td><td colspan="2">$p \rightarrow q$
$q \rightarrow r$
$\therefore p \rightarrow r$</td></tr>
<tr><td>Generalization</td><td>a. p
$\therefore p \vee q$</td><td>b. q
$\therefore p \vee q$</td><td rowspan="2">Proof by Division into Cases</td><td colspan="2" rowspan="2">$p \vee q$
$p \rightarrow r$
$q \rightarrow r$
$\therefore r$</td></tr>
<tr><td>Specialization</td><td>a. $p \wedge q$
$\therefore p$</td><td>b. $p \wedge q$
$\therefore q$</td></tr>
<tr><td>Conjunction</td><td colspan="2">p
q
$\therefore p \wedge q$</td><td>Contradiction Rule</td><td colspan="2">$\neg p \rightarrow \boldsymbol{c}$
$\therefore p$</td></tr>
</table>

Let $a, b,$ and n be integers with $n > 1$. Then

$$ab \equiv [(a \, mod \, n)(b \, mod \, n)](mod \, n)$$

Or, equivalently,

$$ab \, mod \, n = [(a \, mod \, n)(b \, mod \, n)] \, mod \, n$$

In particular, if m is a positive integer, then

$$a^m \equiv [(a \, mod \, n)^m](mod \, n)$$

Or, equivalently,

$$a^m \, mod \, n = [(a \, mod \, n)^n] \, mod \, n$$