Pomona College

Computer Science

Fall 2024: CSCI 181RT
Real-Time Systems in the Real World

Lab 13 - Fin

Wednesday, December 4, 2024
Edmunds Hall 105
1:15 PM - 4:00 PM

Professor Jennifer DesCombes

Lab #13 Preview

- Simple Digital Filter
  - FIR vs IIR
    - https://en.wikipedia.org/wiki/Finite_impulse_response
    - https://en.wikipedia.org/wiki/Infinite_impulse_response
    - https://community.sw.siemens.com/s/article/introduction-to-filters-fir-versus-iir

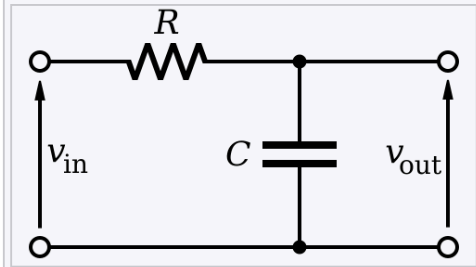
Analog RC Lowpass Filter

Time response [edit]

The time response of a low-pass filter is found by solving the response to the simple low-pass RC filter.

Using Kirchhoff's Laws we arrive at the differential equation[6]

$$v_{\text{out}}(t) = v_{\text{in}}(t) - RC\frac{\mathrm{d}\,v_{\text{out}}}{\mathrm{d}\,t}$$



A simple low-pass RC filter

Step input response example [edit]

If we let $v_{\text{in}}(t)$ be a step function of magnitude V_i then the differential equation has the solution[7]

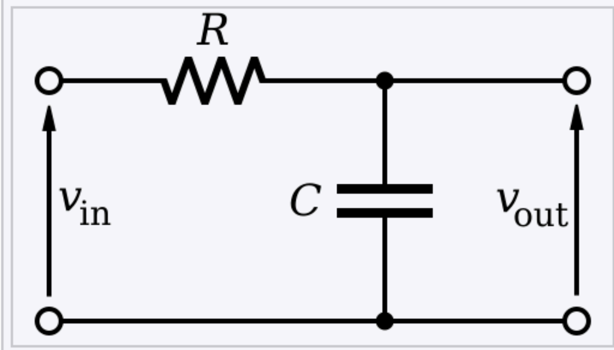
$$v_{\text{out}}(t) = V_i(1 - e^{-\omega_0 t}),$$

where $\omega_0 = \frac{1}{RC}$ is the cutoff frequency of the filter.

- Conceptually, At Any Instant
  - Ir = (Vin - Vout)/R
  - DeltaVc$_{\text{(new)}}$ = Vc + Ir/C

Digital Equivalent of RC Lowpass Filter

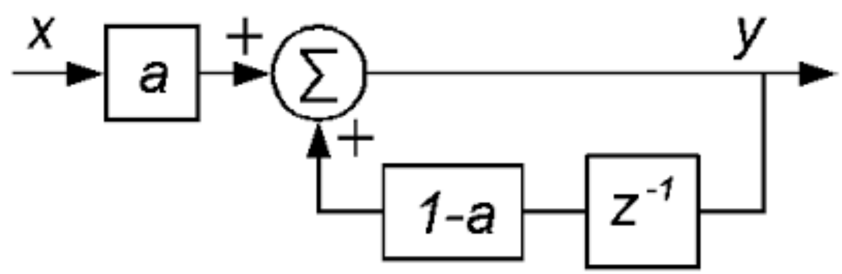
- Conceptually, At Any Instant
  - Ir = (Vin - Vout)/R
  - DeltaVc(new) = Vc + Ir/C
- Discrete Time Steps
  - Gain 'G' Models Effects of R and C
  - Vout[i] = Vout[i-1] +
    (Vin[i] - Vout[i-1]) x G
  - Vout[i] = Vout[i-1] +
    (Vin[i] x G) - (Vout[i-1] x G)
  - Vout[i] = (Vin[i] x G)
    + (Vout[i-1] x (1 - G))



A simple low-pass RC filter

IIR Filter

- Vout[i] = (Vin[i] x G) + (Vout[i-1] x (1 - G))
- Vout[i] = GVin[i] + (1-G)Vout[i-1]



Single Pole IIR Impulse Response

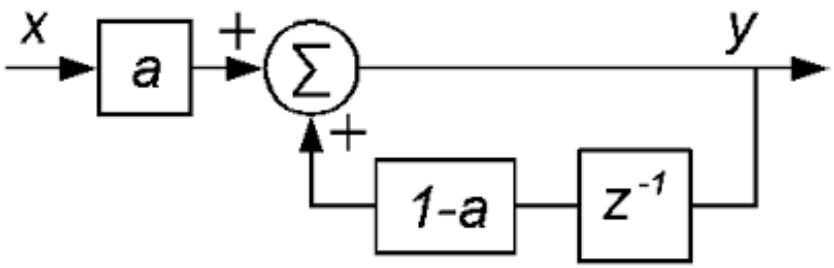
The single pole infinite impulse response (IIR) filter is an incredibly efficient filter!

The impulse response of the single pole IIR is

$$y[n] = \alpha x[n] + (1 - \alpha)y[n-1] \tag{1}$$

The filter uses two weights, α and $1-\alpha$. The weight α is applied to the input signal x[n] and the value is typically small (α < 0.1). The weight $1-\alpha$ is much larger than α and is applied to the feedback y[n-1]. The smaller the α the more impact the feedback has on the output and the smoother (and more narrowband) the filter.

IIR Filter



- Many Multipole Configurations
- Gain Variations Create Different Responses
  - Under-damped, Over-damped
  - Out-of-band Slope is Determined by Number of Stages