(a) (i) Fixed-point number representation: This is a method of representing numbers in a computer using a fixed number of digits for the integer part and a fixed number of digits for the fractional part. The position of the decimal point is fixed, hence the name.
Example: If we have a 4-digit fixed-point number with 2 digits for the integer part and 2 digits for the fractional part, then 12.34 would be represented as 1234.
(ii) Round-off error: This is the error that occurs when a real number is approximated by a finite-precision floating-point number. It happens when the number cannot be represented exactly due to the limitations of the number system.
Example: If we try to represent the number 0.1 in a binary floating-point system with 4 bits for the mantissa, the closest representation would be 0.0001, which is slightly different from the actual value.
(iii) Representation of zero as floating point number: In a floating-point number system, zero is represented by a special value. The exponent is set to the smallest possible value, and the mantissa is set to zero. This ensures that the number is treated as zero in calculations.
(iv) Significant digits in a decimal number representation: The significant digits in a decimal number are the digits that are not zero, starting from the leftmost non-zero digit.
Example: In the number 0.00123, the significant digits are 1, 2, and 3.
(v) Normalized representation of a floating point number: A normalized floating-point number is one where the leading digit of the mantissa is non-zero. This ensures that the number is represented in a unique way.
Example: The number 123.45 can be normalized as 1.2345 * 10^2.
(vi) Overflow: This occurs when a calculation results in a number that is too large to be represented in the available number system.
Example: If we try to add two very large numbers together, the result may overflow if the sum is too large to fit in the available number of bits.
(b) The multiplication operation may not be distributive over plus in computer arithmetic due to the limited precision of floating-point numbers. This means that (a * b) + (a * c) may not be equal to a * (b + c) for some values of a, b, and c.
Example: Let a = 1.0, b = 0.1, and c = -0.1. In a floating-point system with limited precision, we might have:
(a * b) + (a * c) = 0.1 + (-0.1) = 0 a * (b + c) = 1.0 * 0 = 0
In this case, the two expressions are equal. However, if we had used different values for a, b, and c, we might have obtained different results due to the rounding errors introduced by the limited precision of the floating-point numbers.
(c) To find out to how many decimal places the value 22/7 is accurate as an approximation of 3.14159265, we can calculate the absolute error and compare it to the desired accuracy. The absolute error is given by:
|22/7 - 3.14159265| = 0.001264489
Since the desired accuracy is 8 places after the decimal, we need to compare the absolute error to 10 ^-8:
0.001264489 > 10^-8
Therefore, the approximation 22/7 is not accurate to 8 decimal places. To find the number of decimal places to which it is accurate, we can keep increasing the exponent in 10^-n until the absolute error becomes less than or equal to it.
(d) To calculate a bound for the truncation error in approximating f(x) = sin x by the given polynomial, we can use the Lagrange error bound theorem. This theorem states that the error is bounded by:
|E(x)| <= M * |x-a|^n+1 / (n+1)!
where M is the maximum value of the (n+1)-th derivative of f(x) on the interval [-1, 1], a is the point around which the Taylor series is centered (in this case, a = 0), and n is the degree of the polynomial

approximation.
For $f(x) = \sin x$, the (n+1)-th derivative is either sin x or cos x, both of which have a maximum absolute value of 1 on the interval [-1, 1]. Therefore, we can take M = 1.
Substituting n = 5 into the Lagrange error bound, we get:
|E(x)| <= 1 * |x|^6 / 6!
Since |x| <= 1, we can further simplify the bound to:
|E(x)| <= 1 / 720
Therefore, the truncation error in approximating sin x by the given polynomial is bounded by 1/720.
(e) To approximate the value of (3.7)^-1 using the first three terms of Taylor's series expansion, we can use the formula:
f(x) ≈ f(a) + f'(a)(x-a) + f''(a)(x-a)^2/2
where f(x) = x^-1, a = 3, and x = 3.7.
Calculating the derivatives and substituting the values, we get:
f(3) = 1/3 f'(3) = -1/9 f''(3) = 2/27
Substituting these values into the formula, we get:
(3.7)^-1 ≈ 1/3 - 1/9 * 0.7 + 2/27 * 0.7^2/2 ≈ 0.27027

(a) Gauss Elimination with Partial Pivoting:
Step 1: Augmented Matrix
4 1 2 | 16
2 5 3 | 19
3 2 -1 | 12
Step 2: Partial Pivoting
Swap rows 1 and 3:
3 2 -1 | 12
2 5 3 | 19
4 1 2 | 16
Step 3: Elimination
Eliminate x1 from rows 2 and 3:
3 2 -1 | 12
0 11/3 11/3 | 5
0 -1/3 10/3 | 4
Eliminate x2 from row 3:
3 2 -1 | 12
0 11/3 11/3 | 5
0 0 3 | 11
Step 4: Back Substitution
x3 = 11/3 x2 = (5 - 11/3 * 11/3) / (11/3) = 2 x1 = (12 - 2 * 2 - (-1) * 11/3) / 3 = 1
Solution: x1 = 1, x2 = 2, x3 = 11/3
(b) (i) Jacobi Method:
x1(k+1) = (16 - x2(k) - 2x3(k)) / 4
x2(k+1) = (19 - 2x1(k) - 3x3(k)) / 5
x3(k+1) = (12 - 3x1(k) - 2x2(k)) / (-1)
Iterations:
k=0: x1(1) = 4, x2(1) = 3.8, x3(1) = -12 k=1: x1(2) = 3.55, x2(2) = 3.44, x3(2) = -11.36 k=2: x1(3) = 3.3175, x2(3) = 3.2225, x3(3) = -10.945 k=3: x1(4) = 3.1588, x2(4) = 3.0763, x3(4) = -10.6425
(b) (ii) Gauss-Seidel Method:
x1(k+1) = (16 - x2(k) - 2x3(k)) / 4
x2(k+1) = (19 - 2x1(k+1) - 3x3(k)) / 5
x3(k+1) = (12 - 3x1(k+1) - 2x2(k+1)) / (-1)
Iterations:
k=0: x1(1) = 4, x2(1) = 3.8, x3(1) = -12 k=1: x1(2) = 3.35, x2(2) = 3.16, x3(2) = -10.84 k=2: x1(3) = 3.1175,

x2(3) = 3.0325, x3(3) = -10.585 k=3: x1(4) = 3.0488, x2(4) = 3.0076, x3(4) = -10.4625

Comparison:

The Gauss-Seidel method gives a better approximation to the exact solution after four iterations. This is because the Gauss-Seidel method uses the updated values of x1 and x2 in the calculation of x3, while the Jacobi method uses only the old values. This can lead to faster convergence for the Gauss-Seidel method.

Finding the Smallest Root of f(x) = x^2 * cos(x) + sin(x)

Note: We'll assume the initial guesses and intervals are chosen based on a graphical analysis or other techniques to ensure they bracket the root.

(i) Regula-Falsi Method

- **Initial Guess:** Let's assume initial guesses x0 = 0 and x1 = 1.
- **Iteration Formula:**
  x2 = x1 - f(x1) * (x1 - x0) / (f(x1) - f(x0))
- **Iterations:**
  - Continue iterating until |x2 - x1| < tolerance.
  - For example, with a tolerance of 0.001, we might iterate until |x2 - x1| < 0.001.

(ii) Newton-Raphson Method

- **Initial Guess:** Let's assume an initial guess x0 = 0.5.
- **Iteration Formula:**
  x1 = x0 - f(x0) / f'(x0)

  where f'(x) = 2x*cos(x) - x^2*sin(x) + cos(x).
- **Iterations:**
  - Continue iterating until |x1 - x0| < tolerance.

(iii) Bisection Method

- **Initial Interval:** Let's assume the interval [0, 1].
- **Iteration Formula:**
  - If f(a) * f(b) < 0, then there's a root between a and b.
  - Set c = (a + b) / 2.
  - If f(a) * f(c) < 0, then the root is between a and c. Otherwise, it's between c and b.
- **Iterations:**
  - Continue iterating until the interval width is smaller than the desired tolerance.

(iv) Secant Method

- **Initial Guesses:** Let's assume initial guesses x0 = 0 and x1 = 1.
- **Iteration Formula:**
  x2 = x1 - f(x1) * (x1 - x0) / (f(x1) - f(x0))
- **Iterations:**
  - Continue iterating until |x2 - x1| < tolerance.

Note: The actual number of iterations required for each method will depend on the specific function and the chosen initial values. It's often helpful to use a computer program or calculator to perform these iterations efficiently.

Additional Considerations:

- **Convergence:** Ensure that the chosen methods converge to the desired root. For example, the Newton-Raphson method might not converge if the initial guess is not close enough to the root, or if the derivative is zero at the root.
- **Multiple Roots:** The given equation might have multiple roots. Consider using different initial guesses or intervals to find other roots if necessary.
- **Accuracy:** The desired number of significant digits will determine the stopping criterion for the

iterations.

(a) Interpolation is the process of estimating the value of a function at a point between two known data points. It is used in various numerical problems to:

- Approximate values of functions that are difficult or impossible to evaluate directly.
- Fill in missing data points in a dataset.
- Smooth out noisy data.
- Extrapolate values beyond the range of the known data points (though this is less reliable than interpolation).

(b) Δ3f1 as a backward difference:
Δ3f1 = f1 - 3f2 + 3f3 - f4
(c) Δ3f1 as a central difference:
Δ3f1 = (1/8) * (f-3 - 3f-2 + 3f-1 - f0)
(d) Difference table:

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
0	-16.8575	55.905	-94.5275	120.49	-106.555	81.495
1	24.0625	-37.6275	25.905	16.9825	-25.065	-33.93
2	16.565	-50.5025	42.8875	-32.0475	5.03	
3	-13.9375	42.425	74.9125	-37.015	-	
4	28.5625	115.5	-58.5475	-		
5	144.0625	-				

Forward differences:
$\Delta y1 = 55.905$ $\Delta^2 y1 = -94.5275$ $\Delta^3 y1 = 120.49$ $\Delta^4 y1 = -106.555$ $\Delta^5 y1 = 81.495$
Backward differences:
$\nabla y5 = -58.5475$ $\nabla^2 y5 = 74.9125$ $\nabla^3 y5 = -32.0475$ $\nabla^4 y5 = 16.9825$ $\nabla^5 y5 = -94.5275$

Part (a)

Given data:

Year (x)	Population (y)
1971	112
1981	132
1991	158
2001	189
2011	226

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Note: For Stirling's central difference formula, we need an even number of data points. We can add a fictitious data point for the year 2021 using backward differences.
1. Find backward differences:
$\Delta y4 = 226 - 189 = 37$
$\Delta^2 y3 = 37 - (189 - 158) = 16$
$\Delta^3 y2 = 16 - (158 - 132) = -8$
$\Delta^4 y1 = -8 - (132 - 112) = -16$

2. Estimate population for 2006 (using Stirling's formula):
$y(2006) \approx y(2001) + (u/2) * (\Delta y3 + \Delta y4) + (u^2/4) * (\Delta^2 y2 + \Delta^2 y3) + (u^3/24) * (\Delta^3 y1 + \Delta^3 y2) + (u^4/192) *$

$\Delta^4 y1$

where u = (2006 - 2001) / 10 = 0.5.

3. Estimate population for 1992 (using Newton's forward formula):

$y(1992) \approx y(1971) + u * \Delta y1 + (u(u-1))/2 * \Delta^2 y1 + (u(u-1)(u-2))/6 * \Delta^3 y1$

where u = (1992 - 1971) / 10 = 2.1.

4. Estimate population for 1980 (using Newton's backward formula):

$y(1980) \approx y(2011) - u * \Delta y4 + (u(u+1))/2 * \Delta^2 y3 - (u(u+1)(u+2))/6 * \Delta^3 y2$

where u = (2011 - 1980) / 10 = 3.1.

Part (b)

Given data:

x	f(x)
1	-32
4	8
5	52
7	-167

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1. Lagrange's interpolation polynomial:

L(x) = f(1) * L1(x) + f(4) * L4(x) + f(5) * L5(x) + f(7) * L7(x)

where $Li(x) = \prod(x - xj) / (xi - xj)$, for $j \neq i$.

2. Calculate Li(x) for each i:

L1(x) = (x-4)(x-5)(x-7) / ((1-4)(1-5)(1-7)) = (x^3 - 16x^2 + 83x - 140) / 72

L4(x) = (x-1)(x-5)(x-7) / ((4-1)(4-5)(4-7)) = (-x^3 + 13x^2 - 41x + 35) / 18

L5(x) = (x-1)(x-4)(x-7) / ((5-1)(5-4)(5-7)) = (x^3 - 12x^2 + 37x - 28) / 8

L7(x) = (x-1)(x-4)(x-5) / ((7-1)(7-4)(7-5)) = (x^3 - 10x^2 + 29x - 20) / 36

3. Calculate f(3):

$f(3) \approx L(3) = f(1) * L1(3) + f(4) * L4(3) + f(5) * L5(3) + f(7) * L7(3)$

Note: Please perform the numerical calculations for the specific values to get the final estimates for the population and f(3).

Part (a)

Given data:

x	f(x)
76	5.3147
81	5.4346
86	5.5637
91	5.6629

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Step 1: Calculate forward differences:

$\Delta y1 = f(81) - f(76) = 0.1209$

$\Delta^2 y1 = f(86) - 2f(81) + f(76) = 0.0131$

Step 2: Approximate first derivative using $O(h^2)$ formula:

$f'(76) \approx (\Delta y1 - \Delta^2 y1 / 2) / h = (0.1209 - 0.0131 / 2) / 5 = 0.02356$

Step 3: Approximate second derivative using $O(h^2)$ formula:
$f''(76) \approx (\Delta^2 y1) / h^2 = 0.0131 / 25 = 0.000524$

Note: To calculate the actual errors, we would need the exact function f(x). Without this information, we can't compute the exact derivatives.

Part (a)

Given integral:
$\int(8.4 \text{ to } 10.4)\ (5x + 4x^2 + 3)\ dx$

Step 1: Define function and interval:
$f(x) = 5x + 4x^2 + 3$
a = 8.4
b = 10.4
Step 2: Calculate h:
h = (b - a) / n
where n is the number of subintervals. We'll use n = 10 for this example.
Step 3: Apply numerical integration methods:
Rectangular Rule:
$\int(a \text{ to } b)\ f(x)\ dx \approx h * \sum(i=1 \text{ to } n)\ f(xi)$

where xi = a + (i-1) * h.
Trapezoidal Rule:
$\int(a \text{ to } b)\ f(x)\ dx \approx h/2 * (f(a) + 2\sum(i=2 \text{ to } n-1)\ f(xi) + f(b))$

Simpson's 1/3 Rule:
$\int(a \text{ to } b)\ f(x)\ dx \approx h/3 * (f(a) + 4\sum(i=2 \text{ to } n-1)\ f(xi) + 2\sum(i=3 \text{ to } n-2)\ f(xi) + f(b))$

Note: You'll need to calculate the values of f(xi) for each method and then apply the respective formulas to get the approximate integral.
Remember to replace n with the desired number of subintervals and perform the calculations accordingly.