$$\varepsilon_{\alpha\beta}(\omega) = \varepsilon_{\alpha\beta}^{\infty} + \frac{1}{v_0} \sum_{j} \frac{S_{\alpha\beta}^{j}}{\left(\Omega_{\mathbf{0}j}^{\text{SCPH+B}}\right)^2 - \omega^2 - 2\Omega_{\mathbf{0}j}^{\text{SCPH+B}} \Sigma^{\text{B+4ph}}(\omega)}$$

SCPH

SCPH+B

Post-process

Real part of Σ

Consider Tadpole and Loop Get SCPH frequencies ω^{SCPH}

Consider Real part of Bubble Get SCPH+B frequencies $\Omega^{\text{SCPH+B}}$

Imaginary part of Σ

Modeoscillator strength Get imaginary part of Bubble and 4ph

$$\Sigma^{\text{B+4ph}}(\omega) = \text{Im } \Sigma^{\text{B}}[G^{\text{SCPH+B}}, \Phi_3](\omega) + \text{Im } \Sigma^{\text{4ph}}[G^{\text{SCPH+B}}, \Phi_4](\omega)$$

Use Born effective charges and SCPH+B eigenvectors to get ${\cal S}$