# Model Predictive Control

Litterature Notes

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January 7, 2024

## 1 A basic formulation

We assume that state values cannot be measured, thus we need an observer. So we use state estimates $\hat{x}(k|k)$ of $x(k|k)$ indicating that it is a measurement that is based on all states up until time $k$
It is based on $u(k-1)$ and not $u(k)$ since that input has not been determined yet.
$\hat{u}(k+i|k)$ denotes the future values at time $k+i$ on input $u$ which is assumed on time $k$. This means that i is some horizon, and $u(k+j|k), j = 0, 1, \ldots, i-1$ is the inputs at each time step.
we have the cost function:

$$V = \sum_{i=H_w}^{H_p} ||\hat{z}(k+i|k) - r(k+i|k)||^2_{Q(i)} + \sum_{i=0}^{H_u-1} ||\Delta\hat{u}(k+i|k)||^2_{R(i)} \tag{1}$$

where $r(k+i|k)$ is a reference trajectory and $\hat{z}(k+i|k)$ is the controlled outputs. The prediction horizon has length $H_p$ but $H_w$ indicates the prediciton window, which determines when to start penalizing. If $H_w > 1$ then we only penalize from that point forward, as there may be some delay between control inputs and effects. $H_u$ is the control horizon, where $H_u \leq H_p$ and future control differences between $\Delta\hat{u}(k+i|k) = 0$ and $\Delta\hat{u}(k+i|k)$ where $i > H_u$. Note that the cost function in (1) only penalizes changes in $u$ and not $u$ itself. The matrices $Q(i)$ and $R(i)$ are weights and both positive semidefinite $(\cdot) \geq 0$

## 2 Constraints

There are different constraints, which are assumed to hold over the entire control- and prediction horizon.

$$E\ vec(\Delta\hat{u}(k|k), \ldots, \Delta\hat{u}(k+H_u-1|k), 1) \leq vec(0) \tag{2}$$
$$F\ vec(\Delta\hat{u}(k|k), \ldots, \hat{u}(k+H_u-1|k), 1) \leq vec(0) \tag{3}$$
$$G\ vec(\hat{z}(k+H_w|k), \ldots, \hat{z}(k+H_p|k), 1) \leq vec(0) \tag{4}$$

where $E, F$, and $G$ are matrices of suitable dimensions. (2) can be used to represent actuator slew rate (change of input of an actuator), actuator ranges (3), and control variable constraints on $z$ based on (4)