Robot Mobility - Discrete time Systems

Rasmus Pedersen

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Exercise 1

Consider the system

$$\dot{x} = \begin{bmatrix} -5 & -4 \\ 1 & -5 \end{bmatrix} x + \begin{bmatrix} 2 \\ 3 \end{bmatrix} u \tag{1}$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \tag{2}$$

1. Design a feedback control law that places the poles at $\{-1, -2\}$.
2. Design a full order observer for the system placing the observer poles at $\{-6, -9\}$.
3. Simulate the system using an ode function in MATLAB. Use `ode45()` in MATLAB.
4. Introduce a reference and design an integral control law for the system and simulate its behavior.
5. Design reference feed-forward (Use the formulation illustrated in Fig. 8, in John's lecture note) and simulate.
6. Find the bandwidth of the closed loop system, you can use `bandwidth()` in MATLAB. What should the sampling frequency at minimum be, according to theory? and what should it be in practice?
7. Discretize the system with the chosen sampling frequency.
8. Repeat the above steps for the discrete system. Note: when simulating you can now do this in e.g., a `for` loop.
9. Compare the continuous and discrete time simulations.
10. Create a simulation where the plant is continuous and the control is discrete. This can be done in plain MATLAB code or in Simulink.
11. Assume that the input to the system is constrained to be $-0.1 \leq u \leq 0.1$. Design an integral anti-windup strategy.

Exercise 2 - Extra! Kalman Filter

Notice that this exercise is not part of the curriculum, but included for your amusement and eager to learn :-). Let a discrete time system be given by

$$x(k+1) = \begin{bmatrix} 0.3 & -0.2 & -0.8 \\ -0.2 & 0.3 & -0.8 \\ 0 & 0 & 0.9 \end{bmatrix} x(k) + \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} u(k) + w(k) \tag{3}$$

$$y = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix} + v(k) \tag{4}$$

where

$$w(k) \sim \mathcal{N}(0, Q), \quad v(k) \sim \mathcal{N}(0, R), \tag{5}$$

with

$$Q = \begin{bmatrix} 0.7071 & 0 & 0 \\ 0 & 0.7071 & 0 \\ 0 & 0 & 0.7071 \end{bmatrix}, \quad R = \begin{bmatrix} 0.7071 & 0 \\ 0 & 0.7071 \end{bmatrix}. \tag{6}$$

Let the sampling time be given as $T_s = 0.1$ and the input to the system be given by

$$u = \sin(t/T_s) + 2\cos(t/T_s). \tag{7}$$

1. Is the system observable?
2. Simulate the system response for 10 seconds in a for loop. You can define the time vector in MATLAB as `t=[0:Ts:10]`.
3. Design a discrete time Luenberger observer for the system and simulate the response.
4. Design a Kalman filter for the system and simulate the response.
5. Compare the Luenberger observer with the Kalman filter. Use the error between real state and estimates of it.