Robot Mobility - Observer Design

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Exercise 1

Consider the same satellite system from last exercise session, as sketched in Figure 1.

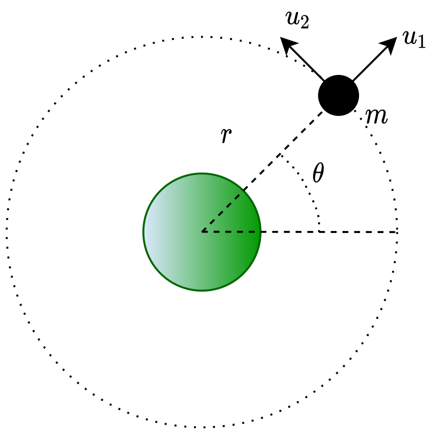

Figure 1: Sketch of satellite in circular orbit.

The satellite is of mass m with thrust in the radial direction u_1 and in the tangential direction u_2. The states and inputs are given by

$$x = \begin{bmatrix} r \\ \theta \\ \dot{r} \\ \dot{\theta} \end{bmatrix}, \quad u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \tag{1}$$

Linearized in the operating point $\bar{x} = \left(\bar{r}, \bar{\theta}, 0, \bar{\dot{\theta}}\right)$, the linear state space system is given by

$$\dot{x} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 3\bar{\dot{\theta}}^2 & 0 & 0 & 2\bar{r}\bar{\dot{\theta}} \\ 0 & 0 & -2\bar{\dot{\theta}}/\bar{r} & 0 \end{bmatrix} x + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1/m & 0 \\ 0 & 1/(m\bar{r}) \end{bmatrix} u \tag{2}$$

The output is given by

$$y = Cx, \tag{3}$$

Where we need to design the output matrix C. Hence, the following exercises:

1. Can we estimate all states of the satellite by measuring only r?
2. Can we estimate all states of the satellite by measuring only $\dot{\theta}$?
3. Can we estimate all states of the satellite by measuring both r and $\dot{\theta}$?
4. What does it take for us to be able to estimate all states?

Exercise 2

Consider the system

$$\dot{x} = \begin{bmatrix} -5 & -4 \\ 1 & -5 \end{bmatrix} x + \begin{bmatrix} 2 \\ 3 \end{bmatrix} u \tag{4}$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \tag{5}$$

1. Is the system controllable and observable?
2. Design a feedback control law that places the poles at $\{-1, -2\}$.
3. Design a full order observer for the system placing the observer poles at $\{-6, -9\}$.
4. Draw a block diagram for an observer based compensator using both L and K.
5. Verify system design by computing the eigenvalues of the closed loop system.
6. Implement the system in Simulink and simulate it's behavior. Choose initial conditions as you like.
7. Compare the system response to the same initial conditions, but with full state feedback.
8. Introduce a reference and design reference feed-forward for both systems.
9. Assume that the input to the system is constrained to be $-0.1 \leq u \leq 0.1$. Design an integral anti-windup strategy.

Exercise 3

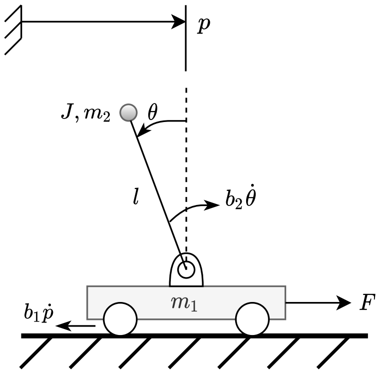

Figure 2: Inverted pendulum on a cart.

Consider the pendulum in Figure 2 with state $x = \begin{bmatrix} p & \theta & \dot{p} & \dot{\theta} \end{bmatrix}^T$. When linearized in the upright position $\bar{x} = (0, 0, 0, 0)$, the linear system matrices are given by

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{gl^2 m_2^2}{J_t M_t - m_2^2 l^2} & \frac{-b_1 J_t}{J_t M_t - m_2^2 l^2} & \frac{-b_2 l m_2 J_t}{J_t M_t - m_2^2 l^2} \\ 0 & \frac{M_t m_2 g l}{J_t M_t - m_2^2 l^2} & \frac{-b_1 l m_2}{J_t M_t - m_2^2 l^2} & \frac{-b_2 M_t}{J_t M_t - m_2^2 l^2} \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ \frac{J_t}{J_t M_t - m_2^2 l^2} \\ \frac{l m_2}{J_t M_t - m_2^2 l^2} \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}. \tag{6}$$

Use the following parameter values: $m_1 = 2$, $m_2 = 1$, $J = 0.01$, $l = 1$, $b_1 = b_2 = 0.1$, $g = 9.81$. Remember that $M_t = m_1 + m_2$ and $J_t = J + m_2 l^2$.

1. Is the system observable?
2. Design a feedback control law for the system.
3. Design an observer for the system.
4. Implement the controller and observer on the non-liner simulation model of the pendulum.
5. Compare the response to the case with full state feedback.
6. Introduce a reference and design an integral control law for tracking the reference.

Exercise 4

Consider the system

$$\dot{x} = \begin{bmatrix} 2 & -3 \\ 4 & -5 \end{bmatrix} x + \begin{bmatrix} 2 \\ 3 \end{bmatrix} u \tag{7}$$

$$y = \begin{bmatrix} -3 & 2 \end{bmatrix} \tag{8}$$

1. Is the system controllable and observable?
2. Design a feedback control law that places the poles at $\{-3, -4\}$.
3. Design a full order observer for the system placing the observer poles at $\{-9, -12\}$.
4. Simulate system's behavior. Choose initial conditions as you like.
5. Compare the system response to the same initial conditions, but with full state feedback.
6. Introduce a reference and design reference feed-forward for both systems.