Robot Navigation: Lecture 5
Lecture Notes

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1 Recap

We looked at odometry: visual, wheel
Map based localistaion.

- Markov/bayssian filter
- Particle filter

There is two update rules:

- Action update:
  $P(x_{n+1} = \sum_{x_k} P(x_{n+1}|x_n, \hat{u_n}) \cdot P(x_n)$
- Perception update: $P(x_{n+1}) = \zeta \cdot P(o_{n+1}|x_{n+1})P(x_{n+1})$

When there is a bar over like this : $\overline{P(x_{n+1})}$ illustrates that this is the result of the action update.
Focussing on the robot model:
It is usually a difference eq, or a differential equation.

$$\dot{x} = g(x, u)$$

$$x_{n+1} = x_n + dT \cdot g(x_n, u_n) + x_n + dT \cdot g(x_n, \hat{u_n} + \Delta u_n)$$

When doing particle filters and assigning weights to the particles, a suitable function is the perception function.

2 Kalman filter Localisation

let x be a random variable with a gaussian distribution with mean μ The estimate $\hat{x}$ is an estimate of x
This is a quality measure of an estimate. $E((x - \hat{x})^2) = E((x - \mu_x + (\mu_x - \hat{x})^2)$
Expanding the estimated values, since the E is a linear operator:

$$\begin{aligned} E((x-\mu_x)^2) + E((\mu_x - \hat{x})^2) + 2E((x-\mu_x)(\mu_x - \hat{x})) \\ = E((x-\mu_x)^2) + t((\mu_x - \hat{x})^2) \end{aligned}$$

2.1 Assumptions

The kalman filter is not optimal.
TODO: Add paranthesis closing
The kalman gain is the middle part.

$$\hat{q} = \hat{q_1} + \frac{\sigma_2^2}{\sigma_1^2 + \sigma_2^2}(\hat{q_2} - \hat{q_1}) \tag{1}$$

Note that in this equation, where σ_1 is much larger, this means that we trust the second distribution (σ_2) which has a lower variance. We have one belief, and one piece of information.
This is only for the onedimensional case.
The n-dimensional case is:

$$\hat{q} = q_1 + P(P + R)^{-1}(q_2 - q_1) \tag{2}$$

where P and R are covariance matrices for q_1 and q_2 respectively.

3 Non linear Kalman filtering

The equations: a linear model

$$x_{n+1} = Ax_n + B(\hat{u_n} + \Delta u_n) = Ax_b n + Bu_n + B \cdot \Delta u_n = Ax_n + Bu_n + \epsilon_n \tag{3}$$

where ϵ is the gross noise.
Then the prediction:

$$\overline{x} = A\overline{x}_n + Bu_n \tag{4}$$

and the measurement noise is given as:

$$y_n = C \cdot x_n + \nu \tag{5}$$

and the kalman

$$\overline{x}_{n+1} + k(y_{n+1} - C\overline{x}_{n+1}) \tag{6}$$

If we compute the error

$$\begin{aligned} \hat{x}_{n+1} - x_{n+1} = A\overline{x}_n - Bu_n - Ax_n + Bu_n - \epsilon_n \\ +KCx_{n+1} + K\nu_{n+1} - KC(A\overline{x}_n + Bu_n) \\ A\overline{x}_n + Bu_n - Ax_n - Bu_n \cdot \epsilon_n + KC(Ax_n + Bu_n + \epsilon_n) + K\nu_{n+1} - KCA\hat{x}_n - KCBu_n \end{aligned} \tag{7}$$