Analysis Work Sheet Final

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1 Sequences

1.1 Q1

Find the formula for the n-th term of the sequence $1, -4, 9, -16, 25$.

Answer: Given $1, -4, 9, -16, 25$, we can clearly see that it's a sequence of squares with an alternating negative sign,

$$a_n = (-1)^{n-1}(n^2), \text{ for } n \geq 1.$$

1.2 Q2

Find a formula for the nth term of the sequence in terms of n, where the sequence is $1, 0, 1, 0, 1, \ldots$.

Answer: The sequence is a periodic one which oscillates from 1 to 0. Let us create a table of values to assist in this,

1	2	3	4	5	6
1	0	1	0	1	0
odd	even	odd	even	odd	even

We can see that when n is even, we get a 0, and else is 1. This implies we will have a formula making use of $n/2$.

Let $a_n = n/2 : a_n = 1/2, 1, 3/2, 2, 5/2, \ldots$. It seems that we get closer to our desired result if we let $a_n = (n+1)/2 : a_n = 1, 3/2, 2, 5/2, 3, \ldots$.

I have just realized that the most suitable function would be $\sin(x)$,

$$a_n = \sin\left(\frac{n\pi}{2}\right)^2.$$

1.3 Q3

Determine if the sequence $\{a_n\}$ converges or diverges. Find the limit if the sequence converges. The sequence is $a_n = 4 + (0.3)^n$

Answer: