# NSF Late Summer Exam 2023

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## Contents



# 1 Question 1

Let $A = \{1, 3, 5, 6, 8\}$, and let $B = \{2, 3, 4, 8, 9\}$

## 1.1 Q1 a

$A \cap B$.

*Answer*: $A \cap B = \{3, 8\}$

## 1.2 Q1 b

$\{b + 2 : b \in B \text{ and } b - 2 \notin A\}$.

*Answer*: $\{4, 6, 11\}$

## 1.3 Q1 c

The number of 3 element subsets of $B$.

*Answer*: For each element, there are 3 possible arrangements (not including order - order is of no concern), and in $B$ there are 5 elements so therefore $3 \times 5 = 15$. ✗

*Real answer*: The problem with my method is that the number of possible sets decrease as we move along,

Given $B = \{2, 3, 4, 8, 9\}$,

Taking 2 first: $\{2, 3, 4\}, \{2, 4, 8\}, \{2, 8, 9\}, \{2, 4, 9\}, \{2, 3, 9\}, \{2, 3, 8\}$,

Taking 3 first: $\{3, 4, 8\}, \{3, 8, 9\}, \{3, 4, 9\}$,

Taking 4 first: $\{4, 8, 9\}$.

10 3-elemented sets possible.

## 1.4 Q1 d

The number of proper subsets of $B$.

*Answer*: The number of proper subsets is also $\mathcal{P}(B)$. There are 5 1-elemented sets, 20 2-elemented sets, 15 3-elemented sets, 10 4-elemented subsets, so therefore $\mathcal{P}(B) = 5 + 20 + 15 + 10 = 60$. ✗

*Real answer*: