Probability and Statistics January Exam 2024

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July 29, 2024

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1 Question One

Assume the number of buses passing through a certain stop in 1 hour follows a Poisson distribution with average $\lambda = 5.5$. Calculate the probability that at least one bus turns up in 1 hour.

Answer: Recall that the Poisson distribution is (?). ✗

Real Answer: the Poisson distribution given $\bar{x} = \mu = \mathbb{E}(X) = \lambda$, and the $\mathbb{P}(X = k)$ is given by

$$\frac{\lambda^k e^{-\lambda}}{k!}.$$

One should consider the fact that the events are assumed to be independent. In light of this, given $\lambda = 5.5$ and $\mathbb{P}(X \geq 1) = 1 - \mathbb{P}(X = 0)$, we get,

$$1 - \frac{5.5^0 e^{-5.5}}{0!} = 0.995913... \approx 0.995,\ \ 99.5\%$$

2 Question Two

You roll a fair dice once

2.1 Q2 a

Calculate the probability of getting the number six.

Answer:

$$\mathbb{P}(X = 6) = \frac{|\{6\}|}{|\Omega|} = \frac{|\{6\}|}{|\{1, 2, 3, 4, 5, 6\}|} = \frac{1}{6}. ✓$$

2.2 Q2 b

You roll a fair dice n times. Calculate the probability of getting at least one six.

Answer: To begin our investigation, let $n = 1 : \mathbb{P}(X = 6) = \frac{1}{6},\ n = 2 : \mathbb{P}(X = 2) = \frac{1}{36},\ n = 3 : \mathbb{P}(X = 6) = \frac{1}{216}$, etc. We can clearly see a climbing in the powers of six where the index linearly increases with n.

We may therefore deduce that for n-times, $\mathbb{P}(X = 6) = 6^{-n}$ ✗.

Real answer: I simply misunderstood the question. They ask at least one six, implying that there maybe more than 1 six. Therefore, we don't want to calculate $\mathbb{P}(X = 6)$, as It's actually not relevant to the question. We want to calculate $\mathbb{P}(\text{At least 1 six})$. We could cleverly set $Y = (\text{number of sixes})$ and then state $\mathbb{P}(Y \geq 1) = 1 - \mathbb{P}(Y = 0)$.

$$1 - \mathbb{P}(Y = 0) = 1 - \left(\frac{5}{6}\right)^n.$$

2.3 Q2 c

Find the values of n such that this probability is greater than or equal to 50%.

Answer: given our formula $1 - \left(\frac{5}{6}\right)^n$, we want,

$$\begin{aligned}
1 - \left(\frac{5}{6}\right)^n &\geq 0.5, \\
\left(\frac{5}{6}\right)^n &\geq 0.5, \\
\ln\left(\left(\frac{5}{6}\right)^n\right) &\geq \ln(0.5), \\
n &\geq \frac{\ln(0.5)}{\ln\left(\left(\frac{5}{6}\right)\right)}, \\
n &\geq 3.8016 \approx 3.8 \approx 4. \checkmark
\end{aligned}$$