Number, Sets and Functions 2023 Exam

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1 Question One

Let $X = \{1, 3, 4, 6, 9\}$ and let $Y = \{2, 3, 5, 8, 9\}$.
Write down each of the following sets. *No justification is needed*,

(a) $X \cup Y$,
answer: $\{1, 2, 3, 4, 5, 6, 8, 9\}$. ✓

(b) $X \triangle Y$,
answer: $\{1, 2, 4, 5, 6, 8\}$. ✓

(c) $\{X \in x : x + 2 \notin X\}$,
answer: $\{3, 6, 9\}$. ✓

(d) $\{y + 2 : y \in Y \wedge y - 2 \in X\}$,
answer: $\{5, 7, 10\}$. ✓

Write down the supremum of each of the following sets

(e) $\{x^2 : -2 \leq x \leq 1\}$,
answer: 1. ✗

Real answer is 4, Simply square -2.

(f) $\left\{\frac{n}{n+1} : n \in \mathbb{N}\right\}$,
answer: $\frac{n}{n+1} = \frac{1}{1+\frac{1}{n}}$,
$\lim_{n\to\infty} \frac{1}{1+\frac{1}{n}} = \frac{1}{1+0} = 1.$ ✓

(g) $\{\sin(x) : x \in \mathbb{Q}\}$,
answer: 1. ✓

2 Question Two

(a) Define precisely what it means for a function $f : A \to B$ to be injective.
answer: Injective means that both A and B are the same, i.e., the domain and codomain are the same. ✗

Real answer: $\forall\ a, b \in A$ if $f(a) = f(b) \implies a = b$.

(b) Define precisely what it means for a function $f : A \to B$ to be surjective.
answer: Surjective means ? ✗

Real answer: $\forall\ b \in B\ \exists\ a \in A : f(a) = b$.

Are the following injective or not?
(c) $f : \mathbb{Z} \to \mathbb{Z}, f(n) = 20n + 22$.
answer: given our definition of injectivity, $\forall\ m, n \in \mathbb{Z}, f(m) = 20m + 22 = 20n + 22 = f(n) \implies m = n$.
$f : \mathbb{Z} \to \mathbb{Z}, f(n) = 20n + 22$ is injective. ✓