SISTEMI DI MODULAZIONE ANALOGICA

Modello sistema

$a(t) \to [MOD] \to s_{Tx}(t) \to [CHANNEL] \to [\oplus w_{Rc}(t)] \to r(t) \to [BPF] \to r_{Mi}(t) \to [DEMOD] \to r_0(t) = a_0(t) + w_0(t)$, $r_{Mi}(t) = s_{Rc}(t) + w_{Rc}(t)$

SNR di uscita Λ_0

$\Lambda_0 = M_{a0}/M_{w0}$

SNR di riferimento Γ_{rif}

B_a banda di $a(t)$

$\Gamma = M_{sRc}/M_{wRc} = M_{sTx}/(N_0 B_a a_{Ch})$

$\Gamma = P_{sRc}/P_{wRc} = P_{sTx}/(KT_{eff,Rc}B_a a_{Ch})$

Prestazioni

$\Lambda_0 \leftrightarrow \Gamma$

MODULAZIONI LINEARI

Implementazione sistema

$a(t) \to [MOD] \to s_{Tx}(t) \to [CHANNEL] \to r(t) \to [DEMOD] \to a_0(t)$

MOD: $a(t) \to [\otimes(\cos(2\pi f_0 t + \varphi_0)] \to a_c(t) \to [BPF] \to s_{Tx}(t)$,

$a_c(t) = a(t)\cos(2\pi f_0 t + \varphi_0)$

$A_c(f) = \frac{1}{2}e^{j\varphi_0}A(f - f_0) + \frac{1}{2}e^{-j\varphi_0}A(f + f_0)$

$s_{Tx}(t) = (a_c * h_{PBF})(t)$

$S_{Tx}(f) = A_c(f)H_{PBF}(f)$

DEMOD: $s_{Tx}(t) \to [\otimes(\cos(2\pi f_0 t + \varphi_1)] \to u(t) \to [LPF] \to a_0(t)$

DSB-SC

Implementazione sistema

MOD: $a(t) \to [\otimes(\cos(2\pi f_0 t + \varphi_0)] \to s_{Tx}(t)$,

$s_{Tx}(t) = a(t)\cos(2\pi f_0 t + \varphi_0)$

$S_{Tx}(f) = \frac{1}{2}e^{j\varphi_0}A(f - f_0) + \frac{1}{2}e^{-j\varphi_0}A(f + f_0)$

DEMOD: $s_{Tx}(t) \to [\otimes(\cos(2\pi f_0 t + \varphi_1)] \to u(t) \to [LPF] \to a_0(t)$,

$a_0(t) = \frac{1}{2}a(t)\cos(\varphi_0 - \varphi_1)$

Banda del sistema B_s

$B_s = 2B_a$

Prestazioni

$\Lambda_0 = \Gamma$

SSB

Implementazione sistema

$\Pi(f) = \frac{1}{2}(1 + \mathrm{sgn}(f))$

$A^{(+)}(f) = A(f)\Pi(f)$, $A^{(-)}(f) = A(f)\Pi(-f)$, $A^{(-)}(f) = [A^{(+)}(-f)]^*$

$SSB_+ \leftrightarrow A^{(+)}(f)$, $SSB_- \leftrightarrow A^{(-)}(f)$

MOD SSB_+: $a(t) \to [\otimes(\cos(2\pi f_0 t + \varphi_0)] \to a_c(t) \to [BPF] \to s_{Tx}(t)$,

$H_{PBF}(f) = \{1$ per $|f| \in (f_0, f_0 + B)$, 0 per $|f| \in (f_0 - B, f_0)$, irrilev. altrove$\}$, $B = B_a$

$S_{Tx}(f) = \frac{1}{2}e^{j\varphi_0} A^{(+)}(f - f_0) + \frac{1}{2}e^{-j\varphi_0} A^{(-)}(f + f_0)$

Filtro di Hilbert

$H^{(h)}(f) = -j\,\mathrm{sgn}(f)$

$a^{(h)}(t) = \mathscr{F}^{-1}(A(f)H^{(h)}(f))$

$\langle a(t), a^{(h)}(t)\rangle = 0$, $M_{a(h)} = M_a$

HILBERT MOD SSB_+:

$(a(t), a(t)) \to (a(t),[H^{(h)}]) \to (a(t), a^{(h)}(t)) \to [\otimes(\cos(2\pi f_0 t + \varphi_0), -\sin(2\pi f_0 t + \varphi_0))] \to \oplus \to s_{Tx}(t)$

$S_{Tx}(f) = \frac{1}{2}[\frac{1}{2}e^{j\varphi_0} A(f - f_0) + \frac{1}{2}e^{-j\varphi_0} A(f + f_0)] - \frac{1}{2}[\frac{1}{2}e^{j\varphi_0} A^{(h)}(f - f_0) + \frac{1}{2}e^{-j\varphi_0} A^{(h)}(f + f_0)]$

$s_{Tx}(t) = \frac{1}{2}a(t)\cos(2\pi f_0 t + \varphi_0) - \frac{1}{2}a^{(h)}(t)\sin(2\pi f_0 t + \varphi_0)$

DEMOD: $s_{Tx}(t) \to [\otimes(\cos(2\pi f_0 t + \varphi_1)] \to u(t) \to [LPF] \to a_0(t)$

$U(f) = \frac{1}{4}e^{j(\varphi_0 - \varphi_1)}A^{(+)}(f) + \frac{1}{4}e^{-j(\varphi_0 - \varphi_1)}A^{(-)}(f) + \frac{1}{4}e^{j(\varphi_0 + \varphi_1)}A^{(+)}(f - 2f_0) + \frac{1}{4}e^{-j(\varphi_0 + \varphi_1)}A^{(-)}(f + 2f_0)$

$a_0(t) = \frac{1}{4}a(t)\cos(\varphi_0 - \varphi_1) - \frac{1}{4}a^{(h)}(t)\sin(\varphi_0 - \varphi_1)$

Banda del sistema B_s

$B_s = B_a$

Prestazioni

$\Lambda_0 = \Gamma$

VSB

Implementazione sistema

MOD: $a(t) \to [\otimes(\cos(2\pi f_0 t + \varphi_0)] \to a_c(t) \to [BPF] \to s_{Tx}(t)$,

$H_{PBF}(f) : H_{PBF}(f + f_0) + H_{PBF}(f - f_0) = K$, $|f| \le B$, $B = B_a$

$S_{Tx}(f) = [\frac{1}{2}e^{j\varphi_0} A(f - f_0) + \frac{1}{2}e^{-j\varphi_0} A(f + f_0)]H_{PBF}(f)$

$H_{PBF}(f)$: $H_{PBF}(f) = \{1$ per $f_0 + \rho B < |f| < f_0 + B$, $\frac{1}{2}[1 + \sin(\frac{1}{2}\pi(|f| - f_0)/\rho B)]$ per $f_0 - \rho B < |f| < f_0 + \rho B$, 0 per $f_0 - \rho B < |f| < f_0 - B$, irrilev. altrove$\}$, $B(1 + \rho)$ banda occupata

DEMOD: $s_{Tx}(t) \to [\otimes(\cos(2\pi f_0 t + \varphi_1)] \to u(t) \to [LPF] \to a_0(t)$

$U(f) = \frac{1}{4}e^{j(\varphi_0 - \varphi_1)}A(f)H_{PBF}(f + f_0) + \frac{1}{4}e^{-j(\varphi_0 - \varphi_1)}A(f)H_{PBF}(f - f_0) + \frac{1}{4}e^{j(\varphi_0 + \varphi_1)}A(f - 2f_0)H_{PBF}(f - f_0) + \frac{1}{4}e^{-j(\varphi_0 + \varphi_1)}A(f + 2f_0)H_{PBF}(f + f_0)$

$\varphi_0 = \varphi_1 \Rightarrow A_0(f) = \frac{1}{4}A(f)[H_{PBF}(f + f_0) + H_{PBF}(f - f_0)] = \frac{1}{4}A(f)K$

Banda del sistema B_s

$B_s = B_a(1 + \rho)$

Prestazioni

$\Lambda_0 = \Gamma$

DSB-TC (AM)

Parametri

Valore min.: $a_m = -\min_t a(t)$

Indice di mod.: $m = a_m/A$

Segnale norm.: $\bar{a}(t) = a(t)/a_m$

Ass. dist. inviluppo $\Leftrightarrow a(t) + A \ge 0 \Leftrightarrow A \ge a_m \Leftrightarrow m \le 1$

Fatt. di forma: $k_f^2 = M_a/a_m^2$

Eff. di mod.: $\eta = \frac{1}{2}M_a/M_{sTx} = m^2k_f^2/(1 + m^2k_f^2)$

Implementazione sistema

MOD: $a(t) \to [\oplus A] \to [\otimes(\cos(2\pi f_0 t + \varphi_0)] \to s_{Tx}(t)$,

$s_{Tx}(t) = [a(t) + A]\cos(2\pi f_0 t + \varphi_0) = A[1 + m\bar{a}(t)]\cos(2\pi f_0 t + \varphi_0)$

DEMOD (coerente):

$s_{Tx}(t) \to [\otimes(\cos(2\pi f_0 t + \varphi_1)] \to u(t) \to [LPF] \to a_0(t)$,

$a_0(t) = \frac{1}{2}[a(t) + A]\cos(\varphi_0 - \varphi_1)$

DEMOD (non coerente):

$s_{Tx}(t) \to [|.|] \to d(t) \to [LPF] \to a_0(t)$,

$d(t) = |s_{Tx}(t)| = [a(t) + A]|\cos(2\pi k f_0 t + \varphi_0)| = 2/\pi + \sum c_k\cos(2\pi 2kf_0 t + \varphi_0)$, $k = -\infty, \ldots, +\infty$, $a_0(t) = (2/\pi)[a(t) + A]$

Banda del sistema B_s

$B_s = 2B_a$

Prestazioni

$\Lambda_0 = \eta\Gamma$

MODULAZIONI ANGOLARI

Parametri

$a(t)$ segnale di informazione

$S_{Tx}(t) = A\cos(\varphi_s)$

Ampiezza: A

φ ist.: $\varphi_s(t)$

f ist.: $f_s(t) = (1/2\pi)d\varphi_s(t)/dt$

φ nat.: $\varphi_N(t) = 2\pi f_0 t + \varphi_0$

f nat.: $f_N(t) = (1/2\pi)d\varphi_N(t)/dt = f_0$

Dev. φ ist.: $\Delta\varphi_s(t) = \varphi_s(t) - \varphi_N(t) = 2\pi\int\Delta f_s(\tau)d\tau$, $\tau = -\infty, \ldots, t$

Dev. f ist.: $\Delta f_s(t) = f_s(t) - f_N(t) = (1/2\pi)d\Delta\varphi_s(t)/dt$

$S_{Tx}(t) = A\cos(\varphi_s) = A\cos(2\pi f_0 t + \varphi_0 + \Delta\varphi_s)$

$a(t)$ segnale di informazione

Param. di mod.: K_F, K_P

Indice di mod.: β_F, β_P

Valore max.: $a_M = \max_t|a(t)| = a_m = -\min_t a(t)$

PM

$\Delta\varphi_s(t) = K_P a(t)$

$S_{Tx}(t) = A\cos(2\pi f_0 t + \varphi_0 + K_P a(t))$

$\beta_P = \max_t|\Delta\varphi_s(t)| = K_P a_M = (K_P\sqrt{M_a})/k_f$

Banda del sistema B_s

$B_s = 2B_a(1 + \beta_P)$

Prestazioni

$\Lambda_0 = k_f^2\beta_P^2\Gamma$

FM

$\Delta f_s(t) = K_F a(t)$

$f_s(t) = f_0 + K_F a(t)$

$\Delta\varphi_s(t) = 2\pi K_F\int a(\tau)d\tau$, $\tau = -\infty, \ldots, t$

$S_{Tx}(t) = A\cos(2\pi f_0 t + \varphi_0 + 2\pi K_F\int a(\tau)d\tau$, $\tau = -\infty, \ldots, t)$

$\beta_F = (1/B)\max_t|\Delta f_s(t)| = K_F a_M/B = (K_F\sqrt{M_a})/Bk_f$

Max dev. di freq.: $\Delta F = \max_t|\Delta f_s(t)| = K_F a_M$

FM Narrowband

$d(t) = \int a(\tau)d\tau$, $\tau = -\infty, \ldots, t$

$D(f) = A(f)/j2\pi f$

$s_{Tx}(t) = A\cos(2\pi f_0 t + \varphi_0 + 2\pi K_F d(t)) = A\cos(2\pi f_0 t + \varphi_0)\cos(2\pi K_F d(t)) - A\sin(2\pi f_0 t + \varphi_0)\sin(2\pi K_F d(t))$

$K_F \ll 1/(2\pi\max_t d(t)) \Rightarrow s_{Tx}(t) = A\cos(2\pi f_0 t + \varphi_0) - A2\pi\sin(2\pi f_0 t + \varphi_0)$

$S_{Tx}(f) = \frac{1}{2}Ae^{j\varphi_0}\delta(f - f_0) + \frac{1}{2}Ae^{-j\varphi_0}\delta(f + f_0) + \frac{1}{2}Ae^{j\varphi_0}K_F A(f - f_0)/(f - f_0) + \frac{1}{2}Ae^{-j\varphi_0}K_F A(f + f_0)/(f + f_0)$

$B_s = 2B_a$

FM Wideband (Formula di Carson)

$B_s = 2B_a(1 + \beta_F) = 2(B_a + \Delta F) = 2B_a$ se $\Delta F \ll 1$

Implementazione sistema

MOD: omissis

DEMOD:

$s_{Tx}(t) \to [limitatore] \to s_L(t) \to [derivatore] \to s_d(t) \to [|.|] \to [LPF] \to a_0(t)$,

$s_L(t) = \cos(2\pi f_0 t + \varphi_0 + 2\pi K_F d(t))$,

$h_d(t) = (1/2\pi)k_d d/dt \leftrightarrow H_d(f) = jk_d f \Rightarrow s_d(t) = (1/2\pi)k_d ds_L(t)/dt = -k_d(f_0 + K_F a(t))\sin(2\pi f_0 t + \varphi_0 + 2\pi K_F d(t))$,

$a_0(t) = k_d|f_0 + K_F a(t)| = k_d f_0 + k_d K_F a(t)$ se $f_0 \gg K_F a_m$

Banda del sistema B_s

$B_s = 2B_a(1 + \beta_F)$

Prestazioni

$\Gamma_{th} = 20(1 + \beta_F)$

$\Lambda_0 = 3k_f^2\beta_F^2\Gamma$ per $\Gamma > \Gamma_{th}$

$\Lambda_0 = 3k_f^2\beta_F^2\Gamma/(1+ \Gamma_{th}/\Gamma)$ per $\Gamma < \Gamma_{th}$

SEGNALI

Banda pratica

Primo zero: $B = \{f > 0, H(f) = 0\}$

Ampiezza (dB): $B = \{f > 0, |H(f)|/H_0 = 10^{-A/20}\}$, con $A = 3, 4, 60$ dB

Energia: $\{f > 0, \int_0^B|H(f)|^2df/\int_0^{+\infty}|H(f)|^2df = p/100\}$, con $p = 90, 99, 99.9$

ESD

$\varepsilon_x(f) = |X(f)|^2$

PSD

$\wp_x(f) = \mathscr{F}(r_x(\tau))$, $r_x(\tau) = E[x(t + \tau)x^*(\tau)]$, $M_x = r_x(0)$, $\wp_{xy}(f) = X(f)Y^*(f)$

$\wp_{xy}(f) = \mathscr{F}(r_{xy}(\tau))$, $r_{xy}(\tau) = E[x(t + \tau)y^*(\tau)]$, $M_{xy} = r_{xy}(0)$

Heaviside conditions

1. $|H(f)| = H_0$, $f \in B$
2. $\arg H(f) = -2\pi f t_0$

MEZZI TRASMISSIVI

Linea

$(\tilde{a}_{Ch}(f))_{dB/km} = (\tilde{a}_{Ch}(f))_{dB/km}\sqrt{(f/f_1)}$

$(a_{Ch}(f))_{dB} = (\tilde{a}_{Ch}(f))_{dB/km}(d)_{km}$

Cavo coassiale

$(\tilde{a}_{Ch}(f))_{dB/km} = 3.45_{E-3}(a^{-1} + b^{-1})(\sqrt{f})/\log_{10}(b/a)$, $2a$ diam int, $2b$ diam ext

Fibra ottica

$\lambda f = c/n$, con n indice di rifrazione del mezzo ($n = 1$ nel vuoto)

Collegamento radio

Ant,Tx(→Ch→)Ant,Rc

Ant,Tx(: $\Phi_0 = P_{Tx}/4\pi d^2$ [W/m²], d distanza, antenna isotropica

$\Phi = \Phi_0 g_{Ant,Tx}$ [W/m²] antenna direzionale

Ant,Rc(: $P_{Rc} = \Phi A_{Ant,Rc}\eta_{Ant,Rc}$, $A_{Ant,Rc}$ area efficace, $\eta_{Ant,Rc}$ fattore di efficienza, condizioni di adattamento

$g_{Ant} = 4\pi A_{Ant}\eta_{Ant}/\lambda^2$

$(a_{Ch})_{dB} = 32.4 + 20\log_{10}(d)_{km} + 20\log_{10}(f_0)_{MHz} - (g_{Ant,Tx})_{dB} - (g_{Ant,Tx})_{dB}$

BIPOLI

$v_i(t), Z_S, Z_L, G(f)$

$v_i(t) \to [g_1] \to v_L(t)$

$P_v = E[v_L(t)i_L(t)] = r_{vLiL}(0) = \int\wp_{vLiL}(f)df = \int\Re[\wp_{vLiL}(f)]df = \int p_v(f)df$, $p_v(f) = \Re[\wp_{vLiL}(f)]$ densità di potenza elettrica

$\wp_{vLiL}(f) = V_L(f)I_L^*(f) = \wp_{vi}(f)Z_L/|Z_S + Z_L|^2$

$p_v(f) = \wp_{vi}(f)R_L/|Z_S + Z_L|^2$

Condizione di adatt per il max trasf di potenza

$Z_S = Z_L^* \Rightarrow p_v(f) = \wp_{vi}(f)/4R_S$

Rumore termico

$\wp_w(f) \simeq 2KTR$, $K = 1.3805_{E-3}$ [J/K] $\Rightarrow p_w(f) \simeq \frac{1}{2}KT$

RETI 2-PORTE

$v_i(t), Z_S, Z_1, Z_2, Z_L, G_1(f), G_2(f), G_L(f)$

$v_i(t)\to[g_1]\to v_1(t)\to[g_2]\to v_2(t)\to[g_L]\to v_L(t)$

Guadagno di potenza della rete

$g(f) = p_{v,out}(f)/p_{v,in}(f)$

Condizione di adatt per il max trasf di potenza

$Z_1 = Z_S^*$ e $Z_L = Z_2^*$

Temperatura di rumore

$p_{w,in}^{(S)}(f) = \frac{1}{2}KT_S(f)$

$p_{w,out}^{(S)}(f) = \frac{1}{2}KT_S(f)g(f)$

$p_{w,out}(f) = p_{w,out}^{(S)}(f) + p_{w,out}^{(A)}(f)$

$p_{w,out}^{(A)}(f) = \frac{1}{2}KT_A(f)g(f)$

$T_{eff,in}(f) = T_S(f) + T_A(f)$

$p_{w,in}(f) = \frac{1}{2}KT_{eff,in}(f)$

$p_{w,out}(f) = \frac{1}{2}KT_{eff,in}(f)g(f)$

Figura di rumore

$F(f) = p_{w,out}(f)/ p_{w,out}^{(A)}(f) = 1 + p_{w,out}^{(A)}(f)/ p_{w,out}^{(S)}(f) = 1 + T_A(f)/T_S(F) = 1 + T_A(f)/T_0$

Rete passiva $\Rightarrow F(f) = a(f)$

Cascata di reti 2-porte

$g(f) = g_1(f)g_2(f)...g_N(f)$

$T_A(f) = T_1(f) + T_2(f)/g_1(f) + ... + T_N(f)/g_1(f)...g_{N-1}(f)$

$F(f) = F_1(f) + (F_2(f)-1)/g_1(f) + ... + (F_N(f)-1)/g_1(f)...g_{N-1}(f)$

SNR out

$\Lambda_M = M_{sL}/M_{wL}$

$\Lambda_P = P_{s,out}/P_{w,out}$

Narrowband $\Rightarrow \Lambda_M = \Lambda_P$

$\Lambda_M = M_{sTx}/N_0Ba_{Ch}$

$\Lambda_P = P_{Tx}/KT_{eff,Rc}Ba_{Ch}$

Link budget

$(\Lambda)_{dB} = (P_{Tx})_{dBm} - (a_{Ch})_{dB} + 114 - 10\log_{10}(T_{eff,Rc}/T_0) - 10\log_{10}(B)_{MHz}$

SISTEMI DI MODULAZIONE DIGITALE

Rumore AWGN

$\underline{w} = [w_1, ..., w_I]$, $w_i \sim \aleph(0, \sigma_I^2) = \aleph(0, N_0/2)$, $i = 1, ..., I$

$p_{\underline{r}|a_n}(\underline{\rho}|n) = p_{\underline{w}}(\underline{\rho} - \underline{s}_m) = (1/(\pi N_0)^{I/2})\exp(-||\underline{\rho} - \underline{s}_m||^2/N_0)$

Regioni di Voronoi

$R_m = \{\underline{\rho} \mid m = \arg\max_{k \in \{1, ..., M\}} D(\underline{\rho};k)\}$

Criterio MAP

$\hat{a}_n = \arg\max_{k \in \{1, ..., M\}} D(\underline{\rho};k)$, $D(\underline{\rho};k) = p_{\underline{r}|a_n}(\underline{\rho}|n)p_m$, $p_m = P[a_n = \alpha_m]$

Rumore AWGN ⇒ per due simboli si e sj adiacenti la soglia si trova nel punto: $\rho_{ij} = (N_0/(2(s_j - s_i)))\ln(p_{an}(\alpha_i)/p_{an}(\alpha_j)) + \frac{1}{2}(s_j + s_i)$

Criterio ML

$\hat{a}_n = \arg\max_{k \in \{1, ..., M\}} p_{\underline{r}|an}(\underline{\rho}|n)$, $p_m = 1/M$, $m = 1, ..., M$

Criterio minima distanza

$\hat{a}_n = \arg\max_{k \in \{1, ..., M\}} d(\underline{\rho}, \underline{s}_n)$, $\underline{w}$ AWGN, $p_m = 1/M$, $m = 1, ..., M$

detector I: $r(t)\to[(\Psi_1,...,\Psi_I)]\to(\frown^{1(t0)},...,\frown^{I(t0)})\to(r_{n,1},...,r_{n,I})\to[||\underline{r} - \underline{s}_m||]\to(D_1,...,D_M)\to[\arg\max_m D_m]\to\hat{a}_n$, $\Psi_i(t) = \Phi^*_i(t_0 - t)$, $i = 1, ..., I$

detector II: $r(t)\to[(\xi_1,...,\xi_M)]\to(\frown^{1(t0)},...,\frown^{I(t0)})\to[\oplus(-\frac{1}{2}E_1,...,-\frac{1}{2}E_M)]\to(U_1,...,U_M)\to[\arg\max_m U_m]\to\hat{a}_n$, $\xi_m = S^*_m(t_0 - t)$, $m = 1, ..., M$

MODULAZIONE BINARIA

Coefficiente di correlazione

$\rho = \langle S_1(t), S_2(t)\rangle/(\sqrt{E_1}\sqrt{E_2})$

Distanza d

$d = d(S_1(t), S_2(t)) = \sqrt{E_1 + E_2 - 2\rho\sqrt{E_1E_2}}$

$d = \sqrt{2E_s(1 - \rho)}$, $E_s = E_1 = E_2$

P[errore sul simbolo] P_e

$P_e = Q(d/2\sigma_I)$

P[errore sul bit] P_{bit}

$P_{bit} = P_e$

$P_{bit} = Q(\sqrt{E_s(1 - \rho)/2\sigma_I^2})$

Segnali antipodali

$\rho = -1$, $I = 1$

Segnali ortogonali

$\rho = 0$, $I = 2$

MODULAZIONE M-ARIA

$(N_{min}/M)Q(d_{min}/2\sigma_I) \le P_e \le (M - 1)Q(d_{min}/2\sigma_I)$, $N_{min} = \#d_{min}$

M-PAM

Segnale $S_m(t)$

$S_m(t) = \alpha_m h_{Tx}(t)$, $\alpha_m = 2m - 1 - M$, $m = 1, ..., M$

Alfabeto A

$a_n \in A = \{\alpha_1, ..., \alpha_m\} = \{-(M-1), -(M-3), ..., (M-3), (M-1)\}$

Base ortogonale $\{\Phi_i\}$

$\Phi_1(t) = h_{Tx}(t)/ \sqrt{E_h}$

Costellazione $\{s_m\}$

$\mathbf{s}_m = [\alpha_m\sqrt{E_h}]$

$E_s = E_h(M^2-1)/3$

Regioni di decisione $\{R_m\}$

$R_1 = \{\mathbf{s}_1 + e \mid e \le \sqrt{E_h}\}$

$R_m = \{\mathbf{s}_m + e \mid |e| \le \sqrt{E_h}\}$, $m = 2, ..., M-1$

$R_M = \{\mathbf{s}_M + e \mid e \ge -\sqrt{E_h}\}$

Implementazione sistema

Tx: $a_n\to[h_{Tx}]\to s_{Tx}(t)$, $s_{Tx}(t) = \sum a_n h_{Tx}(t - nT)$, $n = -\infty, ..., +\infty$, $a_n \in A$

Rc: $r(t)\to[h_{Rc}]\to\frown^{(t0+nT)}\to r_n\to[\text{detector}]\to\hat{a}_n$, $h_{Rc}(t) = h^*_{Tx}(t_0 - t)/\sqrt{E_h}$

$r_n = s_{an} + w_n = a_n\sqrt{E_h} + w_n$

Banda minima B_{min}

$B_{min} = 1/2T$, $h_{Tx}(t) = A\text{sinc}(t/T)$

SNR di riferimento Γ_{rif}

$\Gamma_{rif} = 2E_s/N_0$

P[errore sul simbolo] P_e

$P_e \simeq 2(1-1/M)Q(\sqrt{3\Gamma/(M^2-1)})$

P[errore sul bit] P_{bit}

$P_{bit} = P_e/\log_2 M$

M-QAM ($M = L^2$)

Segnale $S_m(t)$

$S_m(t) = \alpha_{m,I}h_{Tx}(t)\cos(2\pi f_0t) - \alpha_{m,Q}h_{Tx}(t)\sin(2\pi f_0t)$, $\alpha_m = \alpha_{m,I} + j\alpha_{m,Q}$, $m = 1, ..., M$

Alfabeto A

$a_n \in A = \{\alpha_1, ..., \alpha_m\}$, $M = L^2 \Rightarrow \alpha_{m,I}, \alpha_{m,Q} \in \{-(L-1), -(L-3), ..., (L-3), (L-1)\}$

Base ortogonale $\{\Phi_i\}$

$f_0 > B_h$: $\Phi_1(t) = \sqrt{2/E_h}h_{Tx}(t)\cos(2\pi f_0t)$, $\Phi_2(t) = -\sqrt{2/E_h}h_{Tx}(t)\sin(2\pi f_0t)$

Costellazione $\{s_m\}$

$\mathbf{s}_m = \sqrt{E_h/2}[\alpha_{m,I}, \alpha_{m,Q}]$

$E_s = E_h(M-1)/3$

$d_{min} = \sqrt{2E_h}$

Regioni di decisione $\{R_m\}$

$|\{R_{ang}\}| = 4$, $|\{R_{ext}\}| = 4(L - 2)$, $|\{R_{int}\}| = (L - 2)^2$

Implementazione sistema

Tx: $(a_{n,I}, a_{n,Q})\to[(h_{Tx}, h_{Tx})]\to[\otimes(\cos(2\pi f_0t), -\sin(2\pi f_0t))]\to\oplus\to s_{Tx}(t)$, $s_{Tx}(t) = \sum a_{n,I}h_{Tx}(t - nT) \cos(2\pi f_0t) - \sum a_{n,Q}h_{Tx}(t - nT) \sin(2\pi f_0t)$, $n = -\infty, ..., +\infty$, $a_{n,I} = \Re[a_n]$, $a_{n,Q} = \Im[a_n]$ **Rc:** $r(t)\to[\otimes(\cos(2\pi f_0t), -\sin(2\pi f_0t))]\to[(h_{Rc}, h_{Rc})]\to(\frown^{(t0+nT)}, \frown^{(t0+nT)})\to(r_{n,1}, r_{n,2})\to[\text{detector}]\to\hat{a}_n$, $h_{Rc}(t) = h^*_{Tx}(t_0 - t)/\sqrt{E_h/2}$

$\underline{r}_n = \underline{s}_{an} + \underline{w}_n = \underline{a}_n\sqrt{E_h/2} + \underline{w}_n$

Banda minima B_{min}

$B_{min} = 1/T$, $h_{Tx}(t) = A\text{sinc}(t/T)$

SNR di riferimento Γ_{rif}

$\Gamma_{rif} = E_s/N_0$

P[errore sul simbolo] P_e

$P_e \simeq 4(1-1/\sqrt{M})Q(\sqrt{3\Gamma/(M-1)})$

P[errore sul bit] P_{bit}

$P_{bit} = P_e/\log_2M$

M-PSK

Segnale $S_m(t)$

$S_m(t) = h_{Tx}(t)\cos(2\pi f_0t + \varphi_m) = \cos(\varphi_m)h_{Tx}(t)\cos(2\pi f_0t) - \sin(\varphi_m)h_{Tx}(t)\sin(2\pi f_0t)$, $\cos(\varphi_m) = \alpha_{m,I}$, $\sin(\varphi_m) = \alpha_{m,Q}$, $\alpha_m = \alpha_{m,I} + j\alpha_{m,Q}$, $\varphi_m = (\pi/M)(2m - 1)$ $m = 1, ..., M$

Alfabeto A

$a_n \in A = \{\alpha_1, ..., \alpha_m\} \Rightarrow \alpha_{m,I} \in \{\cos(\varphi_m), m = 1, ..., M\}$, $\alpha_{m,Q} \in \{\sin(\varphi_m), m = 1, ..., M\}$

Base ortogonale $\{\Phi_i\}$

$f_0 > B_h$: $\Phi_1(t) = \sqrt{2/E_h}h_{Tx}(t)\cos(2\pi f_0t)$, $\Phi_2(t) = -\sqrt{2/E_h}h_{Tx}(t)\sin(2\pi f_0t$

Costellazione $\{s_m\}$

$\mathbf{s}_m = \sqrt{E_h/2}[\cos(\varphi_m), \sin(\varphi_m)]$

$E_s = E_h/2$

$d_{min} = \sqrt{2E_h}\sin(\pi/M)$

Regioni di decisione $\{R_m\}$

$R_M = \{\underline{s}_M + \underline{e} \mid -\pi/M \le \arg(e_1 + je_2) \le \pi/M\}$, $\underline{e} = [e_1, e_2]$

Implementazione sistema

Tx: $(a_{n,I}, a_{n,Q})\to[(h_{Tx}, h_{Tx})]\to[\otimes(\cos(2\pi f_0t), -\sin(2\pi f_0t))]\to\oplus\to s_{Tx}(t)$, $s_{Tx}(t) = \sum a_{n,I}h_{Tx}(t - nT) \cos(2\pi f_0t) - \sum a_{n,Q}h_{Tx}(t - nT) \sin(2\pi f_0t)$, $n = -\infty, ..., +\infty$, $a_{n,I} = \Re[a_n] \in \{\alpha_{m,I} = \cos(\varphi_m), m = 1, ..., M\}$, $a_{n,Q} = \Im[a_n] \in \{ \alpha_{m,Q} = \sin(\varphi_m), m = 1, ..., M\}$ **Rc:** $r(t)\to[\otimes(\cos(2\pi f_0t), -\sin(2\pi f_0t))]\to[(h_{Rc}, h_{Rc})]\to(\frown^{(t0+nT)}, \frown^{(t0+nT)})\to(r_{n,1}, r_{n,2})\to[\text{detector}]\to\hat{a}_n$, $h_{Rc}(t) = h^*_{Tx}(t_0 - t)/\sqrt{E_h/2}$

$\underline{r}_n = \underline{s}_{an} + \underline{w}_n = \underline{a}_n\sqrt{E_h/2} + \underline{w}_n$

Banda minima B_{min}

$B_{min} = 1/T$, $h_{Tx}(t) = A\text{sinc}(t/T)$

SNR di riferimento Γ_{rif}

$\Gamma_{rif} = E_s/N_0$

P[errore sul simbolo] P_e

$P_e = Q(\sqrt{2\Gamma})$, $M = 2$

$P_e \simeq 2Q(\sqrt{2\Gamma\sin^2(\pi/M)})$, $\Gamma \gg 1$, $M \ge 4$

P[errore sul bit] P_{bit}

$P_{bit} = Q(\sqrt{2\Gamma})$, $M = 2$

$P_{bit} = P_e/\log_2M$, $M > 2$

ORTOGONALE COERENTE, M-FSK

Segnale $S_m(t)$

Ortogonali $\Leftrightarrow f_n = f_0 + m/2T$, $f_0 \gg 1/T$

Segnale $S_m(t)$ (M-FSK non coerente)

$S_m(t) = A\sin(2\pi f_mt + \varphi_m)$ se $0 \le t < T$, $S_m(t) = 0$, altrimenti, $m = 1, ..., M$

Ortogonali $\Leftrightarrow f_n = f_0 + m/T$, $f_0 \gg 1/T$

Alfabeto A

Omissis

Base ortogonale $\{\Phi_i\}$

$\Phi_i(t) = s_i(t)/\sqrt{E_i}$, $i = 1, ..., M$

Costellazione $\{s_m\}$

$\mathbf{s}_m = \sqrt{E_n}\, \underline{\mathbf{e}}_m$

$E_s = E_n = A^2T/2$

$d_{min} = \sqrt{2E_s}$

Regioni di decisione $\{R_m\}$

omissis

Implementazione sistema

omissis

Banda minima B_{min}

$B_{min} = M/2T$

SNR di riferimento Γ_{rif}

$\Gamma_{rif} = 2E_s/MN_0$

P[errore sul simbolo] P_e

$Q(\sqrt{E_s/2\sigma^2}) \le P_e \le (M - 1)Q(\sqrt{E_s/2\sigma^2})$

P[errore sul bit] P_{bit}

$P_{bit} = (M/2)Q(\sqrt{M\Gamma/2})$

BANDA MINIMA E LINK BUDGET

Efficienza spettrale ν

$\nu = R_b/2B_{min}$

SNR di riferimento Γ_{rif}

$\Gamma = M_{sTx}/(N_0B_{min}a_{Ch})$

$\Gamma = P_{sTx}/(KT_{eff,Rc}B_{min}a_{Ch})$

Link budget

$(\Gamma)_{dB} = (P_{Tx})_{dBm} - (a_{Ch})_{dB} + 114 - 10\log_{10}(T_{eff,Rc}/T_0) - 10\log_{10}(B_{min})_{MHz}$

TRASMISSIONE DIGITALE DI SEGNALI ANALOGICI

Fattore di carico k_f

$k_f = \sigma_a/v_{sat}$

SNR di quantizzazione Λ_q

$\Lambda_q = M_a/M_{eq}$

e_q granulare $\Rightarrow M_{eq} \simeq M_{egr} = \Delta^2/12 \Rightarrow \Lambda_q = 3k_f^2L^2$

$(\Lambda_q)_{dB} \simeq 6.02b + 4.77 + 20\log_{10}(k_f)$

A-law

$A = 87.56$, $v_{sat} = 1$

$y = F[a] = \{A|a|/(1 + \ln(A))$ per $0 \le |a| \le 1/A$, $(1 + \ln(A|a|))/(1 + \ln(A))$ per $1/A \le |a| \le 1$

$\text{sgn}[y] = \text{sgn}[a]$

μ-law

$\mu = 255$, $v_{sat} = 1$

$y = F[a] = \ln(1 + \mu|a|)/\ln(1 + \mu)$

$(\Lambda_q)_{dB} = 6.02b + 4.77 - 20\log_{10}[\ln(1 + \mu)] - 10 \log_{10}\{1 + (v_{sat}/\mu\sigma_a)^2 + (v_{sat}/\mu\sigma_a)\sqrt{3}\}$

Λ_{PCM}

$\Lambda_{PCM} = E[a^2(t)]/E[|\tilde{a}(t) - a(t)|^2] = E[a^2(nT_s)]/E[|\tilde{a}_q(nT_s) - a(nT_s)|^2]$

$\Lambda_{PCM} = M_a/(M_{eq} + M_{eBC}) = \Lambda_q/(1 + 4(2^{2b} - 1)P_{bit})$, $\Lambda_q = M_a/(\Delta^2/12)$

SNR Λ_a di una tx analogica lineare con ripetitori analogici

$\Lambda_a = P_{Tx}/kT_0NFsrB = \Gamma_{rif,a}/N$, N = # sez. rip.

SNR Λ_{PCM} di una tx digitale M-PAM con ripetitori analogici/rigenenerator

$\Lambda_{PCM} = f(P_{bit,N})$

$P_{bit,N} = Q(\sqrt{3(\Gamma_{rif,d}/N)/(M^2-1)})2(M - 1)/(M\log_2M)$, N = # sez. rip.

$P_{bit,N} = Q(\sqrt{3\Gamma_{rif,d}/(M^2-1)})2N(M - 1)/(M\log_2M)$, N = # sez. rig.