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Leptogenesis: A non-relativistic study

Leptogenese: Eine nicht-relativistische Betrachtung

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1. Introduction

2. Outline of baryogenesis

One way to describe the observed baryonic asymmetry is by a postulating, that the universe has been in an asymmetric state just from the beginning and that the matter and antimatter is concentrated in big domains throughout the universe, which come into contact just at their outer borders. Techincally there is no reason for the universe not to have started in an asymmetric state, but in that case one would measure high gamma rates due to the matter-antimatter-annihilation right between these distinct regions.
Since there is no such radiation observed, patches of different kinds of matter have to be as big as the presently observable universe. Because this doesn't seem very plausible, the baryonic asymmetry had to arise dynamically from an universe where matter and antimatter existed in the same amount.
Actually in 1967 the Sovjetian physicist Andrei Sakharov postulated the criteria, which have to be met in order for an excess of baryons over anti-baryons to be generated out a fully symmetrical universe.

2.1. Sakharov Conditions

As mentioned above there are three crucial properties of nature, the Sakharov conditions, wich are required to produce a net baryon number greater than zero. These three conditions are:

1. B-violating process(es)
2. C and CP violation
3. Departure from or loss of thermal equilibrium

For a general insight of these three conditions the first one will be skipped, since it is quite obvious, that in an totally symmetric universe there has to be at least one B-violating process in order to cause an inbalance in matter and antimatter.
The general importance of the other two will be discussed in the following.

2.1.1. C and CP-violation

Charge conjugation (C), parity (P) and their combination (CP) are two or more specifically three basic symmetries of the universe. C symmetry states, that physical processes are the same, even after exchanging particles for their respective anti-particles, while P-symmetry guarantees invariance under the transformation $\vec{r} \to -\vec{r}$. CP symmetry then simply is an sequence of a C followed by a P transformation.
To explain why C has to be violated for baryogenesis beeing possible, consider the B-violating reaction

$$X \to Y + B$$

with X and Y particles with B=0 and B representing the excess baryons. This reactions happens with a certain rate, which is, using C as a symmetry, just the same as the reation rate for the conjugate process.

$$\Gamma(X \to Y + B) = \Gamma(\bar{X} \to \bar{Y} + \bar{B}) \tag{2.1}$$

Eq. 2.1 implies, that under C exactly the same amount of baryons and anti-baryons will be produced and therefore no excess baryons are left after the annihilaton. This means C must be violated.
But additionally to this CP violation is essential for baryogenesis. To illustrate why, take a closer look at the also clearly B violating X decay with its two channels:

$$X \to q_L q_L$$
$$X \to q_R q_R$$

with q an arbritrary quark. The subscripts L and R denote the the left - or right-handedness chirality of the decay products. CP then effects each particle as follows

$$X \underset{CP}{\longrightarrow} \bar{X}$$
$$q_L \underset{CP}{\longrightarrow} \bar{q}_R$$
$$q_R \underset{CP}{\longrightarrow} \bar{q}_L$$

So CP doesn't just change matter for anti-matter, but the handedness of the particles as well. So if CP holds as a symmetry the consequences for the reaction rates are:

$$\Gamma(X \to q_L q_L) = \Gamma(\bar{X} \to \bar{q}_R \bar{q}_R) \qquad\qquad \Gamma(X \to q_R q_R) = \Gamma(\bar{X} \to \bar{q}_L \bar{q}_L)$$

Adding these two results in

$$\Gamma(X \to q_L q_L) + \Gamma(X \to q_R q_R) = \Gamma(\bar{X} \to \bar{q}_R \bar{q}_R) + \Gamma(\bar{X} \to \bar{q}_L \bar{q}_L) \tag{2.2}$$

Eq. 2.2 implies, that as long as there are as many particles X as anti-particles $\bar{\text{X}}$ in the initial state of the universe, which is just the starting point of the modell the Sakharov conditions try to describe, there can only be an asymmetry between left and right-handed particles be achieved, but that isn't a baryon asymmetry, which is clearly needed for baryogenesis. CP must be violated.
So the bottom line here is, that the existence of B-violating processes is not sufficient for baryogenesis, but that there also has to be C and also CP violation, since without this kind of symmetry breaking any baryonic excess would be washed out by the corresponding C or CP conjugated process, as shown with the simple examples above.

2.1.2. Departure from thermal equilibrium

The last condition to be met in order for baryogenesis to be achievable is that the the B, C and CP violating processes must occur outside the thermal equilibrium. To illustrate this we first consider the phase space distribution of a species X of quantum particles

$$f(E_X) = \frac{1}{e^{\frac{E_X - \mu_X}{T}} \pm 1} \tag{2.3}$$

The energy E_X and the momentum $\vec{p}_X$ are related via the relativistic energy-momentum-relation $E^2 = \vec{p}^2 + m^2$. μ_X describes the chemical potential of the particle species X, which is an important quantity for describing thermal equilibrium states, as the chemical potentials of two species X and Y, which are in thermal equilibrium are related by $\mu_X = \mu_Y$ or for more species $\sum_i \mu_i = 0$.

Using eq. 2.3 to compute the particle density of a certain particle species one gets

$$n_X = g_X \int \frac{d^3p}{(2\pi)^3}\, f_X(E) \tag{2.4}$$

where g_X denotes the number of inner degrees of freedom of X.
In the non-relativistic limit there holds m $\gg$ E-μ $\gg$ T. With this approximation the denominator of the exponential function in eq. 2.3 gets small compared to the numerator so the exponential itself gets so big that the ± 1 can be neglected, in the non-relativistic limit, you get the same particle density for fermions and bosons. By dividing the ernergy E_X into the rest energy m_X and the kinetic energy E_{kin} and after approximating

$$E_{\text{kin}} \approx \frac{p^2}{2m} \tag{2.5}$$

for non-relativistic particles, integrating according to 2.4 yields

$$n_X = g_X \frac{4\pi}{(2\pi)^3} \int dp\, p^2 e^{\frac{\mu - m_X}{T}} e^{-\frac{p^2}{2m_X T}} = g_X \left(\frac{m_X T}{2\pi}\right)^{\frac{3}{2}} e^{-\frac{m_X - \mu_X}{T}} \tag{2.6}$$

Analogously you get the number density for the corresponding anti-particle $\bar{X}$

$$n_{\bar{X}} = g_{\bar{X}} \left(\frac{m_{\bar{X}} T}{2\pi}\right)^{\frac{3}{2}} e^{-\frac{m_{\bar{X}} - \mu_{\bar{X}}}{T}} \tag{2.7}$$

Now suppose X and its anti-particle $\bar{X}$ with $B_X = -B_{\bar{X}} \neq 0$ are in thermal equilibrium than the condition $\mu_X = \mu_{\bar{X}}$ holds. Comparing eq. 2.6 and 2.7 one sees, that the chemical potential is the only property that could differ for particles and antiparticles. Now using the equilibrium condition for chemical one finally gets

$$n_X = n_{\bar{X}} \tag{2.8}$$

Looking at eq 2.8 it is quite obvious that even with B, C and CP violating any produced excess baryon number B will be washed out in equilibrium by other processes happening in equilibrium. This illustrates the final Sakharov Condition, that next to B, C and CP violation a departure from equilibrium is needed for a dynamic production of excess baryons.
Interresting to note is, that there is quite an easy way of approximatelly determining if reactions take place in thermal equilibrium is by comparing the reaction rate with the expansion of universe, discribed by the Hubble constant H, which isn't actually a constant but changes with time. So if the relation

$$\Gamma \gtrsim H \tag{2.9}$$

holds, the reactions take place fast enough for them to be in equilibrium. This can be made understandable, it is useful to look at this from the rest fram of the particles taking part in the reactions. Then the particles don't notice any expansion of the universe since they move and react to fast with each other, therefore the expansion doesn't really affect the equilibrium state. Otherwise if the reactions occur slower than the universe expands, so if

$$\Gamma < H \tag{2.10}$$

is valid, than the expansions happens fast enough that particles get separated too far from each other, so they can't react anymore and the reactions fall out of equilibrium.

2.2. Baryogenesis in the Standard Modell

Although nowadays there are no records or experimental proofs of baryon number violating processes, that doesn't mean there is a need for physics outside the Standard Modell (SM) of particle physics, at least on a qualitative level.

2.2.1. The $SU(2)_L \times U(1)_Y$ symmetry of the SM

As it turns out the electroweak part of the SM with its $SU(2)_L \times U(1)_Y$ symmetry groups suits best for describing baryogenesis. But before the way this is achieved in the SM is displayed, this section will give a short rundown on the $SU(2)_L \times U(1)_Y$ symmetry found in the SM.
The $U(1)_Y$ symmetry can be represented by the following transformations

$$\Psi \longrightarrow e^{i\frac{Y}{2}\alpha(x)}\Psi$$
$$\bar{\Psi} \longrightarrow e^{-i\frac{Y}{2}\alpha(x)}\bar{\Psi}$$

with $\alpha(x)$ being an arbitrary function. Y denotes the $U(1)_Y$ quantum number, the hypercharge. Because of the exponential nature of this transformation to check for $U(1)_Y$-symmetry the hypercharges of all appearing particles in an Lagrangian must add up to zero.
While all particles are singlets under $U(1)_Y$ transformation this isn't the case for the $SU(2)_L$ transformation. In this case left-handed particles transform like a doublet

$$\begin{pmatrix} u_L \\ d_L \end{pmatrix} \longrightarrow U(x) \begin{pmatrix} u_L \\ d_L \end{pmatrix}$$

where $U(x)=e^{i\theta^i(x)T^i}$ with $T^i=\frac{\sigma^i}{2}$ the isospin and its third component T_3, which is the $SU(2)_L$ quantum number. It should be noticed that u and d in the transformation above stand for all up-type particles (ν,u,c,t) respectively all down-type particles (e,μ,τ,d,s,b). The right-handed particles on the other hand transform as singlets under $SU(2)_L$

$$X_R \longrightarrow X_R$$

where X_R stands for any right-handed SM particle. Using this simple transformation one can easily deduce, that for right-handed particles $T=T_3=0$.
Also there is a simple relation that connects the hypercharge Y, the electrical charge Q and the third component of the isospin T_3.

$$Q = T_3 + \frac{Y}{2} \tag{2.11}$$

2.2.2. Electroweak baryogenesis

As stated above the electroweak sector of the SM has every ingredient needed for successfull baryogenesis. The following discussions will illustrate how the SM satisfies all three Sakharov conditions.

C and CP violation

It is already proven theoretically und experimentally by numerous well-known experiments, like for example the Wu experiment in 1956, that C symmetry is maximally violated by the weak interaction in the leptonic as well as in the hadronic sector. As shown by Kobayashi and Maskawa through expanding the Cabibbo hypothesis and experimentally confirmed, weak interactions in the hadronic sector also violate CP invariance, which manifests as an complex phase in the CKM

quark mixing matrix. In the leptonic sector however the CP violation through a complex phase only got postulated in the PMNS neutrino mixing matrix to try to descripe neutrino oscillations, but this phase still needs to be measured.
Nevertheless the elektroweak part of the SM, more precise the weak interactions, since electromagnetism doesn't violate C or even P, satisfies at least one of the three Sakharov conditions.

B violation

Although the first Sakharov condition, the necessity of baryon number violating processes, seems to be the most obvious, the way these are realised in the SM is a bit more difficult than it seems. Since at the first look the baryonic and, since it is going to play an important role during the following discussion, the leptonic current are conserved

$$\partial^\mu J^B_\mu = 0 \tag{2.12}$$

$$\partial^\mu J^L_\mu = 0 \tag{2.13}$$

one would assume there is no way the SM could produce an baryon asymmetry. However, by considering quantum fluctuation meaning orders higher than just tree level one finds, that the currents for the left- and right-handed parts f_L and f_R respectively, where stands for quarks and leptons equally, aren't conserved and not the same [1]

$$\partial^\mu \bar{f}_L \gamma_\mu f_L = -c_L \frac{g^2}{32\pi^2} F^a_{\mu\nu} \tilde{F}^{a\mu\nu} \tag{2.14}$$

$$\partial^\mu \bar{f}_R \gamma_\mu f_R = +c_R \frac{g^2}{32\pi^2} F^a_{\mu\nu} \tilde{F}^{a\mu\nu} \tag{2.15}$$

where g denotes the gauge coupling, $F^{a\mu\nu}$ the field tensor, $\tilde{F}^{a\mu\nu}$ the dual field tensor and c_L and c_R depend on the representation of f_L and f_R.This behaviour of the currents at quantum levels is known as Adler-Bell-Jackiw or chiralty anomaly. Since $SU(2)_L$ gauge boson only couples with left-handed particles c^W_R=0, while the $U(1)_Y$ gauge boson couples to both hadednesses, but with different strength, therefore $c^Y_R \neq c^Y_L$. Although this section only focuses on electroweak baryogenesis, it is mentionable that with the $SU(3)_c$ gauge bosons of the strong interactions don't produce any chiralty anomaly because they couple with left as well as right-handed particles with the same strenght, so $c^c_R = c^c_L$ and both currents in (2.14) and (2.15) cancel each other out in the case of strong interactions.
Putting this and eqautions 2.12 - 2.15 together, gives a pretty interesting result

$$\partial^\mu J^B_\mu = \partial^\mu J^L_\mu = \frac{n_F}{32\pi^2} \left(-g_w^2 W^a_{\mu\nu} \tilde{W}^{a\mu\nu} g'^2 G^a_{\mu\nu} \tilde{G}^{a\mu\nu} \right) \tag{2.16}$$

with $W^{a\mu\nu}$ and $B^{a\mu\nu}$ the field strenght tensors of the $SU(2)_L$ and $U(1)_Y$ gauge groups and n_F=3 the number of particle families.
Analyzing eq. 2.16 one easily figures out, that although baryon and lepton number are not conserved separatedly the difference B-L of these numbers is very well conserved. Integrating both sides of eq. 2.16 as shown in [1, pp. 15f.] results in

$$\Delta B = \Delta L = n_F \Delta N_{CS} \tag{2.17}$$

where ΔN_{CS} is the difference of so called Chern-Simons numbers. How exatly these numbers are derived and what there integral representation is can also be looked up in [1, 2, 3], but isn't of great interest for this thesis. However one property of these numbers is quite relevant for baryon asymmetry, namely that each integer valued Chern-Simons number describes one distinct

vacuum state of the infinite electroweak vacua with minimal energy, which are separated by an potential barrier. The difference of these numbers of two vacuum states right next to each other is $\Delta N_{CS} = \pm 1$, so changing from one vacuum state N_i to another N_f results in $\Delta N_{CS} \neq 0$ and therefore an change in baryon and lepton number is induced. Also interesting to notice is, since the number of particle families n_F=3 baryon and lepton numbers change at least by three units each.
The last question regarding B violation in the SM is about how such a transition between two vacuum states can be accomplished. One way is through a quantummechanical effect called the instaton, where the system simply tunnels through the barrier between to vacuum states with different Cher-Simons numbers. However 't Hooft, the one showing B vioaltion by the chiral anomaly, also showed [1, Ref. 22,24] that the cross section for such a tunneling process is about

$$\sigma \propto e^{-\frac{4\pi}{\alpha_w}} \sim 10^{-164} \tag{2.18}$$

with $\alpha_w = \frac{g^2}{4\pi} \cong \frac{1}{30}$. This cross section is so small, that such a instaton transition between two vacua probably didn't happen even once during the whole lifetime of the universe.
A second way such an change of vacua can be induced is through the so called sphaleron processes. The requirement for these processes to take place is that the system has enough energy to go over the potential barrier instead of tunneling through. The minimum energy needed, known as the spaleron energy, is about[1, 2]

$$E_{sph} = \frac{4\pi}{g_2} v(T) \cong 8 - 13\text{TeV} \tag{2.19}$$

where λ describes the four-Higgs interaction and temperature dependent quantity v(T) denotes the vacuum expectation value of the Higgs field at the temperature T, which will be important later on.
In fact these kind of processes are quite possible for temperatures above around 100 GeV, however below this temperature the rate of spaleron processes is exponentially suppressed by a Boltzmann factor. It is also mentionable that comparing the sphaleron rate for temperatures above 100 GeV, which are proportional to the fourth power of the temperature [1, p. 19], with the Hubble constant, gives information about when these processes are in thermal equilibrium and numerical evaluations yield that the sphaleron processes are in thermal equilibrium for

$$100\text{GeV} \lesssim T \lesssim 10^{12}\text{GeV}$$

So as shown in section 2.1.2, even though the SM provides the necessary tools for C, CP and B violation, below the temperature of around 10^{12}GeV any produced net baryon number will be washed out and below 100 GeV the temperature isn't even high enough to induce sphaleron processes.

Departure from thermal equilibrium and elecroweak phase transistion

The final question to answer regarding baryogenesis in the SM is how the last Sakharov condition, the departure from thermal equilibrium is realized. The most common way is by using the electroweak phase transition.
This phenomenon heavily relies on the vacuum expation value (VEV) of the $SU(2)_L$ Higgs doublett and its behaviour during the early times of the universe. At the present day the VEV isn't equal to zero, which leads to a gauge symmetry breaking and therefore masses of every massive particle. But it has already be shown [1, Ref. 32], that for high temperatures the VEV of the universe equals zero and the $SU(2)_L \times U(1)_Y$ gauge symmetry is still intact, even at the ground

states. This obviously means, that at some point during the evolution of the universe and at some critical temperature T=T$_c$ the VEV changed from zero to non-zero, or in other words a phase transition from a totally symmetrical phase to a phase with broken symmetry happened at some point. In order to generate a departure from thermal equilibrium for the B violating reaction this transition must be strongly of first order, meaning at T=T$_c$ the VEV changes discontinuously from zero to non-zero.

Just as with cooling steam this process can be imagined with bubbles of phases with broken symmetries forming and expanding inside the phase of unbroken symmetry, just as droplets of water form in the vapor and expand, until the connect and finally cover all space. Now the way this phase transition leads to a baryon asymmetry is as follows.

First of all consinder a thin wall, so that the area where quarks and fermions interact with the walls can be approximated as a step function. Also, to simplify matters, assume that the expansion of the bubbles of broken symmetry is sperical symmetric, so this problem can be reduced to one dimension.

At the start of this baryon asymmetry generating process there is the same amount of particles and anti-particles.

While the bubble expands left- and right-handed quarks and anti-quarks from the unbroken phase hit the bubble wall, get reflected under CP violating processes and change their handedness because of angular momentum conservation and since charge conservation holds (anti-)quarks are only allowed to scatter into (anti-)quarks. The scattering processes are the following

$$q_L \to q_R$$
$$q_R \to q_L$$
$$\bar{q}_L \to \bar{q}_R$$
$$\bar{q}_L \to \bar{q}_R$$

Since these scattering processes are not CP conserving the reflection coefficents are not the same for all of the reactions above.

$$\Delta R = R_{\bar{L}\to\bar{R}} - R_{R\to L} = R_{\bar{R}\to\bar{L}} - R_{L\to R} \tag{2.20}$$

Using CPT invariance yields

$$R_{\bar{L}\to\bar{R}} = R_{L\to R} \tag{2.21}$$
$$R_{\bar{R}\to\bar{L}} = R_{R\to L} \tag{2.22}$$

These relations alone imply that there still is no net baryon number since the differences J$_q^L$ of the fluxes of $\bar{q}_R$ and q$_L$ and the J$_q^R$ of q$_R$ and$\bar{q}_L$ reflected back into the symmetric phase are the same and cancel each other out. But considering that the (B+L) violating sphaleron processes because of their electroweak origin only interact with left-handed quarks and right-handed anti-quarks J$_q^L$ changes while J$_q^R$ stays the same since it only takes right-handed quarks and left-handed antiquarks into account. This leads to a non-zero baryon number and especially if J$_q^L$>0 than there are more left-handed quarks then right-handed anti-quarks and therefore ΔB>0 in the symmetric phase away from the wall. If the bubble than expands over the region of a net baryon number greater zero this B gets frozen in, since in the broken phase the (B+L) violating processes that could wash out the asymmetry are strongly supressed by the Boltzmann factor as stated above.

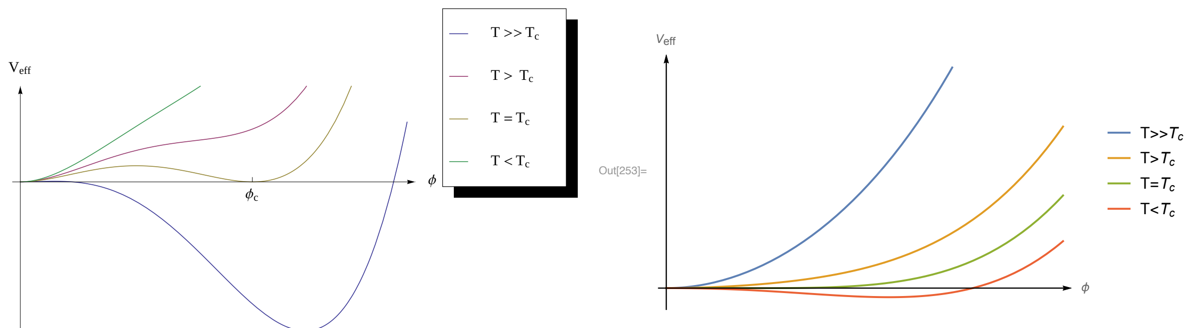
Taking into account that particles from the broken phase can transmit into the symmetric phase and evaluating this quantitative as shown in [1, pp. 36-37] yields the result mentioned above. For a net baryon number greater than zero the CP violating processes at the bubble wall have to act in such way that the current J$_q^L$ is greater than zero as well.

2.2.3. Failures of the SM

Since the SM offers everything needed to describe baryogenesis in the early universe one could naively say that the only thing left is the experimental proof to be delivered.
Having said this recent experiments have shown that the SM alone, despite containing possible B, C and CP violating processes, isn't able to provide an phase transition of strong enough first order or more precisely a phase transition of first order at all, what will be shown in the following.
According to the Landau theory phase transitions are described by the behaviour of a so called order parameter. So for a first order phase transition the order parameter has to change dicontinuously at the cirtical point, while for a second order transition the order parameter has to change drastically as well, but this change occurs continuously. In this case of the electroweak phase transition the order parameter is the expectation value of the Higgs field, denoted as ϕ.
In addtion to this one needs to describe the temperature dependent free energy of the system as a function of the parameter which result in the effective potential $V_{\text{eff}}(\phi, T)$. This potential describes the energy of a system in a state with the Higgs expectation value ϕ. Since in general this state isn't one of minimal energy and because every system prefers to minimize its energy, it changes into a state described by the minimum of the potential where the expaction value of the Higgs is by definitischt the Higgs VEV.
As it is already known, that the Higgs VEV had to change from zero at the big bang to a non-zero value while the universe cooled down to the temperature measured nowadays, so it is just natural to look the change of this value with temperature. The change of the VEV can now happen continunously in which case the system undergoes a second order phase transition or discontinuously what is needed for an first oder transition and especially for elektroweak baryogenesis. Both cases are shown in figure 2.1 for the effective Higgs potential including 1-loop corrections [3]

$$V_{\text{eff}} = D(T^2 - T_0)^2\phi^2 - ET\phi^3 - \frac{1}{4}\lambda\phi^4 \tag{2.23}$$

for different non-zero temperature regimes. D and E are constant factors which aren't of great interest for this discussion and λ describes the already mentioned 4 Higgs self coupling.



(b) Effective Higgs field for second order transition

(a) Effective Higgs field for first order transition

Figure 2.1.: Effective Higgs field for different phase transitions

Analyzing figure 2.1a clearly shows the first order characteristics of the phase transition. While for high temperatures the VEV equals zero for the highly symmetric phase, the potential slowly develops a second minimum for decreasing temperature until at the critical temperature $T = T_c$ there are to energetically degenerated mimima, one at $\phi_1 = 0$ and one at $\phi_2 = \phi_c$, which are separated by an energy barrier. However the system can change from the minimum at ϕ_1 to the one at ϕ_2 via tunneling through the barrier what results in a disontinuous change of the VEV and therefore induces a first order phase transition. While the universe keeps on cooling down

because of the universe's expanding the new minimum is gets energetically lowered while the original one stays at $V_{\text{eff}}=0$ and thus becomes a maximum, leaving an unstable state where once was a symmetric vacuum state.
How this can be imagined in the early universe is, that at some point in space and time the universe tunneled from one minimun to another thus breaking SM symmetry locally and producing a local bubble of broken phase. These bubbles get bigger with time and combine with other bubbles whose production gets much more likely as the lower temperatures lower the barrier between the minima and increase the tunneling probability.
On the other hand figure 2.1b shows how the universe would develop in case of a second order transition. In this case even at $T=T_c$ there are no to degenerated minima but the new one develops gradually while the orignial minimum gradualy becomes the unstabel maximum you also get in figure 2.1a for low temperatures. Since there are no two minima the universe can choose between there is also no bubble formation but instead a continuous condensation throughout the universe what isn't enough to induce baryogenesis.
Now that the two possibilities of the electroweak phase transition were represented the question how and why the SM fails to provide a strong enough first order phase transition still needs to be answered.
To do this it is useful to define a quantity that corresponds to the strenght of the phase transision, which for this cause will be

$$\frac{v_{T_c}}{T_c} \gtrsim 1 \tag{2.24}$$

The reason why 2.24 is a good way to represent the strength of the phase transition is that by using equation 2.19 and the fact that the B+L violating spahleron processes are exponentially surpressed by a Boltzmann factor inside of the phases with broken symmetry one gets for the rate of these spalerons at the critical temperature

$$\Gamma_{\text{spaleron}} \propto e^{-E_{\text{sph}}(T_c)/T_c} \propto e^{-v_{T_c}/T_c} \tag{2.25}$$

So equation 2.25 really shows that the spaleron rates inside the bubbles with a Higgs VEV greater than zero are surpressed exactly by the quantity given 2.24. So in order for these processes to be surpressed adequately has to be at least 1, which results in a surpression factor of roughly 0.36, for the phase transition to be strong enough to cause baryogenesis.
There are various methods to use the condition in 2.25 in order to calculate the Higgs mass and what the biggest mass is the Higgs particle can have in order for a first order phase transition to be possible which results in about m_H<70 GeV [4, pp. 3f.].
This theoretical result together with the experimental discovery that the Higgs mass is greater than 114 GeV [6, pp. 100ff.] clearly shows that the electroweak phase transition and therefore the SM as a whole isn't able to explain how the observed baryonic asymmetry arose during the early times of the universe.
A solution for this problem is expanding the SM in such a way that the new elements are able to explain problems the SM couldn't. One of these expansions results in leptogenesis, what will be the topic of the following sections.

3. Outline of leptogenesis

As stated in the section before the SM alone isn't quite enough to describe the observed baryon abundance, so the SM has to be expanded to that effect that it can describe such phenomena. Although are efforts made to explain direct baryogenesis using GUT theories, there is an much more favored alternative, namely the so called barygenesis via leptogenesis, what this and the following section will be about.

3.1. Expandig the SM

There are experimental reasons why the SM doesn't tell the whole story about our universe, namely the results of neutrino oscillation experiments. Before the discovery of these neutrino oscillations it was accepted that neutrinos are massless and therefore their left-handedness is well defined. But being able to oscillate between different flavours implies that neutrinos aren't massless and therefore are not purely left-handed and even more so that right-handed neutrinos exist. The easisest way to implement right-handed neutrinos into the SM would be to show, that neutrinos are so-called Majorana particles, which, in contrast to Dirac particles, are their own anti-particles. This would mean that the right-handed neutrinos are right-handed antineutrinos at the same time, but it was already shown that latter exist. Theoretically the way to describe Majorana masses would be to exchange the usual mass term including the Higgs field for a Majorana mass terms, that can be written in the following way [8, p. 18], to the SM Lagrangian.

$$\mathcal{L}_M = -\frac{1}{2}\overline{\Psi^C} M^M \Psi$$

where the superscript C stands for the charge conjugated neutrino field defined by

$$\Psi^C \equiv C\gamma_0\Psi^*$$

with the marix C, which is dependend on the representation of the gamma matrices. The Majorana mass M^M is an $\mathrm{n}_F\times\mathrm{n}_F$ matrix with n_F again the number of particle families.
Using the representation of the $\mathrm{U}(1)_Y$ symmetry group given in the previous section, it can easily be seen that in general by using a mass term like in 3.1 to the SM Lagrangian this symmetry is no longer viable.

$$\overline{\Psi^C}\Psi \overset{U(1)Y}{\longrightarrow} \overline{(e^{i\frac{Y}{2}\alpha}\Psi)^C} e^{i\frac{Y}{2}\alpha}\Psi = \overline{e^{-i\frac{Y}{2}\alpha}\Psi^C} e^{i\frac{Y}{2}\alpha}\Psi = e^{i\frac{Y}{2}\alpha}\overline{\Psi^C} e^{i\frac{Y}{2}\alpha}\Psi \neq \overline{\Psi^C}\Psi$$

The relation above shows, that by using Majorana masses of particles with hypercharge Y$\neq$0, like the left-handed neutrinos, one cannot preserve the SM Lagrangians $\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y$ symmetry. That being said right-handed Dirac mass terms for neutrinos have to be added to the Lagrangian in order for it to still preserve its $\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y$ symmetry. Since, like all other right-handed particles in the SM, the right-handed neutrinos form a $\mathrm{SU}(2)_Y$ singlet, which we will call N. Also using the isospin conjugate of the Higgs doublet

$$\tilde{\phi} \equiv i\sigma^2\phi^*$$

the Yukawa term can be written as

$$\mathcal{L}_{\text{N,Yuk}} = h_{ij}\overline{N_i}\tilde{\phi}^\dagger l_j + h^*_{ij}\overline{l_i}\tilde{\phi}N_j \tag{3.1}$$

This interaction term will be of analyzed in more detail in Appendix A.1.
The h_{ij} describe the Yukawa couplings and the l_i the left-handed lepton SU(2) doublets of the Standard modell. It can be shown, that this additional term doesn't violate the symmetries of the SM Lagrangian.
However, as explained above adding left-handed Majorana neutrino mass terms to the SM Lagrangian breaks its symmetry, but since right-handed neutrinos have to be added anyways one can also try to add a right-handed Majorana mass term, too.

$$\mathcal{L}_{N,M} = -\frac{1}{2}\overline{N^C}M^M N \tag{3.2}$$

The mass term in equation 3.2 however doesn't violate the Lagrangian's $SU(2)_L \times U(1)_Y$ symmetry, because for right-handed neutrinos $T=T^3=Y=0$ and therefore the transformations given in the previous section become the trivial identity transfomation. Anyways, the Dirac Lagrangian, so the SM Langrangian without any Majorana mass terms, is obviously invariant under any U(1) transformation, not only under $U(1)_Y$ transformations. The Majorana mass terms on the other hand are only invariant under the exactly this $U(1)_Y$, especially only for particles with $T=T^3=Y=0$ like the right-handed neutrinos while violating other U(1) symmetries. And according to the Noether theorem, every symmetry of a theory results in a conserved current or quantum number, so breaking the U(1) symmetry not assigned to the hypercharge by using Majorana mass terms one certain quantum number, in this case breaking the following $U(1)_l$ symmetry results in a non-conservation of the lepton number.

$$\Psi \xrightarrow{U(1)_L} e^{i\ell\alpha(x)}\Psi \tag{3.3}$$

with the lepton number ℓ of the field Ψ. This seems rather obvious because if Majorana particles are particles and anti-particles at the same time one cannot assing them a distinct lepton number and therefore it is not conserved.
Finally, after putting the Dirac and Majorana mass terms together one ends up with [8, p. 21]

$$\mathcal{L}_{M+D} = \left(\overline{\nu_L^C}, \overline{N}\right)\begin{pmatrix} M_L^M & m^D \\ m^D & M_R^M \end{pmatrix}\begin{pmatrix} \nu_L \\ N^C \end{pmatrix} + h.c. \tag{3.4}$$

The matrix m^D contains the masses of the Dirac neutrinos, so the neutrinos found in nature up until now, while M_L^M and M_R^M describe the masses of the left as well as the right-handed Majorana neutrinos. All of these matrices are of the dimension $n_F \times n_F$ with n_F the number of neutrino flavours. Also, as explained above, since it isn't possible to introduce left-handed Majorana neutrinos to the SM without violating its fundemental symmetry M_L^M has to be equal to zero.

3.2. The seesaw Mechanism

Although the addition of neutrino masses can be described using the mass term 3.4 or rather

$$\mathcal{L}_{M+D} = \left(\overline{\nu_L^C}, \overline{N}\right)\begin{pmatrix} 0 & m^D \\ m^D & M_R^M \end{pmatrix}\begin{pmatrix} \nu_L \\ N^C \end{pmatrix} + h.c. \tag{3.5}$$

there is still a problem, namely why the neutrino masses are many orders of magnitude smaller than those of the other SM particles. This however can be described using the so called seesaw mechanism. In this discussion only the so called type I seesaw mechanism will be presented. By doing so the following two assumtions have to be made:

1. The Dirac masses arise directly from the Higgs mechanism that gives mass to all SM particles, introducing the electroweak mass scale of order $\sim 10^2 - 10^3$ GeV.
2. The Majorana masses are much bigger than the Dirac masses, $\mathrm{m}^D \ll \mathrm{M}^M$. This mass scale arises from GUT's and is of order $\sim 10^{10} - 10^{16}$ GeV.

The subscript R will be dropped from now on since there is no non-zero left-handed Majorana mass and therefore no further distinction is needed.
Now, after diagonalising the mass matrix in 3.5 [9, pp. 2-3], one gets two mass eigenvalues, in particular

$$M_1 \backsimeq -m^D \left(M^M\right)^{-1} \left(m^D\right)^T$$
$$M_2 \backsimeq M^M$$

or for just one neutrino family

$$M_1 \backsimeq -\frac{m_D^2}{M^M} \tag{3.6}$$
$$M_2 \backsimeq M^M \tag{3.7}$$

The negative sign for M_1 comes from the fact that these are just the eigenvalues of the mass matrix given in 3.5, the physical masses are the absolute values of these eigenvalues.
Finding the corresponding eigenstates for each eigenvalue one gets that the eigenstate associated with M_1 is ν, the observable, left-handed light neutrino. One can now easily see that the smallness of the neutrino masses compared to those of all the other SM particles comes from the assumption $\mathrm{m}^D \ll \mathrm{M}^M$. On the other hand however the eigenstate appendant to M_2 is N, the newly added, right-handed heavy neutrino, that will play a crucial role in leptogenesis. The heavy neutrino mass being mostly governed by the Majorana mass implies that the right-handed neutrinos are Majorana particles, which means they are their own anti particles.
Interesting to note as well is how these two masses behave under finetuning. It is quite obvious from equation 3.6 that by raising the large mass scale and as a consequence thereof raising the mass of the heavy neutrino the mass of the light neutrinos gets even lower and vice versa, hence the name seesaw mechanism.

3.3. Leptogenesis and the Sakharov conditions

After the necessary expansion of the SM was performed in the previous section, this section will focus on how the right-handed, heavy neutrinos are able to produce a net, non-zero baryon number, that is how the Sakharov conditions can be fullfilled using this exapanded SM.
In the following discussions the assumption that three heavy right-handed neutrinos exist will be made. In addition it is required that the masses of these neutrions are hierarchical in the sense that $\mathrm{M}_1 \ll \mathrm{M}_{2,3}$ and that only the lightest of these neutrinos actually play a significant role for leptogenesis.
The key ingredient for baryogenesis via leptogenesis is the decay of the heavy, right-handed neutrinos introduced above, that is described by the Yukawa interaction in 3.1.The Feynman diagrams for both decay channels are depicted in figure 3.1. The right-handed neutrinos being

Majorana particles as a result of the seesaw mechanism they don't preserve lepton number and because of this they can decay into leptons as well as anti leptons, as it can be seen in figure 3.1.

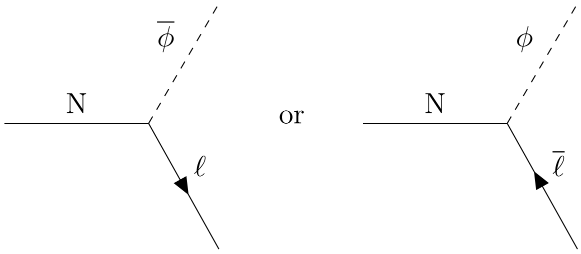


Figure 3.1.: Feynman diagrams for the N-decay

B violation

The B violation in the frame of leptogenesis is achieved in the same way as in direct electorweak baryogenesis via the B+L violating sphaleron processes. Because of the eventually by N decays produced net lepton number this means that the lepton abundance is converted into a baryon abundance by these processes.

C and CP violation

As in direct baryogenesis C violation is already maximally violated SM electroweak interaction, so this is also the case in this slightly extended model. The CP violation for the N_1 decay however is a bit more complicated, since for it to be calculated one has to take the one-loop Feynman diagrams additional to the tree level decay into account. The interference between these diagramms gives than rise to the CP violation needed for successfull baryogenesis. These one-loop diagrams are depicted in figure 3.2.

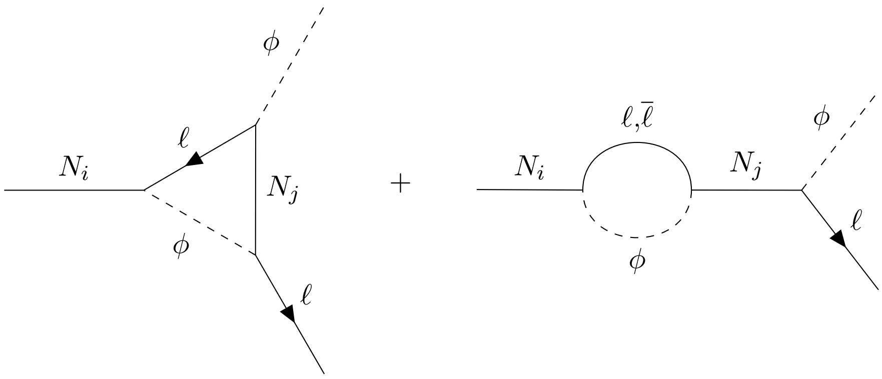


Figure 3.2.: One-loop diagrams for the N-decay

By comparing this figure to [12, Fig. 5.1] one sees that in figure 3.2 one diagramm is missing. This missing diagram hoever doesn't contribute to the CP violation and therefore it was neglected. The distinction between N_i and N_j has to be made since the virtual neutrinos trasmissioned during these processes aren't of the same flavour as the neutrino in the initial state. This means that in order for CP to be violated there has to exist at least an extra neutrino next to the lightest one.
This being said one can calculate the CP asymmetry[12, pp. 24ff.], which is in general defined

as

$$\epsilon = \frac{\Gamma(N_1 \to \overline{\phi}\ell) - \Gamma(N_1 \to \phi\bar{\ell})}{\Gamma(N_1 \to \overline{\phi}\ell) + \Gamma(N_1 \to \phi\bar{\ell})} \tag{3.8}$$

by interfering the tree level decay with the one-loop decays. This results in [12, p. 26]

$$\epsilon \gtrsim 10^{-6} \tag{3.9}$$

Although this value is rather small it isn't zero and therefore the CP symmetry is violated by the decay of heavy right-handed Majorana neutrinos.

Deviation from the equilibrium

The last condition that has to be met for successfull baryogenesis is that the decay of the heavy neutrinos must occur outside of equilibrium. For high enough temperatures, namely for T $\gtrsim$ M_1, the decay of the neutrino is in equilibrium with the inverse decay $\phi\ell \to N$ and no net lepton number can be produced even if the other two conditions hold as shown in sec. 2.1.2. Even if during inflation an abundance of N was produced the lepton asymmetry would washed out by equilibrium processes as soon as the temperature rises up to at least the mass of the heavy neutrino during the reheating phase of the ealrly universe.
However if the temperature drops below M_1 the heavy neutrino inverse decay is exponentially surpressed by a Boltzmann factor because with falling temperature it becomes exponentially more unprobable for a lepton and Higgs to have enough energy to form a heavy neutrino, while the neutrinos themselves can still decay into lepton and Higgs. Because of this the equilibrium density of the neutrinos is also heavily Boltzmann surpressed and if the decay rate is small enough the actual neutrino density can be greater than the equilibrium density and stay greater for an considerable amount of time. This means that decays happening before the actual neutrino density converges to the equilibrium density actually happen outside of equilibrium and a net lepton number can be produced that, because of surpression of inverse decays, won't be washed out and as a consequence thereof will be transformed into a baryon abundance through the B+L violating electroweak sphaleron processes, that will still be active as the heavy neutrino mass at least of order of the upper limit of the temperature range in which sphaleron processes act.
The requirement for the neutrino decay to be slow enough to sustain its out-of-equilibrium state is the following [8, p. 30].

$$\Gamma_D < H \tag{3.10}$$

Γ_D denotes the total decay rate of the neutrinos while H is the expansion rate of the universe. This relation is equivalent to the one given in 2.10 as a requirement for processes to be out of equilibrium.

4. Analytic approximations and calculations

4.1. Rate equations for leptogenesis

Now in order to qualitatively describe leptogenesis one has to consider rate equations for the lepton number and B-L number densities. In a static universe without lepton number violating processes the rate equation would trivialy be

$$\frac{d}{dt}n = 0 \tag{4.1}$$

If one now condsiders a universere expandig with the rate H the rate equation above changes to.

$$\left(\frac{d}{dt} + 3H\right) n = 0 \tag{4.2}$$

Finally including lepton number violating processes the rate equation one obtains for the neutrino number density is

$$\left(\frac{d}{dt} + 3H\right) n_N = -\Gamma_N \left(n_N - n_N^{eq}\right) + \Gamma_{N,B-L} n_{B-L} \tag{4.3}$$

Applying this reasoning to the B-L number density yields

$$\left(\frac{d}{dt} + 3H\right) n_{B_L} = \Gamma_{B-L,N} \left(n_N - n_N^{eq}\right) + \Gamma_{B-L} n_{B-L} \tag{4.4}$$

The coefficient Γ_N denotes how fast the neutrino density equalizes with its equilibrium density, while Γ_{B-L} describes the washout of a net B-L number. $\Gamma_{B-L,N}$ describes how the B-L asymmetry is affected by the deviation of the neutrino density from its equilibrium value and together with $\Gamma_{N,B-L}$ these two coefficients characterize CP violating processes [13, p. 4]. Since both these coefficients describe CP violating processes they must depend on the CP violating parameter ϵ introduced in the section before and it was also seen there that this parameter is small vor heavy neutrino decays and therefore the second term in equation can be neglegted.
The goal now is to determine these coefficients at leas at leading order and the first one will be Γ_N. To get this coefficient one has to integrate equation 4.3 over phase space, resulting in

$$\left(\frac{\partial}{\partial t} - Hp\frac{\partial}{\partial p}\right) f_N = \Gamma_N \left(e^{E_N/T} - f_N\right) \tag{4.5}$$

The whole calculation on how performing the phase space integral can be found in appendix B.1. One might now notice that equation 4.5 differs from equation 4 in [13] by a factor of $\frac{M_N}{E_N}$, that origins in a different normaliziation of the phase space. However, since we operate in a non-relativistic regime $\mathrm{E_N} \approx \mathrm{M_N}$ and therefore this factor is ~ 1 and negligible. Using this argumentation one can also see that $\Gamma_N = \Gamma_0$ with Γ_0 the total decay rate of the heavy neutrinos, wich is governed by the Yukawa interaction term 3.1. It has to be mentioined that for the equilibrium distribution the Boltzmann statistic with neglegted chemical potential was used since the energy of a neutrino $\mathrm{E_N} \approx \mathrm{M_N} \gg \mathrm{T}$ during the phase where the decay happen outside equilibrium and therefore quantum mechanical effects play an insignificant role and can be neglected. Going back to the decay rate, that can be calculated as done in appendix A.1.2, one gets

$$\Gamma_N = \Gamma_0 = \frac{|h_{11}|^2 M_N}{8\pi} \tag{4.6}$$

Since the coefficient Γ_{B-L} refers to the washout of the B-L asymmetry it arises from the inverse decay $l\phi \to$ N and can be calculated via

Integral-herleitung klären

$$\Gamma_{B-L} n_{B-L} = \int \prod_{a=N,\ell,\phi} \frac{d^3 p_a}{2E_a (2\pi)^3} (2\pi^4) \delta(p_\ell + p_\phi - p_N)(f_\ell f_\phi - f_{\bar{\ell}} f_{\bar{\phi}}) \sum |M_0|^2 \tag{4.7}$$

$\sum |M_0|^2 = 16\pi M_N \Gamma_0$ describes the tree level matrix element for exactly this inverse decay summed over all spins of N and isospin components of ℓ. In order to evaluate this integral properly one has to first describe the term $(f_l f_\phi - f_{\bar{l}} f_{\bar{\phi}})$ more precisely. Since we operate at rather hight temperatures one can use Boltzmann statistics for lepton and Higgs regardless and expanding in the chemical potentials up to first order yields.

$$f_\ell f_\phi - f_{\bar{\ell}} f_{\bar{\phi}} \simeq 2e^{-\frac{E_N}{T}} \frac{\mu_l + \mu_\phi}{T} \tag{4.8}$$

As mentioned in [13, p. 7] the chemical potentials are proportional to the B-L number density and by using the coefficients c_l and c_ϕ in order to avoid introducing the exact temperature dependend relations one can also connect n_{B-L} to the lepton and Higgs asymmetries through [13, p. 7]

$$n_\ell - n_{\bar{\ell}} = -c_\ell n_{B-L} \tag{4.9}$$
$$n_\phi - n_{\bar{\phi}} = -c_\phi n_{B-L} \tag{4.10}$$

Now by expanding the number distribution for leptons and Higgs and their respective anti particles in the chemical potentials up to first order one can finally put the chemical potential and n_{B-L} in relation to each other.

Fermi, Bose Verteilung klären

$$\mu_\ell = \frac{3c_l}{T^2} n_{B-L} \tag{4.11}$$
$$\mu_\phi = \frac{3c_l}{2T^2} n_{B-L} \tag{4.12}$$

Using all these results and putting them into relation B.2 one gets the following result for the washout rate $\Gamma_{\text{B-L}}$

$$\Gamma_{\text{B-L}} = \frac{3}{\pi^2} \left(c_\ell + \frac{c_\phi}{2} \right) z^2 K_1(z) \Gamma_0 \tag{4.13}$$

with $K_1(z)$ the modified Bessel funktion of the second kind and

$$z \equiv \frac{M_N}{T} \tag{4.14}$$

what can be seen as an unitless measure for time, since the temperature of the universe decreases over time and therefore z increases. The exact derivation of these results can be looked up in appendix B.2.
Using the relations given in 2.9 and 2.10 it is usefull to define the so called wash out factor K as follows

$$K \equiv \frac{\Gamma_0}{H|_{T=M_N}} \tag{4.15}$$

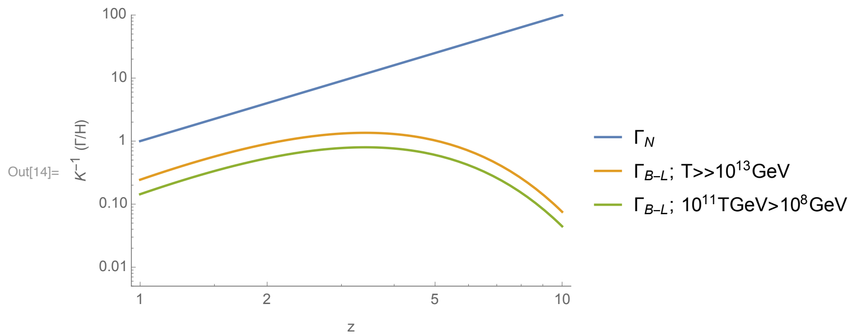

Out[14]=
100
10
1
0.10
0.01
K^{-1} (Γ/H)
1
2
5
10
z
Γ_N
Γ_{B-L}; T>>10^{13}GeV
Γ_{B-L}; 10^{11}TGeV>10^{8}GeV

Figure 4.1.

5. Summary

A. Appendix A

A.1. The Yukawa interaction term for right-handed neutrinos

A.1.1. Feynman rules for the Yukawa interaction

A.1.2. The tree level decay rate for heavy neutrinos

B. Appendix B

B.1. Integrating the rate equation over phase space

$$\left(\frac{d}{dt}+3H\right)n_N = -\Gamma_N\left(n_N - n_N^{eq}\right) + \underbrace{\Gamma_{N,B-L}n_{B-L}}_{\text{negligible}}$$

phase space integral

$$\int\left(\frac{d}{dt}+3H\right)f_N d^3p = -\int\Gamma_N\left(f_N - f_N^{eq}\right)d^3p$$
$$\int\left(\frac{d}{dt}+3H\right)f_N p^2 dp = -\int\Gamma_N\left(f_N - f_N^{eq}\right)^2 dp$$

Using

$$\int p^3\frac{\partial f_N}{\partial p}dp = \left[p^3 f_N\right]_0^\infty - 3\int p^2 f_N dp = -3\int p^2 f_N dp$$
$$3H\int f_N p^2 dp = -H\int p^3\frac{\partial f_N}{\partial p}dp$$

it follows

$$\int\left(\partial_t - Hp\partial_p\right)f_N p^2 dp = -\int\Gamma_N\left(f_N - f_N^{eq}\right)p^2 dp$$
$$\Rightarrow \left(\partial_t - Hp\partial_p\right)f_N = \Gamma_N\left(f_N^{eq} - f_N\right)$$

B.2. Detailed calculation of Γ_{B-L}

First the rather simple derivation of 4.8 shall be displayed here. For the product of two Boltzman factors one has

$$f_\ell^{eq}f_\phi^{eq} = e^{-\frac{E_\ell-\mu_\ell}{T}}e^{-\frac{E_\phi-\mu_\phi}{T}} = e^{-\frac{E_N-\mu_\ell-\mu_\phi}{T}} = e^{-\frac{E_N}{T}}e^{\frac{\mu_\ell+\mu_\phi}{T}}$$

where the relation $E_\ell+E_\phi=E_N$ was used.
Expanding this up to first order in the chemical potential results in

$$f_\ell f_\phi \simeq e^{-\frac{E_N}{T}}\left(1+\frac{\mu_\ell+\mu_\phi}{T}\right)$$

Finally putting this togehter for leptons and Higgs particles and using the relation $\mu_X = -\mu_{\bar{X}}$ for the chemical potentials of a particle X and its anti particle X yields the result presented above.

$$f_l f_\phi - f_{\bar{l}}f_{\bar{\phi}} \simeq e^{-\frac{E_N}{T}}\left(1+\frac{\mu_l+\mu_\phi}{T} - 1 - \frac{\mu_{\bar{l}}+\mu_{\bar{\phi}}}{T}\right) = 2e^{-\frac{E_N}{T}}\frac{\mu_l+\mu_\phi}{T}$$

Now, as already explained, in order to relate the chemicial potentials to the density n_{B-L} one has to expand the left-hand side of 4.9 and 4.10 up to first order in the chemical potentials while using Fermi-Dirac and Bose-Einstein distributions instead of Boltzmann statistics.

Faktor 2 klären

$$\begin{aligned}
n_\ell - n_{\bar\ell} &= \int \frac{d^3p}{(2\pi)^3} f_\ell - f_{\bar\ell} = \\
&= \frac{1}{(2\pi)^3} \int (f_\ell - f_{\bar\ell})\, p^2\, dp\, dcos\theta\, d\phi = \\
&= \frac{1}{2\pi^2} \int \left(f_\ell\,(\mu_\ell = 0) + \frac{e^{E/T}}{\left(e^{E/T}+1\right)^2 T}\mu_\ell - f_{\bar\ell}\,(\mu_{\bar l} = 0) - \frac{e^{E/T}}{\left(e^{E/T}+1\right)^2 T}\mu_{\bar\ell} \right) p^2\, dp = \\
&= \frac{1}{2\pi^2} \int \frac{2e^{E/T}}{\left(e^{E/T}+1\right)^2 T}\mu_\ell : p^2\, dp = \\
&= \frac{\mu_\ell}{\pi^2 T} \underbrace{\int_0^\infty \frac{2e^{E/T}}{\left(e^{E/T}+1\right)^2} E^2\, dE}_{=\frac{\pi^2 T^3}{6}} = \\
&= \frac{\mu_\ell T^2}{6}
\end{aligned}$$

Analogous calculation for the Higgs yields:

$$n_\phi - n_{\bar\phi} = \frac{\mu_l T^2}{3}$$

Solving 4.9 and 4.10 for the chemical potentials results in

Minus klären

$$\begin{aligned}
\mu_l &= \frac{3c_l}{T^2} n_{B-L} \\
\mu_\phi &= \frac{3c_l}{2T^2} n_{B-L}
\end{aligned}$$

Plugging all these results into 4.7 one gets the following relation

$$\begin{aligned}
\Gamma_{B-L} &= \frac{3}{8\pi^4} \left(c_\ell + \frac{c_\phi}{2}\right) \frac{M_N}{T^3}\Gamma_0 \int \prod_{a=N,\ell,\phi} \frac{d^3p_a}{E_a}\delta^4\left(p_\ell + p_\phi - p_N\right) e^{-\frac{E_N}{T}} = \\
&= \frac{3}{8\pi^4} \left(c_\ell + \frac{c_\phi}{2}\right) \frac{M_N}{T^3}\Gamma_0 \int \frac{d^3p_N}{E_N} e^{-\frac{E_N}{T}} \left(\int \frac{d^3p_\ell}{E_\ell}\frac{d^3p_\phi}{E_\phi}\delta^4\left(p_\ell + p_\phi - p_N\right) \right)
\end{aligned}$$

For the sake of minimizing the writing of redundant coefficients we first will evaluate the two body decay phase space in the parentheses above. For simplification the frame of reference used will be the rest frame of the neutrino $\vec{p}_N = 0$.

$$\begin{aligned}
\int \frac{d^3p_\ell}{E_\ell}\frac{d^3p_\phi}{E_\phi}\delta^4\left(p_\ell + p_\phi - p_N\right) &= \int \frac{d^3p_\ell}{E_\ell}\frac{d^3p_\phi}{E_\phi}\delta\left(E_\ell + E_\phi - M_N\right)\delta^3\left(\vec{p}_\ell + \vec{p}_\phi\right) = \\
&= \int \frac{d^3p}{E_\ell}\frac{1}{E_\phi}\delta\left(E_\ell + E_\phi - M_N\right)
\end{aligned}$$

From the first to the second line the using the delta distribution for the particle momenta yields $\vec{p} \equiv \vec{p}_\ell = -\vec{p}_\phi$. Also by introducing E$\equiv$= $E_\ell + E_\phi$ and using spherical coordinates and the following change of integration variables

$$E = E_\ell + E_\phi = \sqrt{M_\ell + p^2} + \sqrt{M_\phi + p^2} \Longrightarrow \frac{dE}{dp} = \frac{p}{E_\ell} + \frac{p}{E_\phi}$$
$$\Longrightarrow dp = \left(\frac{p}{E_l} + \frac{p}{E_\phi}\right)^{-1} dE = \frac{E_\ell E_\phi}{pE_\ell + pE_\phi} dE = \frac{E_\ell E_\phi}{pE} dE$$

one gets

$$4\pi \int \frac{p^2}{pE} \delta\left(E - M_N\right) dE = 4\pi \int \frac{p}{E} \delta\left(E - M_N\right) dE = 4\pi \frac{p}{M_N} = 2\pi \frac{M_N}{M_N} = 2\pi$$

The second to last step uses the delta distribution and the resulting relation E=$\mathrm{M_N}$ while during the last step uses the fact that leptons and Higgs are relativistic and therefore their Energie is E $\approx$ p and in addition that the energy needed for the inverse decay to be possible in the neutrino's rest frame has to be E $\approx p = \frac{M_\mathrm{N}}{2}$. Finally plugging this into the original relation for Γ_{B-L} yields

$$\Gamma_{B-L} = \frac{3}{8\pi^4} \left(c_\ell + \frac{c_\phi}{2}\right) \frac{M_N}{T^3} \Gamma_0 \int \frac{d^3 p_N}{E_N} e^{-\frac{E_N}{T}} \cdot 2\pi \frac{E_N}{M_N} =$$
$$= \frac{3}{\pi^2} \left(c_\ell + \frac{c_\phi}{2}\right) \frac{M_N}{T^3} \Gamma_0 \int_0^\infty \frac{dp_N}{E_N}\, p_N^2 e^{-\frac{E_N}{T}}$$

And with the final change of integration variables

$$x = \sqrt{\frac{p_N^2}{M_N^2} + 1} \Longrightarrow p_N = \sqrt{x^2 - 1} M_N \Longrightarrow dp_N = \frac{x}{\sqrt{x^2 - 1}} M_N$$

one gets

$$\Gamma_{B-L} = \frac{3}{\pi^2} \left(c_\ell + \frac{c_\phi}{2}\right) \frac{1}{T^3} \Gamma_0 \int_1^\infty dx\, \frac{\left(x^2 - 1\right) M_N^2}{M_N \cdot x} \cdot \frac{x}{\sqrt{x^2 - 1}} \cdot M_N e^{-zx} =$$
$$= \frac{3}{\pi^2} \left(c_\ell + \frac{c_\phi}{2}\right) z^3 \Gamma_0 \int_1^\infty dx\, \sqrt{x^2 - 1} e^{-zx} =$$
$$= \frac{3}{\pi^2} \left(c_\ell + \frac{c_\phi}{2}\right) z^2 K_1(z) \Gamma_0$$

where in the last step the definition of the modified Bessel function of the second kind was used

$$K_n(z) = \frac{\sqrt{\pi}}{\Gamma\left(n - \frac{1}{2}\right)} \left(\frac{1}{2} z\right)^n \int_1^\infty dx\, \left(x^2 - 1\right)^{n - \frac{1}{2}} e^{-zx}$$

With $\Gamma(n)$ the gamma function as generalization of the factorial.

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