The Standard Model Higgs Boson

Part of the Lecture Particle Physics II, UvA Particle Physics Master 2011-2012

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Lecturer: Ivo van Vulpen
Assistant: Antonio Castelli

Disclaimer

These are private notes to prepare for the lecture on the Higgs mechanism, part of the of lecture Particle Physics II. The first two sections almost entirely based on the book 'Quarks and Leptons' from the authors F. Halzen & A. Martin. The rest is a collection of material takes from publications and the documents listed below.

Material used to prepare lecture:
- o Quarks and Leptons, F. Halzen & A. Martin (main source)
- o Gauge Theories of the Strong, Weak and Electromagnetic Interactions, C. Quigg
- o Introduction to Elementary Particles, D. Griffiths
- o Lecture notes, Particle Physics 1, M. Merk

Contents

Introduction

1 Symmetry breaking

After a review of the shortcomings of the model of electroweak interactions in the Standard Model, in this section we study the consequences of spontaneous symmetry breaking of (gauge) symmetries. We will do this in three steps of increasing complexity and focus on the principles of how symmetry breaking can be used to obtain massive gauge bosons by working out in full detail the breaking of a local U(1) gauge invariant model (QED) and give the photon a mass.

1.1 Problems in the Electroweak Model

The electroweak model, beautiful as it is, has some serious shortcomings.

1] Local $\mathbf{SU(2)_L \times U(1)_Y}$ gauge invariance forbids massive gauge bosons

In the theory of QuantumElectroDynamics (QED) the requirement of local gauge invariance, i.e. the invariance of the Lagrangian under the transformation $\phi^{'} \to e^{i\alpha(x)}\phi$ plays a fundamental rôle. Invariance was achieved by replacing the partial derivative by a covariant derivative, $\partial_\mu \to \mathcal{D}_\mu = \partial_\mu - ieA_\mu$ and the introduction of a new vector field A with very specific transformation properties: $A^{'}_\mu \to A_\mu + \frac{1}{e}\partial_\mu\alpha$. This Lagrangian for a free particle then changed to:

$$\mathcal{L}_{\rm QED} \; = \; \mathcal{L}_{\rm free} + \mathcal{L}_{\rm int} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

, which not only 'explained' the presence of a vector field in nature (the photon), but also automatically yields an interaction term $\mathcal{L}_{\rm int} = eJ^\mu A_\mu$ between the vector field and the particle as explained in detail in the lectures on the electroweak model. Under these symmetry requirements it is unfortunately not possible for a gauge boson to acquire a mass. In QED for example, a mass term for the photon, would not be allowed as such a term breaks gauge invariance:

$$\frac{1}{2}m_\gamma^2 A_\mu A^\mu = \frac{1}{2}m_\gamma^2(A_\mu + \frac{1}{e}\delta_\mu\alpha)(A^\mu + \frac{1}{e}\delta^\mu\alpha) \neq \frac{1}{2}m_\gamma^2 A_\mu A^\mu$$

The example using only U(1) and the mass of the photon might sounds strange as the photon is actually massless, but a similar argument holds in the electroweak model for the W and Z bosons, particles that we *know* are massive and make the weak force only present at very small distances.

2] Local $\mathbf{SU(2)_L \times U(1)_Y}$ gauge invariance forbids massive fermions

Just like in QED, invariance under local gauge transformations in the electroweak model requires introducing a covariant derivative of the form $D_\mu = \partial_\mu + ig\frac{1}{2}\vec{\tau}\cdot\vec{W}_\mu + ig'\frac{1}{2}YB_\mu$ introducing a weak current, $J^{\rm weak}$ and a different transformation for isospin singlets and doublets. A mass term for a fermion in the Lagrangian would be of the form $-m_f\bar{\psi}\psi$, but such terms in the Lagrangian are not allowed as they are *not* gauge invariant. This is clear when we decompose the expression in helicity states:

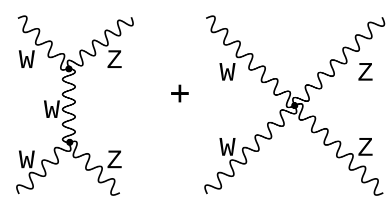
$$\begin{aligned} -m_f\bar{\psi}\psi &= -m_f\left(\bar{\psi_R}+\bar{\psi_L}\right)\left(\psi_L+\psi_R\right) \\ &= -m_f\left[\bar{\psi_R}\psi_L+\bar{\psi_L}\psi_R\right] \qquad , \text{since } \bar{\psi_R}\psi_R=\bar{\psi_L}\psi_L=0 \end{aligned}$$

Since ψ_L (left-handed, member of an isospin doublet, I = $\frac{1}{2}$) and ψ_R (right-handed, isospin singlet, I = 0) behave differently under rotations these terms are not gauge invariant:

$$\begin{aligned} {\psi_L}' \rightarrow \psi_L &= e^{i\alpha(x)T+i\beta(x)Y}\psi_L \\ {\psi_R}' \rightarrow \psi_R &= e^{i\beta(x)Y}\psi_R \end{aligned}$$

3] Violating unitarity

Several Standard Model scattering cross-sections, like WW-scattering (some Feynman graphs are shown in the picture on the right), violate unitarity at high energy as $\sigma(\mathrm{WW}\rightarrow\mathrm{ZZ})\propto \mathrm{E}^2$. This energy dependency clearly makes the theory non-renormalizable.



How to solve the problems: a way out

To keep the theory renormalizable, we need a very high degree of symmetry (local gauge invariance) in the model. Dropping the requirement of the local $\mathrm{SU(2)_L}\times\mathrm{U(1)_Y}$ gauge invariance is therefore not a wise decision. Fortunately there is a way out of this situation:

> Introduce a new field with a very specific potential that keeps the full Lagrangian invariant under $\mathrm{SU(2)_L}\times\mathrm{U(1)_Y}$, but will make the vacuum *not* invariant under this symmetry. We will explore this idea, spontaneous symmetry breaking of a local gauge invariant theory (or Higgs mechanism), in detail in this section.

The Higgs mechanism: - Solves all the above problems
- Introduces a fundamental scalar $\rightarrow$ the Higgs boson !

1.2 A few basics on Lagrangians

A short recap of the basics on Lagrangians we'll be using later.

$$\mathcal{L} = \mathrm{T(kinetic)} - \mathrm{V(potential)}$$

The Euler-Lagrange equation then give you the equations of motion:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = 0$$

For a real scalar field for example:

$$\mathcal{L}_{\text{scalar}} = \frac{1}{2}\left(\partial_\mu \phi\right)\left(\partial^\mu \phi\right) - \frac{1}{2} m^2 \phi^2 \rightarrow \text{Euler-Lagrange} \rightarrow \underbrace{(\partial_\mu \partial^\mu + m^2)\phi = 0}_{\text{Klein-Gordon equation}}$$

In electroweak theory, kinematics of fermions, i.e. spin-1/2 particles is described by:

$$\mathcal{L}_{\text{fermion}} = i\bar{\psi}\gamma_\mu \partial^\mu \psi - m\bar{\psi}\psi \rightarrow \text{Euler-Lagrange} \rightarrow \underbrace{\left(i\gamma_\mu \partial^\mu - m\right)\psi = 0}_{\text{Dirac equation}}$$

In general, the Lagrangian for a real scalar particle (ϕ) is given by:

$$\mathcal{L} = \underbrace{\left(\partial_\mu \phi\right)^2}_{\text{kinetic term}} + \underbrace{C}_{\text{constant}} + \underbrace{\alpha\phi}_{?} + \underbrace{\beta\phi^2}_{\text{mass term}} + \underbrace{\gamma\phi^3}_{\text{3-point int.}} + \underbrace{\delta\phi^4}_{\text{4-point int.}} + \ ... \qquad (1)$$

We can interpret the particle spectrum of the theory when studying the Lagrangian under small perturbations. In expression (1), the constant (potential) term is for most purposes of no importance as it does not appear in the equation of motion, the term linear in the field has no direct interpretation (and should not be present as we will explain later), the quadratic term in the fields represents the mass of the field/particle and higher order terms describe interaction terms.

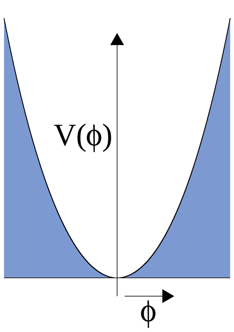
1.3 Simple example of symmetry breaking

To describe the main idea of symmetry breaking we start with a simple model for a real scalar field ϕ (or a theory to which we add a new field ϕ), with a specific potential term:

$$\begin{aligned} \mathcal{L} &= \frac{1}{2}(\partial_\mu \phi)^2 - \mathrm{V}(\phi) \\ &= \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}\mu^2\phi^2 - \frac{1}{4}\lambda\phi^4 \end{aligned} \qquad (2)$$

Note that $\mathcal{L}$ is symmetric under $\phi \rightarrow -\phi$ and that λ is positive to ensure an absolute minimum in the Lagrangian. We can investigate in some detail the two possibilities for the sign of μ^2: positive or negative.

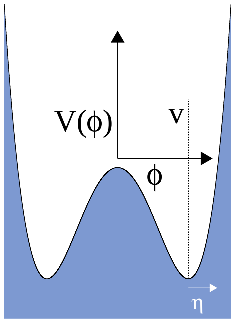
1.3.1 $\mu^2 > 0$: Free particle with additional interactions



To investigate the particle spectrum we look at the Lagrangian for small perturbations around the minimum (vacuum). The vacuum is at $\phi = 0$ and is symmetric in ϕ and using expression (1) we see that the Lagrangian describes a free particle with mass μ that has an additional four-point self-interaction:

$$\mathcal{L} = \underbrace{\frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}\mu^2\phi^2}_{\text{free particle, mass } \mu} \quad \underbrace{-\frac{1}{4}\lambda\phi^4}_{\text{interaction}}$$

1.3.2 $\mu^2 < 0$: Introducing a particle with imaginary mass ?



The situation with $\mu^2 < 0$ looks strange since at first glance it would appear to describe a particle ϕ with an imaginary mass. However, if we take a closer look at the potential, we see that it does not make sense to interpret the particle spectrum using the field ϕ since perturbation theory around $\phi = 0$ will not converge (not a stable minimum) as the vacuum is located at:

$$\phi_0 = \sqrt{-\frac{\mu^2}{\lambda}} = v \quad \text{or} \quad \mu^2 = -\lambda v^2 \tag{3}$$

As before, to investigate the particle spectrum in the theory, we have to look at small perturbations around this minimum. To do this it is more natural to introduce a field η (simply a shift of the ϕ field) that is centered at the vacuum: $\eta = \phi - v$.

Rewriting the Lagrangian in terms of η

Expressing the Lagrangian in terms of the shifted field η is done by replacing ϕ by $\eta + v$ in the original Lagrangian from equation (2):

$$\begin{aligned} \text{Kinetic term:} \quad \mathcal{L}_{\text{kin}}(\eta) &= \frac{1}{2}(\partial_\mu(\eta+v)\partial^\mu(\eta+v)) \\ &= \frac{1}{2}(\partial_\mu\eta)(\partial^\mu\eta) \qquad \text{, since } \partial_\mu v = 0. \end{aligned}$$

$$\begin{aligned} \text{Potential term:} \quad \mathrm{V}(\eta) &= +\frac{1}{2}\mu^2(\eta+v)^2 + \frac{1}{4}\lambda(\eta+v)^4 \\ &= \lambda v^2\eta^2 + \lambda v\eta^3 + \frac{1}{4}\lambda\eta^4 - \frac{1}{4}\lambda v^4 \end{aligned}$$

, where we used $\mu^2 = -\lambda v^2$ from equation (3). Although the Lagrangian is still symmetric in ϕ, the perturbations around the minimum are *not* symmetric in η, i.e. $\mathrm{V}(-\eta) \neq \mathrm{V}(\eta)$. Neglecting the irrelevant $\frac{1}{4}\lambda v^4$ constant term and the leaving only the terms up to η^2 we have as Lagrangian:

$$\begin{aligned} \text{Full Lagrangian:} \quad \mathcal{L}(\eta) &= \frac{1}{2}(\partial_\mu\eta)(\partial^\mu\eta) - \lambda v^2\eta^2 - \lambda v\eta^3 - \frac{1}{4}\lambda\eta^4 - \frac{1}{4}\lambda v^4 \\ &= \frac{1}{2}(\partial_\mu\eta)(\partial^\mu\eta) - \lambda v^2\eta^2 \end{aligned}$$

From section 1.2 we see that this describes the kinematics for a massive scalar particle:

$$\frac{1}{2}m_\eta^2 = \lambda v^2 \rightarrow m_\eta = \sqrt{2\lambda v^2} \quad \left(= \sqrt{-2\mu^2}\right) \qquad \text{Note: } m_\eta > 0.$$

Executive summary on $\mu^2 < 0$ scenario

At first glance, adding a $V(\phi)$ term as in equation (2) to the Lagrangian, implies adding a particle with imaginary mass with a four-point self-interaction. However, when examining the particle spectrum using perturbations around the vacuum we see that it actually describes a massive scalar particle (real, positive mass) with three- and four-point self-interactions. Although the Lagrangian retains it's original symmetry (symmetric in ϕ), the vacuum is not symmetric in the field η: spontaneous symmetry breaking. Note that we have added a single degree of freedom to the theory: a scalar particle.

1.4 Breaking a global symmetry

In an existing theory we are free to introduce an additional complex scalar field: $\phi = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2)$ (two degrees of freedom):

$$\mathcal{L} = (\partial_\mu \phi)^*(\partial^\mu \phi) - \mathrm{V}(\phi) \qquad \text{, with } \mathrm{V}(\phi) = \mu^2(\phi^*\phi) + \lambda(\phi^*\phi)^2$$

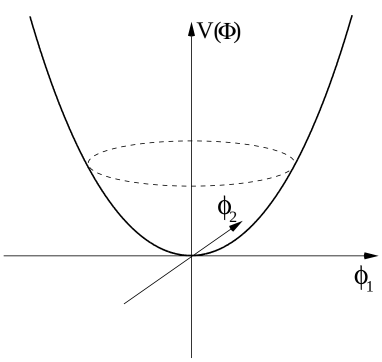
Note that the Lagrangian is invariant under a U(1) global symmetry, i.e. under $\phi' \to e^{i\alpha}\phi$ since $\phi'^*\phi' \to \phi^*\phi e^{-i\alpha}e^{+i\alpha} = \phi^*\phi$.

The Lagrangian in terms of ϕ_1 and ϕ_2 is given by:

$$\mathcal{L}(\phi_1,\phi_2) = \frac{1}{2}(\partial_\mu\phi_1)^2 + \frac{1}{2}(\partial_\mu\phi_2)^2 - \frac{1}{2}\mu^2(\phi_1^2+\phi_2^2) - \frac{1}{4}\lambda(\phi_1^2+\phi_2^2)^2$$

There are again two distinct cases: $\mu^2 > 0$ and $\mu^2 < 0$. As in the previous section, we investigate the particle spectrum by studying the Lagrangian under small perturbations around the vacuum.

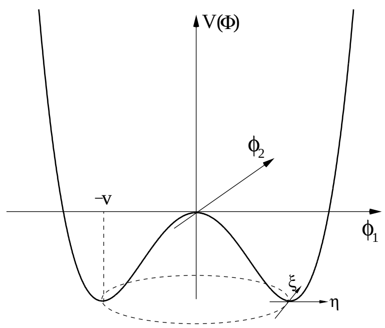
1.4.1 $\mu^2 > 0$



This situation simply describes two massive scalar particles, each with a mass μ with additional interactions:

$$\begin{aligned}\mathcal{L}(\phi_1,\phi_2) \quad = \quad & \underbrace{\frac{1}{2}(\partial_\mu\phi_1)^2 - \frac{1}{2}\mu^2\phi_1^2}_{\text{particle } \phi_1\text{, mass } \mu} + \underbrace{\frac{1}{2}(\partial_\mu\phi_2)^2 - \frac{1}{2}\mu^2\phi_2^2}_{\text{particle } \phi_2\text{, mass } \mu} \\ + \quad & \text{interaction terms}\end{aligned}$$

1.4.2 $\mu^2 < 0$

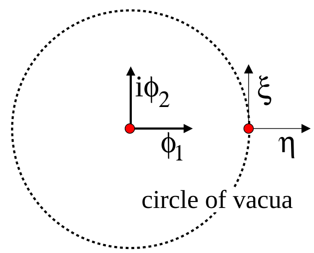


When $\mu^2 < 0$ there is not a single vacuum located at $\left(\begin{smallmatrix}0\\0\end{smallmatrix}\right)$, but an infinite number of vacua that satisfy:

$$\sqrt{\phi_1^2 + \phi_2^2} = \sqrt{\frac{-\mu^2}{\lambda}} = v$$

From the infinite number we choose ϕ_0 as $\phi_1 = v$ and $\phi_2 = 0$. To see what particles are present in this model, the behaviour of the Lagrangian is studied under small oscillations around the vacuum.

When looking at perturbations around this minimum it it natural to define the shifted fields η and ξ, with: $\eta = \phi_1 - v$ and $\xi = \phi_2$, which means that the (perturbations around the) vacuum are described by:



$$\phi_0 = \frac{1}{\sqrt{2}}(\eta + v + i\xi)$$

Using $\phi^2 = \phi^*\phi = \frac{1}{2}[(v+\eta)^2 + \xi^2]$ and $\mu^2 = -\lambda v^2$ we can rewrite the Lagrangian in terms of the shifted fields.

$$\begin{aligned}
\text{Kinetic term:}\quad \mathcal{L}_{\text{kin}}(\eta,\xi) &= \frac{1}{2}\partial_\mu(\eta + v - i\xi)\partial^\mu(\eta + v + i\xi) \\
&= \frac{1}{2}(\partial_\mu\eta)^2 + \frac{1}{2}(\partial_\mu\xi)^2 \quad \text{, since } \partial_\mu v = 0.
\end{aligned}$$

$$\begin{aligned}
\text{Potential term:}\quad \text{V}(\eta,\xi) &= \mu^2\phi^2 + \lambda\phi^4 \\
&= -\frac{1}{2}\lambda v^2[(v+\eta)^2 + \xi^2] + \frac{1}{4}\lambda[(v+\eta)^2 + \xi^2]^2 \\
&= -\frac{1}{4}\lambda v^4 + \lambda v^2\eta^2 + \lambda v\eta^3 + \frac{1}{4}\lambda\eta^4 + \frac{1}{4}\lambda\xi^4 + \lambda v\eta\xi^2 + \frac{1}{2}\lambda\eta^2\xi^2
\end{aligned}$$

Neglecting the constant and higher order terms, the full Lagrangian can be written as:

$$\mathcal{L}(\eta,\xi) = \underbrace{\frac{1}{2}(\partial_\mu\eta)^2 - (\lambda v^2)\eta^2}_{\text{massive scalar particle } \eta} + \underbrace{\frac{1}{2}(\partial_\mu\xi)^2 + 0\cdot\xi^2}_{\text{massless scalar particle } \xi} + \text{higher order terms}$$

We can identify this as a massive η particle and a massless ξ particle:

$$m_\eta = \sqrt{2\lambda v^2} = \sqrt{-2\mu^2} > 0 \quad \text{and} \quad m_\xi = 0$$

Unlike the η-field, describing radial excitations, there is no 'force' acting on oscillations along the ξ-field. This is a direct consequence of the U(1) symmetry of the Lagrangian and the massless particle ξ is the so-called Goldstone boson.

Goldstone theorem:
For each broken generator of the original symmetry group, i.e. for each generator that connects the vacuum states one massless spin-zero particle will appear.

Executive summary on breaking a global gauge invariant symmetry

Spontaneously breaking a continuous global symmetry gives rise to a massless (Goldstone) boson. When we break a *local* gauge invariance something special happens and the Goldstone boson will disappear.

1.5 Breaking a local gauge invariant symmetry: the Higgs mechanism

In this section we will take the final step and study what happens if we break a *local* gauge invariant theory. As promised in the introduction, we will explore it's consequences using a local U(1) gauge invariant theory we know (QED). As we will see, this will allows to add a mass-term for the gauge boson (the photon).

Local U(1) gauge invariance is the requirement that the Lagrangian is invariant under $\phi^{'} \to e^{i\alpha(x)}\phi$. From the lectures on electroweak theory we know that this can be achieved by switching to a covariant derivative with a special transformation rule for the vector field. In QED:

$$\begin{aligned} \partial_\mu &\to D_\mu = \partial_\mu - ieA_\mu && \text{[covariant derivatives]} \\ A_\mu^{'} &= A_\mu + \frac{1}{e}\partial_\mu\alpha && [A_\mu \text{ transformation}] \end{aligned} \tag{4}$$

The local U(1) gauge invariant Lagrangian for a complex scalar field is then given by:

$$\mathcal{L} = (D^\mu\phi)^\dagger (D_\mu\phi) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - V(\phi)$$

The term $\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$ is the kinetic term for the gauge field (photon) and $V(\phi)$ is the extra term in the Lagrangian we have seen before: $V(\phi^*\phi) = \mu^2(\phi^*\phi) + \lambda(\phi^*\phi)^2$.

1.5.1 Lagrangian under small perturbations

The situation $\mu^2 > 0$: we have a vacuum at $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$. The exact symmetry of the Lagrangian is preserved in the vacuum: we have QED with a massless photon and two massive scalar particles ϕ_1 and ϕ_2 each with a mass μ.

In the situation $\mu^2 < 0$ we have an infinite number of vacua, each satisfying $\phi_1^2 + \phi_2^2 = -\mu^2/\lambda = v^2$. The particle spectrum is obtained by studying the Lagrangian under small oscillations using the same procedure as for the continuous global symmetry from section (1.4.2). Because of <u>local</u> gauge invariance some important differences appear. Extra terms will appear in the kinetic part of the Lagrangian through the covariant derivatives. Using again the shifted fields η and ξ we define the vacuum as $\phi_0 = \frac{1}{\sqrt{2}}[(v + \eta) + i\xi]$.

$$
\begin{aligned}
\text{Kinetic term:} \quad \mathcal{L}_{\rm kin}(\eta,\xi) &= (D^{\mu}\phi)^{\dagger}(D_{\mu}\phi) \\
&= (\partial^{\mu}+ieA^{\mu})\phi^{*}(\partial_{\mu}-ieA_{\mu})\phi \\
&= \text{ ... see Exercise 1}
\end{aligned}
$$

$$
\text{Potential term:} \quad V(\eta,\xi) = \lambda v^2\eta^2 \text{ , up to second order in the fields. See section 1.4.2.}
$$

The full Lagrangian can be written as:

$$
\mathcal{L}(\eta,\xi) = \underbrace{\frac{1}{2}(\partial_{\mu}\eta)^2 - \lambda v^2\eta^2}_{\eta\text{-particle}} + \underbrace{\frac{1}{2}(\partial_{\mu}\xi)^2}_{\xi\text{-particle}} \underbrace{-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}e^2v^2A_{\mu}^2}_{\text{photon field}} - \underbrace{evA_{\mu}(\partial^{\mu}\xi)}_{?} + \text{int.-terms} \tag{5}
$$

At first glance: massive η, massless ξ (as before) and also a mass term for the photon. However, the Lagrangian also contains strange terms that we cannot easily interpret: $-evA_{\mu}(\partial^{\mu}\xi)$. This prevents making an easy interpretation.

1.5.2 Rewriting the Lagrangian in the unitary gauge

In a local gauge invariance theory we see that A_{μ} is fixed up to a term $\partial_{\mu}\alpha$ as can be seen from equation (4). In general, A_{μ} and ϕ change simultaneously. We can exploit this freedom, to redefine A_{μ} and remove all terms involving the ξ field.

Looking at the terms involving the ξ-field, we see that we can rewrite them as:

$$
\frac{1}{2}(\partial_{\mu}\xi) - evA^{\mu}(\partial_{\mu}\xi) + \frac{1}{2}e^2v^2A_{\mu}^2 \quad = \quad \frac{1}{2}e^2v^2\left[A_{\mu} - \frac{1}{ev}(\partial_{\mu}\xi)\right]^2 \quad = \quad \frac{1}{2}e^2v^2(A_{\mu}^{'})^2
$$

This specific choice, i.e. taking $\alpha = -\xi/v$, is called the *unitary gauge*. Of course, when choosing this gauge (phase of rotation α) the field ϕ changes accordingly, see first part of section 1.1:

$$
\phi^{'} \rightarrow e^{-i\ \xi/v}\phi \;=\; e^{-i\ \xi/v}\frac{1}{\sqrt{2}}(v+\eta+i\xi) \;=\; e^{-i\ \xi/v}\frac{1}{\sqrt{2}}(v+\eta)e^{+i\ \xi/v} \;=\; \frac{1}{\sqrt{2}}(v+h)
$$

Here we have introduced the real h-field. When writing down the full Lagrangian in this specific gauge, we will see that all terms involving the ξ-field will disappear and that the extra degree of freedom will appear as the mass term for the gauge boson associated to the broken symmetry.

1.5.3 Lagrangian in the unitary gauge: particle spectrum

$$\begin{aligned}\mathcal{L}_{\text{scalar}} &= \left(D^\mu\phi\right)^\dagger\left(D_\mu\phi\right) - V(\phi^\dagger\phi)\\ &= (\partial^\mu + ieA^\mu)\frac{1}{\sqrt{2}}(v+h)\ \ (\partial_\mu - ieA_\mu)\frac{1}{\sqrt{2}}(v+h) - V(\phi^\dagger\phi)\\ &= \frac{1}{2}(\partial_\mu h)^2 + \frac{1}{2}e^2A_\mu^2(v+h)^2 - \lambda v^2h^2 - \lambda v h^3 - \frac{1}{4}\lambda h^4 + \frac{1}{4}\lambda v^4\end{aligned}$$

Expanding $(v+h)^2$ into 3 terms (and ignoring $\frac{1}{4}\lambda v^4$) we end up with:

$$= \underbrace{\frac{1}{2}(\partial_\mu h)^2 - \lambda v^2h^2}_{\substack{\text{massive scalar}\\ \text{particle h}}} + \underbrace{\frac{1}{2}e^2v^2A_\mu^2}_{\substack{\text{gauge field }(\gamma)\\ \text{with mass}}} + \underbrace{e^2vA_\mu^2h + \frac{1}{2}e^2A_\mu^2h^2}_{\substack{\text{interaction Higgs}\\ \text{and gauge fields}}} \underbrace{- \lambda vh^3 - \frac{1}{4}\lambda h^4}_{\substack{\text{Higgs self-}\\ \text{interactions}}}$$

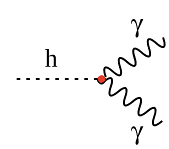
1.5.4 A few words on expanding the terms with $(\mathbf{v}+\mathbf{h})^2$

Expanding the terms in the Lagrangian associated to the vector field we see that we do not only get terms proportional to A_μ^2, i.e. a mass term for the gauge field (photon), but also automatically terms that describe the interaction of the Higgs field with the gauge field. These interactions, related to the mass of the gauge boson, are a consequence of the Higgs mechanism.

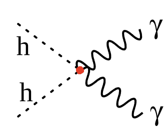
In our model, QED with a massive photon, when expanding $\frac{1}{2}e^2A_\mu^2(v+h)^2$ we get:

1] $\frac{1}{2}e^2v^2A_\mu^2$: the mass term for the gauge field (photon)
Given equation (1) we see that $m_\gamma = ev$.

2] $e^2vA_\mu^2h$: photon-Higgs three-point interaction



3] $\frac{1}{2}e^2A_\mu^2h^2$: photon-Higgs four-point interaction



Executive summary on breaking a local gauge invariant symmetry

We added a complex scalar field (2 degrees of freedom) to our existing theory and broke the original symmetry by using a 'strange' potential that yielded a large number of vacua. The extra degrees of freedom appear in the theory as a mass term for the gauge boson connected to the broken symmetry (m_γ) and a massive scalar particle (m_h).

Exercises lecture 1

Exercise 1: interaction terms

a) Compute the 'interaction terms' as given in equation (5).

b) Are the interaction terms symmetric in η and ξ ?

Exercise 2: Toy-model with a massive photon

a) Derive expression (14.58) in Halzen & Martin.
Hint: you can either do the full computation or, much less work, just insert $\phi = \frac{1}{\sqrt{2}}(v+h)$ in the Lagrangian and keep A_μ unchanged.

b) Show that in this model the Higgs boson can decay into two photons and that the coupling $h \to \gamma\gamma$ is proportional to m_γ.

c) Draw all Feynman vertices that are present in this model.

d) Show that Higgs three-point (self-)coupling, or $h \to hh$, is proportional to $\mathrm{m_h}$.

Exercise 3: the potential part: V$(\phi^\dagger\phi)$

Use in this exercise $\phi = \frac{1}{\sqrt{2}}(v+h)$.

a) The normal Higgs potential: $V(\phi^\dagger\phi) = \mu^2\phi^2 + \lambda\phi^4$
Show that $\frac{1}{2}m_h^2 = \lambda v^2$, where $(\phi_0 = v)$. How many vacua are there?

b) Why is $V(\phi^\dagger\phi) = \mu^2\phi^2 + \beta\phi^3$ not possible ?
How many vacua are there?

Terms $\propto \phi^6$ are allowed since they introduce additional interactions that are not cancelled by gauge boson interactions, making the model non-renormalizable. Just ignore this little detail for the moment and compute the 'prediction' for the Higgs boson mass.

c) Use $V(\phi^\dagger\phi) = \mu^2\phi^2 - \lambda\phi^4 + \frac{4}{3}\delta\phi^6$, with $\mu^2 < 0$, $\lambda > 0$ and $\delta = -\frac{2\lambda^2}{\mu^2}$.
Show that $\mathrm{m}_h(\mathrm{new}) = \sqrt{3}\mathrm{m}_h(\mathrm{old})$, with 'old': m_h for the normal Higgs potential.

2 The Higgs mechanism in the Standard Model

In this section we will apply the idea of spontaneous symmetry breaking from section 1 to the model of electroweak interactions. With a specific choice of parameters we can obtain massive Z and W bosons while keeping the photon massless.

2.1 Breaking the local gauge invariant $\mathbf{SU(2)_L \times U(1)_Y}$ symmetry

To break the $\mathrm{SU(2)_L \times U(1)_Y}$ symmetry we follow the ingredients of the Higgs mechanism:

1) Add an isospin doublet:

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}$$

Since we would like the Lagrangian to retain all its symmetries, we can only add $\mathrm{SU(2)_L \times U(1)_Y}$ multiplets. Here we add a left-handed doublet (like the electron neutrino doublet) with weak Isospin $\frac{1}{2}$. The electric charges of the upper and lower component of the doublet are chosen to ensure that the hypercharge Y=+1. This requirement is vital for reasons that will become more evident later.

2) Add a potential V(ϕ) for the field that will break (spontaneously) the symmetry:

$$V(\phi) = \mu^2(\phi^\dagger\phi) + \lambda(\phi^\dagger\phi)^2 \quad \text{, with } \mu^2 < 0$$

The part added to the Lagrangian for the scalar field

$$\mathcal{L}_{\rm scalar} = (D^\mu\phi)^\dagger(D_\mu\phi) - V(\phi)$$

, where D_μ is the covariant derivative associated to $\mathrm{SU(2)_L \times U(1)_Y}$:

$$D_\mu = \partial_\mu + ig\frac{1}{2}\vec{\tau}\cdot\vec{W}_\mu + ig'\frac{1}{2}YB_\mu$$

3) Choose a vacuum:
We have seen that any choice of the vacuum that breaks a symmetry will generate a mass for the corresponding gauge boson. The vacuum we choose has ϕ_1=ϕ_2=ϕ_4=0 and $\phi_3 = v$:

$$\text{Vacuum} = \phi_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h \end{pmatrix},$$

This vacuum as defined above is neutral since $I = \frac{1}{2}$, $I_3 = -\frac{1}{2}$ and with our choice of $Y = +1$ we have $Q = I_3 + \frac{1}{2}Y$=0. We will see that this choice of the vacuum breaks $\mathrm{SU(2)_L \times U(1)_Y}$,but leaves $\mathrm{U(1)_{EM}}$ invariant, leaving only the photon massless. In writing down this vacuum we immediately went to the unitary gauge (see section 1.5).

2.2 Checking which symmetries are broken in a given vacuum

How do we check if the symmetries associated to the gauge bosons are broken ? Invariance implies that $e^{i\alpha Z}\phi_0 = \phi_0$, with Z the associated 'rotation'. Under infinitesimal rotations this means $(1 + i\alpha Z)\phi_0 = \phi_0 \rightarrow Z\phi_0 = 0$.

What about the $\mathrm{SU(2)_L}$, $\mathrm{U(1)_Y}$ and $\mathrm{U(1)_{EM}}$ generators:

$$\begin{aligned}
\mathrm{SU(2)_L}: \quad \tau_1\phi_0 &= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+h \end{pmatrix} = +\frac{1}{\sqrt{2}}\begin{pmatrix} v+h \\ 0 \end{pmatrix} \neq 0 \rightarrow \text{broken} \\
\tau_2\phi_0 &= \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+h \end{pmatrix} = -\frac{i}{\sqrt{2}}\begin{pmatrix} v+h \\ 0 \end{pmatrix} \neq 0 \rightarrow \text{broken} \\
\tau_3\phi_0 &= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+h \end{pmatrix} = -\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+h \end{pmatrix} \neq 0 \rightarrow \text{broken} \\
\mathrm{U(1)_Y}: \quad Y\phi_0 &= Y_{\phi_0} \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+h \end{pmatrix} = +\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+h \end{pmatrix} \neq 0 \rightarrow \text{broken}
\end{aligned}$$

This means that all 4 gauge bosons (W_1, W_2, W_3 and B) acquire a mass through the Higgs mechanism. In the lecture on electroweak theory we have seen that the W_1 and W_2 fields mix to form the charged W^+ and W^- bosons and that the W_3 and B field will mix to form the neutral Z-boson and photon.

$$\underbrace{W_1 \quad W_2}_{\text{W}^+ \text{ and W}^- \text{ bosons}} \qquad \underbrace{W_3 \quad B}_{\text{Z-boson and } \gamma}$$

When computing the masses of these mixed physical states in the next sections, we will see that one of these combinations (the photon) remains massless. Looking at the symmetries we can already predict this is the case. For the photon to remain massless the $\mathrm{U(1)_{EM}}$ symmetry should leave the vacuum invariant. And indeed:

$$\mathrm{U(1)_{EM}}: \quad Q\phi_0 = \frac{1}{2}(\tau_3 + Y)\phi_0 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+h \end{pmatrix} = 0 \rightarrow \text{unbroken}$$

It is not so strange that $\mathrm{U(1)_{EM}}$ is conserved as the vacuum is neutral and we have:

$$\phi_0^{'} \rightarrow e^{i\alpha Q_{\phi_0}}\phi_0 = \phi_0$$

Breaking of $\mathbf{SU(2)_L \times U(1)_Y}$: looking a bit ahead

1) $\mathrm{W_1}$ and $\mathrm{W_2}$ mix and will form the massive a $\mathrm{W^+}$ and $\mathrm{W^-}$ bosons.
2) $\mathrm{W_3}$ and B mix to form massive Z and massless γ.
3) Remaining degree of freedom will form the mass of the scalar particle (Higgs boson).

2.3 Scalar part of the Lagrangian: gauge boson mass terms

Studying the scalar part of the Lagrangian

To obtain the masses for the gauge bosons we will only need to study the scalar part of the Lagrangian.

$$\mathcal{L}_{\text{scalar}} = (D^\mu\phi)^\dagger(D_\mu\phi) - V(\phi) \tag{6}$$

The $V(\phi)$ term will again give the mass term for the Higgs boson and the Higgs self-interactions. The $(D^\mu\phi)^\dagger(D_\mu\phi)$ terms:

$$D_\mu\phi = \left[\partial_\mu + ig\frac{1}{2}\vec{\tau}\cdot\vec{W}_\mu + ig'\frac{1}{2}YB_\mu\right] \;\; \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+h \end{pmatrix}$$

will give rise to the masses of the gauge bosons (and the interaction of the gauge bosons with the Higgs boson) since, as we discussed in section 1.5.4, working out the $(v+h)^2$-terms from equation (6) will give us three terms:

1) Masses for the gauge bosons ($\propto v^2$)
2) Interactions gauge bosons and the Higgs ($\propto vh$) and ($\propto h^2$)

We are interested in the masses of the vector bosons and will therefore only focus on 1):

$$\begin{aligned}
(D_\mu\phi) &= \frac{1}{\sqrt{2}}\left[ig\frac{1}{2}\vec{\tau}\cdot\vec{W}_\mu + ig'\frac{1}{2}YB_\mu\right]\begin{pmatrix} 0 \\ v \end{pmatrix} \\
&= \frac{i}{\sqrt{8}}\left[g(\tau_1 W_1 + \tau_2 W_2 + \tau_3 W_3) + g^{'}YB_\mu\right]\begin{pmatrix} 0 \\ v \end{pmatrix} \\
&= \frac{i}{\sqrt{8}}\left[g\left(\left(\begin{smallmatrix} 0 & W_1 \\ W_1 & 0 \end{smallmatrix}\right) + \left(\begin{smallmatrix} 0 & -iW_2 \\ iW_2 & 0 \end{smallmatrix}\right) + \left(\begin{smallmatrix} W_3 & 0 \\ 0 & -W_3 \end{smallmatrix}\right)\right) + g^{'}\left(\begin{smallmatrix} Y_{\phi_0}B_\mu & 0 \\ 0 & Y_{\phi_0}B_\mu \end{smallmatrix}\right)\right]\begin{pmatrix} 0 \\ v \end{pmatrix} \\
&= \frac{i}{\sqrt{8}}\begin{pmatrix} gW_3 + g^{'}Y_{\phi_0}B_\mu & g(W_1 - iW_2) \\ g(W_1 + iW_2) & -gW_3 + g^{'}Y_{\phi_0}B_\mu \end{pmatrix}\begin{pmatrix} 0 \\ v \end{pmatrix} \\
&= \frac{iv}{\sqrt{8}}\begin{pmatrix} g(W_1 - iW_2) \\ -gW_3 + g^{'}Y_{\phi_0}B_\mu \end{pmatrix}
\end{aligned}$$

We can then also easily compute $(D^\mu\phi)^\dagger$: $(D^\mu\phi)^\dagger = -\frac{iv}{\sqrt{8}}\left(g(W_1 + iW_2)\ ,\ (-gW_3 + g^{'}Y_{\phi_0}B_\mu)\right)$ and we get the following expression for the kinetic part of the Lagrangian:

$$(D^\mu\phi)^\dagger\,(D_\mu\phi) = \frac{1}{8}v^2\left[g^2(W_1^2 + W_2^2) + (-gW_3 + g^{'}Y_{\phi_0}B_\mu)^2\right] \tag{7}$$

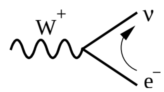
2.3.1 Rewriting $(\mathbf{D}^\mu\phi)^\dagger\,(\mathbf{D}_\mu\phi)$ in terms of physical gauge bosons

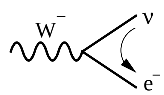
Before we can interpret this we need to rewrite this in terms of W^+, W^-, Z and γ since that are the gauge bosons that are observed in nature.

1] Rewriting terms with $\mathbf{W_1}$ and $\mathbf{W_2}$ terms: charged gauge bosons $\mathbf{W^+}$ and $\mathbf{W^-}$

When discussing the charged current interaction on $\mathrm{SU(2)_L}$ doublets we saw that the charge raising and lowering operators connecting the members of isospin doublets were τ_+ and τ_-, linear combinations of τ_1 and τ_2 and that each had an associated gauge boson: the $\mathrm{W^+}$ and $\mathrm{W^-}$.

$$\begin{aligned}\tau_+ &= \frac{1}{2}(\tau_1 + i\tau_2) = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \\ \tau_- &= \frac{1}{2}(\tau_1 - i\tau_2) = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}\end{aligned}$$





We can rewrite W_1, W_2 terms as W^+, W^- using $W^\pm = \frac{1}{\sqrt{2}}(W_1 \mp iW_2)$. In particular, $\frac{1}{2}(\tau_1 W_1 + \tau_2 W_2) = \frac{1}{\sqrt{2}}(\tau_+ W^+ + \tau_- W^-)$.

Looking at the terms involving W_1 and W_2 in the Lagrangian in equation (7), we see that:

$$g^2(W_1^2 + W_2^2) = 2g^2 W^+ W^- \tag{8}$$

2] Rewriting terms with $\mathbf{W_3}$ and $\mathbf{B_\mu}$ terms: neutral gauge bosons Z and γ

$$(-gW_3 + g'Y_{\phi_0}B_\mu)^2 = (W_3, B_\mu)\begin{pmatrix} g^2 & -gg'Y_{\phi_0} \\ -gg'Y_{\phi_0} & g'^2 \end{pmatrix}\begin{pmatrix} W_3 \\ B_\mu \end{pmatrix}$$

When looking at this expression there are some important things to note, especially related to the role of the hypercharge of the vacuum, Y_{ϕ_0}:

1 Only if $Y_{\phi_0} \neq 0$, the W_3 and B_μ fields mix.

2 If $Y_{\phi_0} = \pm\, 1$, the determinant of the mixing matrix vanishes and one of the combinations will be massless (the coefficient for that gauge field squared is 0). In our choice of vacuum we have $Y_{\phi_0} = +1$ (see Exercise 4 why that is a good idea). In the rest of our discussion we will drop the term Y_{ϕ_0} and simply use its value of 1.

The two eigenvalues and eigenvectors are given by [see Exercise 3]:

eigenvalue		*eigenvector*		
$\lambda = 0$	$\rightarrow$	$\frac{1}{\sqrt{g^2+g'^2}}\begin{pmatrix} g' \\ g \end{pmatrix}$	$= \frac{1}{\sqrt{g^2+g'^2}}(g'W_3 + gB_\mu) = A_\mu$	photon(γ)
$\lambda = (g^2 + g'^2)$	$\rightarrow$	$\frac{1}{\sqrt{g^2+g'^2}}\begin{pmatrix} g \\ -g' \end{pmatrix}$	$= \frac{1}{\sqrt{g^2+g'^2}}(gW_3 - g'B_\mu) = Z_\mu$	Z-boson (Z)

Looking at the terms involving W_3 and B in the Lagrangian we see that:

$$(-gW_3 + g'Y_{\phi_0}B_\mu)^2 = (g^2 + g'^2)Z_\mu^2 + 0 \cdot A_\mu^2 \tag{9}$$

3] Rewriting Lagrangian in terms of physical fields: masses of the gauge bosons

Finally, by combining equation (8) and (9) we can rewrite the Lagrangian from equation (7) in terms of the physical gauge bosons:

$$(D^\mu\phi)^\dagger (D_\mu\phi) = \frac{1}{8}v^2[g^2(W^+)^2 + g^2(W^-)^2 + (g^2+g'^2)Z_\mu^2 + 0\cdot A_\mu^2] \qquad (10)$$

2.4 Masses of the gauge bosons

2.4.1 Massive charged and neutral gauge bosons

As a general mass term for a massive gauge bosons V has the form $\frac{1}{2}M_V^2 V_\mu^\mu$, from equation (10) we see that:

$$\begin{aligned} M_{W^+} = M_{W^-} &= \frac{1}{2}vg \\ M_Z &= \frac{1}{2}v\sqrt{(g^2+g'^2)} \end{aligned}$$

Although since g and g' are free parameters, the SM makes no absolute predictions for $\mathrm{M_W}$ and $\mathrm{M_Z}$, it has been possible to set a lower limit before the W- and Z-boson were discovered (see Exercise 2). The measured values are $M_W = 80.4$ GeV and $M_Z = 91.2$ GeV.

Mass relation W and Z boson:

Although there is no absolute prediction for the mass of the W- and Z-boson, there is a clear prediction on the ratio between the two masses. From discussions in QED we know the photon couples to charge, which allowed us to relate e, g and g' (see Exercise 3):

$$e = g\sin(\theta_W) = g'\cos(\theta_\mathrm{W}) \qquad (11)$$

In this expression θ_W is the Weinberg angle, often used to describe the mixing of the W_3 and B_μ -fields to form the physical Z boson and photon. From equation (11) we see that $g'/g = \tan(\theta_\mathrm{W})$ and therefore:

$$\frac{M_W}{M_Z} = \frac{\frac{1}{2}vg}{\frac{1}{2}v\sqrt{g^2+g'^2}} = \cos(\theta_\mathrm{W})$$

This predicted ratio is often expressed as the so-called ρ-(Veltman) parameter:

$$\rho = \frac{M_W^2}{M_Z^2\cos^2(\theta_W)} = 1$$

The current measurements of the M_W, M_Z and θ_W confirm this relation.

2.4.2 Massless neutral gauge boson (γ):

Similar to the Z boson we have now a mass for the photon: $\frac{1}{2}M_\gamma^2 = 0$, so:

$$M_\gamma = 0. \qquad (12)$$

2.5 Mass of the Higgs boson

Looking at the mass term for the scalar particle, the mass of the Higgs boson is given by:

$$m_h = \sqrt{2\lambda v^2}$$

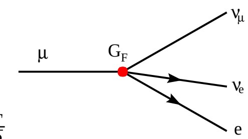
Although v is known ($v \approx 246$ GeV, see below), since λ is a free parameter, the mass of the Higgs boson is **not** predicted in the Standard Model.

Extra: how do we know v ?:

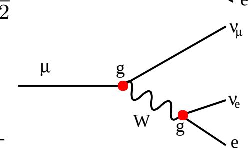
$$\text{Muon decay: } \frac{g^2}{8M_W^2} = \frac{G_F}{\sqrt{2}} \rightarrow v = \sqrt{\frac{1}{\sqrt{2}G_F}}$$

We used $M_W = \frac{1}{2}vg$. Given $G_F = 1.166 \cdot 10^{-5}$, we see that $v = 246$ GeV. This energy scale is known as the electroweak scale.

Fermi: $\propto \frac{G_F}{\sqrt{2}}$



EW: $\propto \frac{g^2}{8M_W^2}$



Exercises lecture 2

Exercise [1]: Higgs - Vector boson couplings

In the lecture notes we focussed on the masses of the gauge bosons, i.e. part 1) when expanding the $((v+h)^2)$-terms as discussed in Section 1.5.4 and 2.3. Looking now at the terms in the Lagrangian that describe the interaction between the gauge fields and the Higgs field, show that the four vertex factors describing the interaction between the Higgs boson and gauge bosons: hWW, hhWW, hZZ, hhZZ are given by:

3-point: $2i\frac{M_V^2}{v}g^{\mu\nu}$ and 4-point: $2i\frac{M_V^2}{v^2}g^{\mu\nu}$, with (V= W,Z).

Note: A vertex factor is obtained by multiplying the term involving the interacting fields in the Lagrangian by a factor i and a factor $n!$ for n identical particles in the vertex.

Exercise [2]: History: lower limits on $\mathrm{M_W}$ and $\mathrm{M_Z}$

Use the relations $e = g\sin\theta_W$ and $G_F = (v^2\sqrt{2})^{-1}$ to obtain lower limits for the masses of the W and Z boson assuming that you do not know the value of the weak mixing angle.

Exercise [3]: $(\mathbf{W}^3_\mu, \mathbf{B}_\mu) \rightarrow (\mathbf{A}_\mu, \mathbf{Z}_\mu)$.

The mix between the W_μ^3 and B_μ fields in the lagr. can be written in a matrix notation:

$$(W_\mu^3, B_\mu) \begin{pmatrix} g^2 & -gg' \\ -gg' & g'^2 \end{pmatrix} \begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix}$$

a) Show that the eigenvalues of the matrix are $\lambda_1 = 0$ and $\lambda_2 = (g^2 + g'^2)$.

b) Show that these eigenvalues correspond to the two eigenvectors:

$$V_1 = \frac{1}{\sqrt{g^2 + g'^2}}(g' W_\mu^3 + g B_\mu) \equiv \mathrm{A}_\mu \quad \text{and} \quad V_2 = \frac{1}{\sqrt{g^2 + g'^2}}(g W_\mu^3 - g' B_\mu) \equiv \mathrm{Z}_\mu$$

Exercise [4]: A closer look at the covariant derivative

The covariant derivative in the electroweak theory is given by:

$$D_\mu = \partial_\mu + ig' \frac{Y}{2} B_\mu + ig \vec{T} \cdot \vec{W}_\mu$$

Looking only at the part involving W_μ^3 and B_μ show that:

$$D_\mu = \partial_\mu + i\mathrm{A}_\mu \frac{gg'}{\sqrt{g'^2 + g^2}} \left(T_3 + \frac{Y}{2} \right) + i\mathrm{Z}_\mu \frac{1}{\sqrt{g'^2 + g^2}} \left(g^2 T_3 - g'^2 \frac{Y}{2} \right)$$

Make also a final interpretation step for the A_μ part and show that:

$$\frac{gg'}{\sqrt{g'^2 + g^2}} = e \quad \text{and} \quad T_3 + \frac{Y}{2} = Q, \text{ the electric charge.}$$

3 Fermion masses, Higgs decay and limits on $\mathrm{m_h}$

In this section we discuss how fermions acquire a mass and use our knowledge on the Higgs coupling to fermion and gauge bosons to predict how the Higgs boson decays. We will also discuss what theoretical information we have on the mass of the Higgs boson.

3.1 Fermion masses

In section 1 we saw that terms like $\frac{1}{2}B_\mu B^\mu$ and $m\bar{\psi}\psi$ were not gauge invariant. Since these terms are not allowed in the Lagrangian, both gauge bosons and fermions are massless. In the previous section we have seen how the Higgs mechanism can be used to accommodate massive gauge bosons in our theory while keeping the local gauge invariance. As we will now see, the Higgs mechanism can also give fermions a mass: 'twee vliegen in een klap'.

Chirality and a closer look at terms like $-m\bar{\psi}\psi$

A term like $-m\bar{\psi}\psi = -m[\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L]$, i.e. a decomposition in chiral states (see exercise 1). Such a term in the Lagrangian in <u>not</u> gauge invariant since the left handed fermions form an isospin doublet, for example $\begin{pmatrix} \nu \\ e \end{pmatrix}_L$ and the right handed fermions form isospin singlets like e_R, they transform differently under $\mathrm{SU(2)_L \times U(1)_Y}$.

$$
\begin{aligned}
\text{left handed doublet} &= \chi_L \rightarrow \chi_L^{'} = \chi_L e^{i\vec{W}\cdot\vec{T}+i\alpha Y} \\
\text{right handed singlet} &= \psi_R \rightarrow \psi_R^{'} = \psi_R e^{i\alpha Y}
\end{aligned}
$$

This means that the term is not invariant under all $\mathrm{SU(2)_L \times U(1)_Y}$ 'rotations'.

Constructing an $\mathrm{SU(2)_L \times U(1)_Y}$ invariant term for fermions

If we could make a term in the Lagrangian that is a *singlet* under $\mathrm{SU(2)_L}$ and $\mathrm{U(1)_Y}$, it would remain invariant. This can be done using the complex (Higgs) doublet we introduced in the previous section. It can be shown that the Higgs has exactly the right quantum numbers to form an $SU(2)_L$ and $U(1)_Y$ singlet in the vertex: $-\lambda_f\bar{\psi}_L\phi\psi_R$, where λ_f is a so-called Yukawa coupling.

Executive summary: - a term: $\propto \bar{\psi}_L\psi_R$ is **not** invariant under $\mathrm{SU(2)_L \times U(1)_Y}$
- a term: $\propto \bar{\psi}_L\phi\psi_R$ is invariant under $\mathrm{SU(2)_L \times U(1)_Y}$

We have constructed a term in the Lagrangian that couples the Higgs doublet to the fermion fields:

$$
\mathcal{L}_{\text{fermion-mass}} = -\lambda_f[\bar{\psi}_L\phi\psi_R + \bar{\psi}_R\bar{\phi}\psi_L] \qquad (13)
$$

When we write out this term we'll see that this does not only describe an interaction between the Higgs field and fermion, but that the fermions will acquire a finite mass if the ϕ-doublet has a non-zero expectation value. This is the case as $\phi_0 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+h \end{pmatrix}$ as before.

3.1.1 Lepton masses

$$
\begin{aligned}
\mathcal{L}_e &= -\lambda_e \frac{1}{\sqrt{2}} \left[(\bar{\nu}, \bar{e})_L \begin{pmatrix} 0 \\ v+h \end{pmatrix} e_R + \bar{e}_R (0, v+h) \begin{pmatrix} \nu \\ e \end{pmatrix}_L \right] \\
&= -\frac{\lambda_e (v+h)}{\sqrt{2}} [\bar{e}_L e_R + \bar{e}_R e_L] \\
&= -\frac{\lambda_e (v+h)}{\sqrt{2}} \bar{e} e \\
&= - \underbrace{\frac{\lambda_e v}{\sqrt{2}} \bar{e} e}_{\text{electron mass term}} - \underbrace{\frac{\lambda_e}{\sqrt{2}} h \bar{e} e}_{\text{electron-higgs interaction}}
\end{aligned}
$$

$$
m_e = \frac{\lambda_e v}{\sqrt{2}} \qquad \frac{\lambda_e}{\sqrt{2}} \propto m_e
$$

A few side-remarks:

1) The Yukawa coupling is often expressed as $\lambda_e = \sqrt{2} \left(\frac{m_e}{v} \right)$ and the coupling of the fermion to the Higgs field is $\frac{\lambda_f}{\sqrt{2}} = \frac{m_f}{v}$, so proportional to the mass of the fermion.

2) The mass of the electron is **not** predicted since λ_e is a free parameter. In that sense the Higgs mechanism does not say anything about the electron mass itself.

3) The coupling of the electron to the electron is **very** small:
The coupling of the Higgs boson to an electron-pair ($\propto \frac{m_e}{v} = \frac{g m_e}{2 M_W}$) is very small compared to the coupling of the Higgs boson to a pair of W-bosons ($\propto g M_W$).

$$
\frac{\Gamma(h \to ee)}{\Gamma(h \to WW)} \propto \frac{\lambda_{eeh}^2}{\lambda_{WWh}^2} = \left(\frac{g m_e / 2 M_W}{g M_W} \right)^2 = \frac{m_e^2}{4 M_W^4} \approx 1.5 \cdot 10^{-21}
$$

3.1.2 Quark masses

The fermion mass term $\mathcal{L}_{\text{down}} = \lambda_f \bar{\psi}_L \phi \psi_R$ (leaving out the hermitian conjugate term term $\bar{\psi}_R \bar{\phi} \psi_L$ for clarity) only gives mass to 'down' type fermions, i.e. only to one of the isospin doublet components. To give the neutrino a mass and give mass to the 'up' type quarks (u, c, t), we need another term in the Lagrangian. Luckily it is possible to compose a new term in the Lagrangian, using again the complex (Higgs) doublet in combination with the fermion fields, that is gauge invariant under $\mathrm{SU(2)_L \times U(1)_Y}$ and gives a mass to the up-type quarks. The mass-term for the up-type fermions takes the form:

$$
\mathcal{L}_{\text{up}} = \bar{\chi}_L \tilde{\phi}^c \phi_R + \text{h.c., with}
$$

$$
\tilde{\phi}^c = -i \tau_2 \phi^* = -\frac{1}{\sqrt{2}} \begin{pmatrix} (v+h) \\ 0 \end{pmatrix} \tag{14}
$$

Mass terms for fermions (leaving out h.c. term):

$$
\begin{aligned}
\text{down-type:} \quad \lambda_d(\bar{u}_L, \bar{d}_L)\phi d_R &= \lambda_d(\bar{u}_L, \bar{d}_L)\begin{pmatrix} 0 \\ v \end{pmatrix} d_R = \lambda_d v\ \bar{d}_L d_R \\
\text{up-type:} \quad \lambda_u(\bar{u}_L, \bar{d}_L)\tilde{\phi}^c d_R &= \lambda_u(\bar{u}_L, \bar{d}_L)\begin{pmatrix} v \\ 0 \end{pmatrix} u_R = \lambda_u v\ \bar{u}_L u_R
\end{aligned}
$$

As we will discuss now, this is not the whole story. If we look more closely we'll see that we can construct more fermion-mass-type terms in the Lagrangian that cannot easily be interpreted. Getting rid of these terms is at the origin of quark mixing.

3.2 Yukawa couplings and the Origin of Quark Mixing

This section will discuss in full detail the consequences of all possible allowed quark 'mass-like' terms and study the link between the Yukawa couplings and quark mixing in the Standard Model: the difference between *mass eigenstates* and *flavour eigenstates*.
If we focus on the part of the SM Lagrangian that describes the dynamics of spinor (fermion) fields ψ, the kinetic terms, we see that:

$$\mathcal{L}_{\text{kinetic}} = i\bar{\psi}(\partial^\mu \gamma_\mu)\psi,$$

where $\bar{\psi} \equiv \psi^\dagger\gamma^0$ and the spinor fields ψ. It is instructive to realise that the spinor fields ψ are the three fermion generations, each consisting of the following five representations:

$$Q^I_{Li}(3,2,+1/3), \quad u^I_{Ri}(3,1,+4/3), \quad d^I_{Ri}(3,1,+1/3), \quad L^I_{Li}(1,2,-1), \quad l^I_{Ri}(1,1,-2)$$

In this notation, $Q^I_{Li}(3,2,+1/3)$ describes an $\mathrm{SU(3)_C}$ triplet, left-handed $\mathrm{SU(2)_L}$ doublet, with hypercharge $Y = 1/3$. The superscript I implies that the fermion fields are expressed in the *interaction (flavour)* basis. The subscript i stands for the three generations. Explicitly, $Q^I_{Li}(3,2,+1/3)$ is a shorthand notation for:

$$Q^I_{Li}(3,2,+1/3) = \begin{pmatrix} u^I_g, u^I_r, u^I_b \\ d^I_g, d^I_r, d^I_b \end{pmatrix}_i = \begin{pmatrix} u^I_g, u^I_r, u^I_b \\ d^I_g, d^I_r, d^I_b \end{pmatrix}, \begin{pmatrix} c^I_g, c^I_r, c^I_b \\ s^I_g, s^I_r, s^I_b \end{pmatrix}, \begin{pmatrix} t^I_g, t^I_r, t^I_b \\ b^I_g, b^I_r, b^I_b \end{pmatrix}.$$

We saw that using the Higgs field ϕ we could construct terms in the Lagrangian of the form given in equation (13). For up and down type fermions for example:

$$
\begin{aligned}
\mathcal{L}_{\text{quarks}} &= -\Lambda_{\text{down}}\bar{\chi}_L\phi\psi_R - \Lambda_{\text{up}}\bar{\chi}_L\tilde{\phi}^c\psi_R + h.c. \\
&= -m_d\ \bar{d}d - m_u\ \bar{u}u
\end{aligned}
$$

, where the interactions between the Higgs and the fermions, the so-called Yukawa couplings, had to be added by hand. If we look however at all possible allowed terms we realise that the Λ's are matrices since in the most general case we have:

$$
\begin{aligned}
-\mathcal{L}_{\text{Yukawa}} &= Y_{ij}\overline{\psi_{Li}}\phi\psi_{Rj} + h.c. \\
&= Y^d_{ij}\overline{Q^I_{Li}}\ \phi\ d^I_{Rj} + Y^u_{ij}\overline{Q^I_{Li}}\ \tilde{\phi}^c\ u^I_{Rj} + Y^l_{ij}\overline{L^I_{Li}}\ \phi\ l^I_{Rj} + h.c. \qquad (15)
\end{aligned}
$$

with

$$\tilde{\phi}^c = i\sigma_2\phi^* = \begin{pmatrix} \overline{\phi}^0 \\ -\phi^- \end{pmatrix}.$$

The matrices Y^d_{ij}, Y^u_{ij} and Y^l_{ij} are arbitrary complex matrices that connect the flavour eigenstates. The thing to note is that we also have terms like $Y_{\rm uc}$ for example, i.e. couplings between different families. Let's look in detail how we should treat these quark mixing terms and how we can construct fields that have a well defined mass, ... the particles in our model.

Writing out the full thing:

Since this is the crucial part of flavour physics, we spell out the term $Y^d_{ij}\overline{Q^I_{Li}}\,\phi\,d^I_{Rj}$ explicitly:

$$\begin{aligned} Y^d_{ij}\overline{Q^I_{Li}}\,\phi\,d^I_{Rj} &= Y^d_{ij}\overline{(u\ \ d)^I_{iL}}\begin{pmatrix}\phi^+\\ \phi\end{pmatrix} d^I_{Rj} = \\ &\begin{pmatrix} Y_{11}\overline{(u\ \ d)^I_L}\begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix} & Y_{12}\overline{(u\ \ d)^I_L}\begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix} & Y_{13}\overline{(u\ \ d)^I_L}\begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix} \\ Y_{21}\overline{(c\ \ s)^I_L}\begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix} & Y_{22}\overline{(c\ \ s)^I_L}\begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix} & Y_{23}\overline{(c\ \ s)^I_L}\begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix} \\ Y_{31}\overline{(t\ \ b)^I_L}\begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix} & Y_{32}\overline{(t\ \ b)^I_L}\begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix} & Y_{33}\overline{(t\ \ b)^I_L}\begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix} \end{pmatrix} \cdot \begin{pmatrix} d^I_R \\ s^I_R \\ b^I_R \end{pmatrix} \end{aligned}$$

After spontaneous symmetry breaking, the following mass terms for the fermion fields arise:

$$\begin{aligned} -\mathcal{L}^{\rm quarks}_{\rm Yukawa} &= Y^d_{ij}\overline{Q^I_{Li}}\,\phi\,d^I_{Rj} + Y^u_{ij}\overline{Q^I_{Li}}\,\tilde{\phi}\,u^I_{Rj} + h.c. \\ &= Y^d_{ij}\overline{d^I_{Li}}\,\frac{v}{\sqrt{2}}\,d^I_{Rj} + Y^u_{ij}\overline{u^I_{Li}}\,\frac{v}{\sqrt{2}}\,u^I_{Rj} + h.c. + \text{interaction terms} \\ &= M^d_{ij}\overline{d^I_{Li}}d^I_{Rj} + M^u_{ij}\overline{u^I_{Li}}u^I_{Rj} + h.c. + \text{interaction terms} \end{aligned}$$

, where we omitted the corresponding interaction terms of the fermion fields to the Higgs field, $\bar{q}qh(x)$. A first step to obtain mass eigenstates, i.e. states with proper mass terms, the matrices M^d and M^u should be diagonalized. We do this with unitary matrices V^d as follows:

$$\begin{aligned} M^d_{\rm diag} &= V^d_L M^d V^{d\dagger}_R \\ M^u_{\rm diag} &= V^u_L M^d V^{u\dagger}_R \end{aligned}$$

Using the requirement that the matrices V are unitary ($V^{d\dagger}_L V^d_L = \mathbb{1}$) the Lagrangian can now be expressed as follows:

$$\begin{aligned} -\mathcal{L}^{\rm quarks}_{\rm Yukawa} &= \overline{d^I_{Li}}\,M^d_{ij}\,d^I_{Rj} + \overline{u^I_{Li}}\,M^u_{ij}\,u^I_{Rj} + h.c. + ... \\ &= \overline{d^I_{Li}}\,V^{d\dagger}_L V^d_L M^d_{ij} V^{d\dagger}_R V^d_R\,d^I_{Rj} + \overline{u^I_{Li}}\,V^{u\dagger}_L V^u_L M^u_{ij} V^{u\dagger}_R V^u_R\,u^I_{Rj} + h.c. + ... \\ &= \overline{d_{Li}}\,(M^d_{ij})_{\rm diag}\,d_{Rj} + \overline{u_{Li}}\,(M^u_{ij})_{\rm diag}\,u_{Rj} + h.c. + ... \end{aligned}$$

, where in the last line the matrices V have been absorbed in the quark states, resulting in the following quark mass eigenstates:

$$\begin{aligned} d_{Li} = (V^d_L)_{ij}d^I_{Lj} \quad d_{Ri} = (V^d_R)_{ij}d^I_{Rj} \\ u_{Li} = (V^u_L)_{ij}u^I_{Lj} \quad u_{Ri} = (V^u_R)_{ij}u^I_{Rj} \end{aligned}$$

Note that we can thus express the quark states as interaction eigenstates $d^I,\ u^I$ or as quark mass eigenstates $d,\ u$.

Rewriting interaction terms using quark mass eigenstates

The interaction terms are obtained by imposing gauge invariance by replacing the partial derivative by the covariant derivate

$$\mathcal{L}_{\text{kinetic}} = i\bar{\psi}(D^{\mu}\gamma_{\mu})\psi \tag{16}$$

with the covariant derivative defined as $D^{\mu} = \partial^{\mu} + ig\frac{1}{2}\vec{\tau}\cdot\vec{W}_{\mu}$, where the τ's are the Pauli matrices and W_i^{μ} and B^{μ} are the three weak interaction bosons and the single hypercharge boson, respectively. We can now write out the charged current interaction between the (left-handed!) quarks:

$$\begin{aligned}
\mathcal{L}_{\text{kinetic, weak}}(Q_L) &= i\overline{Q^I_{Li}}\gamma_{\mu}\big(\partial^{\mu} + \frac{i}{2}gW_i^{\mu}\tau_i\big)Q^I_{Li} \\
&= i\overline{(u\ \ d)^I_{iL}}\gamma_{\mu}\big(\partial^{\mu} + \frac{i}{2}gW_i^{\mu}\tau_i\big)\begin{pmatrix} u \\ d \end{pmatrix}^I_{iL} \\
&= i\overline{u^I_{iL}}\gamma_{\mu}\partial^{\mu}u^I_{iL} + i\overline{d^I_{iL}}\gamma_{\mu}\partial^{\mu}d^I_{iL} - \frac{g}{\sqrt{2}}\overline{u^I_{iL}}\gamma_{\mu}W^{-\mu}d^I_{iL} - \frac{g}{\sqrt{2}}\overline{d^I_{iL}}\gamma_{\mu}W^{+\mu}u^I_{iL} + \ldots
\end{aligned}$$

, where we used $W^{\pm} = \frac{1}{\sqrt{2}}(W_1 \mp iW_2)$, see Section 2.

If we now express the Lagrangian in terms of the quark mass eigenstates $d,\ u$ instead of the weak interaction eigenstates $d^I,\ u^I$, the price to pay is that the quark mixing between families (i.e. the off-diagonal elements) appears in the charged current interaction:

$$\begin{aligned}
\mathcal{L}_{\text{kinetic, cc}}(Q_L) &= \frac{g}{\sqrt{2}}\overline{u^I_{iL}}\gamma_{\mu}W^{-\mu}d^I_{iL} + \frac{g}{\sqrt{2}}\overline{d^I_{iL}}\gamma_{\mu}W^{+\mu}u^I_{iL} + \ldots \\
&= \frac{g}{\sqrt{2}}\overline{u_{iL}}(V^u_L V^{d\dagger}_L)_{ij}\gamma_{\mu}W^{-\mu}d_{iL} + \frac{g}{\sqrt{2}}\overline{d_{iL}}(V^d_L V^{u\dagger}_L)_{ij}\gamma_{\mu}W^{+\mu}u_{iL} + \ldots
\end{aligned}$$

The CKM matrix

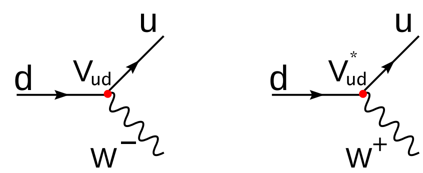
The combination of matrices $(V^d_L V^{u\dagger}_L)_{ij}$, a unitary 3×3 matrix is known under the short-hand notation V_{CKM}, the famous Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix. By convention, the interaction eigenstates and the mass eigenstates are chosen to be equal for the up-type quarks, whereas the down-type quarks are chosen to be rotated, going from the interaction basis to the mass basis:

$$\begin{aligned}
u^I_i &= u_j \\
d^I_i &= V_{\text{CKM}}d_j
\end{aligned}$$

or explicitly:

$$\begin{pmatrix} d^I \\ s^I \\ b^I \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}\begin{pmatrix} d \\ s \\ b \end{pmatrix} \tag{17}$$

From the definition of V_{CKM}, follows that the transition from a down type quark to an up-type quark is described by V_{ud}, whereas the transition from an up type quark to a down-type quark is described by V^*_{ud}. A separate lecture describes in detail how V_{CKM} allows for CP-violation in the SM.



Note on lepton masses

We should note here that in principle a similar matrix exists that connects the lepton *flavour* and *mass* eigenstates. This matrix is known as the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix and, unlike for the quarks, this matrix is diagonal:

$$V_{\text{PMNS}} = \mathbb{1}.$$

3.3 Higgs boson decay

When trying to find the Higgs boson it is important to study details of the coupling of the Higgs boson to fermions and gauge bosons as that determines if and how (often) the Higgs boson is produced and what the experimental signature is. Given that the mass terms for fermions and gauge bosons are intimately linked, see for example Section 3.1.1 and 1.5.4 respectively, these couplings are known. In Section 3.3.3 we list all couplings and as an example we'll compute the decay rate fraction of a Higgs boson into fermions as a function of it's unknown mass in Section 3.3.1.

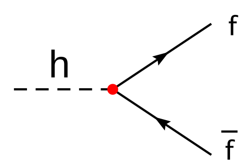
3.3.1 Higgs boson decay to fermions

Now that we have derived the coupling of fermions and gauge bosons to the Higgs field, we can look in more detail at the decay of the Higgs boson.
The general expression for the two-body decay rate:

$$\frac{d\Gamma}{d\Omega} = \frac{|\mathcal{M}|^2}{32\pi^2 s}\ |p_f|\ S, \tag{18}$$

with $\mathcal{M}$ the matrix element, $|p_f|$ the momentum of the produced particles and $S = \frac{1}{n!}$ for n identical particles. In a two-body decay we have $\sqrt{s} = m_h$ and $|p_f| = \frac{1}{2}\beta\sqrt{s}$ (see exercise 2). Since the Higgs boson is a scalar particle, the Matrix element takes a simple form:

$$\begin{aligned} -i\mathcal{M} &= \bar{u}(p_1)\frac{im_f}{v}v(-p_2) \\ i\mathcal{M}^\dagger &= \bar{v}(-p_2)\frac{-im_f}{v}u(p_1) \end{aligned}$$



Since there are no polarizations for the scalar Higgs boson, computing the Matrix element squared is 'easy':

$$
\begin{aligned}
\mathcal{M}^2 &= \left(\frac{m_f}{v}\right)^2 \sum_{s_1,s_2} (\bar{v})_{s_2}(-p_2) u_{s_1}(p_1)(\bar{u})_{s_1}(p_1) v_{s_2}(-p2) \\
&= \left(\frac{m_f}{v}\right)^2 \sum_{s_1} u_{s_1}(p_1)(\bar{u})_{s_1}(p_1) \sum_{s_1} \bar{v}_{s_2}(-p_2) v_{s_2}(-p_2) \\
&= \left(\frac{m_f}{v}\right)^2 \mathrm{Tr}(\not{p}_1 + m_f)\mathrm{Tr}(-\not{p}_2 - m_f) \\
&= \left(\frac{m_f}{v}\right)^2 \left[-\mathrm{Tr}(\not{p}_1 \not{p}_2) - m_f^2 \mathrm{Tr}(\mathbb{1}))\right] \\
&= \left(\frac{m_f}{v}\right)^2 \left[-4p_1 \cdot p_2 - 4m_f^2\right] \\
& \quad \text{use: } s = (p_1 - p_2)^2 = p_1^2 + p_2^2 - 2p_1 \cdot p_2 \text{ and since } p_1^2 = p_2^2 = m_f^2 \\
& \qquad \text{and } s = m_h^2 \text{ we have } m_h^2 = 2m_f^2 - 2p_1 \cdot p_2 \\
&= \left(\frac{m_f}{v}\right)^2 \left[2m_h^2 - 8m_f^2\right] \\
&= \left(\frac{m_f}{v}\right)^2 2m_h^2\beta^2 \qquad\qquad , \text{with } \beta = \sqrt{1 - \frac{4m_f^2}{m_h^2}}
\end{aligned}
$$

Including the number of colours (for quarks) we finally have:

$$\mathcal{M}^2 = \left(\frac{m_f}{v}\right)^2 2m_h^2\beta^2 N_c$$

Decay rate:

Starting from equation (18) and using $\mathcal{M}^2$ (above), $|p_f| = \frac{1}{2}\beta\sqrt{s}$, S=1 and $\sqrt{s} = m_h$ we get:

$$\frac{d\Gamma}{d\Omega} = \frac{|\mathcal{M}|^2}{32\pi^2 s}\ |p_f|\ S \quad = \quad \frac{N_c m_h}{32\pi^2}\left(\frac{m_f}{v}\right)^2 \beta^3$$

Doing the angular integration $\int d\Omega = 4\pi$ we finally end up with:

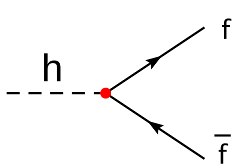
$$\Gamma(h \to f\bar{f}) = \frac{N_c}{8\pi v^2} m_f^2 m_h \beta_f^3.$$

3.3.2 Higgs boson decay to gauge bosons

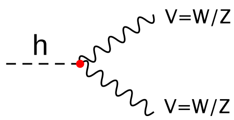
The decay ratio to gauge bosons is a bit more tricky, but is explained in great detail in Exercise 5.

3.3.3 Review Higgs boson couplings to fermions and gauge bosons

A summary of the Higgs boson couplings to fermions and gauge bosons.



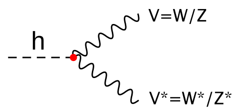
$$\Gamma(h \to f\bar{f}) = \tfrac{N_c}{8\pi v^2}\ m_f^2\ m_h\ \sqrt{1-x} \qquad , \text{with } \ x = \tfrac{4m_f^2}{m_h^2}$$



$$\Gamma(h \to VV) = \frac{g^2}{64\pi M_W^2}\ m_h^3\ \mathcal{S}_{VV}\ (1 - x + \tfrac{3}{4}x^2)\sqrt{1-x}$$

, with $x = \frac{4M_V^2}{m_h^2}$ and $\mathcal{S}_{WW,ZZ} = 1, \frac{1}{2}$.

The decay of the Higgs boson to two off-shell gauge bosons is given by:

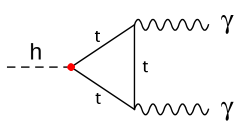


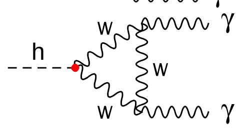
$$\Gamma(h \to VV^*) = \frac{3M_V^4}{32\pi^2 v^4}\ m_h\ \delta'_V\ \mathcal{R}(x) \qquad \text{, with}$$

$$\delta'_W = 1,\ \delta'_Z = \frac{7}{12} - \frac{10}{9}\sin^2\theta_W + \frac{40}{27}\sin^4\theta_W \qquad \text{, with}$$

$$\mathcal{R}(x) = \frac{3(1-8x+20x^2)}{\sqrt{4x-1}}\mathrm{acos}\left(\frac{3x-1}{2x^{3/2}}\right) - \frac{1-x}{2x}(2 - 13x + 47x^2) - \frac{3}{2}(1 - 6x + 4x^2)\ln(x)$$

Since the coupling of the gauge bosons is so much larger than that to fermions, the Higgs boson decays to off-shell gauge bosons even though $M_{V^*} + M_V < 2M_V$. The increase in coupling 'wins' from the Breit-Wigner suppression. For example: at m_h= 140 GeV, the $h \to WW^*$ is already larger than $h \to b\bar{b}$.

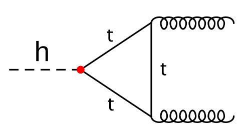




$$\Gamma(h \to \gamma\gamma) = \frac{\alpha^2}{256\pi^3 v^2}\ m_h^3\ \left|\frac{4}{3}\sum_f N_c^{(f)} e_f^2 - 7\right|^2$$

, where e_f is the fermion's electromagnetic charge.

Note: - WW contribution $\approx$ 5 times top contribution
- Some computation also gives $h \to \gamma Z$

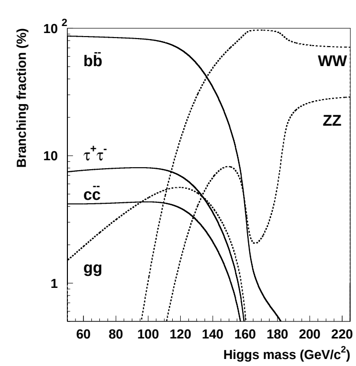


$$\Gamma(h \to \text{gluons}) = \frac{\alpha_s^2}{72\pi^3 v^2}\ m_h^3\ \left[1 + \left(\frac{95}{4} - \frac{7N_f}{6}\right)\frac{\alpha_s}{\pi} + ...\right]^2$$

Note: - The QCD higher order terms are large.
- Reading the diagram from right to left you see the dominant production mechanism of the Higgs boson at the LHC.

3.3.4 Higgs branching fractions

Having computed the branching ratio's to fermions and gauge bosons in Section 3.3.1 and Section 3.3.2 we can compute the relative branching fractions for the decay of a Higgs boson as a function of it's mass. The distribution is shown here.



3.4 Theoretical bounds on the mass of the Higgs boson

Although the Higgs mass is not predicted within the minimal SM, there are theoretical upper and lower bounds on the mass of the Higgs boson if we assume there is no new physics between the electroweak scale and some higher scale called Λ. In this section we present a quick sketch of the various arguments and present the obtained limits.

3.4.1 Unitarity

In the absence of a scalar field the amplitude for elastic scattering of longitudinally polarised massive gauge bosons (e.g. $\mathrm{W_L^+W_L^-} \rightarrow \mathrm{W_L^+W_L^-}$) diverges quadratically with the centre-of-mass energy when calculated in perturbation theory and at an energy of 1.2 TeV this process violates unitarity. In the Standard Model, the Higgs boson plays an important role in the cancellation of these high-energy divergences. Once diagrams involving a scalar particle (the Higgs boson) are introduced in the gauge boson scattering mentioned above, these divergences are no longer present and the theory remains unitary and renormalizable. Focusing on solving these divergences alone also yields most of the Higgs bosons properties. This cancellation only works however if the Higgs boson is not too heavy. By requiring that perturbation theory remains valid an upper limit on the Higgs mass can be extracted. With the requirement of unitarity and using all (coupled) gauge boson scattering processes it can be shown that:

$$m_h \; < \; \sqrt{\frac{4\pi\sqrt{2}}{3G_F}} \quad \sim 700 \;\; \mathrm{GeV/c^2}.$$

It is important to note that this does not mean that the Higgs boson can not be heavier than 700 GeV/c^2. It only means that for heavier Higgs masses, perturbation theory is not valid and the theory is not renormalisable.

This number comes from an analysis that uses a partial wave decomposition for the matrix element $\mathcal{M}$, i.e.:

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi s}\mathcal{M}^2 \;\; \text{, with} \quad \mathcal{M} = 16\pi \sum_{l=0}^{l=\infty}(2l+1)\mathrm{P}_l(\cos\theta)a_l,$$

where P_l are Legendre Polynomial and a_l are spin-l partial waves. Since $(W_L^+W_L^- + Z_L + Z_L + HH)^2$ is well behaved, it must respect unitarity, i.e. $|a_i| < 1$ or $|Re(a_i)| \leq 0.5$. As the largest amplitude is given by:

$$a_0^{\max} = -\frac{G_F m_h^2}{4\pi\sqrt{2}} \cdot \frac{3}{2}$$

this can be transformed into an upper limit on m_h:

$$|a_0| < \frac{1}{2} \rightarrow m_h^2 \; < \; \frac{8\pi\sqrt{2}}{6G_F} \left(= \; \frac{8}{3}\pi v^2 \text{ using } G_F = \frac{1}{\sqrt{2}v^2} \right)$$

$$m_h \; < \; 700 \text{ GeV} \quad \text{using } v = 246 \text{ GeV}.$$

This limit is soft, i.e. it means that for Higgs boson masses > 700 GeV perturbation theory break down.

3.4.2 Triviality and Vacuum stability

In this section, the running of the Higgs self-coupling λ with the renormalisation scale μ is used to put both a theoretical upper and a lower limit on the mass of the Higgs boson as a function of the energy scale Λ.

Running Higgs coupling constant

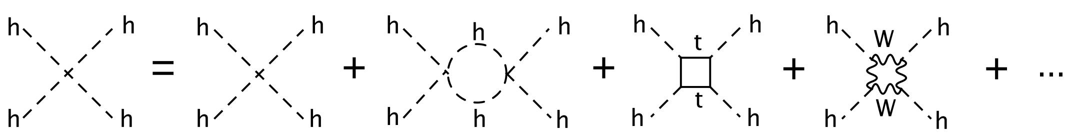
Similar to the gauge coupling constants, the coupling λ 'runs' with energy.

$$\frac{d\lambda}{dt} = \beta_\lambda \qquad \text{, where } t = \ln(Q^2).$$

Although these evolution functions (called β-functions) have been calculated for all SM couplings up to two loops , to focus on the physics, we sketch the arguments to obtain these mass limits by using only the one loop results. At one loop the quartic coupling runs with the renormalisation scale as:

$$\frac{d\lambda}{dt} \;\equiv\; \beta_\lambda \;=\; \frac{3}{4\pi^2}\left[\lambda^2 + \frac{1}{2}\lambda h_t^2 - \frac{1}{4}h_t^4 + \mathcal{B}(g,g')\right] \tag{19}$$

, where h_t is the top-Higgs Yukawa coupling as given in equation (13). The dominant terms in the expression are the terms involving the Higgs self-coupling λ and the top quark Yukawa coupling h_t. The contribution from the gauge bosons is small and explicitly given by $\mathcal{B}(g,g') = -\frac{1}{8}\lambda(3g^2 + g'^2) + \frac{1}{64}(3g^4 + 2g^2g'^2 + g'^4)$. The terms involving the mass of the Higgs boson, top quark and gauge bosons can be understood from looking in more detail at the effective coupling at higher energy scales, where contributions from higher order diagrams enter:



This expression allows to evaluate the value of $\lambda(\Lambda)$ relative to the coupling at a reference scale which is taken to be $\lambda(v)$.

If we study the β-function in 2 special regimes: $\lambda \gg g, g', h_t$ or $\lambda \ll g, g', h_t$, we'll see that we can set both a lower *and* upper limit on the mass of the Higgs boson as a function of the energy-scale cut-off in our theory (Λ):

$$\underbrace{\text{Triviality}}_{\substack{\text{upper bound on } m_h \\ m_h^{\max}(\Lambda)}} \qquad \text{and} \qquad \underbrace{\text{Vacuum stability}}_{\substack{\text{lower bound on } m_h \\ m_h^{\min}(\Lambda)}}$$

3.4.3 Triviality: $\lambda \gg g, g', h_t$ heavy Higgs boson $\rightarrow$ upper limit on m_h

For large values of λ (heavy Higgs boson since $m_h^2 = 2\lambda v^2$) and neglecting the effects from gauge interactions and the top quark, the evolution of λ is given by the dominant term in equation (19) that can be easily solved for $\lambda(\Lambda)$:

$$\frac{d\lambda}{dt} = \frac{3}{4\pi^2}\lambda^2 \qquad \Rightarrow \qquad \lambda(\Lambda) = \frac{\lambda(v)}{1 - \frac{3\lambda(v)}{4\pi^2}\ln\left(\frac{\Lambda^2}{v^2}\right)} \tag{20}$$

Note:

- We now have related λ at a scale v to λ at a higher scale Λ. We see that as Λ grows, $\lambda(\Lambda)$ grows. We should remember that $\lambda(v)$ is related to m_h: $m_h = \sqrt{-2\lambda v^2}$.
- There is a scale Λ at which $\lambda(\Lambda)$ is infinite. As Λ increases, $\lambda(\Lambda)$ increases until at $\Lambda{=}v\exp(2\pi^2/3\lambda(v))$ there is a singularity, known as the Landau pole.

$$\frac{3\lambda(v)}{4\pi^2}\ln\left(\frac{\Lambda^2}{v^2}\right) = 1 \rightarrow \text{At a scale } \Lambda = ve^{2\pi^2/3\lambda(v)} \qquad \lambda(\Lambda) \;\text{ is infinite.}$$

If the SM is required to remain valid up to some cut-off scale Λ, i.e. if we require $\lambda(Q) < \infty$ for all $Q < \Lambda$ this puts a constraint (a maximum value) on the value of the Higgs self-coupling at the electroweak scale (v): $\lambda(v)^{\max}$ and therefore on the maximum Higgs mass since $m_h^{\max} = \sqrt{2\lambda(v)^{\max}v^2}$. Taking $\lambda(\Lambda) = \infty$ and 'evolving the coupling downwards', i.e. find $\lambda(v)$ for which $\lambda(\Lambda) = \infty$ (the Landau pole) we find:

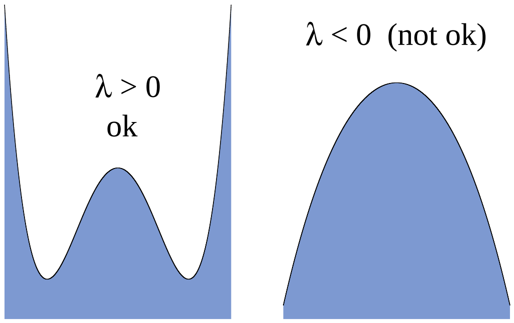
$$\lambda^{\max}(v) = \frac{4\pi^2}{3\ln\left(\frac{\Lambda^2}{v^2}\right)} \qquad \Rightarrow \quad m_h < \sqrt{\frac{8\pi^2 v^2}{3\ln\left(\frac{\Lambda^2}{v^2}\right)}} \tag{21}$$

For $\Lambda{=}10^{16}$ GeV the upper limit on the Higgs mass is 160 GeV/c^2. This limit gets less restrictive as Λ decreases. The upper limit on the Higgs mass as a function of Λ from a computation that uses the two-loop β function and takes into account the contributions from top-quark and gauge couplings is shown in the Figure at the end of Section 3.4.4.

3.4.4 Vacuum stability $\lambda \ll g, g', h_t$ light Higgs boson $\rightarrow$ lower limit on m_h

For small λ (light Higgs boson since $m_h^2 = 2\lambda v^2$), a lower limit on the Higgs mass is found by the requirement that the minimum of the potential be lower than that of the unbroken theory and that the electroweak vacuum is stable. In equation (19) it is clear that for small λ the dominant contribution comes from the top quark through the Yukawa coupling ($-h_t^4$).

$$\begin{aligned}\beta_\lambda &= \frac{1}{16\pi^2}\left[-3h_t^4 + \frac{3}{16}(2g^4 + (g^2 + g'^2)^2)\right] \\ &= \frac{3}{16\pi^2 v^4}\left[2M_W^4 + M_Z^4 - 4m_t^4\right] \\ &< 0.\end{aligned}$$



Since this contribution is negative, there is a scale Λ for which $\lambda(\Lambda)$ becomes negative. If this happens, i.e. when $\lambda(\mu) < 0$ the potential is unbounded from below. As there is no minimum, no consistent theory can be constructed.

The requirement that λ remains positive up to a scale Λ, such that the Higgs vacuum is the global minimum below some cut-off scale, puts a lower limit on $\lambda(v)$ and therefore on the Higgs mass:

$$\frac{d\lambda}{dt} = \beta_\lambda \rightarrow \lambda(\Lambda) - \lambda(v) = \beta_\lambda \ln\left(\frac{\Lambda^2}{v^2}\right) \quad \text{and require } \lambda(\Lambda) > 0.$$

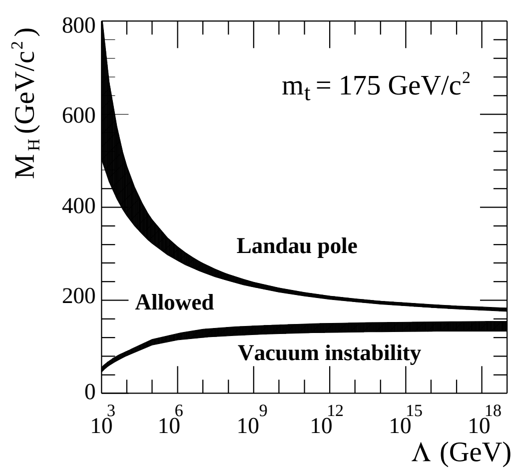
$$\begin{aligned}
\lambda(v) &> \beta_\lambda \ln\left(\frac{\Lambda^2}{v^2}\right) \text{ and } \lambda^{\text{min}}(v) \rightarrow (m_h^{\text{min}})^2 > 2\lambda^{\text{min}}(v)v^2 \text{ , so} \\
m_h^2 &> 2v^2\beta_\lambda \ln\left(\frac{\Lambda^2}{v^2}\right) \\
(m_h^{\text{min}})^2 &= \frac{3}{8\pi^2 v^2}\left[2M_W^4 + M_Z^4 - 4m_t^4\right] \\
&> -493 \ln\left(\frac{\Lambda^2}{v^2}\right)
\end{aligned}$$

Note: This results makes no sense, but is meant to describe the logic. If we go to 2-loop beta-function we get a new limit: $m_h > 130 - 140$ GeV if $\Lambda = 10^{19}$ GeV. A detailed evaluation, taking into account these considerations has been performed. The region of excluded Higgs masses as a function of the scale Λ from this analysis is also shown in the Figure at the end of Section 3.4.4 by the lower excluded region.

Summary of the theoretical bounds on the Higgs mass

In the Figure on the right the theoretically allowed range of Higgs masses is shown as a function of Λ.

For a small window of Higgs masses around 160 GeV/c^2 the Standard Model is valid up to the Planck scale ($\sim 10^{19}$ GeV). For other values of the Higgs mass the Standard Model is only an effective theory at low energy and new physics has to set in at some scale Λ.



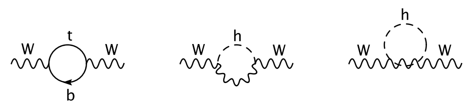
3.5 Experimental limits on the mass of the Higgs boson

3.5.1 Indirect measurements

The electroweak gauge sector of the SM is described by only three independent parameters: g, g' and v. The predictions for electroweak observables, are often presented using three (related) variables that are known to high precision: G_F, M_Z and $\alpha_{\rm QED}$. To obtain predictions to a precision better than the experimental uncertainties (often at the per mill level) higher order loop corrections have to be computed. These higher order radiative corrections contain, among others, contributions from the mass of the top quark and the Higgs boson. Via the precision measurements one is sensitive to these small contributions and thereby to the masses of these particles.

Radiative corrections

An illustration of the possibility to estimate the mass of a heavy particle entering loop corrections is the very good agreement between the estimate of the top quark mass using only indirect measurements and the direct observation.



$$
\begin{array}{lll}
\text{Estimate:} & m_t & = 177.2^{+2.9}_{-3.1}\ \mathrm{GeV/c^2} \\
\text{Measurement:} & m_t & = 173.2 \pm 0.9\ \mathrm{GeV/c^2}
\end{array}
$$

Sensitivity to Higgs boson mass through loop corrections

Apart from the mass of the W-boson, there are more measurements that provide sensitivity to the mass of the Higgs boson. A summary of the measurements of several SM measurements is given in the left plot of Figures 1.
While the corrections connected to the top quark behave as m_t^2, the sensitivity to the mass of the Higgs boson is unfortunately only logarithmic ($\sim \ln m_h$):

$$
\begin{aligned}
\rho &= \frac{\mathrm{M_W^2}}{\mathrm{M_Z^2}\cos\theta_\mathrm{W}}\left[1+\Delta_\rho^{\rm quarks}+\Delta_\rho^{\rm higgs}+...\right] \\
&= \frac{\mathrm{M_W^2}}{\mathrm{M_Z^2}\cos\theta_\mathrm{W}}\left[1+\frac{3}{16\pi^2}\left(\frac{m_t}{v}\right)^2+1-\frac{11\tan\theta_W}{96\pi^2}g^2\ln\left(\frac{m_h}{\mathrm{M_W}}\right)+...\right]
\end{aligned}
$$

The results from a global fit to the electroweak data with only the Higgs mass as a free parameter is shown in the right plot of Figure 1. The plot shows the $\Delta\chi^2$ distribution as a function of m_h. The green band indicates the remaining theoretical uncertainty in the fit. The result of the fit suggests a rather light Higgs boson and it can be summarised by the central value with its one standard deviation and the one-sided (95% CL) upper limit:

$$
m_\mathrm{h} = 95^{+30}_{-24}\,{}^{+74}_{-43}\ \ \mathrm{GeV/c^2} \qquad \text{and} \quad m_\mathrm{h} < 162\ \ \mathrm{GeV/c^2} \quad \text{(at 95\% CL).}
$$

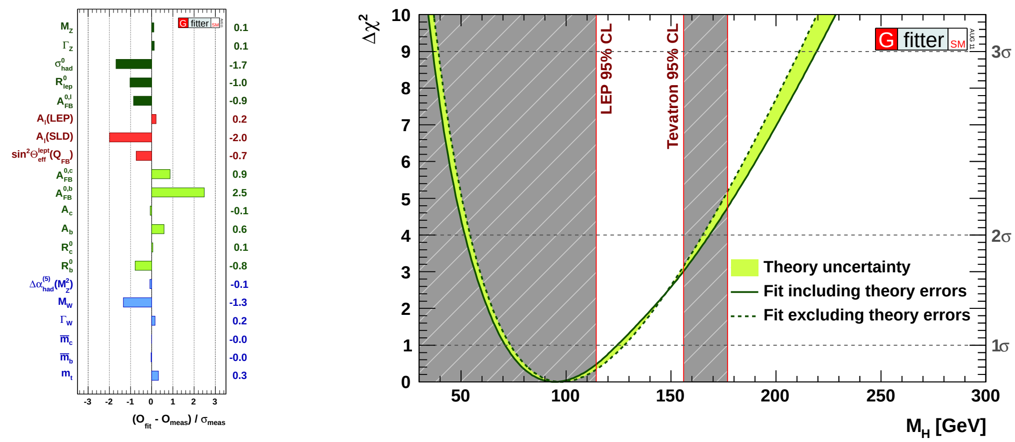


Figure 1: *Status of various SM measurements (left plot) and the $\Delta\chi^2$ distribution as a function of m_h from a global fit with only m_h as a free parameter (right plot).*

3.5.2 Direct measurements

See transparencies from second half of Lecture 4.

Exercises lecture 3

Exercise 1): Show that $\bar{u}u = (\bar{u}_L u_R + \bar{u}_R u_L)$

Exercise 2):
Show that in a two body decay (a heavy particle M decaying into two particles with mass m) the momentum of the decay particles can be written as:

$$|p_f| = \frac{\sqrt{s}}{2}\beta, \text{ with } \beta = \sqrt{1-x} \text{ and } x = \frac{4m^2}{M^2}$$

Exercise 3): Higgs decay into fermions for $m_h = 100$ GeV
Use $m_b = 4.5$ GeV, $m_\tau = 1.8$ GeV, $m_c = 1.25$ GeV

a) Compute $\Gamma(H \rightarrow b\bar{b})$.
b) Compute $\Gamma(H \rightarrow \text{all})$ assuming only decay into the three heaviest fermions.
c) What is the lifetime of the Higgs boson. Compare it to that of the Z boson.

Exercise 4) H&M exercise 6.16:
The helicity states λ of a massive vector particle can be described by polarization vectors. Show that:

$$\sum_\lambda \epsilon_\mu^{(\lambda)*}\epsilon_\nu^{(\lambda)} = -g_{\mu\nu} + \frac{p_\mu p_\nu}{M^2}$$

Exercise 5) Higgs decay to vector bosons
Computing the Higgs boson decay into gauge bosons (W/Z = V), with boson momenta p, q and helicities λ, δ is a bit more tricky. Let's go through it step by step.

a) Draw the Feynman diagram and use the vertex factor you computed last week to show that the matrix element squared is given by:

$$M^2 = \left(\frac{gM_V^2}{M_W}\right)^2 \sum_{\lambda,\delta} g_{\mu\nu}(\epsilon_\lambda^\mu)^*(\epsilon_\delta^\nu)^* g_{\alpha\beta}(\epsilon_\lambda^\alpha)(\epsilon_\delta^\beta)$$

, where λ and δ are the helicity states of the Z bosons.

b) Use your results of exercise 4 and work out to show that:

$$M^2 = \left(\frac{gM_V^2}{M_W}\right)^2 \left[2 + \frac{(p\cdot q)^2}{M_V^4}\right]$$

, where p and q are are the momenta of the two Z bosons.

d) Show that the matrix element can finally be written as:

$$M^2 = \frac{g^2}{4\mathrm{M_W^2}} m_h^4 (1 - x + \frac{3}{4}x^2), \text{ with x} = \frac{4\mathrm{M_V^2}}{m_h^2}$$

e) Show that the Higgs decay into vector bosons can be written as:

$$\Gamma(h \to VV) = \frac{g^2 S_{VV}}{64\pi \mathrm{M_W^2}} m_h^3 (1 - x + \frac{3}{4}x^2)\sqrt{1-x}$$

, with x = $\frac{4M_V^2}{m_h^2}$ and $S_{\mathrm{WW,ZZ}} = 1, \frac{1}{2}$.

f) Compute Γ(h$\rightarrow$ WW) for $m_h = 200$ GeV.
What is the total width (only WW and ZZ decays)? And the lifetime ?

4 Problems with the Higgs mechanism and Higgs searches

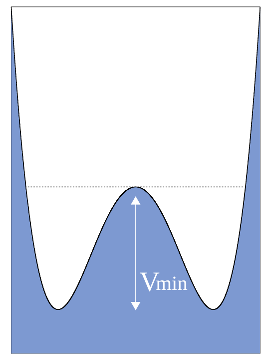
Although the Higgs mechanism cures many of the problems in the Standard Model, there are also several 'problems' associated to the Higgs mechanism. We will explore these problems in this section and very briefly discuss the properties of non-SM Higgs bosons.

4.1 Problems with the Higgs boson

4.1.1 Problems with the Higgs boson: Higgs self-energy

Since the Higgs field occupies all of space, the non-zero vacuum expectation value of the Higgs field (v) will contribute to the vacuum energy, i.e. it will contribute to the cosmological constant in Einstein's equations: $\Lambda = \frac{8\pi G_N}{c^4}\rho_{\text{vac}}$.

Energy density Higgs field:
With $V(\phi^\dagger\phi) = \mu^2\phi^2 + \lambda\phi^4$, The 'depth' of the potential is:



$$\begin{aligned} V_{\text{min}} = V(v) &= \frac{1}{2}\mu^2 v^2 + \frac{1}{4}\lambda v^4 \quad \text{use } \mu^2 = -\lambda v^2 \\ &= -\frac{1}{4}\lambda v^4 \qquad\qquad \text{use } m_h^2 = 2\lambda v^2 \\ &= -\frac{1}{8} m_h^2 v^2 \end{aligned}$$

Note that we cannot simply redefine V_{min} to be 0, or any arbitrary number since quantum corrections will always yields a value like the one (order of magnitude) as given above. The Higgs mass is unknown, but since we have an lower limit on the (Standard Model) Higgs boson mass from direct searches at LEP ($m_h > 114.4$ GeV/c^2) we *can* compute the contribution of the Higgs field to ρ_{vac}.

$$\begin{aligned} \rho_{\text{vac}}^{\text{Higgs}} &= \frac{1}{8} m_h^2 v^2 \\ &> 1 \cdot 10^8 \text{ GeV}^4 \qquad \text{and since GeV} = \frac{1}{r} \\ &> 1 \cdot 10^8 \text{ GeV/r}^3 \qquad \text{(energy density)} \end{aligned}$$

Measured vacuum energy density:
An experiment to measure the energy density in vacuum and the energy density in matter has shown:

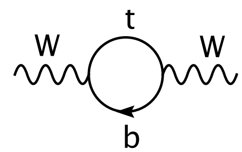
$$\Omega_m \approx 30\% \;\text{ and }\; \Omega_\Lambda \approx 70\% \sim 10^{-46}\text{GeV}^4 \quad \rightarrow \quad \text{empty space is really quite empty.}$$

Problem:
- 10^{54} orders of magnitude mismatch.
- Why is the universe larger than a football ?

4.1.2 Problems with the Higgs boson: the hierarchy problem

In the electroweak theory of the SM, loop corrections give small corrections. In the loops the integration is done over momenta up to a cut-off value Λ.

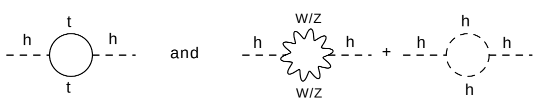
Success of radiative corrections:
When we discussed the sensitivity of the electroweak measurements to the mass of the Higgs boson through the radiative corrections, the example of the prediction of the top quark mass was mentioned:



$$\text{Indirect estimate:} \quad m_t = 178^{+9.8}_{-4.2}\ \mathrm{GeV/c^2}$$
$$\text{Direct result:} \quad m_t = 172.4 \pm 1.2\ \mathrm{GeV/c^2}$$

Failure of radiative corrections:
Also the Higgs propagator receives quantum corrections.

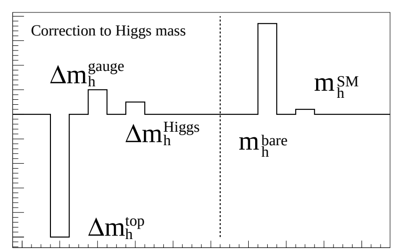
$$m_h = m_h^{\text{bare}} + \Delta m_h^{\text{ferm.}} + \Delta m_h^{\text{gauge}} + \Delta m_h^{\text{Higgs}} + \ldots$$



The corrections from the fermions (mainly form the top quark) are large and, when expressed in terms of the loop-momentum cut-off Λ given by:

$$\left(\Delta m_h^2\right)^{\text{top}} = -\frac{3}{8\pi^2}\lambda_t^2\Lambda^2$$

The corrections from the top quark are not small, but huge and of order Λ. If Λ is chosen as 10^{16} (GUT) or 10^{19} (Planck), and taking the corrections into account (same order of magnitude), it is unnatural for m_h to be of order of $\mathrm{M_{EW}}(\approx v)$.



The hierarchy problem: why is $\mathrm{M_{EW}} \ll \mathrm{M_{PL}}$?

Most popular theoretical solution to the hierarchy problem is the concept of SuperSymmetry, where for every fermion/boson there is a boson/fermion as partner. For example, the top and stop (supersymmetric bosonic partner of the top quark) contributions (almost) cancel. The quadratic divergences have disappeared and we are left with

$$\Delta m_h^2 \propto (m_f^2 - m_S^2) \ln\left(\frac{\Lambda}{m_S}\right).$$

4.2 Higgs bosons in models beyond the SM (SUSY)

When moving to a supersymmetric description of nature we can no longer use a single Higgs doublet, but will need to introduce at least two, because:

A) In the SM we used $\phi/\tilde{\phi}^c$ to give mass to down/up-type particles in $\mathrm{SU(2)_L}$ doublets. In susy models these two terms cannot appear together in the Lagrangian. We need an additional Higgs doublet to give mass to the up-type particles.

B) Anomalies disappear only if in loops $\sum_f \mathrm{Y}_f = 0$. In SUSY there is an additional fermion in the model: the partner for the Higgs boson, the Higgsino. This will introduce an anomaly unless there is a second Higgsino with opposite hypercharge.

$$\phi_1 = \underbrace{\begin{pmatrix} \phi_1^+ \\ \phi_1^0 \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ v_1 \end{pmatrix}}_{\mathrm{Y}_{\phi_1}=+1} \qquad \text{and} \qquad \phi_2 = \underbrace{\begin{pmatrix} \phi_2^0 \\ \phi_2^- \end{pmatrix} \rightarrow \begin{pmatrix} v_2 \\ 0 \end{pmatrix}}_{\mathrm{Y}_{\phi_2}=-1}$$

Number of degrees of freedom in SUSY models:

SM: Add 4 degrees of freedom → 3 massive gauge bosons → 1 Higgs boson (h)
SUSY: Add 8 degrees of freedom → 3 massive gauge bosons → 5 Higgs boson (h, H, A, H^+, H^-)

parameters:$\tan(\beta) = \frac{v_2}{v_1}$and $\mathrm{M_A}$.

Note: - Sometimes people choose α = mixing angle to give h,A, similar to W_3/B_μ-mixing to give Z-boson and photon.
- $\mathrm{M_W} = \frac{1}{2}\sqrt{\mathrm{v}_1^2 + \mathrm{v}_2^2}\ \mathrm{g} \rightarrow \mathrm{v}_1^2 + \mathrm{v}_2^2 = \mathrm{v}^2$ (246 GeV).

Differences SM and SUSY Higgses:

With the new parameters, all couplings to gauge bosons and fermions change:

$$\begin{aligned}
g_{hVV}^{\mathrm{SUSY}} &= g_{hVV}^{\mathrm{SM}}\ \sin(\beta - \alpha) \\
g_{hb\bar{b}}^{\mathrm{SUSY}} &= g_{hb\bar{b}}^{\mathrm{SM}}\ -\frac{\sin\alpha}{\cos\beta} \rightarrow \frac{\Gamma(h \rightarrow b\bar{b})^{\mathrm{SUSY}}}{\Gamma(h \rightarrow b\bar{b})^{\mathrm{SM}}} = \frac{\sin^2(\alpha)}{\cos^2(\beta)} \\
g_{ht\bar{t}}^{\mathrm{SUSY}} &= g_{ht\bar{t}}^{\mathrm{SM}}\ -\frac{\cos\alpha}{\sin\beta} \rightarrow \frac{\Gamma(h \rightarrow t\bar{t})^{\mathrm{SUSY}}}{\Gamma(h \rightarrow t\bar{t})^{\mathrm{SM}}} = \frac{\cos^2(\alpha)}{\sin^2(\beta)}
\end{aligned}$$

To determine if an observed Higgs sparticle is a SM or SUSY Higgs a detailed investigation of the branching fraction is required. Unfortunately, also SUSY does not give a prediction for the lightest Higgs boson mass:

$$\begin{aligned}
m_h^2 &< M_Z^2 + \delta^2 m_{top} + \delta^2 m_X + ... \\
&\leq 130\ \text{GeV}.
\end{aligned}$$

Exercises lecture 4

Exercise 1): b-tagging at LEP.
A Higgs boson of 100 GeV decays at LEP: given a lifetime of a B mesons of roughly 1.6 picoseconds, what distance does it travel in the detector before decaying ? What is the most likely decay distance ?

Exercise 2): H $\rightarrow$ ZZ $\rightarrow$ 4 leptons at the LHC (lepton = e/μ).

a) Why is there a 'dip' in te fraction of Higgs bosons that decays to 2 Z bosons (between 160 and 180 GeV)?

b) How many events H $\rightarrow$ ZZ $\rightarrow$ $e^+e^-\mu^+\mu^-$ muons are produced in 1 fb^{-1} of data for $\mathrm{m_h}$= 140, 160, 180 and 200 GeV ? The expected number of evets is the product of the luminosity and the cross-section: N = $\mathcal{L} \cdot \sigma$

On the LHC slides, one of the LHC experiments shows its expectation for an analysis aimed at trying to find the Higgs boson in the channel with 2 electrons and 2 muons. We concentrate on $\mathrm{m_h}$=140 GeV.

c) What is the fraction of events in which all 4 leptons have been well reconstructed in the detector ? What is the single (high-energy) lepton detection efficiency ? Name reasons why not all leptons are detected.

We do a counting experiment using the two bins around the expected Higgs boson mass (we assume for the moment that the background is extremely well known and does not fluctuate). In a counting experiment a Poisson distribution describes the probabilities to observe x events when λ are expected:

$$\mathrm{P(x|\lambda)} \;=\; \frac{\lambda^{\mathrm{x}}\ \mathrm{e}^{-\lambda}}{\mathrm{x!}}$$

d) Does this experiment expect to be able to discover the $\mathrm{m_h}$=140 GeV hypothesis after 9.3 fb^{-1}.

e) Imagine the data points was the actual measurement after 9.3 fb^{-1}. Can this experiment claim to have discovered the Higgs boson at $\mathrm{m_h}$=140 GeV?

Article Englert & Brout (Augustus 1964)

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BROKEN SYMMETRY AND THE MASS OF GAUGE VECTOR MESONS*

F. Englert and R. Brout
Faculté des Sciences, Université Libre de Bruxelles, Bruxelles, Belgium
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It is of interest to inquire whether gauge vector mesons acquire mass through interaction[1]; by a gauge vector meson we mean a Yang-Mills field[2] associated with the extension of a Lie group from global to local symmetry. The importance of this problem resides in the possibility that strong-interaction physics originates from massive gauge fields related to a system of conserved currents.[3] In this note, we shall show that in certain cases vector mesons do indeed acquire mass when the vacuum is degenerate with respect to a compact Lie group.

Theories with degenerate vacuum (broken symmetry) have been the subject of intensive study since their inception by Nambu.[4-6] A characteristic feature of such theories is the possible existence of zero-mass bosons which tend to restore the symmetry.[7,8] We shall show that it is precisely these singularities which maintain the gauge invariance of the theory, despite the fact that the vector meson acquires mass.

We shall first treat the case where the original fields are a set of bosons φ_A which transform as a basis for a representation of a compact Lie group. This example should be considered as a rather general phenomenological model. As such, we shall not study the particular mechanism by which the symmetry is broken but simply assume that such a mechanism exists. A calculation performed in lowest order perturbation theory indicates that those vector mesons which are coupled to currents that "rotate" the original vacuum are the ones which acquire mass [see Eq. (6)].

We shall then examine a particular model based on chirality invariance which may have a more fundamental significance. Here we begin with a chirality-invariant Lagrangian and introduce both vector and pseudovector gauge fields, thereby guaranteeing invariance under both local phase and local γ_5-phase transformations. In this model the gauge fields themselves may break the γ_5 invariance leading to a mass for the original Fermi field. We shall show in this case that the pseudovector field acquires mass.

In the last paragraph we sketch a simple argument which renders these results reasonable.

(1) Lest the simplicity of the argument be shrouded in a cloud of indices, we first consider a one-parameter Abelian group, representing, for example, the phase transformation of a charged boson; we then present the generalization to an arbitrary compact Lie group.

The interaction between the φ and the A_μ fields is

$$H_{\mathrm{int}} = ieA_\mu \varphi^* \overleftrightarrow{\partial}_\mu \varphi - e^2 \varphi^* \varphi A_\mu A_\mu, \tag{1}$$

where $\varphi = (\varphi_1 + i\varphi_2)/\sqrt{2}$. We shall break the symmetry by fixing $\langle\varphi\rangle \neq 0$ in the vacuum, with the phase chosen for convenience such that $\langle\varphi\rangle = \langle\varphi^*\rangle = \langle\varphi_1\rangle/\sqrt{2}$.

We shall assume that the application of the theorem of Goldstone, Salam, and Weinberg[7] is straightforward and thus that the propagator of the field φ_2, which is "orthogonal" to φ_1, has a pole at $q=0$ which is not isolated.

We calculate the vacuum polarization loop $\Pi_{\mu\nu}$ for the field A_μ in lowest order perturbation theory about the self-consistent vacuum. We take into consideration only the broken-symmetry diagrams (Fig. 1). The conventional terms do not lead to a mass in this approximation if gauge invariance is carefully maintained. One evaluates directly

$$\Pi_{\mu\nu}(q) = (2\pi)^4 ie^2[g_{\mu\nu}\langle\varphi_1\rangle^2 - (q_\mu q_\nu/q^2)\langle\varphi_1\rangle^2]. \quad (2)$$

Here we have used for the propagator of φ_2 the value $[i/(2\pi)^4]/q^2$; the fact that the renormalization constant is 1 is consistent with our approximation.[9] We then note that Eq. (2) both maintains gauge invariance ($\Pi_{\mu\nu}q_\nu = 0$) and causes the A_μ field to acquire a mass

$$\mu^2 = e^2\langle\varphi_1\rangle^2. \quad (3)$$

We have not yet constructed a proof in arbitrary order; however, the similar appearance of higher order graphs leads one to surmise the general truth of the theorem.

Consider now, in general, a set of boson-field operators φ_A (which we may always choose to be Hermitian) and the associated Yang-Mills field $A_{a,\mu}$. The Lagrangian is invariant under the transformation[10]

$$\delta\varphi_A = \textstyle\sum_{a,A}\epsilon_a(x)T_{a,AB}\varphi_B,$$

$$\delta A_{a,\mu} = \textstyle\sum_{c,b}\epsilon_c(x)c_{acb}A_{b,\mu} + \partial_\mu\epsilon_a(x), \quad (4)$$

where c_{abc} are the structure constants of a compact Lie group and $T_{a,AB}$ the antisymmetric generators of the group in the representation defined by the φ_B.

Suppose that in the vacuum $\langle\varphi_{B'}\rangle \neq 0$ for some B'. Then the propagator of $\sum_{A,B'}T_{a,AB'}\varphi_A$

FIG. 1. Broken-symmetry diagram leading to a mass for the gauge field. Short-dashed line, $\langle\varphi_1\rangle$; long-dashed line, φ_2 propagator; wavy line, A_μ propagator. (a)→ $(2\pi)^4 ie^2 g_{\mu\nu}\langle\varphi_1\rangle^2$, (b)→ $-(2\pi)^4 ie^2(q_\mu q_\nu/q^2) \times\langle\varphi_1\rangle^2$.

$\times\langle\varphi_{B'}\rangle$ is, in the lowest order,

$$\left[\frac{i}{(2\pi)^4}\right]\sum_{A,B',C'}\frac{T_{a,AB'}\langle\varphi_{B'}\rangle T_{a,AC'}\langle\varphi_{C'}\rangle}{q^2}$$

$$\equiv\left[\frac{-i}{(2\pi)^4}\right]\frac{(\langle\varphi\rangle T_a T_a\langle\varphi\rangle)}{q^2}.$$

With λ the coupling constant of the Yang-Mills field, the same calculation as before yields

$$\Pi_{\mu\nu}{}^a(q) = -i(2\pi)^4\lambda^2(\langle\varphi\rangle T_a T_a\langle\varphi\rangle)$$

$$\times[g_{\mu\nu} - q_\mu q_\nu/q^2],$$

giving a value for the mass

$$\mu_a{}^2 = -(\langle\varphi\rangle T_a T_a\langle\varphi\rangle). \quad (6)$$

(2) Consider the interaction Hamiltonian

$$H_{\mathrm{int}} = -\eta\bar\psi\gamma_\mu\gamma_5\psi B_\mu - \epsilon\bar\psi\gamma_\mu\psi A_\mu, \quad (7)$$

where A_μ and B_μ are vector and pseudovector gauge fields. The vector field causes attraction whereas the pseudovector leads to repulsion between particle and antiparticle. For a suitable choice of ϵ and η there exists, as in Johnson's model,[11] a broken-symmetry solution corresponding to an arbitrary mass m for the ψ field fixing the scale of the problem. Thus the fermion propagator $S(p)$ is

$$S^{-1}(p) = \gamma p - \Sigma(p) = \gamma p[1-\Sigma_2(p^2)] - \Sigma_1(p^2), \quad (8)$$

with

$$\Sigma_1(p^2) \neq 0$$

and

$$m[1-\Sigma_2(m^2)] - \Sigma_1(m^2) = 0.$$

We define the gauage-invariant current $J_\mu{}^5$ by using Johnson's method[12]:

$$J_\mu{}^5 = -\eta\lim_{\xi\to 0}\bar\psi'(x+\xi)\gamma_\mu\gamma_5\psi'(x),$$

$$\psi'(x) = \exp[-i\textstyle\int_{-\infty}^x \eta B_\mu(y)dy^\mu\gamma_5]\psi(x). \quad (9)$$

This gives for the polarization tensor of the pseudovector field

$$\Pi_{\mu\nu}^{5}(q)=\eta^2\frac{i}{(2\pi)^4}\int \mathrm{Tr}\{S(p-\tfrac{1}{2}q)\Gamma_{\nu5}(p-\tfrac{1}{2}q;p+\tfrac{1}{2}q)\times S(p+\tfrac{1}{2}q)\gamma_{\mu}\gamma_5 -S(p)[\partial S^{-1}(p)/\partial p_{\nu}]S(p)\gamma_{\mu}\}d^4p, \qquad (10)$$

where the vertex function $\Gamma_{\nu5}=\gamma_\nu\gamma_5+\Lambda_{\nu5}$ satisfies the Ward identity[5]

$$q_\nu\Lambda_{\nu5}(p-\tfrac{1}{2}q;p+\tfrac{1}{2}q)=\Sigma(p-\tfrac{1}{2}q)\gamma_5+\gamma_5\Sigma(p+\tfrac{1}{2}q), \qquad (11)$$

which for low q reads

$$q_\nu\Gamma_{\nu5}=q_\nu\gamma_\nu\gamma_5[1-\Sigma_2]+2\Sigma_1\gamma_5 -2(q_\nu p_\nu)(\gamma_\lambda p_\lambda)(\partial\Sigma_2/\partial p^2)\gamma_5. \qquad (12)$$

The singularity in the longitudinal $\Gamma_{\nu5}$ vertex due to the broken-symmetry term $2\Sigma_1\gamma_5$ in the Ward identity leads to a nonvanishing gauge-invariant $\Pi_{\mu\nu}^{5}(q)$ in the limit $q\to 0$, while the usual spurious "photon mass" drops because of the second term in (10). The mass of the pseudovector field is roughly η^2m^2 as can be checked by inserting into (10) the lowest approximation for $\Gamma_{\nu5}$ consistant with the Ward identity.

Thus, in this case the general feature of the phenomenological boson system survives. We would like to emphasize that here the symmetry is broken through the gauge fields themselves. One might hope that such a feature is quite general and is possibly instrumental in the realization of Sakurai's program.[3]

(3) We present below a simple argument which indicates why the gauge vector field need not have zero mass in the presence of broken symmetry. Let us recall that these fields were introduced in the first place in order to extend the symmetry group to transformations which were different at various space-time points. Thus one expects that when the group transformations become homogeneous in space-time, that is $q\to 0$, no dynamical manifestation of these fields should appear. This means that it should cost no energy to create a Yang-Mills quantum at $q=0$ and thus the mass is zero. However, if we break gauge invariance of the first kind and still maintain gauge invariance of the second kind this reasoning is obviously incorrect. Indeed, in Fig. 1, one sees that the A_μ propagator connects to intermediate states, which are "rotated" vacua. This is seen most clearly by writing $\langle\varphi_1\rangle=\langle[Q\varphi_2]\rangle$ where Q is the group generator. This effect cannot vanish in the limit $q\to 0$.

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BROKEN SYMMETRIES, MASSLESS PARTICLES AND GAUGE FIELDS

P. W. HIGGS
Tait Institute of Mathematical Physics, University of Edinburgh, Scotland

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Recently a number of people have discussed the Goldstone theorem [1,2]: that any solution of a Lorentz-invariant theory which violates an internal symmetry operation of that theory must contain a massless scalar particle. Klein and Lee [3] showed that this theorem does not necessarily apply in non-relativistic theories and implied that their considerations would apply equally well to Lorentz-invariant field theories. Gilbert [4], however, gave a proof that the failure of the Goldstone theorem in the nonrelativistic case is of a type which cannot exist when Lorentz invariance is imposed on a theory. The purpose of this note is to show that Gilbert's argument fails for an important class of field theories, that in which the conserved currents are coupled to gauge fields.

Following the procedure used by Gilbert [4], let us consider a theory of two hermitian scalar fields

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$\varphi_1(x)$, $\varphi_2(x)$ which is invariant under the phase transformation

$$\begin{aligned} \varphi_1 &\to \varphi_1 \cos\alpha + \varphi_2 \sin\alpha , \\ \varphi_2 &\to -\varphi_1 \sin\alpha + \varphi_2 \cos\alpha . \end{aligned} \tag{1}$$

Then there is a conserved current j_μ such that

$$\mathrm{i}\left[\int \mathrm{d}^3x\, j_0(x),\ \varphi_1(y)\right] = \varphi_2(y). \tag{2}$$

We assume that the Lagrangian is such that symmetry is broken by the nonvanishing of the vacuum expectation value of φ_2. Goldstone's theorem is proved by showing that the Fourier transform of $\mathrm{i}\langle[j_\mu(x),\ \varphi_1(y)]\rangle$ contains a term $2\pi\langle\varphi_2\rangle\epsilon(k_0)k_\mu\delta(k^2)$, where k_μ is the momentum, as a consequence of Lorentz-covariance, the conservation law and eq. (2).

Klein and Lee [3] avoided this result in the nonrelativistic case by showing that the most general form of this Fourier transform is now, in Gilbert's notation,

$$\text{F.T.} = k_\mu \rho_1(k^2, nk) + n_\mu \rho_2(k^2, nk) + C_3 n_\mu \delta^4(k) , \tag{3}$$

where n_μ, which may be taken as (1, 0, 0, 0), picks out a special Lorentz frame. The conservation law then reduces eq. (3) to the less general form

$$\text{F.T.} = k_\mu \delta(k^2)\rho_4(nk) + [k^2 n_\mu - k_\mu(nk)]\rho_5(k^2, nk) + C_3 n_\mu \delta^4(k) . \tag{4}$$

It turns out, on applying eq. (2), that all three terms in eq. (4) can contribute to $\langle\varphi_2\rangle$. Thus the Goldstone theorem fails if $\rho_4 = 0$, which is possible only if the other terms exist. Gilbert's remark that no special timelike vector n_μ is available in a Lorentz-covariant theory appears to rule out this possibility in such a theory.

There is however a class of relativistic field theories in which a vector n_μ does indeed play a part. This is the class of gauge theories, where an auxiliary unit timelike vector n_μ must be introduced in order to define a radiation gauge in which the vector gauge fields are well defined operators. Such theories are nevertheless Lorentz-covariant, as has been shown by Schwinger [5]. (This has, of course, long been known of the simplest such theory, quantum electrodynamics.) There seems to be no reason why the vector n_μ should not appear in the Fourier transform under consideration.

It is characteristic of gauge theories that the conservation laws hold in the strong sense, as a consequence of field equations of the form

$$j^\mu = \partial_\nu F'^{\mu\nu},$$

$$F_{\mu\nu}' = \partial_\mu A_\nu' - \partial_\nu A_\mu' . \tag{5}$$

Except in the case of abelian gauge theories, the fields A_μ', $F_{\mu\nu}'$ are not simply the gauge field variables A_μ, $F_{\mu\nu}$, but contain additional terms with combinations of the structure constants of the group as coefficients. Now the structure of the Fourier transform of $\mathrm{i}\langle[A_\mu'(x),\ \varphi_1(y)]\rangle$ must be given by eq. (3). Applying eq. (5) to this commutator gives us as the Fourier transform of $\mathrm{i}\langle[j_\mu(x),\ \varphi_1(y)]\rangle$ the single term $[k^2 n_\mu - k_\mu(nk)]\rho(k^2, nk)$. We have thus exorcised both Goldstone's zero-mass bosons and the "spurion" state (at $k_\mu = 0$) proposed by Klein and Lee.

In a subsequent note it will be shown, by considering some classical field theories which display broken symmetries, that the introduction of gauge fields may be expected to produce qualitative changes in the nature of the particles described by such theories after quantization.

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BROKEN SYMMETRIES AND THE MASSES OF GAUGE BOSONS

Peter W. Higgs
Tait Institute of Mathematical Physics, University of Edinburgh, Edinburgh, Scotland
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In a recent note[1] it was shown that the Goldstone theorem,[2] that Lorentz-covariant field theories in which spontaneous breakdown of symmetry under an internal Lie group occurs contain zero-mass particles, fails if and only if the conserved currents associated with the internal group are coupled to gauge fields. The purpose of the present note is to report that, as a consequence of this coupling, the spin-one quanta of some of the gauge fields acquire mass; the longitudinal degrees of freedom of these particles (which would be absent if their mass were zero) go over into the Goldstone bosons when the coupling tends to zero. This phenomenon is just the relativistic analog of the plasmon phenomenon to which Anderson[3] has drawn attention: that the scalar zero-mass excitations of a superconducting neutral Fermi gas become longitudinal plasmon modes of finite mass when the gas is charged.

The simplest theory which exhibits this behavior is a gauge-invariant version of a model used by Goldstone[2] himself: Two real[4] scalar fields φ_1, φ_2 and a real vector field A_μ interact through the Lagrangian density

$$L = -\tfrac{1}{2}(\nabla\varphi_1)^2 - \tfrac{1}{2}(\nabla\varphi_2)^2 - V(\varphi_1^2 + \varphi_2^2) - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}, \quad (1)$$

where

$$\nabla_\mu\varphi_1 = \partial_\mu\varphi_1 - eA_\mu\varphi_2,$$

$$\nabla_\mu\varphi_2 = \partial_\mu\varphi_2 + eA_\mu\varphi_1,$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu,$$

e is a dimensionless coupling constant, and the metric is taken as $-+++$. L is invariant under simultaneous gauge transformations of the first kind on $\varphi_1 \pm i\varphi_2$ and of the second kind on A_μ. Let us suppose that $V'(\varphi_0^2) = 0$, $V''(\varphi_0^2) > 0$; then spontaneous breakdown of U(1) symmetry occurs. Consider the equations [derived from (1) by treating $\Delta\varphi_1$, $\Delta\varphi_2$, and A_μ as small quantities] governing the propagation of small oscillations about the "vacuum" solution $\varphi_1(x) = 0$, $\varphi_2(x) = \varphi_0$:

$$\partial^\mu\{\partial_\mu(\Delta\varphi_1) - e\varphi_0 A_\mu\} = 0, \quad (2a)$$

$$\{\partial^2 - 4\varphi_0^2 V''(\varphi_0^2)\}(\Delta\varphi_2) = 0, \quad (2b)$$

$$\partial_\nu F^{\mu\nu} = e\varphi_0\{\partial^\mu(\Delta\varphi_1) - e\varphi_0 A^\mu\}. \quad (2c)$$

Equation (2b) describes waves whose quanta have (bare) mass $2\varphi_0\{V''(\varphi_0^2)\}^{1/2}$; Eqs. (2a) and (2c) may be transformed, by the introduction of new variables

$$B_\mu = A_\mu - (e\varphi_0)^{-1}\partial_\mu(\Delta\varphi_1),$$

$$G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu = F_{\mu\nu}, \quad (3)$$

into the form

$$\partial_\mu B^\mu = 0, \quad \partial_\nu G^{\mu\nu} + e^2\varphi_0^2 B^\mu = 0. \quad (4)$$

Equation (4) describes vector waves whose quanta have (bare) mass $e\varphi_0$. In the absence of the gauge field coupling ($e = 0$) the situation is quite different: Equations (2a) and (2c) describe zero-mass scalar and vector bosons, respectively. In passing, we note that the right-hand side of (2c) is just the linear approximation to the conserved current: It is linear in the vector potential, gauge invariance being maintained by the presence of the gradient term.[5]

When one considers theoretical models in which spontaneous breakdown of symmetry under a semisimple group occurs, one encounters a variety of possible situations corresponding to the various distinct irreducible representations to which the scalar fields may belong; the gauge field always belongs to the adjoint representation.[6] The model of the most immediate interest is that in which the scalar fields form an octet under SU(3): Here one finds the possibility of two nonvanishing vacuum expectation values, which may be chosen to be the two $Y = 0$, $I_3 = 0$ members of the octet.[7] There are two massive scalar bosons with just these quantum numbers; the remaining six components of the scalar octet combine with the corresponding components of the gauge-field octet to describe massive vector bosons. There are two $I=\frac{1}{2}$ vector doublets, degenerate in mass between $Y=\pm 1$ but with an electromagnetic mass splitting between $I_3=\pm\frac{1}{2}$, and the $I_3=\pm 1$ components of a $Y=0$, $I=1$ triplet whose mass is entirely electromagnetic. The two $Y=0$, $I=0$ gauge fields remain massless: This is associated with the residual unbroken symmetry under the Abelian group generated by Y and I_3. It may be expected that when a further mechanism (presumably related to the weak interactions) is introduced in order to break Y conservation, one of these gauge fields will acquire mass, leaving the photon as the only massless vector particle. A detailed discussion of these questions will be presented elsewhere.

It is worth noting that an essential feature of the type of theory which has been described in this note is the prediction of incomplete multiplets of scalar and vector bosons.[8] It is to be expected that this feature will appear also in theories in which the symmetry-breaking scalar fields are not elementary dynamic variables but bilinear combinations of Fermi fields.[9]

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[4]In the present note the model is discussed mainly in classical terms; nothing is proved about the quantized theory. It should be understood, therefore, that the conclusions which are presented concerning the masses of particles are conjectures based on the quantization of linearized classical field equations. However, essentially the same conclusions have been reached independently by F. Englert and R. Brout, Phys. Rev. Letters 13, 321 (1964): These authors discuss the same model quantum mechanically in lowest order perturbation theory about the self-consistent vacuum.

[5]In the theory of superconductivity such a term arises from collective excitations of the Fermi gas.

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