BARYOGENESIS

Lecture notes

1 Introduction

1.1 What is the Problem

The CPT theorem assures that for any particle species X there exists an antiparticle $\bar{X}$ with exactly the same mass, $m_X = m_X$, and decay width, $\Gamma_X = \Gamma_{\bar{X}}$. The charges associated with particles and antiparticles are opposite, $Q_X = Q_{\bar{X}}$. This striking symmetry would naturally lead us to conclude that the Universe with symmetric initial conditions contains particles and antiparticles in equal number densities, $n_X = n_{\bar{X}}$ at all times. The observed Universe is, however, drastically different. We do not observe any bodies of antimatter within the solar system. Antiprotons p are observed in the cosmic rays but are likely to be produced as secondaries in collisions $pp \to 3p + \bar{p}$ at a rate similar to the observed one, $n_{\bar{p}}/n_p \sim 310^4$. The observational limit on $\bar{n}_{He}/n_{He}$ is similarly of order 10^5. We cannot exclude that the dominance of matter over antimatter is only local and is only realized up to a certain length scale l_B beyond which the picture is reveresed and islands of antimatter are found. The size of our matter domain must, however, be quite large, roughly speaking $l_B \gtrsim 10\,\mathrm{Mpc}$. Indeed, for smaller scales one would expect a significant amount of energetic ”-rays coming from the reactions of annihilation of

$p\bar{p} \to \pi$s followed by the subsequent decays $\pi^0 \to 2\gamma$, which would take place in the boundary area separating the matter and antimatter islands. Another signature for the presence of domains of antimatter would be the distortion of the spectrum of the CMBR. In such a case, the lower bound on l_B might be weaker if voids separate matter and antimatter domains. The voids might be created because of an excessive pressure produced by the annihilations at early stages of the evolution of the Universe or because of low density matter and antimatter in the boundary regions, provided that the baryon asymmetry changes sign locally so that in the boundaries it is zero or very small. All these considerations lead us to conclude that, if domains of matter and antimatter exist in the Universe, they are separated on scales certainly larger than the radius of our own galaxy (3 Kpc) and most probably on scales significantly larger than the Virgo cluster (10-100 Mpc).

1.2 The observed value

The baryon number density does not keep constant during the evolution of the Universe because it scales like $a^{-3}(t)$, where $a(t)$ is the cosmological scale factor. It is therefore convenient to define the baryon asymmetry of the Universe in terms of the quantity

$$\eta = \frac{n_B}{n_\gamma} = \frac{n_b - n_{\bar{b}}}{n_\gamma}. \tag{1}$$

This quantitiy is essential for determining the light element abundances produced at the nucleosynthesis epoch and also the relative strength of the cosmic microwave background (CMB) temperature fluctuation on various large angular scales. The parameter has not been changed since nucleosynthesis. At the relevant energy scales (1 MeV) the baryon asymmetry is conserved if there are no processes which would have produced entropy to change the photon number density. The baryon number density is

$$n_B = \frac{\rho_B}{m_B} = \frac{\Omega_B}{m_B}\rho_c = 1.1 \times 10^5 h^2 \Omega_B \,\mathrm{cm}^3, \tag{2}$$

where $h \sim 0.7$, Ω_B is the baryon density in units of the total density and ρ_c is the total density (assuming a flat universe), while the photon number densiry is

$$n_\gamma = \frac{2\zeta(3)}{\pi^2} T^3 = 415 \left(\frac{T0}{2.7\,\mathrm{K}}\right)^3 \mathrm{cm}^3. \tag{3}$$

Putting the above to Eqs. together, we obtain

$$\eta = 2.7 \times 10^8 \Omega_B h^2 \left(\frac{T_0}{2.7\,\mathrm{K}}\right)^3 , \tag{4}$$

where T_0 is the temperature in K.

How is η determined? We now have three different methods to measure η all agree with each other at the level of one standard deviation (including systematic uncertainties). The most direct estimate is obtained by counting the number of baryons in the universe and comparing the resulting n_b with the number density of the $T = 2.7K$ microwave photon background (CMB), $n_\gamma = 2\zeta(3)T^3/\pi^2 \simeq 420/cm^3$. In fact this not very precise method yields a number for η that is not too far off from the one that comes from the still most accurate determination to date, the theory of primordial nucleosynthesis – a theory that is one of the triumphs of the standard cosmological model (SCM). There the present abundances of light nuclei, p, D, ^{3}He, ^{4}He, etc. are predicted in terms of the input parameter η. Comparison with the observed abundances yields [11]

$$\eta \approx (1.2 - 5.7) \times 10^{-10} . \tag{5}$$

The most precise determination of the baryonic asymmetry of the Univ. (BAU) comes actually from the relative size of the various sizes of the peaks of the CMB power spectrum on different angular scales [8],

$$\eta \simeq (6.1 \pm 0.2) \times 10^{-10} . \tag{6}$$

Can the order of magnitude of the BAU η be understood within the SCM, without further input? The answer is no! The following exercise shows nicely the point; namely, in order to understand (6) the universe must have been baryon-asymmetric already at early times. The usual, plausible starting point of the SCM is that the big bang produces equal numbers of quarks and antiquarks that end up in equal numbers of nucleons and antinucleons if there were no baryon number violating interactions. Let's compute the nucleon and antinucleon densities. At temperatures below the nucleon mass m_N we would have, as long as the (anti)nucleons are in thermal equilibrium,

$$\frac{n_b}{n_\gamma} = \frac{n_{\bar{b}}}{n_\gamma} \simeq \left(\frac{m_N}{T}\right)^{3/2} \exp\left(-m_N/T\right) . \tag{7}$$

The freeze-out of (anti)nucleons occurs when the $N\bar{N}$ annihilation rate $\Gamma_{ann} = n_b < \sigma_{ann}|\mathbf{v}| >$ becomes smaller than the expansion rate (when basically they cannot find each other in order to annihilate, see discussion below). Using $\sigma_{ann} \sim 1/m_\pi^2$ and using eq. (30) we find that this happens at $T \simeq 20$ MeV. Then we have from (7) that at the time of feeze-out $n_b/n_\gamma = n_{\bar{b}}/n_\gamma \simeq 10^{-18}$, which is 8 orders of magnitude below the observed value! In order to prevent $N\bar{N}$ annihilation some unknown mechanism must have operated at $T \gtrsim 40$ MeV, the temperature when $n_b/n_\gamma = n_{\bar{b}}/n_\gamma \simeq 10^{-10}$, and separated nucleons from antinucleons. However, the causally connected region at that time contained only about 10^{-7} solar masses! Hence this separation mechanism were completely useless for generating our universe made of baryons. Therefore the conclusion to be drawn from these considerations is that the universe possessed already at early times ($T \gtrsim 40$ MeV) an asymmetry between the number of baryons and antibaryons.

How does this asymmetry arise? There might have been some (tiny) excess of baryonic charge already at the beginning of the big bang – even though that does not seem to be an attractive idea. In any case, in the context of inflation such an initial condition becomes futile: at the end of the inflationary period any trace of such a condition had been wiped out.

2 Some Basics of Cosmology

2.1 The Standard Model of Cosmology

The current understanding of the large-scale evolution of our universe is based on a number of observations. These include the expansion of the universe and the approximate isotropic and homogeneous matter and energy distribution on large scales. The Einstein field equations of general relativity imply that the metric of space-time shares these symmetry properties of the sources of gravitation on large scales. It is represented by the Robertson-Walker (RW) metric which corresponds to the line element

$$ds^2 = dt^2 - R^2(t)\left\{\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2\sin^2\theta\, d\theta d\phi^2\right\}\,, \tag{8}$$

where (t, r, θ, ϕ) are the dimensionless comoving coordinates and $k = 0, 1, -1$ for a space of vanishing, positive, or negative spatial curvature. Cosmological data are consistent with $k = 0$ [10]. The dynamical variable $R(t)$ is the cosmic scale factor and has dimension of length. The matter/energy distribution on large scales may be modeled by the stress-energy tensor of a perfect fluid, T^{μ}_{ν} =diag$(\rho, -p, -p, -p)$, where $\rho(t)$ is the total energy density of the matter and radiation in the universe and $p(t)$ is the isotropic pressure.

The dynamical equations which determine the time-evolution of the scale factor follow from Einstein's equations. Inserting the metric tensor which is encoded in (8) and the above form of $T_{\mu\nu}$ into these equations one obtains the Friedmann equation

$$H^2 \equiv \left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G_N}{3}\rho - \frac{k}{R} + \frac{\Lambda}{3} \,. \tag{9}$$

Here $H(t) \equiv \dot{R}(t)/R(t)$ is the Hubble parameter which measures the expansion rate of the universe at time t, and Λ denotes the cosmological "constant" at time t. According to the inflationary universe scenario the Λ term played a crucial role at a very early epoch when vacuum energy was the dominant form of energy in the universe, leading to an exponential increase of the scale factor. Recent observations indicate that today the largest component of the energy density of the universe is some dark energy which can also be described by a non-zero cosmological constant [10]. The baryogenesis scenarios that we shall discuss in these lecture notes are associated with a period in the evolution of the early universe where, supposedly, a Λ term in the evolution equation (9) for H can be neglected.

The covariant conservation of the stress tensor $T_{\mu\nu}$ yields another important equation, namely

$$d(\rho R^3) = -p d(R^3) \,. \tag{10}$$

Figure 1: Cartoon of the history of the universe. The slice of a cake, stretched at the top, illustrates the expansion of the universe as it cooled off. Inflation may have ended well below T_{GUT} [6].

This can be read as the first law of thermodynamics: the total change of energy is equal to the work done on the universe, $dU = dA = -pdV$. Moreover, it turns out (see section 2.2) that the various forms of matter/energy which determine the state of the universe during a certain epoch can be described, to a good approximation, by the equation of state

$$p = w\rho \,, \tag{11}$$

where, for instance, $w = 1/3, 0, -1$ if the energy of the universe is dominated by relativistic particles (i.e., radiation), non-relativistic particles, and vacuum energy, respectively.

Integrating (10) with (11) one obtains that the energy density evolves as $\rho \propto R^{-3(1+w)}$. In the radiation-dominated era, $\rho \propto R^{-4}$. Inserting this scaling law into the Friedmann equation, one finds that in this epoch the expansion rate behaves as

$$H(t) \propto t^{-1}. \tag{12}$$

Fig. 1 illustrates the history of the early universe, as reconstructed by the SCM and by the SM of particle physics. The baryogenesis scenarios which will be discussed in sections 5 and 6 apply to some instant in the – tiny – time interval after inflation and before or at the time of the electroweak phase transition. In this era, where the SM particles were massless, the energy of the universe was – according to what is presently known – essentially due to relativistic particles.

2.2 Equilibrium Thermodynamics

As was just mentioned the baryogenesis scenarios which we shall discuss in sections 5 and 6 apply to the era between the end of inflation and the electroweak phase transition. During this period the universe expanded and cooled off to temperatures $T \gtrsim T_{EW} \sim 100$ GeV. For most of the time during this stage the reaction rates of the majority of particles were much faster than the expansion rate of the cosmos. The early universe, which we view as a (dense) plasma of particles, was then to a good approximation in thermal equilibrium. In several situations it is reasonable to treat this gas as dilute and weakly interacting[1]. Let's therefore recall the equilibrium distributions of an ideal gas. Because particles in the early universe were created and destroyed, it is natural to describe the gas by means of the grand canonical ensemble. Consider an ensemble of a relativistic particle species A. Its phase space distribution or occupancy function is given by

$$f_A(\mathbf{p}) = \frac{1}{e^{(E_A - \mu_A)/T_A} \mp 1} \,, \tag{13}$$

[1]This is of course not true in general. The early universe contained, in particular, particles that carried unscreened non-abelian gauge charges. Such a plasma behaves in many ways differently than an ideal gas.

where T_A is the temperature, μ_A is the chemical potential of the species which is associated with a conserved charge of the ensemble, and the minus (plus) sign refers to bosons (fermions). If different species are in chemical equilibrium then their chemical potentials are related. For instance, suppose the particle reaction $A + B \leftrightarrow C$ takes place rapidly. Then the relation $\mu_A + \mu_B = \mu_C$ holds. Take the standard example $e^+ + e^- \leftrightarrow n\gamma$. Because $\mu_\gamma = 0$ we have $\mu_{e^+} = -\mu_{e^-}$.

From (13) one obtains the number density n_A, the energy density ρ_A, the isotropic pressure p_A, and the entropy density s_A. Defining $d\tilde{p} \equiv d^3p/(2\pi)^3$ we have

$$
\begin{aligned}
n_A &= g_A \int d\tilde{p}\, f_A(\mathbf{p})\,, && (14)\\
\rho_A &= g_A \int d\tilde{p}\, E_A(\mathbf{p}) f_A(\mathbf{p})\,, && (15)\\
p_A &= g_A \int d\tilde{p}\, \frac{\mathbf{p^2}}{3E_A} f_A(\mathbf{p})\,, && (16)\\
s_A &= \frac{\rho_A + p_A}{T}\,. && (17)
\end{aligned}
$$

Here $E_A = \sqrt{\mathbf{p^2} + m_A^2}$, where m_A is the mass of A, and g_A denotes the internal degrees of freedom of A; for instance, $g_e = 2$ for the electron and $g_\nu = 1$ for a massless neutrino.

In the following we need these expressions in the ultra-relativistic ($T_A >> m_A$) and nonrelativistic ($T_A << m_A$) limits. Integrating eqs. (7) - (9) one obtains the well-known textbook formulae for n_A, ρ_A, and p_A. For relativistic particles A (and $T_A >> \mu_A$)

$$
\begin{aligned}
n_A &= a_A g_A T_A^3\,, && (18)\\
\rho_A &= b_A g_A T_A^4\,, && (19)\\
p_A &\simeq \rho_A/3\,, && (20)
\end{aligned}
$$

while for nonrelativistic particles the number density becomes exponentially suppressed for decreasing temperature:

$$
\begin{aligned}
n_A &= g_A(\frac{m_A T_A}{2\pi})^{3/2} e^{-(m_A-\mu_A)/T_A}\,, && (21)\\
\rho_A &= n_A m_A\,, && (22)\\
p_A &\simeq n_A T_A << \rho_A\,. && (23)
\end{aligned}
$$

In eqs. (11), (12) a_A and b_A are numbers depending on whether A is a boson or fermion. Eqs. (20), (23) are the equations of state that we used already above.

When considering the total energy density and pressure of all particle species it is useful to express these quantities in terms of the photon temperature T. The corresponding formulae are obtained in a straightforward fashion by summing the respective contributions, taking into account that some species A may have a thermal distribution with a temperature $T_A \neq T$. When the universe was in thermal equilibrium its entropy remained constant. Its entropy density is given by

$$s = \frac{S}{V} = \frac{\rho + p}{T} = \frac{2\pi^2}{45} g_{*s} T^3 \,, \tag{24}$$

where the last equality comes from the fact that s is dominated by the contributions from relativistic particles. During the epoch we are interested in, the factor g_{*s} was equal to the total number of relativistic degrees of freedom g_* [1]. (For $T >> m_{top}$ we have $g_* \simeq 106$ in the SM.) The entropy being constant implies $s \propto R^{-3}$, hence $g_{*s} T^3 R^3 = const$. From this we obtain that in the radiation dominated epoch the temperature of the universe decreased as

$$T \propto R^{-1} \,. \tag{25}$$

From these relations we can draw another important conclusion. Consider the number N_A of some particle species A. Because $N_A \equiv R^3 n_A \propto n_A/s$ this ratio also remained constant, in the absence of "A number" violation and/or entropy production, during the expansion of the universe. Therefore in the context of baryogenesis the relevant quantity is the baryon-to-entropy ratio $n_B/s \equiv (n_b - n_{\bar{b}})/s$, where n_b and $n_{\bar{b}}$ denotes the number density of baryons and antibaryons, respectively. The BAU $\eta \equiv n_B/n_\gamma$ is given in terms of this ratio by $\eta = 1.8 g_{*s} n_B/s$. The relativistic degrees of freedom g_{*s} decreased during the expansion of the early universe. This number and, hence, η remained constant only after the time of e^+e^- annihilation. From then on

$$\eta \simeq 7 \frac{n_B}{s} . \tag{26}$$

2.3 Departures from Thermal Equilibrium

Departures from thermal equilibrium (DTE) were, of course, crucial for the development of the universe to that state that we perceive today. Examples for DTEs include the decoupling of neutrinos, the decoupling of the photon background radiation, and primordial nucleosynthesis. More speculative examples are inflation, first order phase transitions in the early universe (see below), the decoupling of weakly interacting massive particles, and the topic of these lectures, baryogenesis. In any case the DTEs have led to the (light) elements, to a net baryon number of the visible universe, and to the neutrino and the microwave background.

A rough criterion for whether or not a particle species A is in local thermal equilibrium is obtained by comparing reaction rate Γ_A with the expansion rate H.[2] Let $\sigma(A+target \to X)$ be the total cross section of the reaction(s) of A that is (are) crucial for keeping A in thermal equilibrium. Then Γ_A is given by

$$\Gamma_A = \sigma(A + target \to X) n_{target} |\mathbf{v}| \,, \tag{27}$$

where n_{target} is the target density and $\mathbf{v}$ is the relative velocity. Keep in mind that $[\Gamma_A] = (sec)^{-1}$. If

$$\Gamma_A \gtrsim H \,, \tag{28}$$

then the reactions involving A occur rapidly enough for A to maintain thermal equilibrium. If

$$\Gamma_A < H \,, \tag{29}$$

then the ensemble of particles A will fall out of equilibrium. The Hubble parameter $H(t)$ which is relevant for the baryogenesis scenarios to be discussed below is the expansion rate during the radiation dominated epoch. It follows from eqs. (9) and (12) that in this era

$$H = \sqrt{\frac{8\pi G_N}{3}\rho} = 1.66\sqrt{g_*}\frac{T^2}{m_{Pl}} \,, \tag{30}$$

where $m_{Pl} = 1.22 \times 10^{19}$GeV denotes the Planck mass.

Eqs. (28) and (29) constitute a useful rule of thumb that is often quite accurate. It is sufficient for the purpose of these lectures. A proper treatment involves the determination of the time evolution of the particle's phase space

[2]The intuitive argument goes as follows typically $\Gamma_A \sim T^n$, then the number of interaction go like $\int_t^\infty \Gamma dt' \propto \gamma/H$. So that when H is much faster there's not even one interaction per one inverse Hubble time, t_H. However, during that time the plasma further cools by a factor of roughly $\exp[Ht_H]$. This implies that particle A cannot catch up with the Univ. total temp' and the interaction goes our of equilibrium.

distribution f_A which is governed by the Boltzmann equation (cf. for instance [1]). Comparing the number density $n_A(t)$, obtained from solving this equation, with the equilibrium distribution n_A^{eq} (which was discussed above for (non)relativistic particles) one sees whether or not A has decoupled from the thermal bath. Rather than going into details let us sketch in Fig. 2 the behaviour of the ratio

$$Y_A \equiv \frac{n_A}{s} \tag{31}$$

as a function of the decreasing temperature when an ensemble of massive particles A decouples from the thermal bath. In thermal equilibrium Y_A is constant for $T >> m_A$. At later times, when $T \lesssim m_A$, $Y_A \propto (m_A/T)^{3/2} \exp(-m_A/T)$ if the reaction rate still obeys (28). Thus, if A would have remained in thermal equilibrium until today its abundance would be completely negligible. However, if Γ_A becomes smaller than H, the interactions of A "freeze out", and the actual abundance of A deviates from its equilibrium value at temperature T. The larger the $A\bar{A}$ annihilation cross section the smaller the decoupling temperature and the actual abundance Y_A. The further fate of the decoupled species depends on whether or not A is stable. If a (quasi)stable species A – a weakly interacting massive particle – froze out at a temperature T not much smaller than m_A then its abundance today can be significant.

Figure 2: The behaviour of $Y_A \equiv n_A/s$ as a function of decreasing temperature for a massive, (non)relativistic particle species A falling out of thermal equilibrium.

3 The Baryon Asymmetry of the Universe

3.1 The Sakharov Conditions

In the early days of the big bang model η was accepted as one of the fundamental parameters of the model. In 1967, three years after CP violation was discovered by the observation of the decays of $K_L \to 2\pi$, Sakharov pointed out in his seminal paper [16] that a baryon asymmetry can actually arise dynamically during the evolution of the universe from an initial state with baryon number equal to zero if the following three conditions hold:

- baryon number (B) violation,

• C and CP violation,
• departure from thermal equilibrium (i.e., an "arrow of time").
Many models of particle physics have these ingredients, in combination with the SCM. The theoretical challenge has been to find out which of them support (plausible) scenarios that yield the correct order of magnitude of the BAU. Before turning to some of these models, let us briefly discuss the Sakharov conditions. The first one seems obvious – see, however, the remark below. The second requirement is easily understood, noticing that the baryon number operator $\hat{B}$ is odd both under C and CP (see Appendix A). Therefore a non-zero baryon number, i.e., a non-zero expectation value $< \hat{B} >$ requires that the Hamiltonian H of the world violates C and CP. A formal argument for condition three is as follows: First, recall that a system which is in thermal equilibrium is stationary and is described by a density operator $\rho = \exp(-H/T)$. Using $\hat{B}(t) = e^{iHt}\hat{B}(0)e^{-iHt}$ we have

$$< \hat{B}(t) >_T = tr(e^{-H/T}e^{iHt}\hat{B}(0)e^{-iHt}) = tr(e^{-iHt}e^{-H/T}e^{iHt}\hat{B}(0)) =< \hat{B}(0) >_T \,,$$

If the Hamiltonian H is $\Theta \equiv CPT$ invariant, $\Theta^{-1}H\Theta = H$, we get for the quantum mechanical equilibrium average of $\hat{B} \equiv \hat{B}(0)$:

$$\begin{aligned} < \hat{B} >_T &= tr(e^{-H/T}\hat{B}) = tr(\Theta^{-1}\Theta e^{-H/T}\hat{B}) \\ &= tr(e^{-H/T}\Theta\hat{B}\Theta^{-1}) = - < \hat{B} >_T \,, \end{aligned} \tag{32}$$

where we used that $\hat{B}$ is odd under CPT (see Appendix A). Thus $< \hat{B} >_T = 0$ in thermal equilibrium.

Figure 3: A *Gedanken-Experiment* that illustrates two of the three Sakharov conditions.

How the average baryon number is kept equal to zero in thermal equilibrium is a bit tricky, as the following example shows [2]. Consider an ensemble of a heavy particle species X that has 2 baryon-number violating decay modes $X \to qq$ and $X \to \ell\bar{q}$ into quarks and leptons. (Take $q = d$ and $\ell = e$.) Further, assume that there is C and CP violation in these decays such that an asymmetry in the partial decay rates of X and its antiparticle $\bar{X}$ is induced:

$$\Gamma(X \to qq) \;=\; (1+\epsilon)\Gamma_0 \,, \qquad \Gamma(\bar{X} \to \bar{q}\bar{q}) \;=\; (1-\epsilon)\Gamma_0 \,, \tag{33}$$

and there will also be an asymmetry for the other channel. CPT invariance is supposed to hold. Then the total decays rates of X and $\bar{X}$ are equal. In the decays of $X, \bar{X}$ a non-zero baryon number ΔB is generated. The ensemble is supposed to be in thermal equilibrium. One might be inclined to appeal to the principle of detailed balance which would tell us that the inverse decay $qq \to X$ is more likely than $\bar{q}\bar{q} \to \bar{X}$, and the temporary excess $\Delta B \neq 0$ would be erased this way. However, this principle is based on T invariance – but CPT invariance implies that this symmetry is broken because of CP violation. In fact applying a CPT transformation to the above decays, CPT invariance tells us that the inverse decays push ΔB into the same direction as (33):

$$\Gamma(qq \to X) \;=\; (1-\epsilon)\Gamma_0\,, \qquad \Gamma(\bar{q}\bar{q} \to \bar{X}) \;=\; (1+\epsilon)\Gamma_0\,. \tag{34}$$

The elimination of the baryon number ΔB is achieved by the B-violating reactions $qq \to \ell\bar{q}$, $\bar{q}\bar{q} \to \bar{\ell}q$, and the CPT-transformed reactions, where the $X, \bar{X}$ resonance contributions are to be taken out of the scattering amplitudes. It is the unitarity of the S matrix which does the job of keeping $< \hat{B} >_T = 0$ in thermal equilibrium.

The following *Gedanken-Experiment*, sketched in Fig. 3, illustrates two of the three Sakharov conditions [17]. Let's simulate the big bang by taking an empty box and heat it up to a temperature, say, above the nucleon mass. Pairs of particles and antiparticles are produced that start interacting with each other, instable particles decay, etc. The K^0 and $\bar{K}^0$ evolve in time as coherent superpositions of K_L and K_S, and these states have CP-violating decays, for instance the observed non-leptonic modes $K_L \to \pi\pi$, and there is the observed CP-violating charge asymmetry in the semileptonic decays $K_L \to \pi^{\mp}\ell^{\pm}\nu$ [9]. When analyzing the semileptonic decays of K^0 and $\bar{K}^0$ one finds that slightly more $\pi^-\ell^+\nu_\ell$ are produced than $\pi^+\ell^-\bar{\nu}_\ell$, by about one part in 10^3. Hence, although initially there were equal numbers of K^0 and $\bar{K}^0$, their decays produce more π^- than π^+. Yet as long as the system is in thermal equilibrium, CP violation in the reactions including $\pi^+\ell^- \leftrightarrow \pi^+\pi^-\nu_\ell$ and $\pi^-\ell^+ \leftrightarrow \pi^+\pi^-\bar{\nu}_\ell$ will wash out the temporary excess of π^-. However, if a thermal instability is created by opening the box for a while, the excess ℓ^+ from neutral kaon decay have a chance to escape. Then the inverse reactions involving ℓ^+ are blocked to some degree, and a mesonic asymmetry $(N_{\pi^-} - N_{\pi^+}) > 0$ is generated. Of course, we haven't yet produced the real thing, as no B-violating interactions came into play.

In general, the Sakharov conditions are sufficient but not necessary for generating a non-zero baryon number. Each of them can be circumvented in principle [2]. For instance, if H is not CPT invariant, the argumentation of eq. (32) fails. However, such ideas have so far not led to a satisfactory explanation of (6). For the baryogenesis scenarios that will be discussed in sections 5,6 the Sakharov conditions are necessary ones.

4 The SM and the Sakharov Conditions

The standard model of particle physics combined with the SCM has all the ingredients for generating a baryon asymmetry. As we shall see it dramatically fails however on the quantitative level. We shall now discuss how each of these realize in the SM.

4.1 C and CP Violation

C and CP are violated by the charged weak interactions explicitly. C is maximaly violated by the SM weak interaction being chiral. The violation of CP goes through the Kobayashi-Maskawa mechanism as discussed previously. The SM flavor sector contains a single CP phase which was observed to be of order one ($\delta_{\rm CKM} \sim 60^o$).

4.2 B violation

Baryon number violation in the SM, but this is a subtle, non-perturbative effect which is completely negligible for particle reactions in the laboratories at present-day collision energies, but very significant for the physics of the early universe. Let us outline how this effect arises. From experience we know that baryon and lepton number, which are conventionally assigned to quarks and leptons as given in the table, are good quantum numbers in particle reactions in the laboratory.

	q	$\bar{q}$	ℓ	$\bar{\ell}$
B	1/3	-1/3	0	0
L	0	0	1	-1

In the SM this is explained by the circumstance that the SM Lagrangian $\mathcal{L}_{SM}(x)$, with its strong-interaction (QCD) and electroweak parts, has a global $U(1)_B$ and $U(1)_L$ symmetry: $\mathcal{L}_{SM}$ is invariant under the following two sets of global phase transformations of the quark and lepton fields[3] $q = u, ..., t$; $\ell = e, ..., \nu_\tau$:

$$\begin{aligned} q(x) \;&\to\; e^{i\omega/3} q(x)\,, \qquad && \ell(x) \to \ell(x)\,, && (35)\\ \ell(x) \;&\to\; e^{i\lambda}\ell(x)\,, && q(x) \to q(x)\,. && (36) \end{aligned}$$

Applying Noether's theorem we obtain the associated symmetry currents J_μ^B and J_μ^L, which are conserved at the Born level:

$$\partial^\mu J_\mu^B \;=\; \partial^\mu \sum_q \frac{1}{3}\bar{q}\gamma_\mu q \;=\; 0\,, \tag{37}$$

$$\partial^\mu J_\mu^L \;=\; \partial^\mu \sum_\ell \bar{\ell}\gamma_\mu \ell \;=\; 0\,. \tag{38}$$

(The currents are to be normal-ordered.) Thus the associated charge operators

$$\hat{B} \;=\; \int d^3x J_0^B(x)\,, \tag{39}$$

$$\hat{L} \;=\; \int d^3x J_0^L(x) \tag{40}$$

are time-independent. At the level of quantum fluctuations beyond the Born approximation these symmetries are, however, explicitly broken because eqs. (37), (38) no longer hold. This is seen as follows. Decompose the vector current

$$\bar{f}\gamma_\mu f \;=\; \bar{f}_L\gamma_\mu f_L \;+\; \bar{f}_R\gamma_\mu f_R\,, \tag{41}$$

where $f = q, \ell$, into its left- and right-handed pieces. Because of the clash between gauge and chiral symmetry at the quantum level the gauge-invariant chiral currents are not conserved: in the quantum theory the current-divergencies suffer from the Adler-Bell-Jackiw anomaly [21, 22]. For a gauge theory based

[3] Possible right-handed Dirac-neutrino degrees of freedom are of no concern to us here. Majorana neutrinos that lead to violation of lepton number – see Appendix B – would be evidence for physics beyond the SM.

on a gauge group G, which is a simple Lie group of dimension d_G, the anomaly equations for the L- and R-chiral currents $\bar{f}_L\gamma_\mu f_L$ and $\bar{f}_R\gamma_\mu f_R$ read

$$\begin{aligned} \partial^\mu \bar{f}_L\gamma_\mu f_L &= -c_L\frac{g^2}{32\pi^2}F^a_{\mu\nu}\tilde{F}^{a\mu\nu}\,, & (42)\\ \partial^\mu \bar{f}_R\gamma_\mu f_R &= +c_R\frac{g^2}{32\pi^2}F^a_{\mu\nu}\tilde{F}^{a\mu\nu}\,, & (43)\end{aligned}$$

where $F^{a\mu\nu}$ is the (non)abelian field strength tensor ($a = 1, ..., d_G$) and $\tilde{F}^{a\mu\nu} = \epsilon^{\mu\nu\alpha\beta}F^a_{\alpha\beta}/2$ is the dual tensor,[4] g denotes the gauge coupling, and the constants c_L, c_R depend on the representation which the f_L and f_R form. Let us apply (41) - (43) to the above baryon and lepton number currents of the SM where the gauge group is $SU(3)_c\times SU(2)_L\times U(1)_Y$. Because gluons couple to right-handed and left-handed quark currents with the same strength, we have $c_L^{QCD} = c_R^{QCD}$. Therefore J^B_μ has no QCD anomaly. However, the weak gauge bosons $W^a_\mu, a = 1, 2, 3,$ couple only to left-handed quarks and leptons, while the weak hypercharge boson couples to f_L and f_R with different strength. Hence $c_R^W = 0$ and $c_L^Y \neq c_R^Y$. Putting everything together one obtains

$$\partial^\mu J^B_\mu \;=\; \partial^\mu J^L_\mu \;=\; \frac{n_F}{32\pi^2}(-g_w^2 W^a_{\mu\nu}\tilde{W}^{a\mu\nu} + g'^2 B_{\mu\nu}\tilde{B}^{\mu\nu})\,, \qquad (44)$$

where $W^a_{\mu\nu}$ and $B_{\mu\nu}$ denote the $SU(2)_L$ and $U(1)_Y$ field strength tensors, respectively, g' is the $U(1)_Y$ gauge coupling, and $n_F = 3$ is the number of generations.

Eq. (44) implies that $\partial^\mu(J^B_\mu - J^L_\mu) = 0$. Thus the difference of the baryonic and leptonic charge operators $\hat{B} - \hat{L}$ remains time-independent also at the quantum level and therefore the quantum number

B - L is conserved in the SM.

How does B+L number violation come about? We note that the right hand side of eq. (44) can also be written as the divergence of a current K^μ:

$$r.h.s. \text{ of } (44) \;=\; n_F\partial_\mu K^\mu\,, \qquad (45)$$

where

$$K^\mu \;=\; -\frac{g_w^2}{32\pi^2}2\epsilon^{\mu\nu\alpha\beta}W^a_\nu(\partial_\alpha W^a_\beta + \frac{g_w}{3}\epsilon^{abc}W^b_\alpha W^c_\beta) + \frac{g'^2}{32\pi^2}\epsilon^{\mu\nu\alpha\beta}B_\nu B_{\alpha\beta}\,. \qquad (46)$$

[4]We use the convention $\epsilon_{0123} = +1$.

Let's integrate eq. (44), using (45), over space-time. Using Gauß's law we convert these integrals into integrals over a surface at infinity. Let's first do the surface integral for the right-hand side of (44). For hypercharge gauge fields B_μ with acceptable behaviour at infinity, that is, vanishing field strength $B_{\alpha\beta}$, the abelian part of K_μ makes no contribution to this integral. For the non-abelian gauge fields W^a_ν vanishing field strength implies that $2\epsilon_{\mu\nu\alpha\beta}\partial^\alpha W^{a\beta} = -g_w \epsilon_{\mu\nu\alpha\beta}\epsilon^{abc}W^{b\alpha}W^{c\beta}$ at infinity. Using this we obtain

$$\int d^4x\, \partial^\mu K_\mu \;=\; \frac{g_w^3}{96\pi^2}\int_{\partial V_4} dn^\mu\, \epsilon_{\mu\nu\alpha\beta}\epsilon^{abc}W^{a\nu}W^{b\alpha}W^{c\beta}\,. \tag{47}$$

Now we choose the surface ∂V_4 to be a large cylinder with top and bottom surfaces at t_f and t_i, respectively, and let the volume of the cylinder tend to infinity. Because $\partial_\mu K^\mu$ is gauge-invariant, we may choose a special gauge. Choose the temporal gauge condition, $W^a_0 = 0$. Then there is no contribution from the integral over the coat of the cylinder and we obtain

$$\int d^3x dt\, \partial_\mu K^\mu \;=\; N_{CS}(t_f) - N_{CS}(t_i) \equiv \Delta N_{CS}\,, \tag{48}$$

where

$$N_{CS}(t) \;=\; \frac{g_w^3}{96\pi^2}\int d^3x\, \epsilon_{ijk}\epsilon^{abc}W^{ai}W^{bj}W^{ck} \tag{49}$$

is the Chern-Simons number. This integral assigns a topological "charge" to a classical gauge field. Actually N_{CS} is not gauge invariant but ΔN_{CS} is. A nonabelian gauge theory like weak-interaction $SU(2)_L$ is topologically non-trivial, which is reflected by the fact that it has an infinite number of ground states whose vacuum gauge field configurations have different topological charges $\Delta N_{CS} = 0, \pm 1, \pm 2, \ldots$ Imagine the set of gauge and Higgs fields and consider the energy functional $E[field]$ that forms a hypersurface over this infinite-dimensional space. The ground states with different topological charge are separated by a potential barrier. In Fig. 4 a one-dimensional slice through this hypersurface is drawn. The direction in field space has been chosen such that the classical path from one ground state to another goes over a pass of minimal height.

Finally we perform the integral over the left-hand sides of (44) and get the result

$$\Delta\hat{B} \;=\; \Delta\hat{L} \;=\; n_F \Delta N_{CS}\,, \tag{50}$$

Figure 4: The periodic vacuum structure of the standard electroweak theory. The direction in field space has been chosen as described in the text. The schematic diagram shows the energy of static gauge and Highs field configurations $W_\mu^a(\mathbf{x}), \Phi(\mathbf{x})$. The integers are the Chern-Simons number N_{CS} of the respective zero-energy field configuration.

with $\Delta\hat{Q} \equiv \hat{Q}(t_f) - \hat{Q}(t_i)$, $Q = B, L$. Eq. (50) is to be interpreted as follows. As long as we consider small gauge field quantum fluctuations around the perturbative vacuum configuration $W_\mu^a = 0$ the right-hand side of (50) is zero, and B and L number remain conserved. This is the case in perturbation theory to arbitrary order where B- and L-violating processes have zero amplitudes. However, large gauge fields $W_\mu^a \sim 1/g_w$ with nonzero topological charge $\Delta N_{CS} = \pm 1, \pm 2, ...$ exist. As discovered by 't Hooft [23] they can induce transitions at the quantum level between fermionic states $|i, t_i>$ and $|f, t_f>$ with baryon and lepton numbers that differ according to the rule (50):

$$\Delta B \;=\; \Delta L \;=\; n_F \Delta N_{CS}\,. \tag{51}$$

This selection rule tells us that B and L must change by at least 3 units.[5] A closer inspection of the global $U(1)$ symmetries and associated currents shows that, in situations where fermion masses can be neglected, the selection rule can be refined: there is a change in quantum numbers by the same amount for every generation. Thus, e.g., $\Delta L_e = \Delta L_\mu = \Delta L_\tau = \Delta B/3 = \Delta N_{CS}$.

The dominant B- and L-violating transitions are between states $|i, t_i>$ and $|f, t_f>$ where $|\Delta B| = |\Delta L|$ changes by 3 units. At temperature $T = 0$, transitions with $|\Delta B| = |\Delta L| = 3$ are induced by the (anti)instanton [24], a gauge field which connects two vacuum configurations whose topological charge differ by ± 1. When put into the temporal gauge $W_0^a = 0$ then the instanton field $W_i^a(\mathbf{x}, t)$ approaches, for instance, $W_i^a = 0$ at $t_i \to -\infty$ and a topologically non-trivial vacuum configuration with $N_{CS} = 1$ at $t_f \to +\infty$, as indicated in Fig. 4. The corresponding amplitudes $< f, t_f | i, t_i >$ involve 9 left-handed quarks (right-handed $\bar{q}$) – where each generation participates with 3 different color states – and 3 left-handed leptons (right-handed $\bar{\ell}$), one of each generation. One of the possible amplitudes is depicted in Fig. 5. Hence we have, for instance, the anti-instanton induced reaction with

[5] Notice that, even after taking these non-perturbative effects into account, the SM still predicts the proton to be stable.

$\Delta B = \Delta L = -3$:

$$u + d \to \bar{d} + 2\bar{s} + \bar{c} + 2\bar{b} + \bar{t} + \bar{\nu}_e + \bar{\nu}_\mu + \bar{\nu}_\tau \,. \tag{52}$$

Figure 5: An example of a (B+L)-violating standard model amplitude. The arrows indicate the flow of the fermionic quantum numbers.

What is the probability for such a transition to occur? It is clear from Fig. 4 that it corresponds to a tunneling process. Thus it must be exponentially suppressed. The classic computation of ‘t Hooft [23, 25] implies, for energies $E_{c.m.}(ud) \lesssim \mathcal{O}(1\,TeV)$, a cross-section

$$\sigma_{\not B + \not L} \,\propto\, e^{-4\pi/\alpha_w} \,\sim\, 10^{-164} \,, \tag{53}$$

where $\alpha_w = g_w^2/4\pi \simeq 1/30$.

When the standard model is coupled to a heat bath of temperature T, the situation changes. As was first shown in [26] (see also [27]), at very high temperatures $T \gtrsim T_{EW} \sim 100$ GeV the B- and L-violating processes in the SM are fast enough to play a significant role in baryogenesis. In order to understand this we have again a look at Fig. 4. The ground states with different N_{CS} are separated by a potential barrier of minimal height

$$E_{sph}(T) \;=\; \frac{4\pi}{g_w} v_T f(\frac{\lambda}{g_w}) \,, \tag{54}$$

where $v_T \equiv < 0|\Phi|0 >_T$ is the vacuum expectation value (VEV) of the SM Higgs doublet field $\Phi(x)$ at temperature T. At $T = 0$ we have $v_{T=0} = 246$ GeV. The parameter f varies between $1.6 < f < 2.7$ depending on the value of the Higgs self-coupling λ, i.e., on the value of the SM Higgs mass. This yields $E_{sph}(T = 0) \simeq 8 - 13$ TeV. The subscript “sph” refers to the sphaleron, a gauge and Higgs field configuration of Chern-Simons number 1/2 (+ integer) which is an (unstable) solution of the classical field equations of the SM gauge-Higgs sector [28, 29]. These kind of field configurations (their locations are indicated by the dots in Fig. 4) lie on the respective minimum energy path from one ground state to another with different Chern-Simons number. Fig. 4 suggests that the rate of fermion-number non-conserving transitions will be proportional to the Boltzmann factor $\exp(-E_{sph}(T)/T)$

as long as the energy of the thermal excitations is smaller than that of the barrier, while unsuppressed transitions will occur above that barrier.

At this point we recall that the electroweak (EW) $SU(2)_L \times U(1)_Y$ gauge symmetry was unbroken at high temperatures, that is, in the early universe. The critical temperature T_{EW} where – running backwards in time – the transition from the broken phase with Higgs VEV $v_T \neq 0$ to the symmetric phase with $v_T = 0$ occurs is, in the SM, about 100 GeV. (A discussion of this transition will be given in the next section.) Hence the B- and L-violating transition rates of the SM will no longer be exponentially suppressed above this temperature. Detailed investigations have led to the following results:

• In the phase where the EW gauge is broken, i.e., $T < T_{EW} \sim 100$ GeV, the sphaleron-induced $\not{B} + \not{L}$ transition rate per volume V is given by (see, e.g., [4, 30])

$$\frac{\Gamma^{sph}_{\not{B}+\not{L}}}{V} = \kappa_1 \left(\frac{m_W}{\alpha_w T}\right)^3 m_W^4 \exp(-E_{sph}(T)/T)\,, \tag{55}$$

where $m_W(T) = g_w v_T/2$ is the temperature-dependent mass of the W boson and κ_1 is a dimensionless constant.

• The calculation of the transition rate in the unbroken phase is very difficult. On dimensional grounds we expect this rate per volume to be proportional to T^4. Recent investigations [31, 32] yield for $T > T_{EW} \sim 100$ GeV:

$$\frac{\Gamma^{sph}_{\not{B}+\not{L}}}{V} = \kappa_2\, \alpha_w^5 T^4\,, \tag{56}$$

with $\kappa_2 \sim 21$.

By comparing $\Gamma^{sph}_{\not{B}+\not{L}}$ above T_{EW} with the expansion rate H given in (30), we can assess whether the (B+L)-violating SM reactions, which conserve B-L, are fast enough to keep up with the expansion of the early universe in the radiation dominated epoch. From the requirement $\Gamma^{sph}_{\not{B}+\not{L}} >> H$ one obtains that these processes are in thermal equilibrium for temperatures

$$T_{EW} \;\sim\; 100\,\text{GeV} \;<\; T \;\; \lesssim\; 10^{12}\,\text{GeV}\,. \tag{57}$$

This result provides an important constraint on any baryogenesis mechanism which operates above T_{EW}. If the B- and L-violating interactions involved in this mechanism conserve B-L, then any excess of baryon and lepton number generated above T_{EW} will be washed out by the B- and L-nonconserving SM sphaleron-induced reactions. Hence baryogenesis scenarios above T_{EW} must be based on particle physics models that violate also B-L. Examples will be discussed in section 6.

4.3 Deviation from Thermal Equilibrium

A baryogenesis scenario based on the SM requires that the thermal instability must come from the electroweak phase transition: The expansion rate of the universe at temperatures, say, $T \lesssim 10^{12}$ GeV is too slow for causing a departure from local thermal equilibrium: the reaction rates of most of the SM particles, which are typically of the order of $\Gamma \sim \alpha_w^2 T$ or larger, are much larger than the expansion rate (30), even for extremely high temperatures. Thus, $\Delta B \neq 0$ was created at the EW transition it would be – if the phase change was strongly first order – frozen in during the later evolution of the universe, as the B- and L-violating reactions below T_{EW} would be strongly suppressed (see eq. (55) and below). As mentioned in the above the SM has all the required ingredients to satisfy the above requirement but fails at the quantitative level.

Before reviewing the results on the nature of the EW transition in the SM let us recall some basic concepts about phase transitions. Consider Fig. 6 where the pressure versus temperature phase diagram of water is sketched. We concentrate on the vapor $\leftrightarrow$ liquid transition. The curve to the right of the triple point is the so-called vapor-pressure curve. For values of p, T along this line there is a coexistence of the liquid and gaseous phases. A change of the parameters across this curve leads to a first order phase transition which becomes weaker along the curve. The endpoint corresponds to a second order transition. Beyond that point there is a smooth cross-over from the gaseous to the liquid phase and vice versa. The nature of a phase transition can

Figure 6: The phase diagram of water.

be characterized by an order parameter appropriate to the system. For the vapor-liquid transition the order parameter is the difference in the densities of water in the liquid and gaseous phase, $\tilde{\rho} = \rho_{liquid} - \rho_{vapour}$. In the case of a strong first order phase transition the order parameter has a strong discontinuity at the critical temperature T_c where the transition occurs: in the example at hand $\tilde{\rho}$ is very small in the vapor phase but it makes a sizeable jump at T_c because of the coexistence of both phases – see Fig. 7. That's what we need in a successful EW baryogenesis scenario! In case of a second order phase transition the order parameter changes also rapidly in the vicinity of T_c, but the change is continuous. In the cross-over region of the phase diagram the continuous change of $\tilde{\rho}$ as a function of T is less pronounced.

Figure 7: The behaviour of the order parameter $\tilde{\rho}$ in the case of a strong first order and a second order transition.

Figure 8: Dynamics of a first-order liquid-vapor phase transition: Formation and expansion of vapor bubbles.

So far to the statics of phase transitions. As to their dynamics, we know from experience how the first-order liquid-vapor transition evolves in time. Heating up water, vapor bubbles start to nucleate slightly below $T = T_c$ within the liquid. They expand and finally percolate above T_c. This is illustrated in Fig 8. Drawing the analogy to the early universe we should, of course, rather consider the cooling of vapor and its transition to a liquid through the formation of droplets.

A standard theoretical method to determine the nature of a phase transition in a classical system, like the vapor↔liquid or paramagnetic↔ferromagnetic transition is as follows. Let $\mathcal{H} = \mathcal{H}(s)$ be the classical Hamiltonian of the system, where $s(\mathbf{x})$ is a (multi-component) classical field. In the case of water $s(\mathbf{x})$ is the local density, while for a magnetic material $\vec{s}(\mathbf{x})$ denotes the three-component local magnetization. From the computation of the partition function Z we obtain the Helmholtz free energy $F = -T \ln Z$ from which the thermodynamic functions of interest can be derived. In particular we can compute the order parameter $s_{av} =< \sum_{\mathbf{x}} s(\mathbf{x}) >_T$ and study its behaviour as a function of temperature.

The investigation of the static thermodynamic properties of gauge field theories proceeds along the same lines. In the case of the standard electroweak theory the role of the order parameter is played by the VEV of the $SU(2)_L$ Higgs doublet field Φ. This becomes obvious when we recall the following. Experiments tell us that the $SU(2)_L \times U(1)_Y$ gauge symmetry is broken at $T = 0$. For the SM this means that the mass parameter in the Higgs potential must be tuned such that there is a non-zero Higgs VEV. On the other hand it was shown a long time ago [33] that at temperatures significantly larger than, say, the W boson mass the Higgs VEV is zero and the $SU(2)_L \times U(1)_Y$ gauge symmetry is restored. (This will be shown below.) Hence during the evolution of the early universe the Higgs field must have condensed at some $T = T_c$. The order of this phase transition is deduced from the behaviour of the Higgs VEV (and other thermodynamic quantities) around T_c.

Let's couple the standard electroweak theory to a heat bath of temperature T. The free energy $F = -T \ln Z$ is obtained from the Euclidean functional integral

$$F(J,T) \;=\; -T \ln \left[\int_\beta \mathcal{D}[\text{fields}] \exp(-\int_\beta dx (\mathcal{L}_{EW} + J \cdot \Phi)) \right] , \tag{58}$$

where $\mathcal{L}_{EW} = \mathcal{L}_{EW}(\Phi, W_\mu^a, B_\mu, q, \ell)$ denotes the Euclidian version of the electroweak SM Lagrangian, J is an auxiliary external field, $\beta = 1/T$,

$$\int_\beta dx = \int_0^\beta d\tau \int_V d^3x \; , \tag{59}$$

and the subscript β on the functional integral indicates that the bosonic (fermionic) fields satisfy (anti)periodic boundary conditions at $\tau = 0$ and $\tau = \beta$. From the free energy density $F(J,T)/V$ the effective potential $V_{eff}(\phi, T)$ is obtained by a Legendre transformation, where $\phi = \partial F / \partial J|_{J=0}$ is the expectation value of the Higgs doublet field, $\phi =< \Phi >_T$. (Actually in order to compare with numerical lattice calculations it is useful to employ a gauge-invariant order parameter.) Recall that the effective potential $V_{eff}(\phi, T)$ is the energy density of the system in that state $|a >_T$ in which the expectation value $< a|\Phi|a >_T$ takes the value ϕ. Hence by computing the stationary point(s), $\partial V_{eff}(\phi, T)/\partial \phi = 0$, the ground-state expectation value(s) ϕ=$< 0|\Phi|0 >_T$ of Φ at a given temperature T are determined. If at some $T = T_c$ two minima are found then this signals two coexisting phases and a first order phase transition.

5 Electroweak Baryogenesis & the SM Double Failure

5.1 Electroweak Baryogenesis

Let us describe how all the above ingredients requires for a viable model of baryogenesis combined into a coherent picture of producing baryon asymmetry. The required departure from thermal equilibrium is provided by the expansion of the Higgs bubbles, the true vacuum. When the bubble walls pass through a point in space, the classical Higgs fields change rapidly in the vicinity of such a point as do the other fields that couple to those fields. As far as different mechanisms are concerned, the following distinction is made in the literature:

• *Nonlocal Baryogenesis* [61], also called "charge transport mechanism", refers to the case where particles and antiparticles have CP non-conserving interactions with a bubble wall. This causes an asymmetry in a quantum number other than B number which is carried by (anti)particle currents into the unbroken phase. There this asymmetry is converted by the (B+L)-violating sphaleron processes into an asymmetry in baryon number. Some instant later the wall sweeps over the region where $\Delta B \neq 0$, filling space with Higgs fields that obey (79). Thus the B-violating back-reactions are blocked and the asymmetry in baryon mumber persists. The mechanism is illustrated in Fig. 9.

• *Local Baryogenesis* [59, 60] refers to case where the both the CP-violating and B-violating processes occur at or near the bubble walls.

In general, one may expect that both mechanisms were at work and $\Delta B \neq 0$ was produced by their joint effort. Which one of the mechanisms is more effective depends on the shape and velocity of the bubbles; i.e., on the underlying model of particle physics and its parameters.

Figure 9: Sketch of nonlocal electroweak baryogenesis.

We consider for simplicity only the so-called thin wall regime which applies if the mean free path of a fermion, l_ψ, is larger than the thickness l_{wall}. Then the quarks and leptons can be treated as free quasi-particles, interacting only in a small region with a non-trivial Higgs background field.[6] In its most naive form the thin wall approximation implies that we can parameterize the Higgs profile as a step function (in the expanding bubble rest frame). The expansion of the wall is supposed to be spherically symmetric and a 1-dimensional description should work. Fig. 10 shows left-handed and right-handed quarks[7] q_L and q_R incident from the unbroken phase, which hit the moving wall and are reflected by the Higgs bubble into right-handed and

[6]Quasi-particles are elementary excitation of a collective system in our case the thermal bath in the low momenta, small coupling limit. In our context, the "quasi-quark" description includes their interaction with thermal gauge and Higgs bosons in the plasma.

[7]In this subsection the symbols q_L, $\bar{q}_L$, etc. do *not* denote fields but particle states.

left-handed quarks, respectively.

Figure 10: Reflection of left- and right-handed quarks at a radially expanding Higgs bubble. The transmission of (anti)quarks from the broken into the symmetric phase is not depicted.

In the frame where the wall is at rest, the fermion interactions with the bubble wall are described by the Dirac equation following from [74]:

$$\begin{pmatrix} P_L + i\vec{\sigma}\cdot\vec{\partial} & \mathcal{M}\theta(z) \\ \mathcal{M}^\dagger\theta(z) & -(P_R + i\vec{\sigma}\cdot\vec{\partial}) \end{pmatrix} \Psi(z) = 0 \ , \tag{60}$$

where P_L and P_R are the symmetric-phase (outside the bubble) complex momenta of the left- and right-handed quasiparticles, including the imaginary damping terms,[8]

$$\begin{aligned} P_L &= 3(\omega - \Omega_L + i\gamma) && (61)\\ P_R &= -3(\omega - \Omega_R + i\gamma) \ . && (62) \end{aligned}$$

In the hot plasma of the early universe, left- and right-handed quasiparticles acquire distinct thermal masses Ω_L and Ω_R because only left-handed quarks couple to the thermal W bosons. The thermal masses also develop flavor dependence because different flavors couple with different strength to the thermal Higgs bosons. The thermal masses of the left-handed quasiparticles are given explicitly by [75, 72]

$$\Omega_L^2 = \frac{2\pi\alpha_s T^2}{3} + \frac{\pi\alpha_W T^2}{2}\left(\frac{3}{4} + \frac{\sin^2\theta_W}{36} + \frac{M_u^2 + V M_d^2 V^\dagger}{4M_W^2}\right) \ , \tag{63}$$

where the contributions from thermal interactions with gluons, electroweak gauge bosons, and Higgs bosons are all apparent. In this expression, V is the CKM matrix, $M_u = \mathrm{diag}(m_u, m_c, m_t)$, $M_d = \mathrm{diag}(m_d, m_s, m_b)$, and the

[8]The damping rate, γ, of a quasiparticle stands as a measure of its limited quantum coherence. The quasiparticle wave is rapidly damped because the components of the wave are rapidly absorbed by the plasma, and reemitted in a different region of the phase space. This decoherence phenomenon prevents components of the wave from participating in quantum interference over a distance longer than a *coherence length*, ℓ, whose magnitude is proportional to $1/\gamma$. Quantum interference is necessary for the generation of a CP-violating observable.

Yukawa couplings to the Higgs have been related to the masses of the quarks and the W in the broken phase. For right-handed up quarks,

$$\Omega_R^2 = \frac{2\pi\alpha_s T^2}{3} + \frac{\pi\alpha_W T^2}{2}\left(\frac{4\sin^2\theta_W}{9} + \frac{M_u^2}{M_W^2}\right) , \tag{64}$$

while for right-handed down quarks,

$$\Omega_R^2 = \frac{2\pi\alpha_s T^2}{3} + \frac{\pi\alpha_W T^2}{2}\left(\frac{\sin^2\theta_W}{9} + \frac{M_d^2}{M_W^2}\right) . \tag{65}$$

Figure 11: The reflection $q_L \to q_R$ and the P-, CP-, and CPT-transformed process.

Instead of performing this calculation let's make a few general considerations. Let's have a look at the scattering process depicted in Fig. 11, where, in the symmmetric phase $z > z_0$, a left-handed quark q_L (having momentum $k_z < 0$) is reflected at the wall into a right-handed q_R. Notice that conservation of electric charge guarantees that a quark is reflected into a quark and not an antiquark. Angular momentum conservation tells us that q_L is reflected as q_R and vice versa. Also shown are the situations after a parity transformation (followed by a rotation around the wall axis in the paper plane orthogonal to the z axis by an angle π), and subsequent charge conjugation C, and time reversal (T) transformations. The analogous figure can be drawn for antiquark reflection. These figures immediately tell us that if CP were conserved then

$$\mathcal{R}_{L\to R} \;=\; \mathcal{R}_{\bar{R}\to\bar{L}} \;, \qquad \mathcal{R}_{R\to L} \;=\; \mathcal{R}_{\bar{L}\to\bar{R}} \tag{66}$$

would hold. (The subscripts $\bar{R}$, $\bar{L}$ denote right-handed and left-handed antiquarks, respectively.) CPT invariance, which is respected by the particle physics models we consider, implies

$$\mathcal{R}_{L\to R} \;=\; \mathcal{R}_{\bar{L}\to\bar{R}} \;, \qquad \mathcal{R}_{R\to L} \;=\; \mathcal{R}_{\bar{R}\to\bar{L}} \;. \tag{67}$$

The charge transport mechanism [62] works as follows. At some initial time we have equal numbers of quarks and antiquarks in the unbroken phase, in particular equal numbers of q_L and $\bar{q}_R$ and q_R and $\bar{q}_L$, respectively, which hit the expanding bubble wall. Reflection converts $q_L \to q_R$, $\bar{q}_R \to \bar{q}_L$, $q_R \to q_L$, and $\bar{q}_L \to \bar{q}_R$ and the particles move back to the region where the Higgs fields are zero. Because the interaction with the bubble wall is assumed to be CP-violating, the relations (66) for the reflection probabilities no longer hold. Actually, for the CP asymmetry

$$\Delta\mathcal{R}_{CP} \equiv \mathcal{R}_{\bar{L}\to\bar{R}} - \mathcal{R}_{R\to L} = \mathcal{R}_{L\to R} - \mathcal{R}_{\bar{R}\to\bar{L}} \tag{68}$$

to be non-zero it is essential that $m_q(z)$ has a z dependent phase. The reflection coefficients are built up by the coherent superposition of the amplitudes for (anti)quarks to reflect at some point z in the bubble. When the phases vary with z the reflection probabilities $\mathcal{R}_{\bar{L}\to\bar{R}}$ and $\mathcal{R}_{R\to L}$ differ from each other.

Notice that at this stage the net baryon number is still zero. This is because the difference J_q^L of the fluxes of $\bar{q}_R$ and q_L, injected from the wall back into the symmetric phase, is equal to J_q^R which we define as the difference of the fluxes of q_R and $\bar{q}_L$, as should be clear from (67). However, the (B+L)-violating weak sphaleron interactions, which are unsuppressed in the symmetric phase away from the wall, act *only* on the (massless) left-handed quarks and right-handed antiquarks. For instance, the reaction (52) decreases the baryon number by 3 units, while the corresponding reaction with right-handed antiquarks in the initial state increases B by the same amount. Thus if the functional form of the CP-violating part $\mathrm{Im}[m_q(z)M_q^*]$ of the background Higgs field is such that $J_q^L > 0$ then, after the anomalous weak interactions took place, there are more left-handed quarks than right-handed antiquarks. The fluxes of the reflected $\bar{q}_L$ and q_R are not affected by the anomalous weak sphaleron interactions. Adding it all up we see that some place away from the wall a net baryon number $\Delta B > 0$ is produced. Some instant later the expanding bubble sweeps over that region and the associated non-zero Higgs fields strongly suppress the (B+L)-violating back reactions that would wash out ΔB. Thus the non-zero B number produced before is frozen in.

We must also take into account that (anti)particles in the broken phase can be transmitted into the symmetric phase and contribute to the (anti)particle fluxes discussed above. Using CPT invariance and unitarity, we find that the probabilities for transmission and the above reflection probabilities are related:

$$\mathcal{T}_{L\to L} = 1 - \mathcal{R}_{R\to L} = 1 - \mathcal{R}_{\bar{R}\to\bar{L}} = \mathcal{T}_{\bar{L}\to\bar{L}} \,, \tag{69}$$

$$\mathcal{T}_{R\to R} \;=\; 1-\mathcal{R}_{L\to R} \;=\; 1-\mathcal{R}_{\bar{L}\to\bar{R}} \;=\; \mathcal{T}_{\bar{R}\to\bar{R}}\,. \tag{70}$$

We can now write down a formula for the current J_q^L, which we define as the difference of the fluxes of $\bar{q}_R$ and q_L, injected from the wall into the symmetric phase. The contribution from the reflected particles involves the term $\Delta\mathcal{R}_{CP} f_s$ where f_s is the free-particle Fermi-Dirac phase-space distribution of the (anti)quarks in the region $z > z_0$ that move to the left, i.e., towards the wall. The contribution from the (anti)quarks which have returned from the broken phase involves $(\mathcal{T}_{\bar{R}\to\bar{R}} - \mathcal{T}_{L\to L}) f_b = -\Delta\mathcal{R}_{CP} f_b$, where f_b is the phase-space distribution of the transmitted (anti)quarks that move to the right. The reference frame is the wall frame. Notice that f_s and f_b differ because the wall moves with a velocity $v_{wall} \neq 0$ – in our convention from left to right. The current J_q^L is given by

$$J_q^L \;=\; \int_{k_z<0} \frac{d^3k}{(2\pi)^3}\frac{|k_z|}{E}(f_s \;-\; f_b)\Delta\mathcal{R}_{CP}\,, \tag{71}$$

where $|k_z|/E$ is the group velocity. The current is non-zero because two of the three Sakharov conditions, CP violation and departure from thermal equilibrium, are met. The current would vanish if the wall were at rest in the plasma frame – which leads to thermal equilibrium –, because then $(f_s - f_b) = 0$.

The current $J^L = \sum_\psi J_\psi^L$ is the source for baryogenesis some distance away from the wall as sketched above. We skip the analysis of diffusion and of the conditions under which local thermal equilibrium is maintained in front of the bubble wall [62, 64, 65]. This determines the densities of the left-handed quarks and right-handed antiquarks and their associated chemical potentials. The rate of baryon production per unit volume is determined by the equation [64]

$$\frac{dn_B}{dt} \;=\; -n_F\frac{\hat{\Gamma}_{sph}}{2T}\sum_{generations}(3\hat{\mu}_{U_L} + 3\hat{\mu}_{D_L} + \hat{\mu}_{\ell_L} + \hat{\mu}_{\nu_L})\,, \tag{72}$$

where $n_F = 3$, $U = u, c, t$, $D = d, s, b$, $\hat{\Gamma}_{sph}$ is the sphaleron rate per unit volume, which in the unbroken phase is given by eq. (56). Here the $\hat{\mu}_i = \mu_i - \bar{\mu}_i = 2\mu_i$ denote the difference between the respective particle and antiparticle chemical potentials. For a non-interacting gas of massless fermions i, the relation between $\hat{\mu}_i$ and the asymmetry in the corresponding particle and antiparticle number densities is $n_i - \bar{n}_i \simeq g\hat{\mu}_i T^2/12$, where $g = 1$ for a left-handed lepton and $g = 3$ for a left-handed quark because of three colors. In the symmetric phase (72) then reads

$$\frac{dn_B}{dt} \; = \; -6n_F \frac{\hat{\Gamma}_{sph}}{T^3}(3B_L + L_L) \,, \tag{73}$$

where B_L and L_L denote the total left-handed baryon and lepton number densities, respectively. The factor of 3 comes from the definition of baryon number, which assigns baryon number 1/3 to a quark. This equation tells us what we already concluded qualitatively above: baryon rather than antibaryon production requires a negative left-handed fermion number density, i.e., a positive flux J_q^L. The total flux $\sum_\psi J_\psi^L$ determines the left-handed fermion number density. Then eq. (73) yields n_B and, using $s = 2\pi^2 g_{*s} T^3/45$ with $g_{*s} \simeq 110$ a prediction for the baryon-to-entropy ratio is obtained.

5.2 The SM Fails to Produce Deviation from Equilibrium

Let us now discuss the effective potential of the SM. At $T = 0$ the tree-level effective potential is just the classical Higgs potential $V_{tree} = -\mu^2(\Phi^\dagger\Phi) + \lambda(\Phi^\dagger\Phi)^2$. Choosing the unitary gauge, $\Phi^{unitary} = (0, \phi/\sqrt{2})$ with $\phi \geq 0$ we have

$$V_{tree}(\phi) \; = \; -\frac{\mu^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4 \,, \tag{74}$$

where $\lambda > 0$ and, by assumption, $\mu^2 > 0$ in order that the Higgs field is non-zero in the state of minimal energy: $\phi_0 \equiv < 0|\Phi|0 >_{T=0} \equiv v_{T=0}/\sqrt{2} = \sqrt{\mu^2/\lambda}$, and $v_{T=0}$ is fixed by, e.g., the experimental value of the W boson mass to $v_{T=0} = 246$ GeV. The mass of the SM Higgs boson is given by

$$m_H \; = v_{T=0}\sqrt{2\lambda} + \text{quantum corrections} \,. \tag{75}$$

The experiments at LEP2 have established the lower bound $m_H > 114$ GeV [34]. Hence the SM Higgs self-coupling $\lambda > 0.33$.

At $T \neq 0$ the SM effective potential is computed at the quantum level as outlined above. Because the gauge coupling g' and the Yukawa couplings of quarks and leptons $f \neq t$ (t denotes the top quark) to the Higgs doublet Φ are small, the contributions of the hypercharge gauge boson and of $f \neq t$ may be neglected. This is usually done in the literature. Let us first discuss, for illustration, the effective potential computed to one-loop approximation for the now obsolete case of a very light Higgs boson. For high temperatures V_{eff} is given by

$$V_{eff}(\phi, T) \ = \frac{1}{2}a(T^2 - T_1^2)\phi^2 - \frac{1}{3}bT\phi^3 + \frac{1}{4}\lambda\phi^4 \ , \tag{76}$$

where

$$a \ = \ \frac{3}{16}g_w^2 + (\frac{1}{2} + \frac{m_t^2}{m_H^2})\lambda \ , \ b \ = \ 9\frac{g_w^3}{32\pi} \ , \ T_1 \ = \ \frac{m_H}{2\sqrt{a}} \ , \tag{77}$$

and m_t is the mass of the top quark. The term cubic in ϕ is due to fluctuations at $T \neq 0$. If the Higgs boson was light the quartic term would be small. Inspecting eq. (76) we recover the result quoted above that at high temperatures the Higgs field is zero in the ground state. When the temperature is lowered we find that at $T_c = T_1/\sqrt{1 - 2b^2/(9a\lambda)} > T_1$ a first order phase transition occurs: the effective potential V_{eff} has two energetically degenerate minima: one at $\phi = 0$ and the other at

$$v_{T_c} \ \equiv \ \phi_{crit} \ = \ \frac{2b}{3\lambda}T_c \ , \tag{78}$$

separated by an energy barrier, see Fig. 12. At T_c the free energy of the symmetric and of the broken phase are equal; however, the universe remains for a while in the symmetric phase because of the energy barrier. As the universe expands and cools down further, bubbles filled with the Higgs condensate start to nucleate at some temperature below T_c. These bubbles become larger by releasing latent heat, percolate, and eventually fill the whole volume at $T = T_1$. Bubble nucleation and expansion are non-equilibrium phenomena which are difficult to compute.

Figure 12: Behaviour of V_{eff} in the case of a first order phase transition.

Fig. 13 shows the behaviour of $V_{eff}(\phi, T)$ in the case of a second order phase transition. In this case there are no energetically degenerate minima separated by a barrier at $T = T_c$, i.e., no bubble nucleation and expansion. The Higgs field gradually condenses uniformly at $T \lesssim T_c$ and grows to its present value as the system cools off.

The value of the critical temperature depends on the parameters of the respective model and is obtained by detailed computations (see the references given below). Nevertheless, we may use the above formula for T_c for

Figure 13: Behaviour of V_{eff} in the case of a second order phase transition.

a crude estimate and obtain $T_c \sim 70$ GeV for $m_H = 100$ GeV. (For a more precise value, see below.) With eqs. (12) and (30) we then estimate that the EW phase transition took place at a time $t_{EW} \sim 5 \times 10^{-11}$ s after the big bang. This implies that the causal domain, the diameter of which is given by $d_H(t) = 2t$ in the radiation-dominated era, was then of the order of a few centimeters.

Back to baryogenesis. It should be clear now why a strong first order EW phase transition is required. In this case the time scale associated with the nucleation and expansion of Higgs bubbles is comparable with the time scales of the particle reactions. This causes a departure from thermal equilibrium. How is this to be quantified? Let's consider one of the bubbles with $v_T \neq 0$ which, after expansion and percolation, eventually become our world. The bubble must get filled with more quarks than antiquarks such that $n_B/s \sim 10^{10}$ and this ratio remains conserved. This means that baryogenesis has to take place outside of the bubble while the sphaleron-induced (B+L)-violating reactions must be strongly suppressed within the bubble. In order that the sphaleron rate, which in the broken phase is given by eq. (55), $\Gamma^{sph}_{\not B + \not L} \propto \exp\left(-4\pi f v_T / g_w T\right)$, is practically switched off, the order parameter must jump at T_c, from $\phi = 0$ in the symmetric phase to a value v_{T_c} in the broken phase such that

$$\frac{v_{T_c}}{T_c} \gtrsim 1\,. \tag{79}$$

This is the condition for a first order transition to be strong.

In view of the experimental lower bound $m_H^{SM} > 114$ GeV, the formulae (76), (77) for V_{eff} which are valid only for a very light Higgs boson no longer apply. Nevertheless, eq. (78) shows that the discontinuity gets weaker when the Higgs mass is increased. The strength of the electroweak phase transition has been studied for the SM $SU(2)$ gauge-Higgs model as a function of the Higgs boson mass with analytical methods [35], and numerically with 4-dimensional [36] and 3-dimensional [37] lattice methods. These results quantify the qualitative features discussed above: the strength of the phase transition changes from strongly first order ($m_H \lesssim 40$ GeV) to weakly first order as the Higgs mass is increased, ending at $m_H \simeq 73$ GeV [38, 39, 40] where the phase transition is second order (cf. the liquid-vapor transition discussed above). The corresponding critical temperature is $T_c \simeq 110$ GeV [41]. For larger values of m_H there is a smooth cross-over between the symmetric and the broken phase.

Thus the result of the LEP2 experiments, m_H^{SM} >114 GeV, leads to the following conclusion: if the SM Higgs mechanism provides the correct description of electroweak symmetry breaking then the EW phase transition in the early universe does not provide the thermal instability required for baryogenesis. The B-violating sphaleron processes are only adiabatically switched off during the transition from $T > T_c$ to $T < T_c$; they are still thermal for $T \lesssim T_c$. Thus the standard model of particle physics cannot explain the BAU η – irrespective of the role that SM CP violation may play in this game.

5.3 The SM Fails to Induce Sizable CP Violation

The role played in baryogenesis scenarios by the SM source of CP violation, the KM phase $\delta_{\rm CKM}$ is crucial since it yields an excess of LH quark current which biases the sphaleron rate. Recall the following well-known features of KM CP violation. All CP-violating effects, which are generated by the KM phase in the charged weak quark current couplings to W bosons, are proportional to the invariant [70, 71]:

$$J_{CP} \;=\; \prod_{\substack{i>j\\u,c,t}}(m_i^2-m_j^2)\prod_{\substack{i>j\\d,s,b}}(m_i^2-m_j^2) \quad {\rm Im}\,Q\,, \tag{80}$$

where i,j = 1,2,3 are generation indices, m_u, etc. denote the respective quark masses, and ${\rm Im}\,Q$ is the imaginary part of a product of 4 CKM matrix elements, which is invariant under phase changes of the quark fields. There are a number of equivalent choices for ${\rm Im}\,Q$. A standard choice is

$${\rm Im}\,Q = {\rm Im}(V_{ud}V_{cb}V_{ub}^*V_{cd}^*)\,. \tag{81}$$

Inserting the moduli of the measured CKM matrix elements yields $|{\rm Im}\,Q|$ smaller than 2×10^{-5}, even if KM CP violation is maximal; i.e., $\delta_{\rm CKM}=\pi/2$ in the KM parameterization of the CKM matrix. We may write ${\rm Im}\,Q \simeq 2\times10^{-5}\sin\delta_{\rm CKM}$. As far as the SM at temperatures $T\neq0$ is concerned, the CP symmetry can be broken only in regions of space where the gauge symmetry is also broken, or at the boundaries of such regions, because $J_{CP}\neq 0$ requires non-degenerate quark masses. Imagine the EW transition would be first order. The question is then: is the KM source of CP violation strong enough to create a sufficiently large asymmetry $\Delta\mathcal{R}_{CP}$ in the probabilities for reflection of (anti)quarks at the expanding wall as discussed above? It is clear that $\Delta\mathcal{R}_{CP}$ must be proportional to a dimensionless quantity of the form J_{CP}/D, where D has mass dimension 12. Reflection of quarks and antiquarks at a bubble wall is not CKM-suppressed; hence D does not contain small CKM matrix elements. If one recalls that in the symmetric phase the quark masses and thus J_{CP} vanish, it seems reasonable to treat the quark masses (perhaps not the top quark mass) as a perturbation. In the massless limit the mass scale of the theory at the EW transition is then given by the critical temperature $T_c \sim 100$ GeV. Thus one gets for the dimensionless measure of CP violation:

$$d_{CP} \equiv \frac{J_{CP}}{T_c^{12}} \sim 10^{-19} \tag{82}$$

as an estimate of $\Delta\mathcal{R}_{CP}$. A semi-analytical analysis shows that the above naive estimation yield the correct suppression factor, orders of magnitude too small to account for the observed n_B/s [74].

Appendix A

Let $q(\mathbf{x}, t)$ be the Dirac field operator that describes a quark of flavor $q = u, ..., t$, $q^\dagger(\mathbf{x}, t)$ denotes its Hermitean adjoint, and $\bar{q} = q^\dagger\gamma^0$. The baryon number operator (39) is

$$\hat{B} = \frac{1}{3}\sum_q \int d^3x : q^\dagger(\mathbf{x}, t)q(\mathbf{x}, t) : , \tag{83}$$

and the colons denote normal ordering. Let C, P denote the unitary and T the anti-unitary operator which implement the charge conjugation, parity, and time reversal transformations, respectively, in the space of states. Their action on the quark fields is, adopting standard phase conventions,

$$Pq(\mathbf{x}, t)P^{-1} = \gamma^0 q(-\mathbf{x}, t)\,, \tag{84}$$
$$Pq^\dagger(\mathbf{x}, t)P^{-1} = q^\dagger(-\mathbf{x}, t)\gamma^0\,, \tag{85}$$
$$Cq(\mathbf{x}, t)C^{-1} = i\gamma^2 q^\dagger(\mathbf{x}, t)\,, \tag{86}$$
$$Cq^\dagger(\mathbf{x}, t)C^{-1} = iq(\mathbf{x}, t)\gamma^2\,, \tag{87}$$
$$Tq(\mathbf{x}, t)T^{-1} = -i\; q(\mathbf{x}, -t)\gamma_5\gamma^0\gamma^2\,, \tag{88}$$
$$Tq^\dagger(\mathbf{x}, t)T^{-1} = -i\gamma^2\gamma^0\gamma_5 q^\dagger(\mathbf{x}, -t)\,, \tag{89}$$

where γ^0, γ^2, and $\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3$ denote Dirac matrices. Then

$$P : q^\dagger(\mathbf{x},t)q(\mathbf{x},t) : P^{-1} \ = : q^\dagger(-\mathbf{x},t)q(-\mathbf{x},t) : , \tag{90}$$

$$C : q^\dagger(\mathbf{x},t)q(\mathbf{x},t) : C^{-1} \ = : q(\mathbf{x},t)q^\dagger(\mathbf{x},t) : = \ - : q^\dagger(\mathbf{x},t)q(\mathbf{x},t) : , \tag{91}$$

$$T : q^\dagger(\mathbf{x},t)q(\mathbf{x},t) : T^{-1} \ = : q^\dagger(\mathbf{x},-t)q(\mathbf{x},-t) : \ . \tag{92}$$

With these relations we immediately obtain:

$$P\hat{B}P^{-1} \ = \ \hat{B} \, , \tag{93}$$

$$C\hat{B}C^{-1} \ = \ -\hat{B} \, . \tag{94}$$

As shown in section 4 the baryon number operator is time-dependent due to non-perturbative effects. Using translation invariance we have $\hat{B}(t) = e^{iHt}\hat{B}(0)e^{-iHt}$, where H is the Hamiltonian of the system. The operator $\hat{B}(0)$ is even with respect to T and odd with respect to $\Theta \equiv CPT$:

$$\Theta\hat{B}(0)\Theta^{-1} \ = \ -\hat{B}(0) \, . \tag{95}$$

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