Decay of heavy Majorana neutrinos using the full Boltzmann equation including its implications for leptogenesis

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Abstract. We have studied the two-body production and decay of a heavy, right-handed neutrino to two light states using the full Boltzmann equation instead of the usual integrated Boltzmann equation which assumes kinetic equilibrium of all species. Decays and inverse decays are inefficient for thermalising the distribution function of the heavy neutrino and in some parameter ranges there can be very large deviations from kinetic equilibrium. This leads to substantial numerical differences between the two approaches. Furthermore we study the impact of this difference on the lepton asymmetry production during leptogenesis and find that in the strong washout regime the final asymmetry is changed by 15-30% when the full Boltzmann equation is used.

1. Introduction

Leptogenesis is perhaps the most attractive model for generating the matter-antimatter asymmetry in our Universe [1] after inflation. The process generates a lepton asymmetry via the production and subsequent decay of a heavy Majorana neutrino. This lepton asymmetry is partially converted to a baryon asymmetry via sphaleron processes [2] which break both B and L, but conserve $B-L$.

The fraction of $B-L$ that ends up in B by the sphaleron processes is given by $a_{sph} = 28/79$ giving $n_B = a_{sph} n_{B-L} = -a_{sph} n_L$ where the n_{B-L} is created by leptogenesis which gives a non zero value of L (see for instance [3]).

In its simplest form (within the context of the see-saw model) leptogenesis consists of adding three heavy right-handed neutrinos to the standard model. In the hierarchical limit one of these right handed neutrinos, ν_{R_1}, is much lighter than the other two and the leptogenesis mechanism consists essentially of the process

$$\nu_{R_1} \rightarrow \begin{cases} \nu_L + \phi \\ \overline{\nu_L} + \phi \end{cases}, \tag{1}$$

where ν_L is a light, left-handed neutrino and ϕ the Higgs. All light flavours behave identically, so you can think of the other light flavours as included as a factor of 3 in the decay rate. Because of loop corrections there can be CP violation in the decay, usually quantified by the asymmetry parameter ϵ. In the following we denote the heavy neutrino by R, the light by L, and the Higgs by H.

This model has been studied extensively in the literature, including deviations from the hierarchical limit, thermal corrections etc [3–17]. However, all studies have used the integrated Boltzmann equation to follow the evolution of the heavy neutrino number density and the lepton asymmetry. This approach assumes Maxwell-Boltzmann statistics for all particles as well as kinetic equilibrium for the heavy species. This assumption is normally justified in freeze-out calculations where elastic scattering is assumed to be much faster than inelastic reactions. However, in the present context, kinetic equilibrium in the heavy species would have to be maintained by the decays and inverse decays alone. Therefore it is not obvious that the integrated Boltzmann equation is always a good approximation. Furthermore $1 \leftrightarrow 2$ processes are generally inefficient for thermalization compared with $2 \leftrightarrow 2$ processes. For $2 \leftrightarrow 2$ deviations from kinetic equilibrium are always of order 20% or less [18], but for $1 \leftrightarrow 2$ processes they can be very large (see for instance Refs. [19–22] for a case where deviation from equilibrium is extremely large).

In this paper we investigate how the use of the full Boltzmann equation affects the final lepton asymmetry in a simplified model with only decays and inverse decays and resonant scattering. We will return to the point of resonant scattering in due course. We find that when $T \sim m_R$ the difference can be very large. However, at small temperature where the inverse decay dominates the difference decreases in magnitude to about 20%.

2. The Boltzmann equation

Here we study only the two-body decay of a heavy right-handed neutrino to a light neutrino plus a Higgs. We do not include thermal corrections to the particle masses, so that for instance the process $H \to LR$ is not kinematically possible. We assume that the asymmetry, represented by

$$\epsilon = -\frac{\Gamma - \bar{\Gamma}}{\Gamma + \bar{\Gamma}} \tag{2}$$

is small, so that when we calculate anything with R, we can assume identical distributions of L and $\bar{L}$. We consider only initial zero abundance of R. We use only single particle distribution functions, in which case the Boltzmann equation for the heavy species can be written as [4, 23]

$$\begin{aligned}\frac{\partial f_R}{\partial t} - pH\frac{\partial f_R}{\partial p} = \frac{1}{2E_R}\int \frac{d^3p_L}{2E_L(2\pi)^3}\frac{d^3p_H}{2E_H(2\pi)^3}(2\pi)^4\delta^4(p_R - p_L - p_H) \\ \times \left[f_H f_L(1-f_R)(|M_{HL\to R}|^2 + |M_{H\bar{L}\to R}|^2) \right. \\ \left. -f_R(1-f_L)(1-f_H)(|M_{R\to HL}|^2 + |M_{R\to H\bar{L}}|^2)\right],\end{aligned} \tag{3}$$

and for the light neutrino it is

$$\begin{aligned}\frac{\partial f_L}{\partial t} - pH\frac{\partial f_L}{\partial p} = \frac{1}{2E_L}\int \frac{d^3p_R}{2E_R(2\pi)^3}\frac{d^3p_H}{2E_H(2\pi)^3}(2\pi)^4\delta^4(p_R - p_L - p_H) \\ \times \left[-f_H f_L(1-f_R)|M_{HL\to R}|^2 \right. \\ \left. +f_R(1-f_L)(1-f_H)|M_{R\to HL}|^2\right],\end{aligned} \tag{4}$$

with a similar equation for $\bar{L}$. The interesting Boltzmann equations for the present purpose are those for R and for $B-L$. H and $L, \bar{L}$ have gauge interactions which are very fast. This means that H can be described by a distribution in chemical equilibrium, and that $L, \bar{L}$ can be described as distributions in kinetic equilibrium,

$$f_R = (1 + e^{p_R/T})^{-1} \tag{5}$$

$$f_L = (1 + e^{(p_L-\mu)/T})^{-1} \tag{6}$$

$$f_{\bar{L}} = (1 + e^{(p_{\bar{L}}+\mu)/T})^{-1}, \tag{7}$$

with $\mu/T = 3(n_L - n_{\bar{L}})/T^3 + \mathcal{O}((\mu/T)^3)$.

Using CPT-invariance, following the idea of [23], we find

$$\begin{aligned}|M_{R\to HL}|^2 = |M_{H\bar{L}\to R}|^2 = 1-\epsilon \\ |M_{R\to H\bar{L}}|^2 = |M_{HL\to R}|^2 = 1+\epsilon\end{aligned} \tag{8}$$

This simplifies the Boltzmann equations.

For the heavy neutrino the Boltzmann equation can be simplified to [20]

$$\begin{aligned}\frac{\partial f_R}{\partial t} - pH\frac{\partial f_R}{\partial p} \\ = \frac{m_R\Gamma_{\rm tot}}{E_R p_R}\int_{(E_R-p_R)/2}^{(E_R+p_R)/2} dp_H \left[f_H f_L(1-f_R) - f_R(1-f_L)(1-f_H)\right],\end{aligned} \tag{9}$$

where $\Gamma_{\rm tot} = \Gamma_{R\to LH} + \Gamma_{R\to \bar{L}H} = \Gamma + \bar{\Gamma}$ is the total rest frame decay rate. The corresponding equation for $L - \bar{L}$ ‡ is

$$\frac{\partial(f_L - f_{\bar{L}})}{\partial t} - pH\frac{\partial(f_L - f_{\bar{L}})}{\partial p} \tag{10}$$

$$= -\frac{m_R\Gamma}{2E_Lp_L}\int_{\frac{m_R^2}{4p_L}+p_L}^{\infty} dE_R\Big[(f_H + f_R)F^- + \epsilon\Big(-2(1+f_H)f_R + (f_R - f_H(1-2f_R))F^+\Big)\Big], \tag{11}$$

to first order in ϵ and with

$$F^+ = f_L + f_{\bar{L}} = \frac{2}{1+e^{p/T}} + \mathcal{O}((\mu/T)^2) \tag{12}$$

$$F^- = f_L - f_{\bar{L}} = \frac{2e^{p/T}}{(1+e^{p/T})^2}\frac{\mu}{T} + \mathcal{O}((\mu/T)^3) \tag{13}$$

$$\tag{14}$$

Since we have so far only included $2 \leftrightarrow 1$ processes, Eq. 11 suffers from the well-known problem of lepton asymmetry generation even in equilibrium [23]. To remedy this problem the resonant part of the $LH \leftrightarrow \bar{L}H$ must be included. To lowest order in ϵ this amounts to adding the term $2\epsilon(1-f_R)f_HF^+$ in Eq. 11 [8], so that the final form of the equation for the lepton asymmetry is

$$\frac{\partial(f_L - f_{\bar{L}})}{\partial t} - pH\frac{\partial(f_L - f_{\bar{L}})}{\partial p} = -\frac{m_R\Gamma}{2E_Lp_L}\int_{\frac{m_R^2}{4p_L}+p_L}^{\infty} dE_R\Big[(f_H + f_R)(F^- + \epsilon F^+) - 2\epsilon f_R(1+f_H)\Big], \tag{15}$$

This equation does not exhibit any lepton asymmetry generating behaviour in thermal equilibrium because $[(f_H + f_R)(F^- + \epsilon F^+) - 2\epsilon f_R(1+f_H)] = 0$ explicitly.

To first order in μ/T equations Eq. 10 and 16 can be easily integrated numerically for given values of m_R and Γ. If Maxwell-Boltzmann statistics and kinetic equilibrium for R are assumed the equations can be further simplified. The integrated Boltzmann equations are then

$$\dot{n}_R + 3Hn_R = -\langle\Gamma\rangle(n_R - n_{R,\rm eq}) \tag{16}$$

$$\dot{n}_{L-\bar{L}} + 3Hn_{L-\bar{L}} = \epsilon\langle\Gamma\rangle(n_R - n_{R,\rm eq}) + n_{L-\bar{L}}\frac{\Gamma}{4}\mathcal{K}_1(m_R/T)\frac{m_R^2}{T^2}, \tag{17}$$

where $z \equiv \frac{m_R}{T}$ and $\langle\Gamma\rangle = (\Gamma+\bar{\Gamma})\mathcal{K}_1(z)/\mathcal{K}_2(z)$ is the thermally averaged total decay rate.

These equations are the ones normally used in leptogenesis calculations for the simplest case of one massive and one light neutrino. However, compared with Eqs. (10-16) they involve the approximation of assuming Maxwell-Boltzmann statistics and kinetic equilibrium. Particularly for the case where $\Gamma/H(T = m_R) \sim 1$ this approximation is not necessarily good.

‡ Note that we ignore the sphaleron L to B conversion during the decay process because we track only $L - \bar{L}$. Including it would have only a modest effect on our numerical results.

When decays and inverse decays are included (as well as gauge interactions of L and H which establish kinetic equilibrium for $L, \bar{L}$, and H individually, but which conserve $L - \bar{L}$) there are essentially only two interesting parameters. All terms that contribute to the development of densities are proportional to either H or Γ. This competition can be parameterised by the single parameter $K = \Gamma/H(T = m_R)$, the decay parameter. The other important parameter is the net decay asymmetry ϵ.

In the present study we assume that the heavy neutrinos start with zero abundance, i.e. that they are not equilibrated at high temperatures through other interactions. We refer the reader to [8] for a thorough discussion of the various possibilities for initial R abundance.

In Figs. 1-3 we show a calculation of n_R and the net asymmetry for different values of K. At high temperatures the equilibration rate of R is much higher when the full Boltzmann equation is used. The main reason for this can be seen in Fig. 4. When $z < 1$ the low momentum states of R are populated very efficiently, while there is little population of high momentum modes. This in turn means that the inverse decay $HL \to R$ is suppressed relative to the decay process $R \to HL$.

To quantify this, we want to define a momentum dependant inverse decay rate,

$$\Gamma(p_R) \equiv \frac{1}{f_{R,eq}(p_R)} \frac{df_R}{dt}|_{p=p_R} \tag{18}$$

The efficiency with which R states with momentum p_R are produced from the background (ignoring decays and Pauli Blocking) is then given by

$$\begin{aligned} \frac{\Gamma(p_R)}{H} &= \frac{1}{f_{R,eq}H} \frac{df_R}{dt} = \frac{m_R \Gamma}{f_{R,eq} H E_R p_R} \int_{(E_R-p_R)/2}^{(E_R+p_R)/2} dp_H f_H f_L. \\ &= \frac{m_R T \Gamma}{H p_R E_R} \log\left[\sinh\left((E_R+p_R)/2T\right) / \sinh\left((E_R-p_R)/2T\right)\right]. \end{aligned} \tag{19}$$

This function is sharply peaked at low momentum with the following limiting behaviour

$$\frac{\Gamma(p_R)}{H} \to \begin{cases} \dfrac{\Gamma}{H} \coth(m_R/2T) \propto z^2 \coth(z/2) & p_R \to 0 \\ \dfrac{\Gamma m_R}{H p_R^2} \left(p_R + T \log\left(\dfrac{2 p_R T}{m_R^2}\right)\right) & p_R \to \infty \end{cases}. \tag{20}$$

This is a steeply increasing function of z and at low temperatures when $T << m_R$ the distribution of R is in complete equilibrium and $f_R << 1$ so that the Boltzmann approximation is valid. Therefore the curves for $n_R/n_{\rm eq}$ approach each other in Figs. 1-3. Note, however, that the highest p_R modes are always out of equilibrium because of the asymptotic $1/p_R$ term. In fact for given values of z and K there will be a limiting value of p_R above which the distribution will be out of equilibrium. In Fig. 5 we show the value of p_R/T for which $\Gamma(p_R)/H = 1$ as a function of z, for the specific case of $K = 1$. At high temperatures no momentum states are in equilibrium, whereas at low temperatures only progressively higher momentum states are out of equilibrium. This figure corresponds well to what is shown in Fig. 4. For $z = 0.2$ no states are in equilibrium, for $z = 1$ states above $p_R/T \sim 1$ are not yet in equilibrium, and for $z = 5$ almost all states have been populated to their equilibrium value.

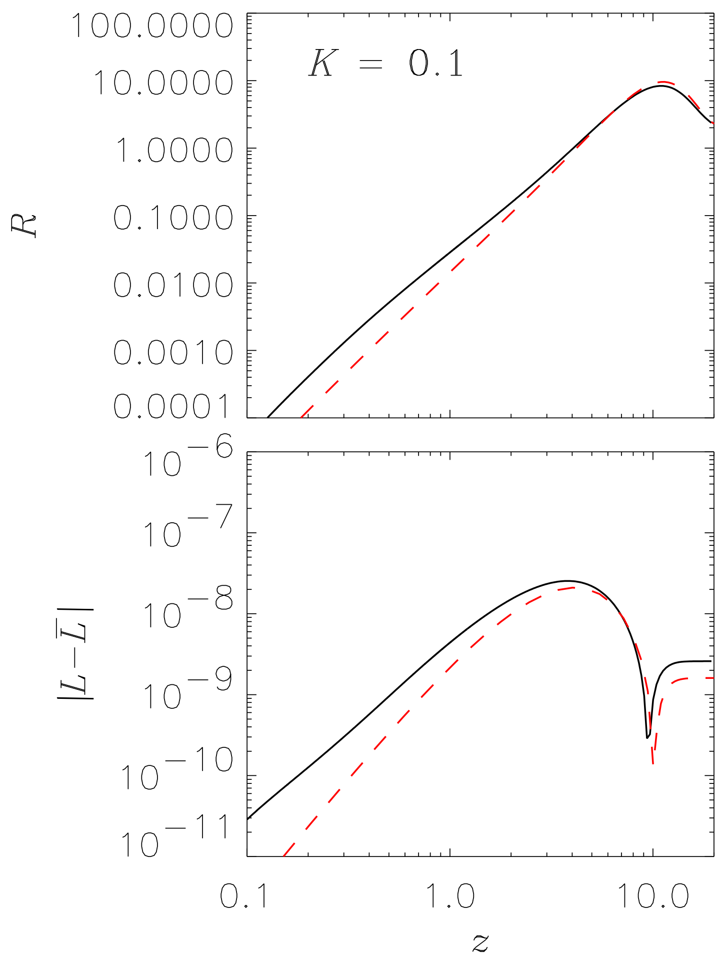

Figure 1. Upper panel: The evolution of $n_R/n_{\rm eq}$ for the full (solid line) and integrated (dotted line) Boltzmann equations. Bottom panel: The evolution of $|n_{L-\bar{L}}|/T^3$ for the same cases. The calculation is for $K = 0.1$ and $\epsilon = 10^{-6}$.

As can be seen in Figs. 1-3 the net asymmetry also grows much more rapidly initially when the full Boltzmann equation is used, and the change of sign occurs at higher temperature. However, at low temperatures the differences are smaller.

In Fig. 6 we show the ratio of the final asymmetry in the two cases

$$\frac{\eta_{\rm full}}{\eta_{\rm int}} = \frac{(n_L - n_{\bar{L}})/n_\gamma|_{\rm full}}{(n_L - n_{\bar{L}})/n_\gamma|_{\rm int}} \tag{21}$$

As can be seen from Fig. 6 there can be a significant difference in the asymmetry between the two approaches. For very high values of K the difference is small because the distribution is kept very close to kinetic equilibrium (for a Fermi-Dirac distribution) throughout the decay. Note, however, that the difference never goes to zero. The reason is that the thermal decay width is changed relative to the Maxwell-Boltzmann approximation when Pauli blocking and stimulated emission factors are included (as also noted by [8]). For smaller values of K the difference increases and can be as large as 50%. However, it should be noted that our framework will break down for small

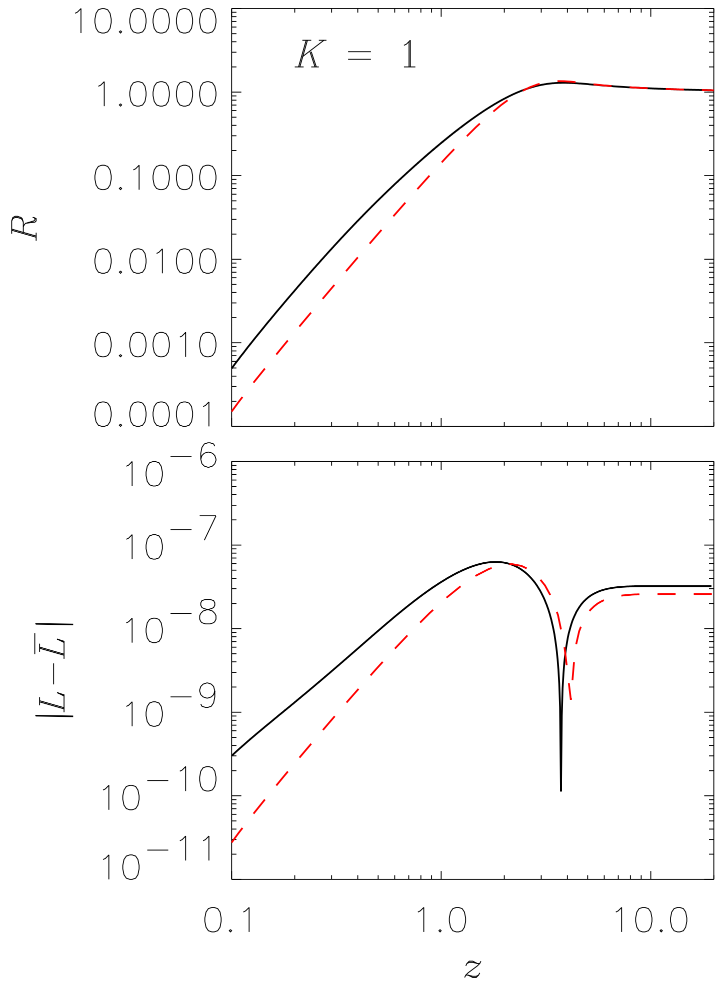

Figure 2. Upper panel: The evolution of $n_R/n_{\rm eq}$ for the full (solid line) and integrated (dotted line) Boltzmann equations. Bottom panel: The evolution of $|n_{L-\bar{L}}|/T^3$ for the same cases. The calculation is for $K = 1$ and $\epsilon = 10^{-6}$.

values of K because we have not included scattering.

Another important point to note is that $\eta_{\rm full}/\eta_{\rm int}$ does not depend on ϵ because the deviation from kinetic equilibrium is governed completely by the total decay rate (i.e. by K), not by the asymmetry, as long as $\epsilon << 1$.

We are only interested in the final value of the asymmetry since, in order to maximise the baryon asymmetry for any given value of ϵ, we want leptogenesis to end before the conversion through sphalerons end.

3. Discussion

We have solved the full Boltzmann equation for decays and inverse decays to follow the generation of lepton asymmetry during leptogenesis. When decays are semi-relativistic the difference between using the full Boltzmann equation and the standard integrated Boltzmann equation can be very large. However, at low temperature where washout

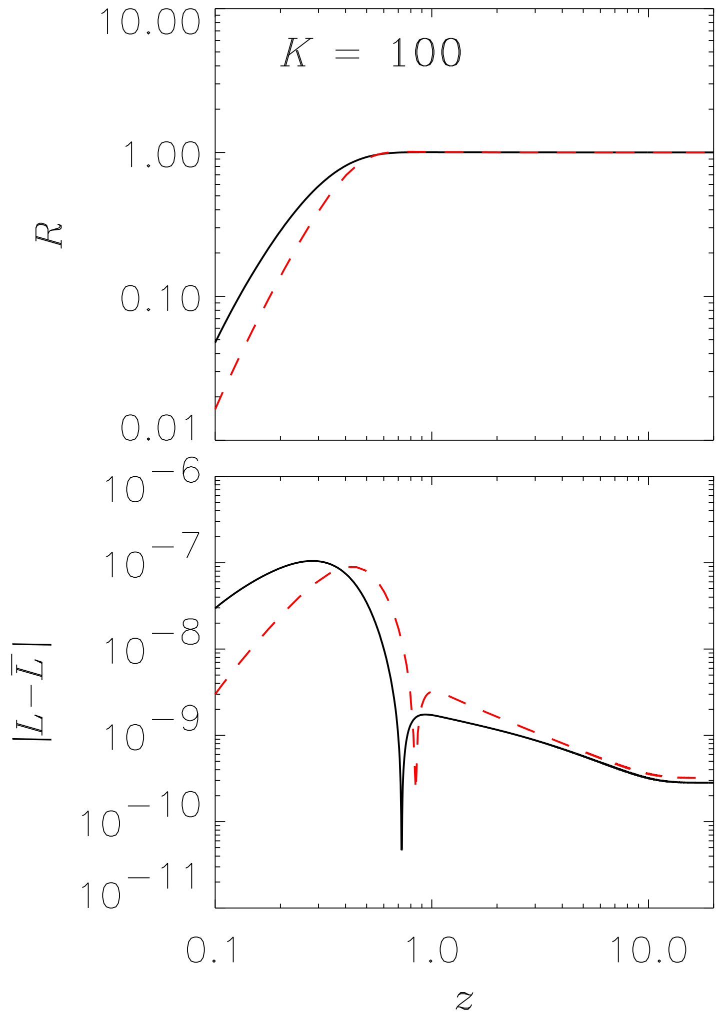

Figure 3. Upper panel: The evolution of $n_R/n_{\rm eq}$ for the full (solid line) and integrated (dotted line) Boltzmann equations. Bottom panel: The evolution of $|n_{L-\bar{L}}/T^3|$ for the same cases. The calculation is for $K = 100$ and $\epsilon = 10^{-6}$.

dominates the difference is relatively modest.

The difference at $K \gtrsim 1$ is less than about 30% between the two approaches. For smaller K the difference can be larger. However, this is the regime where $1 \leftrightarrow 2$ processes do not dominate, i.e. outside the regime in which our framework is valid.

The conclusion is that the Boltzmann approximation yields results in the strong washout regime which are accurate to at least 30%. For $K > 5$ the difference is not larger than 15%.

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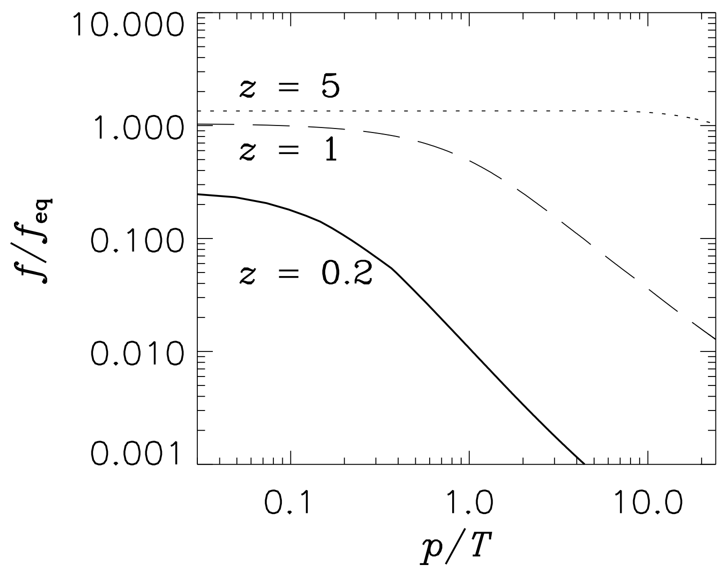

Figure 4. The distribution function of R relative to a chemical equilibrium Fermi-Dirac distribution with the same temperature for different values of z. The calculation is for $K = 1$.

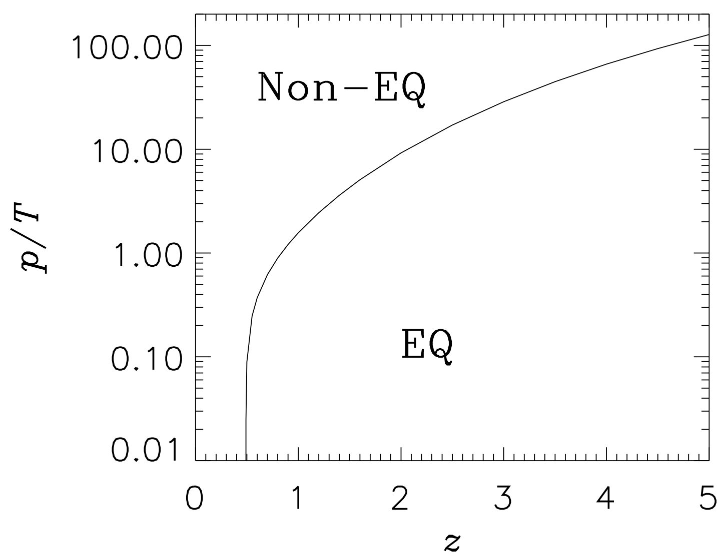

Figure 5. The dividing line between equilibrium and non-equilibrium for $K = 1$. All momentum states above the line are out of equilibrium.

for valuable comments.

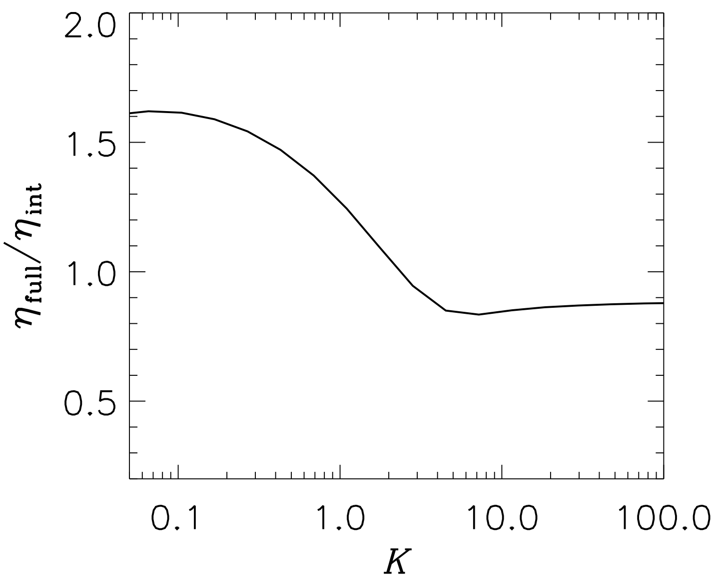

Figure 6. The difference between the asymmetry in the full case and the integrated case as a function of K.

References

[1] M. Fukugita and T. Yanagida, "Baryogenesis Without Grand Unification," Phys. Lett. B **174**, 45 (1986).

[2] V. Kuzmin, V.A. Rubakov and M.E. Shaposhnikov, "On anomalous electroweak baryon-number non-conservation in the early uiniverse," Phys. Lett. **155B**, 36-42 (1985).

[3] W. Buchmuller, P. Di Bari and M. Plumacher, "Leptogenesis for pedestrians," Annals Phys. **315**, 305 (2005) [arXiv:hep-ph/0401240].

[4] M. A. Luty, "Baryogenesis Via Leptogenesis," Phys. Rev. D **45**, 455 (1992).

[5] T. Hambye, Y. Lin, A. Notari, M. Papucci and A. Strumia, "Constraints on neutrino masses from leptogenesis models," Nucl. Phys. B **695**, 169 (2004) [arXiv:hep-ph/0312203].

[6] W. Buchmuller, P. Di Bari and M. Plumacher, "The neutrino mass window for baryogenesis," Nucl. Phys. B **665**, 445 (2003) [arXiv:hep-ph/0302092].

[7] W. Buchmuller and M. Plumacher, "Neutrino masses and the baryon asymmetry," Int. J. Mod. Phys. A **15**, 5047 (2000) [arXiv:hep-ph/0007176].

[8] G. F. Giudice, A. Notari, M. Raidal, A. Riotto and A. Strumia, "Towards a complete theory of thermal leptogenesis in the SM and MSSM," Nucl. Phys. B **685**, 89 (2004) [arXiv:hep-ph/0310123].

[9] G. F. Giudice, M. Peloso, A. Riotto and I. Tkachev, "Production of massive fermions at preheating and leptogenesis," JHEP **9908**, 014 (1999) [arXiv:hep-ph/9905242].

[10] M. Hirsch, M. Malinsky, J. C. Romao, U. Sarkar and J. W. F. Valle, "Thermal leptogenesis in extended supersymmetric seesaw," arXiv:hep-ph/0608006.

[11] S. Blanchet and P. Di Bari, "Leptogenesis beyond the limit of hierarchical heavy neutrino masses," JCAP **0606**, 023 (2006) [arXiv:hep-ph/0603107].

[12] S. Blanchet and P. Di Bari, "Flavor effects on leptogenesis predictions," arXiv:hep-ph/0607330.

[13] A. Abada, S. Davidson, A. Ibarra, F. X. Josse-Michaux, M. Losada and A. Riotto, "Flavour matters in leptogenesis," arXiv:hep-ph/0605281.

[14] T. Ota and W. Rodejohann, "Breaking of L(mu) - L(tau) flavor symmetry, lepton flavor violation and leptogenesis," Phys. Lett. B **639**, 322 (2006) [arXiv:hep-ph/0605231].

[15] E. Nardi, Y. Nir, E. Roulet and J. Racker, "The importance of flavor in leptogenesis," JHEP **0601**,

164 (2006) [arXiv:hep-ph/0601084].

[16] A. Abada, S. Davidson, F. X. Josse-Michaux, M. Losada and A. Riotto, "Flavour issues in leptogenesis," JCAP **0604**, 004 (2006) [arXiv:hep-ph/0601083].

[17] T. Hambye, M. Raidal and A. Strumia, "Efficiency and maximal CP-asymmetry of scalar triplet leptogenesis," Phys. Lett. B **632**, 667 (2006) [arXiv:hep-ph/0510008].

[18] S. Hannestad, "Non-equilibrium effects on particle freeze-out in the early universe," New Astron. **4**, 207 (1999) [arXiv:astro-ph/9903034].

[19] J. Madsen, "Bose Condensates, Big Bang Nucleosynthesis, And Cosmological Decay Of A Phys. Rev. Lett. **69**, 571 (1992).

[20] G. D. Starkman, N. Kaiser and R. A. Malaney, "Mixed dark matter from neutrino lasing," Astrophys. J. **434**, 12 (1994) [arXiv:astro-ph/9312020].

[21] N. Kaiser, R. A. Malaney and G. D. Starkman, "Neutrino lasing in the early universe," Phys. Rev. Lett. **71**, 1128 (1993) [arXiv:hep-ph/9302261].

[22] S. Hannestad and J. Madsen, "A cosmological three level neutrino laser," Phys. Rev. D **55**, 4571 (1997) [arXiv:astro-ph/9702125].

[23] E. W. Kolb and S. Wolfram, "Baryon Number Generation In The Early Universe," Nucl. Phys. B **172**, 224 (1980) [Erratum-ibid. B **195**, 542 (1982)].