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Neutrinos and thermal leptogenesis

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1 Introduction

During the last few decades cosmology has evolved from its relatively speculative origins to an exact science. This has been made possible by new observational data: observations on the large scale structure, supernovae, cosmological microwave background radiation etc. A cosmological "standard model" has been established, which fits observations with great accuracy.

Some of these observations are explained in depth with microphysical understanding, eg., the big bang nucleosynthesis predicts the observed abundances of the light elements with very few free parameters. Other aspects of the model are more general ideas and will require new physics beyond the standard model, before we have a robust quantitative understanding of the observations. These more general ideas would include inflation and dark energy, both which are phenomena whose existence are well established, but whose exact microphysical mechanisms are unknown.

One of these open questions is the birth of the cosmological baryon asymmetry — the problem of baryogenesis. Our universe appears to be filled with baryons and no antibaryons. This is in clear contradiction with our current fundamental microphysical theory, the standard model of particle physics.

The relatively trivial observation that we and everything surrounding us are made of matter and not antimatter has deep implications: It tells us of baryon number violation and broken CP-symmetry, and other physics, beyond the standard model. Hence, the theory of baryogenesis plays a role of a profound interconnection between particle physics and cosmology.

Leptogenesis is a candidate for a theory of baryogenesis. In this scenario heavy Majorana neutrinos decay producing net lepton number. The sphaleron processes, predicted by the standard model, convert some of this net lepton number to baryon number producing a universe with an excess of baryons. This model was originally suggested already in the 1980's, and it has gained momentum in the 1990's when it became clear that the standard electroweak theory was unable to explain the observed baryon asymmetry. Further motivation for leptogenesis has come from the discovery of the neutrino massess when neutrino oscillation was observed.

This basic theory of leptogenesis called *thermal leptogenesis* is the main topic of this work. Several modifications to the theory have been suggested during the last ten years, including Dirac leptogenesis, resonant leptogenesis and Affleck-Dine leptogenesis. Alhough they have many attractive properties, we do not treat these modifications in this work, not in the least because they ruin the somewhat simplistic beauty of the basic thermal theory.

This work does not attempt to duplicate the ever-advancing exact numerical predictions of leptogenesis, requiring advanced finite temperature field theory, quantum statistical theory and numerical methods. Instead, the aim of this work is to introduce the reader to the theory of thermal leptogenesis and to demonstrate, hopefully in an understandable way, the feasibility of leptogenesis to explain the comsological baryon asymmetry.

The contents of tgus wirj fall naturally into three parts: First we review the current observational status on the amount of baryons in our universe and problematize these findings. After that we discuss the data on neutrino masses from oscillation experiments and explore different ways to extend the standard model to include this new discovery. Finally in the latter half of this work we introduce the mechanism of leptogenesis, derive an expression for the prediction of the baryon asymmetry and demonstrate the capability of leptogenesis to quantitatively explain the observed asymmetry. In addition we introduce in the appendix a consistent formulation of Feynman rules for Majorana particles necessary for calculating the parameters of leptogenesis.

On notation and details

In this work it is assumed that the reader is somewhat familiar with standard cosmology, at least with the Friedmann-Robetson-Walker (FRW) model. Also some familiarity with relativistic quantum field theory and statistical mechanics is assumed. The calculations specific to leptogenesis, such as the derivation of the relevant Boltzmann equations, are in more detail.

By the standard model we mean the standard model of particle physics, not that of cosmology, including the theory of QCD and EW-theory, but with massless neutrinos.

Natural units are used throughout the work to ease calculations. Temperatures and masses are consequently given in the units of energy, usually in electronvolts.

2 Cosmological baryon asymmetry

The standard cosmological model has several outstanding questions, the most important ones being the nature of dark matter and dark energy, mechanism of inflation and baryogenesis.

The existence of dark matter was originally suggested to explain the galactic rotation curves; it has also become necessary to explain structure formation. The existence of dark matter is generally accepted, but there are many candidates for the dark matter particle waiting for experimental confirmation.

Dark energy is postulated in order to explain the supernovae observations which suggest that the expansion of the universe has started to accelerate during late times. Dark energy is becoming more and more accepted as an idea, though there are very few credible candidates for the source of this mysterious energy. There are doubts if the supernovae observations could be explained better without dark energy with a more accurate treatment of the non-linear equations of general relativity in our inhomogenous universe [1] and an accurate theory of the propagation of observed light in an inhomogenous medium [2].

The generic idea of inflation, ie., superluminal expansion of the universe in its early days, has been invoked to explain the horizon and the flatness problem. It can also explain the origin of primordial perturbations resulting into the large scale structure we observe. During inflation the energy content of the universe was dominated by the potential of the inflaton field, making the universe expand exponentially. Inflation end as the inflaton decays to other particles heating the universe to the temperature of reheating T_{reh}. After that the universe continues to expand according to a power law. The generic idea of inflation is widely accepted and several specific models have been suggested [3]. However, no consensus exists on the specific mechanism of inflation.

2.1 Baryons and the standard model

In the standard model of particle physics (SM), and relativistic quantum field theory in general, each particle has a counterpart, an antiparticle. Antiparticles are otherwise similar to particles, except that they have opposite quantum numbers — electric, colour, and weak charges. The theory itself is highly symmetrical in respect to the switch between matter and antimatter: particles and antiparticles are featured symmetrically in the standard model Lagrangian.

Baryons are those hadrons (particles made of quarks, bound by the strong interaction described by quantum chromodynamics, QCD), which are made of three quarks. The only baryons relevant to cosmology are obviously the stable ones: protons and neutrons[1]. Along with a sufficient amount of electrons to make the universe electrically neutral, these are the constituents of the familiar everyday matter.

As baryons, neutrons and electrons are described by relativistic quantum field theory, they have their antiparticles: antiproton, antineutron and the positron. Since matter and antimatter have the same mass and similar features, differentiating between them is not altogether facile in experimental context. They have the same mass and other features, eg., spectral lines. To differentiate between matter and antimatter one has to observe the electric charge of the matter or observe annihilation or the lack of it.

From the symmetry of the underlying theory one would then *a priori* expect the universe to have 50% matter and 50% antimatter.

2.2 Baryons in our universe

Earth and everything on it consists completely of matter and no antimatter. From manned missions and interplanetary probes we know that the moon and all the planets in our solar system are made up of only protons, neutrons and electrons. From solar wind we know that also the sun is made of matter. Of objects beyond our solar system we have no physical samples. However,we know cosmic rays originating from our galaxy and well beyond it consist of approximately 90% protons, 9% helium nucleai and 1% electrons, and of very little antimatter.[2] Furthermore, if antimatter would exist in a significant quantity in the universe it would produce a huge amount of γ-radiation when annihilating with normal matter. But no large scale annihilation events have been observed anywhere in our universe.

The most compelling argument for the lack of the existence of antimatter is somewhat more theoretical: It is very difficult to conceive any mechanism, which would separate matter and antimatter in the early universe so completely that they would not annihilate each other and hence produce a universe filled only with radiation. Therefor we must conclude that the visible matter in the universe consists only of matter and that there exists no significant amount of antimatter. That is, to contrast the near perfect symmetry between matter and antimatter in the microphysical theory, the standard model, cosmology has a near-perfect *asymmetry* between matter and antimatter.

[1]Neutrons are of course not stable when free, but have an half-life of about 886 s, however they are stable when bound to nucleai.

[2]Significant amount of antiparticles are however produced as the *secondaries* of cosmic rays as the energetic cosmic rays interact with material in the atmosphere of earth.

To quantify the aforementioned asymmetry, we need to introduce some quantum number which describes the amount of matter. Baryons are counted by introducing the baryon number B, which is defined so that each quark has $B_q = 1/3$ and each antiquark has $B_{\bar{q}} = -1/3$ so that a proton or a neutron has $B_{p,n} = 1$. Similarly we define the lepton number L, which is defined so that each lepton has $L_l = 1$ and each antilepton $L_{\bar{l}} = -1$. Both the baryon and lepton number are experimentally conserved to extremely great precision.

The naive measure of the amount of matter in the universe would be $B + L$, corresponding to protons, neutrons and electrons. However the light, very weakly interacting neutrinos have as well lepton number. Since neutrinos interact so weakly, measuring their number density and the lepton number conserved in them is very difficult — in general we have no obervational data on the lepton number conserved in the cosmic neutrino background. Hence instead of considering $B + L$, we consider only baryons.

The conclusion of the lack of antimatter in the universe can then be formulated as

$$B = N_p + N_n ,$$

where N_p and N_n are the number of protons and neutrons in the universe. Instead of measuring the total baryon number in the visible universe, we usually measure the amount of baryons per photons

$$\eta \equiv \frac{n_B}{n_\gamma} .$$

This has the advantage that if the universe expands adiabatically, with no photon production, this number stays fixed even if the physical density may change. The primary motivation of this work is to explain the measured value of η in a dynamical way.

To know the value of η in the universe, we need to know the abundance of photons and baryons in the contemporary universe. Photon density is easy to estimate: The number density of photons is dominated by the cosmic microwave background (CMB). It is nearly perfect black-body radiation at the temperature $T = 2.725\,\mathrm{K}$. The number density then can be calculated from the distribution function, to be [4]

$$n_\gamma(T) = 2\frac{\zeta(3)}{\pi^2}T^3 .$$

To estimate the number density of baryons we have to somehow measure the total amount of protons and neutrons in some (large) volume. In practice there are three different ways to measure this: direct observation, big-bang nucleosynthesis (BBN) and the CMB.

Direct observation means evaluating the number of baryons by counting the objects on the sky consisting of baryons: First to estimate the amount of baryons in a star and then the amount of stars in the visible universe. Since a large amount of matter is in dark clouds or other objects which are hard to observe, this method is inherently unprecise and unpractical.

Cosmic Microwave Background

Cosmic microwave background radiation was created when the temperature of the universe dropped to the range of binding energies of atoms, and free nucleai and electrons started forming electrically neutral atoms. The universe transformed from opaque to practically transparent and the electromagnetic radiation decoupled forming the CMB. The small perturbations in the otherwise nearly perfect black-body spectrum tells us of the perturbations in the energy density in the early universe.

The power spectrum of the perturbations is well understood in modern cosmology. Since the perturbation spectrum has many features ie. peaks, several independent cosmological parameters can be derived from it. The Wilkinson Microwave Anistropy Probe (WMAP) first year data can be fitted with the standard Λ_{CDM}-model very well, giving as the amount of baryons [5]

$$\eta_{\mathrm{CMB}} = (6.14 \pm 0.25) \times 10^{-10} \,. \tag{2.1}$$

Big-bang nucleosynthesis

In the cosmological standard model the early universe was very hot and dense and strongly interacting particles formed quark-gluon-plasma. As the universe expanded and cooled down, a QCD phase transition occured in which all quarks and gluons were bound to hadrons, most importantly to protons and neutrons. Once the universe cooled further, the protons and those neutrons which had not yet decayed, started to form nucleai; mostly deuterium, tritium and helium. This production of the lightest elements is called the theory of Big-Bang nucleosynthesis which is a well-established part of the standard cosmological model. Within the theory of the standard model of particle physics, BBN can explain and predict with great accuracy the observed abundances of light elements in the universe [5]. Assuming near perfect thermal equilibrium, BBN has two free parameters: the net amount of baryons and leptons. Since the net lepton abundancy can be expected to be of the same magnitude as the et baryon abundancy, and the BBN prediction is not very sensitive to the amount of leptons in the early universe [6], BBN gives a prediction for η. BBN fits observed abundances of the lightest elements with 95% confidence level [7] for

$$\eta_{\mathrm{BBN}} = (4.7 - 6.5) \times 10^{-10} \,.$$

As can be seen from figure 2.1, the BBN and CMB measurement of η agree quite well with each other. For this work, however, the actual decimals are not important, but rather merely the non-zeroness and magnitude of η intrest us. For the rest of this work a crude value

$$\eta = 6 \times 10^{-10}$$

will suffice.

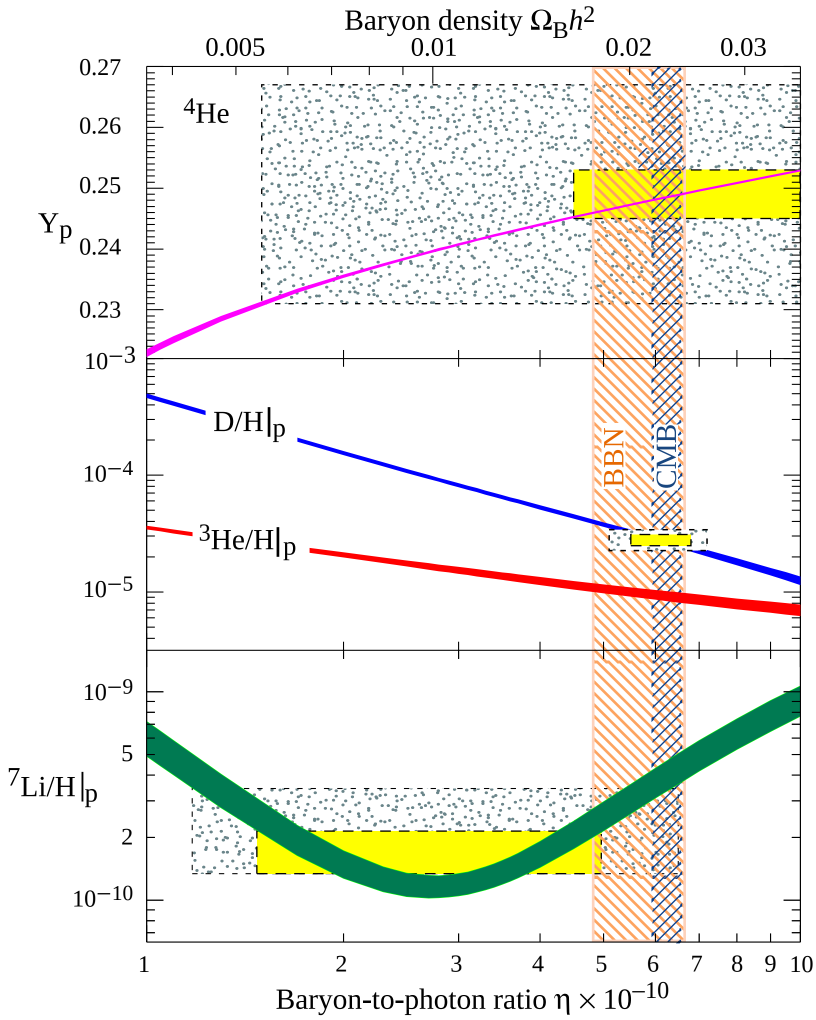


Figure 2.1: The abundances of ^{4}He, D, ^{3}He and ^{7}Li as predicted by the standard model of big-bang nucleosynthesis — the bands the 95% range. Boxes indicate the observed light element abundances (smaller boxes: $\pm 2\sigma$ statistical errors; larger boxes: $\pm 2\sigma$ statistical *and* systematic errors). The narrow vertical band indicates the CMB measure of the cosmic baryon density, while the wider band indicates the BBN concordance range (both at 95% CL). Image courtesy of Particle Data Group.

2.3 Baryogenesis and Sakharov's conditions

If the microphysics of our universe would be completely baryon number conserving then we would have no alternative but to deduce that the excess amount of baryons in our universe is an *initial condition*. The standard model of particle physics nevertheless has baryon number violating processes (see section 4) and most theories beyond the standard model feature also baryon number violation. Hence we do not attribute the value of η to intial conditions, but rather hope to explain its value dynamically. Moreover, in the context of inflationary models it is natural to assume the baryon number to be zero during inflation, and that the baryonic excess is created during or after reheating.

In 1967 Andrei Sakharov published an article [8], in which he problematized the issue of matter-antimatter asymmetry in our universe. He was the first to consider in detail what criteria a theory should fulfill in order to be able to explain the global baryon asymmetry. These criteria were to be necessary but not sufficient to create baryon asymmetry, and are nowadays called *Sakharov's conditions*. These are B-number violation, C and CP violation and departure from thermal equilibrium.

B-number violation

The requirement to violate baryon number is the most obvious of the three Sakharov's conditions: If baryon number should be violated macrophysically, then there must exist a microphysical process which violates it. It is also the condition most easily fulfilled in most candidate theories: Already the standard model has a process which violates B, the *sphaleron process* (see section 4). Also in grand unified and other theories beyond the standard model there usually exists operators which violate baryon number.

C and CP violation

Here C-symmetry refers to *charge conjugation*, ie., replacing the charges of particles with their opposites. CP-symmetry is the combination of C-symmetry and parity transformation, ie., charge conjugation combined with a flip of the sign of all spatial coordinates.

Consider a generic process creating baryon number: $X \to Y + B$ where X and Y are non-baryonic particles. If the theory is C-invariant, the rate of this process must be equal to its C-conjugated processes $\overline{X} \to \overline{Y} + \overline{B}$. If C is not violated, the abundance of X and $\overline{X}$ must be equal, so the total change in baryon number goes

$$\frac{dB}{dt} \propto \Gamma(X \to Y + B) - \Gamma(\overline{X} \to \overline{Y} + \overline{B}) = 0$$

and no excess of baryons can be produced. Though a single microphysical process may violate baryon number, statistically no net baryon number is produced. Hence we conclude that C-symmetry must be violated. This is not difficult, since already the electroweak sector of the standard model breaks C maximally.

However, breaking C is not enough. If CP is conserved, then

$$\Gamma(X \to Y_L + B_L) = \Gamma(\overline{X} \to \overline{Y}_R + \overline{B}_R)$$

and similarly

$$\Gamma(X \to Y_R + B_R) = \Gamma(\overline{X} \to \overline{Y}_L + \overline{B}_L) ,$$

where we use the notation Y_R to be the right-handed projection of the particle Y. From this we get the equality

$$\Gamma(X \to Y_L + B_L) + \Gamma(X \to Y_R + B_R) = \Gamma(\overline{X} \to \overline{Y}_R + \overline{B}_R) + \Gamma(\overline{X} \to \overline{Y}_L + \overline{B}_L)$$

and no net baryon number is produced. At best, an inequality between left-handed and right-handed particles is created.

Departure from thermal equilibrium

In CPT-conserving theory in thermal equilibrium $\langle B \rangle = 0$. This can be shown easily by using the fact that under CPT-transformation the Hamiltonian is invariant and the baryon number operator is odd.

$$\begin{aligned}
\langle B \rangle &= \mathrm{Tr}\, e^{-\beta \hat{H}} \hat{B} \\
&= \mathrm{Tr}\, (CPT)(CPT)^{-1} e^{-\beta \hat{H}} \hat{B} \\
&= \mathrm{Tr}\, e^{-\beta \hat{H}} (CPT)^{-1} \hat{B} (CPT) \\
&= -\mathrm{Tr}\, e^{-\beta \hat{H}} \hat{B} \\
&= -\langle B \rangle
\end{aligned}$$

Since CPT-invariance appears to be a necessity for formulating consistent quantum field theory, thermal equilibrium must be broken somehow to produce baryon asymmetry.

2.4 Candidates for baryogenesis

To explain the cosmic baryon asymmetry, several theories and models have been suggested. The most pleasing alternative has been the electroweak baryogenesis, since it requires no physics beyond the standard model, whereas other scenarios require at least some extension to it.

EW-baryogenesis

The standard model of particle physics, perhaps surprisingly, fulfills all Sakharov's conditions. The electroweak sector of the standard model naturally violates C-symmetry maximally, but it also violates CP-symmetry through the complex phase in the Cabibbo-Kobayashi-Maskawa matrix. Furthermore the standard model also has non-perturbative baryon number violation, explained in section 4. The departure from thermal equilibrium can be facilitated either by a first order electroweak phase transition or an out-of-equilibrium decay of some sufficiently weakly interacting heavy particle.

As the temperature lowers in the expanding universe to the electroweak temperature the Higgs field transitions from the symmetric phase $\langle H \rangle = 0$ to the broken phase $\langle H \rangle = v$. The broken phase will start forming bubbles which will then expand. Near the bubble walls non-equilibrium processes can then produce net baryon and lepton number.

The actual production of net baryon number is however very difficult, at least to facilitate the fairly large observed asymmetry [9]. The standard model CP violation must involve all three generations of particles, making it small at low energies. Furthermore it has been shown that the order of the electroweak phase transition depends on the Higgs mass [10] and current lower limits for its mass rule out a first order electroweak phase transition. Hence EW-baryogenesis is in practice ruled out. Since baryogenesis cannot be explained within the standard model, the existence of baryons in our universe can be considered as evidence for physics beyond the standard model.

GUT-baryogenesis

The standard model describes the interactions of particles by two symmetry groups, $\mathrm{SU}(3)_{\mathrm{QCD}}$ and $\mathrm{SU}(2) \otimes \mathrm{U}(1)_{\mathrm{Y}}$. The motivation for grand unified theories (GUT) is to explain all these interactions by a single large symmetry group, which includes all these groups as its subgroups. Though no definitive GUT theory has been found, there are many different models tossed around with many common properties.

Sakharov's conditions are easily fulfilled in GUTs. They naturally produce baryon and lepton number violation, departure from thermal equilibrium from, eg., heavy particles decaying out of equilibrium, and complex couplings give CP violation in those decays [11].

Any GUT can hence facilitate baryogenesis. The problem is in formulating a GUT, which is consistent with the standard model and the obervational upper limit on proton lifetime. Futhermore in the context of inflation baryogenesis should occur after or during reheating, and hence if $T_{\mathrm{GUT}} \sim 10^{16}$GeV, this poses strict limits on the reheating temperature.

3
Neutrinos

The main topic of this thesis is leptogenesis. To motivate and explain this model we first need to depart on an excursion to the world of neutrinos.

The existence of neutrinos was originally suggested by Wolfgang Pauli in 1930 to uphold the conservation of energy by explaining the unseen energy in beta decay with a new particle that was very difficult to observe. Pauli originally called his new particle the *neutron*, since the particle nowadays called neutron had not yet been found. In fact the name neutrino is a pun by Enrico Fermi on neutron: the Italian word for neutron, *neutrone*, can be interpreted as the augmentative of a word which has also a diminutive: *neutrino*, a tiny neutron.

Neutrinos are the lightest fermions in the standard model and they interact only through the electroweak sector in the standard model. They also come in all three flavours. In the standard model the neutrinos are Weyl-fermions: massless spin-1/2-fermions which have only the left-handed chirality.

3.1 Neutrino oscillation and neutrino masses

The standard model assumption of massless neutrinos is in agreement with all kinematical, ie. direct measurements of neutrino mass.[1]

The existence of non-zero mass for neutrinos, no matter how small, a new fascinating phenomena, neutrino oscillation, possible. Neutrinos interact through the electroweak interaction. Hence they are created and destroyed in interaction eigenstates, but they propagate in mass eigenstates. If these two sets of eigenstates are not the same then neutrinos will oscillate as they propagate, changing from one interaction eigenstate to another. This was originally suggested by Pontecorvo in 1957 [12].

For most practical applications it is sufficient to consider only two flavours of neutrinos [13] ν_e and ν_μ. These flavour states are defined as the interaction eigenstates: ν_e is a neutrino which interacts with electrons through charged

[1] The Karlsruhe tritium neutrino experiment, KATRIN, is expected to start data collection in 2009. KATRIN attempts to measure the electron neutrino mass by measuring the electron spectrum from the β-decay of tritium, and is expected to have sub-eV accuracy. Neutrino oscillation data inidicate that electron's neutrino mass might be of order 10^{-2}eV, which means that even a negative result from KATRIN would not be in disagreement with the oscillation data.

current, ν_μ a neutrino which interacts with muons. These states can be expressed as a linear combination of the mass eigenstates ν_1 and ν_2,

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} . \tag{3.1}$$

A neutrino is created through an electroweak interaction into either of these flavour states, say ν_e. As the neutrino propagates, it evolves in its mass eigenstates,

$$\nu_e(t) = e^{-i\hat{H}t}\nu_e = e^{-iE_1 t}\cos\theta\,\nu_1 + e^{-iE_2 t}\sin\theta\,\nu_2 .$$

Assuming $p \gg m_{1,2}$, we can approximate the energy of the mass eigenstates to be

$$E_{1,2} \simeq p + \frac{m_{1,2}^2}{2p} ,$$

and also $v_\nu \simeq 1$ in natural units, giving $t \simeq l$, where l is the propagated distance. Using these we get

$$\nu_e(t) = e^{-ipt}\left(\cos\theta\, e^{-il\frac{m_1^2}{2p}}\,\nu_1 + \sin\theta\, e^{-il\frac{m_2^2}{2p}}\,\nu_2\right) .$$

Solving ν_1 and ν_2 from eq. (3.1) and inputting them to the previous expression we get

$$\begin{aligned} \nu_e(t) \;=\; & \nu_e e^{-ipt}\left(\cos^2\theta\, e^{-i\frac{m_1^2}{2p}l} + \sin^2\theta\, e^{-i\frac{m_2^2}{2p}l}\right) \\ & + \nu_\mu e^{-ipt}\cos\theta\sin\theta\left(-e^{-i\frac{m_1^2}{2p}l} + e^{-i\frac{m_2^2}{2p}l}\right) . \end{aligned}$$

The absolute squared of the coefficients then gives the propability to find a flavour eigenstate, eg.

$$P(\nu_e \to \nu_\mu) = \sin^2 2\theta \sin^2\left(\frac{l}{4p}|m_1^2 - m_2^2|\right) .$$

In general the oscillation is always sensitive to both the mixing angle θ, and the squared mass difference $\delta m_{12}^2 \equiv |m_1^2 - m_2^2|$.

Different oscillation experiments are sensitive to different oscillations, depending on the energies of the neutrinos oscillating and their propagation distance. Furthermore, since neutrinos interact very weakly a significant flux of neutrinos is required to get any reasonable amount of data.

In solar neutrino oscillation expriments neutrinos which are created in the sun in nuclear processes propagate to earth, where they interact with electrons and these interaction are then measured. Solar neutrino experiments are sensitive to energies of the order $\sim$ MeV, distances 10^{11}m and mass differences of the order $\delta m^2 \gtrsim 10^{-11}\mathrm{eV}^2$ [13].

Atmospheric neutrinos are created when pions are created from cosmic rays in the atmosphere, which then decay to muons and three neutrinos. The atmospheric neutrino oscillation experiments are sensitive to energies of the order $\sim$ GeV, distances of the order $\sim 10^4$km and mass differences of the order $\delta m^2 \gtrsim 10^{-4}\text{eV}^2$.

Since the solar and atmospheric oscillation experiments probe completely different scales, they measure in fact two *different* mass differences, m_{atm} and $m_\odot$. Atmospheric neutrino oscillation experiments have shown that at 90% confidence level [7]

$$1.9 \times 10^{-3}\text{eV}^2 < \delta m^2_{atm} < 3.0 \times 10^{-3}\text{eV}^2 \ .$$

Solar neutrino oscillation data is less consistent, but favours strongly oscillation, with the best fit for the mass difference being

$$\delta m^2_\odot \simeq 6.5 \times 10^{-5}\text{eV}^2 \ .$$

Consequently oscillation experiments indicate *two* mass differences. This is unsurprising since the standard model has three flavours of neutrinos. Although the exact mass differences, hierarchy of the masses, and the size of the mixing angles is still an open question, the oscillation experiments have shown the existence of non-zero neutrino masses.

From cosmological data we have a upper limit to the sum of the masses of all neutrino flavours [14]

$$\sum_i m_{\nu_i} < (0.4 - 1.0)\text{eV} \ .$$

This however is not inconsistent with the oscillation results, since oscillation indicate mass scales of $\sim 10^{-1}$eV and $\sim 10^{-2}$eV.

The interesting aspect of this data is the approximate mass scale: it is several magnitudes smaller than the mass scales of other massive particles of the standard model. Furthermore, the non-zero mass of the neutrinos requires the existence of otherwise undetected right-handed neutrinos. In additioson the different magnitude of the mass scale hints at a different mechanism to generate the masses than the familiar Higgs mechanism of the standard model.

3.2 Mass terms for neutrinos

All charged fermions, that is all fermions except neutrinos, are described in the standard model as Dirac particles. The theory of a Dirac fermion is described by its Lagrangian density

$$\mathcal{L} = \overline{\psi} i \partial\!\!\!/ \psi - m\overline{\psi}\psi + \mathcal{L}_I \tag{3.2}$$

where the $\mathcal{L}_I$ is the interaction part Lagrangian. In the usual notation

$$\partial\!\!\!/ \equiv \partial_\mu \gamma^\mu \ ,$$

where the γ^μ is the set of four, four by four, anticommuting Dirac matrices. In the case of free theory $\mathcal{L}_I = 0$, the classical equations of motion for the spinor field can be found through the usual use of Euler-Lagrange-equations resulting in

$$(i\partial\!\!\!/ - m)\psi = 0 \; .$$

A massive Dirac fermion has four degrees of freedom which correspond to the left-handed fermion, right-handed fermion, left-handed antifermion and right-handed antifermion. The left- and right-handed particles are separated with the projection operators

$$\begin{aligned} P_L &= \frac{1}{2}(1-\gamma^5) \\ P_R &= \frac{1}{2}(1+\gamma^5) \end{aligned}$$

so that $\psi_L \equiv P_L\psi$ and $\psi_R \equiv P_R\psi$. Since $(\gamma^5)^2 = 1$ $P_LP_R = 0$, and the free Lagrangian can be rewritten as

$$\mathcal{L} = \overline{\psi}_L i\partial\!\!\!/ \psi_L + \overline{\psi}_R i\partial\!\!\!/ \psi_R - m\overline{\psi}_L\psi_R - m\overline{\psi}_R\psi_L \tag{3.3}$$

where $\overline{\psi}_L = \overline{\psi}P_R$ and similarly $L \leftrightarrow R$.

For a massive particle, its handedness corresponds to the sign of its spin projected onto its momentum. The spin does not change sign under a boost. However, for any given momentum a frame can always be found where the momentum is pointing to the opposite direction. This means that the handedness of a massive particle is not a Lorentz invariant quantity. For a massless particle one cannot boost to a frame where the momentum is in the opposite direction — there is no rest frame for a massless particle. This can be seen from eq. (3.3) by taking the massless limit $m \to 0$ decoupling the particles' right- and left-handed components.

Next we define the charge conjugation or the C-operation by

$$\psi^C \equiv C\gamma_0\psi^* \; .$$

Here the matrix C depends on the representation of the Dirac gamma matrices.[2] This translates for the barred charge conjugated field

$$\overline{\psi^C} = \psi^T C \; .$$

In some sense the C-operation exchanges particles with antiparticles and vice versa. It also changes the handedness of a particle, as can be seen from

$$C\gamma^0(P_L\psi)^* = CP_R\gamma^0\psi^* = P_RC\gamma^0\psi^* = P_R\psi^* \; .$$

In 1937 Ettore Majorana published a paper (see discussion and review of the original article in Italian in [15]) where he suggested another form of a mass term for the Lagrangian:

$$\begin{aligned} \mathcal{L}_M &= \overline{\psi} i\partial\!\!\!/ \psi - \frac{1}{2}\overline{\psi^C}M^M\psi \\ &= \overline{\psi_L} i\partial\!\!\!/ \psi_L + \overline{\psi_R} i\partial\!\!\!/ \psi_R - \frac{1}{2}\overline{\psi^C_L}M^M\psi_L - \frac{1}{2}\overline{\psi^C_R}M^M\psi_R \end{aligned} \tag{3.4}$$

[2] In Dirac representation $C = i\gamma^2\gamma^0$, in Majorana representation $C = \gamma^0$. In this work where explicit form of the gamma matrices is required we comply with the Dirac representation.

$$\begin{tikzcd} \nu_L \arrow[r, leftrightarrow, "CPT"] \arrow[d, leftrightarrow, "Lorentz"'] & \bar{\nu}_R \arrow[d, leftrightarrow, "Lorentz"] \\ \nu_R \arrow[r, leftrightarrow, "CPT"'] & \bar{\nu}_L \end{tikzcd}$$

(a) A Dirac particle

$$\begin{tikzcd} \nu_L \arrow[r, leftrightarrow, "Lorentz", "CPT"'] & \nu_R \end{tikzcd}$$

(b) A Majorana particle

Figure 3.1: The difference between Dirac and Majorana fermions in a nutshell.

The solutions now fulfill an additional condition — the Majorana condition:

$$\psi^C = \psi \; .$$

Thus the solutions have now only two degrees of freedom: left-handed particles and right-handed particles.

Next consider the U(1)-transformation

$$\psi \to e^{i\alpha}\psi, \; \overline{\psi} \to e^{-i\alpha}\overline{\psi} \; . \tag{3.5}$$

Clearly the Dirac Lagrangian of eq. (3.2) is invariant under this transformation. This symmetry has an associated conserved quantum number. For leptons, this can be interpreted as the lepton number. The Majorana Lagrangian eq. (3.4) however is not invariant under (3.5). This means that the Majorana mass term does not conserve the lepton number.

Why does a different sort of mass term in the Lagrangian ruin the symmetry? Dirac fermions come in four sorts: particles and antiparticles, and both of these as left- and right-handed. Although a Lorentz boost can change the handedness of a particle, a particle stays a particle and an antiparticle stays an antiparticle in all frames; the number of particles and antiparticles consequently stay fixed. Majorana fermions however come only in two sorts: left-handed and right-handed which can be Lorentz boosted to each other. If one chooses for instance the left-handed particle as the *particle* of the theory, then its antiparticle should be its CPT-conjugate, which is the right-handed particle. Hence particles and antiparticles are differentiated only by their handedness which is a Lorentz non-invariant quantity, and the amount of particles vs. antiparticles depends on the chosen frame.

As the Majorana particles have no conserved quantum number, the assignment which particle is matter and which antimatter is arbitrary and irrelevant. In general it is better to say that a Majorana particle is its own antiparticle.

3.3 Standard model EW-sector

The standard model electroweak sector is generated from the EW $\mathrm{SU}(2) \otimes \mathrm{U}(1)$ gauge symmetry group. This group has two charges associated with it: the weak isospin T^3, which is the charge of the SU(2)-group, and the weak hypercharge Y, which is the charge of the U(1)-group.

	ν_L	l_L	ν_R	l_R	ϕ^+	ϕ^0
T^3	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	$\frac{1}{2}$	$-\frac{1}{2}$
Y	$-\frac{1}{2}$	$-\frac{1}{2}$	0	-1	$\frac{1}{2}$	$\frac{1}{2}$
$Q = T^3 + Y$	0	-1	0	-1	1	0

Table 3.1: The SU(2)xU(1) charges of the leptons.

We define the left-handed SU(2) lepton doublets as

$$l \equiv \begin{pmatrix} \nu_e & \nu_\mu & \nu_\tau \\ e^- & \mu^- & \tau^- \end{pmatrix}_L \tag{3.6}$$

with their right-handed singlet counterparts as

$$r \equiv (e^- \;\; \mu^- \;\; \tau^-)_R \, . \tag{3.7}$$

Also, to generate masses, the standard model includes a complex scalar field, the Higgs field

$$\phi \equiv \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} ,$$

which is a SU(2)-doublet. The T^3 and Y -charges of the SM leptons (with the addition of right-handed neutrinos) and the Higgs are described in table 3.1.

The Higgs couples through Yukawa-couplings to the leptons, L and R:

$$\mathcal{L}_Y = f_{ij}\bar{r}_i\phi^\dagger l_j + f^*_{ij}\bar{l}_i\phi r_j \, .$$

Note that each term respects the gauge group symmetry: A doublet is associated with a hermitian conjugate of another doublet, and the total sum of T^3 and Y charges of each term is zero.

In the electroweak symmetry breaking the Higgs field's potential has a minimum at $\phi \neq 0$ so that the field gets a vacuum expectation value (VEV) $\langle\phi\rangle = v \neq 0$. The Yukawa terms then transform into mass terms for the fermions and their couplings to the physical Z^0 and $W^\pm$ bosonic gauge fields. As there are no right-handed neutrinos in the SM, there are no mass terms for the neutrinos.

3.4 Addition of the neutrino masses to the SM

Neutrino oscillation experiments have shown that the neutrinos have a mass difference between generations. This then implies that they must have masses.

To generate a mass for the neutrinos without requiring the existence of new (right-handed) fields would come about by putting in a Majorana mass term for the usual left-handed neutrinos:

$$+\overline{\nu^c_L} M^M \nu_L \tag{3.8}$$

This term however breaks the $SU(2) \otimes U(1)$ invariance. Therefor if neutrinos are massive, the principle of gauge invariance requires the existence of right-handed neutrinos.

The next-to-minimal way to generate the neutrino masses would be then to include a set of right-handed neutrinos and give them Yukawa couplings to L. Thus we define the right-handed SU(2)-singlet

$$N \equiv (\nu)_R = (\nu_e \ \nu_\mu \ \nu_\tau)_R$$

and we write the usual Higgs doublet in another form,

$$\tilde{\phi} \equiv i\sigma_2 \phi^* = \begin{pmatrix} \bar{\phi}^0 \\ -\phi^- \end{pmatrix},$$

which is still a SU(2)-doublet. Using these we can add an additional term to the SM Lagrangian:

$$\mathcal{L}_N = h_{ij}\tilde{\phi}^\dagger \overline{N}_i l_j + h_{ij}^* \bar{l}_i N_j \tilde{\phi} \tag{3.9}$$

As before, the Higgs acquires a VEV breaking the EW-gauge symmetry spontaneously and mass terms and couplings to the gauge fields emerge from the Yukawa couplings. If we denote the VEV of the Higgs by v, the total EW-sector of the Lagrangian reads as

$$\begin{aligned} \mathcal{L}_Y + \mathcal{L}_N \quad = \quad & h_{ij} v \overline{N}_{Ri} \nu_{Lj} + h_{ij} \overline{\nu}_{Li} N_{Rj} H \\ & + (\text{mass terms for charged leptons}) \\ & + (\text{couplings to } Z^0 \text{ and } W^\pm)\,, \end{aligned}$$

where H is the physical Higgs boson. If the h-matrix is diagonal, then we can simply read the Dirac masses of the i:th neutrino to be $m_{Di} = h_{ii} v$.

The gauge invariance prohibited us from adding a Majorana mass term for the left-handed neutrinos, however, nothing stops us from adding such a term for the right-handed (singlet) neutrinos, since they are invariant under the EW $SU(2) \otimes U(1)$:

$$\mathcal{L}_M = -\frac{1}{2}\overline{N^C} M^M N$$

The Majorana mass M^N is in general a $N_g \times N_g$ -matrix in flavour space where N_g is the number of generations.[3]

Using a slightly different notation we can rewrite the Dirac and Majorana mass terms together as a single term:

$$\mathcal{L}^{M+D} = \begin{pmatrix} \overline{\nu_L^C}, & \overline{N} \end{pmatrix} \begin{pmatrix} M_L^M & m^D \\ m^D & M_R^M \end{pmatrix} \begin{pmatrix} \nu_L \\ N^C \end{pmatrix} + \text{h.c.} \tag{3.10}$$

Here the ν should still be a N_g-vector in the flavour space and m^D, M_L^M and M_R^M $N_g \times N_g$ matrices, however, we have suppressed this index and/or consider only a single flavour. As mentioned before, the EW gauge invariance

[3] Here we have assumed that there are as many flavours of right-handed neutrinos as there are left-handed neutrinos. Nothing a priori requires us to assume that, though for the purposes of this work at least two flavours are required.

requires the M_L^M to be zero, so what remains to characterize the neutrino masses are two ($N_g \times N_g$ matrix) parameters.

Since the Majorana mass term is not invariant under the U(1) transformation of eq. (3.5), the new mass term does not conserve the related current. For charged particles this would result into the non-conservation of electric charge, but for neutrinos this is not a problem since they are electrically neutral. On the other hand, this does violate lepton number — in fact the addition of a Majorana mass term is the minimal way to add (classical) lepton number violation to the standard model.

3.5 See-saw mechanism

The Lagrangian of eq. (3.10) has two free parameters: the Dirac mass and the Majorana mass. The Dirac mass is the product of the Higgs' VEV v and the associated Yukawa couplings h_{ij}. Since it has exactly the same form as other Dirac masses of the SM, we would expect it to be at least of the same order of magnitude as the masses of other fermions. However, oscillation and cosmological data indicate smaller mass scale for the neutrinos by several orders of magnitudes.

To solve this hierarchy problem we suppose that both the m^D and $M_R^M \neq 0$ and the mass matrix of 3.10 is thus non-diagonal. (Since we have put $M_L^M = 0$, we can omit the redundant indices and denote the Dirac mass by m and the Majorana mass of the right-handed fields by M.) We however observe mass eigenstates so we diagonalize the mass matrix

$$\begin{vmatrix} -\lambda & m \\ m & M-\lambda \end{vmatrix} = \lambda^2 - M\lambda - m^2 = 0$$

$$\lambda = \frac{M \pm \sqrt{M^2+4m^2}}{2} = \frac{M}{2}\left(1 \pm \left(1 + \frac{4m^2}{M^2} + \mathcal{O}(\frac{m^3}{M^3})\right)\right)$$

where we have assumed that $\frac{m}{M} \ll 1$ and expanded in this small parameter. This results in two mass eigenstates with the eigenvalues

$$m_\nu \simeq -\frac{m^2}{M} \text{ and } m_N \simeq M\ .$$

If one assumes that the Dirac mass term is, in accordance with other mass terms in the SM, $\sim \mathcal{O}(1\text{MeV})$, and that the Majorana mass term is $\sim \mathcal{O}(10^{16}\text{GeV})$, a scale for GUT physics, then one gets as the scale of the lighter neutrinos

$$M_\nu = \mathcal{O}(0.1\text{eV})\ ,$$

which fits the experimental upper limit of the neutrino mass quite nicely. This so called *see-saw mechanism* hence explains naturally away the hierarchy problem related to the smallness of the observed neutrino mass.

It should be pointed out, however, that the see-saw mechanism doesn't explain the size of the neutrino masses — it merely transfers the problem to the larger GUT scale, hoping that possible GUT-scale physics is able to predict the heaviness of the Majorana mass term, and the values of the Dirac mass terms.

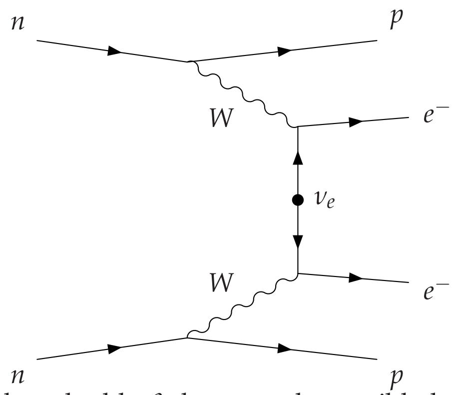


Figure 3.2: Neutrinoless double β-decay made possible by a Majorana mass term.

3.6 Consequences of the Majorana mass terms

The standard model Lagrangian is of course gauge invariant. It can be constructed by specifying the fermion content and then including all the terms allowed by gauge-invariance. The neutrino oscillation experiments require us to add right-handed neutrinos, and when adding them we must add all gauge-invariant terms involving them. Hence the addition of the Majorana mass term can be required merely by consistency.

Set aside the explicit gauge symmetry, the SM Lagrangian possesses additional symmetries. For example the transformation of eq. (3.5) applied only on the leptons or the quarks leaves the Lagrangian invariant. The associated Noether current then says that the lepton and baryon number are conserved.[4]

Our SM extended with the Majorana neutrinos is no longer invariant under the transformation of eq. (3.5). Therefore the Majorana term for the right-handed fields violates the conservation of the associated quantum number, the lepton number.

This violation of lepton number has observational consequences. The most easily observable effect might be neutrinoless beta-decay, which is illustrated in figure 3.2. In the figure the dot in the neutrino line represents the Majorana mass-term responsible for the change of a neutrino to its antineutrino. The process is suppressed by the mass-scale of the heavy neutrino, so the rate for this process is small, but still observable.

[4]This is a global symmetry of the SM Lagrangian, and indeed B and L are both conserved at the tree level. However, next-to-tree level the EW anomaly breaks the B and L, resulting only in $B - L$ left conserved.

4

An interlude: B-number violating sphaleron processes

The standard model is a gauge field theory with a gauge symmetry group $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{SU}(1)$. This symmetry is *required* from the theory and no observable can depend on the choice of gauge. However, the standard model Lagrangian has other, "accidental" symmetries as well — some of them exact and some of them only approximate.

Two exact symmetries of the Lagrangian are the $\mathrm{U}(1)_\mathrm{L}$ and $\mathrm{U}(1)_\mathrm{B}$ symmetries coming from the freedom of rotating the phase of all lepton or quark fields. Hence they have associated conserved currents, the leptonic and baryonic currents J_μ^B and J_μ^L. If we define the quark doublets and singlets in analogue with eq. (3.6) and (3.7) as

$$\begin{aligned} q &\equiv \begin{pmatrix} u & s & t \\ d & c & b \end{pmatrix}_L \\ u &\equiv \begin{pmatrix} u & s & t \end{pmatrix}_L \\ d &\equiv \begin{pmatrix} d & c & b \end{pmatrix}_L , \end{aligned}$$

then the baryonic and leptonic currents are given by

$$\begin{aligned} J_\mu^B &= \frac{1}{3} \left(\bar{q}_i \gamma_\mu q_i - \bar{u}_i \gamma_\mu u_i - \bar{d}_i \gamma_\mu d_i \right) \\ J_\mu^L &= \bar{l}_i \gamma_\mu l_i - \bar{r}_i \gamma_\mu r_i . \end{aligned}$$

These correspond to the baryon and lepton number mentioned in previous chapters. This classical symmetry is responsible for the observational fact that baryon and lepton number are conserved to extremely high precision — in fact no laboratory experiment has recorded violation of baryon or lepton number.

In 1969 it was realised [16] [17] that through the Adler-Bell-Jackiw-triangle-anomaly these symmetries are nevertheless broken, and as a result the baryonic and leptonic currents are anomalous. Their divergences are then given by

$$\partial_\mu J_B^\mu = \partial_\mu J_L^\mu = \frac{N_g}{32\pi^2} \left(g^2 \,\mathrm{Tr}\, W_{\mu\nu} \tilde{W}^{\mu\nu} - g'^2 \,\mathrm{Tr}\, B_{\mu\nu} \tilde{B}^{\mu\nu} \right) , \tag{4.1}$$

where N_g is the number of fermion generations, $W_{\mu\nu}$ and $B_{\mu\nu}$ are the field tensors of the SU(2) and U(1) fields, g and g' are their associated coupling constants and trace is taken over the group index. We have also defined the dual of a field tensor by

$$\tilde{W}^{\mu\nu} \equiv \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}W_{\alpha\beta} .$$

The traces of the field tensors on the RHS of the equation (4.1) can be written as a divergences of two quantities,

$$\begin{aligned} K_\mu &\equiv \epsilon^{\mu\nu\alpha\beta}\left(W^a_{\nu\alpha}A^a_\beta - \frac{1}{3}g\epsilon_{abc}A^a_\nu A^b_\alpha A^c_\beta\right) \\ k_\mu &\equiv X_{\nu\alpha}A'_\beta , \end{aligned}$$

where the one-index tensors A_μ and A'_μ refer to the vector potentials of the SU(2) and U(1) fields. Then we could define a new current which would have a vanishing divegence,

$$\partial_\mu\left(J^\mu_L - \frac{N_g g^2}{32\pi^2}K^\mu + \frac{N_g g'^2}{32\pi^2}k^\mu\right) = 0 .$$

It would be tempting to define this current as the new baryonic and leptonic number, understanding the addition as baryon and lepton number carried by the gauge fields. This is however not possible, since k_μ and K_μ are not gauge invariant.

As we are interested in the time-evolution of the baryon and lepton numbers, we consider their evolution from an inital time $t = 0$ to some point of time by defining the *change* in the baryon number [18]

$$\Delta B(t) \equiv N_g\left[N_{CS}(t) - N_{CS}(0)\right] ,$$

where we use the Chern-Simons numbers of the SU(2) gauge field

$$N_{CS} = \frac{g^2}{32\pi^2}\int \mathrm{d}x^3\epsilon^{\mu\nu\sigma}\left(W^a_{\mu\nu}A^a_\sigma - \frac{g}{3}\epsilon_{abc}A^a_\mu A^b_\nu A^c_\sigma\right) .$$

Even though the definition of K_μ was not gauge invariant, the *change* of the divergence of K_μ is gauge invariant. The different values of the Chern-Simons number correspond to different vacuum configurations of the gauge field. The value of the Chern-Simons number in pure gauge configurations, corresponding to vacuum, is an integer, and thus the change in the baryon (and lepton) number corresponding to the change of the vacuum of the gauge field is $N_g \times$ integer: both the baryon and lepton numbers change in multiples of three. This anomaly induces a 12-fermion operator of the form

$$\sum_{i=1}^{3}(q_iq_iq_il_i) ,$$

coupling to all left-handed fermions. Since the divergences of J^B_μ and J^L_μ are the same, $B - L$ is conserved.

In 1976 't Hooft published an article [19] in which he estimated the rate of these baryon number violating processes. He considered the instanton solution between two separate vacua and calculated the action associated witht the saddle-point configuration between them. This field configuration is called the *sphaleron*, from the Greek word meaning ready to fall, as the saddle-point configuration is inherently unstable. The propability of tunneling between the different vacua is approximately

$$\Gamma \sim e^{-S_{inst}} = e^{-\frac{4\pi}{\alpha}} = \mathcal{O}(10^{-170}) \ .$$

This rate is so infintely small that the sphaleron process is in no contradiction with the practical observation of the lack of violation of B or L.

In 1985 Kuzmin, Rubakov and Shaposhnikov [20] pointed out that in thermal bath the transition can happen instead of quantum mechanical tunneling by classical thermal fluctuations over the potential barrier. If the temperature is above the saddle-point energy between the different vacua then the exponential Boltzmann suppression disappears completely.

The energy of the saddle-point configuration can be estimated by the sphaleron configurations. Below the electroweak phase transition temperature ($T < T_{EW}$) the transition rate per unit volume was found to be

$$\frac{\Gamma_{\text{sph}}}{V} \sim e^{-\frac{M_W}{\alpha T}} \ ,$$

which is still very much suppressed. In the symmetric phase $T > T_{EW}$, however, the transition rate is no longer suppressed, but rather [21]

$$\frac{\Gamma_{\text{sph}}}{V} \sim \alpha^5 \ln \alpha^{-1} T^4 \ .$$

Sphaleron processes can be in equilibrium when the sphaleron rate Γ_{sph} exceeds the expansion rate of the universe H. Comparison of the estimate for Γ_{sph} to H in radiation dominated universe gives as the temperature interval when sphalerons were in equilibrium as [22]

$$100\,\text{GeV} < T < 10^{13}\text{GeV} \ .$$

Following the terminology of literature, we call the entire $B + L$-violating process the sphaleron process, even though the sphaleron specifically refers only to the unstable saddle-point configuration.

5
Leptogenesis

Long before the impossibility of electroweak baryogenesis became apparent, Fukugita and Yanagida in 1986 proposed an alternative [23] for GUT-baryogenesis. Since lower limits on the lifetime of the proton from proton decay experiments limit the GUT-produced B-number violation to be too small for the cosmic baryon asymmetry, they proposed to produce the B-number using the sphaleron process, which is exponentially suppressed at low energies, and therefor does not suffer from the limits of the proton lifetime experiments. To produce the sufficient L for the sphaleron process to be convert into B, they suggested decays of particles with Majorana mass. This is the basic idea for thermal leptogenesis.

5.1 The simple model

To produce successful leptogenesis, we introduce to the standard model three heavy right-handed neutrinos with Majorana mass terms, N_1, N_2 and N_3:

$$\mathcal{L} = \mathcal{L}_{SM} + \bar{N} i \not{\partial} N + M_i \overline{N^C}_i N_i + h_{ij} \bar{N}_i \tilde{\phi}^\dagger l_j + h^*_{ji} \bar{l}_i \tilde{\phi} N_j \,. \tag{5.1}$$

The Yukawa couplings h_{ij} can in general be complex and of the same order than other Yukawa couplings in the SM. We assume that in our simple model the Majorana masses are hierarchical, ie. $M_1 < M_2 < M_3$, and that they are of the GUT scale. This guarantees successfull see-saw mechanism producing left-handed neutrinos with the correct mass scale.

5.2 Sakharov's conditions in Leptogenesis

Like all theories attempting to offer an explanation for baryogenesis, leptogenesis has to fulfill all Sakharov's conditions. In fact the minimal extension of the standard model in eq. (5.1) is sufficient to fulfill Sakharov's condition much more drastically than the bare SM.

B-number violation

Baryon number violation is satisfied in leptogenesis by the same mechanism as in EW baryogenesis, by the $B + L$ violating sphalerons. The sphalerons are in equilibrium in the temperature range $100\,\text{GeV} < T < 10^{13}\text{GeV}$, so the excess lepton number to be converted to baryon number has to be produced at sufficiently high temperatures.

C and CP violation

C violation is satisfied already maximally in the standard model through the electroweak sector, since the EW gauge fields couple only to left-handed fields.

CP violation is featured in the standard model in the complex Yukawa couplings of the fermions.[1] Similarly we assume that the Yukawa couplings of the heavy neutrinos we have introduced to the SM are also complex. These complex phases cannot be absorbed to the redefinition of the lepton fields unlike in plain SM, and they produce CP violation in the decay amplitudes of the heavy neutrinos. This CP violation in the decay amplitude comes from the interference of the one-loop diagrams of the decays and requires the existence of Majorana neutrinos with different masses, as is explained in appendix A.4.

Departure from thermal equilibrium

In leptogenesis the departure from thermal equilibrium is provided, as in many forms of GUT-baryogenesis, by the decays of heavy but relatively weakly interacting particles — in our case the heavy neutrinos. The equilibrium density of the heavy neutrinos drops abruptly as the temperature drops below their mass. If the neutrinos interact weakly enough, they decay so slowly that for some time their number density is significantly larger than the equilibrium density, assuming of course they are in equilibrium at sufficiently high temperatures.

Here the criteria of the interaction to be weak enough is related to the expansion rate of the universe. In practice this requirement is given by

$$\Gamma_D|_{T=M} < H|_{T=M} \,,$$

where Γ_D is the thermally averaged decay rate of the heavy neutrino.

5.3 Thermal history of Leptogenesis

The story of leptogenesis begins right after inflation ends, at the reheating temperature which we assume to be larger than the mass of N_1, or $T_{\text{reh}} > M_1$.

At $\mathbf{T \lesssim T_{\text{reh}}}$ inflation ends and the energy from inflaton decays to other fields. Inverse decays produce large amount of N_1 and their abundance approaches their equilibrium value. The possible lepton asymmetry produced from these inverse decays is quickly washed out by $\Delta L = 1$ and $\Delta L = 2$ scatterings producing a universe near thermal equilibrium with $B = L = 0$.

[1]The complex quark couplings are usually absorbed to the redefinition of the quark fields to produce the CKM-matrix describing the mixing between different flavours of quarks with one complex phase to describe CP-violation.

At $\mathbf{T \sim M_1}$ the equilibrium density of the heavy neutrinos drop suddenly but the neutrinos do not decay fast enough to maintain their equilibrium density.

As $\mathbf{T < M_1}$ the heavy neutrinos decay out-of-equilibrium. Their decays violate CP, and hence produce more antileptons than leptons. As soon as any net lepton number is produced, $\Delta L = 1$ and $\Delta L = 2$ scatterings start pushing the lepton number back to zero competing with the rate of the decays.

When $\mathbf{T < M_1}$ and $\mathbf{T < 10^{13}}$ GeV the sphalerons start to convert the lepton number to baryon number. As temperature lowers sufficiently, decays and washout processes freeze out and the sphalerons are fast enough to keep the baryon number at its equilibrium value compared to the lepton number.

At the EW scale $\mathbf{T \sim T_{EW}}$ the electroweak symmetry breaks spontaneously and all Dirac fermions acquire mass. At the same scale the sphalerons freeze out due to the Boltzmann suppression, and all baryon number violating processes are suppressed from this point on. The baryon number per comoving volume hence freezes out to its current value.

6

Boltzmann equations for leptogenesis

Next we proceed to treat the theory of leptogenesis quantitatively. In principle we should take into account all B- and L-violating processes. In this treatise, however, we consider only decays, inverse decays, $\Delta L = 2$ scatterings and the sphalerons. All rates are treated at the tree level only, except the CP-violating amplitude of the decays ϵ_{D}.

Furthermore we assume $M_1 \ll M_{2,3}$, so that we can safely ignore the decays of the two heaviest Majorana neutrino flavours. Their only contribution to this treatment is their off-shell contributions to the one loop corrections of the decay amplitude, producing the CP-violation in N_1 decay. Thus from here on we refer to the lightest heavy Majorana neutrino as merely N, omitting the subscript.

6.1 Boltzmann equations and formalism

Leptogenesis is inherently an out-of-equilibrium process. For this reason we describe it by Boltzmann equations. In general, the differential equation describing the evolution of the abundance of particle species X is given by

$$\dot{n_X} + 3\frac{\dot{R}}{R} = -\sum [Xa \ldots \leftrightarrow ij \ldots] , \tag{6.1}$$

where

$$\begin{aligned} [Xa \ldots \leftrightarrow ij \ldots] \quad \equiv \quad & \frac{n_X n_a \ldots}{n_X^{\mathrm{eq}} n_a^{\mathrm{eq}} \ldots} \gamma^{\mathrm{eq}}(Xa \ldots \to ij \ldots) \\ & - \frac{n_i n_j \ldots}{n_i^{\mathrm{eq}} n_j^{\mathrm{eq}} \ldots} \gamma^{\mathrm{eq}}(ij \ldots \to Xa \ldots) . \end{aligned}$$

Here γ^{eq} is the space time density of scatterings in thermal equilibrium, which can be understood as a generalization of the decay rate Γ. For a process $Xa \to ij$, the density of scatterings is given by

$$\gamma^{\mathrm{eq}}(na \to ij) \equiv \int \mathrm{d}\vec{p}_X \mathrm{d}\vec{p}_a f_X f_a \int \mathrm{d}\vec{p}_i \mathrm{d}\vec{p}_j (2\pi)^4 \delta^4(P_X + P_a - P_i - P_j) |\mathcal{M}|^2 , \tag{6.2}$$

where the integral measure is defined as

$$\mathrm{d}\vec{p}_X \equiv \frac{\mathrm{d}^3 p_X}{2E_X(2\pi)^3} \,, \tag{6.3}$$

$|\mathcal{M}|^2$ is the transition amplitude squared summed over spin states and f_X is the distribution function. This is generalized to processes involving more particles in the obvious way, by adding more phase-space integrals to eq. (6.2).

The Boltzmann equation of eq. (6.1) is entirely classical. Quantum corrections become relevant only when the mean distance between collisions is shorter than the wavelengths of the particles [11]. The second term of the LHS of equation (6.1) describes the dilution of the number density due to expansion of the universe. The equation can be simplified if, instead of the physical number density, we choose as our variable the number density in a comoving volume. We choose the comoving volume so that it contains one photon at time $t_\star$, where $t_\star$ is an epoch before the onset of leptogenesis, following the notation of [24]:

$$\begin{aligned} N_X(t) &\equiv n_X(t) R_\star(T)^3 \\ R_\star(t_\star) &= (n_\gamma^{\mathrm{eq}}(t_\star))^{-\frac{1}{3}} \end{aligned} \tag{6.4}$$

Since

$$\frac{dN_X}{dt} = \dot{n}_X R^3 + 3 n_X R^2 \dot{R} \,,$$

we can rewrite eq. (6.1) as

$$\frac{dN_X}{dt} = -R_\star(t)^3 \sum [Xa \ldots \leftrightarrow ij \ldots] \tag{6.5}$$

It is often easier to solve the equations as a function of temperature then time. For that end, we define the inverse of temperature in terms of the mass of the heavy neutrino

$$z \equiv \frac{m_N}{T} \,,$$

which should not be confused with the cosmologial redshift which is often denoted by the same symbol.

For radiation, entropy and energy density behave as

$$\begin{aligned} s_\gamma &= \frac{4\rho_\gamma}{3T} \\ \rho_\gamma &= \frac{\pi^2 T^4}{15} \,. \end{aligned}$$

Here γ refers to radiation in general, not specifically to photons. In adiabatic expansion of the universe the comoving entropy density stays constant,

$$\frac{d(sR^3)}{dt} = 0 \,.$$

In the radiation dominated and adiabatically expanding universe we then get from the conservation of entropy

$$0 = \frac{d(s_\gamma R^3)}{dt} = \frac{d}{dt}\frac{4\rho_\gamma R^3}{3T} = \frac{d}{dt}\frac{4\pi^2 T^3 R^3}{45} \;\Rightarrow T \propto \frac{1}{R} \,.$$

Then

$$\frac{dN}{dt} = \frac{dz}{dt}\frac{dN}{dz} = z\frac{d\ln z}{dt}\frac{dN}{dz} = z\frac{d\ln R}{dt}\frac{dN}{dz} = \frac{z}{R}\frac{dR}{dt}\frac{dN}{dz} = zH\frac{dN}{dz}$$

and we can rewrite eq. (6.5) as

$$zH\frac{dN_X}{dz} = -R_\star(t)^3 \sum [Xa\ldots \leftrightarrow ij\ldots] \ . \tag{6.6}$$

6.2 Boltzmann equation for N_N

For the number density of N's, we need to consider only decays, $N \rightarrow l\tilde{\phi}$ and $N \rightarrow \bar{l}\tilde{\phi}^\dagger$, and inverse decays, $l\tilde{\phi} \rightarrow N$ and $\bar{l}\tilde{\phi}^\dagger \rightarrow N$. Thus eq. (6.6) for N_N becomes

$$zH\frac{dN_N}{dz} = -R_\star(t)^3 \left\{ [N \leftrightarrow l\tilde{\phi}^\dagger] + [N \leftrightarrow \bar{l}\tilde{\phi}] \right\} \ .$$

In a decay processes, $N \rightarrow ij$, the expression 6.2 is simplified considerably [32],

$$\gamma^{\mathrm{eq}}(N \rightarrow ij\ldots) = \gamma^{\mathrm{eq}}(ij\ldots \rightarrow N) = n_N^{\mathrm{eq}}\frac{K_1(z)}{K_2(z)}\Gamma_N \ , \tag{6.7}$$

where Γ_N is the decay width of N in its rest frame, K_n is the n'th modified Bessel function and $z \equiv m_N/T$. Here the factor K_1/K_2 is a time dilatation factor, resulting from the fact that not all particles are at rest, but are rather distributed according to their distribution function f_N. In deriving eq. (6.7), we have approximated f_N by Maxwell-Boltzmann distribution, ie., assumed that the density of particles is small enough to make the quantum corrections (stimulated emission, Fermi blocking) insignificant.

Substituting this into eq. (6.5), we get

$$zH\frac{dN_N}{dz} = -R_\star^3(t)\frac{K_1(z)}{K_2(z)}\Gamma_N \left(n_N - n_N^{\mathrm{eq}}\frac{n_i n_j\ldots}{n_i^{\mathrm{eq}} n_j^{\mathrm{eq}}\ldots} \right) \ . \tag{6.8}$$

The system is out of equilibrium because of the decays of the heavy neutrinos, nevertheless it can be assumed that other lepton number consreving scatterings are fast enough to keep the rest of the system in equilbrium — otherwise we wouldn't need the heavy neutrinos to satisfy Sakharov's conditions! Consequently the distributions of other particle species is their equilibrium value, and eq. (6.8) reduces to

$$zH\frac{dN_N}{dz} = -\frac{K_1(z)}{K_2(z)}\Gamma_N \left(N_N - N_N^{\mathrm{eq}} \right) \ .000$$

6.3 Boltzmann equation for N_{B-L}

To derive the Boltzmann equation governing the time-evolution of N_{B-L} we would need take into account all processes violating $B-L$. The only B-violating process is the sphaleron process which does not violate $B-L$. For

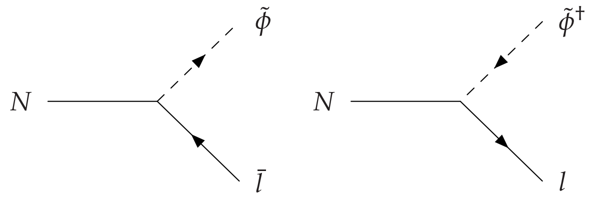


Figure 6.1: The two possible decay channels of the heavy right handed neutrino N.

the lepton number all processes involving the heavy neutrino can violate lepton number, and since they conserve the baryon number, they violate $B-L$.

In literature usually only decays, inverse decays, and $\Delta L = 1$ and $\Delta L = 2$ scatterings at the tree level are taken into account. Even then the derivation of the terms means a lot of effort, aside from solving the equation. Here we neglect the effect of $\Delta L = 1$ scatterings, involving the top quark, and consider only the effect of the lepton sector.

Since we assign no conserved quantum number to the right-handed neutrinos, decays and inverse decays affect the $B-L$ by creating (destroying) a lepton or an antilepton. These are pictured in figure 6.1. $\Delta L = 2$-scatterings are scatterings of leptons and Higgses which are mediated by the right-handed neutrinos. They are all shown at the tree level in figure 6.2.

We start from the general expression for the classical Boltzmann equation eq. (6.6), where we have already transformed into the new variable the amount of $B-L$ per comoving volume N_{B-L}, and expressed this as a function of the variable $z \equiv \frac{M_N}{T}$:

$$\begin{aligned} zH\frac{dN_{B-L}}{dz} &= -R_\star(t)^3\Big\{[N \leftrightarrow l\tilde{\phi}^\dagger] - [N \leftrightarrow \bar{l}\tilde{\phi}] \\ &\quad -2[ll \leftrightarrow \tilde{\phi}\tilde{\phi}] - 2[l\tilde{\phi}^\dagger \leftrightarrow \bar{l}\tilde{\phi}] + 2[\bar{l}\bar{l} \leftrightarrow \tilde{\phi}^\dagger\tilde{\phi}^\dagger]\Big\} \end{aligned} \tag{6.9}$$

Here decays and inverse decays are taken into account by the terms in the RHS in the first line, and $\Delta L = 2$ scatterings are taken into account by the second line. The factor of 2 in the scattering terms comes from the fact that L and also $B-L$ change by a factor of 2 in each scattering with respect to decays.

Using the definition of the [ ]-expression, we can write

$$[N \leftrightarrow l\tilde{\phi}^\dagger] = \frac{n_N}{n_N^{\text{eq}}}\gamma^{\text{eq}}(N \to l\tilde{\phi}^\dagger) - \frac{n_l n_{\tilde{\phi}^\dagger}}{n_l^{\text{eq}} n_{\tilde{\phi}^\dagger}^{\text{eq}}}\gamma^{\text{eq}}(l\tilde{\phi}^\dagger \to N) \tag{6.10}$$

$$[N \leftrightarrow \bar{l}\tilde{\phi}] = \frac{n_N}{n_N^{\text{eq}}}\gamma^{\text{eq}}(N \to \bar{l}\tilde{\phi}) - \frac{n_{\bar{l}} n_{\tilde{\phi}}}{n_{\bar{l}}^{\text{eq}} n_{\tilde{\phi}}^{\text{eq}}}\gamma^{\text{eq}}(\bar{l}\tilde{\phi} \to N)\,. \tag{6.11}$$

We know that CP is violated, ie., all the gammas in the RHS of the equations 6.10 and 6.11 are not equal. However, from CPT invariance, we know

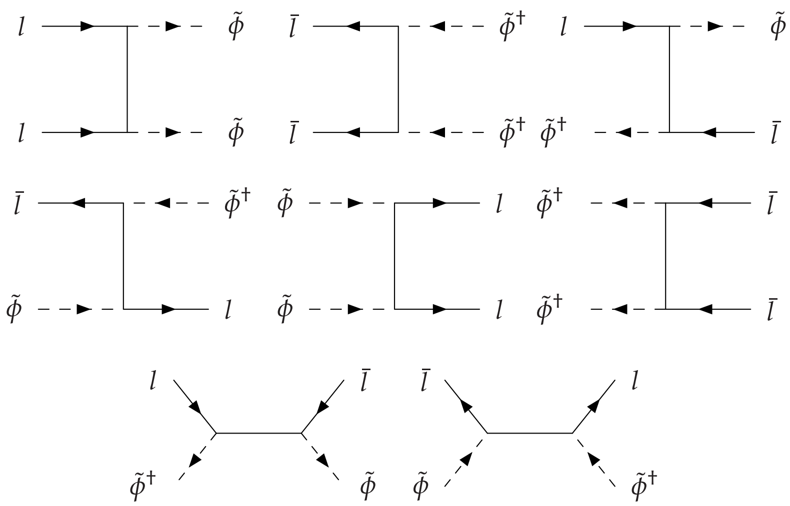


Figure 6.2: The $\Delta L = 2$ processes contributing to washout considered for the equation of $B - L$.

that

$$\gamma^{\mathrm{eq}}(N \to l\tilde{\phi}^\dagger) = \gamma^{\mathrm{eq}}(\bar{l}\tilde{\phi} \to N) \equiv (1 + \epsilon_{\mathrm{D}})\frac{\gamma_{\mathrm{D}}}{2} \tag{6.12}$$

$$\gamma^{\mathrm{eq}}(N \to \bar{l}\tilde{\phi}) = \gamma^{\mathrm{eq}}(l\tilde{\phi}^\dagger \to N) \equiv (1 - \epsilon_{\mathrm{D}})\frac{\gamma_{\mathrm{D}}}{2}\ , \tag{6.13}$$

where we have reparametrized the γ's by introducing the total decay rate γ_{D}, and ϵ_{D} which measures the amount of CP-violation in the decays.

When deriving the Boltzmann equation for N_N we approximated all other abundances of particles to be their equilibrium value, except the right-handed neutrino. Now since we are producing an excess amount of L, we take also the abundance of lepton doublets to be out-of-equilibrium. Hence we parameterize

$$\frac{n_l}{n_l^{\mathrm{eq}}} = 1 + X_l \tag{6.14}$$

$$\frac{n_{\bar{l}}}{n_{\bar{l}}^{\mathrm{eq}}} = 1 + X_{\bar{l}}\ , \tag{6.15}$$

where X_l and $X_{\bar{l}}$ are now first-order small in ϵ_{D}.[1]

Using equations (6.12) - (6.15) we can rewrite equations (6.10) and (6.11)

$$[N \leftrightarrow l\tilde{\phi}^\dagger] - [N \leftrightarrow \bar{l}\tilde{\phi}] = \epsilon_{\mathrm{D}}\gamma_{\mathrm{D}}(\frac{n_N}{n_N^{\mathrm{eq}}} + 1) - \frac{\gamma_{\mathrm{D}}}{2}(X_l - X_{\bar{l}})\ , \tag{6.16}$$

where we have already neglected second order terms in ϵ_{D}.

[1]This is not necessarily clear at this stage. However, it is evident that the departure of the number of (anti-)leptons from its equilibrium value is minute compared to the equilibrium value.

Next we consider the $\Delta L = 2$ -terms in eq. (6.9). We again approximate the $n_{\tilde{\phi}}$ to be its equilibrium value, and write

$$\begin{aligned}
[ll \leftrightarrow \tilde{\phi}\tilde{\phi}] &= (1+X_l)^2\gamma^{\rm eq}(ll \to \tilde{\phi}\tilde{\phi}) - \gamma^{\rm eq}(\tilde{\phi}\tilde{\phi} \to ll) && (6.17)\\
[l\tilde{\phi}^\dagger \leftrightarrow \bar{l}\tilde{\phi}] &= (1+X_l)\gamma^{\rm eq}(l\tilde{\phi}^\dagger \to \bar{l}\tilde{\phi}) - (1+X_{\bar{l}})\gamma^{\rm eq}(\bar{l}\tilde{\phi} \to l\tilde{\phi}^\dagger) && (6.18)\\
[\bar{l}\bar{l} \leftrightarrow \tilde{\phi}^\dagger\tilde{\phi}^\dagger] &= (1+X_{\bar{l}})^2\gamma^{\rm eq}(\bar{l}\bar{l} \to \tilde{\phi}^\dagger\tilde{\phi}^\dagger) - \gamma^{\rm eq}(\tilde{\phi}^\dagger\tilde{\phi}^\dagger \to \bar{l}\bar{l})\ , && (6.19)
\end{aligned}$$

When we calculate the $\Delta L = 2$ processes at tree level, we get CP-conserving amplitudes, and we can write

$$\begin{aligned}
&\gamma^{\rm eq}(ll \to \tilde{\phi}^\dagger\tilde{\phi}^\dagger) = \gamma^{\rm eq}(\tilde{\phi}^\dagger\tilde{\phi}^\dagger \to ll) = \gamma^{\rm eq}(\bar{l}\bar{l} \to \tilde{\phi}\tilde{\phi}) = \gamma^{\rm eq}(\tilde{\phi}\tilde{\phi} \to \bar{l}\bar{l}) \equiv \gamma_t\\
&\gamma^{\rm eq}(l\tilde{\phi}^\dagger \to \bar{l}\tilde{\phi}) = \gamma^{\rm eq}(\bar{l}\tilde{\phi} \to l\tilde{\phi}^\dagger) \equiv \gamma_s
\end{aligned}$$

where t and s refer to t- and s-channels in Mandelstam variables.[2]

By comparing figures 6.1 and 6.2 we notice that the last two Feynman diagrams of the $\Delta L = 2$-scatterings look like a decay followed by an inverse decay. Indeed, $l\tilde{\phi}^\dagger \leftrightarrow \bar{l}\tilde{\phi}$ can be mediated by an on-shell N. However, since we have included decays and inverse decays already from γ_s, we must now subtract these on-shell contributions, to avoid over-counting.

The contributions to γ_s due to a (real) on-shell N as the mediating particle are

$$\begin{aligned}
\gamma^{\rm eq}_{on-shell}(l\tilde{\phi}^\dagger \to \bar{l}\tilde{\phi}) &= \gamma^{\rm eq}(l\tilde{\phi}^\dagger \to N)BR(N \to \bar{l}\tilde{\phi})\\
\gamma^{\rm eq}_{on-shell}(\bar{l}\tilde{\phi} \to l\tilde{\phi}^\dagger) &= \gamma^{\rm eq}(\bar{l}\tilde{\phi} \to N)BR(N \to l\tilde{\phi}^\dagger)\ ,
\end{aligned}$$

where BR refers to the branching ratio:

$$\begin{aligned}
BR(N \to \bar{l}\tilde{\phi}) &= \frac{1-\epsilon_{\rm D}}{2}\\
BR(N \to l\tilde{\phi}^\dagger) &= \frac{1+\epsilon_{\rm D}}{2}\ .
\end{aligned}$$

From this we get the subtracted amplitude

$$\begin{aligned}
[l\tilde{\phi}^\dagger \leftrightarrow \bar{l}\tilde{\phi}]^{\rm sub} &= (1+X_l)\left(\gamma_s - (1-\epsilon_{\rm D})^2\frac{\gamma_{\rm D}}{4}\right)\\
&\quad -(1+X_{\bar{l}})\left(\gamma_s - (1+\epsilon_{\rm D})^2\frac{\gamma_{\rm D}}{4}\right) && (6.20)\\
&= \left(\gamma_s + \frac{\gamma_{\rm D}}{4}\right)(X_l - X_{\bar{l}}) + \gamma_{\rm D}\epsilon_{\rm D} && (6.21)\\
&\quad -\frac{\gamma_{\rm D}\epsilon_{\rm D}}{2}(X_l + X_{\bar{l}})\ , && (6.22)
\end{aligned}$$

where the last term is already second order small so we can ignore it.

Combining equations (6.16), (6.17), (6.19) and (6.20) into eq. (6.9) we get

$$zH\frac{dN_{B-L}}{dz} = -R_\star^3(t)\left\{\epsilon_{\rm D}\gamma_{\rm D}\left(\frac{n_N}{n_N^{\rm eq}} - 1\right) - (X_l - X_{\bar{l}})(\gamma_{\rm D} + 2\gamma_s + 4\gamma_t)\right\}\ ,$$

[2]The γ_s actually includes both t- *and* s-channel processes. However, γ_t includes only t-channel effects and hence the danger of confusion is avoided.

where we have again neglected terms which are second order in ϵ_D.[3]

The term $X_l - X_{\bar{l}}$ corresponds to the total lepton number:

$$X_l - X_{\bar{l}} = 1 + X_l - (1 + X_{\bar{l}}) = n_l - n_{\bar{l}} = N_L R_\star(t)^3 .$$

Using this, we can rewrite the Boltzmann equation for N_{B-L} to give

$$zH\frac{dN_{B-L}}{dz} = -\frac{\epsilon_D \gamma_D}{n_N^{eq}}\left(N_N - N_N^{eq}\right) + N_L\left(\gamma_D + 2\gamma_s + 4\gamma_t\right) .$$

Using eq. (6.7) we can rewrite the total decay rate γ_D in terms of the decay width Γ_N to get

$$zH\frac{dN_{B-L}}{dz} = -\epsilon_D \Gamma_N \frac{K_1(z)}{K_2(z)}\left(N_N - N_N^{eq}\right) + N_L\left(\gamma_D + 2\gamma_s + 4\gamma_t\right) \tag{6.23}$$

To solve the eq. (6.23), we need to relate L to $B-L$, in principle by writing the Boltzmann equation for L separately. In practice, we can consider temperatures where sphaleron processes are suppressed and thus $B-L=-L$, or temperatures where the sphalerons are in equilibrium and we can relate $B-L$ and L by a constant factor.

6.4 Simplifying notation

N_L and N_{B-L} can be related to each other depending on the temperature. At a high enough temperature sphaleron processes are suppressed and no lepton number is converted to baryon number, so we can write $N_{B-L} = -N_L$. At temperatures where sphalerons are in equilibrium, some of the lepton number leaks to the baryon number through sphalerons and we can write $N_L = -a_{sph}N_{B-L}$, where $0 < a_{sph} < 1$, measuring the effectiveness of the sphalerons. This relation will be calculated when in equilibrium in more detail in section 7.4.

To keep our notation consistent with the literature [25] [26], we rewrite the Boltzmann equations now as

$$\begin{aligned} \frac{dN_N}{dz} &= -D\left(N_N - N_N^{eq}\right) && (6.24)\\ \frac{dN_{B-L}}{dz} &= -\epsilon_D D\left(N_N - N_N^{eq}\right) - WN_{B-L} , && (6.25) \end{aligned}$$

where we have multiplied both sides with zH and defined new constants

$$\begin{aligned} D &= \frac{\Gamma_N}{zH}\frac{K_1(z)}{K_2(z)}\\ W &= a_{sph}\frac{\gamma_D + 2\gamma_s + 4\gamma_t}{zH} . \end{aligned}$$

Here a_{sph} is conversion factor between L and $B-L$, which at $T \gg 10^{13}$Gev is -1 and at $T \ll 10^{13}$Gev its value will be calculated at section 7.4.

[3]This is different from [24], where the factor of the lepton number is $1/2\gamma_D + 2\gamma_s + 2\gamma_t$

Inputting the energy density for radiation

$$\rho_{\rm rad} = \frac{\pi^2}{30} g_* T^4$$

into the Friedmann equation we get an expression for the Hubble parameter in the early, radiation dominated era

$$H = \sqrt{\frac{8\pi^3 g_*}{90}} \frac{T^2}{M_{\rm Pl}} = \sqrt{\frac{8\pi^3 g_*}{90}} \frac{m_N^2}{M_{\rm Pl}} \frac{1}{z^2} \ .$$

We then define

$$K = \frac{\Gamma_N}{H(z=1)} \ ,$$

and rewrite the expression for D as

$$D = \frac{\Gamma_N}{zH(z=1)} \frac{H(z=1)}{H(z)} \frac{K_1(z)}{K_2(z)} = zK \frac{K_1(z)}{K_2(z)} \ .$$

After a short calculation (see appendic A.3) we see that Γ_N is proportional to the *effective neutrino mass*

$$\tilde{m}_1 \equiv \frac{(hh^\dagger)_{11} v^2}{M}$$

and since K is dimensionless, we define the *equilibrium neutrino mass*

$$m_* \equiv \frac{K}{\tilde{m}_1} \ .$$

Using the explicit form for H and Γ_N from the appendix, we get

$$m_* \simeq 10^{-2} \text{eV} \ .$$

7 Solving the baryon asymmetry

In the previous chapter we have derived the Boltzmann equations relevant for leptogenesis. These describe the evolution of the number density of the heavy neutrinos and $B - L$-number. Next we need to *solve* these equations and then relate that solution to the observable, η.

7.1 Analytical solution for the Boltzmann equation for N_N

Equation 6.24 is a first order linear inhomogeneous equation. The homogeneous equation

$$\frac{dN_N}{dz} = -DN_N$$

has the solution

$$N_N = N_N^0 e^{-\int_{z_i}^{z} \mathrm{d}z'\, D} \ .$$

To solve the inhomogenous equation, we let the initial N^0 depend on time,

$$\frac{dN_N}{dz} = -DN_N^0 e^{-\int_{z_i}^{z} \mathrm{d}z'\, D} + \frac{dN_N^0}{dz} e^{-\int_{z_i}^{z} \mathrm{d}z'\, D} = -D(N_N - N_N^{\mathrm{eq}}) \ ,$$

which then gives the solution for N_N^0

$$N_N^0 = \int_{z_i}^{z} \mathrm{d}z'\, DN_N^{\mathrm{eq}} e^{\int_{z_i}^{z'} \mathrm{d}z''\, D} \ .$$

The final solution for N_N is then

$$N_N = \int_{z_i}^{z} \mathrm{d}z'\, DN_N^{\mathrm{eq}} e^{-\int_{z'}^{z} \mathrm{d}z''\, D} + N_N^{\mathrm{i}} e^{-\int_{z_i}^{z} \mathrm{d}z'\, D} \ ,$$

where N_N^{i} is the initial value of the abundance of the Majorana neutrinos.

7.2 Analytical solution for the Boltzmann equation for N_{B-L}

The Boltzmann equation for N_{B-L}, eq. 6.25, is, like the Boltzmann equation for N_N, a first order linear inhomogenous differential equation. Again, the homogenous equation

$$\frac{dN_{B-L}}{dz} = -WN_{B-L}$$

has the solution

$$N_{B-L} = N^0_{B-L} e^{-\int_{z^{\mathrm{i}}}^{z} \mathrm{d}z' \, W} \; .$$

To solve the inhomogenous equation we write $N^0_{B-L}(z)$ and get

$$\begin{aligned}
\frac{dN_{B-L}}{dz} &= -WN^0_{B-L} e^{-\int_{z^{\mathrm{i}}}^{z} \mathrm{d}z' \, W} + \frac{dN^0_{B-L}}{dz} e^{-\int_{z^{\mathrm{i}}}^{z} \mathrm{d}z' \, W} \\
&= -\epsilon_{\mathrm{D}} D(N_N - N_N^{B-L}) - WN_{B-L} \\
&= \epsilon_{\mathrm{D}} \frac{dN_N}{dz} - WN_{B-L} \\
N^0_{B-L} &= \epsilon_{\mathrm{D}} \int_{z^{\mathrm{i}}}^{z} \mathrm{d}z' \, \frac{dN_N}{dz} e^{\int_{z^{\mathrm{i}}}^{z'} \mathrm{d}z'' \, W} \; ,
\end{aligned}$$

and from this we finally get

$$N_{B-L} = \epsilon_{\mathrm{D}} \int_{z^{\mathrm{i}}}^{z} \mathrm{d}z' \, \frac{dN_N}{dz} e^{-\int_{z'}^{z} \mathrm{d}z'' \, W} + N^{\mathrm{i}}_{B-L} e^{-\int_{z^{\mathrm{i}}}^{z} \mathrm{d}z' \, W} \; ,$$

where N^{i}_{B-L} is the initial value of $B-L$.

Following the notation of [26] and others, we define the *efficieny factor* κ so that

$$N_{B-L} = -\frac{3}{4}\epsilon_{\mathrm{D}}\kappa + N^{\mathrm{i}}_{B-L} e^{-\int_{z^{\mathrm{i}}}^{z} \mathrm{d}z' \, W} \; .$$

The efficiency factor measures the amount of produced lepton asymmetry surviving until recombination, 1 being the maximal case and 0 the minimal. The factor $3/4$ is chosen for the normalization of κ so that $\kappa = 1$ in the case when the intial abundance of the neutrinos is the thermal abundance and no washout is present.

7.3 The equilibrium number density of N

We calculate N_N^{eq} as a function of z. The physical number density n_N^{eq} we get by integrating the distribution function over the momentum space. We make here again the Maxwell-Boltzmann approximation, so we calculate

$$\begin{aligned}
n_N^{\mathrm{eq}} &= g \int \frac{\mathrm{d}^3 k}{(2\pi)^3} e^{-\beta\sqrt{k^2+m^2}} \\
&= \frac{g}{2\pi^2} \int_0^\infty \mathrm{d}k \, k^2 e^{-\beta\sqrt{k^2+m^2}} \\
&= \frac{g}{2\pi^2} \int_m^\infty \mathrm{d}z \, z\sqrt{z^2-m^2} e^{-\beta z} \; .
\end{aligned}$$

Partially integrating this we get

$$\begin{aligned}
n_N^{\mathrm{eq}} &= \frac{g}{2\pi^2} \left\{ \Big|_m^\infty \frac{1}{3}(x^2-m^2)^{\frac{3}{2}} e^{-\beta x} + \beta \int_m^\infty \frac{1}{3}(x^2-m^2)^{\frac{3}{2}} e^{-\beta x} \mathrm{d}x \right\} \\
&= \frac{g}{2\pi^2} \frac{\beta m^4}{3} \int_1^\infty (x^2-1)^{\frac{3}{2}} e^{-(\beta m)x} \, \mathrm{d}x \; .
\end{aligned}$$

Using the integral expression for the modified Bessel functions of the second kind

$$K_n(z) = \frac{\sqrt{\pi}}{\Gamma(n+\frac{1}{2})}\left(\frac{z}{2}\right)^n \int_1^\infty e^{-zx}(x^2-1)^{n-\frac{1}{2}}\mathrm{d}x \, ,$$

we get

$$\begin{aligned} n_N^{\mathrm{eq}} &= \frac{g}{2\pi^2}\frac{\beta m^4}{3}\left(\frac{2}{\beta m}\right)^2 \frac{\Gamma(\frac{5}{2})}{\sqrt{\pi}}K_2(\beta m) \\ &= \frac{g}{2\pi^2}T^3 z^2 K_2(z) \, . \end{aligned}$$

To derive N_N^{eq} from n_N^{eq} we recall the definition of N from eq. 6.4

$$N_X(t) = n_X(t)R_\star(T)^3 \, ,$$

where $R_\star(T)^3$ is a volume with a single photon before the onset of leptogenesis. The equilibrium density of photons (massless bosons with no chemical potential) is given by

$$n_\gamma^{\mathrm{eq}} = 2\frac{T^3}{\pi^2}\zeta(3) \, .$$

For the Majorana neutrinos (massive fermions with no chemical potential) the equilibrium density is

$$n_N^{\mathrm{eq}} = 2\frac{T^3}{\pi^2}\left[\frac{3}{4}\zeta(3) + \mathcal{O}(\frac{m_N}{T})\right] \, .$$

Therefore at temperatures high enough $m_N \ll T$ the ratio of the equilibrium densities is

$$\frac{n_N^{\mathrm{eq}}}{n_\gamma^{\mathrm{eq}}} \simeq \frac{3}{4} \, .$$

Requiring that $N_N^{\mathrm{eq}} \propto n_N^{\mathrm{eq}}$ and that $N_N^{\mathrm{eq}} \simeq 3/4$ at high T (small z), we arrive[1] at

$$N_N^{\mathrm{eq}} = \frac{3}{8}z^2 K_2(z) \, .$$

7.4 Taking sphalerons into account: Relating the $B-L$-asymmetry to B-number

To relate the $B-L$-number to the B-number, we need to investigate which processes were in equilibrium during leptogenesis, and then relate these processes to the chemical potentials of the relevant particle species and finally to their number densities.

[1]Here we need the behaviour of $K_2(z)$ as $z \to \infty$. In general the modified Bessel functions of the second kind can be written as a power series, using

$$K_n(z) = \frac{1}{2}\left(\frac{z}{2}\right)^{-n}\sum_{k=0}^{n-1}\frac{(n-k-1)!}{k!}\left(\frac{z^2}{4}\right)^k + (-1)^{n+1}\ln\left(\frac{z}{2}\right) \, .$$

Putting $n=2$ we then get

$$\lim_{z\to 0} z^2 K_2(z) = \lim_{z\to 0}(2+\mathcal{O}(z)) = 2 \, .$$

At high enough temperatures $T > 10^{13}$GeV the sphalerons were not fast enough to be in equilibrium. As the universe cools down the sphalerons then start to convert the excess lepton number to excess baryon number. Here we consider the temperatures in the range of $T_{EW} < T < 10^{13}$GeV.

The Yukawa interactions achieve thermal equilibrium if $T > m$ where m is the mass of the heaviest particle in the Yukawa interaction. Before the EW-symmetry is broken at $T \sim T_{EW}$, all fermions are massless, so the Yukawa interactions are suppressed only by the Higgs mass. Reading off from the Yukawa sector of the SM, we get the reactions

$$\begin{aligned} \overline{q}\tilde{\phi}u + \text{h.c.}: &\qquad q_i \leftrightarrow \tilde{\phi}u_j \\ \overline{q}\phi d + \text{h.c.}: &\qquad q_i \leftrightarrow \phi d_j \\ \overline{l}\phi e + \text{h.c.}: &\qquad l_i \leftrightarrow \phi e_j \ , \end{aligned}$$

These produce the equivalent relations for the chemical potentials of the different particle species

$$\begin{aligned} \mu_{q_i} + \mu_\phi - \mu_{u_j} &= 0 \\ \mu_{q_i} - \mu_\phi - \mu_{d_j} &= 0 \\ \mu_{l_i} - \mu_\phi - \mu_{e_j} &= 0 \ , \end{aligned}$$

where we have already replaced $\mu_{\tilde{\phi}}$ with $-\mu_\phi$, following from the definition $\tilde{\phi} \equiv i\sigma_2\phi^*$.

QCD is a non-abelian gauge theory and has instanton solutions. These correspond to the sphaleron processes, coupling to all $\mathrm{SU}(3)_{\mathrm{QCD}}$-charged particles. When in equilibrium, these QCD-instantons give rise to another relation between the chemical potentials:

$$\sum_i \left(2\mu_{q_i} - \mu_{u_i} - \mu_{d_i}\right) = 0$$

EW-sphalerons are also instanton-solutions, coupling to all SU(2)-charged particles. When in equilibrium, sphalerons produce the additional constraint

$$\sum_i \left(3\mu_{q_i} + \mu_{l_i}\right) = 0 \ .$$

Since the observed universe is electrically neutral to great precision, and electrical charge is conserved, we require also the primordial plasma to be electrically neutral:

$$\rho_{\text{charge}} = \sum_i q_i n_i = 0$$

To relate this expression with physical number densities to chemical potentials, we use the small-$\beta\mu$-expansion of the number density,

$$n - \overline{n} = \frac{1}{6} g T^3 \begin{cases} \beta\mu + \mathcal{O}((\beta\mu)^3) & \text{for fermions} \\ 2\beta\mu + \mathcal{O}((\beta\mu)^3) & \text{for bosons} \end{cases} \ ,$$

where β is the inverse temperature and g the number of degrees of freedom. We then get as the total electric charge of the plasma, assuming gauge boson contribution to be zero, to be

$$\begin{aligned}
\rho_{\text{charge}} &= N_g \sum_i \left(\frac{2}{3} n_{q_i} - \frac{1}{3} n_{q_i} + \frac{2}{3} n_{u_i} - \frac{1}{3} n_{d_i} - n_{l_i} - n_{e_i} \right) + n_\phi \\
&\simeq \frac{1}{6} g T^2 N_g \sum_i \left(3\frac{1}{3} \mu_{q_i} + 3\frac{2}{3} \mu_{u_i} - 3\frac{1}{3} \mu_{d_i} - \mu_{l_i} - \mu_{e_i} + \frac{2}{N_g} \mu_\phi \right) = 0 \,,
\end{aligned}$$

where the quark chemical potentials are multiplied by the colour factor of 3 to take into account the three different possible colours of the quarks, and the factor of 2 for the Higgs contribution comes from the fact that it is a boson.

The Yukawa-interactions couple different flavours of leptons and quarks to leptons and quarks, ie., producing effectively processes like

$$\begin{aligned}
q_i \leftrightarrow \tilde{\phi} u_j \leftrightarrow q_k \\
l_i \leftrightarrow \phi e_j \leftrightarrow l_k \,.
\end{aligned}$$

These processes ensure that the chemical potentials of different generations of leptons and quarks have the same chemical potential, ie., that

$$\begin{aligned}
\forall i,j: \;\; \mu_{q_i} = \mu_{q_j} &\equiv \mu_q \\
\mu_{u_i} = \mu_{u_j} &\equiv \mu_u \\
\mu_{d_i} = \mu_{d_j} &\equiv \mu_d \\
\mu_{l_i} = \mu_{l_j} &\equiv \mu_l \\
\mu_{e_i} = \mu_{e_j} &\equiv \mu_e \,.
\end{aligned}$$

Using these relations, we arrive to six equations with six variables:

$$\begin{aligned}
\mu_q + \mu_\phi - \mu_u &= 0 \\
\mu_q - \mu_\phi - \mu_d &= 0 \\
\mu_l - \mu_\phi - \mu_e &= 0 \\
2\mu_q - \mu_u - \mu_d &= 0 \\
3\mu_q + \mu_l &= 0 \\
\mu_q + 2\mu_u - \mu_d - \mu_l - \mu_e + \frac{2}{N_g} \mu_\phi &= 0 \,.
\end{aligned}$$

Only five of these equations are linearly independent: the SU(3)-instanton equation can be derived by summing the first and second equations. We ignore that equation and are left with five equations and six variables. From these we can solve five chemical potentials in terms of a single chemical potential — we choose this to be μ_l. After some algebra, we get

$$\begin{aligned}
\mu_\phi &= \frac{4N_g}{6N_g+3}\mu_l \\
\mu_q &= -\frac{1}{3}\mu_l \\
\mu_u &= \frac{2N_g-1}{6N_g+3}\mu_l \\
\mu_d &= -\frac{6N_g+1}{6N_g+3}\mu_l \\
\mu_e &= \frac{2N_g+3}{6N_g+3}\mu_l \ .
\end{aligned}$$

Calculating then the baryonic and leptonic chemical potentials

$$\begin{aligned}
\mu_B &= \frac{N_g}{2}\left(2\mu_q+\mu_u+\mu_d\right) = -N_g\frac{2}{3}\mu_l \\
\mu_L &= \frac{N_g}{2}\left(2\mu_l+\mu_e\right) = N_g\frac{14N_g+9}{12N_g+6}\mu_l \ ,
\end{aligned}$$

and using these we can finally relate B, L and $B-L$ to be

$$\begin{aligned}
B &= \frac{8N_g+4}{22N_g+13}(B-L) \\
L &= -\frac{14N_g+9}{22N_g+13}(B-L) \ .
\end{aligned}$$

For the standard model the number of generations is $N_g = 3$, so we get

$$\begin{aligned}
B &= \frac{28}{79}(B-L) \\
L &= -\frac{51}{79}(B-L) \ ,
\end{aligned}$$

from which we can finally read off

$$a_{\text{sph}} = \frac{51}{79} \ .$$

7.5 Relating N_{B-L} to observed baryon asymmetry

We normalized N_{B-L} to be the number density of baryons minus leptons in a comoving volume with a single photon before the onset of leptogenesis. Observational data, however, gives a value only for η, which is the amount of baryons divided by the amount of photons *today*. To relate η to N_{B-L}, we need to take into account the amount of baryon asymmetry N_B produced from given N_{B-L} and the dilution of the normalization by an increase in the number of photons in the universe.

From section 7.4 we know that if sphalerons are in equilibrium, $B = \frac{28}{79}(B-L)$. When temperature decreases sufficiently, all lepton number violating processes are suppressed, and as temperature continues to fall, the sphalerons are also suppressed, resulting in the freeze-out of the given number density of baryon and lepton number, so we can use the above relation in all temperatures after leptogenesis.

$$\eta = \frac{N_B^{\mathrm{f}}}{N_\gamma^{\mathrm{f}}} = \frac{28}{79}\frac{N_{B-L}^{\mathrm{f}}}{N_\gamma^{\mathrm{f}}} = -\frac{21}{79}\frac{\epsilon_{\mathrm{D}}\kappa}{N_\gamma^{\mathrm{f}}}$$

In an adiabatically expanding universe the number density of photons in a comoving volume stays constant. When massive particles annihilate, they produce more photons. As a result the amount of photons grows as the degrees of freedom get suppressed, giving the final number density for photons to be [24]

$$N_\gamma^{\mathrm{f}} = \frac{g_\star}{g_0} N_\gamma^{\mathrm{i}} \,,$$

where $g_\star$ and g_0 are the effective number of degrees of freedom at the reference time $t_\star$ before the onset of leptogenesis and after recombination. The effective number of degrees of freedom is defined as

$$g_{\mathrm{eff}} \equiv \sum_{\mathrm{bos}} g + \frac{7}{8}\sum_{\mathrm{fer}} g \,.$$

To the normal value of $g_\star$ for the standard model we have to add the new degrees of freedom coming from the additional Majorana neutrinos. This gives

$$g_\star = g^{\mathrm{SM}} + g^{\mathrm{Maj}} = 106.75 + \frac{7}{8} \times 3 \times 2 = 112 \,,$$

and for the relation this gives [24] $g_\star/g_0 = 2387/86$. For η we get then the expression

$$\eta = -\frac{22}{79}\frac{\epsilon_{\mathrm{D}}\kappa}{\frac{2387}{86}N_\gamma^{\mathrm{i}}} = -\frac{172}{17143}\epsilon_{\mathrm{D}}\kappa \simeq -10^{-2}\epsilon_{\mathrm{D}}\kappa \,. \tag{7.1}$$

8 Numerical estimates

We arrived at the analytical expressions for η arising from thermal leptogenesis. The expression was dependent on the Yukawa couplings and Majorana masses of the heavy neutrinos via several parameters. The experimental data is far too inconclusive to give good limits on the parameters of the theory and hence for η. Instead, in this chapter we discuss approximations which enable us to arrive to some estimate for η. Finally also numerical solutions for the Boltzmann equations are presented.

8.1 Ignoring washout

If $K \ll 1$ then $\Gamma_N \ll H(z=1)$ and the decays of the heavy neutrinos occur very late, at $z \gg 1$. At these low temperatures the washout effects become less and less significant. Therefore, if we take the limit $K \ll 1$, we might ignore washout completely and set $W = 0$. Then we can solve the efficiency factor analytically:

$$\kappa = -\frac{4}{3}\int_{z_i}^{z} \mathrm{d}z' \frac{dN_N}{dz'} = -\frac{4}{3}\left(N_N(z) - N_N^{\mathrm{i}}\right)$$

Assuming all the neutrinos decay we get for the final value (corresponding to very large values of z) for κ to be

$$\kappa = \frac{4}{3}N_N^{\mathrm{i}} .$$

Hence at this limit the final value of the baryon asymmetry is highly dependent on the initial conditions and the theory loses its predictability completely.

By setting $W = 0$, we have not only ignored $\Delta L = 2$-scatterings but also inverse decays. To improve the result, we need to consider the dynamic generation of the neutrinos through inverse decays, ie., we need to solve the Boltzmann equations with the initial condition $N_N^{\mathrm{i}} = 0$ taking into account also inverse decays.

8.2 Ignoring $\Delta L = 2$-scatterings

To ignore the $\Delta L = 2$-scatterings we need to replace the washout term W with a washout term with contribution only from the inverse decays. This can be written as [25]

$$W_{\mathrm{ID}} = \frac{1}{2}\frac{\Gamma_{\mathrm{ID}}}{Hz}\frac{K_1(z)}{K_2(z)} .$$

The inverse decay width, Γ_{ID}, is related to the decay width by the equilibrium number densities of the heavy neutrinos and lepton doublets,

$$\Gamma_{\mathrm{ID}} = \Gamma_N \frac{N_N^{\mathrm{eq}}(z)}{N_l^{\mathrm{eq}}} .$$

For leptons $N_l^{\mathrm{eq}} = 3/4$ at the high temperatures we are considering, while for heavy neutrinos the number density, calculated in section 7.3, is

$$N_N^{\mathrm{eq}} = \frac{3}{8}z^2 K_2(z) .$$

Combining these resuls with the definition of D, we get

$$W_{\mathrm{ID}} = \frac{1}{4}\frac{zK_1(z)\Gamma_N}{H} = \frac{1}{4}z^2 D K_2(z) = \frac{1}{2}D\frac{N_N^{\mathrm{eq}}}{N_l^{\mathrm{eq}}} .$$

Replacing the general washout term W with W_{ID}, we arrive at the Boltzmann equations and their solutions with only decays and inverse decays:

$$\begin{aligned}
\frac{dN_N}{dz} &= -D\left(N_N - N_N^{\mathrm{eq}}\right) \\
\frac{dN_{B-L}}{dz} &= -\epsilon_{\mathrm{D}} D\left(N_N - N_N^{\mathrm{eq}}\right) - W_{\mathrm{ID}} N_{B-L} \\
\kappa &= -\frac{4}{3}\int_{z^{\mathrm{i}}}^{z} \mathrm{d}z' \frac{dN_N}{dz'} e^{-\int_{z'}^{z} \mathrm{d}z'' W_{\mathrm{ID}}}
\end{aligned}$$

Using this simplified set of equations the final amount of baryon asymmetry can be solved in terms of only two parameters: ϵ_{D}, signifying the amount of CP-violation, and K, signifying the strength of the decay compared to the mass of the heavy neutrino.

8.3 Numerical solutions for κ

The form of the solution for the efficiency factor depends greatly on the parameter K, dividing the phase space into two distinct regions where good analytical approximations can be derived: the so called *strong washout regime* and the *weak washout regime*, corresponding to $K \gg 1$ and $K \ll 1$. In the intermediate regimes numerical methods have to be used.

In figure 8.1 the numerical results from [25] are plotted for the approximation where $\Delta L = 2$-scatterings are ignored, ie., only inverse decays contribute to washout. In the weak washout regime the result is highly dependent on the intial value of N_N, as expected, but at the medium-to-strong washout the theory becomes very predictive. The current neutrino oscillation data would seem to favour this regime: if the mass differences measured in the oscillation

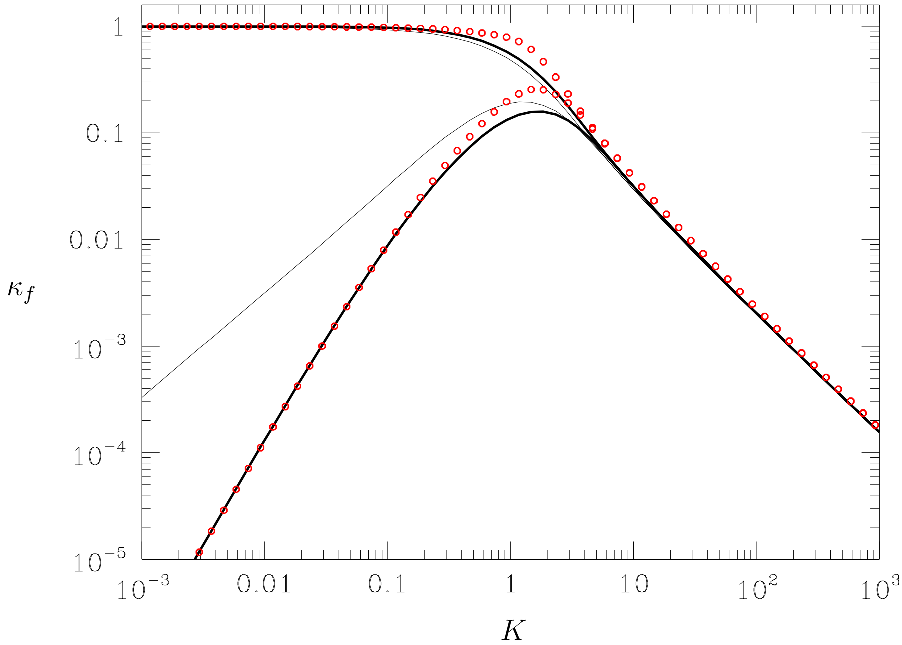


Figure 8.1: The numerical solutions for κ when only decays and inverse decays are included. The thick solid lines correspond to the numerical solutions with different initial abundance. Image taken from [25].

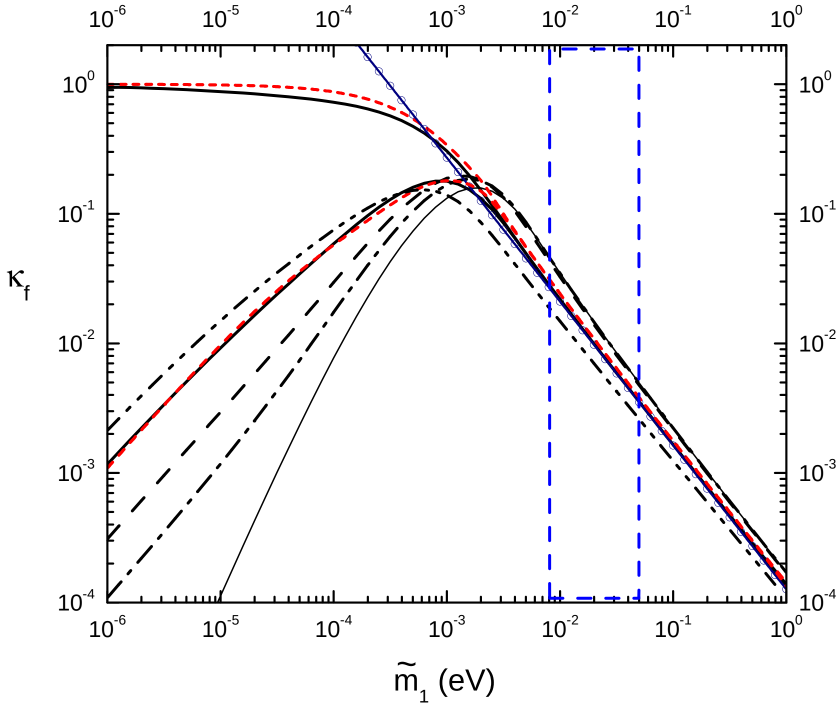


Figure 8.2: The numerical solutions for κ where scatterings have been included. Different lines illustrate different values of the effective Higgs mass in the range $10^{-10}\ldots 1$. The dashed box indicates the range $(m_{\text{sol}}, m_{\text{atm}})$. That part of the parameter space would seem to favour $\kappa \sim 10^{-2}$. Here the horizontal axis is the effective neutrino mass $\tilde{m}_1$ which is $\tilde{m}_1 \simeq 10^{-3}\text{eV} \times \text{K}$. Image taken from [25].

experiments are assumed to reflect the scale of the light neutrino masses, then $K \sim 10 \ldots 10^2$. This would give $\kappa \sim 10^{-1} \ldots 10^{-3}$.

If scatterings are also taken into account, the value for κ doesn't change drastically. In the figure 8.2 the numerical solutions for κ from [25] are plotted with scatterings. In the weak washout regime the theory is again unpredictive, but in the regime favoured by the current oscillation data $\kappa \sim 10^{-2}$.

8.4 Numerical estimates for η

The final value of η depends on two factors, κ and ϵ_{D}. The value of κ can be given within a magnitude, if current neutrino oscillation data is taken seriously. The story with ϵ_{D} is however more involved.

The exact expression for ϵ_{D} at one loop can be calculated (see appendix A.4) in terms of the Yukawa couplings and Majorana masses of the heavy neutrinos. To arrive at a numerical value for ϵ_{D}, however, crude approximations must be used. Assuming that the ratio of the heavy neutrino masses have similar value as other SM particles, eg., $M_1/M_3 \sim m_u/m_t \sim 10^{-5}$, the expression for the ϵ_{D} can be approximated as [27]

$$|\epsilon_{\mathrm{D}}| \sim 10^{-6} \,. \tag{8.1}$$

On the other hand assuming hierarchical left-handed neutrino masses an upper limit can be derived for ϵ_{D} [24]:

$$|\epsilon_{\mathrm{D}}| \leq \frac{3}{16\pi} \frac{M_1 \sqrt{\delta m_{\mathrm{atm}}}}{v^2}$$

The upper bound of this relation is often saturated in models with hierarchical neutrino masses.

Assuming the rough estimate in eq. (8.1) gives at least the right magnitude one can estimate the value for η from eq. (7.1) to be

$$|\eta| \sim 10^{-2} \times 10^{-2} \times 10^{-6} \sim 10^{-10} \,.$$

Although there are uncertainties of a magnitude or two in both the estimate for κ and for ϵ_{D}, this estimate for η gives a prediction with exactly the observed magnitude. Even if the theory is far from giving exact numerical predictions, this crude estimate demonstrates the potential of the theory to explain the cosmological baryon asymmetry.

9 Conclusions

In chapter 2 we reviewed the current best estimates for the ratio of baryons and photons in the universe from the observations from CMB and BBN giving a concordant value of $\eta \simeq 6 \times 10^{-10}$, with no measurable amount of antibaryons. Instead of attributing this asymmetry to initial conditions we attempted to explain this through a dynamical mechanism producing the observed value. The standard model of particle physics is unable to explain the observed asymmetry and as a consequence we need to extend the standard model in some way to arrive at a successful theory of leptogenesis.

In chapter 3 the current data on neutrino masses from neutrino oscillation experiments was introduced. These masses were naturally introduced to the SM via the see-saw mechanism. An integral part of this mechanism is the Majorana masses for the right-handed neutrinos, which introduce lepton number violation to the SM. Combined with baryon number violation already present in the bare SM — the sphalerons of the EW-theory — we have sufficient ingredients for a successful theory of baryogenesis through leptogenesis: the production of net lepton number via the decay of the heavy right-handed neutrinos and the conversion of this net lepton number to baryon number through the sphalerons.

In chapters 5-7 we introduced the theory of leptogenesis in more detail and derived the relevant Boltzmann equation necessary for the quantitative treatment of the theory.

The final amount of baryons was shown to be related to the produced lepton asymmetry, which depends on the properties of the heavy neutrino. This final baryon asymmetry produced by leptogenesis was derived in terms of two factors: the amount of CP-violation in the decay of the heavy neutrinos and efficiency factor measuring the amount of washout of the produced lepton number.

The efficiency factor could be solved numerically from the relevant Boltzmann equations with an accuracy of one order of magnitude, given the neutrino mass scales indicated by the oscillation experiments.

The amount of CP-violation in the decays is hard to estimate since it depends on the complex phases of the Yukawa couplings of the heavy neutrinos. Crude estimates can be however performed, and these combined with the best fits for the efficiency factor give the observed magnitude for η. This shows that leptogenesis is indeed a viable scenario for the production of the cosmological baryon asymmtery. Future experiments on neutrinos will hopefully give more exact estimates for the parameters of leptogenesis, resulting in a more precise prediction.

In summa, we have presented a feasible theory of baryogenesis. The theory is in many ways attractive. It is a minimal extension of the SM requiring only the addition of right-handed neutrinos, which is necessary to explain the oscillation data, and Majorana masses for them. Unlike many models of baryogenesis, the parameters of leptogenesis can relatively readily be related to observable quantitites. This gives leptogenesis falsifiability necessary for a scientific hypothesis.

Thermal leptogenesis is beginning to reach maturity. Its predictions match observation well, although there is far too much phase space to draw any definite conclusions. Future experiments will hopefully draw light on the parameters of leptogenesis. If experiments rule out thermal leptogenesis, then many modifications of the basic scenario can still offer reasonable explanation of the cosmic baryon asymmetry.

A

Calculating parameters with Majorana particles

The process of producing excess baryons into the universe through the machinery of leptogenesis was parameterised using the CP violating measuring ϵ_{D}, the heavy neutrino's total decay width γ_{D}, the effective mass of the lightest neutrino $\tilde{m}_1$, and the mass of the heavy neutrino M. Here we relate these parameters to the fundamental parameters of the theory — the Yukawa coupling constants and the Majorana masses.

We first introduce the necessary machinery *sans* renormalization. After that we calculate explicitly the simplest parameter γ_{D} and present results for ϵ_{D}.

A.1 The Lagrangian with three heavy neutrinos

To produce successful leptogenesis, we introduced to the standard model three heavy, right-handed Majorana neutrinos. This resulted to the Lagrangianarag

$$\mathcal{L} = \mathcal{L}_S M + \bar{N} i \partial\!\!\!/ N + M_i \overline{N^C}_i N_i + h_{ij} \bar{N}_i \tilde{\phi}^\dagger l_j + h^*_{ji} \bar{l}_i \tilde{\phi} N_j \,. \tag{A.1}$$

Here i and j refer to flavour index. We have also chosen the Majorana masses M_i to be real and diagonal. This can be done without limitation, since if M is complex and non-diagonal, we can always find the diagonalizing matrix of M and use that to redfine new wavefunctions, Ns, so that M' is real and diagonal.

To calculate observables from this Lagrangian, one uses perturbation theory. The Feynman rules derived from this Lagrangian are of course the familiar standard model ones, plus additional terms corresponding to the Majorana neutrinos.

A.2 Feynman rules for Majorana particles

To calculate the relevant parameters for the Boltzmann equations, one needs to calculate the matrix elements of given processes. This is accomplished usually using perturbation theory: the amplitudes of the processes are found to certain accuracy as a series in the interaction parameters — in our case the Yukawa couplings and Majorana masses. For a specific process, $\langle f|i\rangle$, we can draw several (usually infinite) number of Feynman diagrams, and using Feynman rules we evaluate the graphs giving the largest contributions.

The Feynman rules for scalar theory and for Dirac fermions is standard content in any book on quantum field theory or particle physics. Feynman rules for Majorana fermions are however more involved, and are usually given only for theories involving exclusively Majorana fermions and scalars (see eg. [28]). This is unfortunate, since the physically well-motivated theories have usually both Majorana and Dirac fermions with interactions mixing these two.

The heart of the problem is the self-charge-conjugacy of Majorana particles, $\psi^c = \psi$. For Dirac fermions charge conjugation transforms left-handed particles to right-handed antiparticles, and therefore reverses the direction of the arrow indicating the flow of fermion number. For Majorana fermions charge conjugation merely changes the handedness of the particle, and thus no proper flow of fermion number can be assigned to Majorana fermions.[1] This means that we can draw more graphs than with mere Dirac fermions.

Feynman rules for theories involving both Dirac and Majorana fermions have been derived, among others, in [29] and [30], but in these formulations one gets Feynman rules with many different vertices and also several different propagators for the Majorana particles. Also these formulations can have ambiguity in the relative sign between two different graphs. Instead, we follow the formulation of [31], where we have only one propagator and two separate expressions for a given Majorana-Dirac-scalar vertex.

We begin with the Lagrangian describing a single Majorana-particle:

$$\mathcal{L} = \overline{\psi_R} i\partial\!\!\!/ \psi_R + \frac{m}{2}\left(\overline{\psi_R}(\psi_R)^c + (\overline{\psi_R}(\psi_R)^c)^\dagger\right) \tag{A.2}$$

Next we define a new field,

$$\chi \equiv \psi_R + (\psi_R)^c = \psi_R + (\psi^c)_L \ ,$$

which then satisfies the Majorana condition,

$$\chi^C = \chi.$$

We can then rewrite the Lagrangian of eq. (A.2)

$$\mathcal{L} = \frac{1}{2}\overline{\chi}(i\partial\!\!\!/ - m)\chi \ ,$$

which is of the form familiar from the Dirac Lagrangian. For interaction terms one rewrites

$$\begin{aligned} \psi_R &= P_R\chi \\ \overline{\psi_R} &= \overline{P_R\chi} = \overline{\chi}P_L \ , \end{aligned}$$

[1] This of course corresponds to the lack of U(1)-symmetry of the Majorana fermions. For this reason they cannot carry any conserved quantum number.

so a Yukawa-interaction with a Dirac fermion (here denoted by Ψ) merely gets a projection operator

$$\mathcal{L}_{\text{int}} = \overline{\psi_R}\Gamma\Psi + \overline{\Psi}\Gamma^*\psi_R = \overline{\chi}P_L\Gamma\Psi + \overline{\Psi}\Gamma^*P_R\chi$$

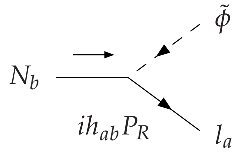
Using the self-conjugacy of χ, we can rewrite these vertex terms also in the form

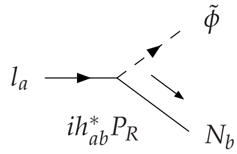
$$\mathcal{L}_{\text{int}} = \overline{\Psi^c}\Gamma P_L\chi + \overline{\chi}\Gamma^*P_R\Psi^c\ .$$

As we can see each Majorana-Dirac-Yukawa-term in the Lagrangian can be written in two different forms: one including the Majorana field and the Dirac field and one including the Majorana field and the *charge conjugate* of the Dirac field.

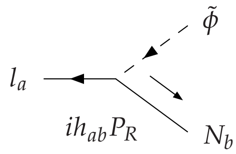
Next we apply this to a theory of a right-handed neutrino with Yukawa interactions of eq. (3.9). We identify χ with N, and then first rewrite the interaction term in two different forms:

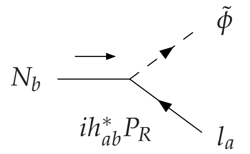
$$\bar{l}_a h_{ab}\tilde{\phi}P_R N_b \quad + \quad \overline{N}_b h^*_{ab}\tilde{\phi}^\dagger P_L l_a$$



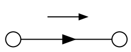


$$= \quad \overline{N}_b h_{ab}\tilde{\phi}P_R l^c_a \quad + \quad \overline{l^c}_a h^*_{ab}\tilde{\phi}^\dagger P_L N_b$$





Here the arrow on the leg of the lepton denotes as usual the fermion number flow, and the small arrow above the line the flow of momentum. From these vertices we can always form a graph where the direction of arrows of the fermion flow in the Dirac propagators and legs and the momentum flow in the Majorana propagators and legs is uninterrupted. Thus we need only one kind of propagator for the Majorana fermion. The propagators for Dirac and Majorana fermions are then the same:



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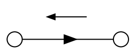
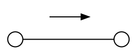
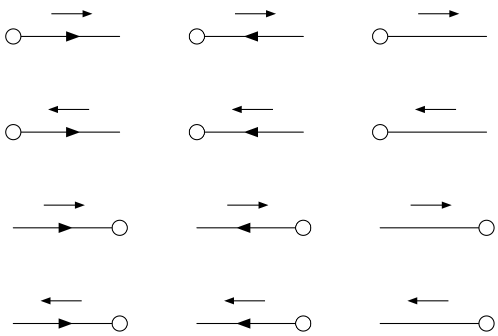


$iS(-p)$



$iS(p)$

Finally, to complete our Feynman rules, we need to list the factors arising from external legs in the graphs. As shown in [31] these give terms



A.3 The decay amplitude Γ_N

To calculate the thermal rate of decays of the heavy neutrino γ_{D}, we can use the result of eq. (6.7) to reduce the problem to the calculation of the non-thermal decay width in the rest frame of the neutrino. Hence we need to calculate Γ_N, as used already in section 6.2.

Γ_N is the total decay width of the heavy neutrino. It has two decay channels, to a lepton and an anti-Higgs doublet and to an antilepton and a Higgs doublet. For decay process, $A \to B + C$, the differential decay width, ie. the decay width of to a specific point in final phase space, is defined as

$$\mathrm{d}\Gamma = \frac{1}{2M}|\mathcal{M}|^2\, \mathrm{d}\vec{p}_B\, \mathrm{d}\vec{p}_C\, (2\pi)^4\delta^4(p_A - p_B - p_C) \tag{A.3}$$

where, again, the integral measure is defined as in eq. 6.3. To get the total decay width of A, we need to integrate over the final phase space (including summing over possible final spin and color states). We start by calculating the amplitude $\mathcal{M}$ at the tree level — one-loop corrections are relevant only for ϵ_{D}. Since the final state in the two different decay channels of N are different, the two amplitudes do not interfere and we can simply calculate them separatly and add only their squares.

The amplitude of a first right-handed neutrino at rest with spin s to decay to the α-component of an anti-Higgs and a j-flavour lepton doublet with spin s' and momentum p is given by

$$i\mathcal{M}_1 = \overline{l_a^\alpha}(p_l, s') i h_{a1} P_R N_1(p_N, s)\ .$$

Here $l_a^\alpha(p_l, s')$ refers to a lepton of flavour a, doublet index α, spin s' and momentum p_l.

Taking the absolute squared of this we get

$$|\mathcal{M}_1|^2 = \overline{l_a^\alpha}(p_l, s')\, h_{a1}\, P_R\, N_1(p_N, s)\, \overline{N_1}(p_N, s)\, P_L\, h_{a1}\, l_a^\alpha(p_l, a)$$

Since we are calculating the decay of the right-handed neutrino to *any* final state, we need to take the sum over the final spin, flavour and doublet indices, s', a and α. In addition, since we are calculating the decay width for unpolarized neutrinos, we need to take the *average* over the initial spin. We then take the trace of this (scalar), and get

$$\begin{aligned}
|\mathcal{M}_1|^2 &= \frac{1}{2}\sum_{j,\alpha,s,s'} h_{a1}h^*_{a1}P_R\, N_1(p_n,s)\,\overline{N_1}(p_N,s)P_L\, l^\alpha_a(p_l,s')\,\overline{l^\alpha_a}(p_l,s') \\
&= \frac{1}{2}2(h^\dagger h)_{11}\ \mathrm{Tr}\ P_R(\not{p}_N + M)P_L\not{p}_l \\
&= (h^\dagger h)_{11}\ \mathrm{Tr}\ P_L(\not{p}_N + M)\not{p}_l \\
&= (h^\dagger h)_{11}\frac{1}{2}\ \mathrm{Tr}\ \not{p}_N\not{p}_l \\
&= 2(h^\dagger h)_{11}(p_N p_l)
\end{aligned}$$

Since we are in the rest frame of the right-handed neutrino the spacial part of its four-momentum is zero, $p_N = (M, \vec{0})$. The lepton is massless, so the length of its four-momentum is zero, $p_l = (p_l, \vec{p}_l)$. Using these we get

$$|\mathcal{M}_1|^2 = 2(hh^\dagger)_{11}\, M\, p_l\ .$$

Since we are computing the total decay width, we need to take into account also the possibility of the right-handed neutrino decaying into an antilepton and a Higgs. This gives otherwise similar calculation, but now the other interaction term contributes, that is, $i\mathcal{M} \propto h$, not h^*.

$$i\mathcal{M}_2 = \overline{N_1}(p_N,s)ih_{1i}P_L l^\alpha_a(p_l,s')$$

Again we need to average over the initial spin s and sum over s', α and i:

$$\begin{aligned}
|\mathcal{M}_2|^2 &= \frac{1}{2}\sum_{s,s',\alpha,a} h_{1a}h^*_{1a}\overline{N_1}(p_N,s)P_L l^\alpha_a(p_l,s')\overline{l^\alpha_a}(p_l,s')P_R N_1(p_N,s) \\
&= (hh^\dagger)_{11}\ \mathrm{Tr}\ (\not{p}_N - M)P_L(\not{p}_l)P_R \\
&= (hh^\dagger)_{11}\ \mathrm{Tr}\ P_R\not{p}_N\not{p}_l \\
&= 2(hh^\dagger)_{11}\, M\, p_l
\end{aligned}$$

At tree level, the amplitudes of $N \to \tilde{\phi}^\dagger l$ and $N \to \tilde{\phi}\bar{l}$ are hence equal and the CP-violating inequality is found only at the one-loop level.

Next we put the total decay amplitude into the definition of the decay width in eq. (A.3). We have already summed over all possible final spins and flavours — only the momentum integrals are then left:

$$\Gamma = \int \mathrm{d}\Gamma = \int \frac{\mathrm{d}^3 p_l}{(2\pi)^3 2E_l}\frac{\mathrm{d}^3 p_\phi}{(2\pi)^3 2E_\phi}\frac{(2\pi)^4}{2M}4(hh^\dagger)_{11}\, M\, p_l\, \delta(M - p_l - p_\phi)\delta(\vec{p}_l + \vec{p}_\phi)$$

We first integrate the p_ϕ-integral using the 3-momentum delta-function, and replace the E_l and E_ϕ with their respective momentum,

$$= \frac{(hh^\dagger)_{11}}{2(2\pi)^2}\int \frac{\mathrm{d}^3 p_l}{p_l}\delta(M - 2p_l)$$

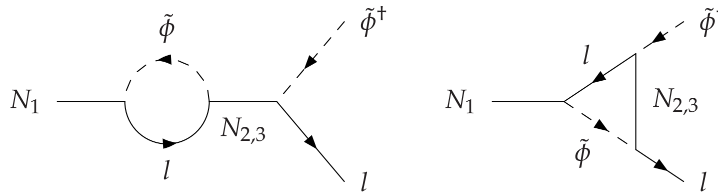


Figure A.1: The one loop corrections to the decay of the first heavy neutrino producing the CP-violating ϵ_{D}.

and then integrate the final integral using the last delta-function:

$$\begin{aligned}\Gamma_N &= \frac{(hh^\dagger)_{11}}{2\pi}\int_0^\infty \mathrm{d}p_l\, p_l\, \delta(M-2p_l) \\ &= \frac{(hh^\dagger)_{11}}{2\pi}\int_0^\infty \frac{\mathrm{d}x\, x}{4}\delta(M-x) \\ &= \frac{(hh^\dagger)_{11}}{8\pi}M\end{aligned}$$

A.4 The measure of CP-violation: ϵ_{D}

When calculating the decay amplitude Γ_N we calculated only at tree level and summed over the both decay channels, $N \to l\tilde{\phi}^\dagger$ and $N \to \bar{l}\tilde{\phi}$. As ϵ_{D} measures the difference of the decay propability to different channels, we would have to reproduce the calculation of Γ_N without the summation. However, already in that calculation it turned out that at the tree level both channels are equivally propable.

The highest order contribution to difference in the branching ratios comes from the one-loop vertex and self-energy corrections to the decay graph. These corrections are displayed for the process $N \to l\tilde{\phi}^\dagger$ are displayed in figure A.1. Similar graphs can be drawn for the other decay channel by merely inverting the flow of the lepton and the Higgs.

It is interesting to note that the non-zero corrections come from the graphs where the additional virtual particle in the vertex and self-energy correction is a Majorana neutrino, but *not the lightest one*. To produce CP-violation in the decay of N the existence of a heavier Majorana neutrino is required.

The values of ϵ_{D} calculated in the literature (see eg. [32], [33], [34], [35] and many others) differ from wach other in details, but seem to converge towards a unifying expression for ϵ_{D}. Assuming hierarchical neutrino masses ($M \ll M_{2,3}$) it can be given as [26]

$$\epsilon_{\mathrm{D}} \simeq \frac{3}{16\pi}\frac{1}{(hh^\dagger)_{11}}\sum_{i=2,3}\mathrm{Im}\left[\left(\mathrm{hh}^\dagger\right)^2_{\mathrm{i1}}\right]\frac{\mathrm{M_1}}{\mathrm{M_i}}\,.$$

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