Leptogenesis - A non-relativistic study

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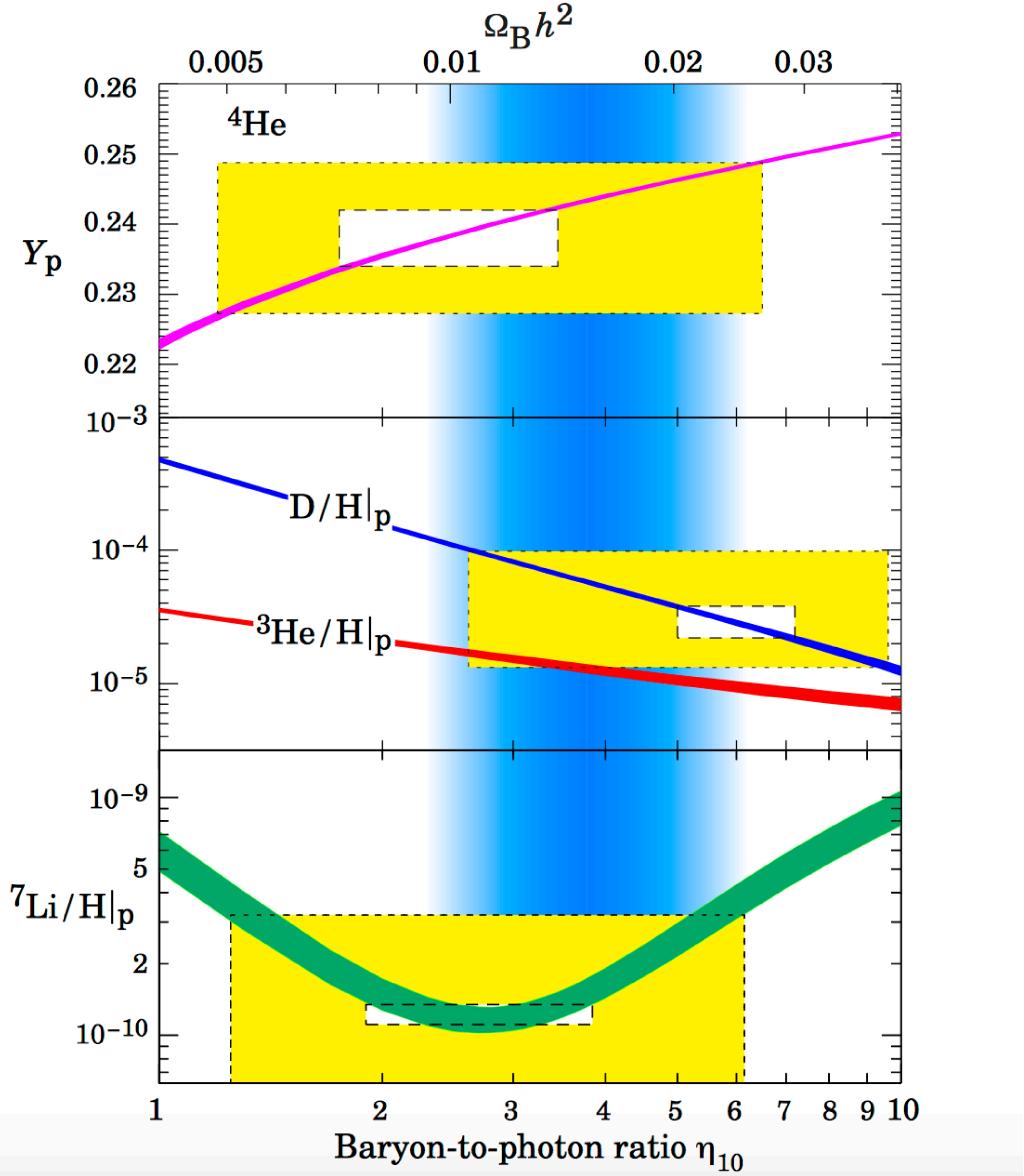
The problem

- Observed matter-antimatter asymmetry:

$$\eta = \frac{n_b - n_{\bar{b}}}{n_\gamma} \simeq \frac{n_b}{n_\gamma}$$

- Observed and predicted values do not coincide:
    - CMB measurement
    - BBN measurement

$$\eta_{obs} = (6.1 \pm 0.16) \times 10^{-10} \quad \longleftrightarrow \quad \eta_{pred} \simeq 10^{-18}$$



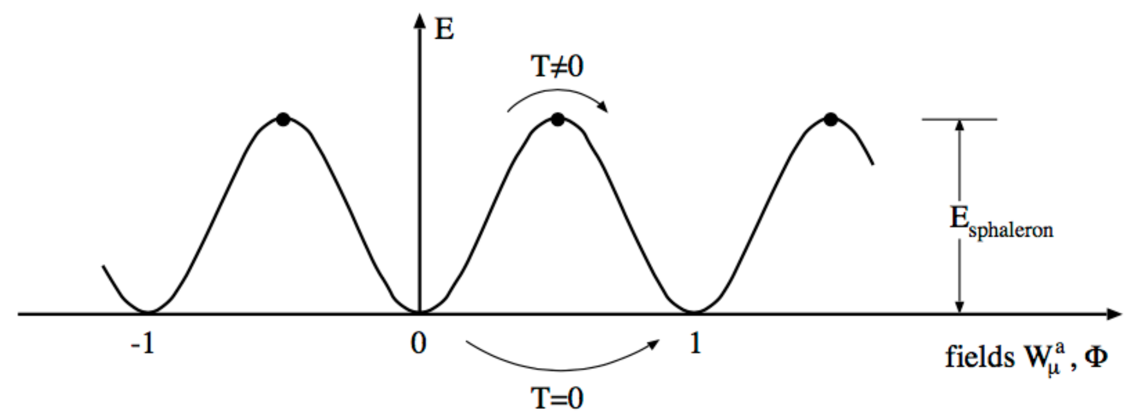
[1] S. Sarkar, *Measuring the baryon content of the universe: BBN versus CMB*, astro- ph/0205116.

Sakharov conditions

- Matter-antimatter asymmetry has to arise dynamically
- Three conditions have to be met:
  - B violation
  - C and CP violation
  - Departure from thermal equilibrium

Electroweak Baryogenesis

- B violation through non-perturbative effects:
  - instanton
  - sphaleron



- C and CP violation in the electroweak sector
- Departure from thermal equilibrium during the electroweak phase transition

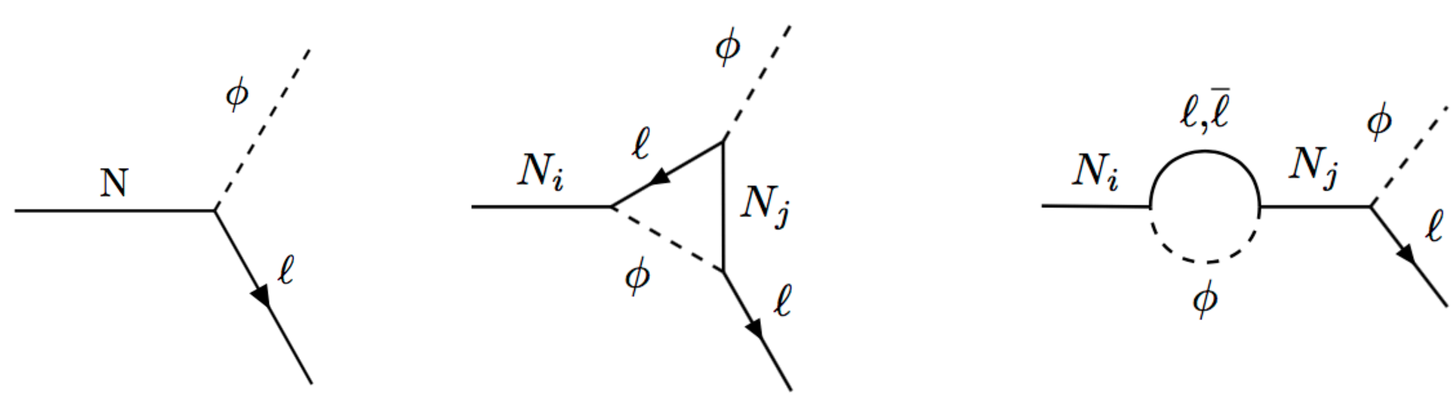
[2] W. Bernreuther, *CP violation and baryogenesis*, Lect. Notes Phys. **591** (2002) 237 [arXiv:hep-ph/0205279].

Leptogenesis solution

- Introduction of heavy Majorana neutrinos and a corresponding Yukawa coupling term:

$$\mathcal{L}_{\text{N,Yuk}} = h_{ij}\overline{N_i}\tilde{\phi}^\dagger l_j + h^*_{ij}\overline{l_i}\tilde{\phi}N_j$$

- Decays of Majorana neutrinos are fermion number violating:



[3] M. Fukugita and T. Yanagida, *Baryogenesis Without Grand Unification*, Phys. Lett. B **174** (1986) 45.

Realization of the Sakharov conditions

- Sphaleron processes converting net fermion number into net baryon number
- Extra CP violation because of complex Yukawa couplings
- Out-of-equilibrium decays for $T < M_N$

Considered model

- Assumptions made:
  - Hierarchical neutrino masses
  - One flavor limit
- Boltzmann equations read:

$$\left(\frac{d}{dt} + 3H\right) n_N = \Gamma_N \left(n_N^{eq} - n_N\right) + \Gamma_{N,u} \left(u - u^{eq}\right)$$

$$\left(\frac{d}{dt} + 3H\right) n_{B-L} = \Gamma_{B-L,N} \left(n_N - n_N^{eq}\right) - \Gamma_{B-L,u} \left(u - u^{eq}\right) - \Gamma_{B-L} n_{B-L}$$

$$\left(\frac{d}{dt} + 5H\right) u = \Gamma_u \left(u^{eq} - u\right)$$

[4] D. Bödeker and M. Wörmann, *Non-relativistic leptogenesis*, JCAP **1402** (2014) 016 [arXiv:1311.2593 [hep-ph]]

Solving the rate equations

- Define:

$$\lim_{z\to\infty} \frac{n_{B-L}}{n_\gamma^{eq}} = \frac{3}{4}\epsilon\kappa_f \quad ; \quad z \equiv \frac{M_N}{T} \quad ; \quad K \equiv \left.\frac{\Gamma_0}{H}\right|_{T=M_N}$$

- Rewrite rate equations:

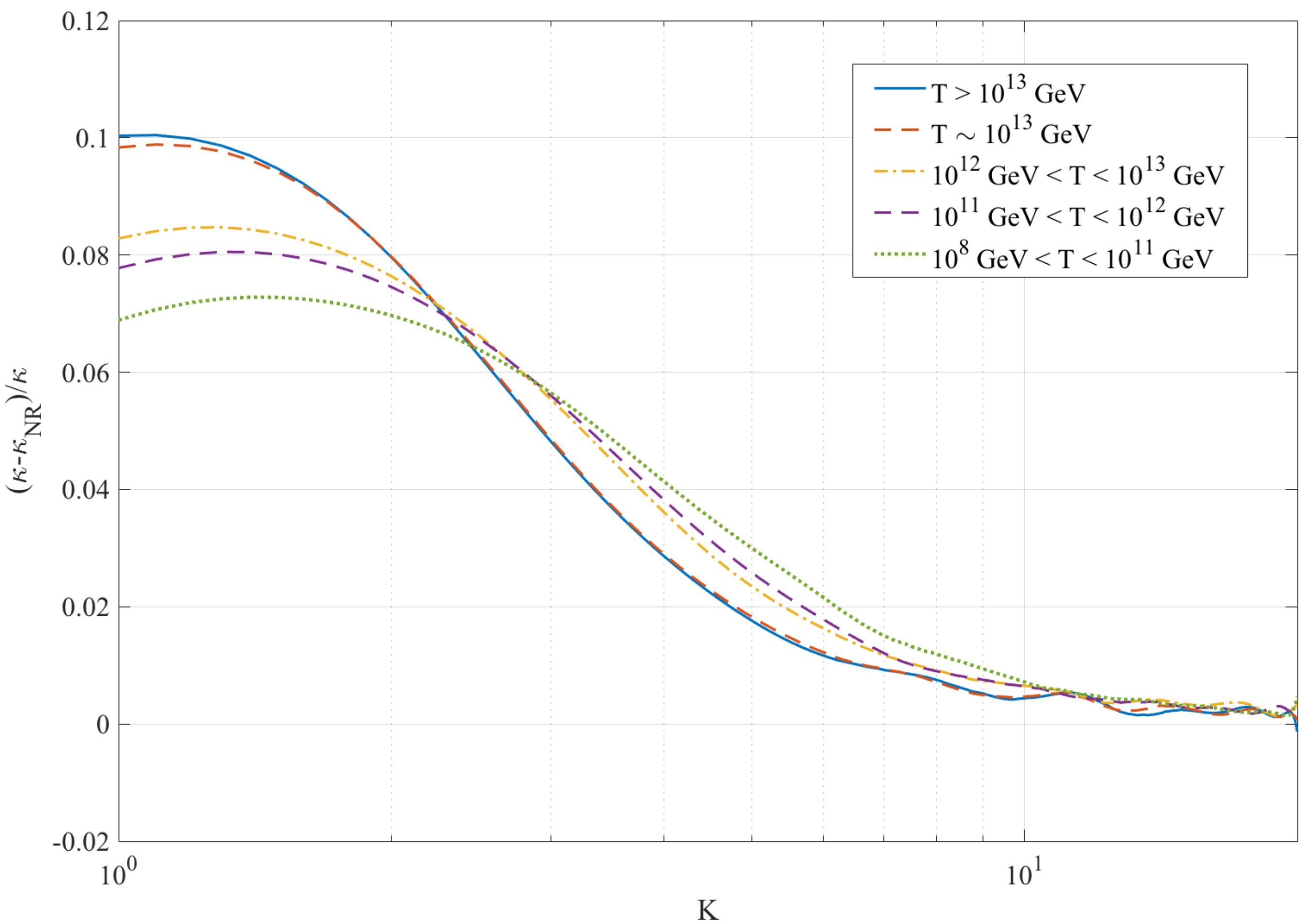
$$\frac{dX_u}{dz} = -zK\left(X_u - X_u^{eq}\right)$$

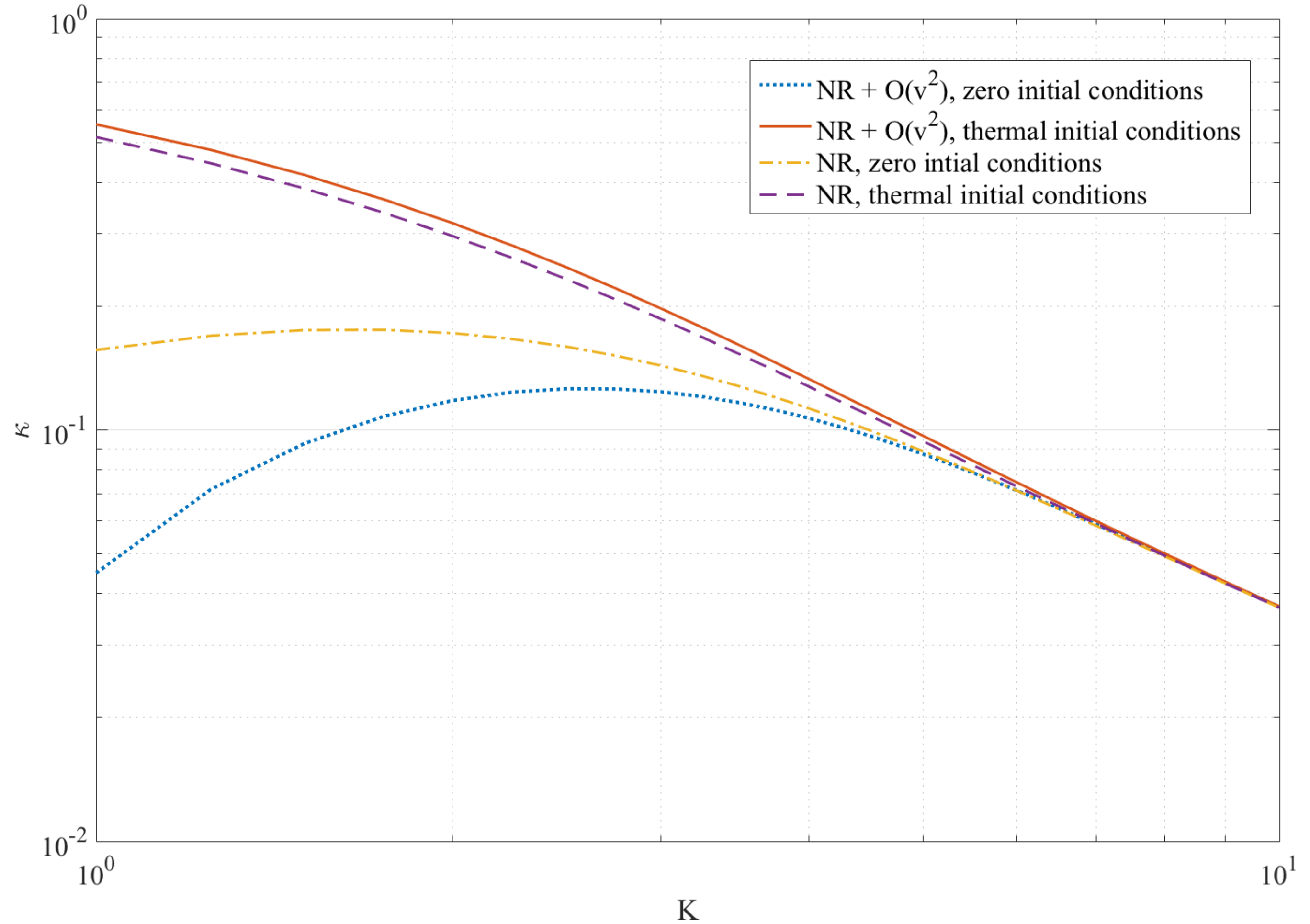
$$\frac{dX_N}{dz} = -zK\left(X_N - X_N^{eq}\right) + \frac{K}{z}\left(X_u - X_u^{eq}\right)$$

$$\frac{d\kappa}{dz} = \frac{2\pi^2}{3\zeta(3)} zK\left[\left(X_N - X_N^{eq}\right) - \frac{1}{z^2}\left(X_u - X_u^{eq}\right)\right] - \frac{3}{\pi^2}\left(c_\ell + \frac{c_\phi}{2}\right) z^3 K_1(z) K\kappa$$

$$X_N^{eq} = \frac{1}{\pi^2} z^2 K_2(z) \qquad X_u^{eq} = \frac{3}{2\pi^2} z^3 K_3(z)$$

Relativistic versus non-relativistic





Conclusion

- Leptogenesis could propose a solution to the matter-antimatter asymmetry problem
- Non-relativistic approximation is viable for the strong washout regime
- Experimental proof has yet to be given
  - $0\nu\beta\beta$-decay to show neutrinos are Majorana fermions

Numerical calculation

```
clc


cl = 344/537;
cp = 52/179;

a = 0.1646; %= 1/pi^2*besselk(2,1)
b = 1.0793; %= 3/(2*pi^2)*besselk(3,1);

z = linspace(1,20);     % Range of z over which rate equations are solved
y0 = [0 0 0];           % Zero initial conditions for X_(B-L), X_N, X_u
y1 = [0 a b];           % Thermal intial conditions for X_(B-L), X_N, X_u
x0 = [0 0];             % Zero initial conditions for X_(B-L), X_N
x1 = [0,a];             % Thermal intial conditions for X_(B-L), X_N

 A11=zeros(1,37);       % Matrices of zeroes for later storage of data
 A21=zeros(1,37);
 A31=zeros(1,37);
 A41=zeros(1,37);


% The rate equations are solved for each value of K in a range from 1 to 10
% in steps of 0.25.
% The last calculated data point for each K gets stored in the matrices
% defined above

for K=1:0.25:10

    [z1,N1] = ode15s(@(z,N) density_Rel(z,N,K,cl,cp),z,y0);
    [z2,N2] = ode15s(@(z,N) density_Rel(z,N,K,cl,cp),z,y1);
    A11(1,int16(K*4-3)) = N1(end,1);
    A21(1,int16(K*4-3)) = N2(end,1);
    [z3,N3] = ode15s(@(z,N) density_nonCorr(z,N,K,cl,cp),z,x0);
    [z4,N4] = ode15s(@(z,N) density_nonCorr(z,N,K,cl,cp),z,x1);
    A31(1,int16(K*4-3)) = N3(end,1);
    A41(1,int16(K*4-3)) = N4(end,1);
end
```

```
% Obtained results get smoothed out in order to suppress noise

quinticMA1 = sgolayfilt(A11, 10, 31);
quinticMA2 = sgolayfilt(A21, 10, 31);
quinticMA3 = sgolayfilt(A31, 10, 31);
quinticMA4 = sgolayfilt(A41, 10, 31);

% Smoothed results are plotted against K

figure(1)
 loglog((1:0.25:10),quinticMA1)
 hold on
 loglog((1:0.25:10),quinticMA2)
 loglog((1:0.25:10),quinticMA3)
 loglog((1:0.25:10),quinticMA4)
 hold off
 xlabel('K')
 ylabel('\kappa')



 % This function defines the rate equation with relativistic corrections
 % included
 %5.4737=2*pi^2/(3*zeta(3))

 function ndot = density_Rel(z,N,K,cl,cp)
    nE = 1/pi^2*z^2*besselk(2,z); % X_N in equilibrium
    uE = 3/(2*pi^2)*besselk(3,z)*z^3; % X_u in equilibrium

    ndot=[5.4737*z*K*((N(2)-nE)-1/z^2*(N(3)-uE))-K*z^3*besselk(1,z)...
        *3/pi^2*(cl+cp/2)*N(1);...
        -K*z*(N(2)-nE)+K/z*(N(3)-uE);...
        -K*(N(3)-uE)*z];
 end
```

```
% This function defines the rate equation without relativistic corrections
% included

function ndot = density_nonCorr(z,N,K,cl,cp)
    nE = 1/pi^2*z^2*besselk(2,z); % X_N in equilibrium

    ndot=[z*5.4737*K*(N(2)-nE)-3/pi^2*(cl + cp/2)*z^3*besselk(1,z)*...
        K*N(1);...
         -K*(N(2)-nE)*z];
end
```