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CP asymmetry in Majorana neutrino decays

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Abstract

We study CP asymmetries in lepton-number violating two-body scattering processes and show how they are related to CP asymmetries in the decays of intermediate massive Majorana neutrinos. Self-energy corrections, which do not contribute to CP asymmetries in two-body processes, induce CP violating couplings of the intermediate Majorana neutrinos to lepton-Higgs states. We briefly comment on the implications of these results for applications at finite temperature. © 1998 Published by Elsevier Science B.V. All rights reserved.

1. Introduction

Decays of heavy Majorana neutrinos may be responsible for most of the cosmological baryon asymmetry [1]. As detailed studies have shown, the observed asymmetry $n_B/s \sim 10^{-10}$ is naturally obtained for theoretically well motivated patterns of neutrino masses and mixings, without [2–4] and with [5,6] supersymmetry.

CP asymmetries in heavy particle decays are conventionally evaluated from the interference between tree diagrams and one-loop vertex corrections [7]. In addition interference terms with self-energy corrections have been considered for several models [8,9], which may have large effects in some cases [10]. However, the correct treatment of self-energy contributions for a decaying particle is not obvious. The naive prescription leads to a well-defined result for the CP asymmetry, yet the individual partial decay widths are infinite.

In this paper we shall investigate this problem in the case of heavy Majorana neutrinos, which are obtained as mass eigenstates if right-handed neutrinos are added to the standard model. Since they are unstable, they cannot appear as in- or out-states of S-matrix elements. Rather, their properties are defined by appropriate S-matrix elements for stable particles [11]. For such scattering processes one may define CP asymmetries for which the resonance contributions, at least in some approximation, can be used to define CP asymmetries for decays of the intermediate unstable particles.

For applications at finite temperature the separation of two-body scattering processes in resonance contributions and remainder is crucial [12]. We shall work this out in the case of heavy Majorana neutrinos using a resummed propagator for the intermediate heavy neutrinos. It turns out that for CP asymmetries of two-body processes various cancellations occur. The asymmetry for the full propagator vanishes identically [1]. Away from resonance poles, the

[1] In a previous version of this paper we concluded incorrectly that therefore self-energy corrections can be neglected at finite temperature.

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entire CP asymmetry vanishes to leading order [13]. This further emphasizes the importance to analyse CP asymmetries of two-body processes in the resonance region. In the following we shall study this in detail and comment on possible implications at finite temperature.

2. Self-energy and vertex corrections

We consider the standard model with three additional right-handed neutrinos. The corresponding Lagrangian for Yukawa couplings and masses of charged leptons and neutrinos reads

$$\mathscr{L}_Y = \overline{l_L}\,\phi\,\lambda_l^*\; e_R + \overline{l_L}\,\tilde{\phi}\,\lambda_\nu^*\;\nu_R - \tfrac{1}{2}\,\overline{\nu_R^C}\,M\,\nu_R + \text{h.c.}\,, \tag{1}$$

where $l_L = (\nu_L, e_L)$ is the left-handed lepton doublet and $\phi = (\varphi^+, \varphi^0)$ is the standard model Higgs doublet. λ_l, λ_ν and M are 3×3 complex matrices in the case of three generations. One can always choose a basis for the fields ν_R such that the mass matrix M is diagonal and real with eigenvalues M_i. The corresponding physical mass eigenstates are then the three Majorana neutrinos $N_i = \nu_{R\,i} + \nu_{R\,i}^C$. At tree level the propagator matrix of these Majorana neutrinos reads

$$i S_0(q)\,C^{-1} = \frac{i}{\not{q} - M + i\epsilon}\,C^{-1}\,, \tag{2}$$

where C is the charge conjugation matrix. This propagator has poles at $q^2 = M_i^2$ corresponding to stable particles, whereas the physical Majorana neutrinos are unstable. This is taken into account by summing self-energy diagrams in the usual way, which leads to the resummed propagator

$$i S(q)\,C^{-1} = \frac{i}{\not{q} - M - \Sigma(q)}\,C^{-1}\,. \tag{3}$$

At one-loop level the two diagrams in Fig. 1 yield the self energy

$$\Sigma_{\alpha\beta}^{ij}(q) = (\not{q}P_R)_{\alpha\beta}\,\Sigma_R^{ij}(q^2) + (\not{q}P_L)_{\alpha\beta}\,\Sigma_L^{ij}(q^2)\,, \tag{4}$$

where $P_{R,L} = \frac{1}{2}(1 \pm \gamma_5)$ are the projectors on right- and left-handed chiral states. Σ_R and Σ_L are the contributions of the diagrams Figs. (1a) and (1b), respectively. They can be written as products of a complex function $a(q^2)$ and a hermitian matrix K,

$$\Sigma_L(q^2) = \left(\Sigma_R(q^2)\right)^T = a(q^2)\,K\,, \quad K = \lambda_\nu^\dagger \lambda_\nu\,. \tag{5}$$

$a(q^2)$ is given by the usual form factor $B_0(q^2,0,0)$ [14], whose finite part reads in the $\overline{\text{MS}}$-scheme,

$$a(q^2) = \frac{1}{16\pi^2}\left(\ln\frac{|q^2|}{\mu^2} - 2 - i\pi\Theta(q^2)\right). \tag{6}$$

For simplicity we will often omit the argument of a in the following, however one should keep in mind that a depends on q^2.

According to Eqs. (3) and (4) the resummed propagator $S(q)$ satisfies

$$\left[\not{q}\left(\left(1 - \Sigma_R(q^2)\right)P_R + \left(1 - \Sigma_L(q^2)\right)P_L\right) - M\right] S(q) = 1\,. \tag{7}$$

The fermion propagator $S(q)$ consists of four chiral parts

$$S(q) = P_R\,S^{RR}(q^2) + P_L\,S^{LL}(q^2) + P_L\,\not{q}\,S^{LR}(q^2) + P_R\,\not{q}\,S^{RL}(q^2)\,. \tag{8}$$

Inserting this decomposition into Eq. (7), and multi-

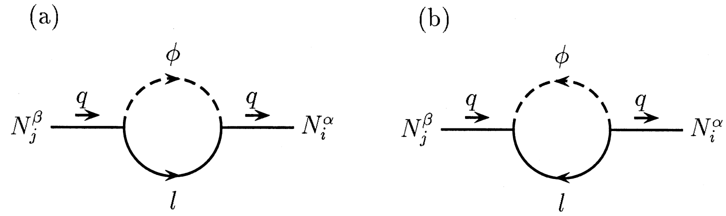

Fig. 1. Leading order contributions to the self-energy of the heavy Majorana neutrinos.

plying the resulting equation from the left and the right with chiral projectors $P_{R,L}$, yields a system of four coupled linear equations for the four parts of the propagator. The solution reads

$$S^{RR}(q^2) = \left[\left(1 - \Sigma_L(q^2)\right) \frac{q^2}{M} \left(1 - \Sigma_R(q^2)\right) - M \right]^{-1}, \tag{9}$$

$$S^{LR}(q^2) = \frac{1}{M}\left(1 - \Sigma_R(q^2)\right) S^{RR}(q^2), \tag{10}$$

$$S^{LL}(q^2) = \left[\left(1 - \Sigma_R(q^2)\right) \frac{q^2}{M} \left(1 - \Sigma_L(q^2)\right) - M \right]^{-1}, \tag{11}$$

$$S^{RL}(q^2) = \frac{1}{M}\left(1 - \Sigma_L(q^2)\right) S^{LL}(q^2). \tag{12}$$

As discussed below, the diagonal elements of $S(q)$ have approximately the usual Breit-Wigner form.

In addition to the self-energy we need the one-loop vertex function. The two expressions for the coupling of N to $\bar{l},\bar{\phi}$ (Fig. 2a) and N to l,ϕ (Fig. 2b) can be written as

$$\begin{aligned} \bar{\Gamma}^{ji}_{\beta\alpha,ab}(q,p) = &+ i\epsilon_{ab}\Big(\left(KMb(q,p)\lambda_\nu^T\right)_{ji} q_\mu \\ &+ \left(KMc(q,p)\lambda_\nu^T\right)_{ji} p_\mu \Big) \\ &\times \left(C\gamma^\mu P_L\right)_{\beta\alpha}, \end{aligned} \tag{13}$$

$$\begin{aligned} \Gamma^{ij}_{\alpha\beta,ab}(q,p) = &- i\epsilon_{ab}\Big(\left(\lambda_\nu^* Mb(q,p)K\right)_{ij} q_\mu \\ &+ \left(\lambda_\nu^* Mc(q,p)K\right)_{ij} p_\mu \Big) \\ &\times \left(P_R\gamma^\mu\right)_{\alpha\beta}. \end{aligned} \tag{14}$$

Here $b(q,p)$ and $c(q,p)$ are diagonal matrices whose

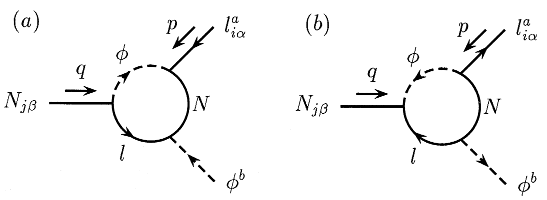

Fig. 2. One-loop corrections to the couplings of heavy Majorana neutrinos N_j to anti-lepton Higgs states (a) and lepton Higgs states (b).

elements are given by the standard form factors C_0 and C_{12} [14],

$$\begin{aligned} b_k(q,p) = &\frac{1}{16\pi^2}\Big(C_0(-p-q,q,M_k,0,0) \\ &+ C_{12}(-p-q,q,M_k,0,0)\Big), \end{aligned} \tag{15}$$

$$\begin{aligned} c_k(q,p) = &\frac{1}{16\pi^2}\Big(C_0(-p-q,q,M_k,0,0) \\ &+ 2C_{12}(-p-q,q,M_k,0,0)\Big). \end{aligned} \tag{16}$$

Since we shall only consider amplitudes with massless on-shell leptons, the terms proportional to c_k will not contribute. We shall only need the imaginary part of b_k which is given by

$$\mathrm{Im}\left\{b_k(q^2)\right\} = \frac{1}{16\pi\sqrt{q^2}\,M_k} f\left(\frac{M_k^2}{q^2}\right) \Theta(q^2), \tag{17}$$

where the function f is defined as

$$f(x) = \sqrt{x}\left(1 - (1+x)\ln\left(\frac{1+x}{x}\right)\right). \tag{18}$$

Transition matrix elements. The two lepton-number violating and the two lepton-number conserving processes are shown in Fig. 3a–d. Consider first the contributions of the full propagator, where the full vertices are replaced by tree couplings. The four scattering amplitudes read

$$\begin{aligned} &\langle \bar{l}^d_j(p')\bar{\phi}^e(q-p')|l^a_i(p)\phi^b(q-p)\rangle \\ &= +i\epsilon_{ab}\epsilon_{de}\left(\lambda_\nu^T\right)_{lj}\left(\lambda_\nu^T\right)_{ki}\left(CP_L v(p')\right)^T \\ &\quad\times S^{LL}_{lk}(q)C^{-1}\left(CP_L u(p)\right), \end{aligned} \tag{19}$$

$$\begin{aligned} &\langle l^d_j(p')\phi^e(q-p')|\bar{l}^a_i(p)\bar{\phi}^b(q-p)\rangle \\ &= +i\epsilon_{ab}\epsilon_{de}\left(\lambda_\nu^\dagger\right)_{lj}\left(\lambda_\nu^\dagger\right)_{ki}\left(\bar{u}(p')P_R\right) \\ &\quad\times S^{RR}_{lk}(q)C^{-1}\left(\bar{v}(p)P_R\right)^T, \end{aligned} \tag{20}$$

$$\begin{aligned} &\langle l^d_j(p')\phi^e(q-p')|l^a_i(p)\phi^b(q-p)\rangle \\ &= -i\epsilon_{ab}\epsilon_{de}\left(\lambda_\nu^\dagger\right)_{lj}\left(\lambda_\nu^T\right)_{ki}\left(\bar{u}(p')P_R\right) \\ &\quad\times S^{RL}_{lk}(q)\not{q}C^{-1}\left(CP_L u(p)\right), \end{aligned} \tag{21}$$

$$\begin{aligned} &\langle \bar{l}^d_j(p')\bar{\phi}^e(q-p')|\bar{l}^a_i(p)\bar{\phi}^b(q-p)\rangle \\ &= -i\epsilon_{ab}\epsilon_{de}\left(\lambda_\nu^T\right)_{lj}\left(\lambda_\nu^\dagger\right)_{ki}\left(CP_L v(p')\right)^T \\ &\quad\times S^{LR}_{lk}(q)\not{q}C^{-1}\left(\bar{v}(p)P_R\right)^T. \end{aligned} \tag{22}$$

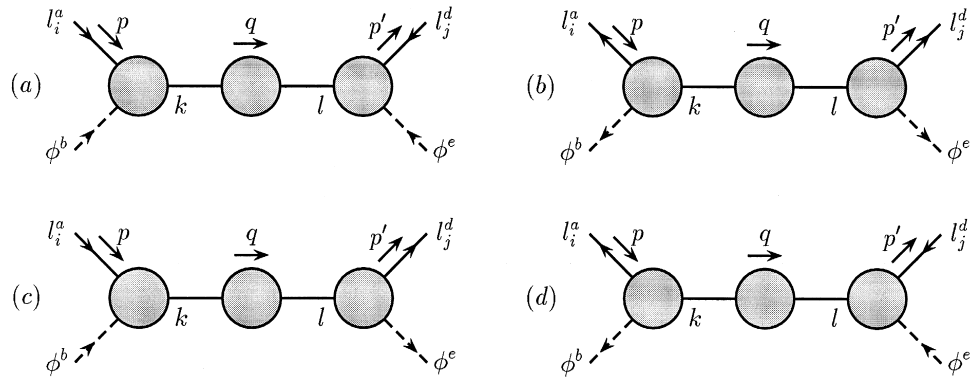

Fig. 3. s-channel contributions to lepton-Higgs scattering, including full propagators and vertices.

Here a,b,d,e denote the SU(2) indices of lepton and Higgs fields and i,j,k,l are generation indices. The relative signs follow from Fermi statistics.

We are particularly interested in the contributions of a single heavy neutrino to the scattering amplitudes. In order to determine these contributions we have to find the poles and the residues of the propagator matrix. Here an unfamiliar complication arises due to the fact that the self-energy matrix is different for left- and right-handed states. Hence, the different chiral projections of the propagator matrix are diagonalized by different matrices.

S^{LL} and S^{RR} are symmetric complex matrices, since $\Sigma_L(q^2) = \left(\Sigma_R(q^2)\right)^T$. Hence, S^{LL} and S^{RR} can be diagonalized by complex orthogonal matrices V and U, respectively,

$$S^{LL}(q^2) = V^T(q^2)MD(q^2)V(q^2),$$
$$S^{RR}(q^2) = U^T(q^2)MD(q^2)U(q^2). \tag{23}$$

Splitting the self-energy into a diagonal and an off-diagonal part,

$$\Sigma_L(q^2) = \Sigma_D(q^2) + \Sigma_N(q^2), \tag{24}$$

one finds

$$D^{-1}(q^2) = q^2\left(1 - \Sigma_D(q^2)\right)^2 - M^2 + \mathcal{O}\left(\Sigma_N^2\right). \tag{25}$$

One can easily identify real and imaginary parts of the propagator poles. The pole masses are given by

$$\overline{M}_i^2 = Z_{Mi} M_i^2, \quad Z_{Mi} = \left(1 + \frac{K_{ii}}{8\pi^2}\left(\ln\frac{M_i^2}{\mu^2} - 2\right)\right), \tag{26}$$

and the widths are $\Gamma_i = K_{ii} M_i/(8\pi)$. In the vicinity of the poles the propagator has the familiar Breit-Wigner form

$$D_i(q^2) \simeq \frac{Z_i}{q^2 - \overline{M}_i^2 + i\overline{M}_i \Gamma_i},$$
$$Z_i = \left(1 + \frac{K_{ii}}{8\pi^2}\left(\ln\frac{M_i^2}{\mu^2} - 1\right)\right). \tag{27}$$

We can now easily write down the contribution of a single resonance N_l with spin s to the lepton-Higgs scattering amplitudes. Suppressing spin indices for massless fermions, one has

$$\begin{aligned}&\langle \bar{l}_j^d(p')\overline{\phi}^e(q-p')|l_i^a(p)\phi^b(q-p)\rangle_l \\ &\quad = \langle \bar{l}_j^d(p')\overline{\phi}^e(q-p')|N_l(q,s)\rangle iD_l(q^2) \\ &\qquad \times \langle N_l(q,s)|l_i^a(p)\phi^b(q-p)\rangle,\end{aligned} \tag{28}$$

$$\begin{aligned}&\langle l_j^d(p')\phi^e(q-p')|\bar{l}_i^a(p)\overline{\phi}^b(q-p)\rangle_l \\ &\quad = \langle l_j^d(p')\phi^e(q-p')|N_l(q,s)\rangle iD_l(q^2) \\ &\qquad \times \langle N_l(q,s)|\bar{l}_i^a(p)\overline{\phi}^b(q-p)\rangle,\end{aligned} \tag{29}$$

$$\begin{aligned}&\langle l_j^d(p')\phi^e(q-p')|l_i^a(p)\phi^b(q-p)\rangle_l \\ &\quad = \langle l_j^d(p')\phi^e(q-p')|N_l(q,s)\rangle_{\mathrm{LC}}\, iD_l(q^2) \\ &\qquad \times \langle N_l(q,s)|l_i^a(p)\phi^b(q-p)\rangle,\end{aligned} \tag{30}$$

$$\begin{aligned}&\langle \bar{l}_j^d(p')\overline{\phi}^e(q-p')|\bar{l}_i^a(p)\overline{\phi}^b(q-p)\rangle_l \\ &\quad = \langle \bar{l}_j^d(p')\overline{\phi}^e(q-p')|N_l(q,s)\rangle_{\mathrm{LC}}\, iD_l(q^2) \\ &\qquad \times \langle N_l(q,s)|\bar{l}_i^a(p)\overline{\phi}^b(q-p)\rangle.\end{aligned} \tag{31}$$

Here the subscript LC distinguishes an amplitude defined by a lepton-number conserving process from the same amplitude defined by a lepton-number violating process. From Eqs. (9)–(12) and (23) one finds

$$\langle N_l(q,s)|l_i^a(p)\phi^b(q-p)\rangle = +i\epsilon_{ab}\big(V(q^2)\lambda_\nu^T\big)_{li}\bar{u}_s(q,M_l)P_L u(p)\,, \tag{32}$$

$$\langle \bar{l}_j^d(p')\bar{\phi}^e(q-p')|N_l(q,s)\rangle = -i\epsilon_{de}\big(\lambda_\nu V^T(q^2)\big)_{jl}\bar{v}_s(q,M_l)P_L v(p')\,, \tag{33}$$

$$\langle N_l(q,s)|\bar{l}_i^a(p)\bar{\phi}^b(q-p)\rangle = -i\epsilon_{ab}\big(U(q^2)\lambda_\nu^\dagger\big)_{li}\bar{v}(p)P_R v_s(q,M_l)\,, \tag{34}$$

$$\langle l_j^d(p')\phi^e(q-p')|N_l(q,s)\rangle = +i\epsilon_{de}\big(\lambda_\nu^* U^T(q^2)\big)_{jl}\bar{u}(p')P_R u_s(q,M_l)\,, \tag{35}$$

$$\langle l_j^d(p')\phi^e(q-p')|N_l(q,s)\rangle_{\mathrm{LC}} = +i\epsilon_{de}\left(\lambda_\nu^*\frac{1}{M}\big(1-\Sigma_L(q^2)\big)V^T(q^2)M\right)_{jl} \times\bar{u}(p')P_R u_s(q,M_l)\,, \tag{36}$$

$$\langle \bar{l}_j^d(p')\bar{\phi}^e(q-p')|N_l(q,s)\rangle_{\mathrm{LC}} = -i\epsilon_{de}\left(\lambda_\nu\frac{1}{M}\big(1-\Sigma_R(q^2)\big)U^T(q^2)M\right)_{jl} \times\bar{v}_s(q,M_l)P_L v(p')\,, \tag{37}$$

where we have used the identity $C\bar{v}_s^T(p)=u_s(p)$.

Eqs. (32) and (33) describe the coupling of the Majorana field N to the lepton fields l_i and the Higgs field ϕ, and Eqs. (34) and (35) give the couplings of N to the charge conjugated fields $\bar{l}_i$ and ϕ^*. In the case of CP conservation, one has $\lambda_{\nu ij}=\lambda_{\nu ij}^*$, which implies $K=K^T$ and therefore

$$\Sigma_L(q^2)=\Sigma_R(q^2)\,,\quad V(q^2)=U(q^2)\,. \tag{38}$$

This yields

$$\langle N_l(q,s)|l_i^a(p)\phi^b(q-p)\rangle = \langle N_l(\tilde{q},s)|\bar{l}_i^a(\tilde{p})\bar{\phi}^b(\tilde{q}-\tilde{p})\rangle\,, \tag{39}$$

with $\tilde{q}=(q_0,-\boldsymbol{q})$, $\tilde{p}=(p_0,-\boldsymbol{p})$, as required by CP invariance.

The amplitudes given in Eqs. (32)–(35) have been obtained from the lepton-number violating processes Figs. (3a) and (3b). The lepton-number conserving processes Figs. (3c) and (3d) yield the amplitudes given in Eqs. (36) and (37). The consistent definition of an on-shell contribution of a single heavy Majorana neutrino to the two-body scattering amplitudes requires that the transition amplitudes extracted from lepton-number conserving and lepton-number violating processes are consistent. This implies

$$\langle l_j^d(p')\phi^e(q-p')|N_l(q,s)\rangle = \langle l_j^d(p')\phi^e(q-p')|N_l(q,s)\rangle_{\mathrm{LC}}\,, \tag{40}$$

$$\langle \bar{l}_j^d(p')\bar{\phi}^e(q-p')|N_l(q,s)\rangle = \langle \bar{l}_j^d(p')\bar{\phi}^e(q-p')|N_l(q,s)\rangle_{\mathrm{LC}}\,. \tag{41}$$

From Eqs. (33), (35), (36) and (37) it is clear that these relations are fulfilled if the mixing matrices $V(q^2)$ and $U(q^2)$ satisfy certain consistency relations. Assuming that the matrix λ_ν has an inverse, one reads off

$$U_{ij}(M_i^2)=\left(MV(M_i^2)\big(1-\Sigma_R(M_i^2)\big)\frac{1}{M}\right)_{ij}, \tag{42}$$

$$V_{ij}(M_i^2)=\left(MU(M_i^2)\big(1-\Sigma_L(M_i^2)\big)\frac{1}{M}\right)_{ij}. \tag{43}$$

The matrices V and U are determined by the requirement that the expressions (cf. Eqs. (23))

$$V(q^2)\big(S^{LL}(q^2)\big)^{-1}V^T(q^2) = V(q^2)\left(\big(1-\Sigma_R(q^2)\big)\times\frac{q^2}{M}\big(1-\Sigma_L(q^2)\big)-M\right)V^T(q^2)\,, \tag{44}$$

$$U(q^2)\big(S^{RR}(q^2)\big)^{-1}U^T(q^2) = U(q^2)\left(\big(1-\Sigma_L(q^2)\big)\frac{q^2}{M}\big(1-\Sigma_R(q^2)\big)-M\right)\times U^T(q^2)\,, \tag{45}$$

are diagonal on-shell, i.e., at $q^2=M_i^2$. Using $\Sigma_L=\Sigma_D+\Sigma_N$, and writing

$$V(q^2)=1+v(q^2)\,,\quad v(q^2)=-v^T(q^2)\,, \tag{46}$$

$$U(q^2)=1+u(q^2)\,,\quad u(q^2)=-u^T(q^2)\,, \tag{47}$$

a straightforward calculation yields

$$v_{ij}(q^2)=w_{ij}(q^2)\big(M_i\Sigma_{Nji}(q^2)+M_j\Sigma_{Nij}(q^2)\big)\,, \tag{48}$$

$$u_{ij}(q^2) = w_{ij}(q^2)\left(M_i \Sigma_{Nij}(q^2) + M_j \Sigma_{Nji}(q^2)\right), \tag{49}$$

where

$$w_{ij}(q^2)^{-1} = (M_i - M_j)\left(1 + \frac{M_i M_j}{q^2}\right) - 2a(q^2)(M_i K_{jj} - M_j K_{ii}). \tag{50}$$

These equations give the matrices V and U to leading order in Σ_N. They are meaningful as long as the matrix elements of Σ_N are small compared to those of w^{-1}.

Inserting Eqs. (48) and (49) in Eqs. (42) and (43), one finds that the consistency conditions for the mixing matrices V and U are fulfilled to leading order in Σ_N. We conclude that the contribution of a single heavy neutrino to two-body scattering processes can indeed be consistently defined. The pole masses are given by Eq. (26) and the couplings to lepton-Higgs initial and final states are given by Eqs. (32)–(35).

3. CP asymmetry in heavy neutrino decays

It is now straightforward to evaluate the CP asymmetry in the decay of a heavy Majorana neutrino,

$$\epsilon_i = \frac{\Gamma(N_i \to l\phi) - \Gamma\left(N_i \to \bar{l}\bar{\phi}\right)}{\Gamma(N_i \to l\phi) + \Gamma\left(N_i \to \bar{l}\bar{\phi}\right)}. \tag{51}$$

From Eqs. (33) and (35) one obtains for the partial decay widths, including mixing effects,

$$\Gamma_M\left(N_i \to \bar{l}\bar{\phi}\right) \propto \sum_j |\left(\lambda_\nu V^T(M_i^2)\right)_{ji}|^2, \tag{52}$$

$$\Gamma_M(N_i \to l\phi) \propto \sum_j |\left(\lambda_\nu^* U^T(M_i^2)\right)_{ji}|^2. \tag{53}$$

To leading order in λ_ν^2 this yields the asymmetry (cf. Eqs. (46), (47)),

$$\epsilon_i^M = \frac{1}{K_{ii}} \mathrm{Re}\left\{\left(u(M_i^2)K\right)_{ii} - \left(v(M_i^2)K^T\right)_{ii}\right\}. \tag{54}$$

Using Eqs. (48)–(50) and (6), one finally obtains

$$\epsilon_i^M = -\frac{1}{8\pi} \sum_j |w_{ij}(M_i^2)|^2 \left(M_i^2 - M_j^2\right) \frac{M_j}{M_i} \times \frac{\mathrm{Im}\left\{K_{Nij}^2\right\}}{K_{ii}}. \tag{55}$$

Consider first the case where differences between heavy neutrino masses are large, i.e., $|M_i - M_j| \gg |\Gamma_i - \Gamma_j|$. Eq. (55) then simplifies to

$$\epsilon_i^M = -\frac{1}{8\pi} \sum_j \frac{M_i M_j}{M_i^2 - M_j^2} \frac{\mathrm{Im}\left\{K_{Nij}^2\right\}}{K_{ii}}. \tag{56}$$

This is the familiar CP asymmetry due to flavour mixing [9]. It has previously been obtained by considering directly the self-energy correction to the Majorana neutrino decay, without any resummation. The CP asymmetry ϵ_i reaches its maximum for $|M_i - M_j| \sim |\Gamma_i - \Gamma_j|$, where the perturbative expansion breaks down.

Interesting is also the limiting case where the heavy neutrinos become mass degenerate. From Eq. (55) it is obvious that the CP asymmetry vanishes in this limit. The vanishing of the CP asymmetry for mass degenerate heavy neutrinos is expected on general grounds, since in this case the CP violating phases of the matrix K can be eliminated by a change of basis.

The CP asymmetry due to the vertex corrections is easily obtained using Eqs. (13), (14), (17) and (18). The partial decay widths corresponding to the full vertex read

$$\Gamma_V\left(N_i \to \bar{l}\bar{\phi}\right) \propto \sum_j |\left(\lambda_\nu(1 - MbK^T M)\right)_{ji}|^2, \tag{57}$$

$$\Gamma_V(N_i \to l\phi) \propto \sum_j |\left(\lambda_\nu^*(1 - MbKM)\right)_{ji}|^2. \tag{58}$$

For the corresponding CP asymmetry (51) one obtains the familiar result

$$\epsilon_i^V = -\frac{1}{8\pi} \sum_j \frac{\mathrm{Im}\left\{K_{Nij}^2\right\}}{K_{ii}} f\left(\frac{M_j^2}{M_i^2}\right), \tag{59}$$

where the function $f(x)$ has been defined in Eq. (18).

4. CP asymmetries in two-body processes

Let us now consider the CP asymmetries in two-body processes. Here we have to take into account

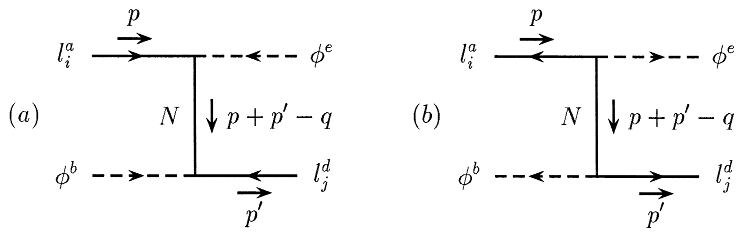

Fig. 4. u-channel contributions to lepton-Higgs scattering.

the s-channel amplitudes shown in Figs. (3a) and (3b), with vertex functions up to one-loop, and the two u-channel amplitudes depicted in Fig. 4a,b. For the u-channel amplitudes vertex and self-energy corrections can be omitted to leading order since the absorptive parts vanish.

In the following we shall evaluate various contributions to the CP asymmetry

$$\epsilon \equiv \frac{\Delta|\mathscr{M}|^2}{2|\mathscr{M}|^2} \equiv \frac{|\mathscr{M}(\bar{l}\bar{\phi}\to l\phi)|^2-|\mathscr{M}(l\phi\to\bar{l}\bar{\phi})|^2}{|\mathscr{M}(\bar{l}\bar{\phi}\to l\phi)|^2+|\mathscr{M}(l\phi\to\bar{l}\bar{\phi})|^2}\,, \tag{60}$$

where we always sum over generations in initial and final states. There are contributions from the full s-channel propagator, $\Delta|\mathscr{M}|_s^2$, from the interference between s-channel amplitudes at tree-level and with one-loop vertex corrections, $\Delta|\mathscr{M}|_{s,\Gamma}^2$, the interference between tree-level s-channel and u-channel amplitudes, $\Delta|\mathscr{M}|_{s,u}^2$, and the interference between s-channel with one-loop vertex corrections and u-channel amplitudes, $\Delta|\mathscr{M}|_{u,\Gamma}^2$.

Consider first the CP asymmetry ϵ_s due to the full propagator. The contribution of a single intermediate neutrino N_i is (cf. (28), (32), (33))

$$|\mathscr{M}_i(l\phi\to\bar{l}\bar{\phi})|_s^2 \propto |D_i(q^2)|^2 \sum_j |(V(q^2)\lambda_\nu^T)_{ij}|^2 \sum_k |(\lambda_\nu V^T(q^2))_{ki}|^2\,. \tag{61}$$

Comparison with Eq. (52) yields immediately

$$|\mathscr{M}_i(l\phi\to\bar{l}\bar{\phi})|_s^2 \propto |D_i(q^2)|^2 \Gamma_M\left(N_i\to\bar{l}\bar{\phi}\right)^2. \tag{62}$$

Similarly, one has for the charge conjugated process

$$|\mathscr{M}_i(\bar{l}\bar{\phi}\to l\phi)|_s^2 \propto |D_i(q^2)|^2 \Gamma_M(N_i\to l\phi)^2. \tag{63}$$

The corresponding CP asymmetry is, as expected, twice the asymmetry in the decay due to mixing,

$$\epsilon_s^{(i)} = \frac{\Delta|\mathscr{M}_i|_s^2}{2|\mathscr{M}_i|_s^2} \simeq 2\epsilon_i^M\,. \tag{64}$$

It is very instructive to compare the contribution of a single resonance with the CP asymmetry ϵ_s for the full propagator. Due to the structure of the propagators S^{LL} and S^{RR} it is difficult to evaluate ϵ_s exactly. However, one may easily calculate ϵ_s perturbatively in powers of Σ_N, like the mixing matrices $V(q^2)$ and $U(q^2)$ in the previous section.

The full propagator (cf. (7)) reads to first order in Σ_N,

$$\begin{aligned} S(q) &= S_D(q) \\ &\quad + S_D(q)\,\not{q}\left[\Sigma_N^T(q^2)P_R + \Sigma_N(q^2)P_L\right]S_D(q) \\ &\quad + \dots\,, \end{aligned} \tag{65}$$

where (cf. (25))

$$S_D(q) = \left[\not{q}\left(1-\Sigma_D(q^2)\right)+M\right]D(q^2)\,. \tag{66}$$

It is now straightforward to calculate the matrix elements of two-body processes, summed over generations in initial and final states,

$$\begin{aligned} &|\mathscr{M}(l\phi\to\bar{l}\bar{\phi})|_s^2 \\ &\quad = 16\,p\cdot p'\,q^2\Bigg(\frac{1}{2q^2}\mathrm{tr}\left[KMD(q^2)K^TMD^*(q^2)\right] \\ &\qquad + \mathrm{Re}\Big\{\mathrm{tr}\Big[KMD(q^2)\Sigma_N^T(q^2)\left(1-\Sigma_D(q^2)\right) \\ &\qquad \times D(q^2)K^TMD^*(q^2) \\ &\qquad + KD(q^2)\left(1-\Sigma_D(q^2)\right)\Sigma_N(q^2) \\ &\qquad \times MD(q^2)K^TMD^*(q^2)\Big]\Big\} + \dots\Bigg)\,. \end{aligned} \tag{67}$$

This yields for the sum and the difference of the CP conjugated matrix elements,

$$2|\mathcal{M}|_s^2 = 16\, p\cdot p' \sum_{i,j} A_{ij} + \dots\,, \tag{68}$$

$$\Delta|\mathcal{M}|_s^2 = -16\, p\cdot p' \sum_{i,j} \left(B_{ij} + C_{ij}\right) + \dots\,, \tag{69}$$

where

$$A_{ij} = \mathrm{Re}\left\{K_{ij}^2 M_i M_j D_j(q^2) D_i^*(q^2)\right\}, \tag{70}$$

$$B_{ij} = i\,\mathrm{Im}\left\{K_{Nij}\right\}^2 M_i M_j D_j(q^2) D_i^*(q^2), \tag{71}$$

$$C_{ij} = 4q^2 \mathrm{Re}\Big\{ia(q^2)\mathrm{Im}\left\{K_{Nij}^2\right\} M_i M_j \left(1 - \Sigma_D(q^2)_i\right) \times K_{ii} D_j(q^2) |D_i(q^2)|^2\Big\}. \tag{72}$$

For $q^2 \simeq M_i^2$ the expressions A_{ij} and C_{ij} are dominated by the contribution of a single resonance N_i,

$$A_{ii} \simeq K_{ii}^2 M_i^2 |D_i(q^2)|^2\,, \tag{73}$$

$$C_{ij} \simeq \frac{1}{4\pi} \mathrm{Im}\left\{K_{Nij}^2\right\} \frac{M_i^3 M_j}{M_i^2 - M_j^2} K_{ii} |D_i(q^2)|^2\,. \tag{74}$$

From Eqs. (56), (73) and (74) one reads off that the sum over C_{ij} yields precisely the contribution of the resonance N_i to the CP asymmetry,

$$-\frac{\sum_j C_{ij}}{A_{ii}} = -\frac{1}{4\pi} \sum_j \frac{M_i M_j}{M_i^2 - M_j^2} \frac{\mathrm{Im}\left\{K_{Nij}^2\right\}}{K_{ii}} = 2\epsilon_i^M\,. \tag{75}$$

The second contribution to the CP asymmetry ϵ_s is due to the sum over B_{ij} (cf. Eq. (71)). B_{ij} involves two different propagators $(i \neq j)$ and corresponds to an interference term. Using $D_j^{*\,-1}(q^2) = q^2 - M_j^2 - 2a^*(q^2) q^2 K_{jj}$ and $2q^2 \mathrm{Im}\{a(q^2)\} K_{ii} = -\mathrm{Im}\{D_i^{-1}(q^2)\}$, one can rewrite C_{ij} to leading order in λ_ν^2 as follows:

$$C_{ij} = -i\,\mathrm{Im}\left\{K_{Nij}^2\right\} M_i M_j D_j^{*\,-1}(q^2) / D_i^{-1}(q^2) \times |D_i(q^2)|^2 |D_j(q^2)|^2\,. \tag{76}$$

Comparing Eqs. (71) and (76) it is obvious that the sum of both terms, i.e., the CP asymmetry ϵ_s corresponding to the full propagator, is identically zero! The pole contribution is cancelled by the interference of the pole term with an off-shell propagator.

The contribution to the CP asymmetry $\Delta|\mathcal{M}|_{s,\Gamma}^2$ can be computed in a similar manner. The diagrams Fig. (3a) and (3b) yield two contributions for the two vertices. After some algebra one obtains the result (cf. (15))

$$\Delta|\mathcal{M}|_{s,\Gamma}^2 = -64\, p\cdot p'\, q^2 \sum_{i,j,k} D_{ijk} + \dots\,, \tag{77}$$

$$D_{ijk} = \mathrm{Im}\left\{K_{ik} K_{jk} K_{ij}\right\} \mathrm{Im}\left\{b_k(q^2)\right\} \times M_k M_j D_i(q^2) D_j^*(q^2)\,. \tag{78}$$

For $q^2 \simeq M_i^2$, one reads off that the sum over D_{ijk} yields, as expected, twice the vertex CP asymmetry,

$$\epsilon_{s,\Gamma}(M_i^2) \simeq \frac{\sum_k D_{iik}}{A_{ii}} = -\frac{1}{4\pi} \sum_k \frac{\mathrm{Im}\left\{K_{Nik}^2\right\}}{K_{ii}} f\left(\frac{M_k^2}{M_i^2}\right) = 2\epsilon_i^V\,. \tag{79}$$

A result very similar to Eqs. (77), (78) is obtained for the asymmetry $\Delta|\mathcal{M}|_{s,u}^2$, the interference between tree-level s-channel and u-channel amplitudes. One finds $(u = (q - p - p')^2)$,

$$\Delta|\mathcal{M}|_{s,u}^2 = -32\, p\cdot p'\, q^2 \sum_{i,j,k} E_{ijk} + \dots\,, \tag{80}$$

$$E_{ijk} = \mathrm{Im}\left\{K_{ik} K_{jk} K_{ij}\right\} \mathrm{Im}\left\{a(q^2)\right\} \times M_k M_j D_i(q^2) D_j^*(q^2) D_k^*(u)\,. \tag{81}$$

Integrating the expressions over phase space and using

$$\int_{-q^2}^{0} du \frac{2\,p\cdot p'}{u - M_k^2} = \frac{q^2\sqrt{q^2}}{M_k} f\left(\frac{M_k^2}{q^2}\right), \tag{82}$$

one finds the cancellation

$$\int_{-q^2}^{0} du \left(\Delta|\mathcal{M}|_{s,\Gamma}^2 + \Delta|\mathcal{M}|_{s,u}^2\right) = 0\,. \tag{83}$$

Finally, we have to consider the CP asymmetry $\Delta|\mathcal{M}|_{u,\Gamma}^2$. A straightforward calculation yields

$$\Delta|\mathcal{M}|_{u,\Gamma}^2 = -32\, p\cdot p'\, q^2 \sum_{i,j,k} F_{ijk} + \dots\,, \tag{84}$$

$$F_{ijk} = \mathrm{Im}\left\{K_{ik} K_{jk} K_{ji}\right\} \mathrm{Im}\left\{b_k(q^2)\right\} \times M_k M_j D_i(q^2) D_j^*(u)\,. \tag{85}$$

After integration over u the resulting matrix $\overline{F}_{ijk}$ is antisymmetric in the indices j and k. As a consequence, the asymmetry $\Delta|\mathscr{M}|^2_{u,\Gamma}$ is identically zero.

As we have seen, the total CP asymmetry vanishes to leading order in λ_ν^2. This result has previously been obtained in [13]. It follows from unitarity and CPT invariance. The considered T-matrix elements satisfy the unitarity relation

$$2\,\mathrm{Im}\langle l\phi|T|l\phi\rangle = \langle l\phi|T^\dagger T|l\phi\rangle\,. \tag{86}$$

If, in perturbation theory, the leading contribution to the right-hand side is given by two-particle intermediate states, one has

$$\begin{aligned}&\sum_l \langle l\phi|T^\dagger T|l\phi\rangle \\ &\quad = \sum_{l,l'}\left(|\langle l'\phi|T|l\phi\rangle|^2 + |\langle \bar{l'}\bar{\phi}|T|l\phi\rangle|^2\right) \\ &\quad\quad + \ldots\,.\end{aligned} \tag{87}$$

CPT invariance implies

$$\langle l'\phi|T|l\phi\rangle = \langle \bar{l}\bar{\phi}|T|\bar{l'}\bar{\phi}\rangle\,. \tag{88}$$

From Eqs. (86)–(88) one then obtains

$$\sum_{l,l'}\left(|\langle \bar{l'}\bar{\phi}|T|l\phi\rangle|^2 - |\langle l'\phi|T|\bar{l}\bar{\phi}\rangle|^2\right) + \ldots = 0\,. \tag{89}$$

In [13] it was concluded that away from resonance poles, where ordinary perturbation theory holds, the CP asymmetry (89) vanishes to order λ_ν^6. Corrections due to four-particle intermediate states are $\mathscr{O}(\lambda_\nu^8)$. In this paper we have developed a resummed perturbative expansion in powers of Σ_N which is also valid for $s \simeq M_i^2$. The same argument then implies that in this case the CP asymmetry (89) vanishes to order λ_ν^2 with corrections $\mathscr{O}(\lambda_\nu^4)$.

The nature of the cancellation is different for different subprocesses. For the full propagator, the CP asymmetry vanishes identically for fixed external momenta. Interference contributions between various s-channel and u-channel amplitudes cancel after phase space integration. In applications at finite temperature the standard practice [12] is to treat in the Boltzmann equations resonance contributions and the remaining two-body cross sections differently. This procedure yields for the CP asymmetry of the decaying heavy neutrino N_i the sum of mixing and vertex contribution, $\epsilon_i = \epsilon_i^M + \epsilon_i^V$. However, the generation of a lepton asymmetry is an out-of-equilibrium process and one may worry to what extent the result is affected by interference terms which are neglected. It therefore appears important to study systematically corrections to the Boltzmann equations [15].

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