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# Výběr funkcí

Pro výběr funkcí jsem se poradil s Chat GPT. Kladl jsem důraz na zadané požadavky:

- Multimodální
- Spojitá na celém rozsahu
- Ohodnotitelná v reálném čase
- Nemá fraktální charakter, či jiné „patologie“

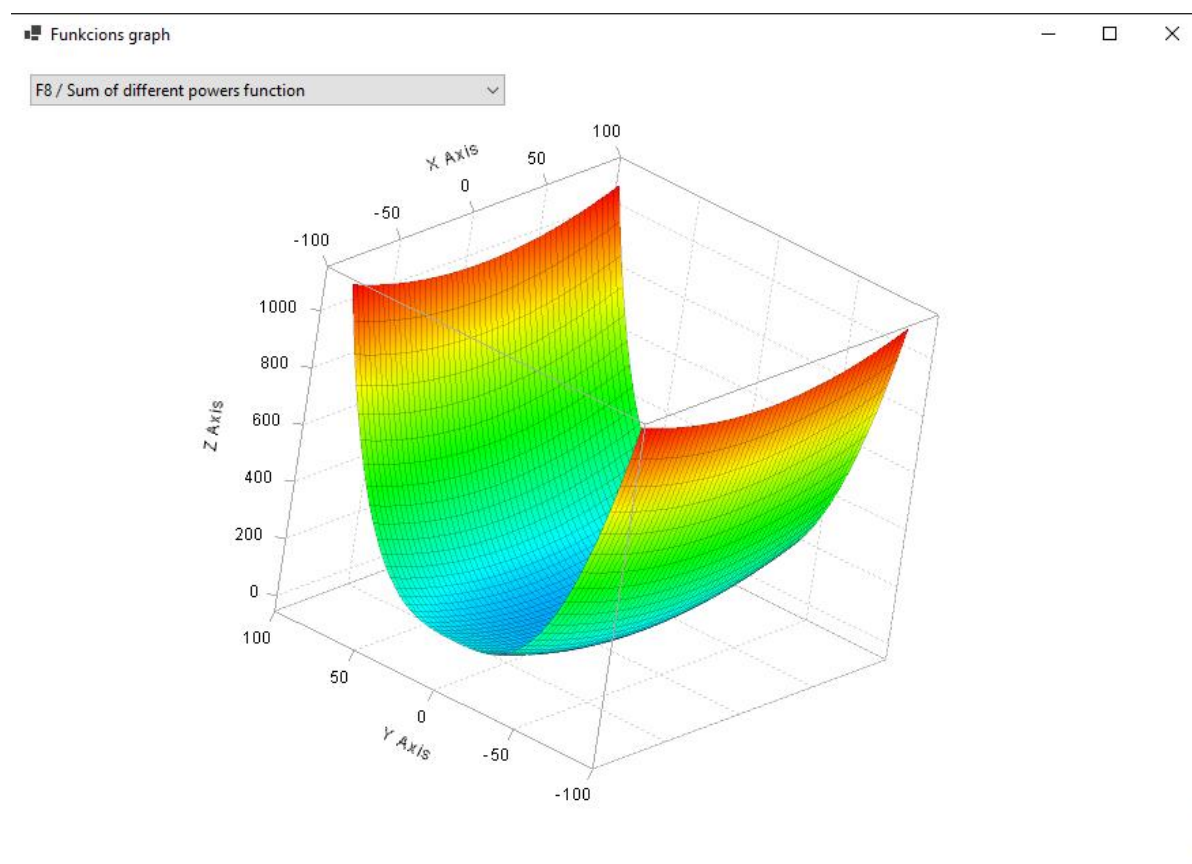
I když použití CHAT-GPT hodně pomohlo, bylo potřeba dohledat zdroje a ověřit fakta. Celou historii chatu si lze prohlédnout zde: <https://chat.openai.com/share/1ee8ec28-67a5-44c5-beb7-ec8868512922>

# Vizualizace funkcí

Pro zobrazení funkcí v **2D** a **3D**, jsem vytvořil pomocný program, ve kterém je možné interaktivně zobrazovat zvolené funkce.

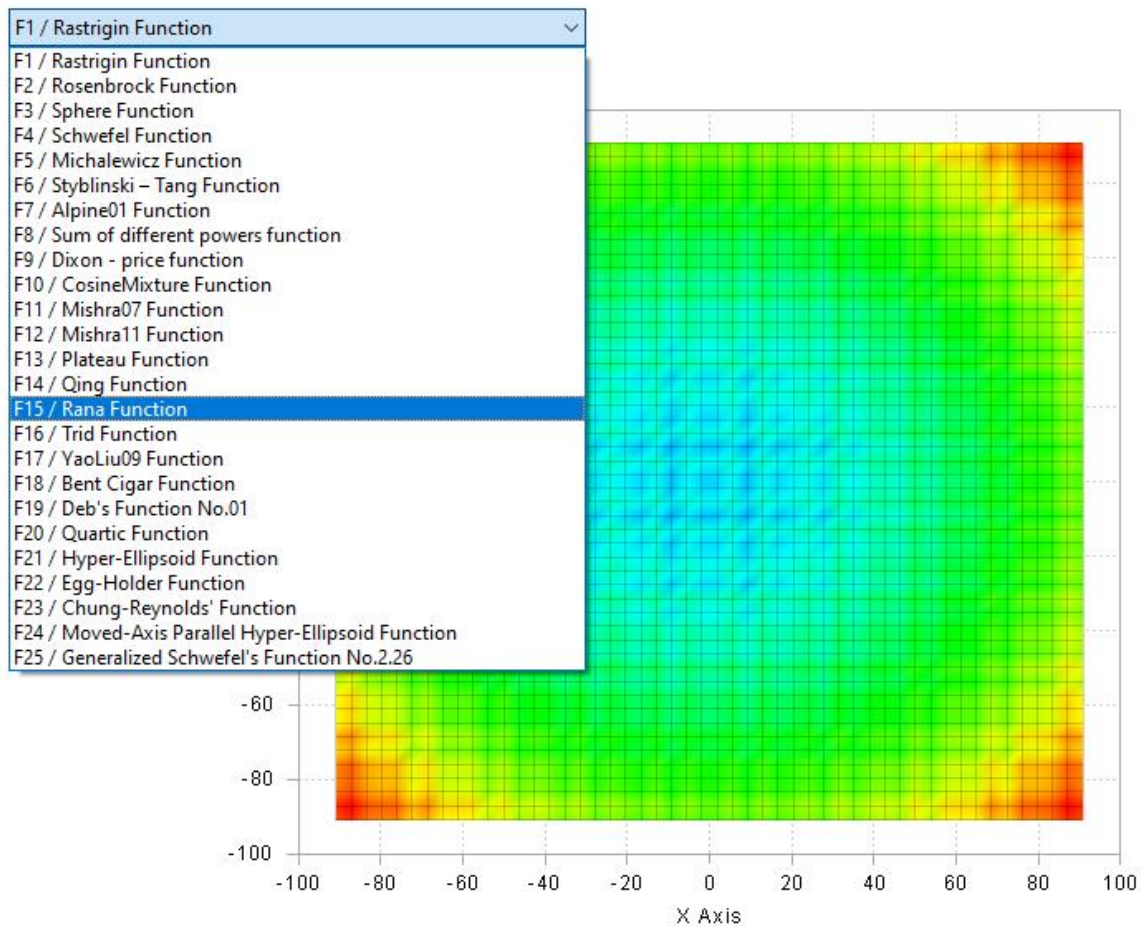
Program podporuje posouvání jednotlivých os a manipulaci s prostorem. Ve výchozím stavu zobrazuje funkci ve 2D (heat mapu).

Při zobrazení funkcí pracuji se zadaným rozsahem: -100, +100.



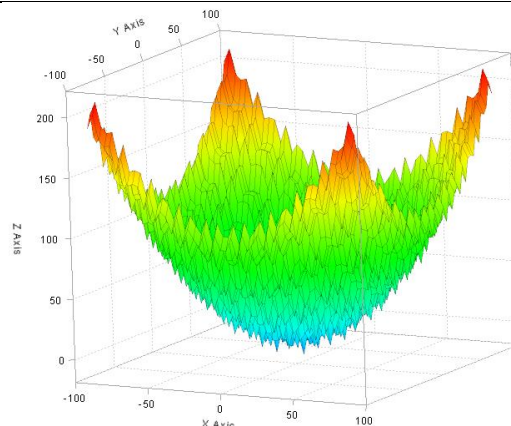
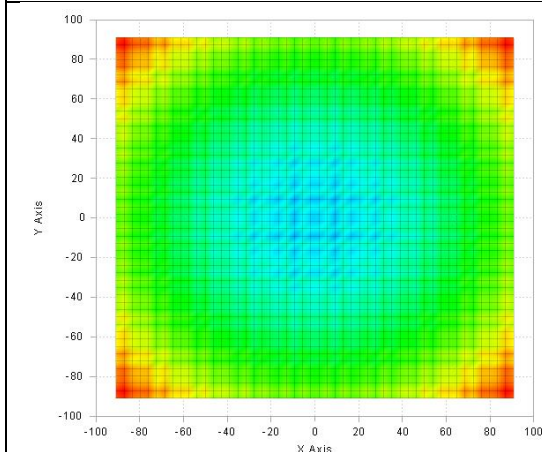
V pomocném programu se nachází výběrový seznam, který obsahuje seznam funkcí, které lze dynamicky prohlížet. Kompletní seznam funkcí, včetně vzorce a zdroje je detailně popsán níže.

#### ■ Funktions graph



# Seznam vybraných funkcí

F1/ Rastrigin function

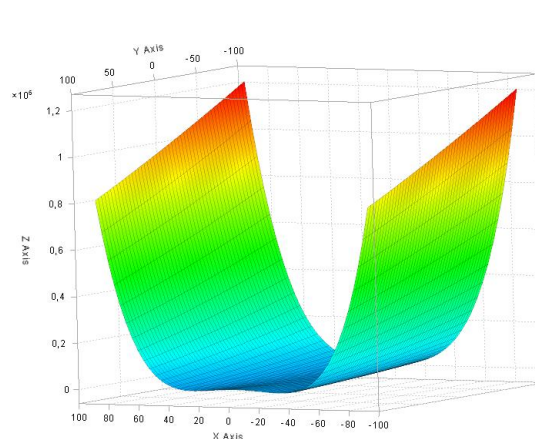
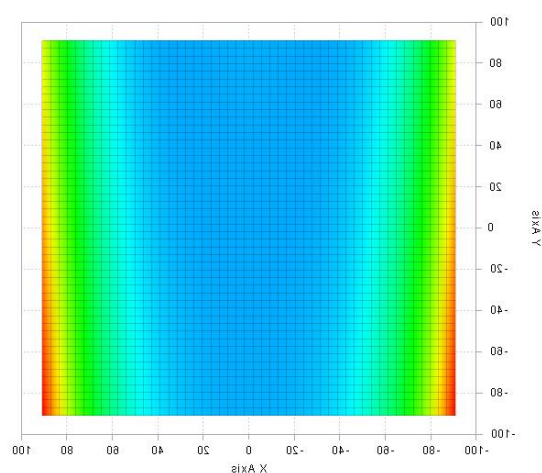


$$f(\mathbf{x}) = An + \sum_{i=1}^n [x_i^2 - A \cos(2\pi x_i)]$$

[https://en.wikipedia.org/wiki/Rastrigin\\_function](https://en.wikipedia.org/wiki/Rastrigin_function)

Poznámka:  $a = 10$

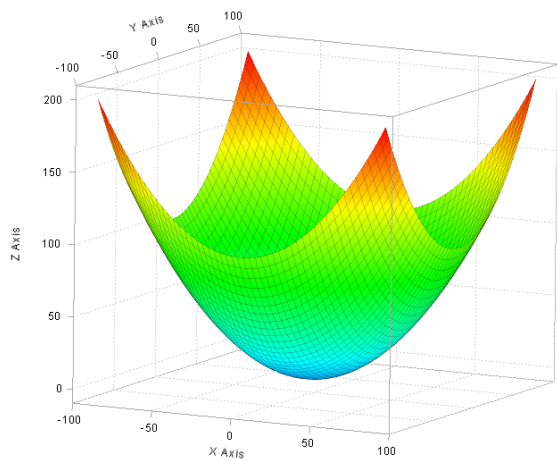
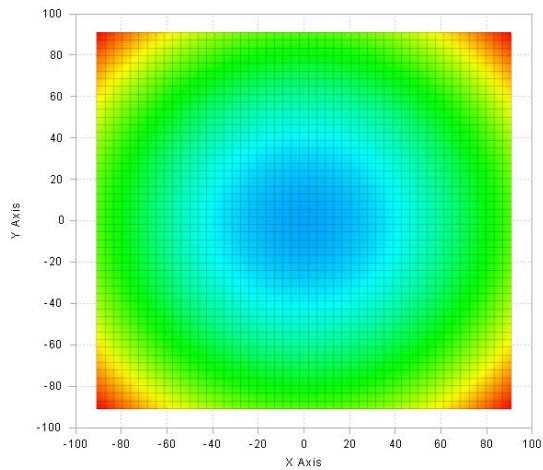
F2 / Rosenbrock function



$$f(\mathbf{x}) = f(x_1, x_2, \dots, x_N) = \sum_{i=1}^{N/2} [100(x_{2i-1}^2 - x_{2i})^2 + (x_{2i-1} - 1)^2] .$$

[https://en.wikipedia.org/wiki/Rosenbrock\\_function](https://en.wikipedia.org/wiki/Rosenbrock_function)

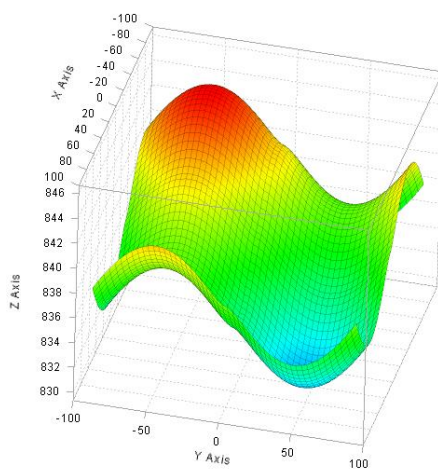
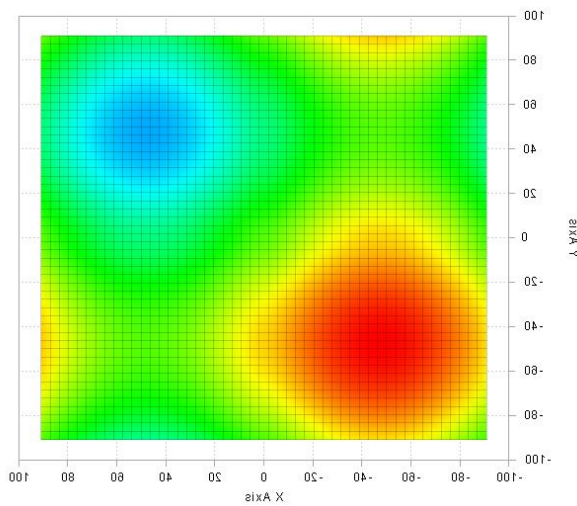
### F3 / Sphere function



$$f(\mathbf{x}) = \sum_{i=1}^d x_i^2$$

<https://www.sfu.ca/~ssurjano/spheref.html>

### F4 / Schwefel function

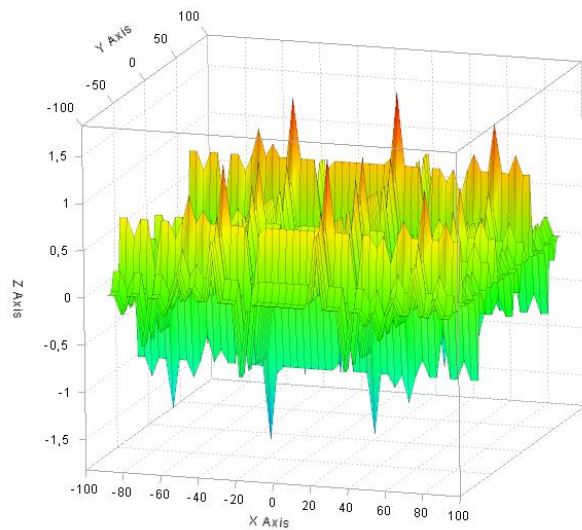
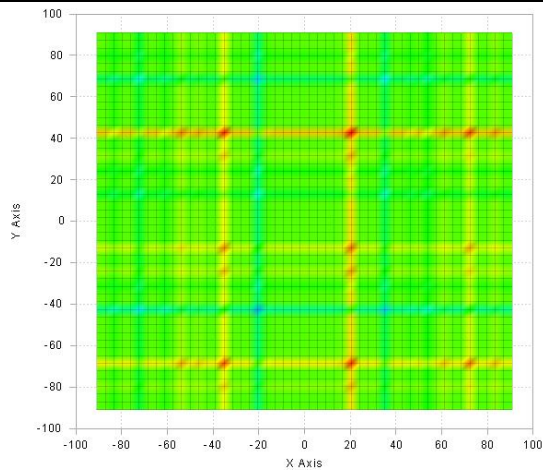


$$f(\mathbf{x}) = 418.9829d - \sum_{i=1}^d x_i \sin(\sqrt{|x_i|})$$

<https://www.sfu.ca/~ssurjano/schwef.html>



### F5 / Michalewicz function

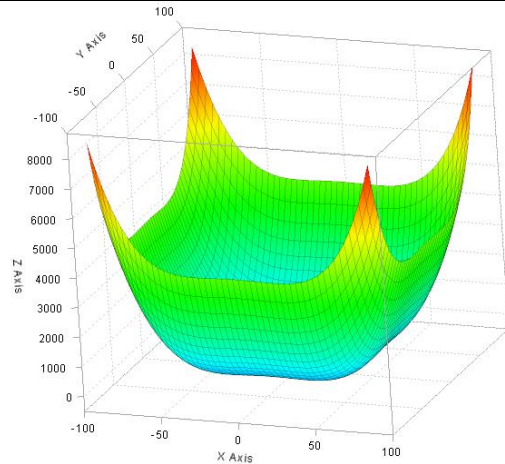
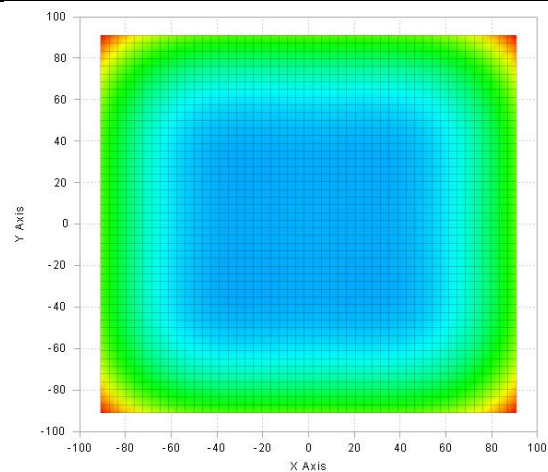


$$f(\mathbf{x}) = - \sum_{i=1}^d \sin(x_i) \sin^{2m} \left( \frac{ix_i^2}{\pi} \right)$$

<https://www.sfu.ca/~ssurjano/michal.html>

Poznámka:  $m=10$

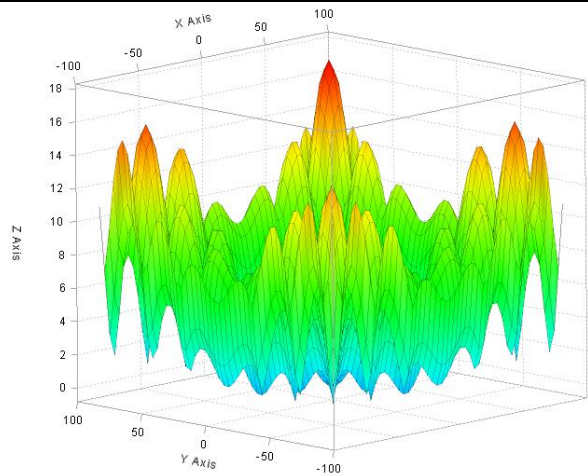
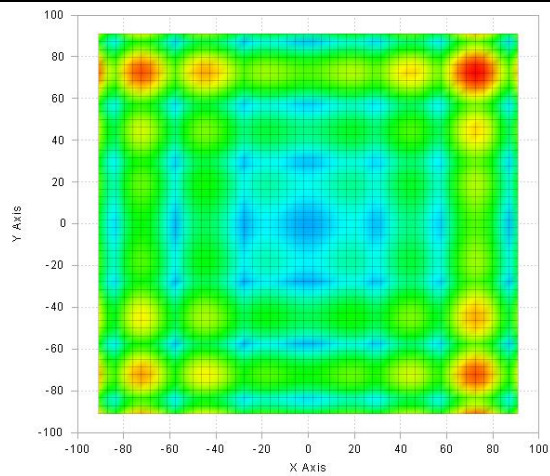
### F6 / Styblinski – Tang function



$$f(\mathbf{x}) = \frac{1}{2} \sum_{i=1}^d (x_i^4 - 16x_i^2 + 5x_i)$$

<https://www.sfu.ca/~ssurjano/stybtang.html>

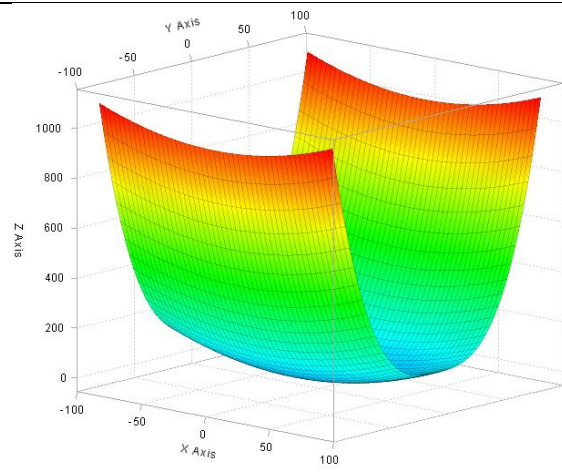
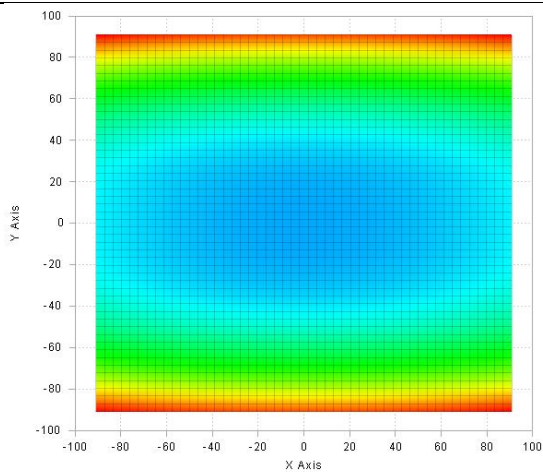
### F7 / Alpine01



$$f_{\text{Alpine01}}(\mathbf{x}) = \sum_{i=1}^n |x_i \sin(x_i) + 0.1x_i|$$

[https://infinity77.net/global\\_optimization/test\\_functions\\_nd\\_A.html#go\\_benchmark.Alpine01](https://infinity77.net/global_optimization/test_functions_nd_A.html#go_benchmark.Alpine01)

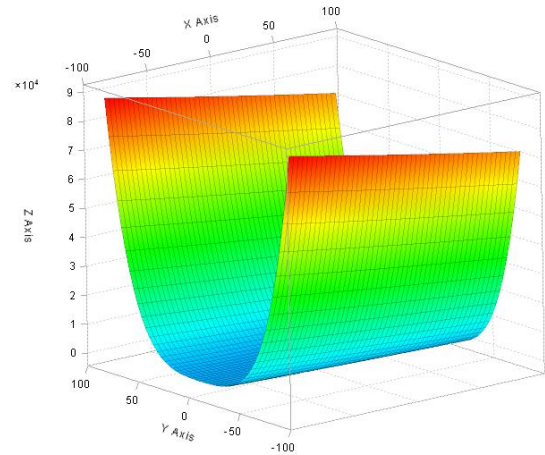
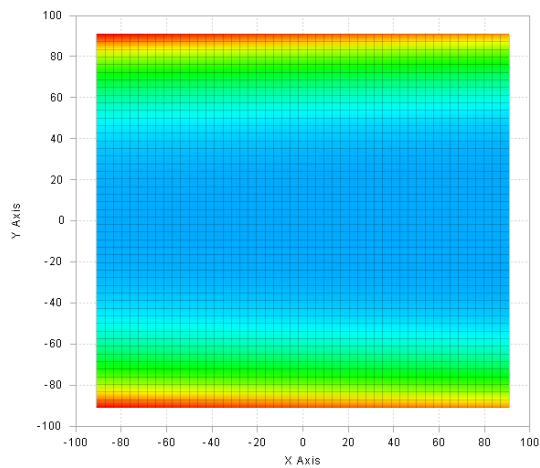
### F8 / Sum of different powers function



$$f(\mathbf{x}) = \sum_{i=1}^d |x_i|^{i+1}$$

<https://www.sfu.ca/~ssurjano/sumpow.html>

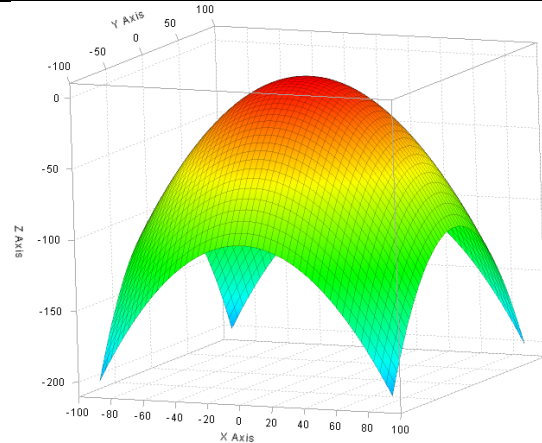
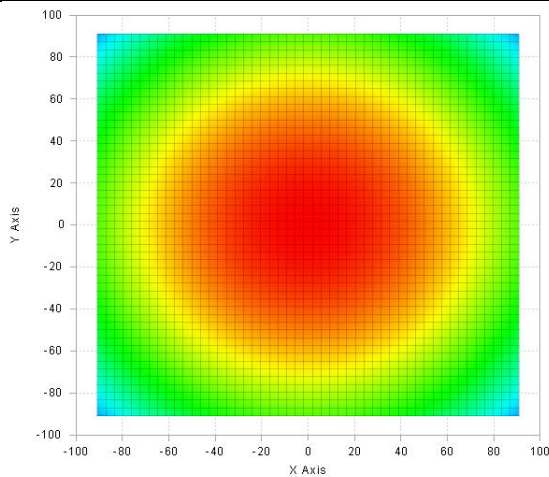
### F9 / Dixon-price function



$$f(\mathbf{x}) = (x_1 - 1)^2 + \sum_{i=2}^d i (2x_i^2 - x_{i-1})^2$$

<https://www.sfu.ca/~ssurjano/dixonpr.html>

### F10 / CosineMixture Function

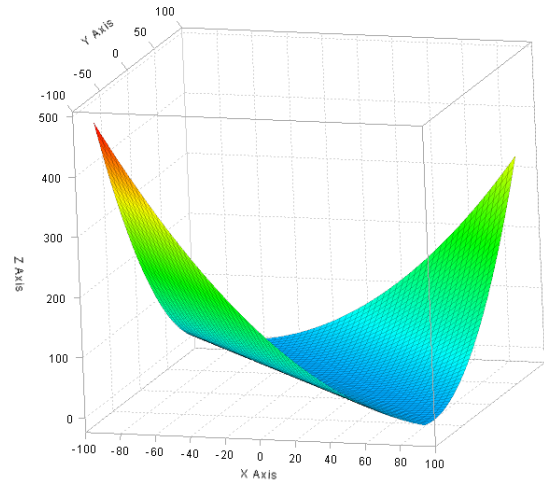
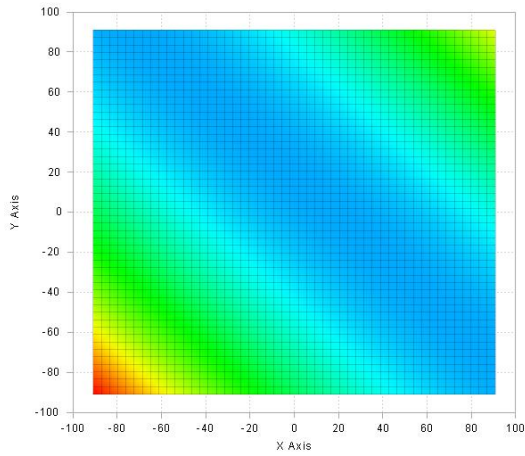


$$f_{\text{CosineMixture}}(\mathbf{x}) = -0.1 \sum_{i=1}^n \cos(5\pi x_i) - \sum_{i=1}^n x_i^2$$

[https://infinity77.net/global\\_optimization/test\\_functions\\_nd\\_C.html](https://infinity77.net/global_optimization/test_functions_nd_C.html)



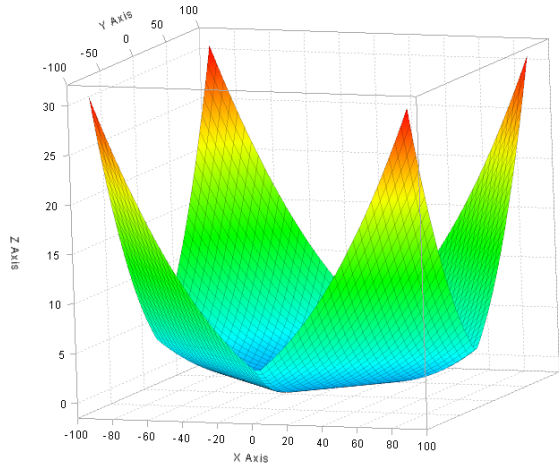
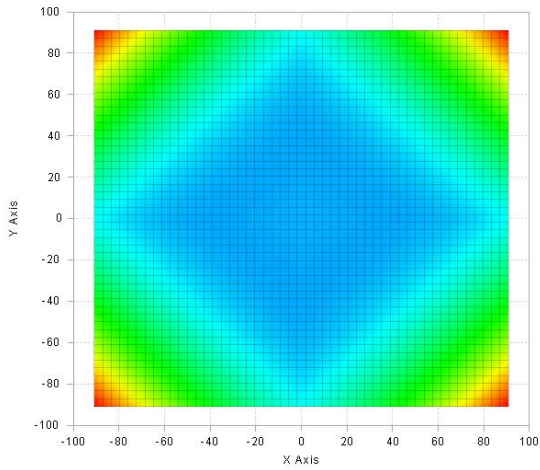
### F11 / Mishra07 Function



$$f_{\text{Mishra07}}(\mathbf{x}) = \left[ \prod_{i=1}^n x_i - n! \right]^2$$

[https://infinity77.net/global\\_optimization/test\\_functions\\_nd\\_M.html](https://infinity77.net/global_optimization/test_functions_nd_M.html)

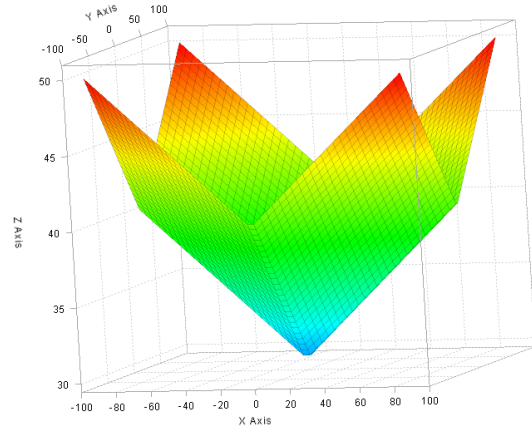
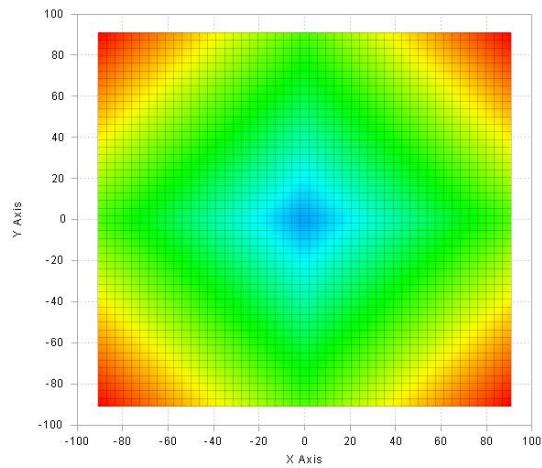
### F12 / Mishra11 Function



$$f_{\text{Mishra11}}(\mathbf{x}) = \left[ \frac{1}{n} \sum_{i=1}^n |x_i| - \left( \prod_{i=1}^n |x_i| \right)^{\frac{1}{n}} \right]^2$$

[https://infinity77.net/global\\_optimization/test\\_functions\\_nd\\_M.html](https://infinity77.net/global_optimization/test_functions_nd_M.html)

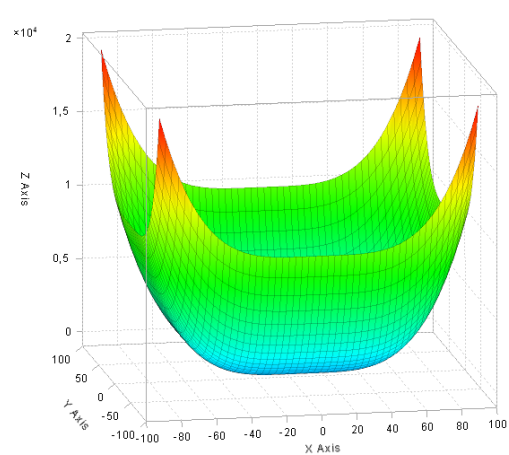
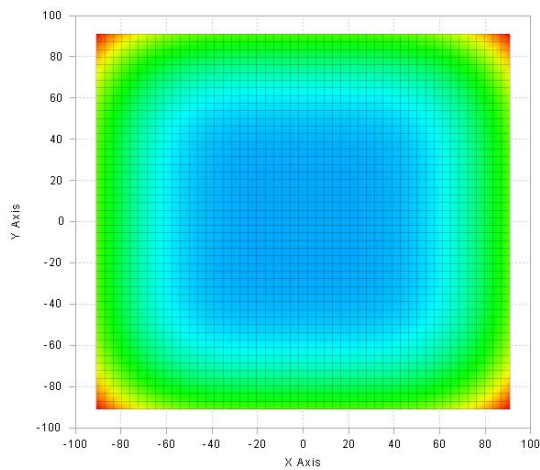
### F13 / Plateau Function



$$f_{\text{Plateau}}(\mathbf{x}) = 30 + \sum_{i=1}^n \lfloor x_i \rfloor$$

[https://infinity77.net/global\\_optimization/test\\_functions\\_nd\\_P.html](https://infinity77.net/global_optimization/test_functions_nd_P.html)

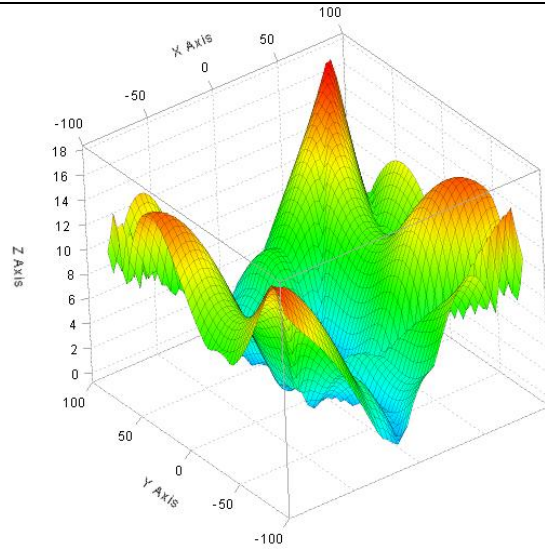
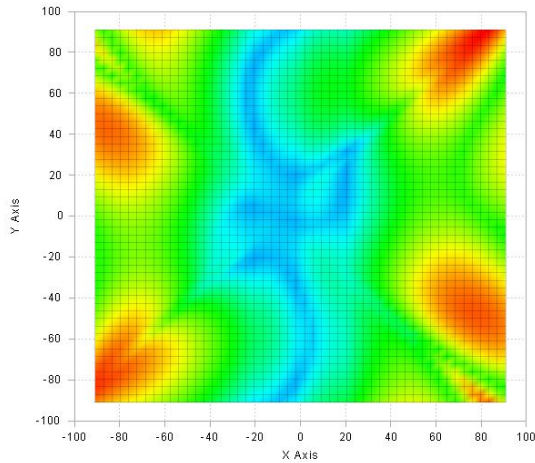
### F14 / Qing Function



$$f_{\text{Qing}}(\mathbf{x}) = \sum_{i=1}^n (x_i^2 - i)^2$$

[https://infinity77.net/global\\_optimization/test\\_functions\\_nd\\_Q.html](https://infinity77.net/global_optimization/test_functions_nd_Q.html)

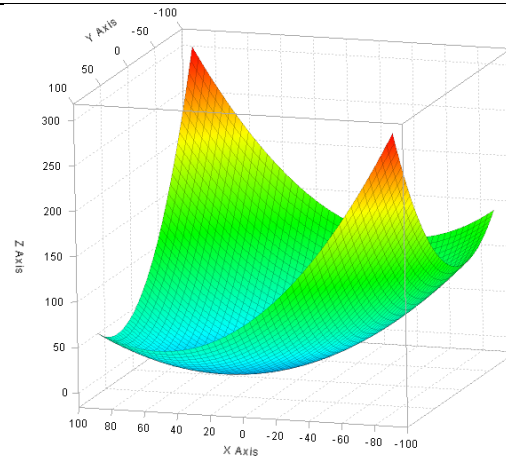
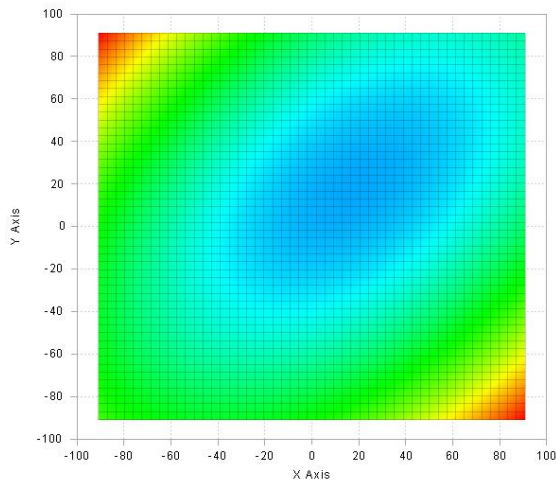
### F15 / Rana Function



$$f_{\text{Rana}}(\mathbf{x}) = \sum_{i=1}^n \left[ x_i \sin \left( \sqrt{|x_1 - x_i + 1|} \right) \cos \left( \sqrt{|x_1 + x_i + 1|} \right) + (x_1 + 1) \sin \left( \sqrt{|x_1 + x_i + 1|} \right) \cos \left( \sqrt{|x_1 - x_i + 1|} \right) \right]$$

[https://infinity77.net/global\\_optimization/test\\_functions\\_nd\\_R.html](https://infinity77.net/global_optimization/test_functions_nd_R.html)

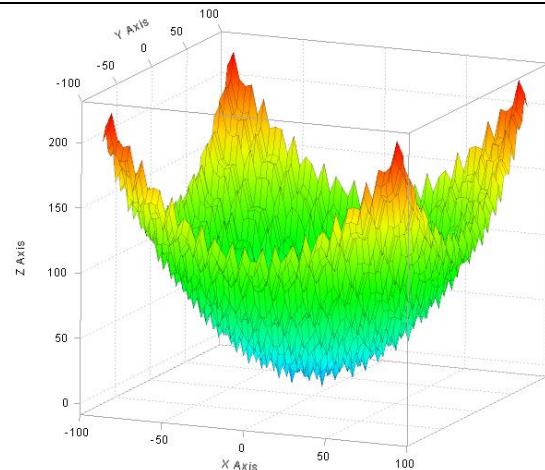
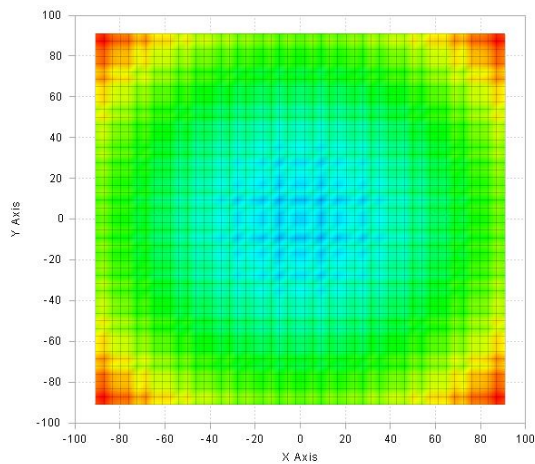
### F16 / Trid Function



$$f_{\text{Trid}}(\mathbf{x}) = \sum_{i=1}^n (x_i - 1)^2 - \sum_{i=2}^n x_i x_{i-1}$$

[https://infinity77.net/global\\_optimization/test\\_functions\\_nd\\_T.html](https://infinity77.net/global_optimization/test_functions_nd_T.html)

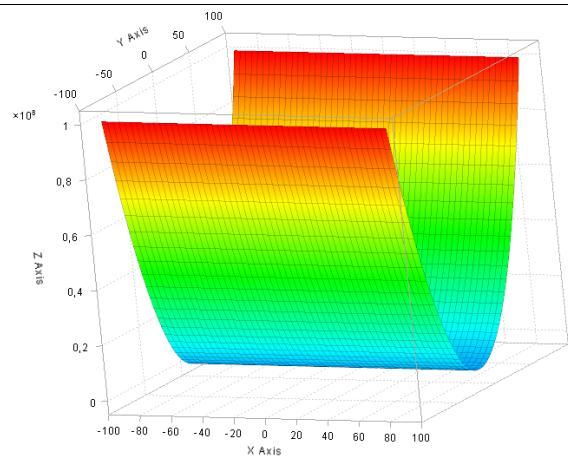
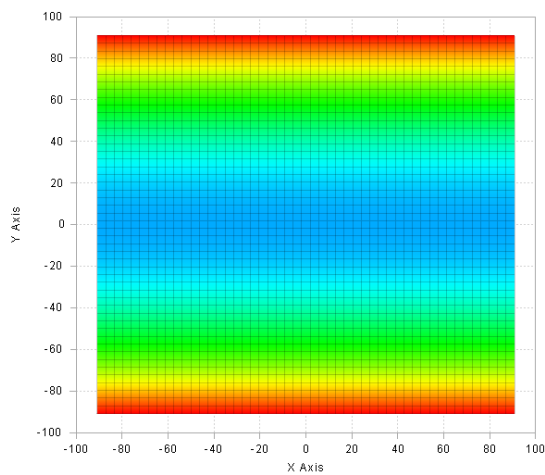
### F17 / YaoLiu09 Function



$$f_{\text{YaoLiu09}}(\mathbf{x}) = \sum_{i=1}^n [x_i^2 - 10 \cos(2\pi x_i) + 10]$$

[https://infinity77.net/global\\_optimization/test\\_functions\\_nd\\_Y.html](https://infinity77.net/global_optimization/test_functions_nd_Y.html)

### F18 / Bent Cigar Function

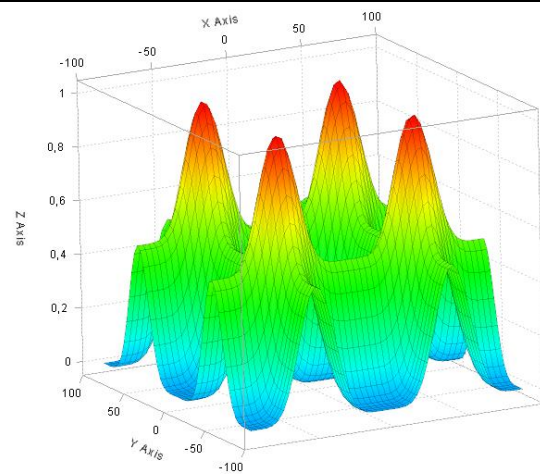
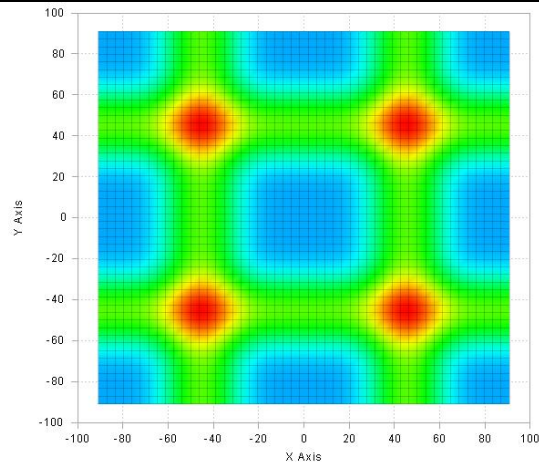


$$f(X) = x_1^2 + 10^6 \sum_{i=2}^n x_i^2$$

<https://al-roomi.org/benchmarks/unconstrained/n-dimensions/164-bent-cigar-function>



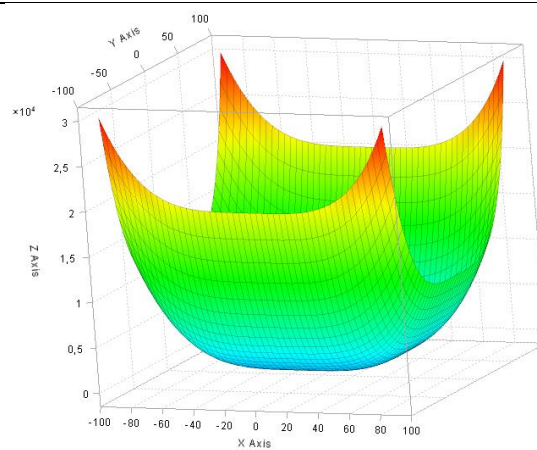
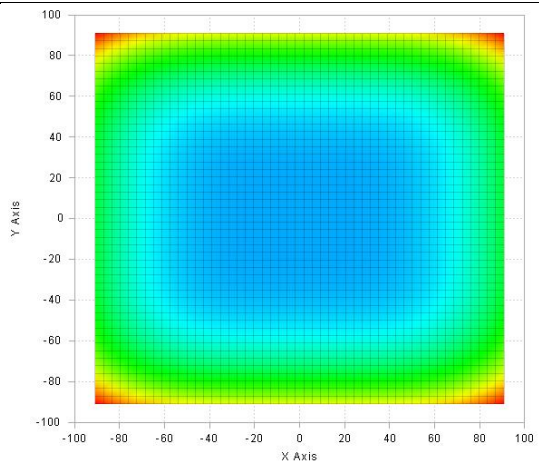
### F19 / Deb's Function No.01



$$f(X) = -\frac{1}{n} \sum_{i=1}^n \sin^6(5\pi x_i)$$

<https://al-roomi.org/benchmarks/unconstrained/n-dimensions/231-deb-s-function-no-01>

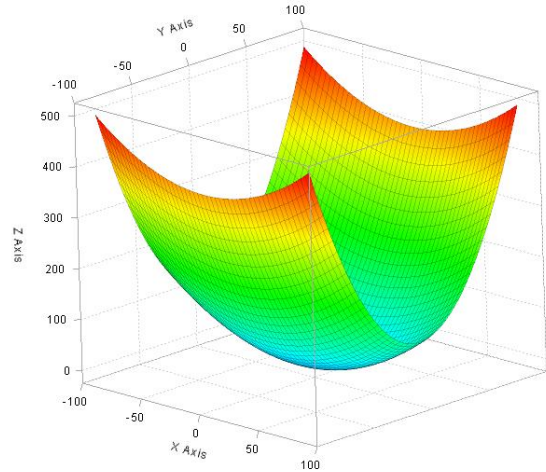
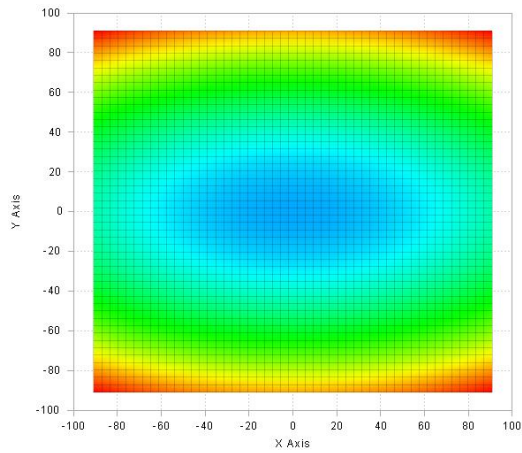
### F20 / Quartic Function



$$f(X) = \sum_{i=1}^n ix_i^4$$

<https://al-roomi.org/benchmarks/unconstrained/n-dimensions/161-quartic-or-modified-4th-de-jong-s-function>

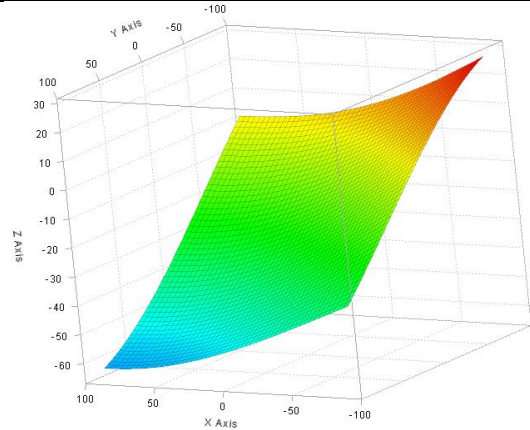
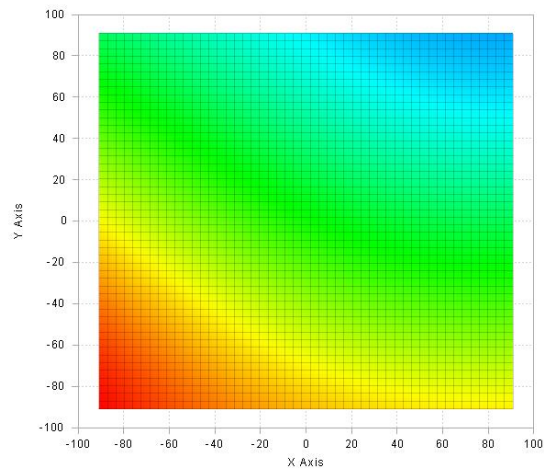
### F21 / Hyper-Ellipsoid Function



$$f(X) = \sum_{i=1}^n i^2 x_i^2$$

<https://al-roomi.org/benchmarks/unconstrained/n-dimensions/177-hyper-elipsoid-function>

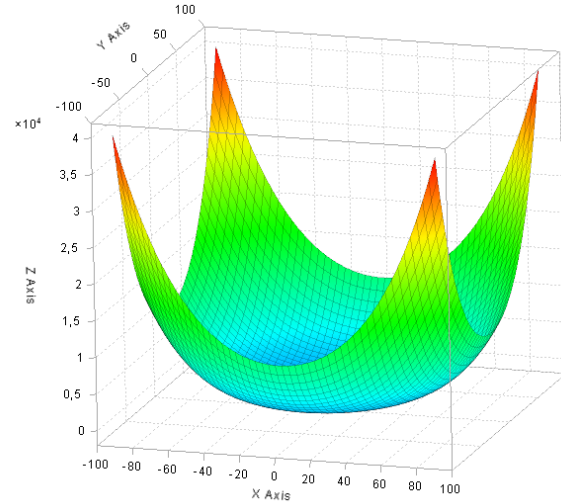
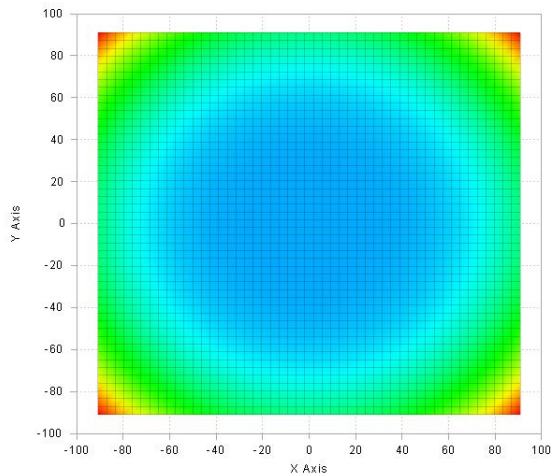
### F22 / Egg-Holder Function



$$f(X) = \sum_{i=1}^{n-1} \left[ -x_i \sin\left(\sqrt{|x_i - x_{i+1} - 47|}\right) - (x_{i+1} + 47) \sin\left(\sqrt{|0.5x_i + x_{i+1} + 47|}\right) \right]$$

<https://al-roomi.org/benchmarks/unconstrained/n-dimensions/187-egg-holder-function>

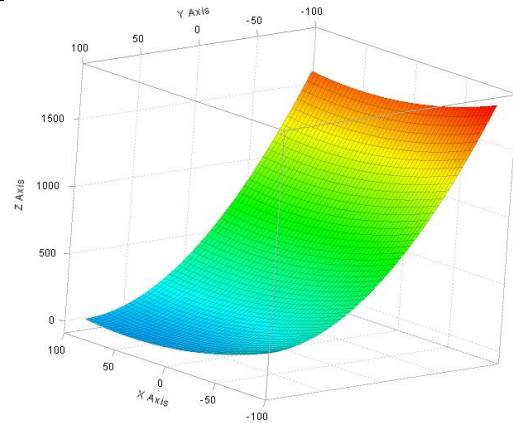
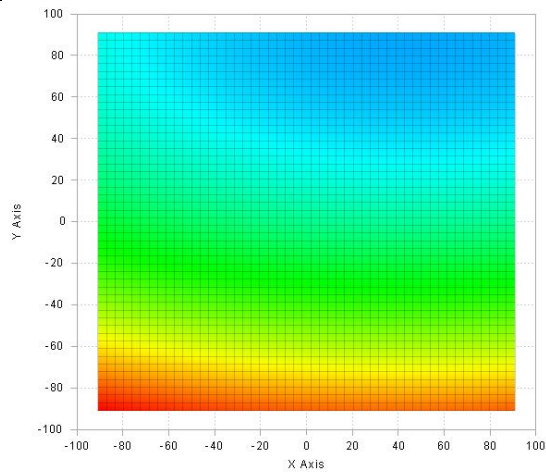
### F23 / Chung-Reynolds' Function



$$f(X) = \left( \sum_{i=1}^n x_i^2 \right)^2$$

<https://al-roomi.org/benchmarks/unconstrained/n-dimensions/165-chung-reynolds-function>

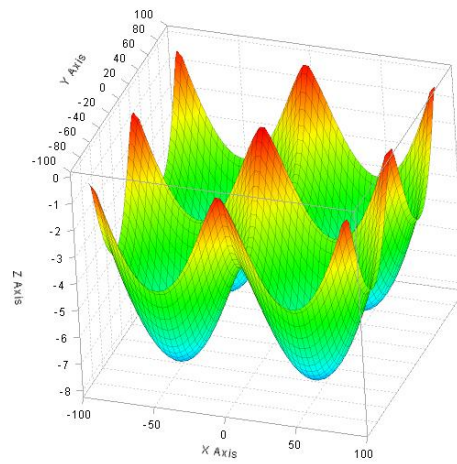
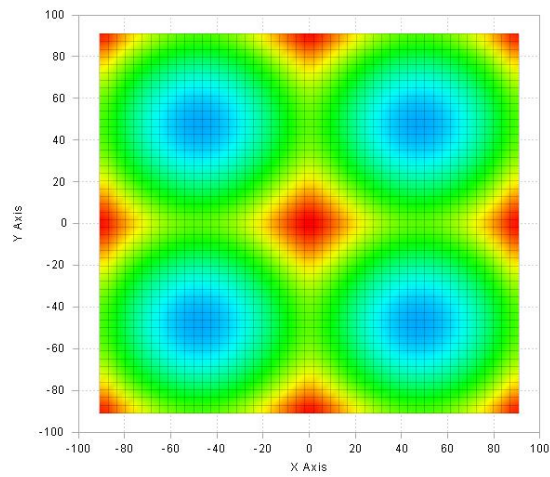
### F24 / Moved-Axis Parallel Hyper-Ellipsoid Function



$$f(X) = \sum_{i=1}^n \left[ i \cdot (x_i - 5 \cdot i) \right]^2$$

<https://al-roomi.org/benchmarks/unconstrained/n-dimensions/230-moved-axis-parallel-hyper-ellipsoid-function>

## F25 / Generalized Schwefel's Function No.2.26



$$f(X) = - \sum_{i=1}^n \left[ x_i \sin \left( \sqrt{|x_i|} \right) \right]$$

<https://al-roomi.org/benchmarks/unconstrained/n-dimensions/176-generalized-schwefel-s-problem-2-26>



## Výkonnost algoritmů

<b>D2</b>					
	<b>DE rand/1/bin</b>	<b>DE rand/1/bin</b>	<b>PSO</b>	<b>SOMA all-to-one</b>	<b>SOMA all-to-all</b>
F1	5,10499	0,00011	439,96329	37,56791	934,58595
F2	1,04798	0,03313	754475,85764	2624,43259	11501,15901
F3	0,00798	1,05E-18	0,58890	67,35834	619,82922
F4	441,99961	710,69584	757,38693	734,60592	752,72985
F5	-1,92734	-1,85146	-0,85063	-1,51783	-1,73317
F6	-75,88305	-78,33233	42803,61156	8882,75714	92,28626
F7	0,01964	2,37E-05	1,64004	1,45208	5,84538
F8	0,04851	2,17E-13	5313,94613	151,00540	623,74021
F9	0,09043	3,93E-13	53305,89747	232,73424	1638,09992
F10	-121076,58724	-19250,34552	-15056,07603	-18514,96740	-17469,99015
F11	0,00077	0	220,71148	0,10146	37,04600
F12	2,63E-20	0	0,00790	5,31E-08	8,91E-08
F13	30,22046	30	35,72549	49,49137	45,58325
F14	0,18138	1,61E-22	281774,87560	31568,22893	62563,14761
F15	-421,11678	-180,292	-90,91300	-140,25166	-92,72711
F16	-1,95542	-2	1068,99284	163,81973	358,75464
F17	4,02214	0,00027	303,64137	64,10757	105,72650
F18	1,54E-06	6,65E-159	50,48377	4,04079	13,41171
F19	1,23E-05	1,12E-06	0,00113	0,00874	0,00362
F20	0,00144	5,32E-52	119141,41918	65754,07972	10357,69405
F21	0,42417	1,26E-23	451,63477	504,94781	495,96390
F22	-393,11544	-199,68403	-125,97661	-110,40750	-120,92315
F23	0,01178	1,31E-49	48254,24256	26352,06893	2182434,42509
F24	2,92E-09	0	15,88628	1,50621	0,26467
F25	-382,10855	-127,26996	-93,28772	-84,17066	-106,69759

# D10

	DE rand/1/bin	DE rand/1/bin	PSO	SOMA all-to-one	SOMA all-to-all
F1	3092,49691	35,91955	10587,59974	13178,05193	12718,54770
F2	136170881,05350	14071,35010	5356949625,63218	2488005843,5554	2350489822,56355
F3	1945,65171	61,08770	11698,95763	12829,90848	15368,60540
F4	3445,25351	3741,18801	4011,47729	3848,61876	3857,93014
F5	-4,97345	-4,09196	-3,26070	-2,95892	-2,68004
F6	493554,55468	-268,15271	21557775,49210	18489176,90937	14834365,23885
F7	37,45389	4,08058	161,23707	139,92353	72,40820
F8	1487598646,89094	80185,52967	15938242187109,8	54040256486129,6	4,87505E+15
F9	68488835,38806	2571,74175	2971542652,32211	1744528616,42272	1983854121,07814
F10	-261585,40097	-68033,41475	-50536,42403	-57431,91465	-51448,35550
F11	4785657785296,75	0	2,87926E+26	5,34460E+22	3,40929E+19
F12	0,15848	0,00017	9,92708	7,85963	4,32743
F13	150,28968	40,46836	316,11860	271,77868	354,64150
F14	928136,49778	123,46525	71685848,02000	21264604,94842	92797114,56907
F15	-1167,49736	-910,62955	-222,32432	-606,67692	-634,16009
F16	1200,86224	-177,25260	6113,41694	9841,34292	7374,57993
F17	2149,84467	69,62447	13881,01979	8984,23465	11936,50010
F18	1311455667,41084	10822749,53324	15148979604,1896	7319781836,06107	3266363875,70407
F19	0,56414	1,01433	1,83126	1,79547	0,76916
F20	4494098,11677	427,85832	168546260,66823	147457986,77282	231685465,81444
F21	2254,05027	34,91895	15609,66421	7640,11630	17501,36625
F22	-979,67387	-662,88915	-303,86712	-356,24242	-473,98366
F23	8228542,22386	215,05079	227253457,43412	86842997,70977	134988967,08905
F24	46890,52059	1616,66145	276563,98148	173677,67371	253243,72497
F25	-762,93157	-483,31243	-390,17314	-328,00327	-201,35106

# D30

	DE rand/1/bin	DE rand/1/bin	PSO	SOMA all-to-one	SOMA all-to-all
F1	33190,99826	1830,52780	58906,23861	57127,52298	72182,02526
F2	9751248165,58860	20318746,48014	28322545052,42540	30546141834,87400	34433365644,84870
F3	31952,79206	1592,23671	66118,55963	69349,36425	75462,24759
F4	11365,50872	11735,40090	12218,79251	12075,92629	12305,46255
F5	-7,06547	-7,44789	-6,31840	-6,59490	-4,67331
F6	27314585,25368	92271,79812	127423825,31453	133738556,77518	151724243,99448
F7	364,79095	54,89980	607,22099	482,57560	565,64447
F8	4,60925E+34	7,15529E+22	5,55101E+45	4,75622E+48	4,97750E+49
F9	53993931824,55770	136543015,03358	324763739138,39700	221066209829,28100	232897494081,26200
F10	-474188,77448	-157798,97092	-145476,66688	-138617,63574	-130144,73037
F11	1,77825E+58	0	1,64963E+84	3,50349E+78	4,28709E+84
F12	6,02592	0,20251	51,90654	39,53759	49,32805
F13	791,69923	189,86238	1265,33523	1180,65954	1221,28511
F14	106880268,74010	119075,36516	351010988,82717	217443284,73263	276734445,61221
F15	-1587,31913	-1088,09986	-487,27182	-588,73719	-576,84209
F16	19783,84727	1060,44486	57509,58248	51740,28332	43293,61067
F17	29125,05185	1462,52726	68606,37740	60772,43677	68174,57458
F18	30764341427,04540	1501066319,30036	67024336044,21860	59731344414,55880	73632474769,86400
F19	5,55713	6,73824	9,06382	8,76917	7,36981
F20	1044873391,31101	973267,54054	4470915905,39053	4735750857,31344	3960678371,11058
F21	27701,59567	1169,74561	65470,17593	66312,76945	73844,48937
F22	-1761,32450	-1415,15037	-432,08684	-315,76247	-458,43129
F23	986537838,34587	2751255,06944	4542754677,81263	4166003251,36463	4872658864,42770
F24	50299371,36464	50204009,28362	72583145,04290	89317368,68520	76864374,05322
F25	-1174,51181	-771,75083	-428,08629	-516,73602	-357,73157

## Pořadí algoritmů

D2					
	DE rand/1/bin	DE rand/1/bin	PSO	SOMA all-to-one	SOMA all-to-all
F1	2	1	4	3	5
F2	2	1	5	3	4
F3	2	1	3	4	5
F4	1	2	5	3	4
F5	1	2	5	4	3
F6	2	1	5	4	3
F7	2	1	4	3	5
F8	2	1	5	3	4
F9	2	1	5	3	4
F10	1	2	5	3	4
F11	2	1	5	3	4
F12	2	1	5	3	4
F13	2	1	3	5	4
F14	2	1	5	3	4
F15	1	2	5	3	4
F16	2	1	5	3	4
F17	2	1	5	3	4
F18	2	1	5	3	4
F19	2	1	3	5	4
F20	2	1	5	4	3
F21	2	1	3	5	4
F22	1	2	3	5	4
F23	2	1	4	3	5
F24	2	1	5	4	3
F25	1	2	4	5	3
<b>Průměr</b>	1,76	1,24	4,44	3,6	3,96



# D10

	DE rand/1/bin	DE rand/1/bin	PSO	SOMA all-to-one	SOMA all-to-all
	2	1	4	3	5
F1	2	1	3	5	4
F2	2	1	5	4	3
F3	2	1	3	4	5
F4	1	2	5	3	4
F5	1	2	3	4	5
F6	2	1	5	4	3
F7	2	1	5	4	3
F8	2	1	3	4	5
F9	2	1	5	3	4
F10	1	2	5	3	4
F11	2	1	5	4	3
F12	2	1	5	4	3
F13	2	1	4	3	5
F14	2	1	4	3	5
F15	1	2	5	4	3
F16	2	1	3	5	4
F17	2	1	5	3	4
F18	2	1	5	4	3
F19	1	3	5	4	2
F20	2	1	4	3	5
F21	2	1	4	3	5
F22	1	2	5	4	3
F23	2	1	5	3	4
F24	2	1	5	3	4
F25	1	2	3	4	5
<b>Průměr:</b>	1,72	1,32	4,36	3,68	3,92

# D30

	DE rand/1/bin	DE rand/1/bin	PSO	SOMA all-to-one	SOMA all-to-all
F1	2	1	4	3	5
F2	2	1	3	4	5
F3	2	1	3	4	5
F4	1	2	4	3	5
F5	2	1	4	3	5
F6	2	1	3	4	5
F7	2	1	5	3	4
F8	2	1	3	4	5
F9	2	1	5	3	4
F10	1	2	3	4	5
F11	2	1	4	3	5
F12	2	1	5	3	4
F13	2	1	5	3	4
F14	2	1	5	3	4
F15	1	2	5	3	4
F16	2	1	5	4	3
F17	2	1	5	3	4
F18	2	1	4	3	5
F19	1	2	5	4	3
F20	2	1	4	5	3
F21	2	1	3	4	5
F22	1	2	4	5	3
F23	2	1	4	3	5
F24	2	1	3	5	4
F25	1	2	4	3	5
Průměr:	1,76	1,24	4,08	3,56	4,36

## Průměrný rank

Napříč dimenzemi					
	DE rand/1/bin	DE rand/1/bin	PSO	SOMA all-to-one	SOMA all-to-all
F1	2,000	1,000	3,667	3,667	4,667
F2	2,000	1,000	4,333	3,667	4,000
F3	2,000	1,000	3,000	4,000	5,000
F4	1,000	2,000	4,667	3,000	4,333
F5	1,333	1,667	4,000	3,667	4,333
F6	2,000	1,000	4,333	4,000	3,667
F7	2,000	1,000	4,667	3,333	4,000
F8	2,000	1,000	3,667	3,667	4,667
F9	2,000	1,000	5,000	3,000	4,000
F10	1,000	2,000	4,333	3,333	4,333
F11	2,000	1,000	4,667	3,333	4,000
F12	2,000	1,000	5,000	3,333	3,667
F13	2,000	1,000	4,000	3,667	4,333
F14	2,000	1,000	4,667	3,000	4,333
F15	1,000	2,000	5,000	3,333	3,667
F16	2,000	1,000	4,333	4,000	3,667
F17	2,000	1,000	5,000	3,000	4,000
F18	2,000	1,000	4,667	3,333	4,000
F19	1,333	2,000	4,333	4,333	3,000
F20	2,000	1,000	4,333	4,000	3,667
F21	2,000	1,000	3,333	4,000	4,667
F22	1,000	2,000	4,000	4,667	3,333
F23	2,000	1,000	4,333	3,000	4,667
F24	2,000	1,000	4,333	4,000	3,667
F25	1,000	2,000	3,667	4,000	4,333
<b>Průměr</b>	1,747	1,267	4,293	3,613	4,080

## Porovnání algoritmů s největším rozdílem

Upřímně nad tímhle bodem si pořád lámu hlavu...

Jak plyne z výsledků výše, tak diferenční evoluce vyhrává na plné čáře. Snažil jsem se to „prokokovat“ a jsou zkrátka robustní oproti PSO a SOMĚ. Naopak PSO a SOMA jsou lehce efektivnější ve více dimenzích, ale čekal jsem, že rozdíl bude větší...

Výkonost jsem bral jako minimem z 30 opakování (možná se měl brát průměr?).

Dělám na tom celý den a výsledky jsou zavádějící. Můžu tento bod případně doplnit později?

# Friedmanův test

Formálně se Friedmanův test provádí následujícím způsobem:

## 1. Hypotézy:

- **Nulová hypotéza (H0):** Neexistuje žádný rozdíl mezi algoritmy.
- **Alternativní hypotéza (H1):** Existuje alespoň jedna statisticky významná odlišnost mezi algoritmy.

## 2. Výpočet Friedmanova statistického testu:

- **DE rand/1/bin:** 1,747
- **DE best/1/bin:** 1,267
- **PSO:** 4,293
- **Soma all to one:** 3,613
- **Soma all to all:** 4,080

$$\chi^2 = \frac{12n}{k(k+1)} \left( \sum_{j=1}^k R_j^2 - \frac{k(k+1)^2}{4} \right)$$

$$\chi^2 = \frac{12 \times 25}{5 \times 6} \left( (1.747)^2 + (1.267)^2 + (4.293)^2 + (3.613)^2 + (4.080)^2 - \frac{5 \times (5+1)^2}{4} \right)$$

Kde  $n$  je počet funkcí (25) a  $k$  je počet algoritmů (5).

$$\chi^2 = \frac{12 \times 25}{5 \times (5+1)} \left( \frac{(1.747)^2 + (1.267)^2 + (4.293)^2 + (3.613)^2 + (4.080)^2}{25} - \frac{5 \times (5+1)^2}{4} \right)$$

Po provedení výpočtu získáme hodnotu Friedmanovy statistiky  $\chi^2$ .

$$\chi^2 = \frac{12 \times 25}{5 \times 6} \left( \frac{3.058309 + 1.604489 + 18.439849 + 13.077609 + 17.04864}{25} - \frac{5 \times 36}{4} \right)$$

$$\chi^2 = \frac{300}{30} \left( \frac{52.228896}{25} - \frac{180}{4} \right)$$

$$\chi^2 = 10 (2.08915584 - 45)$$

$$\chi^2 = 10 \times (-42.91084416)$$

$$\chi^2 = -429.1084416$$



### 3. Přijetí nebo zamítnutí nulové hypotézy:

Pro přijetí nebo zamítnutí nulové hypotézy využijeme kritickou hodnotu  $\chi^2$  pro daný počet algoritmů a funkcí a zvolenou hladinu významnosti  $\alpha$ .

Kritická hodnota pro  $k = 5$  algoritmů a  $n = 25$  funkcí při hladině významnosti  $\alpha = 0.05$  je  $\chi_{krit}^2 = 11.070$ .

Pokud  $\chi^2 \leq \chi_{krit}^2$ , pak nemáme dostatek důkazů k zamítnutí nulové hypotézy. V opačném případě bychom zamítli nulovou hypotézu a přijali hypotézu alternativní. V našem případě  $-429.1084416 \leq -11.070$ , takže zamítáme nulovou hypotézu.

### 4. Závěr:

- Zamítáme nulovou hypotézu a máme důkaz o existenci rozdílu mezi algoritmy.
- Existuje alespoň jeden algoritmus, který se odlišuje v průměrném výkonu od ostatních algoritmů.

## Zdroje

- Zdroje k funkcím jsou uvedené v [seznamu funkcí](#).
- Využití LLM je podporováno? Okay... Velkou část problematiky jsem řešil s CHAT GPT. Kompletní výpis zde: <https://chat.openai.com/share/f853bc79-0336-4bb7-8d3e-988a4c86b013>