17.   RESISTIVITY THEORY

17.1   Introduction

The whole of resistivity logging is based upon a few very important equations which are introduced in this section. The equations, which are known as the Archie Equations, relate the resistivity of a formation to the resistivity of the fluids saturating a formation, the porosity of the formation and the fractional degree of saturation of each fluid present. As always, the story begins with Ohm's Law.

17.2   Basic Definitions and Ohm's Law

Ohm's Law states that the current flowing from point A to point B in a conductor I is proportional to the difference in electrical potential ΔE between point A and point B. The constant of proportionality is called the electrical conductance c. Current is measured in amperes (A), potential difference in volts (V), and conductance in siemens (S).

Hence, we can write

$$I = c \, \Delta E \tag{17.1}$$

We also define an electrical resistance r, which is the inverse of conductance.

$$r = \frac{1}{c} \tag{17.2}$$

Resistance is measured in ohms (Ω). Hence, we can rewrite Eq. (17.1) as

$$I = \frac{\Delta E}{r} \tag{17.3}$$

Thus, if we take a cylindrical rock sample with two flat faces A and B, and set a potential difference $\Delta E = E_A - E_B$ between its end faces, a current I will flow through the rock from face A to face B (Fig. 17.1). If we measure the current and the potential difference, we can calculate the resistance of the rock sample using Eq. (17.1).
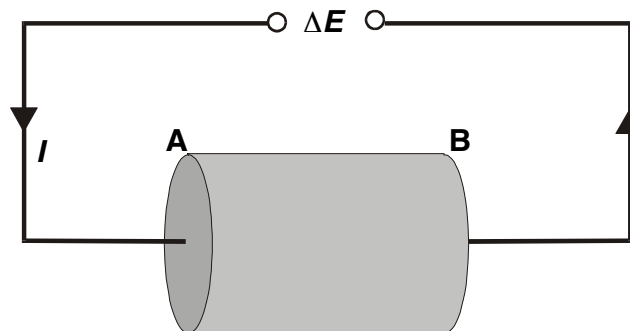

Figure 17.1 Ohm's Law for a rock sample.

- If the resistance is high, a given potential difference ΔE will only give a small current I.
- If the resistance is low, a given potential difference ΔE will give a high current I.

The value of resistance is a property of the material which describes how much the material resists the passage of a current for a given applied potential difference.

Imagine that the size of our rock sample now changes.

- If the length of the sample is doubled, one can see that the resistance of the sample to the passage of a current should also double.
- If the area perpendicular to the current flow doubles (the area of the end face in this example), there is twice the material for the current to pass through, the resistance of the sample to the passage of the current should therefore fall to a half of what is was before.

So the resistance (and therefore conductance) depend upon the size of the sample.

If we take the resistance per unit length and area, we can remove the effect of the dimensions of the sample. The value we obtain is then only a function of the property of the material and not its dimensions. The resistance per unit length and area is called the *resistivity R*, and can be expressed as

$$R = \frac{\Delta E}{I} \frac{A}{L} \qquad \textbf{(17.4)}$$

where: R = the resistance of the sample (Ωm or ohm.m)
       ΔE = the potential difference across the sample (volts, V)
       I = the current flowing through the sample (amperes, A)
       A = the cross-sectional area of the sample perpendicular to the current flow (m^2)
       L = the length of the sample (m).

Note that a conductivity C can also be defined as the reciprocal of the resistivity R, and therefore

$$C = \frac{1}{R} = \frac{I}{\Delta E} \frac{L}{A} \qquad \textbf{(17.5)}$$

where: C = the conductivity of the sample (S/m).

In petrophysical logging of electrical rock properties there are two main types of tool. One type measures resistivity directly, and the result is given in ohm.m (Ω.m). The other type measures conductivity directly, and the result is given in either siemens per metre (S/m), or more often in milli-siemens per metre (mS/m). The two measurements are, of course, measuring the same property of the rock, and can be interconverted using

$$C\left(\text{S/m}\right) = \frac{1}{R\left(\text{ohm.m}\right)} \qquad \textbf{(17.6)}$$

$$C\left(\text{mS/m}\right) = \frac{1000}{R\left(\text{ohm.m}\right)} \qquad \textbf{(17.7)}$$

17.3   Resistivity of Rocks

Reservoir rocks contain the following constituents

- Matrix material   High resistivity
- Formation waters   Low resistivity
- Oil   High resistivity
- Gas   High resistivity
- Water-based mud filtrate   Low resistivity
- Oil-based mud filtrate   High resistivity

All have a high electrical resistivity (*electrical insulators*) except the formation water and water-based mud filtrate, which are good *electrical conductors* and have a low electrical resistivity.

The resistivity of the reservoir rocks therefore depends only upon the water or water based mud filtrate occupying its pore space.

Uninvaded Formations. For uninvaded formations, the measured bulk resistivity of the rock depends only upon the amount of the aqueous formation fluids present in the rock, and the resistivity of those aqueous fluids. Since the amount of formation fluids depends both on porosity ϕ and water saturation S_w, we can say that the resistivity of the formation R_t depends upon porosity ϕ, water saturation S_w, and the resistivity of the formation water R_w. This resistivity is called the *true resistivity of the formation*. It is the resistivity of the formation in the uninvaded zone, where the rock contains some saturation of oil S_o, gas S_g, and water S_w, and where $S_o+S_g+S_w=1$.

The aim is to use knowledge of the resistivity of the formation, together with independent knowledge of the porosity and resistivity of the formation waters, to calculate S_w, and hence enable ourselves to calculate the STOOIP.

Typical values of R_t range from 0.2 to 2000 ohm.m (5000 to 0.5 mS/m).

The uninvaded zone of formations is commonly only measured directly by the most deeply penetrating electrical logging tools. The shallower investigating tools measure the invaded zone. Hence, if one wants a resistivity reading for use in STOOIP calculations, one should always chose the deepest penetrating electrical tool of those that have been run.

Invaded Zones. In most cases there is an invaded zone, where the formation fluids have been disturbed by the drilling fluid. The resistivity of the formation in this zone depends upon the resistivity of the mud filtrate R_{mf}, the resistivity of any remaining formation water R_w, the saturation of the mud filtrate S_{XO}, the saturation of the remaining formation water S_w (if any), and the porosity of the rock ϕ.

If these values and the depth of invasion are known, the resistivity measured in the invaded zone can be corrected to account for the presence of the mud filtrate.

The symbols used in electrical logging in the invaded borehole environment are shown in Fig. 17.2.
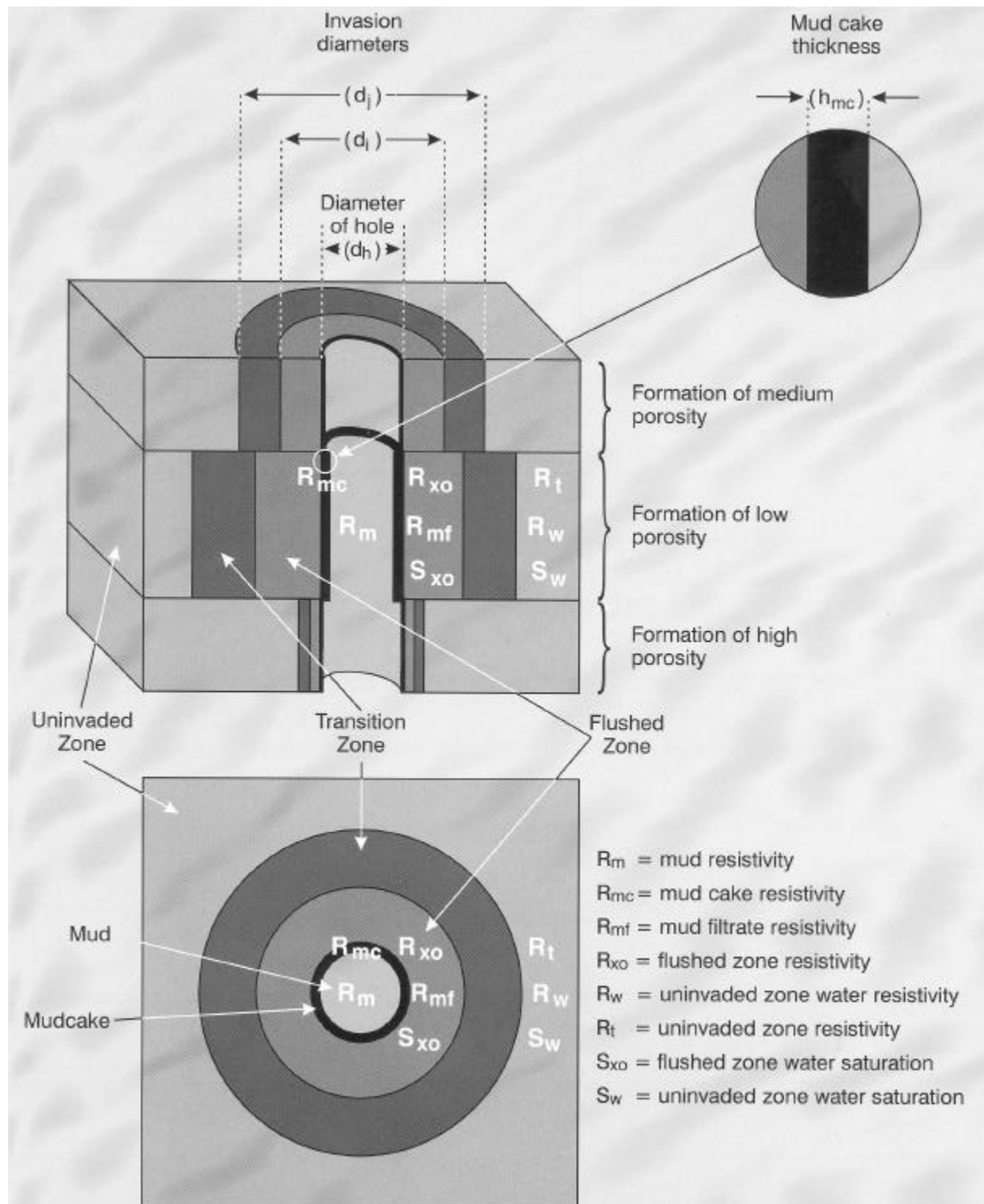
Figure 17.2 Borehole invasion, and the symbols used.

17.3   Temperature and Pressure

Calculating Formation Temperature. The resistivity of formation fluids and water-based drilling muds varies greatly with temperature, but little with pressure. The temperature in a borehole can be

found directly from modern Horner corrected temperature logs up the entire borehole, or more traditionally, from the geothermal gradient obtained from Horner corrected BHT measurements.

In the former case the temperature is given directly at a given depth. However, this is not used to calculate the mud resistivity as the newer temperature logs also measure directly the mud resistivity as it makes the temperature log.

In the latter, and more common, case the BHT allows us to calculate a mean geothermal gradient for the borehole. The formation temperature can then either be calculated directly, or obtained with a nomogram such as that given in Figure 17.3.
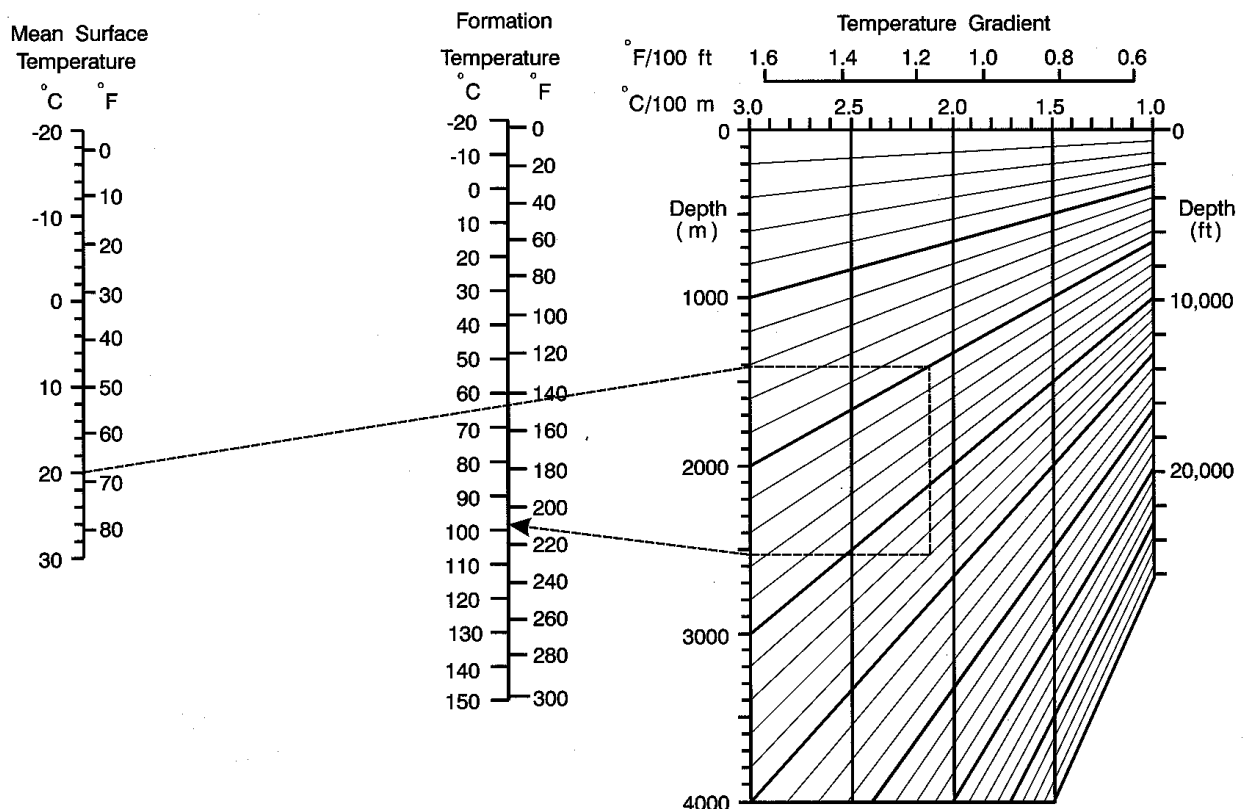

Figure 17.3 The formation temperature nomogram.

The Effect of Salt Composition. The resistivity of the formation fluids depends upon the concentration and type of salts dissolved in it. We know the concentration and type of dissolved solids in the formation water usually from chemical analysis of samples obtained by the RFT. The resistivity of this solution at a given temperature and pressure can be obtained by making up a synthetic brine to the recipe indicated by the chemical analysis of the RFT sample, or more often, by the use of equations relating the composition to the fluid resistivity.

For simplicity, we express the dissolved salts in a solution as an NaCl equivalent. Figure 17.4 shows a chart expressing multipliers *K* as a function of total concentration of solids for each individual ionic contributor to the salts in solution. We simply multiply the relevant multiplier by the solid

concentration for each component and sum the results. The sum is the total NaCl equivalent concentration in ppm, and this can be used to describe the concentration of the solution.
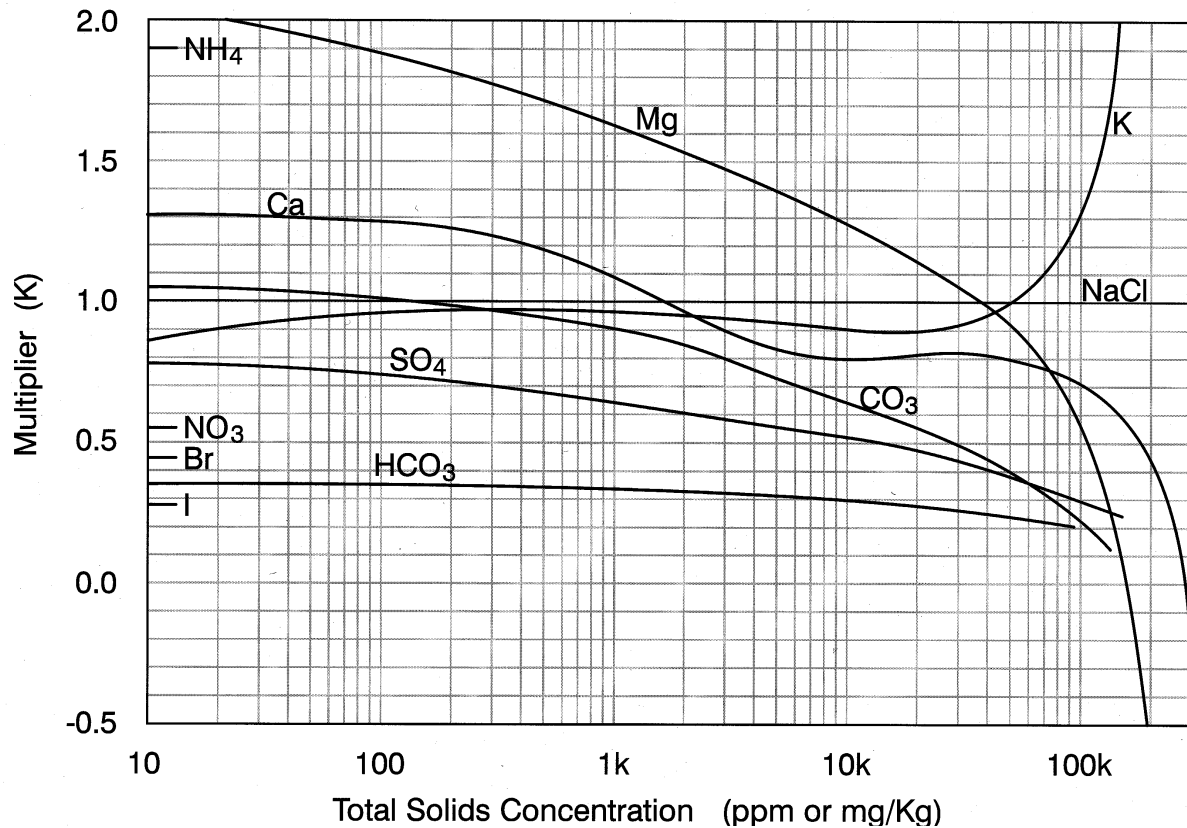

Figure 17.4 The NaCl equivalent plot.

For example, a solution contains 20,000 ppm NaCl, 10,000 ppm KCl and 1000 ppm MgSO$_4$. The multipliers are Na(1.00), Cl(1.00), K(0.9), Mg(1.63), SO$_4$(0.64). The total NaCl equivalent is 20,000×1+20,000×1+10,000×0.9+10,000×1+1000×1.63+1000×0.64 = 30,635 ppm NaCl.

The Variation of Formation Fluid Resistivity with Temperature. The resistivity of aqueous formation fluids varies significantly with temperature. The increase is approximately 4% per degree centigrade. Clearly, if the BHT is 200°C, the increase in the resistivity of the formation fluids can be as much as 800%. This will cause the resistivities measured with downhole tools to increase steadily down the borehole. To avoid this added complication all downhole resistivity measurements are corrected to read the resistivity they would read if the entire borehole was at some constant temperature (24°C/75°F is often used). To do this we need to convert the raw resistivity data. This is done automatically by the wireline logging company using a chart or an equation that represents the chart. Figure 17.5 shows a typical chart for this type of correction. It is based upon Hilchie's equation for the variation of the resistivity of aqueous fluids with temperature. Hilchie's equation relates the resistivity of an NaCl solution at one temperature to that at another.

$$R_{wT2} = R_{wT1}\left(\frac{T_1 + X}{T_2 + X}\right) \quad \text{where} \quad X = 10^{-(0.340396 \times \log_{10} R_{wT1} - 0.641427)} \tag{17.8}$$

where: R_{wT1} = the resistivity of the fluid at temperature T_1
R_{wT2} = the resistivity of the fluid at temperature T_2
T_1 = the temperature T_1
T_2 = the temperature T_2
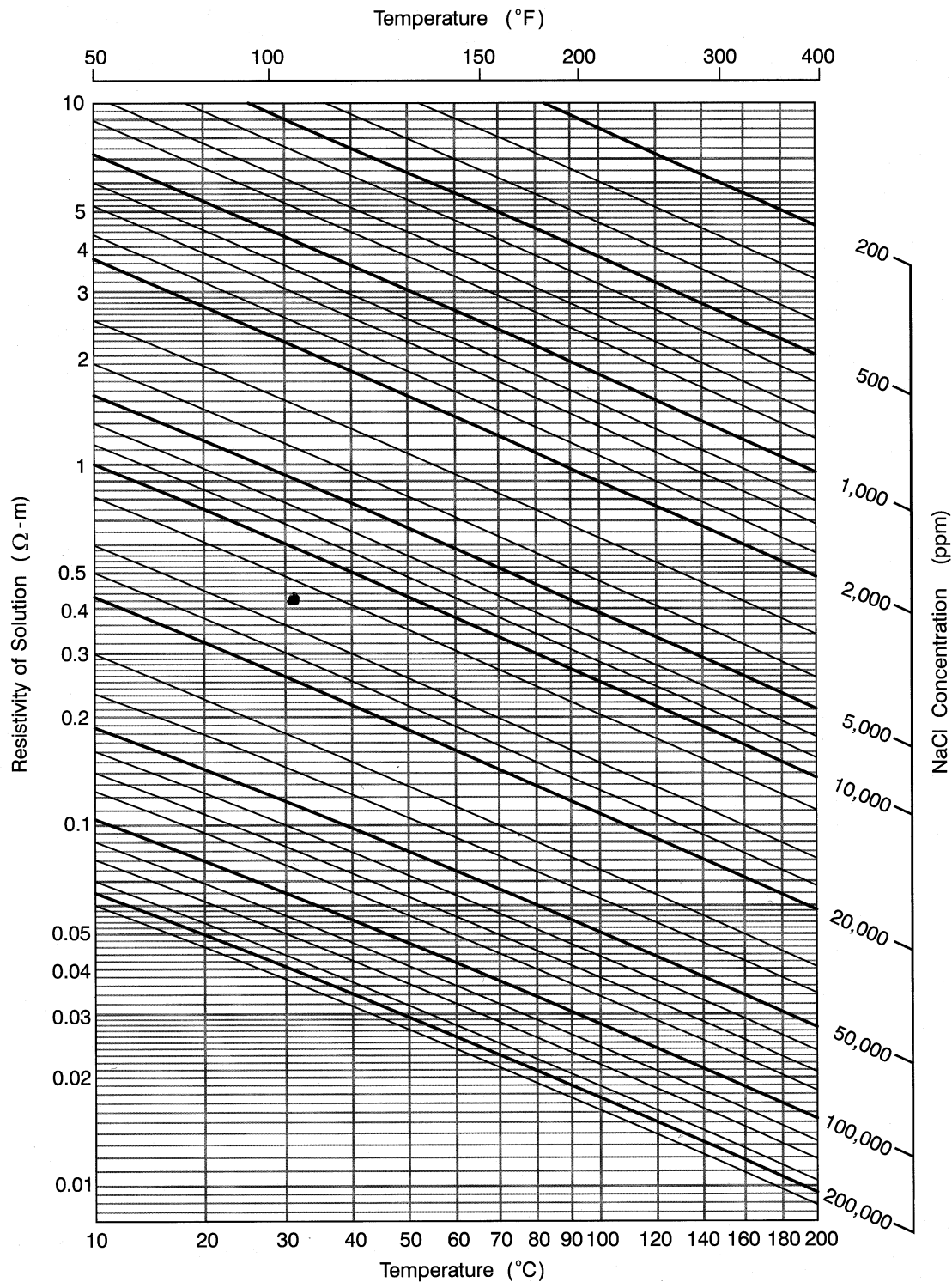
Temperature   (°F)

Figure 17.5 Resistivity of NaCl fluids as a function of temperature.

The Variation of Drilling Mud Resistivity with Temperature. The resistivity of the drilling mud is measured by many tools and is available directly for calculation at a given depth. Tools that measure the drilling mud resistivity include most resistivity tools, the newer temperature tools and some RFT tools.

The Variation of Mudcake and Mud Filtrate Resistivity with Temperature. The mud filtrate and mudcake resistivities may be obtained from the mud density (mud weight) and the resistivity of the drilling mud using the nomogram in Fig. 17.6. Note that this nomogram is valid for 24°C/75°F.
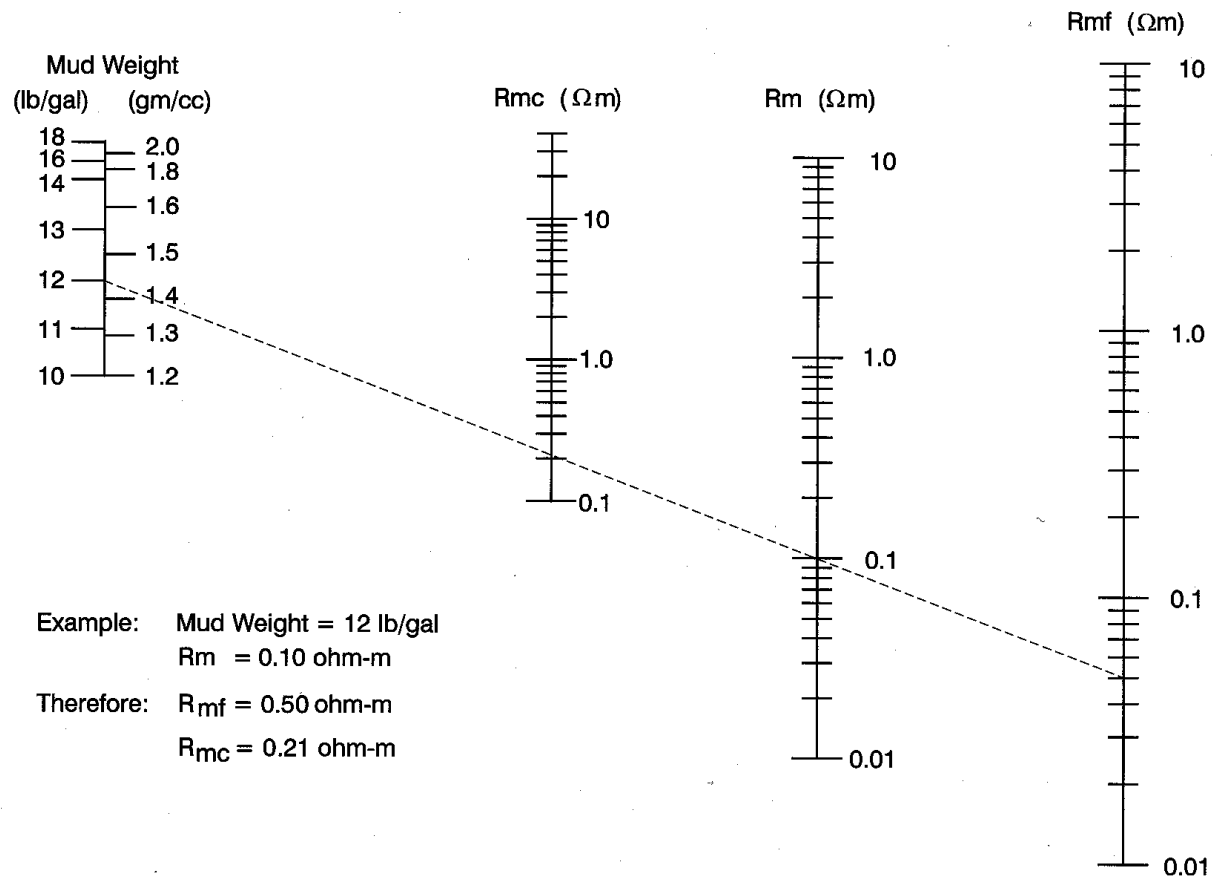

Figure 17.6 Mudcake and mud filtrate resistivity nomogram.

Once the various corrections for temperature have been carried out for the formation fluids and the mud-derived material, and the log is at 24°C/75°F equivalent, we can carry out calculations on the resistivity data along the borehole comparing the responses at different depths without regard to the fact that the formation fluids contain dissolved salts other than NaCl, and that the resistivity of the fluid is actually very sensitive to temperature (depth) changes.

17.4   Formation Factor (Archie's First Law)

In the late 1920s and early 1930s Archie carried out a series of experiments to analyze the relationship between the resistivity of rock samples saturated with various resistivity fluids. He used clean sandstones and carbonates which were 100% saturated with aqueous solutions of varying concentration and hence resistivity. His results are empirical (derived from experimental work on particular samples) and hence should not be assumed to have theoretical rigour. However, we are now beginning to see how the theory of the electrical properties of rocks can give rise to the Archie equations.

Archie observed that the bulk resistivity of a rock R_o fully saturated with an aqueous fluid of resistivity R_w is directly proportional to the resistivity of the fluid

$$R_o = F \ R_w \qquad\qquad\qquad (17.9)$$

The constant of proportionality F is called the *Formation Factor* and describes the effect of the presence of the rock matrix.

It can be immediately seen that $F = 1.00$ for a rock with 100% porosity, i.e., no matrix, just 100% fluid.

If we take 100% fluid and slowly add grains of rock, the porosity decreases. However, the insulating grains of rock have negligible conductivity (infinite resistivity) compared to the conducting fluid. Hence, R_o will increase, which implies that F is always greater than unity in a porous medium such as a rock. In real rocks F takes values usually between 20 and 500. Note that formation factor has no units because it is the ratio of two resistivities.

One way of interpreting the formation factor is therefore as a factor that describes the extent to which electrically insulating mineral grains 'dilute' the conducting fluid, making the bulk material more resistive than the fluid alone. The formation factor includes both the effect of the variable porosity and the effect of the tortuous pathways that the current is forced to take through the conducting fluid due to the presence of the insulating rock grains. It can be seen, therefore, that the formation factor is related to the porosity of the rock and the connectivity of the pore spaces. The natural complexity of pore systems in rocks means that the formation factor cannot be expressed simply as a function of porosity and connectivity in a theoretically rigorous way.

Archie examined the way that the formation factor changes from rock to rock, and noticed that the following rule commonly holds true

$$F = \phi^{-m} \qquad\qquad \textbf{Archie's First Law} \qquad\qquad (17.10)$$

where: m =   the cementation index/factor/exponent (no units).

Equations (17.9) and (17.10) are often combined and called Archie's first law. The resulting equation is

$$R_o \ = R_w \ \phi^{-m} \qquad\qquad\qquad (17.11)$$

The cementation index is the factor that describes the increase in resistivity that results from the insulating mineral grains forcing the current to take tortuous pathways through the conducting fluid.

The cementation factor has a theoretical value of unity for uniform pores that penetrate the rock directly from one side of the sample to the other (i.e., direct tubes of pore space), and is zero for a rock with 100% porosity (i.e., no grains to get in the way of the fluid flow). No other values of the cementation factor are able to be defined in a purely theoretical way in rocks due to the complexity of the way that pore spaces are arranged.

In real rocks the cementation index usually varies between 1.0 and 3.0.

Values between 1.0 and 1.4 are associated with igneous and metamorphic rocks that contain fractures. Fractures are a form of porosity that is localized and well connected, and hence approximates to the situation where we had uniform tubes of porosity going through the sample.

Values between 1.4 and 2.0 are found in sandstones, with the higher values found in more consolidated sandstones, where the current flow paths are more tortuous.

Values between 2.0 and 2.6 are typical for carbonates, and represent a greater degree of tortuosity in the current flow that is found in carbonates because much of the porosity in carbonates is unconnected (e.g., vugs).

Both the formation factor and the cementation exponent can be measured on core plugs in the laboratory. This is done in the following way for a single plug.

*   Clean and dry the rock sample.
*   Measure the porosity of the rock sample with helium or with a fluid saturation technique.
*   Saturate the rock with a conductive fluid if not already done in the porosity determination step.
*   Measure the bulk resistivity of the rock saturated with the fluid.
*   Measure the resistivity of the fluid that saturates the rock in a separate vessel.
*   Rearrange and apply Eq.(17.9) to obtain the formation factor.
*   Rearrange and apply Eq. (17.11) to obtain the cementation exponent.
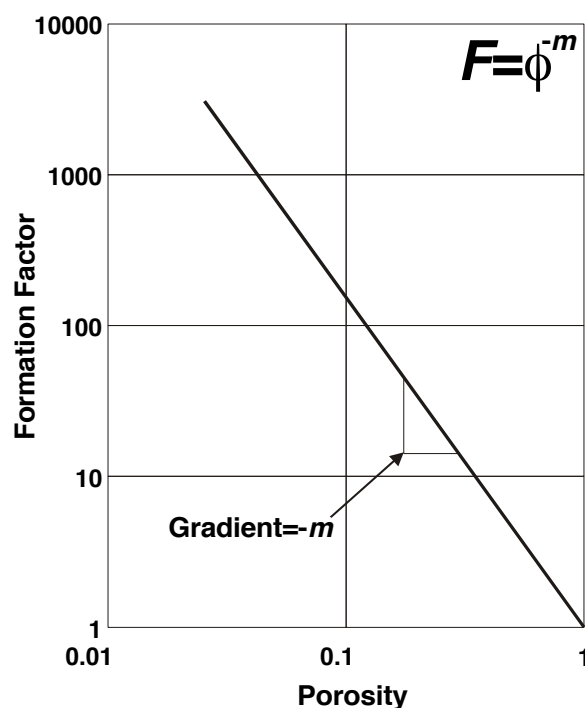
If there is a suite of such measurements from core plugs from a particular formation, a mean cementation exponent can be obtained graphically by plotting F against ϕ on log-log graph paper, which gives a straight line for a given lithotype, which intersects $F=1$ when $\phi=1$, and with a gradient equal to $-m$. This is shown in Fig. 17.7.

Note that sometimes a constant '*a*' is placed before the porosity term, and so $F=a$ when $\phi=1$. However, there is no physical justification for this term. It arises from applying a best fit engineering equation to F versus porosity data, and should be avoided.

Figure 17.7 Cementation exponent from a formation factor-porosity cross-plot.

$$F=\phi^{-m}$$

Graph with y-axis labelled **Formation Factor** (ranging from 1 to 10000) and x-axis labelled **Porosity** (ranging from 0.01 to 1). Line labelled **Gradient=-*m***.
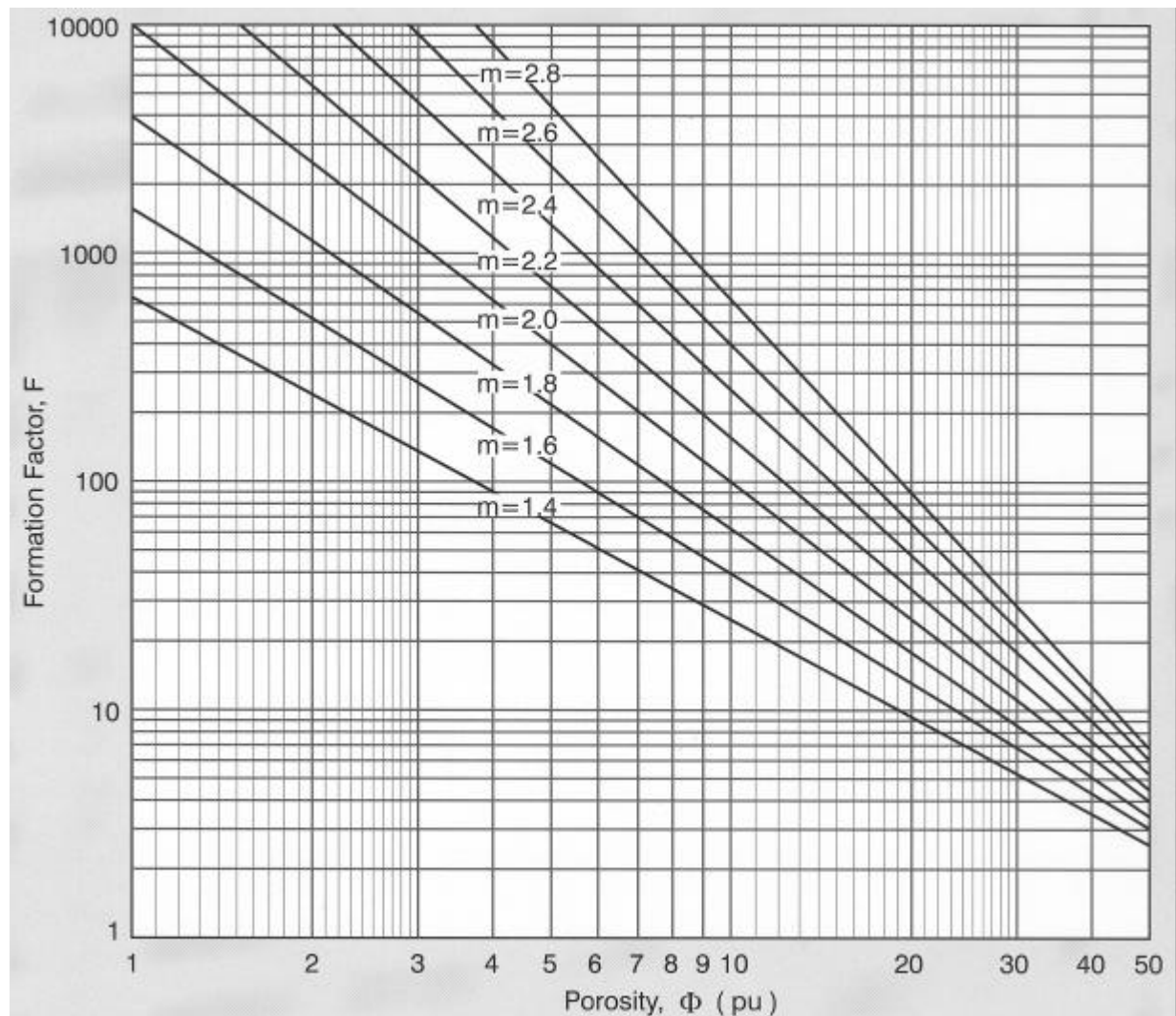
Figure 17.8 Cementation exponent from a formation factor-porosity cross-plot.

Figure 17.8 shows a formation factor-porosity cross-plot for various values of the cementation exponent.

It should be mentioned that there are a range of equations used in the oil industry to calculate the formation factor. Archie's equation is the most flexible, and the others are simply specific cases of it that have been shown to work empirically for a given type of rock. In all cases the equations have been derived from fitting a best line to a set of real data. The empirical relationships this provides, therefore includes values of *a* which are non-unity. Also, one should be extremely careful to only apply the equations when you are sure that they are valid. This often means going back to the original papers to see what type of rocks the equations have been derived from. By comparison the use of Archie's equation with known cementation exponent is more reliable, and should ALWAYS be used if the cementation exponent is known.

The Humble Formula. This is applied to soft formations and to clean sandstones with a sucrosic texture.

$$F = 0.62 \times \phi^{-2.15} \tag{17.12}$$

The Soft Formation Formula.

$$F = 0.81 \times \phi^{-2} \tag{17.13}$$

The Low Porosity Carbonate Formula. Valid for low porosity clean carbonates with no fracturing.

$$F = \phi^{-m} \qquad \text{where} \quad m = 1.87 + \frac{0.019}{\phi} \tag{17.14}$$

17.5 Partial Water Saturation (Archie's Second Law)

Archie also examined the work of other investigators who did experiments on the resistivity of partially saturated sandstones. He observed that the bulk resistivity of a rock R_t partially saturated with an aqueous fluid of resistivity R_w is directly proportional to the resistivity of the rock when fully saturated with the same fluid, i.e.,

$$R_t = I \, R_o \tag{17.15}$$

The constant of proportionality I is called the *resistivity index* and describes the effect of partial desaturation of the rock.

- If the rock is fully saturated, $I=1.00$.
- If the rock is full of dry air (i.e., not saturated with a conductive fluid), $I \rightarrow \infty$.

The resistivity index therefore varies between unity and infinity depending upon the degree of saturation of the rock.

Archie observed that the following relationship exists empirically for sandstones

$$I = S_w^{-n} \qquad\qquad \textbf{Archie's Second Law} \quad \textbf{(17.16)}$$

where: S_w = the fractional water saturation of the rock
   I = the resistivity index
   n = the saturation exponent.

Again, the last two equations can be combined into a form which is usually referred to as Archie's second law

$$R_t = R_o \, S_w^{-n} \tag{17.17}$$

The saturation exponent normally has a range of values from 1.8 to 2.0, however much lower and much higher values have been found.

The value of the saturation exponent can be obtained from laboratory experiments on core samples. The procedure is as follows for a single core sample:

* Follow the procedure to measure the resistivity of a sample saturated completely with a conductive fluid as outlined in the last section.
* Replace in a step-wise manner some of the conductive fluid in the rock with a non-conductive fluid (e.g., gas) allowing for equilibrium to be attained at each step.
* Measure the resistivity of the sample when equilibrium is attained at the end of each step.
* Calculate from measurements of the evolved fluids and prior knowledge of the pore volume of the sample, the conductive fluid saturation in the sample at the end of each step.
* Rearrange and apply Eq. (17.15) to calculate the saturation index.
* Rearrange and apply Eq. (17.17) to calculate the saturation exponent.

These experiments are extremely time consuming as one has to wait a long time for the samples to come to equilibrium.

As before, a mean saturation exponent can be calculated from such measurements on a suite of cores. In this case the resistivity index I is plotted against the water saturation S_w, again on log-log paper (Fig. 17.9). The result is a straight line intersecting $I=1$ when $S_w=1$, and with a gradient equal to $-n$.
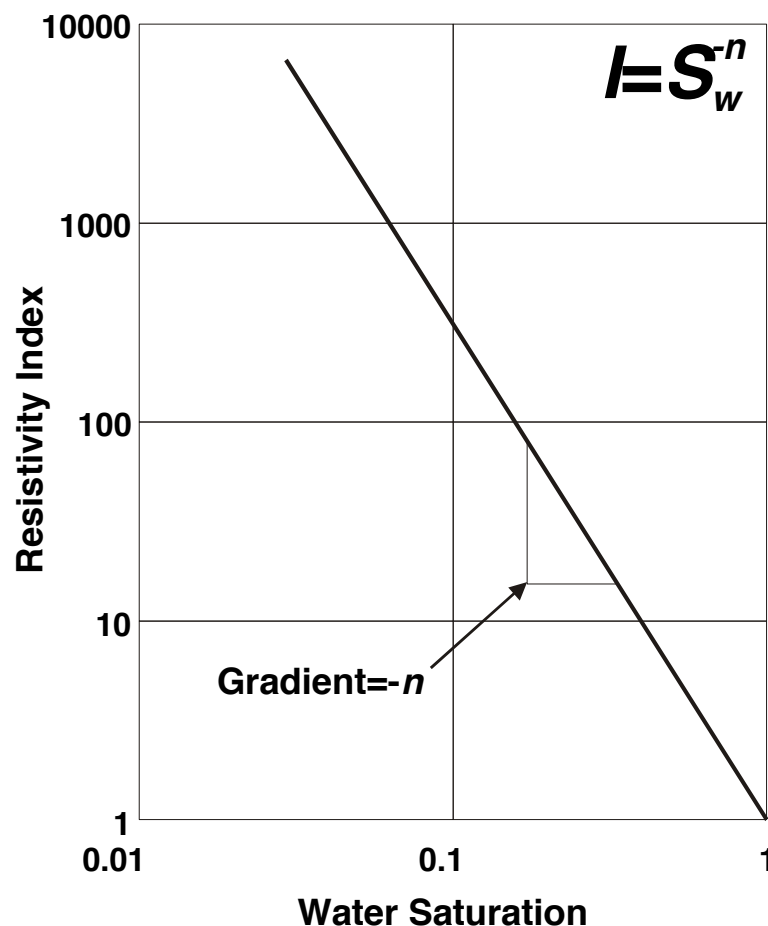

Figure 17.9 Saturation exponent from a resistivity index-water saturation cross-plot.

17.6 Combining Archie's Laws

The two equations for each of the Archie laws can be combined into one controlling equation. Combining Eq. (17.11) and (17.17) gives

$$R_t = R_w \phi^{-m} S_w^{-n} \tag{17.18}$$

We are ultimately interested in calculating the water saturation, S_w, so we rearrange the equation to give

$$S_w = \sqrt[n]{\frac{R_w \phi^{-m}}{R_t}} = \sqrt[n]{\frac{R_w F}{R_t}} = \sqrt[n]{\frac{R_o}{R_t}} \tag{17.19}$$

Table 17.1 summarizes the sources of the parameters that go into this equation to calculate the water saturation.

Table 17.1 Sources of data for calculation of water saturation.

Parameter	Source
R_t	• Deep investigation resistivity tool
R_w	• From SP log • Calculated from water zone • Measured on RFT sample
ϕ	• Sonic tool • Formation density tool • Neutron tool
m	• Measured in laboratory • Guessed
n	• Measured in laboratory • Guessed

17.7 The Effect of Errors in Resistivity Calculations

The accurate determination of the water saturation is key to being able to calculate an accurate value for the amount of oil in place. Errors of a few percent in the determination of the water saturation result in errors worth billions of dollars when transferred into errors in the determination of STOOIP.

In attempting to reduce errors we ensure that the five parameters in Eq. (17.19) are measured using independent methods. Laboratory determined m and n values are the best ones to take, however early in a reservoirs life these are not available, and so guesses are used instead.

Table 17.2 shows the propagation of errors in Eq. (17.19) for the calculation of water saturation.

Table 17.2 Propagation of errors in water saturation calculations.

Data			
	-20%	Base Case	+20%
R_t	32	40	48
R_w	0.32	0.4	0.48
ϕ	0.18	0.2	0.22
m	1.8	2	2.2
n	1.8	2	2.2
Saturations from changing individual parameters			
R_t	0.56	0.50	0.46
R_w	0.45	0.50	0.55
ϕ	0.56	0.50	0.45
m	0.43	0.50	0.59
n	0.46	0.50	0.53
Saturations from changing combinations of m and n parameters			
	m-20%	m	m+20%
n-20%	0.39	0.46	0.55
n	0.43	0.50	0.59
n+20%	0.46	0.53	0.62
Saturations from changing combinations of resistivity parameters			
	R_t-20%	R_t	R_t+20%
R_w-20%	0.50	0.45	0.41
R_w	0.56	0.50	0.46
R_w+20%	0.61	0.55	0.50
Saturations from changing all parameters			
Worst Case Low	0.28		
Base Case	0.50		
Worst Case High	0.82		

Note that 20% underestimations and overestimations of m and n lead to an underestimation and overestimation of the water saturation by 0.11 and 0.12 respectively. These are huge errors when progressed through to the STOOIP calculation. Clearly there is a case here for doing m and n determinations on core.

Note also that the same degree of underestimation or overestimation in R_t and R_w leads to no further error as these errors cancel out.

The worst case scenarios for over and underestimation with 20% errors on the input parameters are 0.32 and 0.22 respectively. These errors will most probably be larger than the recoverable oil saturation in the reservoir. Clearly, the parameters that go to calculate the water saturation must be derived very carefully indeed.

17.8   The Hingle Plot

Theory. This plot is based on Eq. (17.18). The objective is to obtain a linear cross-plot of the R_t data measured by the resistivity tool and the ϕ data measured by one of the porosity tools. To do this all of the exponents of Eq. (17.18) are multiplied by $-1/m$ to give

$$R_t^{-1/m} = R_w^{-1/m} \ \phi \ S_w^{+n/m} \tag{17.20}$$

Because in any given reservoir we can take R_w, m and n as constant, and because we will apply the equation for selected values of S_w, Eq. (17.20) becomes

$$R_t^{-1/m} = \ B\phi \tag{17.21}$$

where B is a constant. The Hingle graph paper is designed such that the y-axis represents $R_t^{-1/m}$ so that R_t can be entered directly in the plot. This implies that a different form of graph paper is needed for each value of m. The x-axis on the Hingle grid is porosity on a linear scale.

Figure 17.10 shows a Hingle plot.

Application. The use of the Hingle plot is as follows. For any given reservoir zone carry out the following steps:

- Construct the 100% water saturation (S_w=1) line. The first point on this line is automatically available, as R_t is infinite when ϕ=0, and this point plots in the bottom left hand corner of the Hingle grid. The second point is calculated with knowledge of R_w for the reservoir. Equation (17.11) is used to calculate R_o knowing R_w for the reservoir, for the value of m relevant to the Hingle grid, and at any value of ϕ (the higher the better for accuracy). For example, in Fig. 17.10, the m value is 2, and if R_w=0.4 ohm.m, we can say that at the arbitrary porosity of ϕ =0.2, the value of R_o=10 ohm.m. The R_o, ϕ point can be plotted on the grid and joined with the first point by a straight line. This is the water line, and represents how R_o varies with porosity when the rock is fully saturated with water.

- Other lines for partial water saturations can now be constructed. Their first point is always in the bottom left hand corner of the Hingle grid because R_t is always infinite when ϕ=0 no matter what the water saturation. The second point is calculated from Eq. (17.17) at a given arbitrary porosity assuming or knowing the value of n and calculating R_t from the relevant R_o, which is available from the water line. For a particular partial saturation line (S_w=0.5, say) the R_t, ϕ point can be plotted on the grid and joined with the first point by a straight line. This is the S_w=0.5 line, and represents how R_t varies with porosity when the rock is 50% saturated with water.

- A fan of partial saturation lines can be constructed in this way, say for every 10% increment in water saturation. A large number of porosity and R_t pairs are now extracted from the logs and plotted on the graph. It is immediately obvious how much water saturation is present on average, and the water saturation for particular points (relating to a particular depth) can be estimated from the graph by interpolation between the iso-saturation lines.
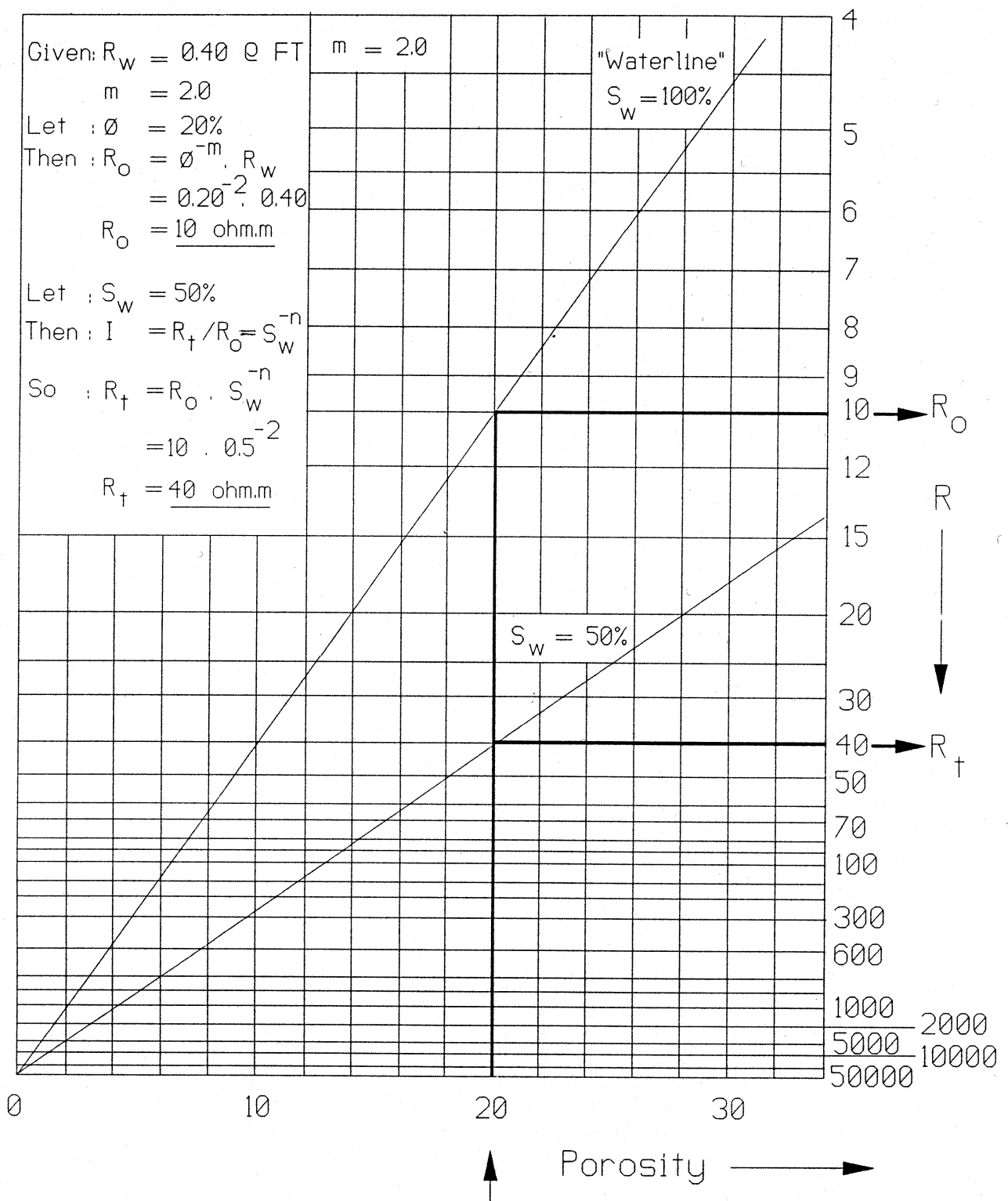
Figure 17.10 The Hingle plot.

17.9   The Pickett Plot

Theory. The Pickett Plot is also based on Eq. (17.18).

In a water-bearing formation we can write from Eq. (17.11)

$$R_o = R_w \, \phi^{-m} \qquad\qquad\qquad \textbf{(17.22)}$$

which, when rearranged becomes

$$\log R_o = \log R_w - m \log \phi \qquad\qquad\qquad \textbf{(17.23)}$$

So a plot of $\log R_o$ against $\log \phi$ gives a straight line. The value on the y-axis is equal to $\log R_o$ when $\phi=1$, and the slope of the line is $-m$.

In a hydrocarbon-bearing formation we can write from Eqs. (17.11) and (17.15)

$$R_t = I \, R_o = I \, R_w \, \phi^{-m} \qquad\qquad\qquad \textbf{(17.24)}$$

which, when rearranged, becomes

$$\log R_t = \log I + \log R_w - m \log \phi \qquad\qquad\qquad \textbf{(17.25)}$$

which is the same straight line as described by Eq. (17.23), with the same gradient, but with a parallel shift equal to $\log I$.

Application. The Pickett Plot plots the formation resistivity R_t against the porosity on a log-log scale. The data form straight lines with a gradient equal to $-m$. Hence, the cementation exponent can be calculated. If one has data in the water zone of the reservoir, Eqs. (17.22) and (17.23) hold true, and the value on the y-axis when the line intersects $\phi=1$, gives $\log R_w$ from which R_w can be calculated. The line is called the water line.

If one has data in the oil-bearing zone, and the value of Rw is known, the value on the y-axis when the line intersects $\phi=1$, gives $\log I + \log R_w$ from which I can be calculated if R_w is known. If the saturation exponent is then known, we can use the I value to calculate the water saturation.

Alternatively, we can establish the water line and construct iso-saturation lines with the same gradient that are offset from the water line by values of $\log I$ that represent increments in water saturation. Plotting the formation resistivity and porosity values from logs on this plot then allows the mean saturation in the reservoir to be judged, and particular values of water saturation at a given depth can be calculated can be approximated by interpolation between the iso-saturation lines.

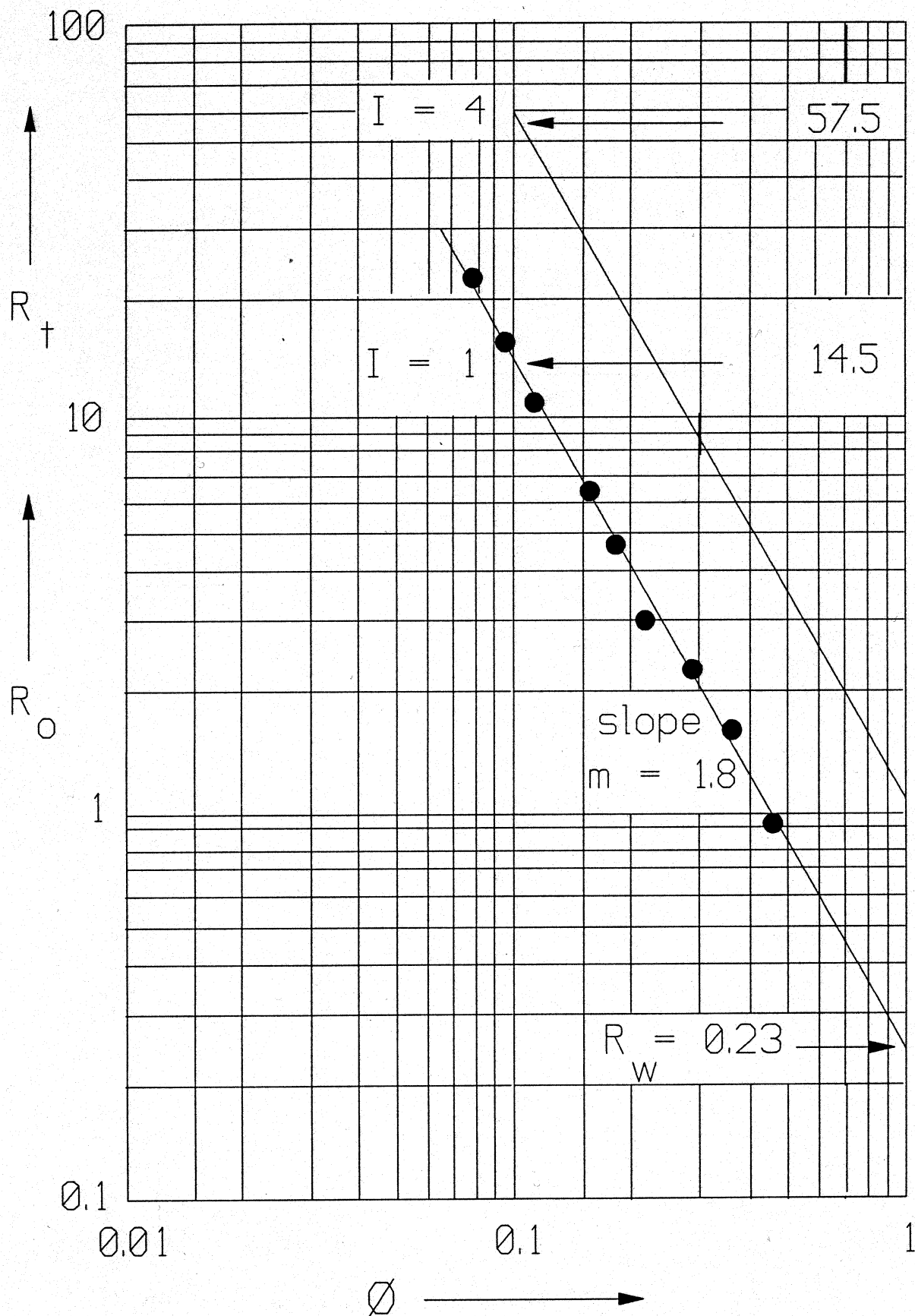Figure 17.11 shows a Pickett plot example.

Figure 17.11 The Pickett Plot.

17.10   Saturation of Moveable Hydrocarbons

Equation 17.17 describes how the ratio of the resistivity of the formation containing partly non-conductive oil and partly conductive water to the resistivity when the formation contains 100% conducting water (*I*) is related to the saturation of water in the formation when it contains both fluids.

We apply this equation to the reservoir zone (i.e., the uninvaded zone deep in the formation) and can calculate the water saturation in the reservoir, and hence calculate the amount of oil in the reservoir.

However, we can also apply the equation in the flushed zone if we are using water based muds. Before invasion, we had a water saturation S_w and an oil saturation S_o. In the flushed zone after invasion, the water has been replaced with mud filtrate, and some of the oil has also been replaced by mud filtrate. The oil that has been replaced by mud filtrate is the *mobile oil*, i.e., that oil which was able to be pushed out of the way by the invading mud filtrate. We want to calculate the saturation of oil which the mud filtrate was able to push further into the formation, because if it was mobile enough to be moved in that way, the likelihood is that it will be mobile enough to be produced easily from the reservoir.

If we apply Eq. (17.17) in the flushed zone we get

$$R_{xoor} = R_{xomf}\ S_{xo}^{-n}$$

(**17.26**)

Hence,

$$S_{xo} = \sqrt[n]{\frac{R_{xomf}}{R_{xoor}}}$$

(**17.27**)

Here R_{xomf} is the resistivity of the formation flushed zone containing nothing but 100% mud filtrate, i.e., invasion has replaced all the oil and water, while R_{xoor} is the resistivity of the flushed zone containing residual oil.

The value S_{xo} is the saturation of water based mud filtrate in the invaded zone. Hence the saturation of oil that was moved by the invasion is $(S_{xo} - S_w)$, and the volume of moveable oil per unit volume of rock is $\phi\,(S_{xo} - S_w)$. We can use this modified value of moveable hydrocarbons in the reservoir to calculate a lower, and more accurate value for moveable STOOIP by inserting $(S_{xo} - S_w)$ in place of $(1 - S_w)$ in Eqs. (1.2) and (1.4).