18. Rekursion

a)

b)

c)

d)

20. Verschiebung des Parameter- und Wertebereichs

a) $A = -5$

$$\begin{aligned} f(n) &= 2f(\lfloor\frac{n+3}{2}\rfloor) - 5 \\ g(n) &= f(n) + A \\ &= f(n) - 5 \\ &= 2f(\lfloor\frac{n+3}{2}\rfloor) - 5 - 5 \\ &= 2(f(\lfloor\frac{n+3}{2}\rfloor) - 5) \\ &= 2(g(\lfloor\frac{n+3}{2}\rfloor)) \end{aligned}$$

b) $B = 3$

$$\begin{aligned} g(n) &= 2g(\lfloor\frac{n+3}{2}\rfloor) \\ g(n+3) &= 2g(\lfloor\frac{n+6}{2}\rfloor) \\ &= 2g(\lfloor\frac{n}{2}\rfloor + 3) \\ \Rightarrow h(n) &= g(n+3) \\ &= 2h(\lfloor\frac{n}{2}\rfloor) \end{aligned}$$

(Test-)Werte berechnen...

$$\begin{aligned} \Rightarrow h(0) &= g(0+3) = f(1) - 5 = 1 \\ h(1) &= 2h(0) = 2 \\ h(2) &= 2h(1) = 4 \\ h(3) &= 2h(1) = 4 \\ h(4) &= 2h(2) = 8 \\ h(5) &= 2h(2) = 8 \\ h(6) &= 2h(3) = 8 \\ h(7) &= 2h(3) = 8 \\ h(8) &= 2h(4) = 16 \end{aligned}$$

Formel für $h(n) = 2^{\lfloor \log_2 n + 1 \rfloor}$ ergibt sich aus den Werten.

$$\begin{aligned} h(n) &= 2^{\lfloor \log_2 n \rfloor} = 2^{\lfloor \log_2 \frac{n}{2} + 1 \rfloor} \\ &= 2 * 2^{\lfloor \log_2 \frac{n}{2} \rfloor} \\ &= 2 * 2^{\lfloor \log_2 n - 1 \rfloor} \\ &= 2^{\lfloor \log_2 n \rfloor} \end{aligned}$$

Formel für

$$\begin{aligned} g(n) &= h(n) - 3 = 2^{\lfloor \log_2 n \rfloor} - 3 \\ f(n) &= g(n) + 5 = h(n) + 2 = 2^{\lfloor \log_2 n \rfloor} + 2 \end{aligned}$$

c)

$$q(n) = q(\lfloor \frac{n+3}{2} \rfloor) + 1 (\text{für} n > 4), q(1) = q(2) = q(3) = 1$$